

Quantum Technology

PROGRAMME

TEACHER GUIDANCE

The physics of GPS – Trilateration

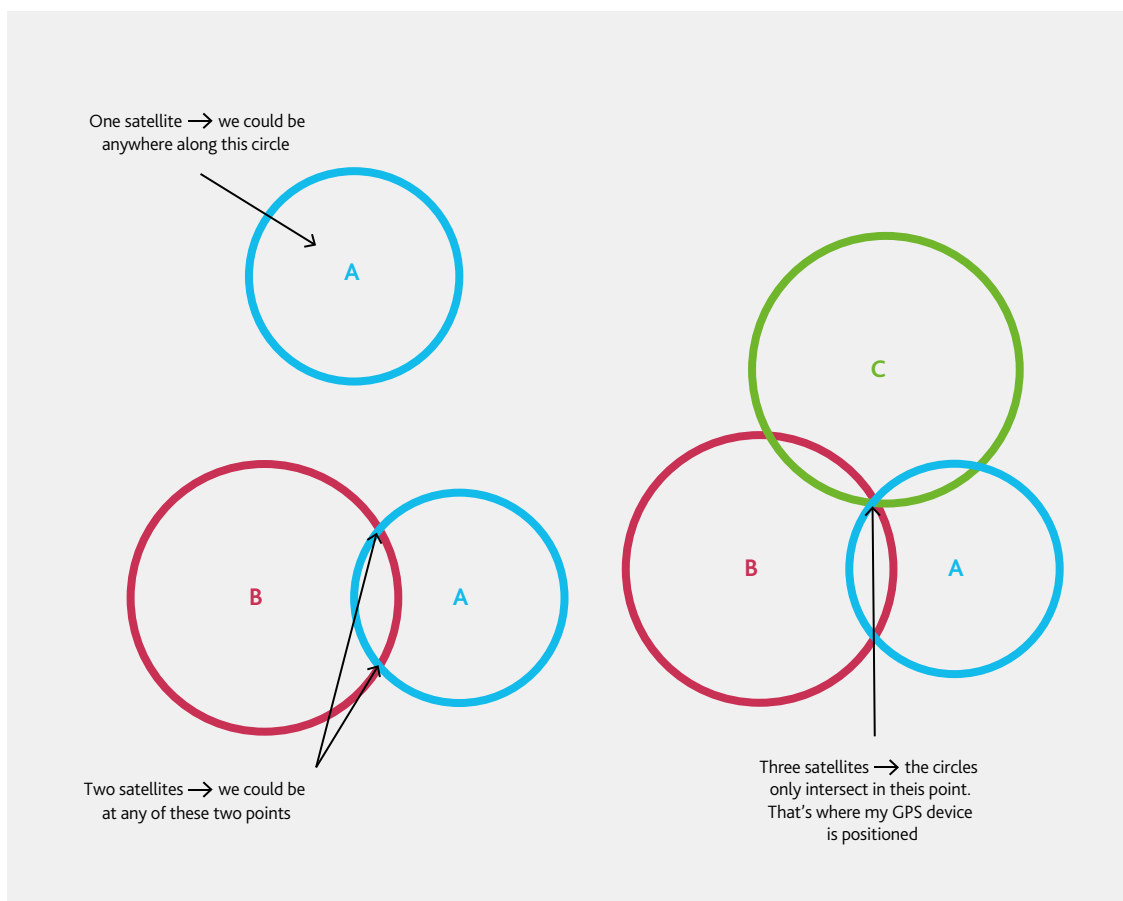
1 BACKGROUND

There are many Global Positioning System (GPS) satellites orbiting the Earth. These are used to locate your position on the planet. GPS positioning works on fairly simple principles, but the installation and application of such a system requires incredible precision.

Two main mathematical ideas underpin the GPS positioning network. Trilateration is the first concept and it is based on finding the position of a GPS device from three distances. The second idea is the relationship between the speed of the signal (speed of light, $c = 299,792,458\text{ms}^{-1}$), the time taken for the signal to travel and the distance travelled. All that is needed for this second concept is the equation:

$$\text{Distance travelled (m)} = \text{Speed (ms}^{-1}\text{)} \times \text{Time (s)}$$

Trilateration works by finding your position on Earth once the location of GPS satellites orbiting the Earth and their distance from your location are known. Since we cannot physically measure the distance of these satellites directly, we need to use the known speed of the signal sent by the GPS satellites and the time the signals were sent. This is quite easy, because satellites send out electromagnetic signals constantly. If our GPS receiver detected the signal from only one satellite, all we could tell is that we could be anywhere on the surface of a sphere of radius equal to the calculated distance from the satellite. If we only received signals from two satellites (satellite A and B, for example), we could tell that we are somewhere along the circle drawn by the intersections of the spheres described by the two signals. But with a third satellite signal we can tell the exact location of our device, because the three spheres will intersect in one point only. We can simplify this concept in 2D, using circles rather than spheres (this is what your activity will model).



To increase the accuracy of its position the GPS receiver receives and analyses signals from at least four GPS satellites at any one time, so this system is fine-tuned to give the best results possible.

The need for accurate measurements of time

The signals sent out by GPS satellites need to carry very precise and accurate time stamps in order to synchronise the GPS receiver on Earth and determine the exact location of the device. This amazingly accurate timekeeping is achieved by the atomic clocks carried by the GPS satellites, which currently use the exact frequency of the microwave spectral line emitted by atoms of cesium-133 as a reference. Since frequency is the number of cycles per second, the use of atomic clocks has given us a new definition of the second as:

“the duration of 9,192,631,770 cycles of microwave light absorbed or emitted by the hyperfine transition of cesium-133 atoms in their ground state undisturbed by external fields”.

If we used ordinary clocks, our positioning calculations would be way off target. Of course, the clocks inside our mobile devices are not atomic clocks, but they constantly synchronise with the atomic clocks of GPS satellites, as they receive their signals. In fact, each signal sent from a satellite carries the position of the satellite and the time at which it was sent.



2 OBJECTIVES

- ▶ Develop an understanding of atomic clocks
- ▶ Practise calculations with standard form numbers
- ▶ Understand how trilateration works

3 ADVANCED PREPARATION

For the trilateration class demonstration:

- ▶ 3 x mm-accurate tape measures, at least 5m in length
- ▶ Small coloured markers (sticky dots)
- ▶ Whiteboard and markers (for recording results where students can see them)

Watch the video suggested in question 4 before the lesson and ensure you have access to YouTube.

4 INTRODUCTION

In this activity you will watch a video that explains the principles of GPS trilateration. Then, you will simulate how a GPS device finds its position on Earth using the signals from four GPS satellites. Finally, you will be able to answer some questions about how GPS works and why atomic clocks precision is important in this process.

5 ACTIVITY

This activity encourages students to use their knowledge and understanding of A level Physics to explore the principles of atomic clocks and their application in GPS technology. The students will consider the accuracy of current and future atomic clocks and practise the use of standard form calculations, as well as simulating how GPS works through a trilateration class demonstration.

An activity sheet with a suggested video and associated questions is included in this activity.

6 PLENARY

Discuss the answers of the students and encourage them to offer their solutions from the whiteboard. Some of the questions and the nature of the activity encourage students to think about possible sources of error in current GPS technology. This section could be linked to the work they have done on endorsed practicals. Useful points for discussion are:

- ▶ use of standard form numbers
- ▶ how trilateration works and the need for corrections due to relativistic effects
- ▶ how quantum physics has enabled us to achieve the most precise measurements (of time) ever recorded in human history



7 EXTENSION

Research on the next generation of GPS satellites and positioning systems that do not require satellites, and their potential impact on our lives.

8 TEACHER INFORMATION

There is a PowerPoint containing three slides that should help you illustrate the concept of trilateration and the current and predicted future accuracy of GPS positioning systems. You could use this presentation to introduce the concept of trilateration in the background information in this resource, or after the trilateration class demonstration explained below.

Class demonstration of trilateration:

- ▶ 3 x mm-accurate tape measures, at least 5m in length
 - ▶ Small coloured markers (sticky dots)
 - ▶ Whiteboard and markers (for recording results where students can see them)
- 1 Give three students each a mm-accurate tape measure of 5m length.
 - 2 Ask the three students to stand around four to five metres apart (if possible) from each other in a triangle. Each places a marker (such as a small sticky 'dot' on the floor) where they stand (this also works well with separation of only 2m).
 - 3 They then meet at some point in between and place another dot there. They then measure (to nearest mm) the distance from their dot to that point. A discussion can be had about how the distance is known because the speed of light is known and the signals are timestamped, allowing a receiver to calculate distance. Also, the receiver 'knows' where each satellite (or dot) is at any given time, so can combine this information to trilaterate.
 - 4 A discussion of the importance of time accuracy should follow – each GPS satellite is equipped with an atomic clock which measures time with great accuracy. However, the receiver doesn't contain an atomic clock, so how can accurate 'travel time' measurements be made? Answer: a fourth GPS satellite is used as a clock by the GPS receiver.
 - 5 Then, introduce some uncertainty. Ask each of them to either add or subtract 20mm from their measurement (without telling the others), and then to meet again using their tape measures. Discuss how this uncertainty would be introduced if we didn't have accurate clocks on the satellites. They can proceed to use $\pm 20\text{mm}$ and find the area of uncertainty.
 - 6 By using more accurate clocks we can reduce the uncertainty. The strontium clocks are 1,000 times more sensitive than older caesium clocks. Instead of $\pm 20\text{mm}$ accuracy we would have $\pm 0.02\text{mm}$ accuracy!
 - 7 In real terms this means that the accuracy of GPS can be improved from five to ten metres to a few centimetres, improving navigation for autonomous cars etc.
 - 8 It is also worth pointing out that the biggest source of error isn't the clocks – ask students what it might be. Answer: the ionosphere that extends from around 60 to 2,000km in altitude, and which affects the speed of radio waves (by refraction). Some military research agencies are working on stand-alone navigations systems, using accurate clocks, gyroscopes and accelerometers that do not use satellites at all – they can't be jammed and they don't suffer from atmospheric effects.

See 'Answers' for the solutions to the students' worksheet.



9 LINKS TO FURTHER RESOURCES

The following videos could be watched before, during or after the activity as a further source of information:

<https://www.youtube.com/watch?v=loRQINFzT0k>

10 ANSWERS

- 1 The quartz crystal of a commercial quartz clock vibrates at a frequency of 32,768Hz. Estimate the uncertainty on the distance between a GPS receiver and a GPS satellite, if we used a quartz clock on GPS satellites.

We could calculate the period of one oscillation and use that to calculate the uncertainty of the distance between a GPS satellite and a receiver.

$$T = 1/f = 1/32768 = 3.05 \times 10^{-5} \text{ s} \rightarrow \text{uncertainty on distance} = T \times c = 3.05 \times 10^{-5} \text{ s} \times 299792458 \text{ ms}^{-1} = 9149 \text{ m}$$

So, if we take the distance travelled by an electromagnetic signal during a single oscillation of the quartz crystal we would have an uncertainty of $\pm 4.6 \text{ km}$.

- 2 In reality the stability of the quartz resonator and its driving circuit increases the accuracy of quartz clocks and an ordinary wristwatch will lose less than 0.5 s each day. Estimate the uncertainty for every second travelled by an electromagnetic signal if GPS satellites carried such quartz clocks on board.

0.5 s 'lost' per day means $0.5 / (24 \times 60 \times 60) = 5.8 \times 10^{-6} \text{ s}$ 'lost' every second, ie a distance = $299792458 \text{ ms}^{-1} \times 5.8 \times 10^{-6} \text{ s} = 1735 \text{ m}$ uncertainty each second travelled at the speed of light.

- 3 The atomic clocks being developed at the Quantum Technology Hubs can use the atomic frequency of Rb-87 which is $f = 6.8 \times 10^9 \text{ Hz}$. Estimate the uncertainty on the distance between a GPS receiver and GPS satellite fitted with such an atomic clock.

We could calculate the period of one oscillation and use that to calculate the uncertainty of the distance between a GPS satellite and a receiver.

$$T = 1/f = 1/(6.8 \times 10^9 \text{ Hz}) = 1.47 \times 10^{-10} \text{ s} \rightarrow \text{uncertainty on distance} = T \times c = 1.47 \times 10^{-10} \text{ s} \times 299792458 \text{ ms}^{-1} = 0.044 \text{ m}$$

So, if we take the distance travelled by an electromagnetic signal during a single oscillation of the atomic frequency of Rb-87 we would have an uncertainty of $\pm 2.2 \text{ cm}$.



- 4 The accuracy of current GPS positioning is about 10m. Is this positioning accuracy suitable for self-driving cars? Explain your answer.

10m is quite a large distance, if we consider that self-driven cars need to know precisely when to approach a bend on the road, turn around a corner, etc... If you have ever driven around a roundabout using a sat nav, you might have noticed that your car's position is not updated very accurately and you might have missed the occasional turn. So, 10m accuracy is not ideal for self-driven cars.

- 5 Watch this video goo.gl/DfX2c4 from the Perimeter Institute and explain why the measurements of time from atomic clocks in GPS satellites need to be corrected for...

Special relativity:

Special relativity shows that clocks moving faster than an observer run slower than the observer's clock. GPS satellites orbit around the Earth at a speed of 14,000km/h. So, the atomic clocks on GPS satellites fall behind clocks on Earth by $7\mu\text{s}$ per day.

And general relativity:

General relativity shows that gravity affects time, so that a clock on Earth (where the gravitational field from the Earth is stronger than around the orbits of GPS satellites) will run slower than a clock in space. GPS satellites are 20,000km above the surface of the Earth and the atomic clocks on them run $45\mu\text{s}$ faster than clocks on Earth per day.

Combining the effects of special and general relativity means that atomic clocks on GPS satellites need to be calibrated to run $45\mu\text{s} - 7\mu\text{s} = 38\mu\text{s}$ slower than they would normally operate to remain synchronised with atomic clocks on Earth.

- 6 Current Cs-133 atomic clocks have an accuracy of about 2ns per day. Calculate how long it would take one of these clocks to lose 1s.

$$2\text{ns} = 2.0 \times 10^{-9} \text{ s} \quad \rightarrow \quad 1\text{s} / 2.0 \times 10^{-9} \text{ s} = 500 \text{ million days} = 1,369,863 \text{ years}$$

- 7 New technologies explored at the Quantum Hubs are using Yb atoms in new generation atomic clocks which are estimated to be capable of an accuracy of 10^{-18} s. Use this figure to estimate how long it would take one of these atomic clocks to lose 1s.

10^{-18} s per day means that 1s would be lost $10^{-18} \text{ s} = 10^{18} \text{ days} = 2.7 \times 10^{15} \text{ years}$ which is much longer than the age of the universe estimated by the Big Bang theory.