# Kinetic theory

## UK Space Agency

The UK Space Agency is at the heart of UK efforts to explore and benefit from space.  It is responsible for all strategic decisions on the UK civil space programme and provides a clear, single voice for UK space ambitions.

The Agency is responsible for ensuring that the UK retains and grows a strategic capability in the space-based systems, technologies, science and applications. It leads the UK's civil space programme in order to win sustainable economic growth, secure new scientific knowledge and provide benefits to all citizens.

## ESA

From the beginnings of the 'space age', Europe has been actively involved in spaceflight. Today it launches satellites for Earth observation, navigation, telecommunications and astronomy, sends probes to the far reaches of the Solar System, and cooperates in the human exploration of space.

Space is a key asset for Europe, providing essential information needed by decision-makers to respond to global challenges. Space provides indispensable technologies and services, and increases our understanding of our planet and the Universe. Since 1975, the European Space Agency (ESA) has been shaping the development of this space capability.

By pooling the resources of 22 Member States, ESA undertakes programmes and activities far beyond the scope of any single European country, developing the launchers, spacecraft and ground facilities needed to keep Europe at the forefront of global space activities.

## National Space Academy

Established in 2011 and led by the National Space Centre, the National Space Academy is now the UK's largest space education and skills development programme for secondary and further education. Its team includes some of the country's best science teachers, project scientists and engineers who deliver masterclasses and intensive teacher training for thousands of students and teachers across the UK every year. Internationally the Academy works extensively with the European Space Agency, the UAE Space Agency, and it also leads the UK's ongoing space education and skills development work with China.

## Astro Academy Principia

A unique education programme developed by the UK's National Space Academy for the UK Space Agency and ESA (European Space Agency), Astro Academy: Principia uses a suite of demonstrations filmed by ESA astronaut Tim Peake aboard the ISS during his six month Principia mission to explore topics from secondary physics and chemistry curricula.  The programme is made up of stand-alone teaching films, downloadable video clips, downloadable files that can be used with the free-to-use dynamical analysis software programme "Tracker", written teacher guides and links to more than 30 further teaching activities.

## Principia

Tim's mission to the International Space Station, called 'Principia', used the unique environment of space to run experiments as well as try out new technologies for future human exploration missions. Tim was the first British ESA astronaut to visit the Space Station where he spent six months as part of the international crew.

# Kinetic theory of gases and pressure

## Introduction

Tim Peake's six-month mission on the International Space Station (ISS) was only possible because the space station is a pressurised environment. In this teacher guide, we explore the origin of pressure in gases from both a molecular perspective and also in terms of atmospheric pressure experienced on the surface of planets or natural satellites which have atmospheres, such as Titan. The oscillatory modes of molecular vibration can be modelled at a simple level using principles of harmonic motion and Tim's demonstrations scale up very visually how individual molecules can behave at sub-microscopic levels.

Tim's demonstrations visually demonstrate at a larger scale the way this oscillation can occur in individual molecules.

## 1) Kinetic theory and random motion

In the kinetic theory of gases, pressure modelling can be done by considering momentum changes as molecules collide with the sides of a container within which a gas is enclosed. In first-order modelling, the molecules are simplified to be considered as:

Clips:

V1 Kinetic theory box triboelectric
V2 Kinetic theory box top view

- Having negligible volume compared to the volume of the container
- Having no intermolecular interactions which act at a distance apart from during each collision
- Having only elastic collisions
- Having the same mass (all molecules)

Tim's demonstration clearly shows the random motion of the balls as well as the range of ball velocities and changes in momentum after each collision – illustrating the behaviour of real gas molecules.
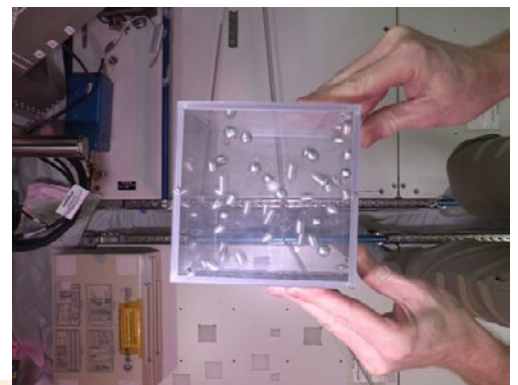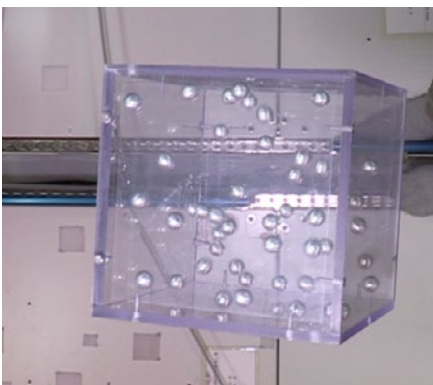


Tim conducts collisions demonstration with small spheres acting as analogues for molecules in a gas.

## 2) Real gases: Extra degrees of freedom

Real gas molecules may, in addition to having translational kinetic energy, have other motions which will contribute to the overall kinetic energy of the

molecule. Tim's demonstrations with two masses attached to either end of a spring clearly show analogues of this behaviour.

**Vibrational motion** – atoms within molecules can move with periodic motion leading to various types of vibrational motion of the molecule, which can include stretching, rocking, bending, wagging, and twisting. This motion is affected by the configuration of the molecule, the mass of the atoms and the strength of the bonds within the molecule.

Tim's compression of the balls together and subsequent release clearly shows a wagging motion of the balls in relation to the centre of the spring. This motion is down to the spring being compressed unevenly across its cross-section, even when Tim tries to ensure even compression. This shows the limits to the analogy with vibrational motion of diatomic molecules, since diatomic molecules only have one vibrational mode, stretching. In addition to this vibrational mode of motion, translational motion is also seen as the ball and spring assembly moves across the Columbus module.

Translational motion can clearly be seen

**Rotational motion** – with various levels of complexity possible and multiple axes of rotation. Every different rotation type can be classically modelled through application of the principles of moments of inertia – applied to the particular molecular shape and axis of rotation being considered.

The image to the left shows Tim using a sphere containing lots of M and M sweets – similar to "Smarties". After agitating the sphere we can see the smarties moving in random directions and with a range of speeds – as with the cube demonstrations, this is representing the TRANSLATIONAL motion of gas atoms/molecules.

However, we can also see that the smarties have ROTATIONAL motion in addition to the translational motion.

Looking carefully at clip V5, in the folders, we can see a variety of different modes.

**The Ideal Gas Equation pV = nRT and kinetic modelling**

The fundamental macroscopic (large-scale) parameters of a gas that can be measured are

- Pressure (measured in pascals, or $N/m^2$)
- Volume (measured in $m^3$)
- Temperature (measured on the absolute scale in kelvin where 0°C = 273K, 100°C = 373K etc)

These three quantities are linked through the classical gas laws and the IDEAL GAS EQUATION:

pV = nRT

- n = number of MOLES of the gas (one MOLE = Avogadro's number of atoms/molecules = $6.02 \times 10^{23}$)

- R = MOLAR GAS CONSTANT, a measure of the kinetic energy/temperature link for one mole's worth of molecules/particles, R = $8.314 m^2 kg\ s^{-2}\ K^{-1}\ mol^{-1}$

- T = absolute temperature in kelvin

For the molecular scale modelling as opposed to macroscopic (considering the small nature of particles rather than masses of them at once), the ideal gas equation is often written as

pV = NkT

- p = pressure (Pa)
- V = volume ($m^3$)
- N = total absolute number of particles (rather than grouping them into moles)
- T = temperature in K
- k = BOLTZMANN's CONSTANT = $1.38 \times 10^{-23} J/K$

## Boltzmann's constant, kinetic theory and kinetic energy

Boltzmann's constant is the molar gas constant divided by Avogadro's number.

It gives a direct link between energy for a single particle and the temperature of the system of which the particle is a part.

**The kinetic energy of a gas molecule can be considered to have elements with translational, rotational and vibrational kinetic energy.**

In classical modelling, each extra mode of freedom will contribute an extra term to the kinetic energy of the molecule:

**Kinetic energy of molecule = kinetic energy due to translational motion + ∑(kinetic energy due to rotational modes) + ∑(kinetic energy due to vibrational modes)**


## Translational kinetic energy

The absolute temperature is a measure of the mean kinetic energy of translation, per molecule, of a system of molecules:

**$E_k = \frac{3}{2}kT$**

- k = Boltzmann's constant ($1.38 \times 10^{-23}$ J/K)

- T = temperature in kelvin


In this modelling, each of the x, y and z modes of translation will contribute kT/2 to the total kinetic energy term. *Therefore, the mean translational kinetic energy of a particle in a system at temperature T is $\frac{1}{2}kT$*


## Rotational and vibrational modes

In classical modelling, **each individual extra mode – of vibration or rotation - will contribute an extra kT/2 term to the total kinetic energy of the molecule.**

So, for a hydrogen molecule which is moving in three dimensions, rotating about one axis through the middle of the main covalent bond and which is also vibrating with an "in and out" motion, the total kinetic energy of the molecule in this particular state of translation, rotation and vibration will be $\frac{5}{2}kT$.

## 3) Triboelectricity – localised charging of the polycarbonate box at impact points during collisions

After several cycles of demonstrations using the kinetic box equipment, it was noticed that after agitation of the box, the small metal spheres would collide with and eventually stick to the sides of the polycarbonate box after only a few seconds. As Tim explains in the experiment, this sort of behaviour was not seen in previous demonstrations using other equipment on board the ISS.
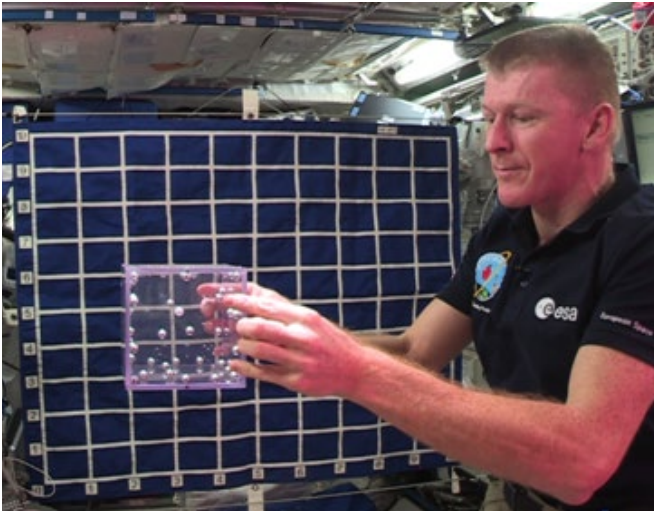


The effect became more pronounced as more shaking sequences of the box were conducted.

Note that during these sequences, the single white ball kept colliding with the sides of the container and moving across the box even when the metal spheres were stuck to the sides. Also note how some of the metal spheres can be seen oscillating on the inside surface of the box about a mean position.

## Explanation

There are a number of possible contributing factors – some of which are still under investigation.

However, it is likely that the observed phenomena were due to the **triboelectric effect**.

1. When the box was being shaken vigorously, the metal spheres and the single white sphere all sustained multiple contacts with the inside surface of the polycarbonate box. This particular polycarbonate material – lexan – is a DIELECTRIC (electrical insulator material) with a relative permittivity ($\varepsilon_r$) of 2.92. The metal spheres are electrical conductors ($\varepsilon_r = 1$) and the white plastic ball is also an insulator with similar permittivity to the lexan.

2. Due to the triboelectric effect, these multiple contacts cause the material of the lexan box to become electrically charged – locally, at the point of each impact . The triboelectric effect is the localised charging that occurs when materials of differing electrical

permittivity are in contact with each other. *In 2000, UCLA scientists demonstrated the effect through multiple collisions with hollow gold spheres and various electrically insulating materials, clearly demonstrating that localised triboelectric charging between metals and insulators is possible\**

3. When the metal spheres hit the surface of the box the localised electric charge at the contact point on the box induces an opposite charge on the metal sphere. This results in the metal sphere sticking to the surface of the box, due to the electrostatic forces between them.

4. Since the effect is very small, it is not noticeable during the initial active shaking phase of the box. However triboelectric charging effects have been demonstrated to be cumulative and although the charging tends to be localised at the point of each impact, multiple impacts on a surface can lead to larger localised charging effects being produced. In addition, the charge accumulated can take hours or days to dissipate depending on the materials. Even if Tim conducted these experiments some time apart, there could still be a residual charge density on the surface, which would simply be increased as further investigations were carried out. This may explain why the effects were more noticeable after several cycles of active shaking.

5. The metal balls were seen to be oscillating about their mean position because the electrical charge produced triboelectrically on the box surface is a localised effect at the point of contact – not throughout the box itself.  Therefore, whenever the metal ball started to move away from this point, subsequent free electron flow across the surface of the metal ball towards the original point of contact would create a localised attractive restoring force that would draw it back. This can be clearly seen in the non-ballistic trajectories of the balls immediately following impact – an observation that would only be possible in a microgravity environment.

6. The plastic ball (white) remained unaffected and continued moving freely across the box and colliding with multiple surfaces. The very similar electrical permittivities of lexan and the plastic ball material also mean that the triboelectric charging effect between the plastic ball and the lexan during each impact is much less than between the lexan and the metal balls – resulting in much smaller localised charge production on the surface of the box – if any. Being an electrical insulator it was unable to be polarised at the point of collision between it and the box and therefore no attractive force was exerted at the point of contact.

*See January 2000, American Physical Society: "Correlation between Charge Transfer and Stick-Slip Friction at a Metal-Insulator Interface" - Budakian, R. and Putterman, S. J

# 4) Circular motion in 1-d: harmonic oscillator demonstrations

The dynamical behaviour of an object performing simple harmonic motion (for example a pendulum which has small displacement from its equilibrium position, or a mass oscillating on the end of a spring which obeys Hooke's Law) is replicated if a system in which an object is moving in a circle at constant velocity is analysed in one plane. Viewed in one plane, the displacement, velocity and acceleration of the object moving in a circle at constant speed at any time exhibit the same behaviour as these parameters for a system exhibiting simple harmonic motion (SHM) behaviour.

In the centripetal forces film (and as seen in the screen shot to the right), Tim filmed a ball rolling around a polycarbonate hoop.

In the image below you can see a screen shot from a motion tracking program called "Tracker" (see tracker guide and tracker videos in additional resources). Viewed from this angle we see the circular motion dynamics reduced to a 1-d plane – with the system exhibiting the same displacement, velocity and acceleration behaviour over one complete cycle as a system undergoing SHM.
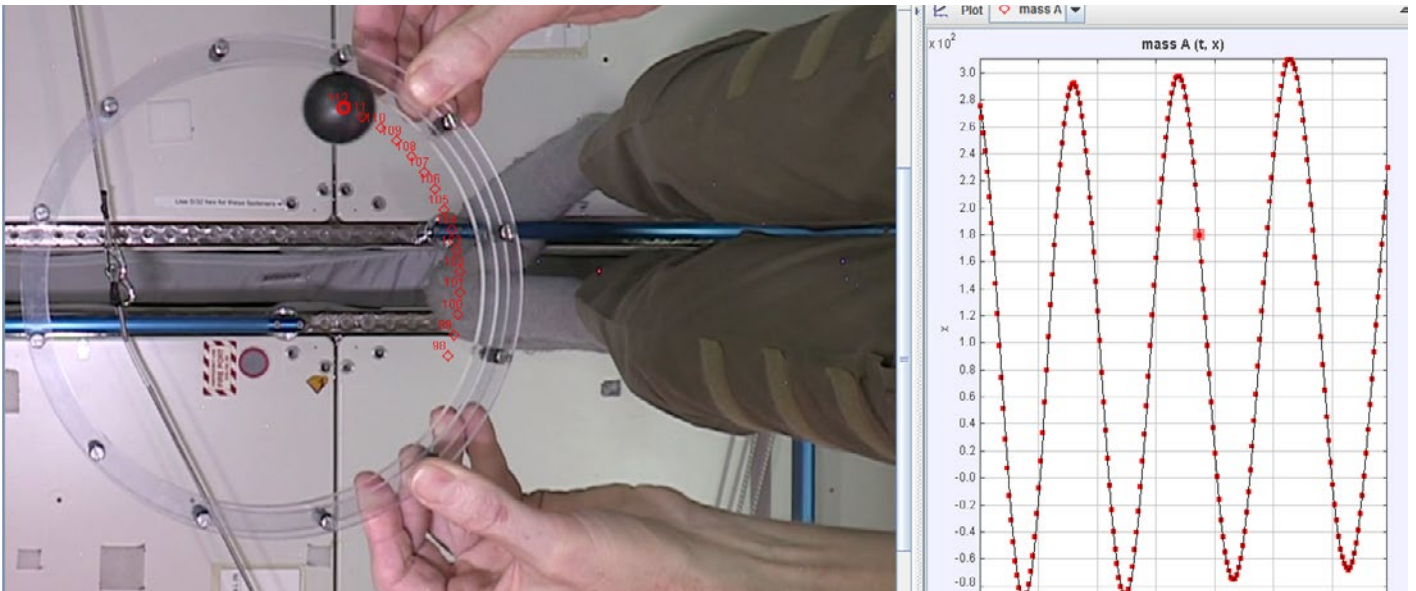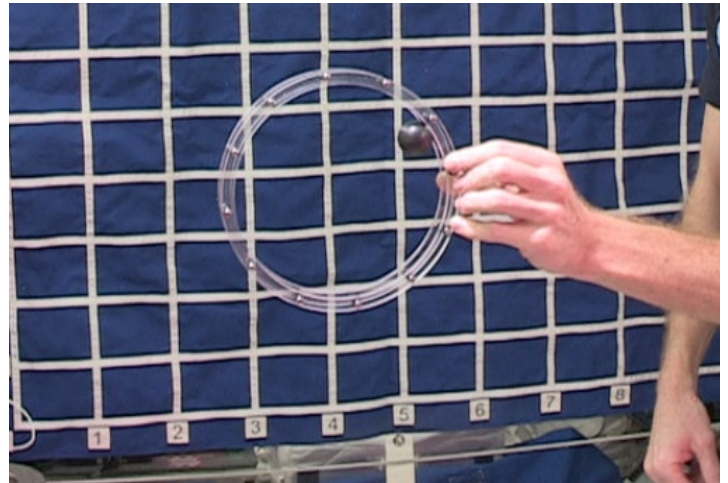




Using tracker software, the circular motion of the ball in the track can be reduced to a 1-d plane and the SHM relationship can clearly be seen

# Ground-based experiments

## Card and a cup of Water

**Curriculum Links:** Air pressure

**Key Stage:** 3, 4 and 5

**Equipment List:**
Straight sided glass
Laminated card or coaster
Water
Food colouring (optional)



Cup of water, card or coaster and ruler

## Procedure:

- Fill the glass almost full with water. You may wish to add some food colouring to the water to enhance the effect.
- Place the laminated card on top of the glass. Holding your hand flat over the card, pick up the glass and turn it upside down, keeping your flat hand on the card.
- Remove your hand from the card and observe what happens. You may wish to hold this over your head (or that of a willing volunteer). If you do this do not add food colouring to the water in case it does spill.

## Expected Outcomes:

The laminated card should not fall down, it should remain in place, holding the water in to the glass despite the fact that the weight of the water is pulling it down towards the ground.

Billions of tiny collisions occur every second between individual gas molecules and the underside of the card. This means that the gases in the air underneath the card are exerting a significant upward force on the card, a force large enough to support the weight of the water.

In addition to the air pressure acting on the card, surface tension and adhesion play a role. Surface tension is created due to the way the water molecules orient themselves. Water is a polar molecule, a molecule that has a positive end and a negative end. As a result there is a small attractive force between each water molecule. This force strengthens the **surface tension** in water, where water molecules stick together creating an elastic-like surface layer.

**Adhesion** occurs because water molecules, having a positive and negative end, are also attracted to other materials. In this demonstration, the water molecules are attracted to the paper, adhering to it, while continuing to keep the surface tension with the other water molecules. This helps to keep the laminated card in place.

So, the card doesn't fall off due to the air pressure, and it doesn't slide off thanks to surface tension and adhesion in the water.

# Mass on a spring

**Curriculum Links:** Simple harmonic motion

**Key Stage:** 5

**Equipment List:**

Clamp stand
Boss
Clamp
Spring
Mass hanger
500g mass
G clamp



Mass hanger with masses attached to spring suspended from clamp stand

## Procedure:

- Use the G clamp to fix the clamp stand to the desk. Use the boss to attach a clamp to the top of the stand and suspend the spring from this point.
- Add the mass hanger to the free end of the spring and load it with 200g of mass. You may wish to clamp a ruler in place beside the mass to mark the equilibrium position.
- Pull the mass down a small amount and ask students what forces are acting on the mass and in what direction. Now compress the spring by the same distance as the displacement and ask students the same question.
- Finally, pull the mass down a small amount (no more than 1cm) and allow it to oscillate about its equilibrium position.
- You may wish to ask students to time the period of one oscillation.
- Repeat the procedure with increasing masses up to 500g and ask students to comment on the relationship between oscillation period and mass.

## Expected Outcomes:

- The mass will oscillate about its equilibrium position with a period that depends on the size of the mass and the stiffness, or spring constant of the spring - a classic example of simple harmonic motion, or SHM.
- When a spring with spring constant k and length l has a mass m attached to the bottom it extends by an extension e, this is called the static extension and is the new equilibrium point. The tension in the spring is balanced by the weight. We can represent this as:

   **T= ke = mg**
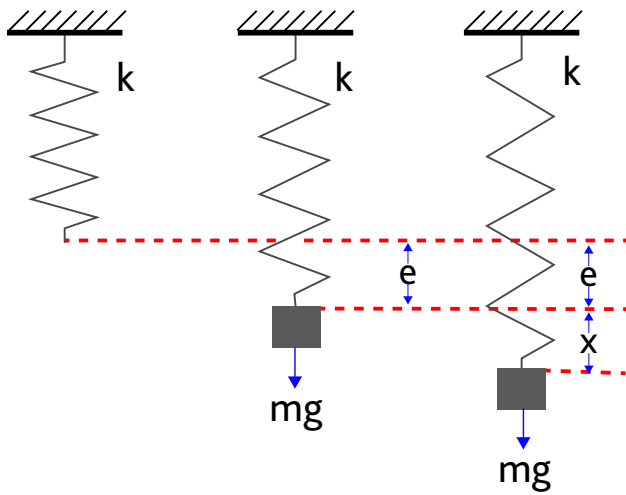
If the mass is pulled down by a displacement x and released it will undergo SHM.

The net upwards force will be:  $F = -(k(e + x) - mg)$

So  $F = -(ke + kx) - mg)$

Since $ke = mg$ this becomes  $F = -(mg + kx - mg)$

And simplifies to:  $F = -kx$

Since both  $F = ma$ and $a = -\omega^2 x$ (for SHM)  we have that  $F = -m\omega^2 x$  the equation now becomes:  $-m\omega^2 x = -kx$

From SHM we have $\omega = 2\pi f$ which gives $m(2\pi f)^2\, x = kx$

Rearranging for gives us:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

And since $f = \dfrac{1}{T}$  the equation becomes:  $T = 2\pi\sqrt{\dfrac{m}{k}}$

In a single oscillation there is a constant interchange between potential and kinetic energy leading to a sinusoidal relationship between displacement, velocity and acceleration.

This oscillation can then be compared to the side-on view of Tim's microgravity ball on a track demonstration and the reduction of circular motion to 1 dimension clearly shows the SHM relationship.
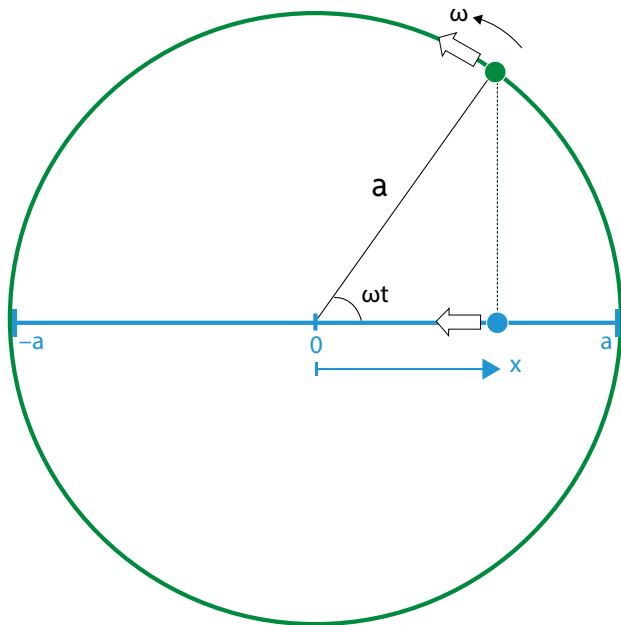
From simple trigonometry we can see that the projection on the x-axis is given by:
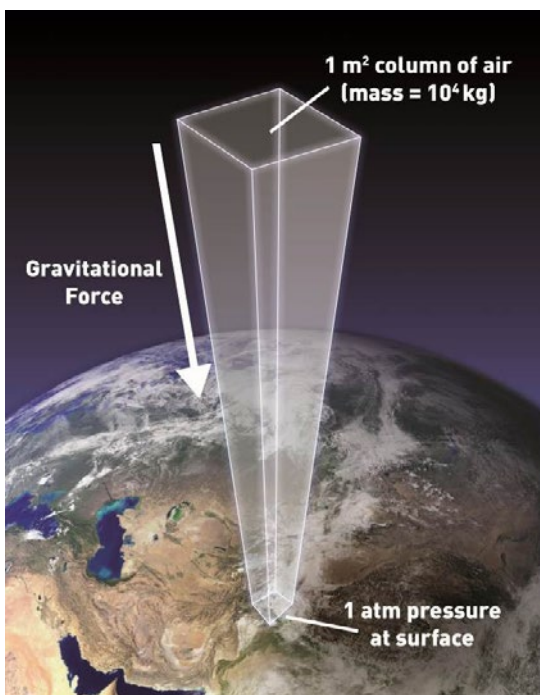
$$x = a \cos(\omega t)$$

And so $v_x = \dfrac{dx}{dt} = -\omega a \sin(\omega t)$

Then $a_x = \dfrac{dv}{dt} = -\omega^2 a \cos(\omega t)$

Which is of the form $a = -\omega^2 x$, showing that the projection is simple harmonic.

# Space contexts – planetary atmospheres compared
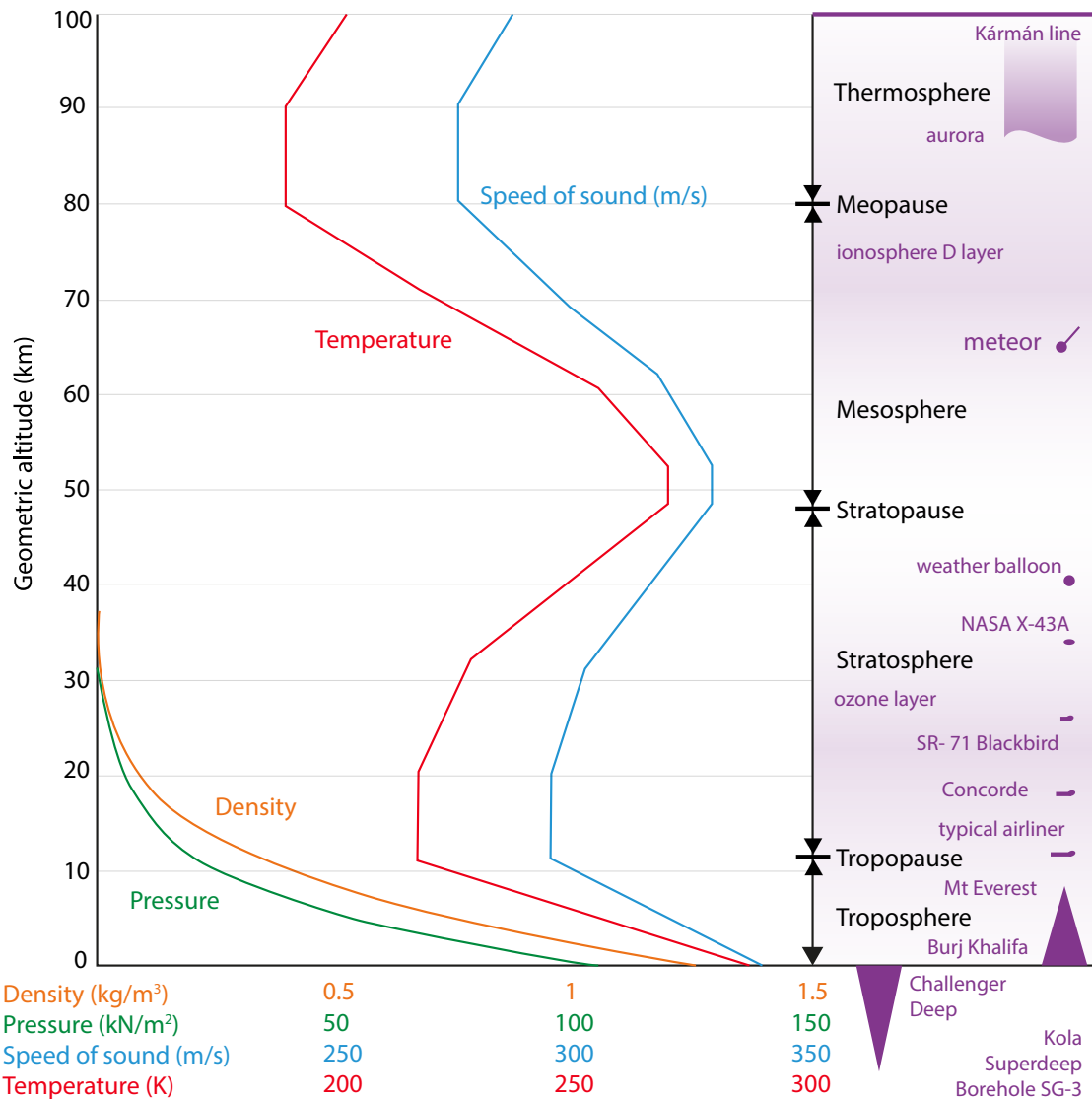
## The Origin of Earth's atmospheric pressure



1 m² column of air (mass = $10^4$ kg)

Gravitational Force

1 atm pressure at surface

A column of air above a metre squared area on the Earth weighs 10 tonnes

Atmospheric pressure on Earth is created by the weight of atmosphere above us which extends to an altitude of well over 100km - the internationally accepted boundary of space (although nearly 90% of the total mass of the atmosphere is within 10km of the Earth's surface).

Atmospheric pressure varies at a given location with the passage of weather systems, and decreases exponentially with altitude. The standard value is 101,325 Pa (1013.25mb), also known as 1 atmosphere (atm). This means that every square metre of the Earth's surface at sea level is sitting under 10 tonnes of atmospheric mass.

This chart (taken from US standard atmospheric data released in 1962 and courtesy of NOAA/ wikicommons) shows how several fundamental parameters change with altitude in the Earth's atmosphere:



| Density (kg/m³) | 0.5 | 1 | 1.5 |
| Pressure (kN/m²) | 50 | 100 | 150 |
| Speed of sound (m/s) | 250 | 300 | 350 |
| Temperature (K) | 200 | 250 | 300 |

- As altitude in the atmosphere increases above the surface, temperature initially drops at a LAPSE RATE of around 6.5°C per 1000m because the lower atmosphere is heated from below by solar radiation that has been absorbed by the Earth and re-radiated.

- The coldest part of the lower atmosphere is the tropopause (the boundary between troposphere and stratosphere) - where the effect of ground heating upwards is minimal. This boundary is where the jet stream occurs: high-velocity horizontal winds racing through the atmosphere.

- Above this altitude, in the **STRATOSPHERE**, temperature increases with altitude due to incident UV absorption.

- The stratosphere is a cool, stable region of rarefied gases and there is seldom mixing between tropospheric and stratospheric constituents.

# Pressure variation with altitude – physics modelling

As we ascend in the atmosphere, the air pressure diminishes, as there is less weight of air above.

If we assume an ideal gas is behaving HYDROSTATICALLY, then for a particular layer of gas at a given altitude the downward force of its weight and the downward force exerted by pressure in the layer above is equal to the upward force exerted by pressure in the layer below. It's possible to show that the following relation holds

$$P = P_0\, e^{-(mgh/kT)}$$

- $P$ = pressure at altitude h metres (Pa)
- $P_0$ = sea level or mean datum pressure (Pa)
- m = mean mass of a particle in the atmosphere (kg)
- g = acceleration due to gravity ($m/s^2$)
- h = height above sea level or mean datum
- k = Boltzmann's constant
- T = Absolute temperature in kelvin

For Earth, with an approximate atmospheric composition of 80% $N_2$ and 20% $O_2$, we find a mean molar mass of 28.8g ($N_2$ =28, $O_2$=32) which needs to be expressed in kg. The Earth value translates to a mean molecular mass of 28.8 times that of a hydrogen atom = $4.8 \times 10^{-26}$ kg. Using this equation is as straightforward as the more familiar uses of exponentials with regards to radioactive decay and capacitance:

Radioactivity $N = N_0 e^{-\lambda t}$

Capacitance $V = V_0 e^{-(t/CR)}$

**Steps in the proof – suggested mathematical derivation problem for students**
- Consider a box of atmosphere, area A, height dx, Volume dV = Adx
- Increase in pressure dP is weight/area = Nmg/A where N = number of particles
- So rate of pressure change with height = dP/dx = Nmg/Adx = Nmg/dV
- Since PV=NkT, PdV = NkT
- So rearrange with expression for dV in terms of P and substitute into dP/dx equation, remembering that dP/dx will be negative for increasing x as in the atmosphere, as height increases, pressure decreases.
- Integrate within limits

Of course the model is inaccurate in that it assumes the atmosphere is ISOTHERMAL; at the same temperature throughout. In fact, as we ascend in the atmosphere, temperature drops at the LAPSE RATE of 6.5°C per 1000m until we reach the coldest part of the atmosphere – the tropopause (boundary between troposphere and stratosphere).  Above this altitude, in the STRATOSPHERE, temperature increases with altitude due to incident UV absorption, resultant molecular dynamic excitation and subsequent heating.

However, since most of the mass of the Earth's atmosphere is in the lower troposphere, use of a mean temperature for the whole stratosphere will yield results close to those actually measured.

Taking the sea-level value for pressure given above, a mean atmospheric temperature of 0°C and the height of Mount Everest as 8850m, we can show that the pressure on its summit is about one-third of the sea-level value, and calculate it in pascal.

## Venus: mean surface pressure nearly 100 times that of Earth's

Venus is probably the most hostile planetary surface environment anywhere in the Solar System. With surface temperatures of the order of 480°C (and little variation in day and night) and a surface pressure of 90+ Earth atmospheres (equivalent to nearly 1000 newtons on every square centimetre) even the most robust planetary landers have only survived to return data for a matter of minutes or, at best, a few hours.
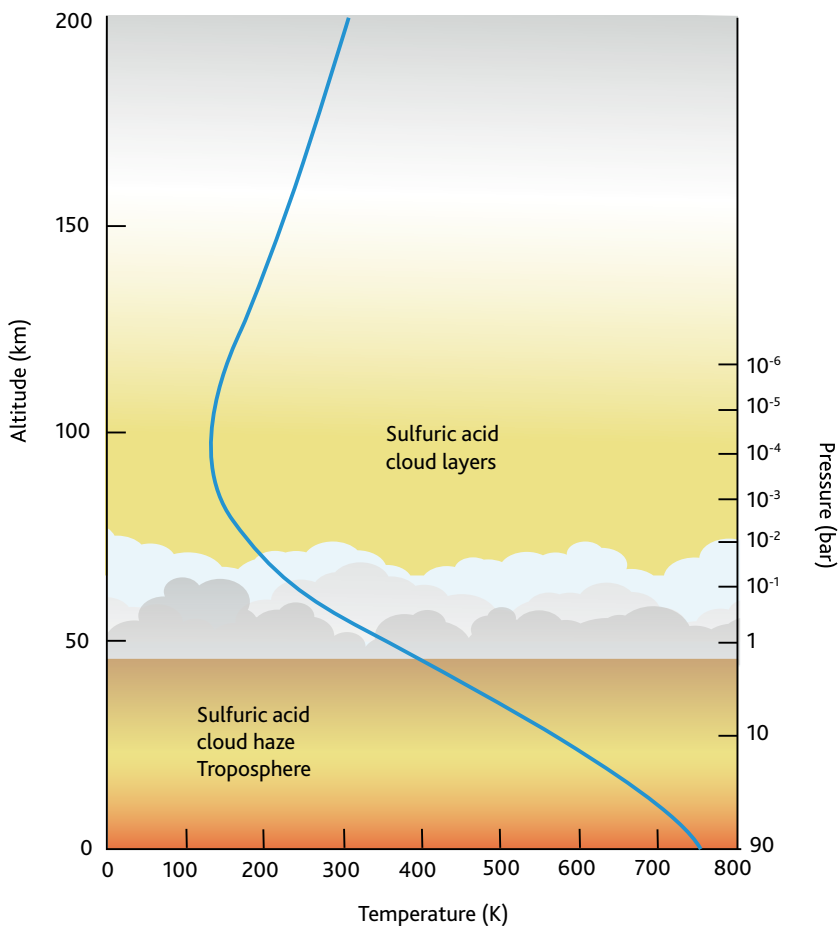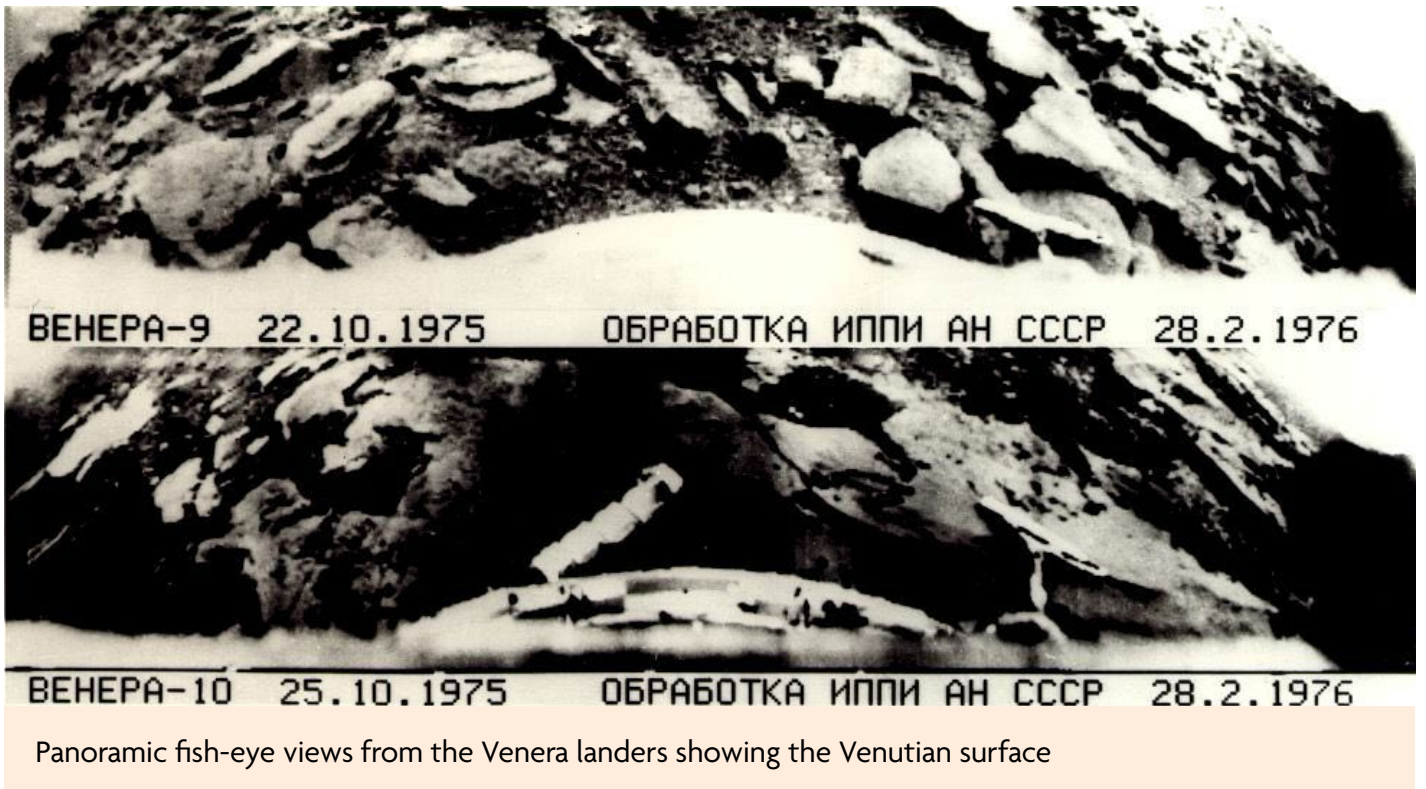


This cross-sectional pressure and temperature profile through the Venusian atmosphere highlights the exponential pressure increase with depth in the atmosphere
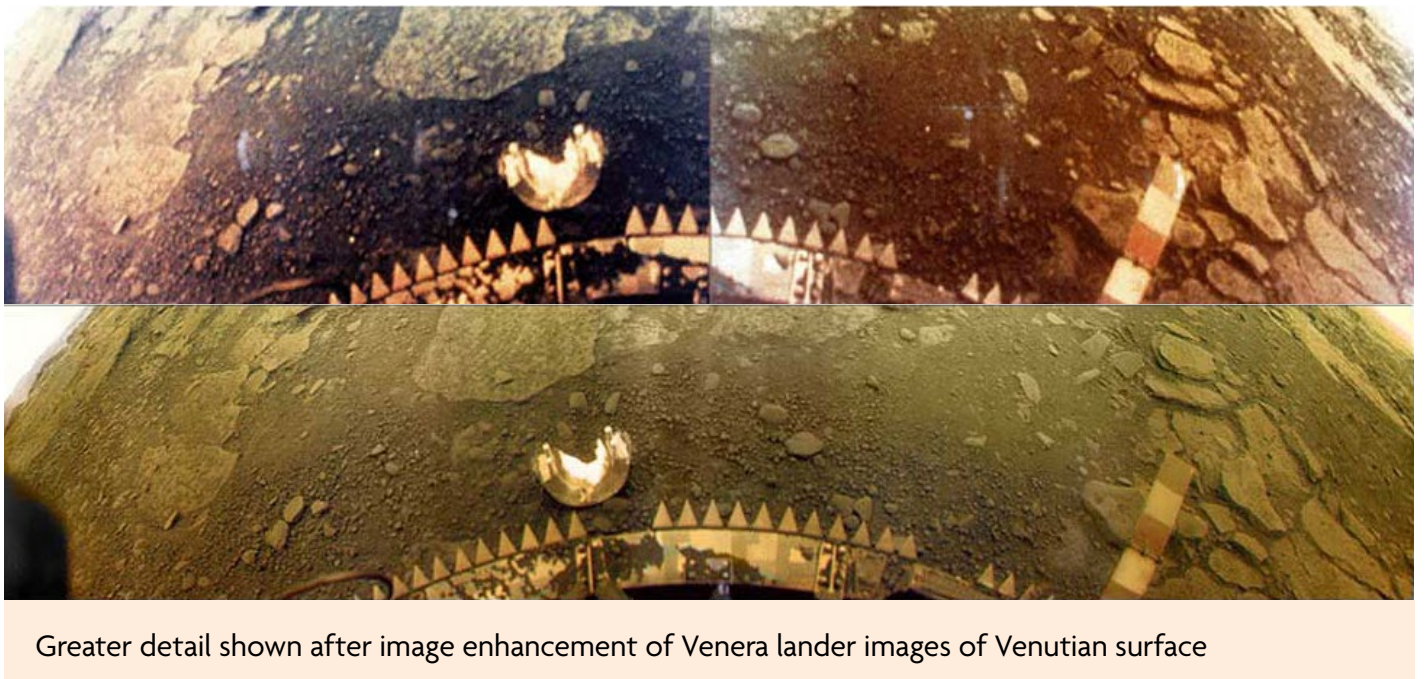
1 bar = 100 000N/m$^2$
1 atm (standard atmosphere at Earth's surface) = 1013.25mb

Our images of the surface date back to the 1970s and 1980s. The Soviet Venera series of landers survived to return surface data, with Veneras 9, 10, 13, and 14 each returning one panoramic fish-eye view before succumbing to the harsh environment:



ВЕНЕРА-9   22.10.1975   ОБРАБОТКА ИППИ АН СССР   28.2.1976

ВЕНЕРА-10   25.10.1975   ОБРАБОТКА ИППИ АН СССР   28.2.1976

Panoramic fish-eye views from the Venera landers showing the Venutian surface

This Venera 13 raw image (1982) at top has been enhanced to bring out further details:



Greater detail shown after image enhancement of Venera lander images of Venutian surface

## Mars: mean surface pressure less than 1/100th that of Earth's

On Mars, the atmosphere has less than 1% of the sea-level pressure on Earth. Martian pressures are given with reference to a Martian imaginary sea-level called the mean datum because Mars has no oceans at present.

Mars' atmosphere is so insubstantial compared to Earth's that even in Martian summer where occasionally the surface temperature in a few low-lying locations might reach 20°C, the lapse rate in the immediate vicinity of the surface is much larger than on Earth. The Martian atmosphere's heat retention capacity is so low that even two or three metres above our surface in midsummer, it would be below 0°C.
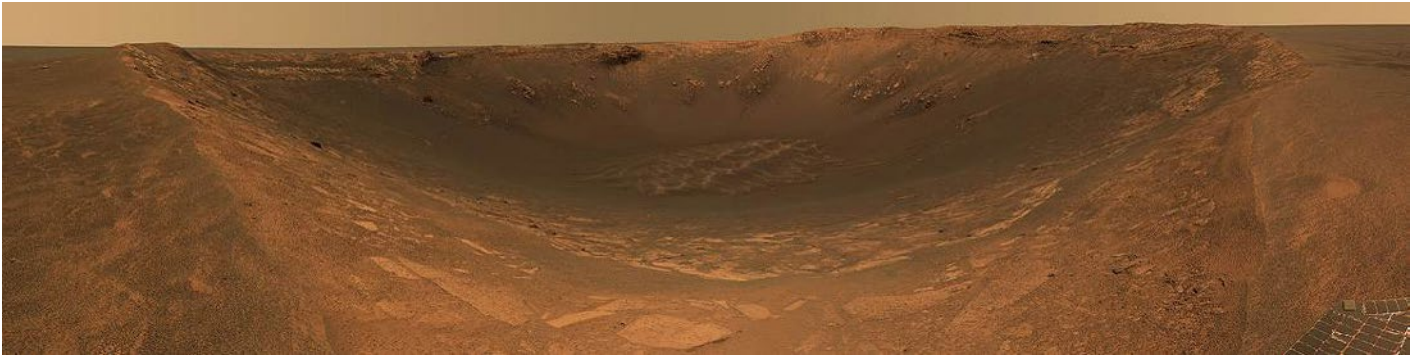


The above image shows a true-colour panorama of the Endurance crater as explored by MER-B (Opportunity).

The image below shows surface pressure data from NASA's Pathfinder mission (1997):



Mars Pathfinder lander measured surface pressure
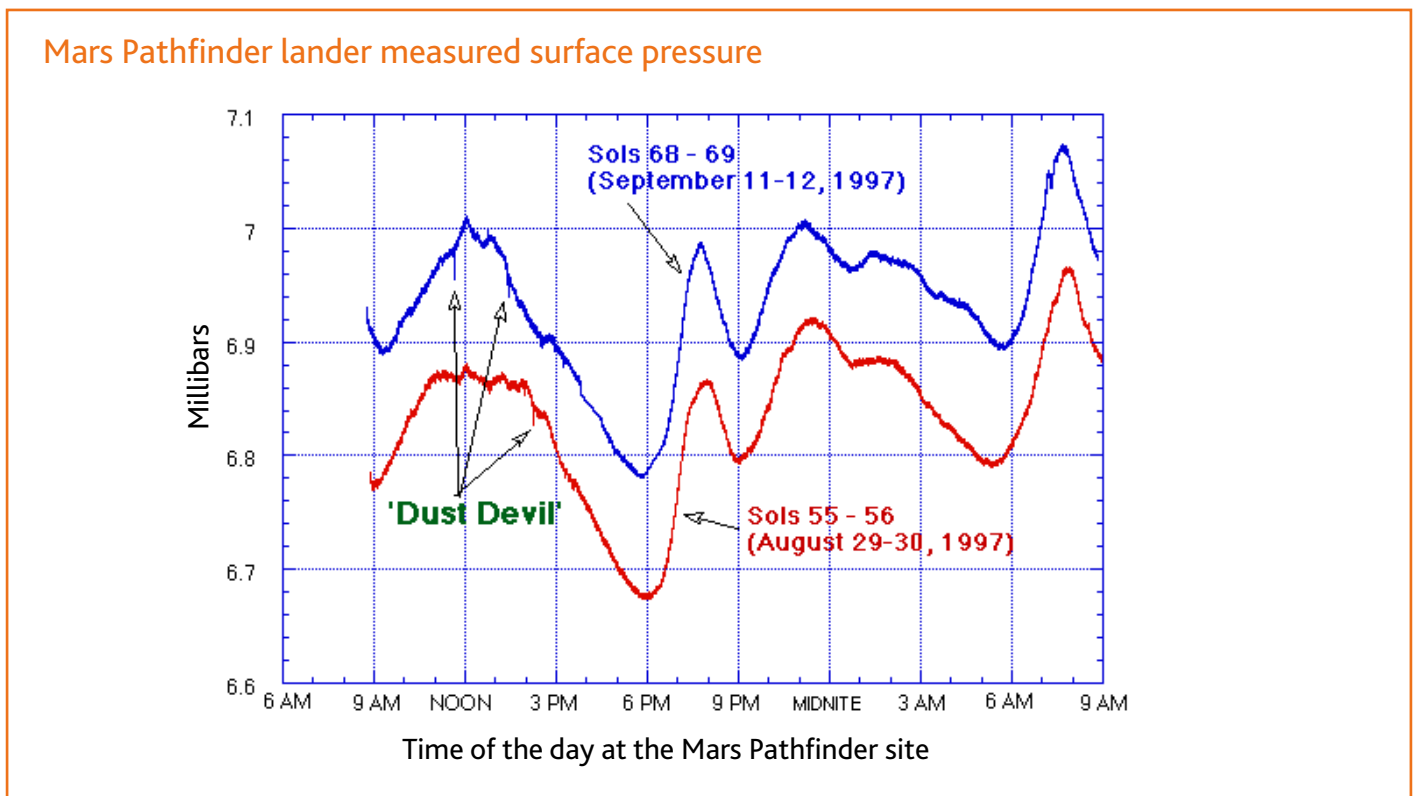
For reference, Earth's mean pressure is 1013.25mb.

## Pluto's atmosphere: a surface pressure of 1 pascal

In 2015 NASA's New Horizons mission flew past Pluto nine years after its launch from Earth. The images below are amongst the highest resolution obtained during the flyby and include backlit images of atmospheric scattering of sunlight in the Plutonian atmosphere.

New Horizons confirmed that the surface pressure on Pluto at the time of flyby was approximately one-hundred-thousandth that on Earth's surface - indicating a mean Pluto surface pressure of 1 pascal due to the very low total atmospheric mass content and very weak gravitational field strength in comparison to Earth.



Light scattering makes the Plutonian atmosphere visible from New Horizons
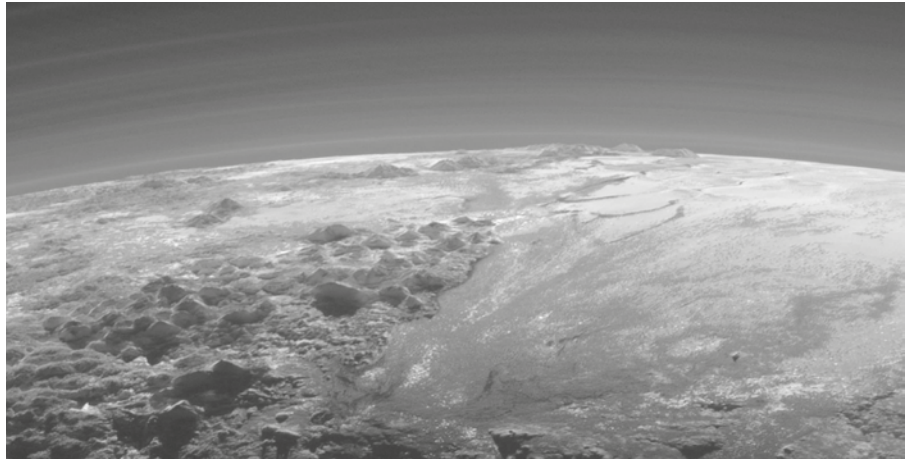


New Horizons sent back images of the Plutonian atmosphere. Credit: NASA

## Spacesuits and misconceptions about human exposure to the vacuum of space

During the Principia mission, Tim was using two very different spacesuit designs:



Tim in his Sokol KV-2 suit. Credit: ESA.

The Sokol KV-2 suit, worn for launch and landing, is designed as an IVA (Intra-Vehicle Activity) suit – for use inside a spacecraft only. Its primary purpose is to keep an astronaut alive in the event of a spacecraft depressurisation during launch or landing - and also for emergency use if toxicity in the ISS atmosphere means that the crew have to retreat to their Soyuz spacecraft "lifeboats" in which case they will seal themselves in their Sokol suits.

The Sokol suit is not designed for EVA (extra-vehicular activity), or "spacewalking" – although a little-known fact is that it can be used with extra protective equipment for an extreme emergency contingency EVA; for example an emergency crew transfer between two spacecraft if not enough EVA suits are available.

The suit Tim wore for his spacewalk was the US "EMU" suit – which, in addition to providing a safe breathing and thermal environment for an astronaut like the IVA suits, is also designed for greater mobility when pressurised and has extra layers of protection against micrometeoroids. It has a self-contained life support system enabling up to nine hours of EVA use in each spacewalk.


Tim wearing the EMU suit during his spacewalk.
Credit: ESA/NASA

## Misconceptions about human exposure to vacuum

Unlike many Hollywood portrayals, the human body, if exposed to the Martian or vacuum environment unprotected, would NOT explode. In vacuum chamber accidents, humans have survived exposure to vacuum of up to one minute without the explosion of the internal organs or skin or eyeballs. The human skin is stronger than expected and would function to some degree as a pressure suit for a short period of time. Blood would not boil through the skin but dissolved gases in the bloodstream would start cascading out of solution – resulting in lethal ebullism in a short period of time.
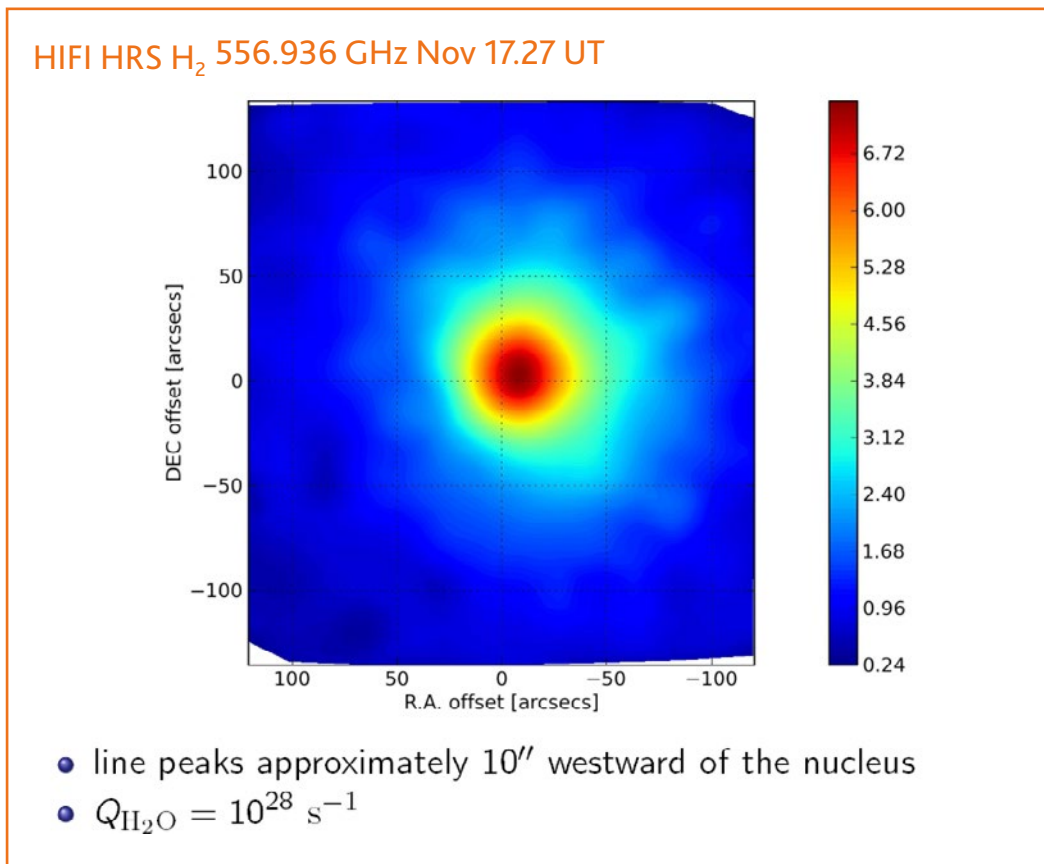
In addition, explosive outgassing of air from the lungs at overpressures corresponding to $10N/cm^2$ (normal atmospheric pressure) would be enough to blast out weaker human front teeth.

In 1966 Johnson Space Center engineer James LeBlanc was testing a spacesuit in a vacuum chamber when his suit decompressed at an equivalent altitude of 250,000ft (to all intents and purposes a vacuum). His last memory was of the saliva boiling off his tongue as he lost consciousness. The chamber was flooded with air within 14 seconds and LeBlanc made a full recovery within days.

However, a lot of misconceptions confuse diving decompression accidents with spacesuit failures. In the early 1980s a decompression accident (the Byford Dolphin incident) resulted in a number of divers being explosively decompressed from a pressure of 9 atm (900,000Pa) to 1 atm in less than a second. This resulted in the explosive disintegration of one of the diver's bodies. However, a spacesuit decompression would result in a pressure gradient far less steep and hence the human body would most likely retain integrity.

## Isotopic ratios, harmonic oscillators and water in comets - Comet Hartley 2 and Herschel

ESA's Herschel infrared space telescope conducted far-infrared spectroscopic observations of Comet Hartley 2 in 2010 and observed the emission of vast quantities of water from its nucleus as seen in the image below.



HIFI HRS $H_2$ 556.936 GHz Nov 17.27 UT

- line peaks approximately $10''$ westward of the nucleus
- $Q_{H_2O} = 10^{28} \text{ s}^{-1}$

The emission rate of $10^{28}$ molecules of water per second means that the nucleus was losing around 300kg per second of mass at this point.

## Heavy water, harmonic oscillators and the origin of Earth's water content

As Tim demonstrated in the Columbus module, molecules can exhibit vibrational modes of behaviour which can be modelled in a simple manner as harmonic oscillators. The frequency of oscillation will depend upon:

- The strength of the chemical bonds between the atoms in the molecule
- The masses of the atoms within the molecule

In deuterated, or "heavy" water, there is one "normal" hydrogen atom in a water molecule and one normal hydrogen atom in a water molecule, and one deuterium atom, which has twice the mass of a hydrogen atom

This means that the vibrational modes and frequencies of a deuterated water molecule can be distinguished from those of normal water through INFRARED SPECTROSCOPY.

The infrared spectroscopy measurements of Comet Hartley 2 conducted by the Herschel infrared space telescope enabled estimates to be made of the ratio of deuterium to hydrogen within the water – and it was found that the water content of this particular comet has, unlike other comets observed, an identical ratio to the water content of Earth's oceans. This provided the first direct evidence that supports the theory that Earth's original water content came from the same source as some comets.

From ESA's science website:

With an extra neutron in one of the hydrogen components of the molecule, heavy water produces a smaller spectral peak.
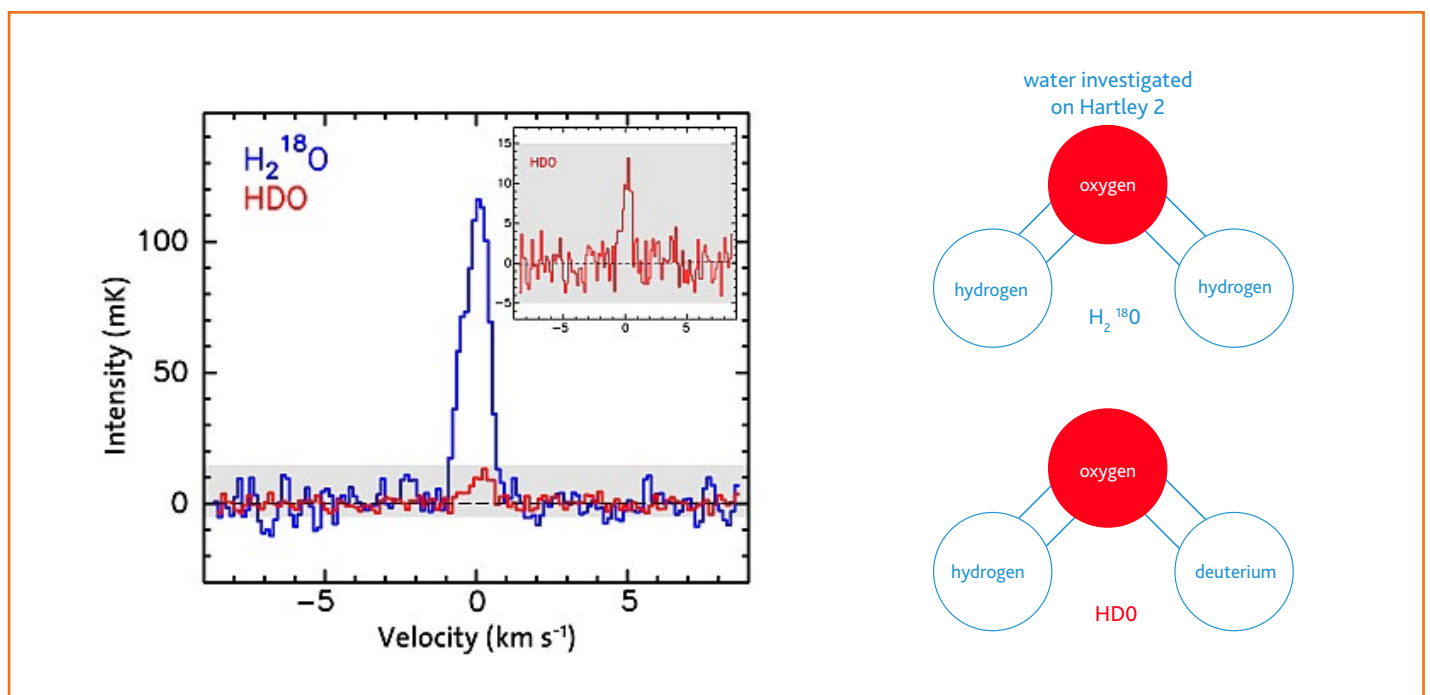


This graph shows the relative signatures of two different 'flavours' - technically known as isotopologues - of water in the spectrum of comet 103P/Hartley 2. The spectrum has been acquired with the HIFI instrument on board Herschel during the comet's approach to Earth in late 2010.

The blue curve refers to the isotopologue $H_2^{18}O$, composed of two hydrogen (H) and one oxygen ($^{18}O$) atoms, and the red curve shows the much weaker signature of another isotopologue, semi-heavy water (HDO), where one of the hydrogen atoms appears in its isotopical form of deuterium (D, or $^2H$). $H_2^{18}O$ was used as a proxy for 'ordinary' water ($H_2^{16}O$) since it was detectable by HIFI on the comet and allowed researchers to observe the relative amounts of hydrogen and deuterium for this study. The HDO feature is also shown, in greater detail, in the inset to the top right of the graph.

From the relative abundance of these two molecules in various Solar System bodies, astronomers can infer the deuterium-to-hydrogen ratio (D/H) in water. By comparing the D/H ratio measured across the Solar System to that measured in Earth's oceans, it is possible to constrain the nature of the bodies that contributed most to delivering water to our planet. The data acquired with Herschel on comet 103P/Hartley 2 represent the first evidence for Earth-like water in a comet, and suggest that comets may have played an important role in bringing water to our planet.
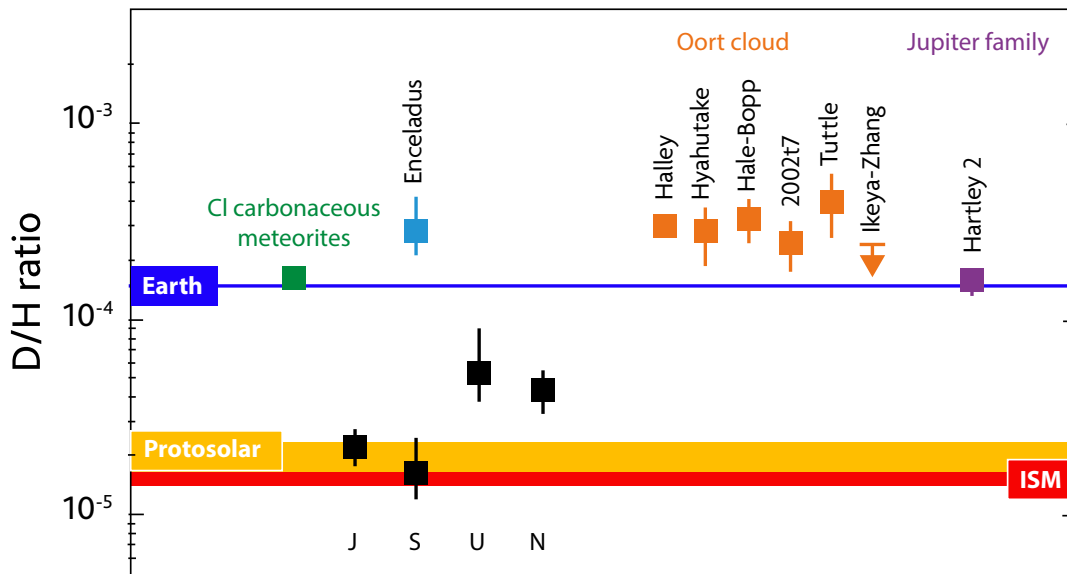


This graph displays the different values of the deuterium-to-hydrogen ratio (D/H) in water observed in various bodies in the Solar System.

The horizontal blue line shows the value of the ratio in Earth's oceans, which has been determined to be $1.56 \times 10^{-4}$. The green square shows the value of the ratio measured in CI carbonaceous chrondrites, a class of meteorites found on Earth, which are believed to originate in the outer asteroid belt.

The six orange data points (five squares and an arrow indicating an upper limit) show the value of the D/H ratio in water observed in comets belonging to the Oort Cloud, which is twice as high as that of Earth-like water. The blue square indicates the value measured on Saturn's moon, Enceladus, which is also higher than that of Earth-like water.

The purple square on the right side of the graph shows the value of the D/H ratio in water observed in the Jupiter-Family comet 103P/Hartley 2, which is very similar to the value measured in Earth's oceans. This measurement is based on data acquired with the HIFI spectrometer on board Herschel in late 2010. As reported by Hartogh et al., 2011, these data represent the first evidence of Earth-like water detected in a comet.

The lower part of the graph shows the value of the D/H ratio measured in molecular hydrogen in the atmosphere of the giant planets of the Solar System (Jupiter, Saturn, Uranus, Neptune; black squares) and an estimate of the typical value in molecular hydrogen for the protosolar material (broad, yellow horizontal line) and in atomic hydrogen in the local interstellar medium (red horizontal line).

## 1. Real Gases

Ideal gas molecule collisions assume that the molecules are point-like masses and that no forces act between the molecules themselves. We can try to account for these for real gas molecule collisions by amending the ideal gas equation of state from
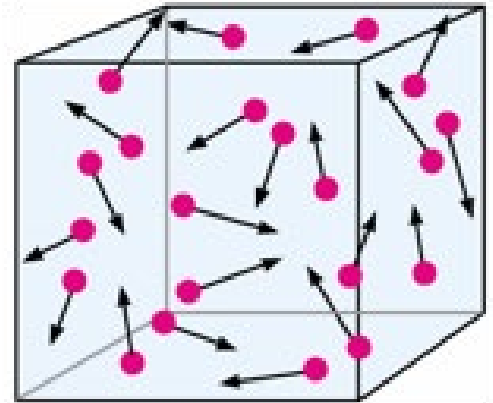
$$PV = nRT$$

where   P = pressure
        V = volume
        n = moles of gas
        R = ideal gas law constant (a proportionality constant)
        T = temperature

To   $\left(P + \dfrac{n^2a}{V^2}\right)(V - nb) = nRT$

Notice how "corrections" are being made to the pressure term and the volume term.

Since collisions of real gases involve attractive forces between the molecules, the term $n^2a/V^2$ is correcting for the interactions of these molecules. The value of a is a constant, and must be experimentally determined for each gas.

Since real gas molecules have a volume, the nb term is correcting for the excluded volume. The value of b is a constant, and must be determined experimentally for each gas.

a.  1.00 mole of carbon dioxide gas at 373K occupies 536mL at 50 atmospheres of pressure. What is the calculated value of the pressure using

   (i) The ideal gas equation?

   (ii) The real gas equation?

   Constants for carbon dioxide are a = 3.61 $L^2$ atm $mol^{-2}$; b = 0.0428 L $mol^{-1}$ and R = 0.08206 L atm $mol^{-1}K^{-1}$

b. Using the real equation, calculate the temperature of 20.0 moles of helium in a 10.0 litre cylinder at 120 atmosphere pressure.

Constants for Helium are a = 0.0341 L$^2$ atm mol$^{-2}$; b = 0.0237 L mol$^{-1}$

c. Compare this value with the temperature calculated from the ideal gas equation

## 2. Gas Law Challenging Question (Oxbridge)

You copy the following paragraph from a Martian physics textbook: "1 snorf of an ideal gas occupies a volume of 1.35 zaks. At a temperature of 22 glips, the gas has a pressure of 12.5 klads. At a temperature of –10 glips, the same gas now has a pressure of 8.7 klads."

Determine the temperature of absolute zero in glips.
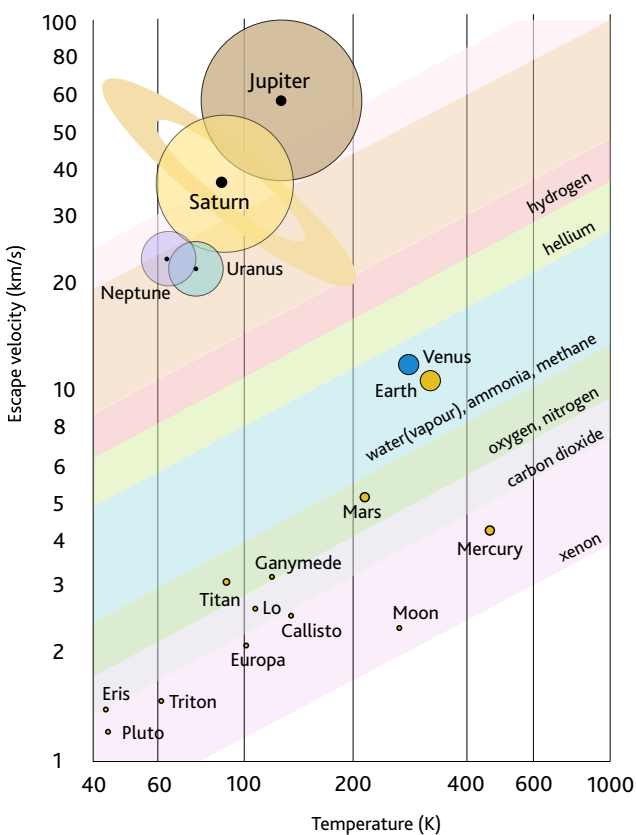


Escape velocities for different Solar System bodies plotted against their surface temperatures. Objects are drawn to scale, the data points are the black dots at their centres.
Source: Wikipedia.

## 3. Leaky Atmospheres

When a gas is at a certain temperature not all of the molecules in that gas are moving at the same velocity. While the majority of the molecules are moving at or near the speed that corresponds to the temperature of the gas, some will hardly be moving at all and some small fraction of them will be moving at very high speeds.

The plot below shows the speed distributions for oxygen and hydrogen molecules in the Earth's exosphere (the upper layer where molecules are gravitationally bound but where the density is too low for them to have gas collisions)

The escape velocity at the surface of a planet can be shown to be $\sqrt{(2gR)}$

(i) Calculate the escape velocity from the Earth's surface (R = 6370km)

(ii) Use the graph to explain why a substantial fraction of hydrogen molecules leak from the Earth's atmosphere whilst only a negligible fraction of oxygen molecules leak away

(iii) Using the relationship between the mean kinetic energy of gas molecules $\frac{1}{2} mv^2_{rms}$ and absolute temperature T show that the rms velocity is given by $v_{rms} = \sqrt{(3RT/M)}$ where R is the molar gas constant and M is the molar mass of the gas.

If the root-mean-square (rms) speed of a gas is greater than about 15 to 20 percent of the escape speed of a planet, virtually all of the molecules of that gas will escape the atmosphere of the planet.

(iv) At what temperature is $v_{rms}$ for $O_2$ equal to 15 percent of the escape speed for Earth?

(v) At what temperature is $v_{rms}$ for $H_2$ equal to 15 percent of the escape speed for Earth?

(vi) Temperatures in the upper atmosphere reach 1000K. How does this help account for the low abundance of hydrogen in Earth's atmosphere?

(vii) Compute the temperatures for which the rms speeds of $O_2$ and $H_2$ are equal to 15 percent of the escape speed at the surface of the moon, where g is about one-sixth of its value on Earth and R = 1738km. How does this account for the absence of an atmosphere on the moon?

## 4. Leaky Planets Question

The escape speed for gas molecules in the atmosphere of Mars is 5.0kms[-1] and the surface temperature of Mars is typically 0°C. Calculate the rms speeds for:
(a) $H_2$,
(b) $O_2$,
(c) $CO_2$ at this temperature

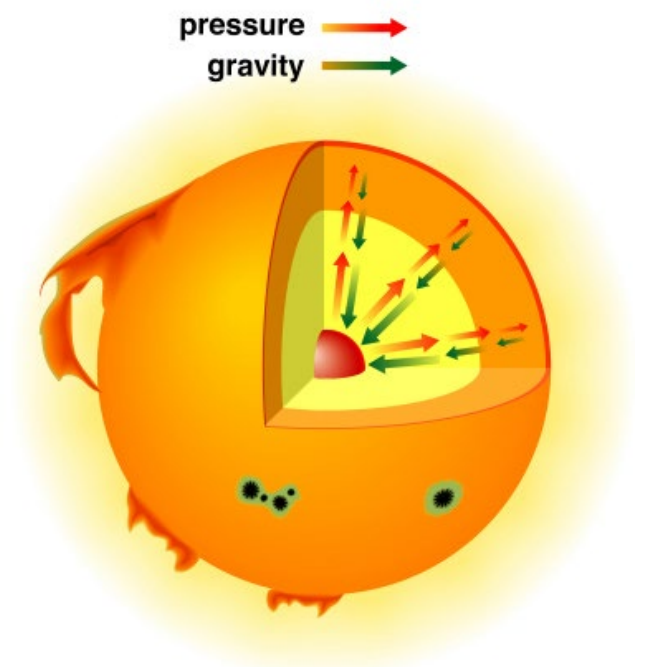(d) Are $H_2$, $O_2$, and $CO_2$ likely to be found in the atmosphere of Mars?

The escape speed for gas molecules in the atmosphere of Jupiter is 60km/s and the surface temperature of Jupiter is typically –150°C. Calculate the rms speeds for:
(a) $H_2$,
(b) $O_2$,
(c) $CO_2$ at this temperature

(d) Are $H_2$, $O_2$, and $CO_2$ likely to be found in the atmosphere of Jupiter?

## 5. Supporting the Sun

To a first approximation, the Sun consists of a gas of equal numbers of protons and electrons. The temperature at the centre of the Sun is about $1 \times 10^7$ K, and the density of the Sun is about $1 \times 10^5$ kgm[-3]. Since the temperature is so high, the protons and electrons are separate particles (rather than being joined together to form hydrogen atoms).



pressure
gravity

Pressure and gravity within the sun.
Credit: www.physast.uga.edu

(a) Estimate the pressure at the centre of the Sun.

(b) Estimate the rms speeds of the protons and the electrons at the centre of the Sun.

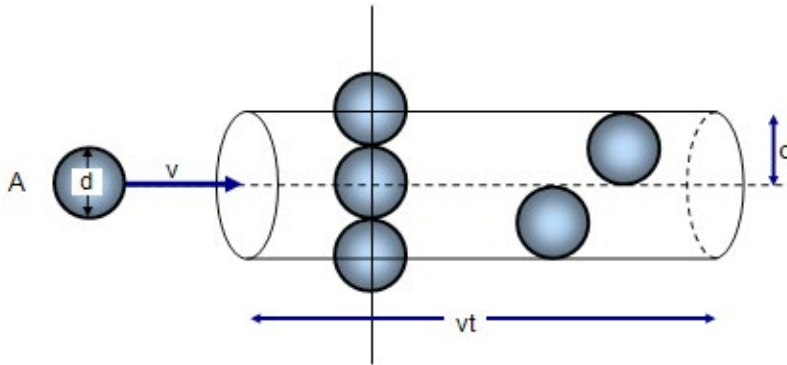# 6. Mean Free Path

The mean free path λ is the average distance a molecule travels in the gas between collisions.

The molecules in a gas are assumed to be in continuous random motion making collisions with each other and with the walls of the container. The mean distance that the molecules travel between one collision and the next is called the mean free path of the molecules.



Consider a molecule A with an effective diameter d travelling with a velocity v (see above).

In a time t the molecule will travel a distance vt and collide with $\pi d^2 vtn$ molecules, where n is the number of molecules per unit volume.

The molecule collides with all molecules whose centres are within a cylinder of length vt and cross-sectional area $\pi d^2$

The mean free path is the total distance covered (vt) divided by the number of collisions. Therefore the mean free path us given by:

Mean free path $\lambda = vt/[\pi d^2 nvt] = 1/[\pi n d^2]$

This proof assumes that all the other molecules remain at rest. If we consider the case of moving molecules the formula must be modified and becomes:

Mean free path $\lambda = 1/\sqrt{2}[\pi n d^2]$

a.      Intergalactic space is nearly a vacuum: there is on average approximately one hydrogen atom per cubic centimetre.

The diameter of a hydrogen atom is about 0.1nm.

i.  Estimate the mean free path of a hydrogen atom under these conditions
ii.  Find the rms speed of the hydrogen atoms at temperature 2.7K
iii. Use (i) and (ii) to estimate the average time between collisions in years

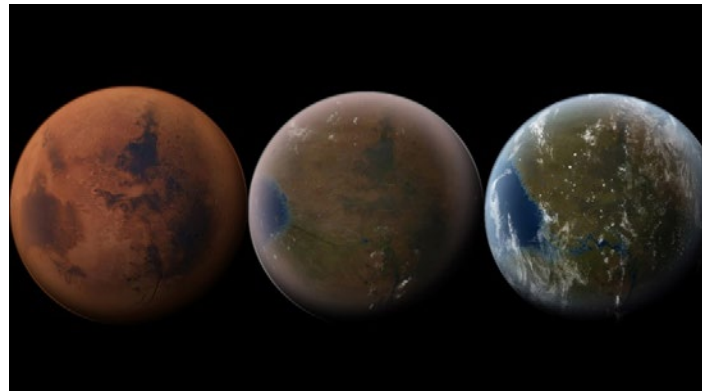b. Show that the mean free path for a molecule in an ideal gas at temperature T and pressure P is given by

$$\lambda = \frac{kT}{\sqrt{2}P\pi d^2}$$

c. State-of-the-art vacuum equipment can attain pressures as low as $7.0 \times 10^{-11}$Pa. Suppose that a chamber contains helium at this pressure and at room temperature (300K). Estimate the mean free path and the collision time for helium in the chamber. Assume the diameter of a helium atom is $1.0 \times 10^{-10}$m.

## 7. Terraforming Mars

Terraforming is the process of transforming a hostile environment into one suitable for human life. Being that Mars is the most Earth-like planet, it is the best candidate for terraforming.

One way on making the Martian atmosphere more hospitable to human life would be to increase the atmospheric density to a level similar to that found on Earth, eliminating the need for a pressure suit.



What Martian terraforming might look like. Credit: http://bigthink.com/videos/the-cheapest-way-to-terraform-mars-2

One idea is that coal could be transported from Earth to Mars and $CO_2$ released through combustion.

Avogadro's constant = $6.02 \times 10^{23}$ and the molar mass of $CO_2$ is 44g.

a. Given that the Martian atmosphere has a height of 60.0km and the radius of Mars is 3390km calculate the volume of the Martian atmosphere.

b. Given that the average temperature of the Martian atmosphere is 218K and the current Martian pressure is 600Pa, by assuming the Martian atmosphere is an ideal gas, calculate the number of gas molecules in the current Martian atmosphere.

c. Calculate the number of gas molecules that would be required in the same volume of Martian atmosphere to increase the Martian atmospheric pressure to 101kPa (the same as the Earth's atmospheric pressure).

d. Calculate the mass of carbon dioxide required

e.  Given that hard coal produces 0.34kg of $CO_2$ per kWh and that 0.49kg of coal generates a kWh such that 0.49kg of coal burned releases 0.34kg of $CO_2$ determine the mass of carbon (coal) which will need to be transported to Mars.

f.  Calculate the energy required to transport this mass of coal to Mars if the spacecraft needs to reach Earth escape velocity of 11.2km/s

g.  SpaceX Falcon Heavy, the world's most powerful rocket, has a payload capability to Mars of 13,200kg. How many trips would be needed to transport the coal required to terraform Mars?

## 1. Real Gases

Ideal gas molecule collisions assume that the molecules are point-like masses and that no forces act between the molecules themselves. We can try to account for these for real gas molecule collisions by amending the ideal gas equation of state from

$$PV = nRT$$

where  P = pressure
V = volume
n = moles of gas
R = ideal gas law constant (a proportionality constant)
T = temperature

To  $(P + \dfrac{n^2a}{V^2})(V - nb) = nRT$

Notice how "corrections" are being made to the pressure term and the volume term.

Since collisions of real gases involve attractive forces between the molecules, the term $n^2a/V^2$ is correcting for the interactions of these molecules. The value of a is a constant, and must be experimentally determined for each gas.

Since real gas molecules have a volume, the nb term is correcting for the excluded volume. The value of b is a constant, and must be determined experimentally for each gas.

a.  1.00 mole of carbon dioxide gas at 373K occupies 536mL at 50 atmospheres of pressure. What is the calculated value of the pressure using

   (i) Ideal gas equation

   $P = nRt /V = 1 \times 8.31 \times 373/5.36 \times 10^{-4} = 5.8 \times 10^6 Pa$ (i.e. $5.8 \times 10^6/1.01 \times 10^5 = 57.1$ atm)

   (ii) Real gas equation
   Constants for carbon dioxide are a = 3.61 $L^2$ atm $mol^{-2}$; b = 0.0428 L $mol^{-1}$ and
   R = 0.08206 L atm $mol^{-1}K^{-1}$

   $P = \dfrac{nRT}{(V- nb)} - \dfrac{(n^2a)}{V^2}$

   $P = (1 \times 0.08206 \times 373) /(536 \times 10^{-3} - 1 \times 0.0428) - (1^2 \times 3.61) /(536 \times 10^{-3})^2 = 49.5$ atm

Using the real equation, calculate the temperature of 20.0 moles of helium in a 10.0 litre cylinder at 120 atmosphere pressure.

Constants for Helium are a = 0.0341 $L^2$ at $mol^{-2}$; b = 0.0237 L $mol^{-1}$

$$T= \frac{\left(P+ \frac{n^2\,a}{V^2}\right) \times (V-nb)}{nR}$$

T = ((120 + (20² x 0.0341) / 10²) x (10 − 20 x 0.0237)) / (20 x 0.08206) = 697K

b. Compare this value with the temperature calculated from the ideal gas equation

PV = nRT
T = PV/nR =(120 x 1.01 x 10⁵ x 10 x 10⁻³) / 20 x 8.31 = 729K
Or, using atmospheres and litres, T = PV/nR = (120X10)/(20X0.08206) = 731K

## 2. Gas Law Challenging Question (Oxbridge)

You copy the following paragraph from a Martian physics textbook: "1 snorf of an ideal gas occupies a volume of 1.35 zaks. At a temperature of 22 glips, the gas has a pressure of 12.5 klads. At a temperature of –10 glips, the same gas now has a pressure of 8.7 klads."

Determine the temperature of absolute zero in glips.

The gas is ideal therefore P (in klads) is proportional to temperature (in glips) and we assume constant volume. So a graph of P vs T produces a straight line with the equation given by;

Gradient, m = $\frac{12.5 - 8.7}{22 - -10}$ = 0.118

So, 12.5 = 0.118 x 22 + c
giving intercept, c = 9.9

This gives:

P = $0.118T_0$ + 9.9

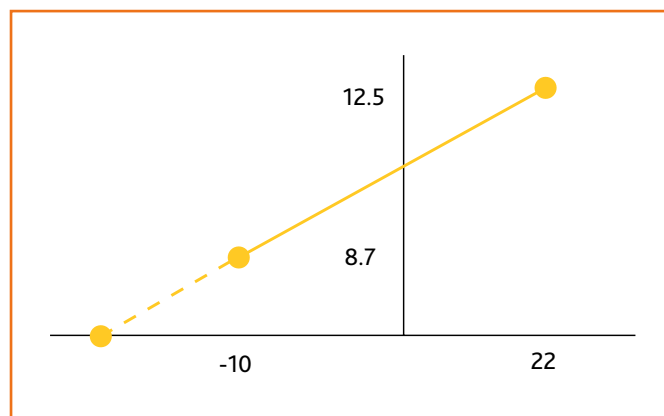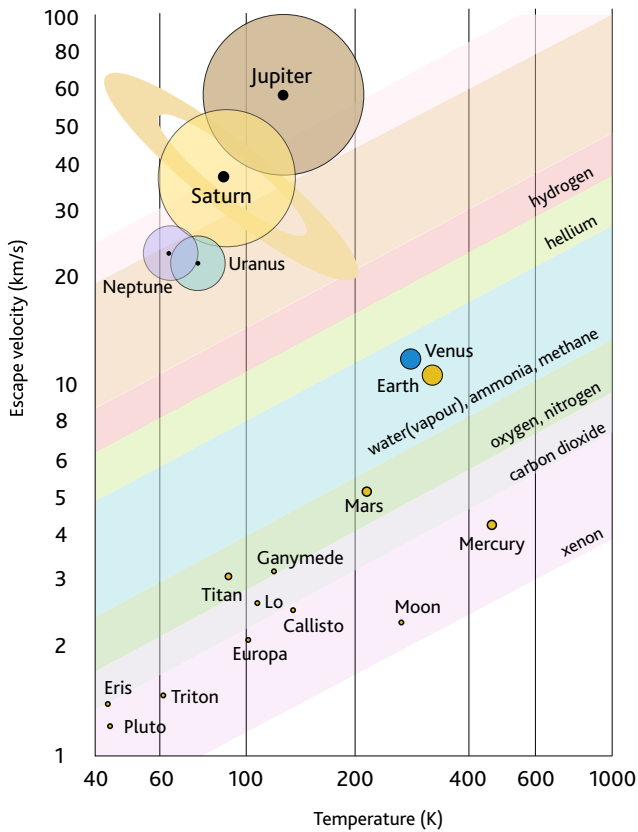Solving for P = 0 gives 0 = $0.118T_0$ + 9.9

So $T_0$ = -83 glips

Or, using basically the same method but using X for absolute temperature:
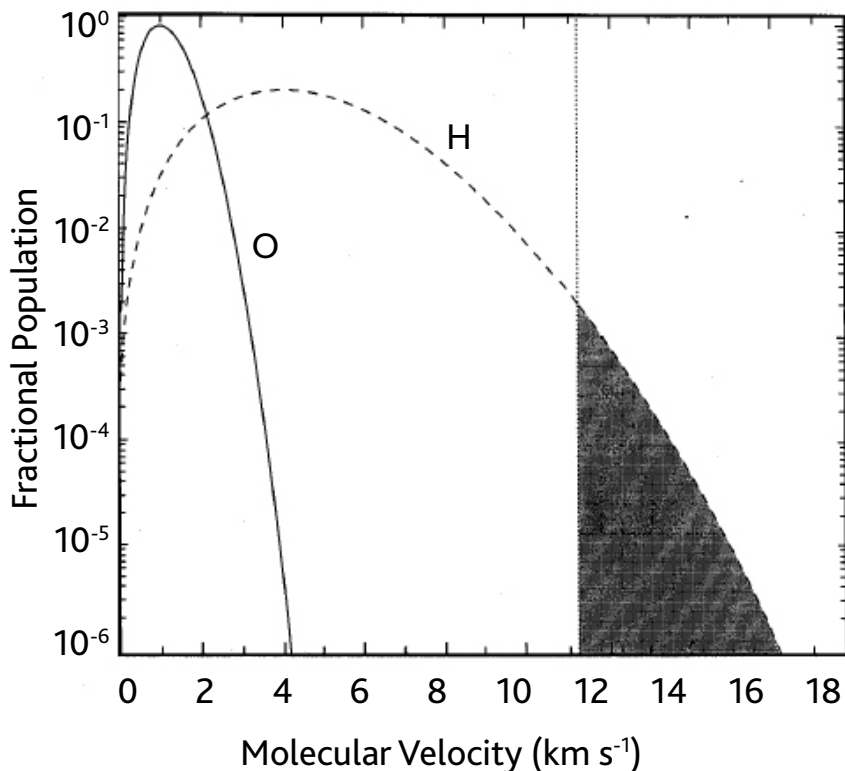(12.5-8.7)/32 = 12.5/(22+X)  gives X = -83 glips

# 3. Leaky Atmospheres

When a gas is at a certain temperature not all of the molecules in that gas are moving at the same velocity. While the majority of the molecules are moving at or near the speed that corresponds to the temperature of the gas, some will hardly be moving at all and some small fraction of them will be moving at very high speeds.

The plot below shows the speed distributions for oxygen and hydrogen molecules in the Earth's exosphere (the upper layer where molecules are gravitationally bound but where the density is too low for them to have gas collisions)

The escape velocity at the surface of a planet can be shown to be $\sqrt{(2gR)}$

(i) Calculate the escape velocity from the Earth's surface (R = 6370km)

$v = \sqrt{(2gR)} = (2 \times 9.81 \times 6370 \times 10^3) = 11,200ms^{-1}$

(ii) Use the graph to explain why a substantial fraction of hydrogen molecules leak from the Earth's atmosphere whilst only a negligible fraction of oxygen molecules leak away

A significant fraction of hydrogen molecules have the required escape velocity (shaded area), whilst virtually zero oxygen molecules have the required velocity

(iii) Using the relationship between the mean kinetic energy of gas molecules $\frac{1}{2} mv^2_{rms}$ and absolute temperature T show that the rms velocity is given by $v_{rms} = \sqrt{(3RT/M)}$ where R is the molar gas constant and M is the molar mass of the gas.

From k.e. theory we have $\frac{1}{2} mv^2_{rms} = \frac{3}{2}KT$

Which gives $v^2_{rms} = 3KT/m$ and since $K = R/N_A$

Using Molar mass, $M=mN_A/n$, this becomes $v^2_{rms} = 3RT/M$

If the root-mean-square (rms) speed of a gas is greater than about 15 to 20 percent of the escape speed of a planet, virtually all of the molecules of that gas will escape the atmosphere of the planet.

(iv) At what temperature is $v_{rms}$ for $O_2$ equal to 15 percent of the escape speed for Earth?
$v_{rms}=\sqrt{(3RT/M)}$

$0.15 \times 11,200 = ((3 \times 8.31 \times T) / 32 \times 10\text{-}3)^{½}$
$T = (0.15 \times 11,200)^2 \times 32 \times 10^{-3} / (3 \times 8.31)$ which is approximately 3600K

(v) At what temperature is $v_{rms}$ for $H_2$ equal to 15 percent of the escape speed for Earth?

$T = (0.15 \times 11,200)^2 \times 2.02 \times 10^{-3} / (3 \times 8.31)$ which is approximately 230K

(vi) Temperatures in the upper atmosphere reach 1000K. How does this help account for the low abundance of hydrogen in Earth's atmosphere?

Hydrogen is lighter so a greater fraction will be found in the upper atmosphere. Temperature at this height is higher so a greater fraction of hydrogen molecules reach escape velocity.

(vii) Compute the temperatures for which the rms speeds of $O_2$ and $H_2$ are equal to 15 percent of the escape speed at the surface of the moon, where $g$ is about one-sixth of its value on Earth and $R$ = 1738km. How does this account for the absence of an atmosphere on the moon?

$v = \sqrt{(2gR)} = (2 \times 1/6 \times 9.81 \times 1738 \times 10^3) = 2384ms^{-1}$

$T_O = (0.15 \times 2384)^2 \times 32 \times 10^{-3} / (3 \times 8.31) = 164K$

$T_H = (0.15 \times 2384)^2 \times 2 \times 10^{-3} / (3 \times 8.31) = 10.3K$

g on the moon is lower so escape velocity is lower and so a larger fraction of molecules exceed the Moon's escape velocity

## 4. Leaky Planets Question

The escape speed for gas molecules in the atmosphere of Mars is 5.0kms$^{-1}$ and the surface temperature of Mars is typically 0°C. Calculate the rms speeds for:

(a) $H_2$,
(b) $O_2$,
(c) $CO_2$ at this temperature

(d) Are $H_2$, $O_2$, and $CO_2$ likely to be found in the atmosphere of Mars?

$v_{rms} = \sqrt{(3RT/M)}$

$v_{rms, H} = \sqrt{(3 \times 8.31 \times 273 / 2 \times 10^{-3})} = 1845ms^{-1}$

$v_{rms, O} = \sqrt{(3 \times 8.31 \times 273 / 32 \times 10^{-3})} = 461ms^{-1}$

$v_{rms, CO} = \sqrt{(3 \times 8.31 \times 273 / 44 \times 10^{-3})} = 393ms^{-1}$

20% of 5kms$^{-1}$ = 1000ms$^{-1}$, so $O_2$ and $CO_2$ should be present but not $H_2$

The escape speed for gas molecules in the atmosphere of Jupiter is 60km/s and the surface temperature of Jupiter is typically –150°C. Calculate the rms speeds for

(a) $H_2$,
(b) $O_2$,
(c) $CO_2$ at this temperature

(d) Are $H_2$, $O_2$, and $CO_2$ likely to be found in the atmosphere of Jupiter?

$v_{rms, H} = \sqrt{(3 \times 8.31 \times 123 / 2 \times 10^{-3})} = 1240ms^{-1}$

$v_{rms, O} = \sqrt{(3 \times 8.31 \times 123 / 32 \times 10^{-3})} = 310ms^{-1}$

$v_{rms, CO} = \sqrt{(3 \times 8.31 \times 123 / 44 \times 10^{-3})} = 264ms^{-1}$

20% of $60kms^{-1} = 12000ms^{-1}$, so all gases should be present as only a small fraction of molecules will have speeds above this value

## 5. Supporting the Sun

pressure ⟶
gravity ⟶

To a first approximation, the Sun consists of a gas of equal numbers of protons and electrons. The temperature at the centre of the Sun is about $1 \times 10^7$ K, and the density of the Sun is about $1 \times 10^5$ kgm$^{-3}$. Since the temperature is so high, the protons and electrons are separate particles (rather than being joined together to form hydrogen atoms).

a.  Estimate the pressure at the centre of the Sun.

Assume we can model the plasma as an ideal gas and then take the pressure of 1 m³

Use P = nRT/V

Pressure and gravity within the sun.
Credit: www.physast.uga.edu

1 m³ has a mass of $10^5$ Kg

Mole of protons has a mass of $6.02 \times 10^{23} \times 1.67 \times 10^{-27} = 1.00 \times 10^{-3}$Kg, so we have $10^5 / 10^{-3} = 10^8$ moles

And, in addition, if we consider electrons, then n = $2 \times 10^8$ mol ($1 \times 10^8$ mol of electrons and $1 \times 10^8$ mole of protons)

P = $(2 \times 10^8 \times 8.31 \times 1 \times 10^7) / 1 = 2\times10^6$Pa
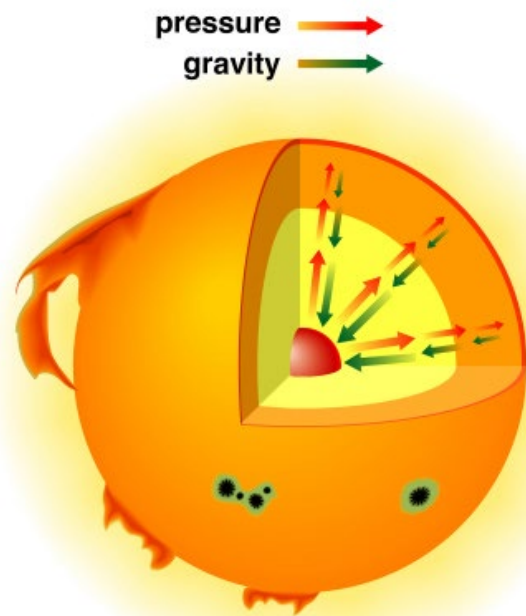
b. Estimate the rms speeds of the protons and the electrons at the centre of the Sun

$v_{rms} = \sqrt{(3RT/M)}$
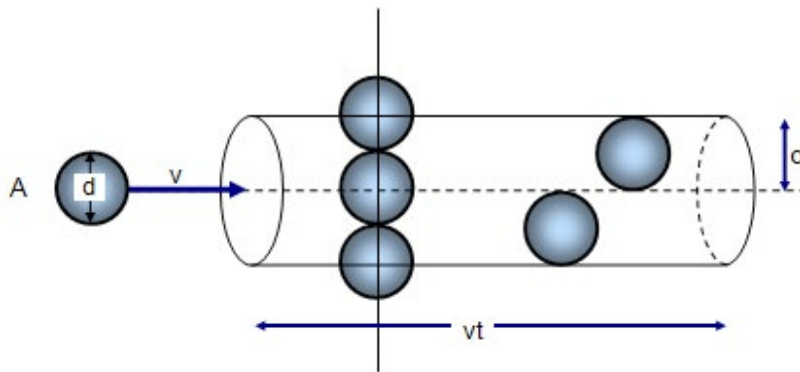
$v_{rms,\ protons} = \sqrt{(3 \times 8.31 \times 1 \times 10^7 / 1 \times 10^{-3})} = 5 \times 10^5 ms^{-1}$

$v_{rms,\ electrons} = \sqrt{(3 \times 8.31 \times 1 \times 10^7 / 5 \times 10^{-4} \times 1 \times 10^{-3})} = 2 \times 10^7 ms^{-1}$ assuming mass of electron is approximately 1/2000 mass of proton

## 6. Mean Free Path

The mean free path $\lambda$ is the average distance a molecule travels in the gas between collisions.

The molecules in a gas are assumed to be in continuous random motion making collisions with each other and with the walls of the container. The mean distance that the molecules travel between one collision and the next is called the mean free path of the molecules.



Consider a molecule A with an effective diameter d travelling with a velocity v (see above).

In a time t the molecule will travel a distance vt and collide with $\pi d^2 vtn$ molecules, where n is the number of molecules per unit volume.

The molecule collides with all molecules whose centres are within a cylinder of length vt and cross-sectional area $\pi d^2$

The mean free path is the total distance covered (vt) divided by the number of collisions. Therefore the mean free path us given by:

Mean free path $\lambda = vt/[\pi d^2 nvt] = 1/[\pi nd^2]$

This proof assumes that all the other molecules remain at rest. If we consider the case of moving molecules the formula must be modified and becomes:

Mean free path $\lambda = 1/\sqrt{2}[\pi nd^2]$

a.  Intergalactic space is nearly a vacuum: there is on average approximately one hydrogen atom per cubic centimetre.

The diameter of a hydrogen atom is about 0.1nm.

i.  Estimate the mean free path of a hydrogen atom under these conditions

$\lambda = 1/\sqrt{2}[\pi nd^2]$ assuming hydrogen atoms are in motion

$\lambda = 1/\sqrt{2}[\pi \times 1 \times 10^6 \times (0.1 \times 10^{-9})^2] = 2.3 \times 10^{13} m$

ii. Find the rms speed of the hydrogen atoms at temperature 2.7K

$v_{rms, H} = \sqrt{(3 \times 8.31 \times 2.7 / 2.02 \times 10^{-3})} = 180 ms^{-1}$ (to 2 s.f.)

iii. Use (i) and (ii) to estimate the average time between collisions in years

$t = 2.3 \times 10^{13} / 180 = 1.28 \times 10^{11}$ s = 4050 years (working to 2 s.f.)


b.  Show that the mean free path for a molecule in an ideal gas at temperature T and pressure P is given by

$$\lambda = \frac{kT}{\sqrt{2}P\pi d^2}$$

$\lambda = 1/\sqrt{2}[\pi nd^2]$ and PV = NkT

n = N/V  and  V = NkT/P  so  n = P/kT

and substitute for n into first equation gives $\lambda = kT/\sqrt{2}[\pi Pd^2]$

b. State-of-the-art vacuum equipment can attain pressures as low as $7.0 \times 10^{-11}$ Pa. Suppose that a chamber contains helium at this pressure and at room temperature (300K). Estimate the mean free path and the collision time for helium in the chamber. Assume the diameter of a helium atom is $1.0 \times 10^{-10}$ m.

$\lambda = kT / \sqrt{2}P\pi d^2$

$\lambda = (1.38 \times 10^{-23} \times 300) / \sqrt{2} \times 7 \times 10^{-11} \times \pi \times (1 \times 10^{-10})^2 = 1.3 \times 10^9$ m
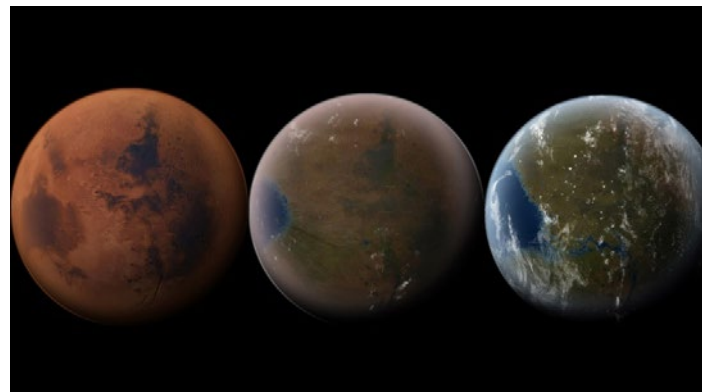
$v_{rms} = \sqrt{(3RT/M)}$

$v_{rms} = \sqrt{(3 \times 8.31 \times 300 / 4 \times 10^{-3})} = 1400$ ms$^{-1}$ (to 2 s.f.)

$t = 1.3 \times 10^9 / 1400 = 9.3 \times 10^5$ s (working with 2 s.f.)

## 7. Terraforming Mars

Terraforming is the process of transforming a hostile environment into one suitable for human life. Being that Mars is the most Earth-like planet, it is the best candidate for terraforming.

One way on making the Martian atmosphere more hospitable to human life would be to increase the atmospheric density to a level similar to that found on Earth, eliminating the need for a pressure suit.



What Martian terraforming might look like.
Credit: http://bigthink.com/videos/the-cheapest-way-to-terraform-mars-2

One idea is that coal could be transported from Earth to Mars and $CO_2$ released through combustion.

Avogadro's constant = $6.02 \times 10^{23}$ and the molar mass of $CO_2$ is 44g.

a. Given that the Martian atmosphere has a height of 60.0km and the radius of Mars is 3390km calculate the volume of the Martian atmosphere.

$V = 4/3\pi R^3 - 4/3\pi r^3 = 4/3\pi(R^3 - r^3)$

$V = 4/3\pi((3450 \times 10^3)^3 - (3390 \times 10^3)^3) = 1.72 \times 10^{20} - 1.63 \times 10^{20} = 9.00 \times 10^{18}$ m$^3$

b. Given that the average temperature of the Martian atmosphere is 218K and the current Martian pressure is 600Pa, by assuming the Martian atmosphere is an ideal gas, calculate the number of gas molecules in the current Martian atmosphere.

$PV = NkT$

$N = (600 \times 9.00 \times 10^{18}) / (1.38 \times 10^{-23} \times 218) = 1.79 \times 10^{42}$

c. Calculate the number of gas molecules that would be required in the same volume of Martian atmosphere to increase the Martian atmospheric pressure to 101kPa (the same as the Earth's atmospheric pressure).

$N = (101 \times 10^3 \times 9.00 \times 10^{18}) / (1.38 \times 10^{-23} \times 218) = 3.02 \times 10^{44}$

So need $3.02 \times 10^{44} - 1.79 \times 10^{42}$ extra molecules

So need $3.00 \times 10^{44}$ extra molecules

d. Calculate the mass of carbon dioxide required

Need $3.00 \times 10^{44} / 6.02 \times 10^{23}$ moles $= 4.98 \times 10^{20}$ mols $= 4.98 \times 10^{20} \times 44 \times 10^{-3} = 2.19 \times 10^{19}$ kg

e. Given that hard coal produces 0.34kg of $CO_2$ per kWh and that 0.49kg of coal generates a kWh such that 0.49kg of coal burned releases 0.34kg of $CO_2$ determine the mass of Carbon (coal) which will need to be transported to Mars.

Amount of coal $= (2.19 \times 10^{19} / 0.34) \times 0.49 = 3.2 \times 10^{19}$kg (to 2 s.f)

f. Calculate the energy required to transport this mass of coal to Mars if the spacecraft needs to reach Earth escape velocity of 11.2km/s

K.e. $= \frac{1}{2} mv^2 = 0.5 \times 3.2 \times 10^{19} \times (11.2 \times 10^3)^2 = 2.01 \times 10^{27}$J

g. SpaceX Falcon Heavy, the world's most powerful rocket, has a payload capability to Mars of 13,200kg. How many trips would be needed to transport the coal required to terraform Mars?

Number of trips $= 3.2 \times 10^{19} / 13,200 = 2.42 \times 10^{15}$