


Two Latin square designs are the same if you can use substitution and reflection to make them look identical.


* Use a Latin square design to test 4 different varieties of wheat. How many really different designs can you find?


This Latin square design is used to test
 three varieties of rhubarb.
Adapt the experiment so that each variety is also tested for three different fertilisers: $A, B$ and $C$.

These are called Graeco-Latin squares.

The problem is much more difficult with 4 crop varieties and 4 fertilisers.
There are 4 really different possible Latin square designs - and for some of them, there is no Graeco-Latin solution.

For each design: either find a solution or prove that it is impossible.

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Amazing fact: For order 11, there are 5363937773277371298119673540771840 different Graeco-Latin squares! Proved in 2005 by McKay and Wanless



## Description

Experimental design for agriculture relies heavily on Latin and Graeco-Latin squares to cope with different fertility patterns found in different parts of a given experimental growing space.

## Resources

Squared paper, scissors.

## Activity 1: Three types of rhubarb

## Activity 2: Four types of wheat

## Activity 3: Graeco-Latin designs

These three activities lead into one another and form a continuous thread of development. Sudoku has made Latin squares more readily recognisable. A whole class discussion about the need for Latin square designs in field experiments will provide a good introduction to Three types of rhubarb. (West Yorkshire is well known for the forced rhubarb crop grown in the area known as The Rhubarb Triangle.) Draw out the wide range of factors which might affect crop yield in different parts of a field - yield will be affected by height, slope, wind, drainage and so on and all these factors vary across a given experimental field area and need to be controlled for in any field experiment. All of the twelve designs can be obtained from each other by substituting one variety for another or by vertical reflection of the whole design. Trial and error makes a useful starting strategy but a more systematic approach is needed in order to find all twelve possible layouts. This activity should conclude with a discussion about what makes one design really different from another in order to prepare for Four types of wheat. Here the task is to find the four solutions that are really different from each other.

## The mathematics



Graeco-Latin designs is a more difficult task and requires the pupils to grapple with notions of proof. Finding the one possible Graeco-Latin square of order 3 provides a straightforward introduction to the problem but finding a complete solution for order 4 is a significant challenge.
For pupils who become intrigued by it, the problem can provide the opportunity for an extended piece of work. Results and mathematical arguments can be displayed in poster format and presented to the rest of the class. The amazing fact can be used to generate work on very large numbers - How could we say this number? If we could draw one solution in 5 minutes, how long would it take to draw them all? A cut-up sheet is provided to support those who are finding the problem too hard and need a more supported way into solving it.

All three activities engage the pupils in systematic analysis, the need for careful recording and proof. Four types of wheat also draws attention to rotation and reflection.

