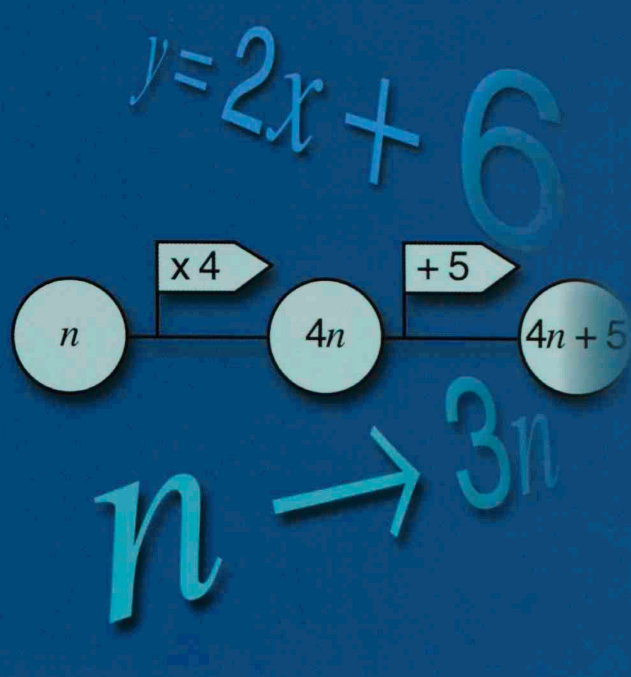
MATHEMATICS

Algebra Makes Sense

Fully photocopiable

- Suitable for Key Stages 3 and 4
- Progression through Levels 4 – 7
- Encourages mathematical thinking



SMILE StandAlone Resource

SMILE MATHEMATICS

Algebra Makes Sense

SMILE
MATHEMATICS

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Contents

Activities in approximate order of difficulty.

Introduction

Introduction

This activity book includes 20 activities focusing on algebra, all of which are photocopiable within the purchasing establishment.

The activities address different aspects of algebra and are appropriate for a range of learning styles. They are arranged by National Curriculum Assessment Levels (4 – 7) in order to aid differentiation and facilitate their easy incorporation into any scheme of work. Within each level the activities are arranged in order of difficulty.

Each activity is also referenced to the Framework for Teaching Mathematics. References to the Framework have been taken from the Yearly Teaching Programmes and page numbers refer to the appropriate Supplement of Examples. The symbol (K) is used to identify the Key Objectives for each year.

The teachers' notes and resource sheets

The activities are organised in double-page spreads. The left-hand page of each spread includes teachers' notes, offering advice on appropriate ways of using the activity and any additional equipment required. Ideas to enrich students' mathematical thinking are also included. The resource sheet on the right-hand page can be photocopied for students to use. Teachers may also photocopy the resource sheet onto acetate and view with an OHP to facilitate whole class discussion.

The role of the teacher

Many of the activities are suitable for whole class lessons and contain much scope for discussion. However, all the materials have been designed to contain sufficiently clear instructions to be accessible to students with minimal teacher support. This makes them ideal for use within whole class differentiated topic work as the teacher is left free to discuss structural issues within algebra with individuals or groups as they arise. Full answers are given for each activity. A reference grid gives teachers guidance on how best to integrate the use of the MicroSMILE Algebra software pack to enhance students' understanding.

The Appendix

The two resource sheets contained within this section have been designed to save students' time. The Mapping Jigsaw Resource Sheet provides a template to create their own mapping jigsaw, and the Algebra Game/Trail Resource Sheet provides an easy-to-use recording sheet.

MicroSMILE Algebra Software Pack

The activities included in this publication are complemented by the MicroSMILE Algebra software pack, a standalone resource which can be purchased separately from SMILE Mathematics. When used in conjunction with Algebra Makes Sense, the two resources can be even more effective in developing algebraic skills. MicroSMILE Algebra consists of 5 programs:

- **Algebra Magic** encourages collecting like terms and simplifying expressions.
- **Equation Buster** develops students' skills in substituting and solving equations.
- **Equations Resource** is a resource program that aims to demonstrate different approaches to solving linear equations.
- **Mapping Match** looks at the relationships between mappings, term-to-term rules, position-to-term rules and algebraic expressions.
- **Predict** challenges students to look for rules connecting 2, 3 and 4 variables.

To help your planning, the table below shows which programs from MicroSMILE Algebra can be used in conjunction with activities from Algebra Makes Sense.

Algebra Makes Sense		MicroSMILE Algebra
Level 4	Think of a Number	Equations Resource
Level 5	Matching Mappings	Mapping Match
	Number Problems	Equations Resource
	Solve It	Equations Resource
Level 6	Algebra Game	Equation Buster
	Mapping Diagrams	Mapping Match
	Random Code	Equation Buster
	Algebra Problems	Equations Resource
Level 7	Equivalent Expressions	Algebra Magic
	Whose Rule is Correct?	Predict & Mapping Match
	Algebra Trail	Equation Buster
	Letters for Lengths	Algebra Magic & Equation Buster
	Fibonacci-type Sequences	Equations Resource & Equation Buster

Mapping Jigsaws

Teachers notes

Type

Suitable for individual, small group or whole class.

Objective

To use problem solving strategies to complete jigsaws which involve simple mappings involving addition, subtraction, multiplication and division.

Framework for Teaching Mathematics

Understand addition, subtraction, multiplication and division as they apply to whole numbers and decimals; know how to use the laws of arithmetic and inverse operations. (Y7 p. 82)

Express simple functions at first in words then using symbols; represent them in mappings. (Y7 p. 160)

Resources

One copy of the resource sheet (consumable) for each student or each group. Scissors and glue.

Description

The activity consists of two jigsaws, Jigsaw A and Jigsaw B. Jigsaw A uses simple addition and subtraction mappings, whilst Jigsaw B uses simple addition, subtraction, multiplication and division mappings.

Students are required to cut out the 12 squares from Jigsaw A. There are two squares that require the students to complete the missing number so that each number is correctly mapped. Students should be encouraged to complete this jigsaw and glue the completed jigsaw into their books before cutting out Jigsaw B.

Jigsaw B consists of 24 squares. These squares will make two jigsaws. By careful sorting, students will realise that although both of the jigsaws make 3 by 4 rectangles, one is '3 by 4' and the other is '4 by 3'. Students should be encouraged to glue their completed jigsaws into their books.

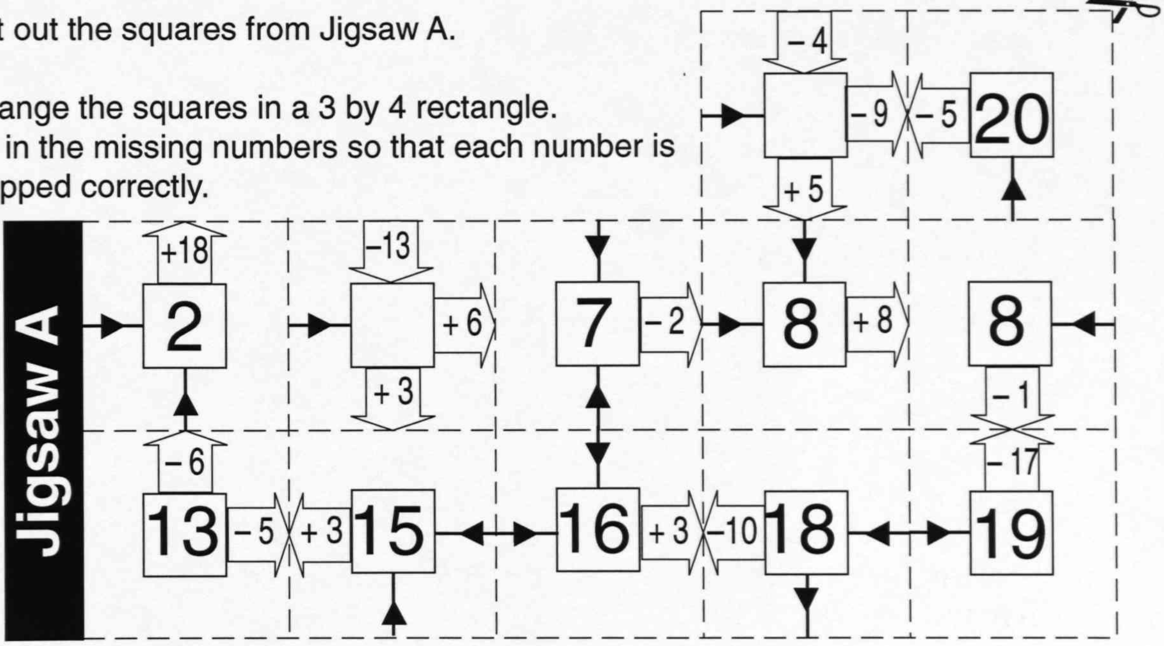
Enriching Mathematical Thinking

Students can be encouraged to create their own mapping jigsaws, using the Mapping Jigsaw Resource Sheet on page 56. This is an excellent homework activity. Students can then swap their jigsaws (already cut up) and solve each other's puzzle. This can be used as a starting activity for a follow-up lesson.

Mapping Jigsaws

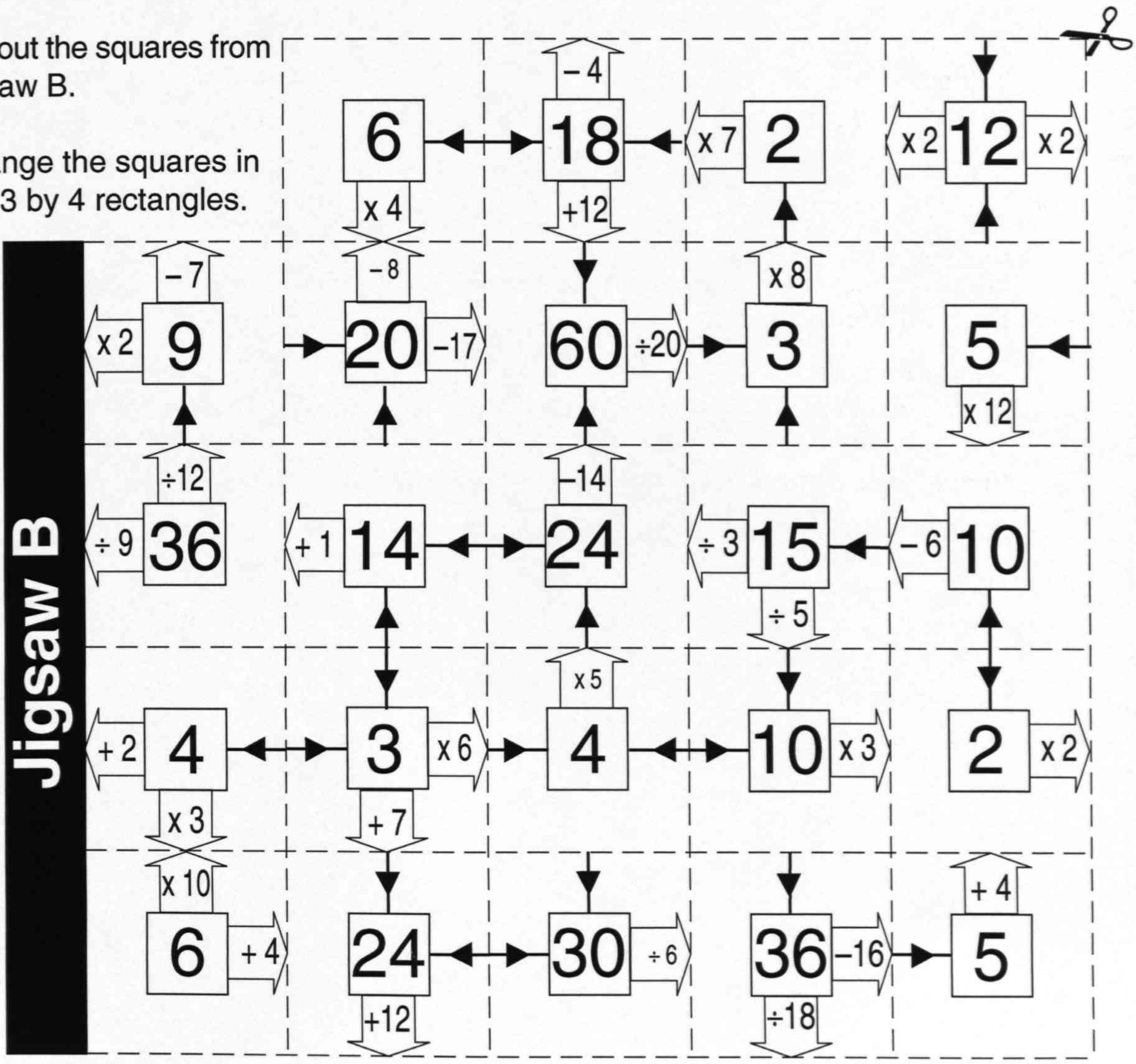
Cut out the squares from Jigsaw A.

Arrange the squares in a 3 by 4 rectangle.
Fill in the missing numbers so that each number is mapped correctly.



Cut out the squares from Jigsaw B.

Arrange the squares in two 3 by 4 rectangles.



Function Codes

Teachers notes

Type

Suitable for individual, small group or whole class.

Objective

To use simple functions to decode words and messages.

Framework for Teaching Mathematics

Express simple functions at first in words then using symbols. (Y7 p. 160)

Given inputs and outputs, find the function. (Y7 p. 162)

Resources

One copy of the resource sheet for each student.

Description

In the activity, consecutive numbers starting at 1 represent the letters of the alphabet. A simple function has then been used to create a code. The inverse of this function is used to decode the message.

The activity starts with an example to show how the function 'divide by 2' is used to decode a message. Students are then given two messages to decode by investigating which of the six given functions has been used. The decoded message will only make sense when the correct function has been used. By the same process, students are then required to decode four mathematical words.

The code used in the challenge uses a different function from the six that are used in the rest of the activity.

Enriching Mathematical Thinking

Students can be encouraged to create their own codes by using a similar technique. This provides an excellent follow-up activity that can also be used for homework. Teachers can then use the students' experience of creating these codes to discuss inverse functions. Students should also be encouraged to discuss which functions are impossible to use for coding.

Function Codes

Using the function 'divide by 2', the message below is decoded as 'hello'.

16 10 24 24 30

16 →	divide by 2 →	8 ⇒	h
10 →	divide by 2 →	5 ⇒	e
24 →	divide by 2 →	12 ⇒	l
24 →	divide by 2 →	12 ⇒	l
30 →	divide by 2 →	15 ⇒	o

Decode Message 1 by using the function 'divide by 2'.

8 18 44 18 8 10   4 50   40 46 30

Decode Message 2 by using the function 'subtract 2'.

22 25 17   11 21   7 24 7 16

Functions

- divide by 2 and add 1
- add 6
- subtract 3
- divide by 3
- add 1 and then divide by 2
- divide by 4

Decode these mathematical words by using the correct function.
Choose from the 6 functions above.

7 3 6 6 3 9 8

37 33 41 1 35 9

20 16 38 8

4 72 20 4

3 36 21 15 6 54 3

Challenge

The message below is decoded as mathematics.

42 6 63 27 18 42 6 63 30 12 60

What function is used to decode the message?

Decode Key

1	a
2	b
3	c
4	d
5	e
6	f
7	g
8	h
9	i
10	j
11	k
12	l
13	m
14	n
15	o
16	p
17	q
18	r
19	s
20	t
21	u
22	v
23	w
24	x
25	y
26	z

Mapping Machines

Teachers notes

Type

Suitable for individual, small group or whole class.

Objective

To explore the effect of simple functions on numbers.

Framework for Teaching Mathematics

Express simple functions at first in words then using symbols. (Y7 p. 160)

Resources

One copy of the resource sheet for each student.

Description

The activity initially looks at the output of the mapping/machine 'double' when different numbers are used as inputs. Students investigate the output of the 'double' mapping/machine on other inputs and are encouraged to record their findings in a mapping diagram.

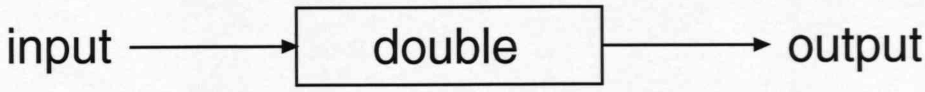
Students then explore the output of different mappings/machines, including machines which involve two mappings, on different input numbers. Students confident with this activity should be encouraged to challenge themselves by choosing fractions, decimals and negative numbers as their inputs. At the end of the session, ask students for their mapping and output and ask what the input would be.

Enriching Mathematical Thinking

Students can be encouraged to create their own mapping machines for homework. Teachers can use some of the students' own examples to discuss the inverse of mappings by asking students which mapping would map these outputs back on to the input numbers.

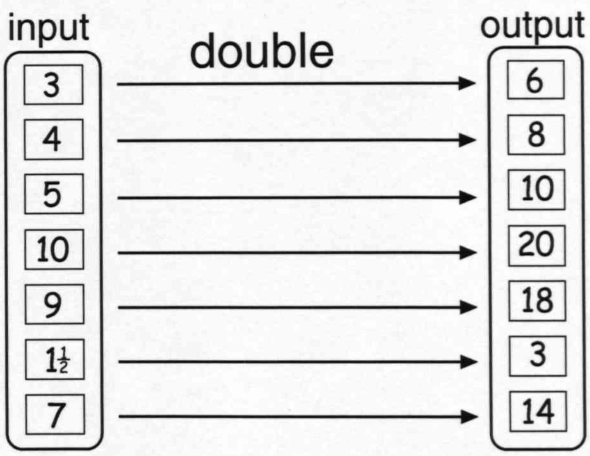
Mapping Machines

This is a 'double' machine.

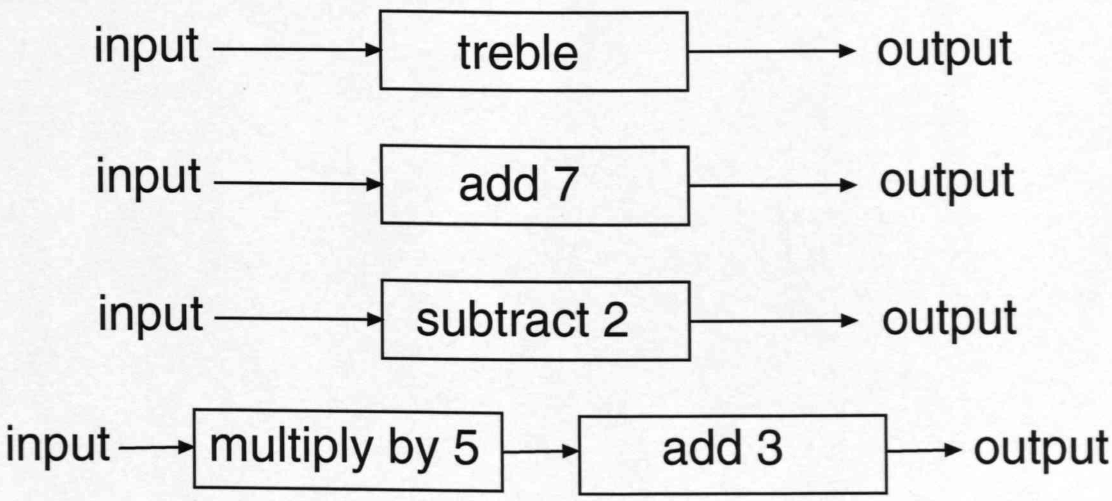


If you input 3 into the 'double' machine, the output will be 6.

The mapping diagram below shows what the 'double' machine does to some other input numbers.



- Draw a mapping diagram to show what the 'double' machine does to some different input numbers.
- Draw different mapping diagrams to show what happens to 3, 4, 5 and some other input numbers when you use the following machines:



Teachers notes

Straws and Triangles

Type

Suitable for individual, small group or whole class.

Objective

To explore sequences of numbers created from patterns involving straws and triangles.

Framework for Teaching Mathematics

Generate and describe simple integer sequences. (Y7 p. 146)

Generate sequences from simple practical context. (Y7 p. 154)

Begin to find a simple rule for the *n*th term of some simple sequences. (Y7 p. 156)

Resources

One copy of the resource sheet for each student. Isometric dotty paper may help students with drawing their patterns for triangles and hexagons and square dotty paper for drawing patterns for squares.

Description

The activity initially looks at the sequence of numbers generated by patterns using straws to create triangles. Students are encouraged to draw other patterns to create a single row of triangles and to record the number of triangles created and the number of straws used. Students record their results in a mapping diagram and are asked to identify a rule or function that will map the number of triangles to the number of straws.

Once students have identified a rule or function, students should then test their rule on given numbers of triangles and numbers of straws by drawing. Students are then encouraged to investigate other sequences generated by patterns involving other shapes made from straws.

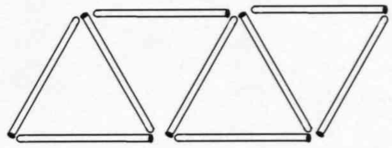
Enriching Mathematical Thinking

The investigation of other sequences generated by patterns involving straws and other shapes can be an excellent homework activity. The sequences created by the students' patterns can be a good starting activity for a follow-up lesson. Invite students to draw an example of their pattern and then show the start of a mapping diagram, mapping the number of shapes completed to the number of straws needed. You could challenge the class to find the inverse for their rules or functions by asking them how many triangles there would be for a given number of straws. Students may need help in identifying the inverse of their rule, especially if it is formulated in words, and this can lead to valuable class discussion.

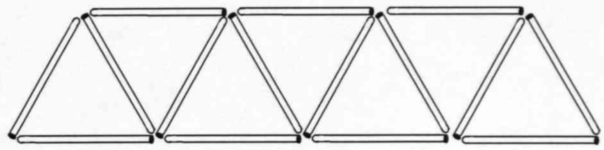
Straws and Triangles

In this pattern straws have been used to create triangles.

There are 4 triangles.
There are 9 straws.



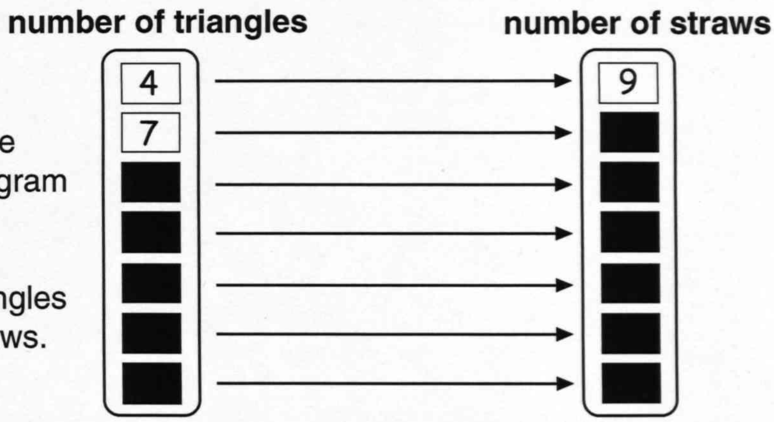
There are 7 triangles.
How many straws are there?



1. Draw 5 more patterns for a single row of triangles.

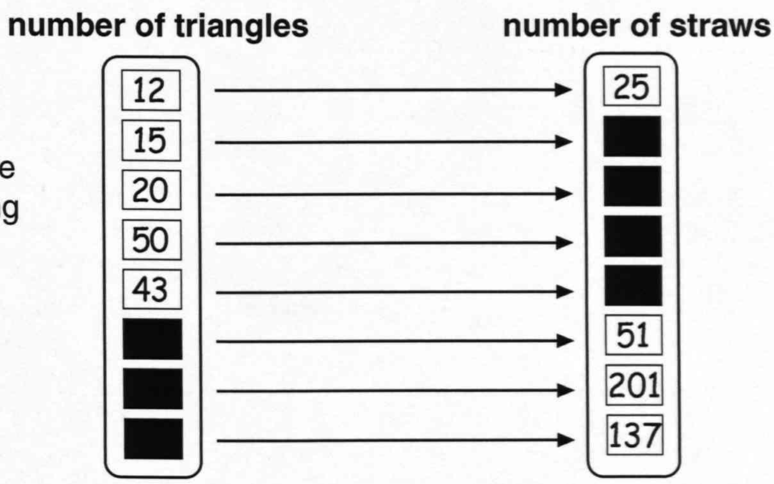
Copy and complete the following mapping diagram showing:

- the number of triangles
- the number of straws.



2. If you know the number of triangles, what is the rule for calculating the number of straws?

Copy and complete the mapping diagram using the rule.



3. Draw mapping diagrams and find the rules for other patterns made from straws, for example, straws arranged to make squares, pentagons and hexagons.

Think of a Number

Teachers notes

Type

Suitable for a pair or the whole class working in pairs.

Objective

To use 'think of a number' type problems to generate algebraic statements.

Framework for Teaching Mathematics

Construct and solve simple linear equations. (Y7 p. 122)

Resources

One copy of the resource sheet (consumable) for each pair. Scissors.

Description

There are five 'think of a number' problems. The first part of the resource sheet contains instructions for the activity. It may be useful to talk through these instructions before the students embark on the activity.

This activity works best with students working in groups of two pairs. Let each pair cut out the five 'think of a number' problems and work together to find a solution to each problem. When each pair is confident that they have been able to explain why their answers work for all five 'think of a number' problems, they should then compare their explanations with another pair. During the plenary session ask pairs of students to share their explanation with the rest of the class. Use their explanation as an opportunity to demonstrate how to represent a variable and construct a linear equation.

Enriching Mathematical Thinking

Students should be encouraged to invent their own 'think of a number' problems and try to explain why they work. This is an excellent homework activity. The students' own 'think of a number' problems can provide a starting activity for a follow-up lesson.

Think of a Number

For each 'think of a number' problem:

1. Try the problem with different numbers.
2. Try the problem on another person.
3. What do you notice?
4. Can you explain why?

Invent your own 'think of a number' problem.

A

Think of a number.
Add 5.
Multiply by 2.
Subtract 8.
Divide by 2.
Subtract the number you first thought of.
What number are you left with?

B

Think of a number.
Add 2.
Multiply by 3.
Subtract 6.
Divide by 3.
Subtract the number you first thought of.
What number are you left with?

C

Think of a number.
Multiply by 4.
Subtract 8.
Divide by 2.
Multiply by 3.
Divide by 6
Add 2.
What number are you left with?

D

Think of a number.
Divide by 2.
Add 3.
Multiply by 2.
Add 4.
Subtract the number you first thought of.
What number are you left with?

E

Think of a number.
Add 5.
Multiply by 4.
Subtract 20.
Divide by 2.
What number are you left with?

Matching Mappings

Teachers notes

Type

Suitable for an individual, or for a pair or the whole class working in pairs.

Objective

To identify which functions were used to create mapping diagrams.

Framework for Teaching Mathematics

Substitute positive integers into simple linear expressions and formulae. (Y7 p. 138)

Express simple functions in words, then using symbols; represent them in mappings. (Y7 p. 160)

Substitute integers into simple formulae. (K) (Y8 p. 139)

Resources

One copy of the resource sheet (consumable) for each pair of students. Scissors and glue.

Description

Students identify which of the three functions was used to create a given mapping diagram. This initial activity could be used as an introduction to the whole class.

The main part of the activity requires students to match mapping diagrams to the correct functions by cutting out the functions and mapping diagrams. Students can either record the matching pairs by copying them into their books or alternatively they can stick the matched ones into their books. One function and one mapping diagram will not match. In order to complete the activity students are required to write down the missing function and create a mapping diagram. Mapping diagram D and F provides a useful discussion point for the plenary session highlighting subtraction and multiplication of negative numbers.

Enriching Mathematical Thinking

The functions used to create mapping diagrams D and E can be used as examples of ‘many to one’ mappings which have no inverse. Students can be asked to write down other functions and matching mapping diagrams for homework. Students can use their examples as a starting activity for the next lesson, e.g. by showing a mapping diagram and asking the rest of the class to find the matching function.

Matching Mappings

Which of the three functions below was used to create this mapping diagram?

Function
double

Function
multiply by 3

Function
add 3

Mapping diagram

3	6
7	10
1	4
63	66
2½	5½

Match each of the functions at the bottom of the page to the correct mapping diagram below.

- You will find that one function does not match a mapping diagram. Draw a mapping diagram to match this function.
- You will find that one mapping diagram does not match any of the functions. Write down the function that was used to create this mapping diagram.

Mapping diagram A

5	15
2	12
9	19
63	73
-10	0

Mapping diagram B

5	13
7	17
2	7
4	11
9	21

Mapping diagram C

5	30
2	12
7	42
10	60
2½	15

Mapping diagram D

4	16
7	49
10	100
-1	1
1	1

Mapping diagram E

12	2
8	3
13	3
5	0
15	0

Mapping diagram F

3	5
7	1
5½	2½
9	-1
4	4

Function	Function	Function
subtract from 8	**the remainder when divided by 5**	**add 10**
Function	Function	Function
subtract 2 then double	**square**	**multiply by 6**

Number Problems

Teachers notes

Type

Suitable for students working individually or for a pair or the whole class working in pairs.

Objective

To use 'think of a number' type problems to generate and solve simple linear equations.

Framework for Teaching Mathematics

Use letter symbols to represent unknown numbers or variables. (Y7 p. 112)

Understand that algebraic operations follow the same conventions and order as arithmetic operations. (Y7 p. 114)

Construct and solve simple linear equations. (Y7 p. 122)

Know that algebraic operations follow the same conventions and order as arithmetic operations. (Y8 p. 115)

Construct and solve linear equations with integer coefficients using appropriate methods. (Y8 p. 123)

Resources

One copy of the resource sheet (consumable) for each pair. Scissors.

Description

The activity starts with an initial number problem and demonstrates two possible methods which students could use to solve it. According to the ability of the group and their prior experience at solving equations, teachers may decide to concentrate on using one method. There are six problems for students to solve. This activity works best with students working in groups of two pairs, each pair attempting to solve all the problems and then discussing their solutions and methods used with the other pair. It does not matter in which order the problems are tackled, but students may find it easier to discuss their work if the problems are cut out and shared. Once the group has solved all of the problems, it is worthwhile encouraging the students to present one of their solutions to the rest of the class.

Enriching Mathematical Thinking

If teachers have decided to concentrate on one method to solve the problems, a good plenary session would be to analyse why the other method works.

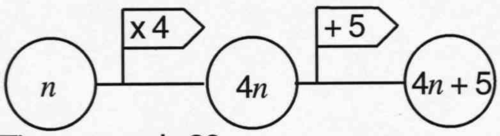
Number Problems

> I think of a number, then multiply it by 4, and then add 5.
> The answer is 29. What number did I start with?

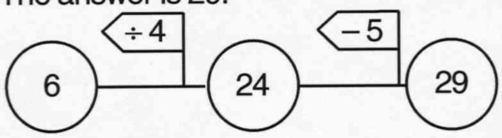
Here are two different approaches to solving the problem.

Using flags

Let n stand for the number.



The answer is 29.



So $n = 6$

Using equations

Let n stand for the number.

Multiply by 4.	$4n$
Add 5	$4n + 5$
The answer is 29.	$4n + 5 = 29$
Subtract 5 from both sides	$4n = 24$
Divide both sides by 4	$n = 6$

So $n = 6$

Therefore the starting number is 6.

Solve the following problems. Explain your answers.

A

I think of a number, then multiply it by 3, then add 6. The answer is 33.

What number did I start with?

B

I think of a number, then multiply it by 7, then add 5. The answer is 61.

What number did I start with?

C

I think of a number, then multiply it by 8, then add 3. The answer is 35.

What number did I start with?

D

I think of a number, then multiply it by 6, then add 9. The answer is 51.

What number did I start with?

E

I think of a number, then add 4, then multiply it by 10. The answer is 50.

What number did I start with?

F

I think of a number, then add 5, then multiply it by 3. The answer is 33.

What number did I start with?

Complete the Mappings

Teachers notes

Type

Suitable for an individual, a small group or the whole class.

Objective

To explore simple functions expressed algebraically, and to complete and record results in a mapping diagram.

Framework for Teaching Mathematics

Substitute positive integers into simple linear expressions and formulae. (Y7 p. 138)

Express simple functions in words, then using symbols; represent them in mappings. (Y7 p. 160)

Substitute integers into simple formulae. (K) (Y8 p. 139)

Express simple functions in symbols; represent mappings expressed algebraically. (Y8 p. 161)

Resources

One copy of the resource sheet (consumable) for each student.

Description

There are four simple functions, expressed algebraically, that students use to complete the mapping diagrams. For each function, students need to use the inverse of the function in order to complete the mapping diagram. There are then three completed mapping diagrams where students have to identify five mistakes. There is one mistake in the first mapping diagram and two mistakes in each of the remaining two. The final three functions are more challenging.

Enriching Mathematical Thinking

A good starting or plenary activity is to discuss the meaning of inverses, i.e. the inverse of an operation takes you back to where you started. Use everyday contexts such as breaking an egg, adding 6 to a number, pumping up a tyre, lighting a match, turning an empty mug upside down, dividing a number by 2, etc. Emphasise that some operations have inverses and some do not.

Some students may require support in using functions such as $x \rightarrow 8 - x$ and $x \rightarrow 20 - x$. It is worth using these two functions in the plenary session and discussing their inverses. You might also want to ask students to discuss mathematical functions which have no inverses, e.g. $x \rightarrow [x]$, $x \rightarrow x^2$.

Complete the Mappings

This mapping diagram shows what happens to numbers when using the function 'add 4'.

1. Complete the mapping diagram for the function 'add 4'.

x	$x+4$
7	11
19	23
5	
22	
35	
$2\frac{1}{2}$	
	92

2. Complete these mapping diagrams.

x	$x+2$
2	
5	
$8\frac{1}{2}$	
23	
	51

x	$x-6$
2	
5	
$8\frac{1}{2}$	
23	
	51

x	$3x$
2	
5	
$8\frac{1}{2}$	
23	
	51

3. These completed mapping diagrams contain 5 mistakes.
Find and correct the mistakes.

x	$2x+2$
2	6
5	12
$8\frac{1}{2}$	19
23	46
$24\frac{1}{2}$	51

x	$8-x$
2	6
5	13
$8\frac{1}{2}$	$-\frac{1}{2}$
23	15
-43	51

x	$x \div 2 + 1$
2	6
5	3.5
$8\frac{1}{2}$	$5\frac{1}{4}$
23	$12\frac{1}{2}$
101	51

4. Complete these mapping diagrams.

x	$2x-3$
7	
3.5	
	$8\frac{1}{2}$
21	
	20

x	$20-x$
12	
	5
$5\frac{1}{2}$	
	23
0	

x	x^2+1
3	
-1	
	50
2.5	
5	

Solve It

Teachers notes

Type

Suitable for an individual, a small group or the whole class.

Objective

To solve simple equations by using trial and improvement methods.

Framework for Teaching Mathematics

Begin to multiply a single term over a bracket (integer coefficient). (Y7 p. 116)

Solve linear equations with integer coefficients (unknown on one side only) using appropriate methods. (Y7 p. 122)

Solve linear equations with integer coefficients (unknown on one side only, with and without brackets) using appropriate methods. (Y8 p. 123)

Substitute integers into simple formulae. (K) (Y8 p. 139)

Description

The activity provides students with a method of solving equations by trial and improvement. Students are given a mapping and are asked to solve two equations based on that mapping.

Students are given a further three mappings and the associated flag diagrams to help them solve the equations. In the final three equations, student must create their own flag diagrams in order to solve the equations.

Enriching Mathematical Thinking

Drawing the graph of the initial mapping given in the activity can be used in the plenary session to help students make the link between solving the equation and its graphical solution. Getting students to draw the graph of a linear mapping of their choice and using it to solve equations is a good homework activity.

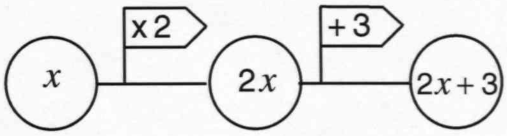
Solve It

$x \to 2x+3$

If $2x+3=26$ then find the value of x.

	x	$\longrightarrow$ $2x+3$	
Try	10	23	too small
	11	25	too small
	12	27	too big
	11.5	26	correct

So $x = 11.5$

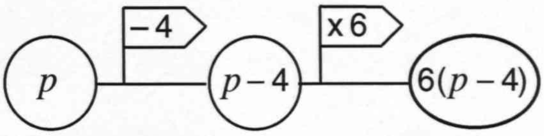


Using the same method, find the value of x if:

$2x+3=39$ $\quad$ $2x+3=145$

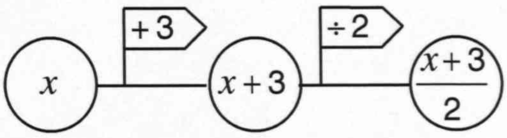
$p \to 6(p-4)$

If $6(p-4)=36$ then find the value of p.



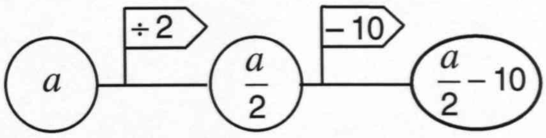
$x \to \frac{x+3}{2}$

If $\frac{x+3}{2}=6$ then find the value of x.



$a \to \frac{a}{2}-10$

If $\frac{a}{2}-10=13$ then find the value of a.



Solve the following equations, drawing flag diagrams to help you.

$9(p+3)=36$ $\quad$ $9(p+3)=27$ $\quad$ $3(y-2)+15=30$

Algebra Match

Teachers notes

Type

Suitable for a pair or the whole class working in pairs.

Objective

To use everyday language to help students to identify the order of operations within an algebraic expression. To help students to appreciate the need for accurate use of language.

Framework for Teaching Mathematics

Use letter symbols to represent unknown numbers or variables. (Y7 p. 112)

Understand that algebraic operations follow the same conventions and order as arithmetic operations. (Y7 p. 114)

Express simple functions in words. (Y7 p. 160)

Know that algebraic operations follow the same conventions and order as arithmetic operations. (Y8 p. 115)

Express simple functions in symbols. (Y8 p. 161)

Resources

One copy of the resource sheet (consumable) for each pair. Scissors.

Description

There are ten algebraic expressions and ten statements. Students are required to match the correct statement to the algebraic expression. One statement and one algebraic expression will be left over. Students must then write a statement/algebraic expression for the un-matched algebraic expression/statement.

A good introduction to this activity is to write three different numbers on the board and invite students to make as many different results using the three numbers. For example, if the numbers 3, 4 and 5 are used, possible results are $4 \div (3 + 5) = 1/2$, $(3 + 5) \div 4 = 2$. Ask students to describe how they reached their totals by using words such as 'then', 'divide into', 'divided by'. With students working in pairs, get them to match the expressions with the correct statements. Students should substitute numbers for letters in order to check that they are correct.

Enriching Mathematical Thinking

Writing a statement and algebraic expression for the unmatched algebraic expression and statement is an excellent homework activity. Discussing their results will provide a rich starting activity for the next lesson.

Algebra Match

Match the statements with the algebraic expressions.

One statement and one algebraic expression do not match.
Write a matching statement/algebraic expression for each.

Statements	Algebraic expressions
Multiply b by 2 <u>then</u> subtract from a.	$a(b-c)$
Subtract c from a <u>then</u> multiply by b.	$\frac{a}{c}-b$
Add b to a <u>then</u> divide into c.	$\frac{ab}{c}$
Multiply a by b <u>then</u> divide by c.	$\frac{c}{a+b}$
Divide c by b <u>then</u> multiply by a.	abc
Multiply a by c <u>then</u> divide into b.	$b(a-c)$
Add b to a <u>then</u> divide by c.	$a-2b$
Multiply ab by c.	$\frac{b}{ac}$
Multiply a by 4 <u>then</u> subtract c.	$\frac{a+b}{c}$
Subtract c from b <u>then</u> multiply by a.	$\frac{c}{b}\times a$

Teachers notes

Algebra Game

Type

Suitable for a pair or the whole class working in pairs.

Objective

To substitute numbers into simple expressions in order to progress through a grid made of hexagons.

Framework for Teaching Mathematics

Know that algebraic operations follow the same conventions and order as arithmetic operations. (Y8 p. 115)

Substitute integers into simple formulae. (K) (Y8 p. 139)

Express simple functions in symbols. (Y8 p. 161)

Substitute numbers into expressions and formulae. (Y9 p. 139)

Resources

One copy of the resource sheet and one dice for each pair. Each student requires a small counter. One copy of the Recording Sheet for Algebra Game/Trail for each student, (page 57). (Optional).

Description

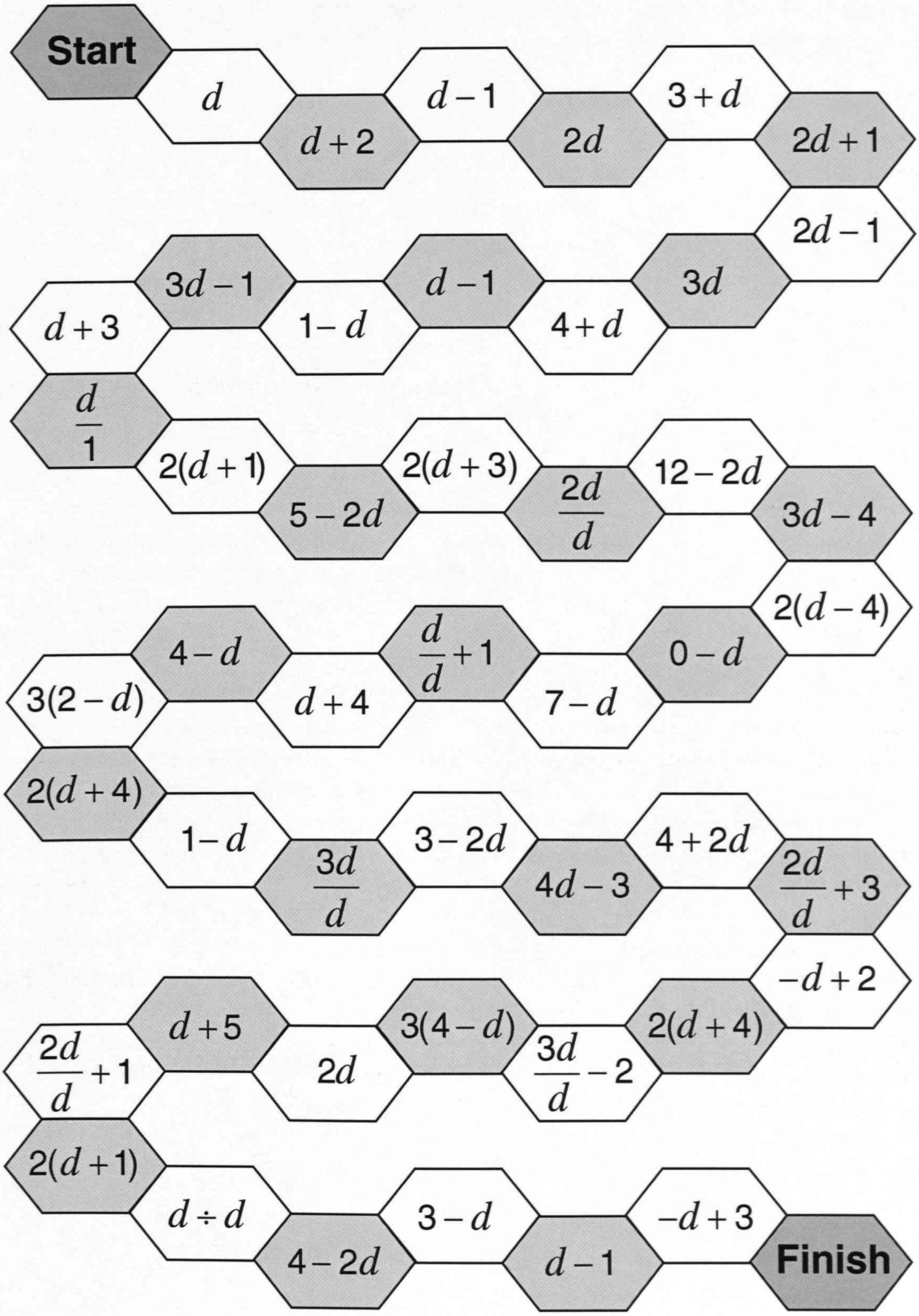
Students place their counter on the [Start] hexagon. Students, working in pairs, take it in turns to roll the dice and then move on that number of hexagons. After the initial throw students must then substitute the value shown on the dice for d in the algebraic expression. Students must then move their counter the number of hexagons on the board equivalent to the value of the algebraic expression. If the value of the expression is positive, students move forward, if the value of the expression is negative, then students must go backwards. Students should be encouraged to record their calculations on the Recording Sheet for Algebra Game. Once a student has announced the value of their expressions, the other student can challenge if they believe it is incorrect. If the challenge is upheld, the student misses a go. For an incorrect challenge the student forfeits one turn. The student who reaches the [Finish] hexagon first is the winner. Students do not need the exact number to finish.

Enriching Mathematical Thinking

At the end of the game, or as a homework activity, let students swap their Recording Sheet for Algebra Game/Trail and check their calculations. Ask students to investigate what is the furthest hexagon from the finish where they can win in one throw, or, what is the quickest route (least number of throws) to win? Algebra Trail at level 7 (page 41) is a harder version of Algebra Game.

Algebra Game

- Take turns to roll the dice.
- Substitute the number on the dice into the algebraic expression.
- Move the counter forward or backwards the resulting number of hexagons.



Teachers notes

Mapping Diagrams

Type

Suitable for an individual, a small group or the whole class.

Objective

To generate and use simple sequences of numbers, recorded in a mapping diagram to create an expression for the *n*th term.

Framework for Teaching Mathematics

Generate and describe integer sequences. (Y8 p. 145)

Generate terms of a linear sequence using term-to-term and position-to-term definitions of the sequence, on paper and using a spreadsheet or graphical calculator. (Y8 p. 147)

Begin to use linear expressions to describe the *n*th term of an arithmetic sequence, justifying its form by referring to the activity or practical context from which it was generated. (Y8 p. 155)

Express simple functions in symbols; represent mappings expressed algebraically. (Y8 p. 161)

Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence, on paper and using ICT. (K) (Y9 p. 149)

Generate sequences from practical contexts and write an expression to describe the *n*th term of an arithmetic sequence. (K) (Y9 155)

Resources

One copy of the resource sheet for each student.

Description

The first four mappings are created from practical contexts. In the first mapping, the *n*th term is given so that students can see the connection between the mapping diagram, the term-to-term rule and position-to-term rule. Some students may need some additional support to generate the *n*th term in the next three mappings. Students' answers to the last two questions can provide valuable discussion points for the plenary session with an additional challenge of finding two different mappings which could describe $5 \rightarrow 25$ and writing each one in the form of $n \rightarrow$.

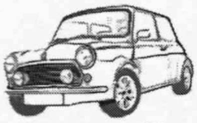
Enriching Mathematical Thinking

Asking students to create their own mapping diagrams and generating the *n*th term in the sequence is an excellent homework activity. If students are stuck for ideas, get them to look back at the first four questions and replace bugs for spiders, squares and hexagons for triangles, etc. Students' examples will provide an excellent starting activity to link the *n*th term to the term-to-term rule and position-to-term rule.

Mapping Diagrams

Copy and complete the mapping diagrams below.

1. How many tyres?

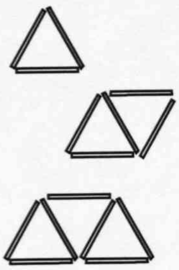


Cars		Tyres
1	→	5
2	→	10
3	→	15
4	→	
5	→	
12	→	
100	→	
n	→	$5n$

2. How many legs?

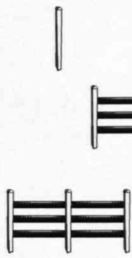
Bugs		Legs
1	→	6
2	→	12
3	→	18
4	→	
5	→	
12	→	
100	→	
n	→	

3. How many straws?



Triangles		Straws
1	→	3
2	→	5
3	→	7
4	→	
5	→	
12	→	
100	→	
n	→	

4. How many rails?



Posts		Rails
1	→	0
2	→	3
3	→	6
4	→	
5	→	
12	→	
100	→	
n	→	

5.

0	→	0
1	→	4
2	→	8
3	→	12
4	→	16
50	→	
n	→	

6.

0	→	1
1	→	5
2	→	9
3	→	13
4	→	17
50	→	
n	→	

7.

0	→	0
1	→	$\frac{1}{2}$
2	→	1
3	→	$1\frac{1}{2}$
4	→	2
50	→	
n	→	

8. Which of these mappings could describe $3 \longrightarrow 9$?

(a) $n \rightarrow 3n$ (b) $n \rightarrow n+6$ (c) $n \rightarrow 4n+3$
(d) $n \rightarrow n^2$ (e) $n \rightarrow 3(n-1)$ (f) $n \rightarrow 12-n$

9. Find three different mappings which could describe $4 \longrightarrow 12$.

Random Code

Teachers notes

Type

Suitable for an individual, a small group or the whole class.

Objective

To solve simple equations to find a code and then to use the code to decode a message.

Framework for Teaching Mathematics

Simplify or transform linear expressions by collecting like terms. (K) (Y8 p. 117)

Construct and solve linear equations with integer coefficients (unknown on either or both sides, without and with brackets) using appropriate methods. (Y8 p. 123)

Substitute integers into simple formulae. (K) (Y8 p. 139)

Construct and solve linear equations with integer coefficients using appropriate methods. (K) (Y9 p. 123)

Substitute numbers into expressions and formulae. (Y9 p. 139)

Resources

One copy of the resource sheet (consumable) for each student.

Description

A code is created using numbers 1 to 27. Students are given clues to the code in the form of equations. Students must solve the equations in order to find the code. The code must then be used to decode a message. It is worth pointing out to students that the number 27 indicates a space and that students will find it easier to crack the code if they solve the equations alphabetically.

Enriching Mathematical Thinking

A good introduction to this activity is to use a basic code where $a = 1$, $b = 2$, $c = 3$, and so on and ask students to find a word with the value 100 (numeracy), or a name of a student with the least value. An excellent homework activity is to challenge students to create their own random code. Students can then swap their equations and message with other students as a starting activity for the next lesson.

Random Code

a	b	c	d	e	f	g	h	i	j	k	l	m	n
5		**23**				**14**		**12**			**18**		

o	p	q	r	s	t	u	v	w	x	y	z	
17			**9**			**22**						**27**

Complete the random code using the algebra clues.

$b+7=15$

$d-1=10$

$2e=30$

$2f+2=40$

$3h-2=19$

$j=3h-1$

$k-1=r$

$m=k-b$

$\frac{1}{2}n=13$

$\frac{1}{7}p=3$

$4q=q+12$

$s=q^2$

$t=am+14$

$v=a^2$

$w=\dfrac{a^2}{v}$

$x=\frac{2}{3}r$

$y=\dfrac{a^2+1}{2}$

$z=\dfrac{p}{h}$

Decode this message using the random code.

23	9	15	5	24	15	27	13	17	22	9	27	17	1	26	27

2	15	16	16	5	14	15

Algebra Problems

Teachers notes

Type

Suitable for a small group or the whole class working in groups.

Objective

To provide a range of methods for solving everyday problems using algebra.

Framework for Teaching Mathematics

Know that algebraic operations follow the same conventions and order as arithmetic operations. (Y8 p. 115)

Construct and solve simple linear equations with integer coefficients (unknown on either or both sides, without and with brackets) using appropriate methods. (Y8 p. 123)

Express simple functions in symbols. (Y8 p. 161)

Construct and solve linear equations with integer coefficients (without and with brackets, negative signs anywhere in the equation, positive or negative solution) using appropriate methods. (K) (Y9 p. 123)

Resources

One copy of the resource sheet (consumable) for each group of students. Scissors.

Description

The activity starts with an initial problem and demonstrates two possible methods for students to use to solve it. According to the ability of the group and their prior experience at solving equations, teachers may decide to concentrate on using one method. There are six problems for students to solve. This activity works best with students working in groups of two pairs, each pair attempting to solve all the problems and then discussing their solutions and methods used with the other pair.

Enriching Mathematical Thinking

If teachers have decided to concentrate on one method to solve the problems, a good plenary session would be to analyse why the other method works.

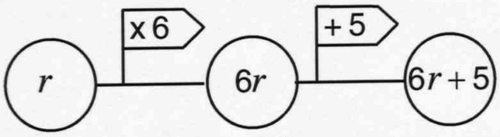
Algebra Problems

Six bags of rice and 5kg of potatoes together weigh 23kg.
What is the weight of one bag of rice?

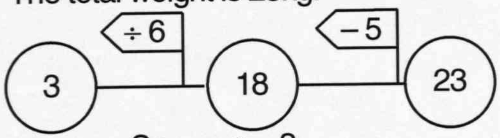
Here are two different approaches to solving the problem.

Using flags

Let r stand for the weight of a bag of rice.



The total weight is 23kg.



So $r = 3$

Using equations

Let r stand for the weight of a bag of rice.

Multiply by 6. $6r$

Add 5. $6r + 5$

The total weight is 23kg. $6r + 5 = 23$

Subtract 5 from both sides. $6r = 18$

Divide both sides by 6. $r = 3$

So $r = 3$

Therefore the weight of one bag of rice is 3kg.

Solve the following problems.

A

Eight similar books and a 60p pen together cost £64.60.

What is the cost of one book?

B

Nelson bought one pair of jeans costing £44, a jumper costing £36 and 4 pairs of socks.
The total bill was £90.
How much was one pair of socks?

C

One side of a rectangle is 5 centimetres longer than the other side.
If the perimeter is 30 centimetres, what is the length of each side?

D

Barbara and her three children went swimming. The children were half price.
The total cost was £6.
What was the cost for one child?

E

The total age of two twins and their mother is 55 years. The mother is 33 years old.

What is the age of the twins?

F

In a triangle, the first angle is 10 degrees smaller than the second angle. The third angle is three times as large as the first angle.
What are the sizes of the angles?

Algebra Sort

Teachers notes

Type

Suitable for a pair or the whole class working in pairs.

Objective

To use everyday language to help students to identify the order of operations within an algebraic expression.

Framework for Teaching Mathematics

Know that algebraic operations follow the same conventions and order as arithmetic operations. (Y8 p. 115)

Express simple functions in symbols. (Y8 p. 161)

Distinguish the different roles played by letters in equations, identities, formulae and functions. (Y9 p. 113)

Resources

One copy of the resource sheet (consumable) for each pair. Scissors.

Description

There are ten statements and ten algebraic expressions. Students are required to match each algebraic expression to the correct statement. Two statements and two algebraic expressions will be left over. Students must then write statements/algebraic expressions for the unmatched algebraic expressions/statements.

A good introduction to this activity is to write four different numbers on the board and invite students to make as many different results using the four numbers. For example, if the numbers 3, 4, 5 and 6 are used, possible results are:

$(4 \times 6) \div (3 + 5) = 3$, $(3 + 5) \div (6 - 4) = 4$, etc.

Ask students to describe how they reached their totals by using words such as 'then', 'divide into', 'divided by', 'product of'. With students working in pairs, get them to match the expressions with the correct statements. Students should substitute numbers for letters in order to check that they are correct.

Enriching Mathematical Thinking

Writing statements and algebraic expressions for the unmatched algebraic statements and expressions is an excellent homework activity. Discussing their results will provide a rich starting activity for the next lesson.

Algebra Sort

Match the statements on the left-hand side with the correct algebraic expressions on the right-hand side.
Two statements and two algebraic expressions do not match.
Write matching algebraic expressions/statements for each.

The product of a and c is divided by b.	$\frac{a-b}{c}$
Divide a by c and then divide by b.	$\frac{c}{a-b}$
Subtract b from a and then divide into c.	$\frac{a+b}{c+d}$
Multiply a by c then divide into b.	$\frac{a+b}{c}$
Subtract b from a then divide by c.	$\frac{cd}{a+b}$
Divide the sum of a and c into b.	$\frac{b}{a+c}$
The sum of a and b is divided by the sum of c and d.	$\frac{a}{c}+b$
a is added to b then divided into the product of c and d.	$\frac{b}{ac}$
Add a to b and divide by the product of c and d.	$\frac{a}{c}+\frac{b}{d}$
The sum of a divided by c and b divided by d.	$\frac{ac}{b}$

Equivalent Expressions

Teachers notes

Type

Suitable for a pair or the whole class working in pairs.

Objective

To use substitution and algebraic manipulation to identify equivalent expressions.

Framework for Teaching Mathematics

Construct and solve linear equations with integer coefficients (with and without brackets, negative signs anywhere in the equation, positive or negative solutions). (K) (Y9 p.123)

Use formulae from mathematics and other subjects; substitute numbers into expressions and formulae; derive a formula and, in simple cases, change its subject. (Y9 p. 139)

Resources

One copy of the resource sheet (consumable) for each pair of students. Scissors and glue.

Description

There are 30 algebraic expressions. Students are required to group the algebraic expressions into 10 equivalent groups. As the activity may well take students more than a lesson to complete, the expressions have been divided into two sets. The second set can be used as an excellent homework activity. Students can decide whether to record their work in their books or alternatively stick the equivalent expressions in their books, recording their substitution of values for each expression.

A good introduction to this activity is to work through the initial example and then ask the students to write other algebraic expressions which are equivalent to $3(2a^2 + 1)$. Students can then check their expression by substituting a suitable value for a.

Enriching Mathematical Thinking

Ask students to write down three expressions which are not equivalent but which give the same result when the same value is substituted into each. For example, $a + 2$, $2a + 2$ and $3a + 2$ will all give the same value when $a = 0$. Then challenge students to find three expressions which are not equivalent but which give the same results when two values are substituted into each. For example, $2a^2 + 2$, $2a + 2$ and $2a^3 + 2$ will all give the same value when $a = 0$ and $a = 1$.

Equivalent Expressions

These three expressions are equivalent.

$6a^2+3$ | $3(2a^2+1)$ | $4a^2+3+3a+2a^2-3a$

Check that they are equivalent by substituting values for a.

For example let $a=7$.

6 x 7^2 + 3	3(2 x 7^2 + 1)	4 x 7^2 + 3 + 3 x 7 + 2 x 7^2 – 3 x 7
6 x 49 + 3	3(2 x 49 + 1)	4 x 49 + 3 + 3 x 7 + 2 x 49 – 21
= 294 + 3	= 3(98 + 1)	= 196 + 3 + 21 + 98 – 21
= 297	= 297	= 297

For each group of 15 expressions below:

- Cut out the expressions and match them in groups of three equivalent expressions.
- Check your groups of three equivalent expressions by substituting values for m and x.

$2m-m^2$	$\frac{1}{2}(10m+10)$	$3m^2-2$
$m(m+1)+m$	$3m^2+2m-2-2m$	$5(m+1)$
$\frac{1}{2}(6m^2-4)$	$-m(m-2)$	$(m+1)^2-1$
$2m(2m-1)$	$5m+5$	m^2+2m
$4m^2-2m$	$m^2+2m-2m^2$	$2(2m^2-m)$

$x(\frac{x}{2}+3)$	$0.5x^2+3x$	$3x-21$
$6x+3x^2$	$3(x-8)+3$	$\frac{x}{2}+2$
$3(2x+x^2)$	$2x+6$	$3(x-7)$
$2(x+3)$	$\frac{1}{2}(x+2)(x+4)-4$	$2(x+2)+2$
$x-0.5x+2$	$\frac{1}{2}(x+4)$	$3x(2+x)$

Whose Rule is Correct?

Teachers notes

Type
Suitable for an individual, a small group or the whole class.

Objective
To generate algebraic formulae from practical contexts and to recognise equivalent algebraic expressions.

Framework for Teaching Mathematics
Construct and solve linear equations with integer coefficients (with and without brackets, negative signs anywhere in the equation, positive or negative solutions). (K) (Y9 p.123)

Generate terms of a sequence using term-to-term and position to term definitions of the sequence. (K) (Y9 p.149)

Generate sequences from practical contexts and write an expression to describe the *n*th term of an arithmetic sequence; (K) (Y9 p.155)

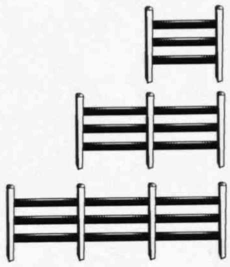
Resources
One copy of the resource sheet for each student.

Description
The initial problem looks at how four students describe the relationship between rails and posts in words. Each student's description can then be used to express the relationship in algebra. Once all the algebraic expressions are completed, students must then find the description that does not describe the relationship between the rails and posts. The second problem challenges students to find the relationship between straws and circles.

Enriching Mathematical Thinking
As an additional activity, for further classwork or homework, students can draw their own sequences made up of straws and rectangles/squares. They can then express the relationship between the number of straws and number of shapes in different ways and then use algebra to express the rule to show that they are equivalent.

Whose Rule is Correct?

Four students, Karen, Joe, Nikki and Rajan are working on problems where they have to find rules. They have each found a rule connecting the number of rails and the number of posts.



Number of rails (r)	Number of posts (p)
3	2
6	3
9	4

Karen's rule

To find the number of rails, take the number of posts, minus 1, then multiply by 3.

Joe's rule

To find the number of rails, take the number of posts, minus 3 then multiply by 3.

Nikki's rule

To find the number of rails, take the number of posts, multiply by 2, minus 3, then add the number of posts.

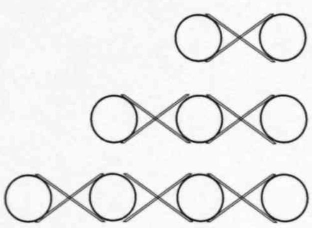
Rajan's rule

To find the number of rails, take the number of posts, multiply by 3 then minus 3.

Karen's rule can be writen in algebra as $r = 3(p-1)$.

- Write Joe's, Nikki's and Rajan's rule using algebra.
- One of the rules is incorrect. Which one?

The four students, Karen, Joe, Nikki and Rajan have each found a rule connecting the number of straws with the number of circles.



Number of straws (s)	Number of circles (c)
2	2
4	3
6	4

Karen's rule

To find the number of straws, take the number of circles, multiply by 2, then minus 2.

Joe's rule

To find the number of straws, take the number of circles, minus 1, then multiply by 2.

Nikki's rule

To find the number of straws, take the number of circles, multiply by 4, then minus 2, then divide by 2.

Rajan's rule

To find the number of straws, take the number of circles, multiply by 4, divided by 2, then minus 2.

Write each student's rule in algebra and find which one is incorrect.

Algebra Trail

Teachers notes

Type
Suitable for a pair or the whole class working in pairs.

Objective
To substitute numbers into algebraic expressions in order to progress through a grid made of hexagons.

Framework for Teaching Mathematics
Use index notation for integer powers and simple instances of the index laws. (Y9 p. 115)

Substitute numbers into expressions and formulae. (Y9 p. 139)

Expand the product of two linear expressions of the form $x + n$. (Y9 able pupils p.121)

Resources
One copy of the resource sheet and one dice for each pair. Each student requires a small counter. One copy of the Recording Sheet for Algebra Game/Trail for each student (page 57). (Optional).

Description
Students place their counter on the [Start] hexagon. Students, working in pairs, take it in turns to roll the dice and then move on that number of hexagons. After the initial throw students must then substitute the value shown on the dice for d in the algebraic expression and round to the nearest integer. Students must then move their counter the number of hexagons on the board equivalent to the value of the algebraic expression. If the value of the expression is positive, students move forward; if the value of the expression is negative, then students must go backwards. Students should be encouraged to record their calculations on the Recording Sheet for Algebra Game/Trail (page 57). Once a student has announced the value of their expressions, the other student can challenge if they believe it is incorrect. If the challenge is upheld, the student misses a go. For an incorrect challenge the challenger forfeits one turn. The student who reaches the [Finish] hexagon first is the winner. Students do not need the exact number to finish.

Enriching Mathematical Thinking
The Recording Sheet for Algebra Game/Trail can be found on page 57. At the end of the game, or as a homework activity, let students swap their recording sheets and check their calculations. Ask students to find which hexagon they should land on to guarantee a win on the next move.

Algebra Trail

- Take turns to roll the dice.
- Substitute the number on the dice into the algebraic expression.
- If the resulting number is a fraction or decimal, round to the nearest whole number.
- Move the counter forward or backwards the resulting number of hexagons.

Start

$-(-d)$

$2(d+1)$

$3(d-1)$

$d \div 2$

$3(d-2)$

$2d \div 3$

$\frac{2}{d}+\frac{3}{d}$

$3d$

$4+d$

d^2-1

$5-d^2$

$3d-1$

d^2-2

$\frac{4+d}{d}$

$(1-d)(d+1)$

$5-2d$

$2(d+3)$

$\frac{2d}{d}$

$12-2d$

$\frac{2}{d}+\frac{3}{d}$

$3d-4$

$(1-d)(d+1)$

$2(d-4)$

$-d(d-3)$

$7-d$

$\frac{d}{d}+1$

$\frac{3}{d}-\frac{2}{d}$

$3(d^2-d)$

$2(d+4)$

$4-d^2$

$\frac{d^2}{3d}$

$3-2d$

$4d-3$

$\frac{2d}{d}+3$

$2(d+4)$

$\frac{4d}{d^2}-2$

$-(4-d)$

$2d \div 3$

$\frac{(d+5)}{d^2}$

$\frac{2d}{d^2}+1$

$(6-d)^2$

$5d \div d$

$4-2d$

$\frac{7-d}{d-7}$

$\frac{4-d^2}{d+1}$

$-d+3$

Finish

Letters for Lengths

Teachers notes

Type

Suitable for an individual, pairs, a small group or the whole class.

Objective

To solve problems involving area and perimeter of rectangles, using algebra.

Framework for Teaching Mathematics

Use index notation for integer powers and simple instances of the index laws. (Y9 p. 115)

Construct and solve linear equations with integer coefficients (with and without brackets, negative signs anywhere in the equation, positive or negative solutions). (K) (Y9 p. 123)

Use formulae from mathematics and other subjects; substitute numbers into expressions and formulae. (Y9 p. 139)

Resources

One copy of the resource sheet for each student.

Description

In the activity, the lengths of each side of the rectangle are given in algebraic terms. A useful introduction to the activity is to ask students to explain why the area is $2x^2$ square in the initial example. The five problems are in order of difficulty and provide students with the opportunity to use algebra to solve open-ended problems. This activity works best with students working in groups of two pairs, each pair attempting to solve all the problems and then discussing their solutions and algebraic expressions with the other pair and getting them to note any differences in the approach. In problems 2, 3 and 4, students are asked to find possible values for the variables. This can lead on to a discussion on limiting the values initially to positive integers. Some students may need support with the Letters for Length – Challenge.

Enriching Mathematical Thinking

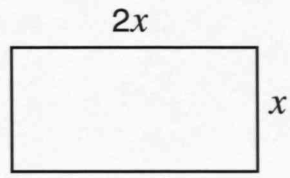
Use the Letters for Length – Challenge shape in the plenary, but replace $5x$ with another variable such as y. Then ask the students to find four different integer values for x and y of four isosceles trapeziums with the same area, where the area is less than 40 square units.

Letters for Lengths

This rectangle has length $2x$ units and width x units.

The area of this rectangle can be written as $2x^2$ square units.

If the area of the rectangle is 24.5 square units, then the value of x is 3.5 units.



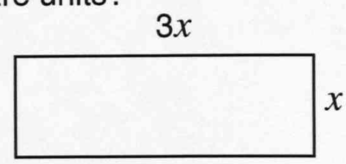
Solve the following problems.

Letters for Length problem 1.

What is the value of x if the area of this rectangle is:

(a) 12 square units?

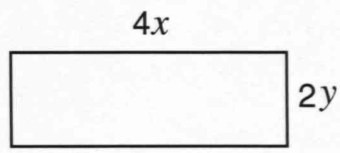
(b) 3 square units?



Letters for Length problem 2.

The area of this rectangle is 48 square units.

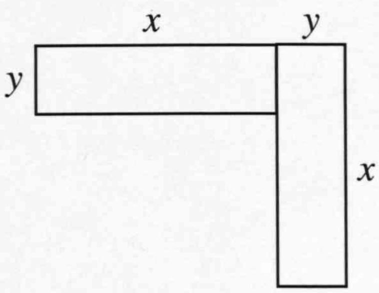
Give some possible values for x and y.



Letters for Length problem 3.

The area of this shape, made from two congruent rectangles, is 28 square units.

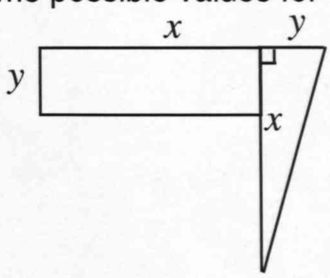
Give some possible values for x and y.



Letters for Length problem 4.

The area of this shape, made from a rectangle and right-angled triangle, is 36 square units.

Give some possible values for x and y.

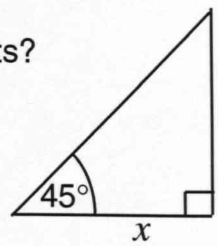


Letters for Length problem 5.

What is the value of x if the area of this right-angled isosceles triangle is:

(a) 8 square units?

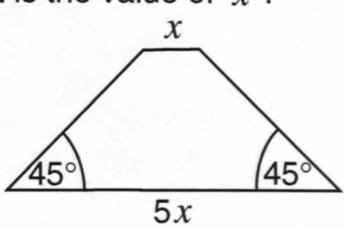
(b) 40.5 square units?



Letters for Length - Challenge

The area of this isosceles trapezium is 54 square units.

What is the value of x ?



Fibonacci-type Sequences

Teachers notes

Type

Suitable for an individual, pairs, a small group or the whole class.

Objective

To investigate possible 'end' numbers in parts of Fibonacci-type sequences using algebra.

Framework for Teaching Mathematics

Solve substantial problems by breaking them into simple tasks, using a range of efficient techniques, methods and resources. (K) (Y9 p.29)

Construct and solve linear equations with integer coefficients (with and without brackets, negative signs anywhere in the equation, positive or negative solutions). (K) (Y9 p. 123)

Substitute numbers into expressions and formulae. (Y9 p. 139)

Generate sequences from practical contexts and write an expression to describe the *n*th term of an arithmetic sequence. (K) (Y9 p. 155)

Resources

One copy of the resource sheet for each student.

Description

A good introduction to this activity is to show students pictures of pine cones and sunflowers and ask what these have to do with the Fibonacci Sequence. A pine cone is formed of spirals. There are 13 spirals going anticlockwise and 8 going clockwise. An average sized seed head of a sunflower has 55 spirals going clockwise and 89 spirals going anticlockwise, smaller sunflowers have only 34 and 55 spirals, larger ones have 89 and 144. These numbers all appear in the Fibonacci Sequence. The main activity is about the end numbers of parts of Fibonacci-type sequences. Let students work in pairs to find possible values for the missing numbers. Then let students share their solutions and strategies with the rest of the class. If the strategies that the students have used do not involve algebra, it is worth challenging them to find the missing numbers of the last two sequences if the starting number is unknown.

Enriching Mathematical Thinking

For students who have confidently explored the end numbers of Fibonacci-type sequences, challenge them to explore the following :

Is it possible to have the 1st and 5th term of a Fibonacci-type sequence equal?

Is it possible to have the 1st and 6th term of a Fibonacci-type sequence equal?

How would you describe the sequence if the 1st and 3rd or the 1st and 4th terms were equal?

Fibonacci-type Sequences

The mathematician Leonardo Fibonacci (1175 –1250) is best remembered for his Fibonacci numbers and the Fibonacci Sequence.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

This sequence, in which each number is the sum of the two preceding numbers, appears in many different areas of mathematics and science.

Here is part of a Fibonacci-type sequence where each number is the sum of the two preceding numbers. The end number is **41**.

2, 7, 9, 16, 25, 41

Here is part of another Fibonacci-type sequence.
The end number is **52**.

8, ■, ■, ■, 52

What are the missing numbers?

Here is part of another Fibonacci-type sequence.
The end number is **52**.

8, ■, ■, 52

What are the missing numbers?

Here is part of another Fibonacci-type sequence.
The end number is **52**.

■, ■, ■, 52

What could the missing numbers be?

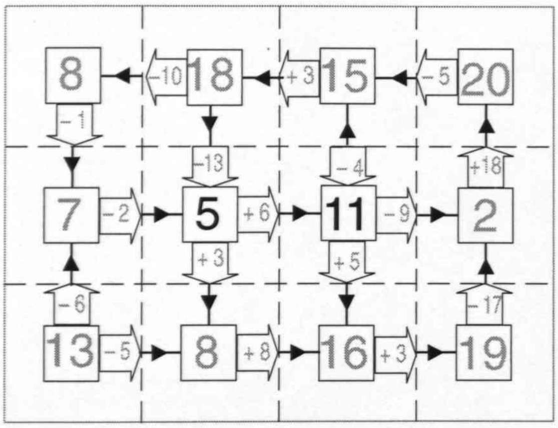
Investigate

Investigate the possible end numbers for parts of other Fibonacci-type sequences.

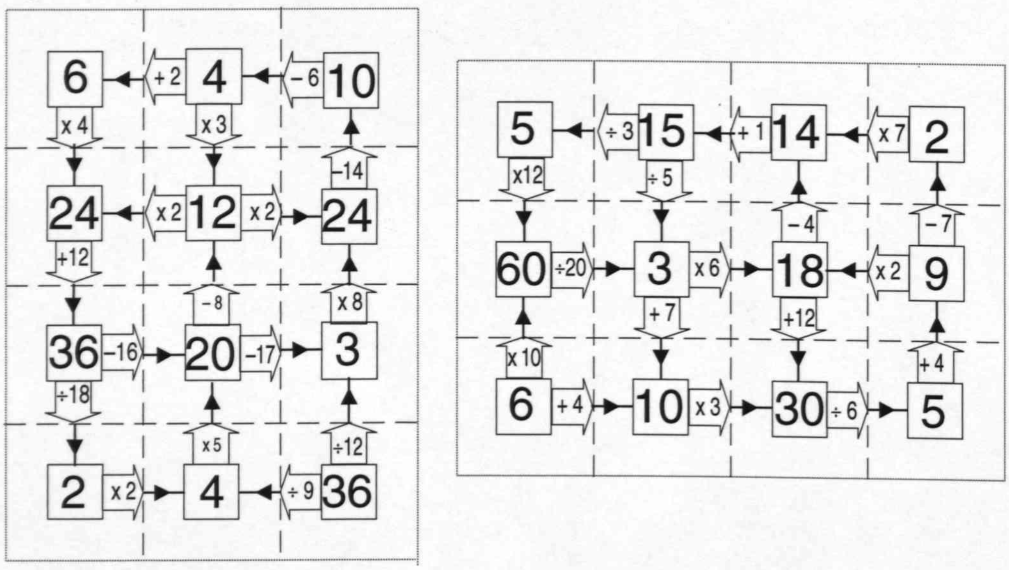
Answers

Mapping Jigsaws

Jigsaw A



Jigsaw B



Function Codes

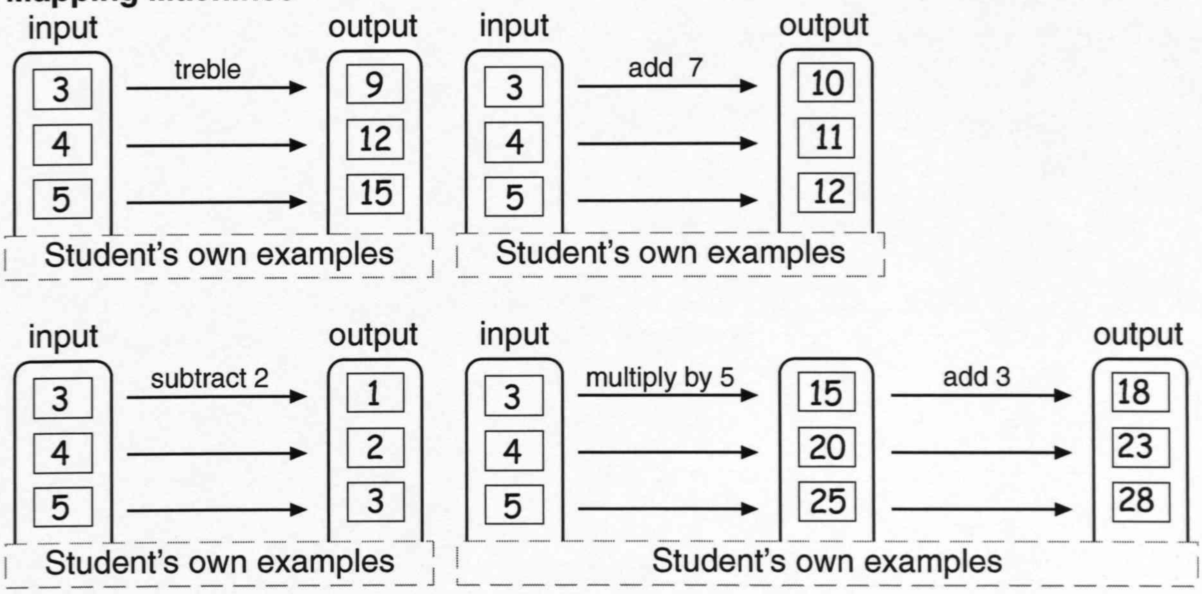
Message 1 The function 'divide by 2' decodes the message to read 'divide by two'.
Message 2 The function 'subtract 5' decodes the message to read 'two is even'.

The five mathematical words are:
'Algebra' (using the function 'divide by 3').
'Area' (using the function 'divide by 4').
'Kite' (using the function 'divide by 2 and add 1').
'Million' (using the function 'add 6').
'Square' (using the function 'add 1 and then divide by 2').

Challenge
The function 'divide by 3 and subtract 1' is used to decode 42 6 63 27 18 42 6 63 30 12 60.

Answers

Mapping Machines

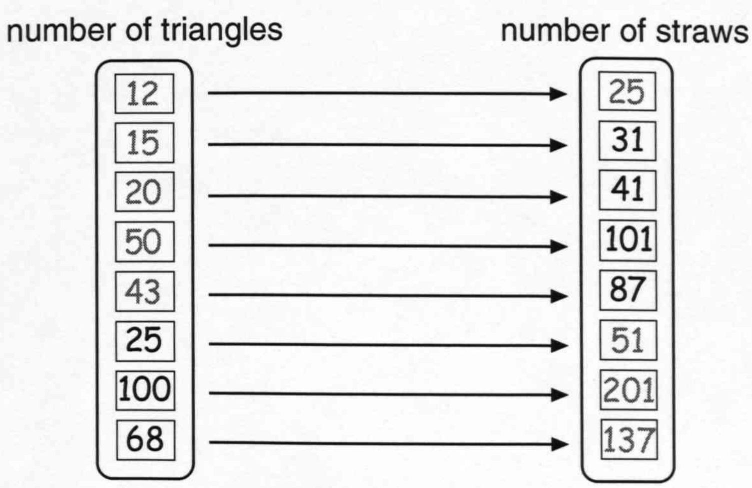


Straws and Triangles

7 triangles require 15 straws.

1. Choice of numbers will vary, but each input for 'number of triangles' will be mapped on to the output for number of straws by the function 'multiply by 2, then add 1'.

2.



3. For single rows of squares, the function used will be 'multiply by 3, then add 1'.

   For single rows of pentagons, the function used will be 'multiply by 4, then add 1'.

   For single rows of hexagons, the function used will be 'multiply by 5, then add 1'.

Answers

Think of a Number

Let x represent the number first thought of.

A The answer will always be 1.

$$\frac{2(x+5)-8}{2}-x=1$$

B The answer will always be 0.

$$\frac{3(x+2)-6}{3}-x=0$$

C The answer will always be the number first thought of.

$$\frac{3\left(\frac{4x-8}{2}\right)}{6}+2=x$$

D The answer will always be 10.

$$2\left(\frac{x}{2}+3\right)+4-x=10$$

E The answer will always be 2 times the number first thought of.

$$\frac{4(x+5)-20}{2}=2x$$

Answers

Answers

Matching Mappings

The function 'add 3' was used to create the first mapping.

Mapping diagram A matches the function 'add 10'.
Mapping diagram C matches the function 'multiply by 6'.
Mapping diagram D matches the function 'square'.
Mapping diagram E matches the function 'the remainder when divided by 5'.
Mapping diagram F matches the function 'subtract from 8'.

The function without a matching mapping diagram is 'subtract 2 then double'.

The mapping diagram without a matching function is mapping diagram B.
This matches with the function 'multiply by 2 then add 3' or equivalent.

Number Problems

A The number is 9.
B The number is 8.
C The number is 4.
D The number is 7.
E The number is 1.
F The number is 6.

Complete the Mappings

1. $x \longrightarrow x+4$

x	$x+4$
7	11
19	23
5	**9**
22	**26**
35	**39**
$2\frac{1}{2}$	**$6\frac{1}{2}$**
88	92

2. $x \longrightarrow x+2$

x	$x+2$
2	**4**
5	**7**
$8\frac{1}{2}$	**$10\frac{1}{2}$**
23	**25**
49	51

$x \longrightarrow x-6$

x	$x-6$
2	**-4**
5	**-1**
$8\frac{1}{2}$	**$2\frac{1}{2}$**
23	**17**
57	51

$x \longrightarrow 3x$

x	$3x$
2	**6**
5	**15**
$8\frac{1}{2}$	**$25\frac{1}{2}$**
23	**69**
17	51

Answers

Complete the Mappings (continued)

3.

x	$2x+2$	
2	6	
5	12	
$8\frac{1}{2}$	19	
23	46	X
$24\frac{1}{2}$	51	
23	48	

x	$8-x$	
2	6	
5	13	X
$8\frac{1}{2}$	$-\frac{1}{2}$	
23	15	X
-43	51	
5	3	
23	-15	

x	$x \div 2 + 1$	
2	6	X
5	3.5	
$8\frac{1}{2}$	$5\frac{1}{4}$	
23	$12\frac{1}{2}$	
101	51	X
2	2	
100	51	

4.

x	$2x-3$
7	**11**
3.5	**4**
$5\frac{3}{4}$	$8\frac{1}{2}$
21	**39**
$11\frac{1}{2}$	20

x	$20-x$
12	**8**
15	5
$5\frac{1}{2}$	**$14\frac{1}{2}$**
-3	23
0	**20**

x	x^2+1
3	**10**
-1	**2**
7	50
2.5	**7.25**
5	**26**

Solve It

$2x+3=39$
$x=18$

$2x+3=145$
$x=71$

$6(p-4)=36$ — $p=10$

$\frac{x+3}{2}=6$ — $x=9$

$\frac{a}{2}-10=13$ — $a=46$

$9(p+3)=36$ — $p=1$

$9(p+3)=27$ — $p=0$

$3(y-2)+15=30$ — $y=7$

Answers

Answers

Algebra Match

Statements	Algebraic expressions
Multiply b by 2 <u>then</u> subtract from a.	$a-2b$
Subtract c from a <u>then</u> multiply by b.	$b(a-c)$
Add b to a <u>then</u> divide into c.	$\frac{c}{a+b}$
Multiply a by b <u>then</u> divide by c.	$\frac{ab}{c}$
Divide c by b <u>then</u> multiply by a.	$\frac{c}{b}\times a$
Multiply a by c <u>then</u> divide into b.	$\frac{b}{ac}$
Add b to a <u>then</u> divide by c.	$\frac{a+b}{c}$
Multiply ab by c.	abc
Subtract c from b <u>then</u> multiply by a.	$a(b-c)$

$4a-c$ matches the statement: Multiply a by 4 <u>then</u> subtract c.

"Divide c into a <u>then</u> subtract b" matches the algebraic expression $\frac{a}{c}-b$

Answers

Answers

Algebra Game
No answers required.

Mapping Diagrams

1. $4 \rightarrow 20$
$5 \rightarrow 25$

$12 \rightarrow 60$
$100 \rightarrow 500$
$n \rightarrow 5n$

2. $4 \rightarrow 24$
$5 \rightarrow 30$

$12 \rightarrow 72$
$100 \rightarrow 600$
$n \rightarrow 6n$

3. $4 \rightarrow 9$
$5 \rightarrow 11$

$12 \rightarrow 25$
$100 \rightarrow 201$
$n \rightarrow 2n + 1$

4. $4 \rightarrow 9$
$5 \rightarrow 12$

$12 \rightarrow 33$
$100 \rightarrow 297$
$n \rightarrow 3n - 3$
or $n \rightarrow 3(n - 1)$

5. $50 \rightarrow 200$
$n \rightarrow 4n$

6. $50 \rightarrow 201$
$n \rightarrow 4n+1$

7. $50 \rightarrow 25$
$n \rightarrow n \div 2$

8. Mappings (a), (b), (d) and (f) all map $3 \rightarrow 9$.

9. Many possible answers, e.g. $n \rightarrow 3n$, $n \rightarrow 16 - n$, etc.

Random Code

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*
5	**8**	23	**11**	**15**	**19**	14	**7**	12	**20**	**10**	18	**2**	**26**

o	*p*	*q*	*r*	*s*	*t*	*u*	*v*	*w*	*x*	*y*	*z*	
17	**21**	**4**	9	**16**	**24**	22	**25**	**1**	**6**	**13**	**3**	27

The message says: "create your own message".

Algebra Problems
A One book costs £8.00.
B One pair of socks cost £2.50
C One side is 5 centimetre, the other side is 10 centimetres.
D One child cost £1.20.
E The twins are 11 years old.
F The smallest angle is 34°, the second angle is 44° and the largest angle is 102°.

Answers

Answers

Algebra Sort

Statements		Algebraic expressions
The product of a and c is divided by b.	→	$\frac{ac}{b}$
Subtract b from a and then divide into c.	→	$\frac{c}{a-b}$
Multiply a by c then divide into b.	→	$\frac{b}{ac}$
Subtract b from a then divide by c.	→	$\frac{a-b}{c}$
Divide the sum of a and c into b.	→	$\frac{b}{a+c}$
The sum of a and b is divided by the sum of c and d.	→	$\frac{a+b}{c+d}$
a is added to b then divided into the product of c and d.	→	$\frac{cd}{a+b}$
The sum of a divided by c and b divided by d.	→	$\frac{a}{c}+\frac{b}{d}$

The two unmatched statements and their algebraic expressions are:

Divide a by c and then divide by b.	→	$\frac{a \div c}{b}$ or $\frac{a}{bc}$
Add a to b and divide by the product of c and d.	→	$\frac{a+b}{cd}$

The two unmatched algebraic expressions and their statements are:

$\frac{a+b}{c}$	→	The sum of a and b, then divided by c. (or equivalent)
$\frac{a}{c}+b$	→	Divide a by c and then add to b.(or equivalent)

Answers

Answers

Equivalent Expressions

$2m - m^2$	=	$-m(m-2)$	=	$m^2 + 2m - 2m^2$
$m(m+1) + m$	=	$m^2 + 2m$	=	$(m+1)^2 - 1$
$\frac{1}{2}(6m^2 - 4)$	=	$3m^2 - 2$	=	$3m^2 + 2m - 2 - 2m$
$5(m+1)$	=	$5m + 5$	=	$\frac{1}{2}(10m + 10)$
$4m^2 - 2m$	=	$2m(2m-1)$	=	$2(2m^2 - m)$
$6x + 3x^2$	=	$3(2x + x^2)$	=	$3x(2+x)$
$2(x+3)$	=	$2x + 6$	=	$2(x+2) + 2$
$\frac{x}{2} + 2$	=	$x - 0.5x + 2$	=	$\frac{1}{2}(x+4)$
$0.5x^2 + 3x$	=	$x(\frac{x}{2} + 3)$	=	$\frac{1}{2}(x+2)(x+4) - 4$
$3x - 21$	=	$3(x-7)$	=	$3(x-8) + 3$

Whose Rule is Correct?

Joe's rule $r = 3(p-3)$
Nikki's rule $r = 2p - 3 + p$
Rajan's rule $r = 3p - 3$
Joe's rule is incorrect because $3(p-3) = 3p - 9$.
All the other rules can be rearranged to show that they are equal to $r = 3p - 3$.

Karen's rule $s = 2c - 2$
Joe's rule $s = 2(c-1)$
Nikki's rule $s = (4c - 2) \div 2$
Rajan's rule $s = 4c \div 2 - 2$
Nikki's rule is incorrect because $(4c - 2) \div 2 = 2c - 1$.
All the other rules can be rearranged to show that they are equal to $s = 2c - 2$.

Algebra Trail

No answers required.

Letters for Length

1. (a) $x = 2$ units
(b) $x = 1$ unit

2. Possible integer solutions are:
$x = 1$ unit and $y = 6$ units
$x = 3$ units and $y = 2$ units
$x = 6$ units and $y = 1$ units

3. Possible integer solutions are:
$x = 1$ unit and $y = 14$ units
$x = 2$ units and $y = 7$ units

4. Possible integer solutions are:
$x = 1$ unit and $y = 24$ units
$x = 2$ units and $y = 12$ units
$x = 3$ units and $y = 8$ units
$x = 4$ units and $y = 6$ units

5. (a) $x = 4$ units
(b) $x = 9$ units

Challenge
$x = 3$ units

Answers

Fibonacci-type Sequences

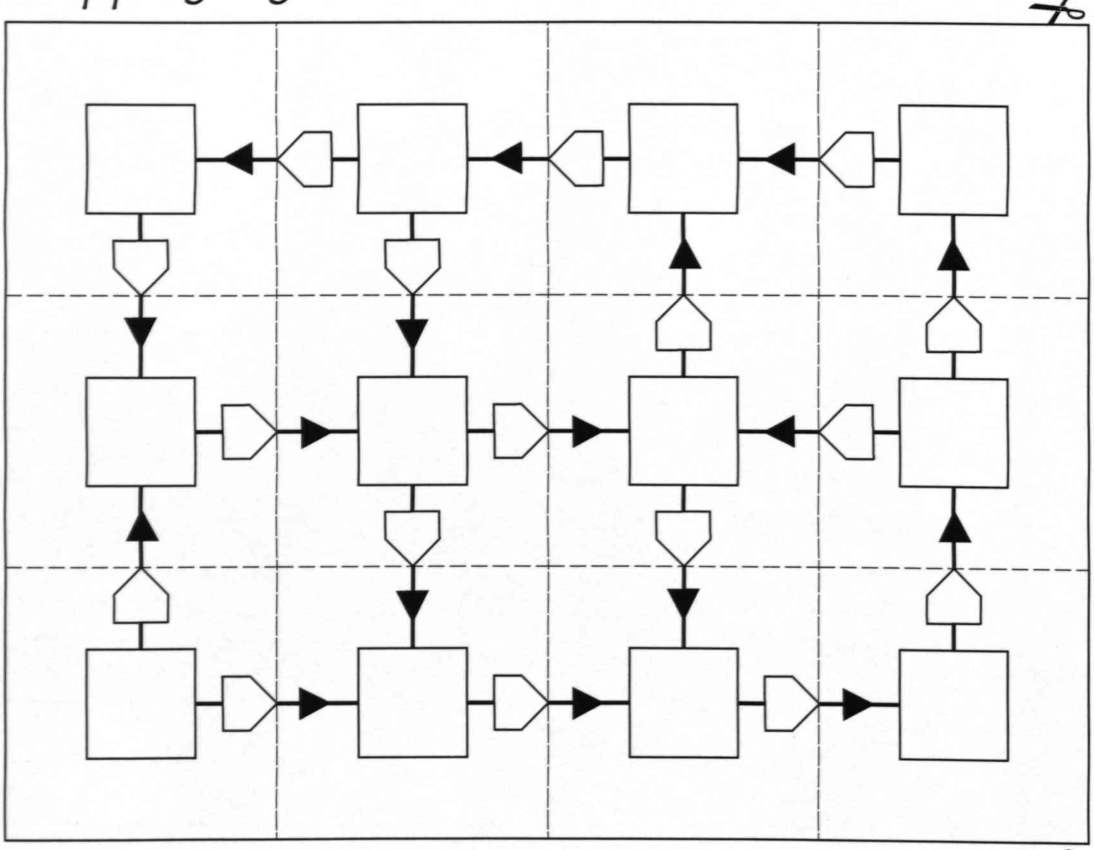
8, **12**, **20**, **32**, 52

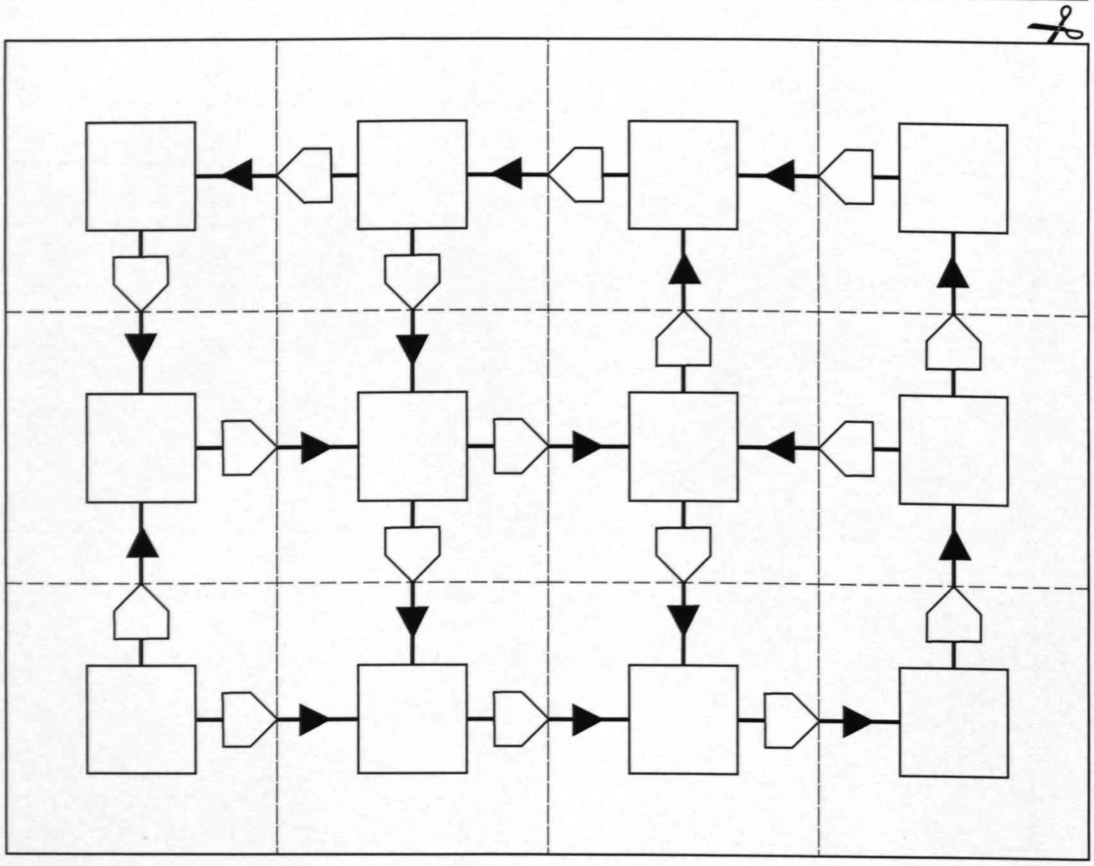
8, **22**, **30**, 52

Possible positive integer values for a and b for the following Fibonacci-type sequence a, b, $a+b$, 52, where 52 = $a+2b$ are:

$a=2$ & $b=25$
$a=4$ & $b=24$
$a=6$ & $b=23$
$a=8$ & $b=22$
$a=10$ & $b=21$
$a=12$ & $b=20$
$a=14$ & $b=19$
$a=16$ & $b=18$
$a=18$ & $b=17$
$a=20$ & $b=16$
$a=22$ & $b=15$
$a=24$ & $b=14$
$a=26$ & $b=13$
$a=28$ & $b=12$
$a=30$ & $b=11$
$a=32$ & $b=10$
$a=34$ & $b=9$
$a=36$ & $b=8$
$a=38$ & $b=7$
$a=40$ & $b=6$
$a=42$ & $b=5$
$a=44$ & $b=4$
$a=46$ & $b=3$
$a=48$ & $b=2$
$a=50$ & $b=1$

Mapping Jigsaws Resource Sheet





Appendix

Recording Sheet for Algebra Game/Trail

Use this sheet to record your workings.

Number on the dice.

For example

First move

Algebraic expression	d	Workings	Move
$2(d-1)$	4	$2(d-1) = 2(4-1) = 2 \times 3 = 6$	6

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Algebra Makes Sense

Introduction

Algebra Makes Sense contains 20 activities made up of student resource sheets and teachers' notes, all of which are photocopiable within the purchasing establishment.

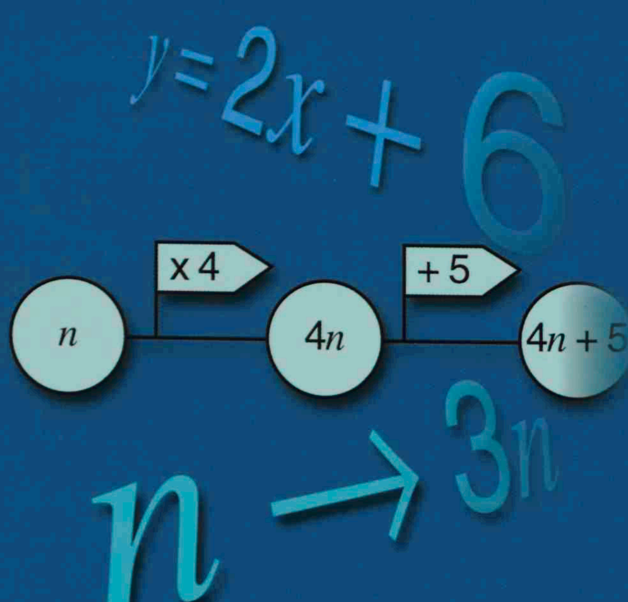
The activities address different aspects of algebra and are appropriate for a range of learning styles. They are arranged by National Curriculum Assessment Levels (4 – 7) in order to aid differentiation and facilitate their easy incorporation into any scheme of work. Within each level the activities are arranged in order of difficulty. Each activity is also referenced to the Framework for Teaching Mathematics.

Format of the activities

Each activity is accompanied by teachers' notes. The teachers' notes offer advice on appropriate ways of using the activities and any additional equipment required.

The role of the teacher

Many of the activities are suitable for whole class lessons and contain much scope for discussion. Ideas to enrich students' mathematical thinking are also included. All the materials have been designed to contain sufficiently clear instructions to be accessible to students with minimal teacher support. This makes them ideal for use within whole class differentiated topic work as the teacher is left free to discuss structural issues within algebra with individuals or groups as they arise. Full answers are given for each activity.



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