Answers • Answers • Answers

0001
to
0500

SMILE MATHEMATICS
Isaac Newton Centre for
Professional Development
108A Lancaster Road
London W11 1QS
Tel 0171-221 8966

SMILE
MATHEMATICS

SMILE

ANSWERS

0001 - 0500

Answers

0001
to
0500

SMILE
MATHEMATICS

SMILE Mathematics
108a Lancaster Road
London
W11 1QS

Tel: 0171-221 8966
Fax: 0171-243 1570

This book contains answers to all the SMILE activities between 0001-0500, in numerical order.

As well as giving the answers there are also:

- explanations about how solutions have been arrived at,
- hints or prompts if you get stuck,
- ideas for extending some activities.

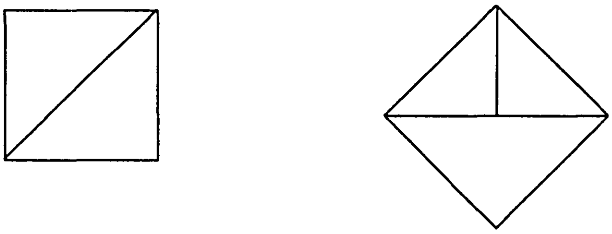
Use this book after you have completed each activity, so that you have immediate feedback on your work. You will remember the work more clearly and be able to identify any difficulties or misconceptions more easily. If you have made errors, look through your work again to see if you can spot where you have made an error. If you then do not understand why your answer is incorrect always seek help from your teacher so that she can help you to clarify any mis-understandings.

You can also use this book while you are working on an activity as it contains hints if you get stuck, or want to know how continue.

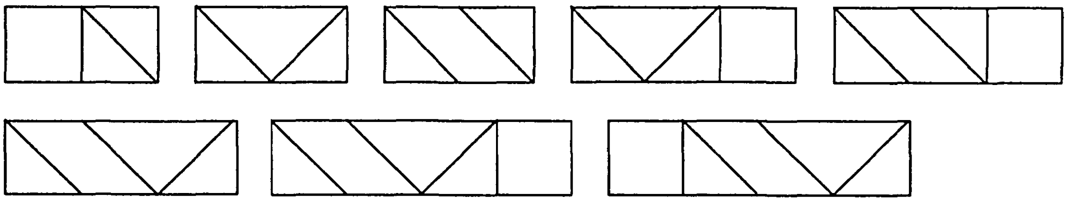
Remember, using the answer book to check your work or to help you if you are stuck is not cheating.

0005 Tangram 1

1. Here are two ways to make a square using some of the pieces.



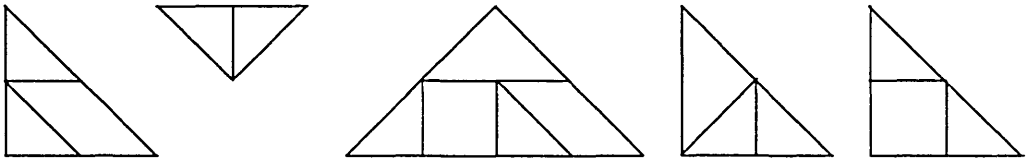
2.



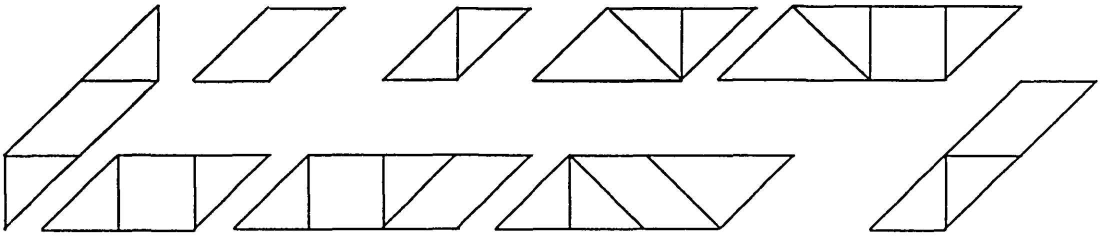
3. The 2 small triangles fit exactly over the square, the large triangle and the parallelogram . . . .
so these 3 pieces must all have the same area.

0006 Tangram 2

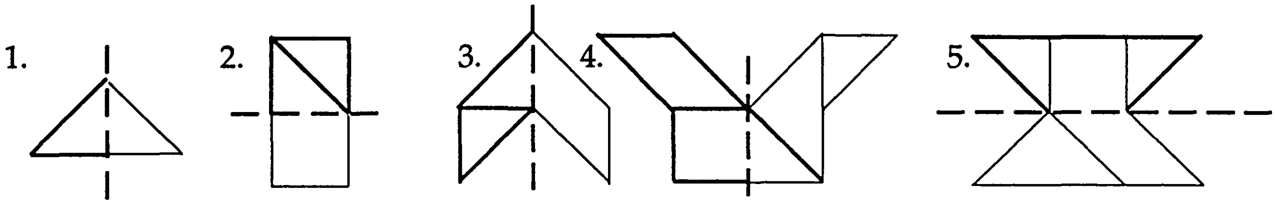
1.



2.



0007 Tangram 3

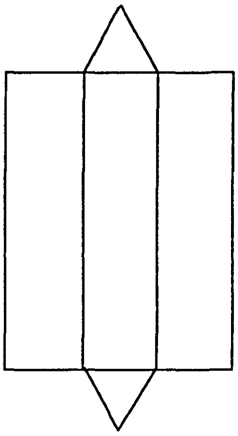


There are other ways of making these shapes, as you may have found.

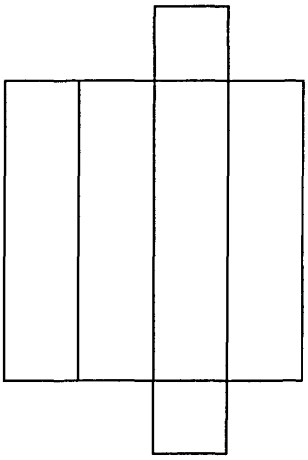
0008 Prisms and Pyramids

These are some of the nets you might have drawn:

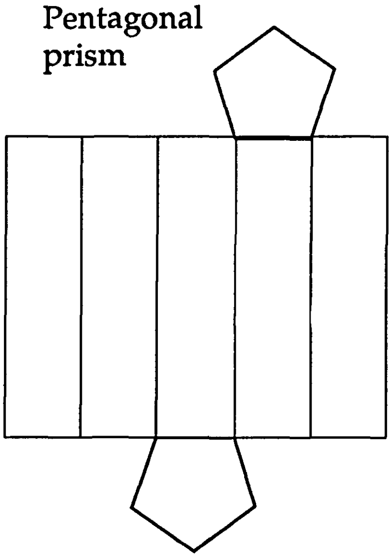
Triangular prism



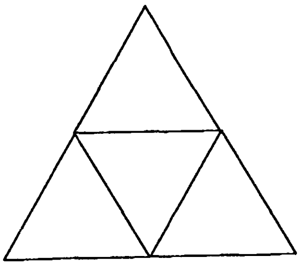
Square prism



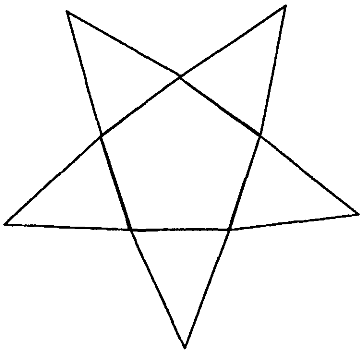
Pentagonal prism



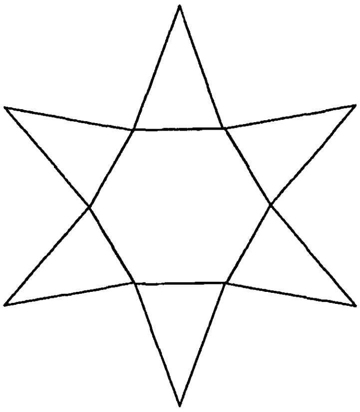
Triangular-based pyramid (tetrahedron)



Pentagonal-based pyramid



Hexagonal-based pyramid



0009 Fraction Dominoes

Once you have played the game, ask your teacher to check that you have matched the fraction dominoes correctly.

0010 Base three introduction

1. 3 units = 1 long
2. 3 longs = 1 flat
3. 3 flats = 1 block
4. 6 units = 2 longs
5. 6 longs = 2 flats
6. 6 flats = 2 blocks
7. 9 units = 3 longs = 1 flat
8. 9 longs = 3 flats = 1 block
9. 9 flats = 3 blocks = 1 long block

1. 14 units = 1 flat, 1 long, 2 units
2. 4 units = 1 long, 1 unit
3. 7 units = 2 longs, 1 unit
4. 8 flats = 2 blocks, 2 flats
5. 6 flats = 2 blocks
6. 6 longs = 2 flats
7. 5 flats = 1 block, 2 flats
8. 3 longs = 1 flat
9. 7 flats = 2 blocks, 1 flat
10. 10 flats = 3 blocks, 1 flat = 1 long block, 1 flat

0017 Introduction to Base Two

- 2 units make 1 long
- 2 longs make 1 flat
- 2 flats make 1 block

1. 4 units = 2 longs = 1 flat
2. 6 units = 3 longs = 1 flat, 1 long
3. 4 longs = 2 flats = 1 block
4. 3 units = 1 long, 1 unit

5. 1 flat, 1 long
6. 1 flat, 1 unit
7. 1 flat, 1 long, 1 unit
8. 1 block
9. 1 long
10. 1 flat
11. 1 flat, 1 unit
12. 1 flat, 1 unit
13. 1 block
14. 1 block, 1 long, 1 unit
15. 1 block, 1 long, 1 unit
16. 1 block, 1 unit
17. 1 block, 1 long, 1 unit

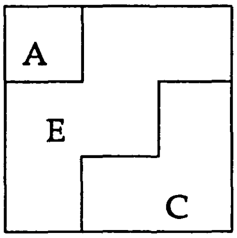
0022 Area 1

1. 3cm^2
2. 5cm^2
3. 10cm^2
4. 7cm^2
5. 7cm^2
6. 12cm^2
7. 12cm^2
8. 17cm^2
9. 9cm^2
10. 14cm^2

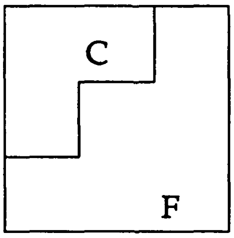
0023 Area 2

1. A = 1cm^2 B = 2cm^2 C = 3cm^2 D = 4cm^2 E = 5cm^2 F = 6cm^2

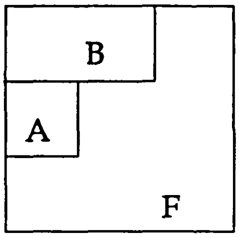
3.



or

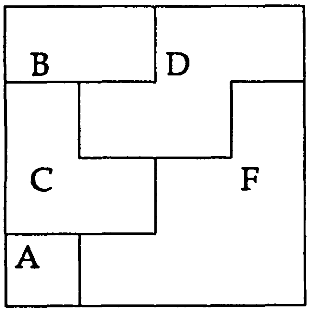


or

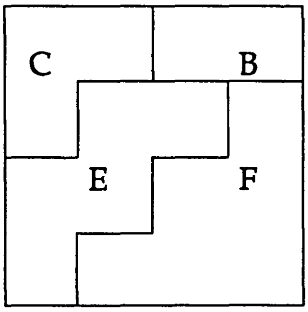


The area of the square is 9cm^2

4.



or



The area of the square is 16cm^2

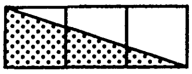
5. A 5cm x 5cm square has an area of 25cm^2 and the total area of the 6 pieces is only 21cm^2. So there are not enough pieces to cover the square.

0024 Area 3

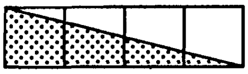
The area of the shapes are all the same because what has been taken away has been added back on.

0025 Area 4

The shaded area is 1½ cm^2



The shaded area is 2cm^2



1. A = 4cm^2 B = 5cm^2 C = 4½cm^2 D = 5cm^2 E = 4cm^2
F = 4½cm^2 G = 5cm^2 H = 5cm^2 I = 3½cm^2 J = 4cm^2

0027 Number Squares 1

1.

1	2	3
3	4	7
4	6	10

2.

5	6	11
7	8	15
12	14	26

3.

2	5	7
7	9	16
9	14	23

4.

4	9	13
4	7	11
8	16	24

5.

8	8	16
3	5	8
11	13	24

6.

5	5	10
9	9	18
14	14	28

7.

7	9	16
6	8	14
13	17	30

8.

8	4	12
2	9	11
10	13	23

9.

7	7	14
3	5	8
10	12	22

10.

6	6	12
3	7	10
9	13	22

11.

7	5	12
5	7	12
12	12	24

12.

7	8	15
9	10	19
16	18	34

0028 Number Square 2

1.

5	1	6
2	7	9
7	8	15

2.

6	2	8
2	3	5
8	5	13

3.

6	5	11
4	5	9
10	10	20

4.

8	5	13
3	6	9
11	11	22

5.

6	4	10
6	7	13
12	11	23

6.

5	7	12
9	3	12
14	10	24

7.

6	10	16
9	2	11
15	12	27

8.

14	0	14
13	5	18
27	5	32

9.

5	10	15
3	2	5
8	12	20

10.

4	9	13
8	7	15
12	16	28

11.

8	9	17
9	4	13
17	13	30

12.

7	14	21
6	4	10
13	18	31

0030 Number Squares 4

1.

8	3	7	18
4	6	2	12
5	9	1	15
17	18	10	45

2.

5	9	4	18
6	3	5	14
5	7	8	20
16	19	17	52

3.

7	6	3	16
6	3	7	16
3	7	6	16
16	16	16	48

4.

4	2	8	14
3	9	2	14
5	4	3	12
12	15	13	40

5.

8	6	5	19
6	5	8	19
5	8	6	19
19	19	19	57

6.

9	12	15	36
9	12	15	36
9	12	15	36
27	36	45	108

7.

5	6	3	14
4	3	1	8
5	1	5	11
14	10	9	33

8.

7	1	2	10
1	4	3	8
5	1	2	8
13	6	7	26

9.

5	2	5	12
0	4	1	5
1	0	2	3
6	6	8	20

0031 Find the Number 1

1. $4 + \boxed{4} = 8$

2. $6 + \boxed{8} = 14$

3. $4 + \boxed{8} = 12$

4. $\boxed{7} + 8 = 15$

5. $\boxed{8} + 9 = 17$

6. $\boxed{13} + 6 = 19$

7. $\boxed{5} - 3 = 2$

8. $\boxed{15} - 6 = 9$

9. $\boxed{12} - 5 = 7$

10. $\boxed{15} - 9 = 6$

11. $13 - \boxed{9} = 4$

12. $17 - \boxed{8} = 9$

13. $7 + \boxed{13} = 20$

14. $\boxed{33} - 13 = 20$

15. $20 - \boxed{7} = 13$

16. $18 + \boxed{7} = 25$

17. $24 - \boxed{10} = 14$

18. $\boxed{32} - 15 = 17$

19. $43 + \boxed{17} = 60$

20. $\boxed{48} - 19 = 29$

0033 Find the Number 3

1. $4 \times \boxed{6} = 24$
2. $7 \times \boxed{5} = 35$
3. $\boxed{4} \times 8 = 32$
4. $\boxed{5} \times 5 = 25$
5. $9 \times \boxed{8} = 72$
6. $\boxed{4} \times 11 = 44$
7. $36 \div \boxed{4} = 9$
8. $\boxed{42} \div 6 = 7$
9. $90 \div \boxed{9} = 10$
10. $\boxed{56} \div 7 = 8$
11. $\boxed{40} \div 8 = 5$
12. $49 \div \boxed{7} = 7$
13. $6 \times \boxed{9} = 54$
14. $\boxed{12} \times 5 = 60$
15. $121 \div \boxed{11} = 11$
16. $\boxed{8} \times 6 = 48$
17. $63 \div \boxed{7} = 9$
18. $96 \div \boxed{12} = 8$
19. $\boxed{132} \div 11 = 12$
20. $\boxed{4} \times 22 = 88$

0034 Find the Number 4

1. 9 + 6 =13 **False**
2. 12 + 7 = 19 **True**
3. 18 + 9 = 27 **True**
4. 36 ÷ 4 = 9 **True**
5. 24 – 18 = 5 **False**
6. 8 x 9 = 72 **True**
7. 63 ÷ 7 = 9 **True**
8. 48 ÷ 8 = 8 **False**
9. ½ of 5 = 10 **False**
10. ½ of 98 = 49 **True**
11. 2 ÷ 4 = ½ **True**
12. ½ of 4 = 2 **True**

13. $\boxed{13} + 12 = 25$
14. $\boxed{31} - 12 = 19$
15. $76 \div \boxed{76} = 1$
16. $9 \times \boxed{11} = 99$
17. $\tfrac{1}{2}$ of $\boxed{6} = 3$
18. $\tfrac{1}{2}$ of $\boxed{30} = 15$
19. $\boxed{2\tfrac{1}{2}} \div 5 = \tfrac{1}{2}$

20 $12 \div \boxed{24} = \tfrac{1}{2}$

0035 Squares and Triangle

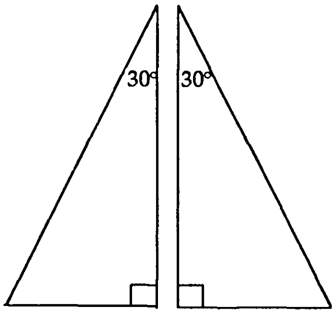
Learn the names of the shapes you have made. Check you understand what a right angle is.

0039 About Angles

1. A right angle is 90 degrees (90°).

3. The 3 small angles together make 90°.
   So each small angle must be 30°.

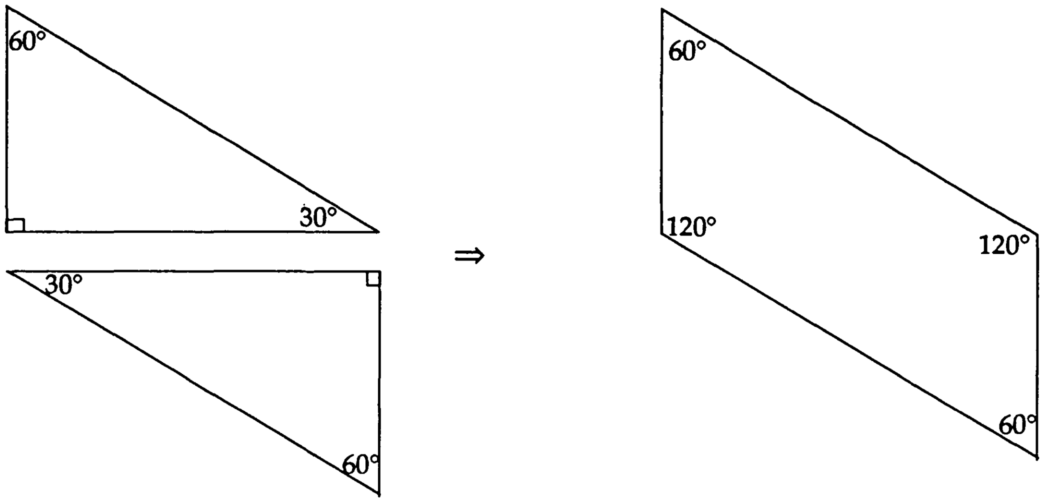
5. The shape is a rectangle.
   Each corner is a right angle (90°).
   So the third angle must be 60°.

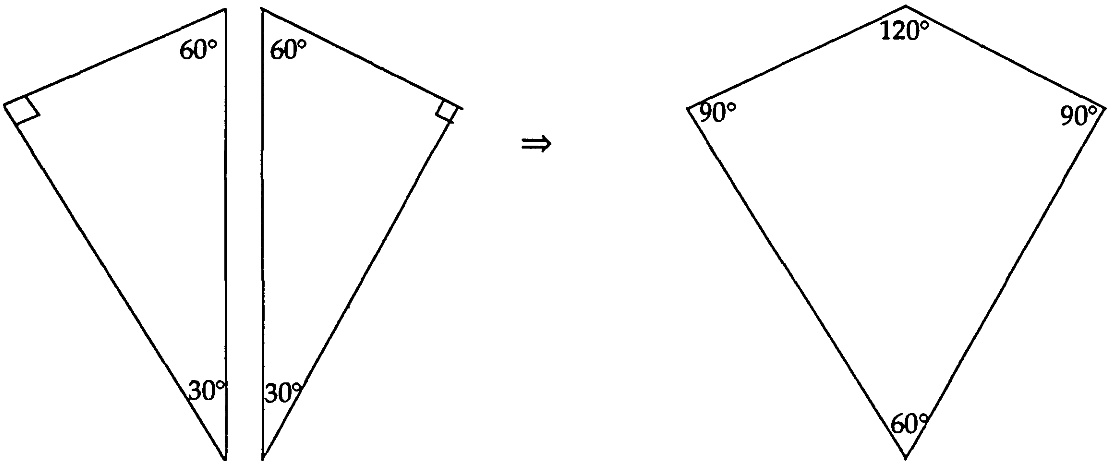


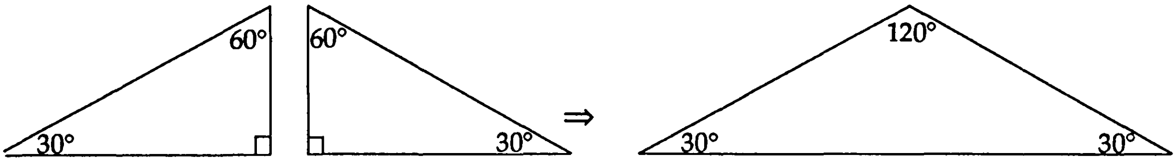
continued/

0039 About Angles (cont)

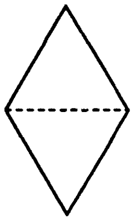
6.



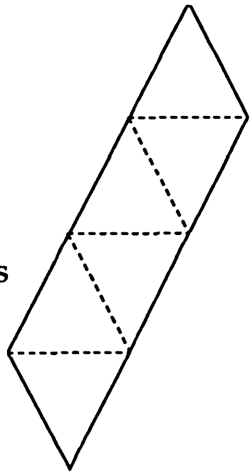




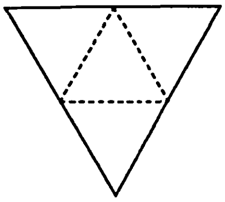
0040 Equilateral Triangle



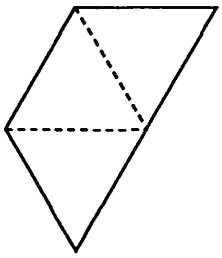
Rhombus



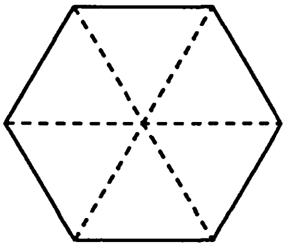
Parallelogram



Equilateral Triangle



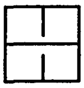
Trapezium



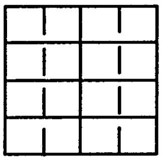
Hexagon

0046 Domino

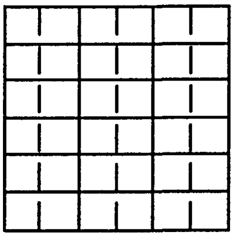
1. Two dominoes make a square.



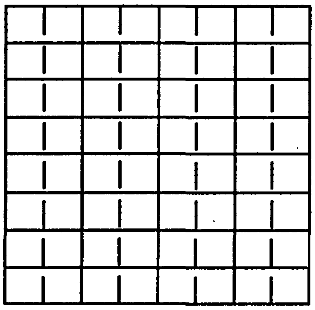
2. These squares can be made from dominoes.



4 x 4



6 x 6



8 x 8

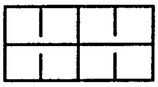
'Odd' squares cannot be made from dominoes: 3 x 3, 5 x 5, 7 x 7 etc.
All these squares have an odd number of small squares.
Dominoes are made from 2 small squares and so when dominoes are put together, there must be an even number of small squares.

3. a) The first 4 dominoes are 1 x 2, 2 x 4, 3 x 6 and 4 x 8.
The 4th domino is 8 squares long.

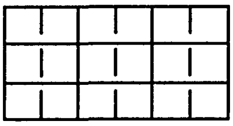
   b) If a domino is 10cm wide, it is 20cm long.

4.

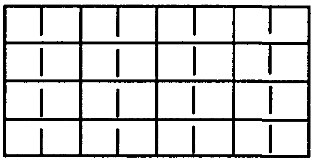
1 x 2
1 domino



2 x 4
4 dominoes



3 x 6
9 dominoes

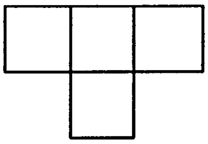


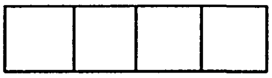
4 x 8
16 dominoes

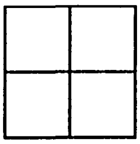
A 5 x 10 domino would need 25 small dominoes.
A 6 x 12 domino would need 36 small dominoes.

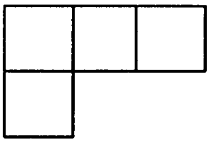
0048 Tetromino

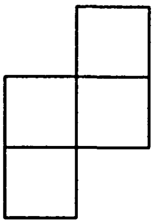
These are the 5 different tetrominoes.

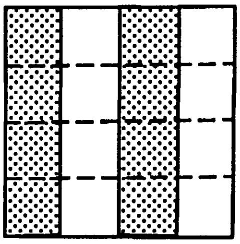


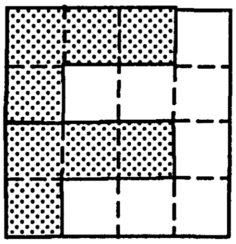


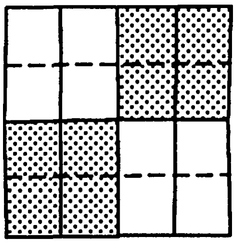




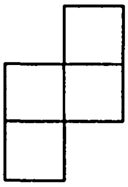




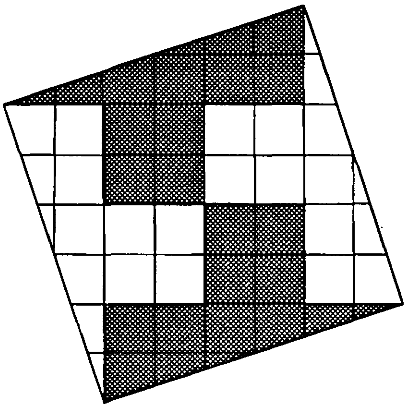


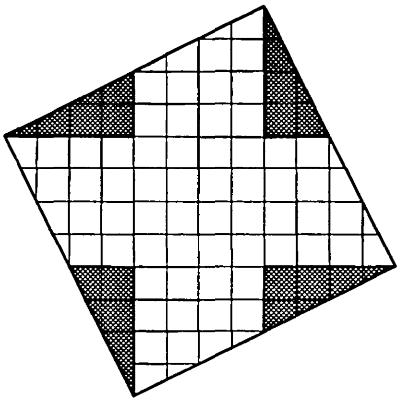


This tetromino will not make a square.



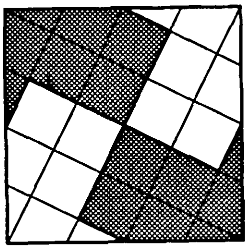
0050 Dissection 1



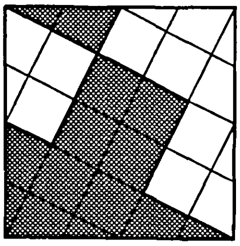


0051 Dissection 2

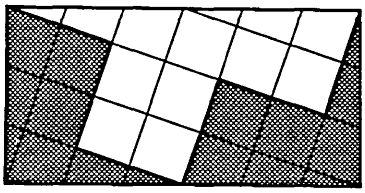
1.



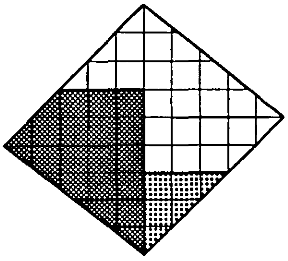
2.

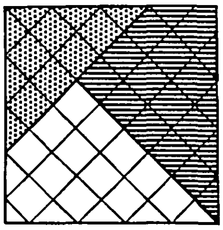


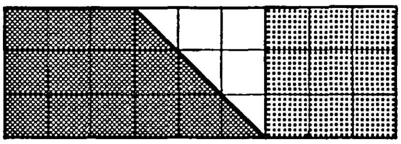
3.



0052 Dissection 3

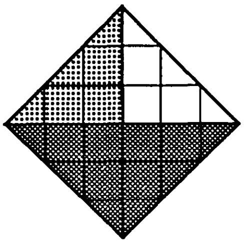




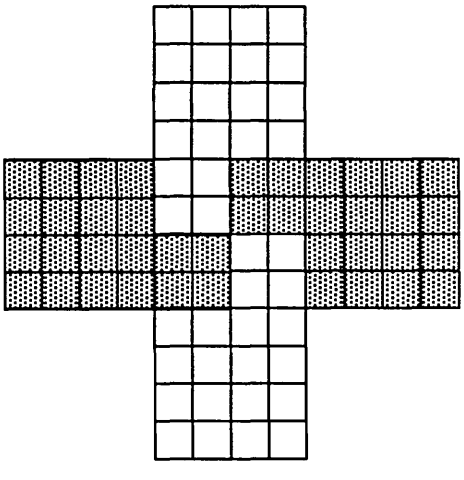


0053 Dissection 4

1.



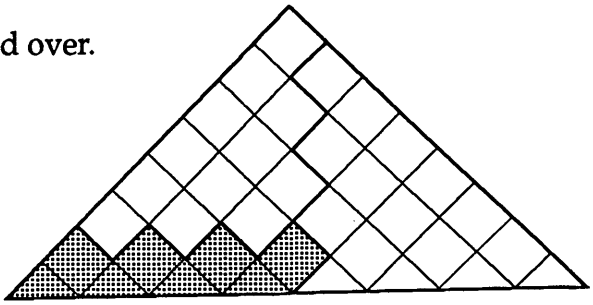
3.



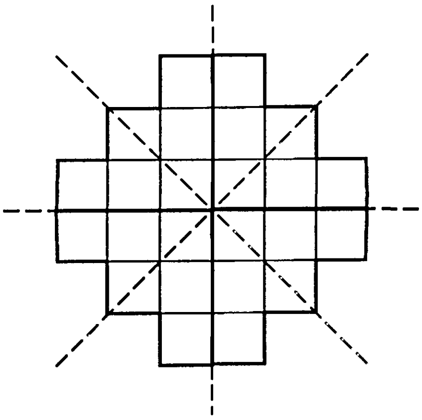
2. Any rectangle can be dissected this way to make a square.

0054 Dissection 5

1. The shaded piece needs to be turned over.



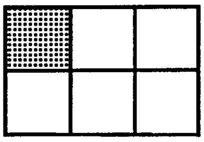
2.



This is an example of a shape made from the 4 outside pieces.
It has 4 lines of symmetry.

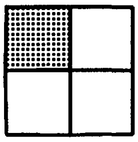
0057 Fractions 3

1.



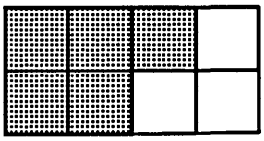
$\frac{1}{6}$

2.



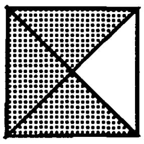
$\frac{1}{4}$

3.



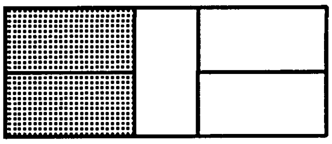
$\frac{5}{8}$

4.



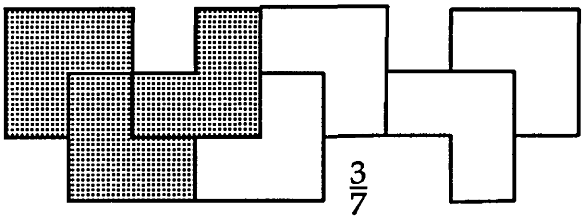
$\frac{3}{4}$

5.



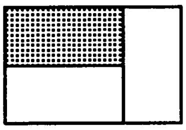
$\frac{2}{5}$

6.



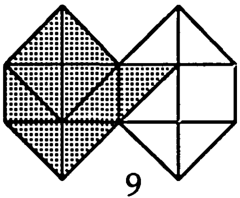
$\frac{3}{7}$

7.



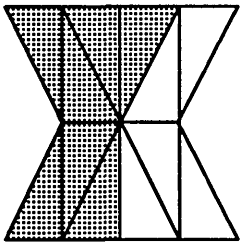
$\frac{1}{3}$

8.



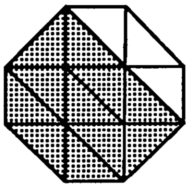
$\frac{9}{16}$

9.



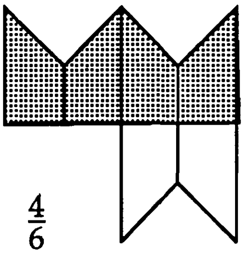
$\frac{7}{12}$

10.



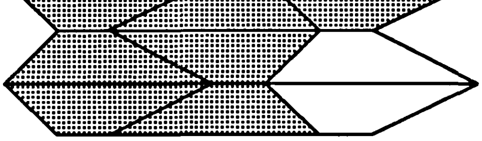
$\frac{11}{14}$

11.



$\frac{4}{6}$

12.



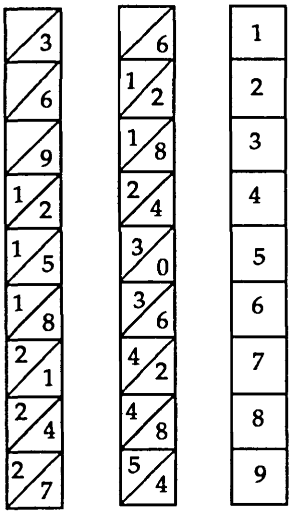
$\frac{7}{9}$

It doesn't matter which parts you have shaded so long as you have shaded the right number each time.

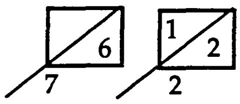
0058 Fractions 4

	Example	1	2	3	4	5	6	7	8	9	10
Number of equal parts	10	3	4	6	8	12	6	10	5	9	8
Fraction shaded with lines	$\frac{3}{10}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{6}$	$\frac{2}{8}$	$\frac{3}{12}$	$\frac{1}{6}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{2}{9}$	$\frac{1}{8}$
Fraction shaded black	$\frac{5}{10}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{4}{8}$	$\frac{5}{12}$	$\frac{2}{6}$	$\frac{7}{10}$	$\frac{1}{5}$	$\frac{2}{9}$	$\frac{4}{8}$
Fraction shaded altogether	$\frac{8}{10}$	$\frac{2}{3}$	$\frac{2}{4}$	$\frac{4}{6}$	$\frac{6}{8}$	$\frac{8}{12}$	$\frac{3}{6}$	$\frac{8}{10}$	$\frac{2}{5}$	$\frac{4}{9}$	$\frac{5}{8}$
Fraction unshaded	$\frac{2}{10}$	$\frac{1}{3}$	$\frac{2}{4}$	$\frac{2}{6}$	$\frac{2}{8}$	$\frac{4}{12}$	$\frac{3}{6}$	$\frac{2}{10}$	$\frac{3}{5}$	$\frac{5}{9}$	$\frac{3}{8}$

0066 Napier's Rods

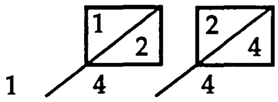


1.



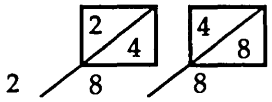
36 x 2 = 72

2.



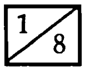
36 x 4 = 144

3.



36 x 8 = 288

4. 520
5. 266
6. 2190
7. 12320
8. 6920
9. 75375
10. 68236
11. 10854
12. 110352

13. Each rod shows a times table.
e.g. the third square down on rod 6 is  because 6 x 3 = 18.

14.
- Long multiplication would be difficult because the index rod only goes up to 9.
- A number which has a digit more than once would be difficult because you would need more than one set of rods, e.g. 63636 x 5.

The illustration on the front of 0066 is an antique drawing of Napier's Rods. The unknown person who drew the picture many years ago made 3 mistakes. Can you find them?

0068 Accurate Measuring

a) PQ = 40mm PQ = 4cm
b) QR = 75mm QR = 7.5cm

1. DE = 1.5cm
2. DE = 15mm
3. AB = 3cm
4. AB = 30mm
5. AD = 6cm
6. AD = 60mm
7. CE = 3.5cm
8. CE = 35mm
9. GE = 2.5cm
10. GE = 25mm
11. JD = 9cm
12. JD = 90mm
13. CA = 4cm = 40mm
14. DH = 7cm = 70mm
15. CF = 5cm = 50mm
16. EB = 4.5cm = 45mm
17. BG = 7cm = 70mm
18. EH =5.5cm=55mm
19. GF = 1cm = 10mm
20. GC = 6cm = 60mm

0069 The Cardioid

The word cardiac means "to do with a heart".
A cardioid is heart-shaped.

0070 Isometric Drawing

Show your isometric drawings to your teacher.

0071 Envelopes

If you enjoyed this activity you could make a similar pattern with a needle and thread.

0072 Angles of a Quadrilateral

For all your quadrilaterals the four angles should make a complete turn.

1. The angles of a quadrilateral add up to 4 right-angles.
2. The angles of a quadrilateral add up to 360 degrees.
3. A = 71°
4. B = 58°
5. C = 108°
6. D = 120°

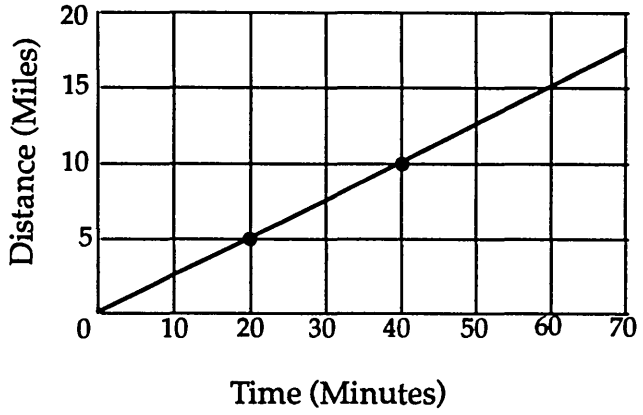
0073 Time/Distance Graph

1.
   a) 5 miles
   b) 25 miles
   c) 7 $^{1}/_{2}$ miles
   d) 15 miles
   e) 22 $^{1}/_{2}$ miles
2.
   a) 20mins
   b) 40mins
   c) 50mins
   d) 5mins
   e) 35mins

continued/

0073 Time/Distance Graph (cont)

After 40 minutes the cyclist has travelled 10 miles, so after 20 minutes she has travelled 5 miles.



3\. a) 15 miles
b) 12 $^{1}/_{2}$ miles
c) 7 $^{1}/_{2}$ miles
d) 6 $^{1}/_{4}$ miles

4\. 15 miles per hour.

5\. a) 1 hour
b) 30mins
c) 10mins
d) 25mins

6\. 22 $^{1}/_{2}$ miles

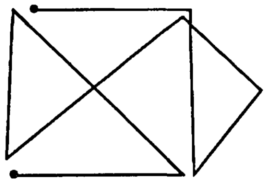
0074 Sum and Product

		Sum	Product
6	5	11	30
7	3	10	21
2	9	11	18
4	8	12	32
5	6	11	30
3	7	10	21
8	8	16	64
10	11	21	110
9	4	13	36
4	11	15	44
6	8	14	48
9	7	16	63
12	12	24	144
8	9	17	72
11	12	23	132

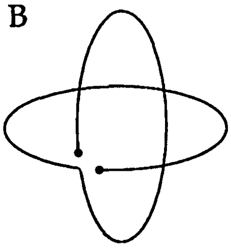
0075 Networks

Networks A, B, C and D are traversable; networks E, F, G and H are not traversable. These diagrams show how the traversable networks might be drawn.

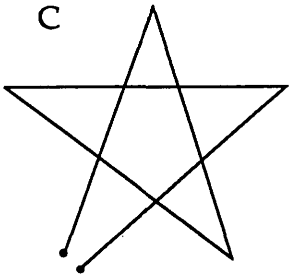
A



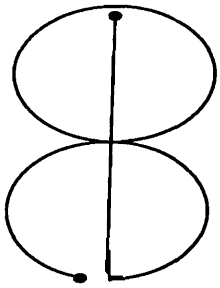
B



C



D



0079 Decimal Dominoes

Once you have played the game, ask your teacher to check that you have matched the decimal dominoes correctly.

0085 Calculator Problems

1. 286 marbles: 237 not green
   199 not black
2. Spent 45p. 5p change.
3. Earned £81. £19 more to earn.
4. Spent 63p. 37p change.
5. a) 31p with 69p change
   b) 26p with 74p change
   c) 87p with 13p change
   d) 28p with 72p change
   e) 68p with 32p change
6. Because the prices on the card are so low.

0090 More Calculator Problems

1. 4740 heart beats per hour.
2. 4296km
3. 37654 dinners are served.
4. £1372
5. 8760 hours in a year (8784 in a leap year)
6. 525600 minutes in a year.
7. 184016 sweets.
8. 170079 words.
9. 2983p or £29.83.
10. All your answers will be different, but look at your answer to number 5 as a guide. In 12 years there are 105048 hours (3 leap years).

0092 Harder Calculator Problems

1. £6.64
2. 3.57km
3. 77p
4. £1.48
5. £38.34
6. £18.60
7. 18.9cm
8. 1133.1gm or 1.1331kg
9. 23.5p or $23^{1}/_{2}$p
10. 6.698415km

0098 The Plaited Cube

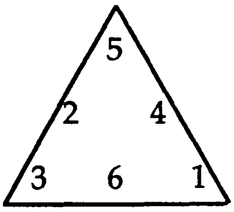
If you plaited your cube correctly, you should have all the shaded squares on the outside.

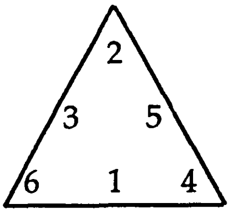
0099 Sum and Product Again

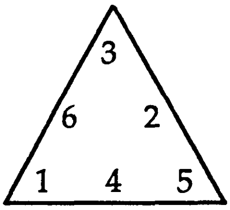
		Sum	Product
3	4	7	12
4	5	9	20
5	6	11	30
6	7	13	42
7	6	13	42
8	5	13	40
9	4	13	36
10	3	13	30
11	2	13	22
12	5	17	60
10	11	21	110
9	8	17	72
8	8	16	64
7	12	19	84
11	11	22	121

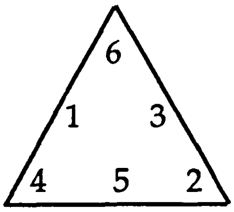
0104 Number Puzzle 1

1. There are 12 'different' answers:



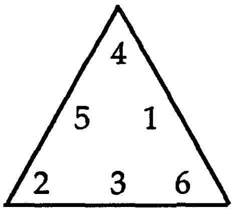


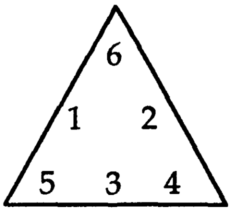


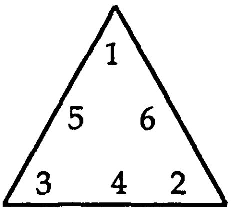


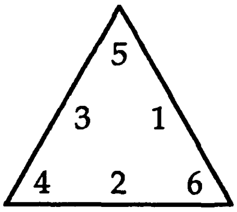
continued/

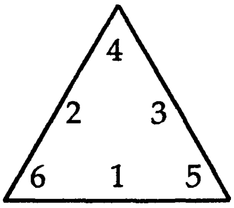
0104 Number Puzzle 1 (cont)

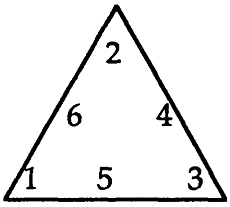


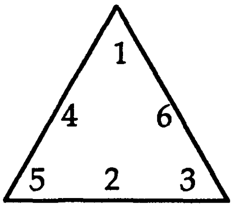


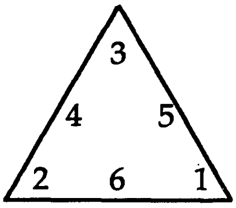




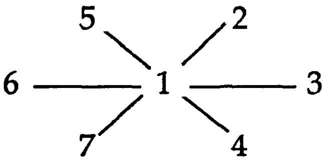


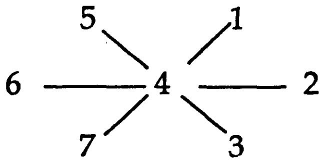


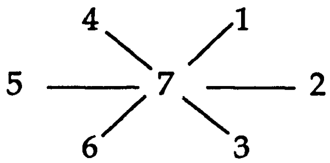




2.

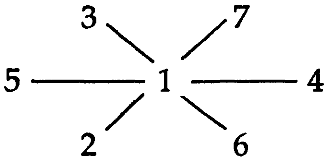




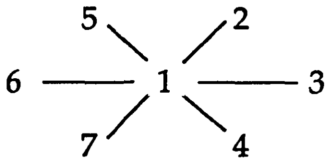


There are many variations on these answers. As long as you have the correct number in the centre and the pairs of the opposite numbers are the same, your answers are correct.

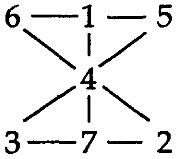
e.g.

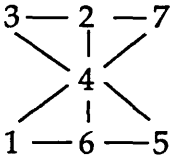


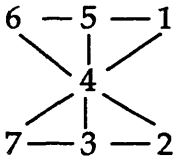
is the same as



3.

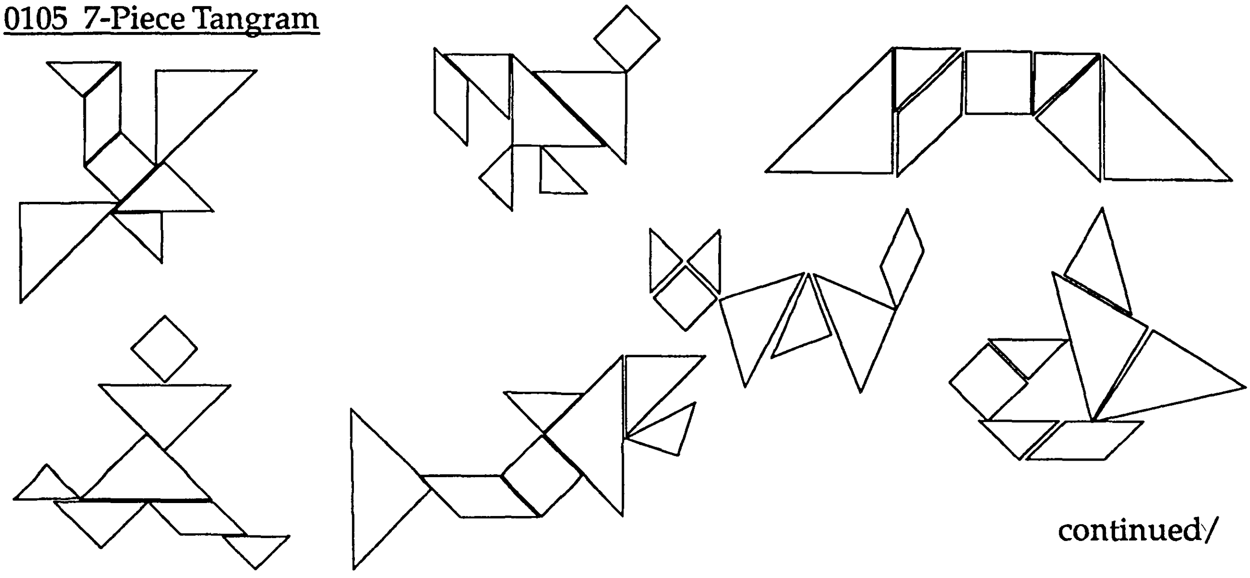






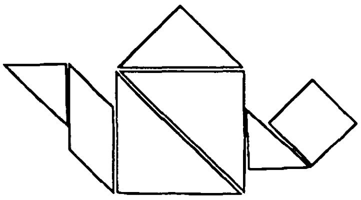
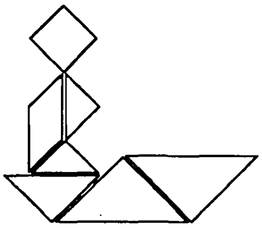
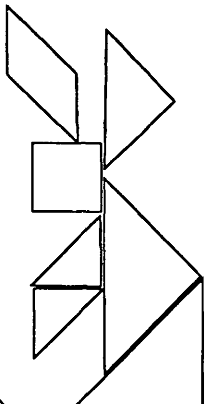
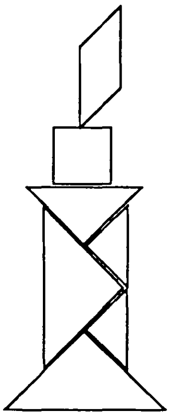
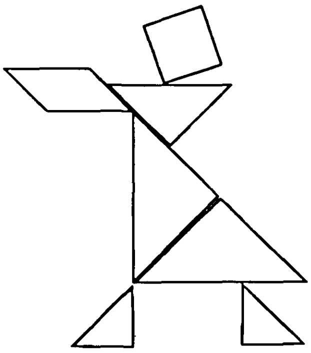
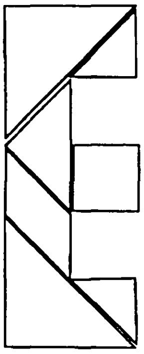
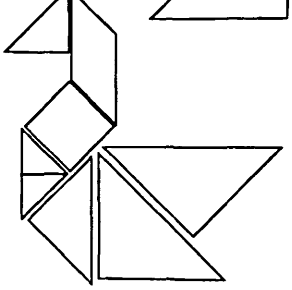
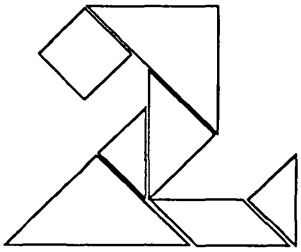
Again, there are variations of these answers.

0105 7-Piece Tangram



continued/

0105 7-Piece Tangram (cont)

0114 Nines

1.

1	2	3	4	5	6	7	8	**9**	10
11	12	13	14	15	16	17	**18**	19	20
21	22	23	24	25	26	**27**	28	29	30
31	32	33	34	35	**36**	37	38	39	40
41	42	43	44	**45**	46	47	48	49	50
51	52	53	**54**	55	56	57	58	59	60
61	62	**63**	64	65	66	67	68	69	70
71	**72**	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	**90**
91	92	93	94	95	96	97	98	**99**	100

2.
9
18
27
36
45
54
63
72
81
90
99

3.
0 + 9 = 9
10 + 8 = 18
20 + 7 = 27
30 + 6 = 36
40 + 5 = 45
50 + 4 = 54
60 + 3 = 63
70 + 2 = 72
80 + 1 = 81
90 + 0 = 90
100 – 1 = 99

4.
10 – 1 = 9
20 – 2 = 18
30 – 3 = 27
40 – 4 = 36
50 – 5 = 45
60 – 6 = 54
70 – 7 = 63
80 – 8 = 72
90 – 9 = 81
100 – 10 = 90
110 – 11 = 99

5.
9 gives 9 = 9
18 gives 1 + 8 = 9
27 gives 2 + 7 = 9
36 gives 3 + 6 = 9
45 gives 4 + 5 = 9
54 gives 5 + 4 = 9
63 gives 6 + 3 = 9
72 gives 7 + 2 = 9
81 gives 8 + 1 = 9
90 gives 9 + 0 = 9
99 gives 9 + 9 = 18 = 1 + 8 = 9

0115 Columns

4.
2 → 2 = 2
12 → 1 + 2 = 3
22 → 2 + 2 = 4
32 → 3 + 2 = 5
42 → 4 + 2 = 6
52 → 5 + 2 = 7
62 → 6 + 2 = 8
72 → 7 + 2 = 9
82 → 8 + 2 = 10
92 → 9 + 2 = 11

5.
4 → 4 = 4
14 → 1 + 4 = 5
24 → 2 + 4 = 6
34 → 3 + 4 = 7
44 → 4 + 4 = 8
54 → 5 + 4 = 9
64 → 6 + 4 = 10
74 → 7 + 4 = 11
84 → 8 + 4 = 12
94 → 9 + 4 = 13

6. Show your own number pattern to your teacher.

0116 Jumpers

Did you manage to score less than 20?

0118 Who's Last?

The player who goes 2nd can always win.
She must always make up the number to 5,

i.e. if the 1st player takes 1 counter, she takes 4
if the 1st player takes 2 counters, she takes 3
if the 1st player takes 3 counters, she takes 2
and if the 1st player takes 4 counters, shet takes 1

The rule is exactly the same with 12 counters and with 26 counters.
The 2nd player can always win.

0119 Area and Perimeter

2.

Rectangle	Height (cm)	Width (cm)	Area (cm^2)	Perimeter (cm)
A	1	7	7	16
B	2	6	12	16
C	3	5	15	16
D	4	4	16	16
E	5	3	15	16
F	6	2	12	16
G	7	1	7	16

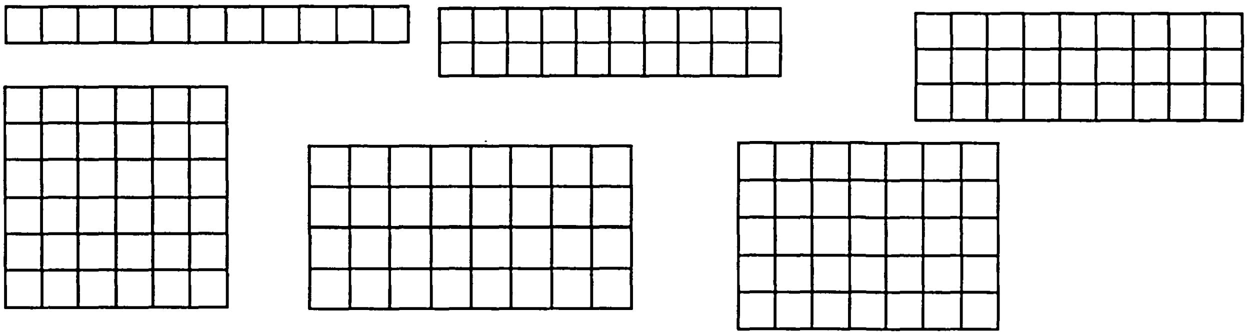
3. All the rectangles have the same perimeter (16cm)

4. Rectangle D has the largest area.

5. Rectangles A and G have the smallest area.

0120 Chocolate Areas

These are the 6 rectangles which have a perimeter of 24cm. They are not drawn full size. They do not contain the same amount of chocolate.



I would choose the square because it has the largest area.

With a rectangle with a perimeter of 20cm.

- I would choose a square again.
- The length of its sides are 5cm.
- Its area is 25cm^2.

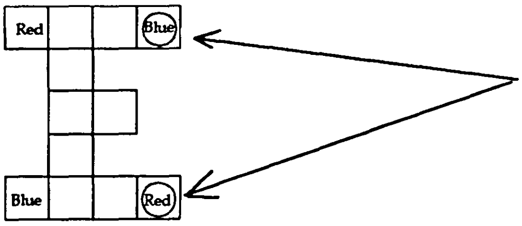
0121 100 Square Patterns

4. 4 → 4 = 4
13 → 1 + 3 = 4
22 → 2 + 2 = 4
31 → 3 + 1 = 4

5. 7 → 7 = 7
16 → 1 + 6 = 7
25 → 2 + 5 = 7
34 → 3 + 4 = 7
43 → 4 + 3 = 7
52 → 5 + 2 = 7
61 → 6 + 1 = 7

6. Show your own number pattern to you teacher.

0123 Peg Puzzle



The secret of this puzzle is to get these counters in these squares first.

0124 Coloured Counter Puzzle

A red counter may have to move back to its starting position to complete this puzzle. Show you teacher how you solved this puzzle.

0125 Noughts and Crosses

Which game did you enjoy the most?

0126 Frogs Puzzle

Hint: You always need to keep the colours separate.

This shows the first three moves.

	R	R	R		B	B	B
1st move	R	R		R	B	B	B
2nd move	R	R	B	R		B	B
3rd move	R	R	B	R	B		B

The least number of moves for 3 red counters to swap positions with 3 blue counters is 15.

0127 Escape

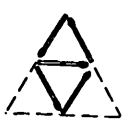
Red can always win, whether red starts or whether blue starts.

0129 Sixteen Counter Puzzle

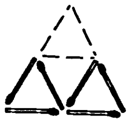
Were you able to improve your score? Write down the least number of moves.

0131 Matchstick Puzzle

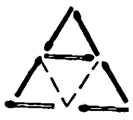
Puzzle 1



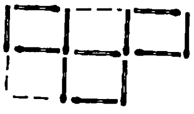
Puzzle 2



Puzzle 3



Puzzle 4



0133 Out of Line

Puzzle 1

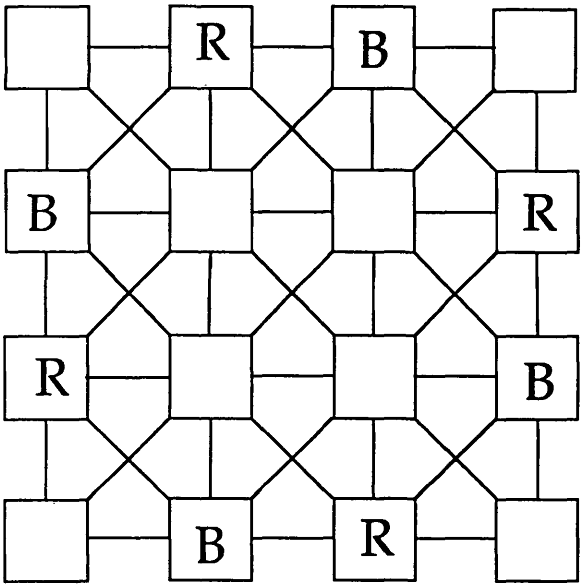
The 4 counters must be either on the squares marked R or on the squares marked B for puzzle 2.

continued/

0133 Out of Line (cont)

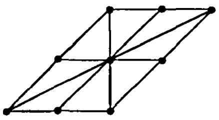
Puzzle 2

You may have the blue and the red counters switched.

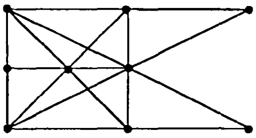


0134 Pegboard Puzzles

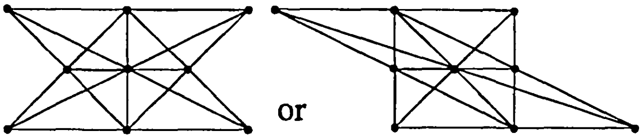
Puzzle 1



Puzzle 2



Puzzle 3



0136 Pegboard Games

Which game did you prefer to play?

0140 Go

Did it matter who started the game?

0142 Volumes 1

1. a) 8cm^3
   b) 24cm^2 (6 x 4cm^2)

2 a) 27cm^3
   b) 54cm^2 (6 x 9cm^2)

3. a) 64cm^3
   b) 96cm^2 (6 x 16cm^2)

continued/

0142 Volumes 1 (cont)

4. a)

Length of each side of cube	Area of each face of cube (cm^2)	Surface area of cube (cm^2)	Volume of cube (cm^3)
1	1	6	1
2	4	24	8
3	9	54	27
4	16	96	64
5	25	150	125
6	36	216	216
7	49	294	343
8	64	384	512

b) The numbers in the 4th column are called cube numbers.

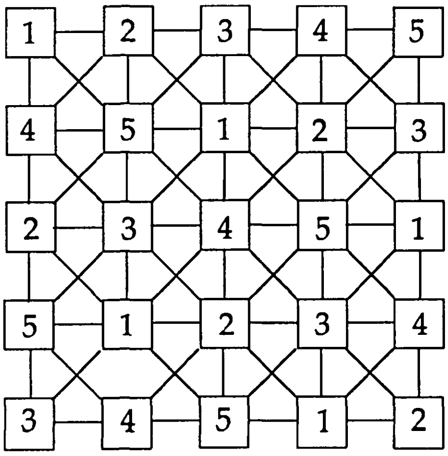
0143 Volumes 2

Three pyramids make a cube.
If the volume of each pyramid is about 42cm^3, the volume of the cube should be about 126cm^3 (42 x 3).

The side of the cube should measure 5cm and so the volume is 5 x 5 x 5 = 125cm^3.
If this is exact do you think the volume of the pyramid was more than 42cm^3 or less than 42cm^3?

0144 All Out of Line

This is the answer for puzzle 3. The numbers stand for colours. For puzzles 1 and 2 you need any one or two of the numbers.



1	2	3	4	5
4	5	1	2	3
2	3	4	5	1
5	1	2	3	4
3	4	5	1	2

0145 Tetraflexagon

Were you able to colour the completed tetraflexagon so that four different faces appear as you flex the model?

0151 More 100 Square Patterns

4. $3 \longrightarrow 3 = 3 \longrightarrow 3$

   $14 \longrightarrow 1 + 4 = 5 \longrightarrow 5$

   $25 \longrightarrow 2 + 5 = 7 \longrightarrow 7$

   $36 \longrightarrow 3 + 6 = 9 \longrightarrow 9$

   $47 \longrightarrow 4 + 7 = 11 \longrightarrow 1 + 2 = 2$

   $58 \longrightarrow 5 + 8 = 18 \longrightarrow 1 + 3 = 4$

   $69 \longrightarrow 6 + 9 = 15 \longrightarrow 1 + 5 = 6$

   $80 \longrightarrow 8 + 0 = 8 \longrightarrow 8$

5. $11 \longrightarrow 1 + 1 = 2 \longrightarrow 2$

   $22 \longrightarrow 2 + 2 = 4 \longrightarrow 4$

   $33 \longrightarrow 3 + 3 = 6 \longrightarrow 6$

   $44 \longrightarrow 4 + 4 = 8 \longrightarrow 8$

   $55 \longrightarrow 5 + 5 = 10 \longrightarrow 1 + 0 = 1$

   $66 \longrightarrow 6 + 6 = 12 \longrightarrow 1 + 2 = 3$

   $77 \longrightarrow 7 + 7 = 14 \longrightarrow 1 + 4 = 5$

   $88 \longrightarrow 8 + 8 = 16 \longrightarrow 1 + 6 = 7$

   $99 \longrightarrow 9 + 9 = 18 \longrightarrow 1 + 8 = 9$

6. Show your own pattern to your teacher.

0153 Decimal Calculations

1. £125.56

2. £2.37

3. $\frac{1}{4} = 0.25$ $\quad \frac{3}{8} = 0.375$ $\quad \frac{2}{5} = 0.4$

   $\frac{7}{12} = 0.583333 = 0.58\dot{3}$ $\quad \frac{2}{3} = 0.\dot{6}$ $\quad \frac{2}{7} = 0.\dot{2}8571\dot{4}$

   The order is : $\frac{1}{4};\ \frac{2}{7};\ \frac{3}{8};\ \frac{2}{5};\ \frac{7}{12};\ \frac{2}{3}$.

4. $\frac{1}{11} = 0.090909\ldots = 0.\dot{0}\dot{9}$ $\quad \frac{2}{11} = 0.181818\ldots = 0.\dot{1}\dot{8}$ $\quad \frac{3}{11} = 0.272727\ldots = 0.\dot{2}\dot{7}$

   $\frac{4}{11} = 0.\dot{3}\dot{6}$ $\quad \frac{5}{11} = 0.\dot{4}\dot{5}$ $\quad \frac{6}{11} = 0.\dot{5}\dot{4}$ $\quad \frac{7}{11} = 0.\dot{6}\dot{3}$ $\quad \frac{8}{11} = 0.\dot{7}\dot{2}$ $\quad \frac{9}{11} = 0.\dot{8}\dot{1}$ $\quad \frac{10}{11} = 0.\dot{9}\dot{0}$

   They are all recurring decimals. The digits of the decimal part are multipes of 9.

5. 45 paces

6. £24

7. £72.72

continued/

0153 Decimal Calculations (cont)

8. 5p (to the nearest penny)

9. $160.70 (to the nearest cent)

10. 11 839ft (to the nearest foot)

11. 67.57mph (to 2 decimal places)

12. 3.31cm (to 2 decimal places)

0154 Square Root Calculator

Guess	Square it	Answer accurate enough?
4	4 x 4 = 16	No, too small.
5	5 x 5 = 25	No, too large.
4.5	4.5 x 4.5 = 20.25	No, too large
.		
.		
.		

If your calculator has a 10-digit display: √20 = 4.472135955

√32 = 5.656854249

If you worked with a computer you could get answers which were accurate to many more decimal places. In theory you can always find a more accurate answer than the one you have, i.e. there is no precise answer to √20 or √32.

Which square roots do have precise answers?

0155 Calculator Trial and Error

If your calculator has 10-digit display, the two numbers which add up to 10 and whose product is 20 are 7.236067977 and 2.763932023.

As in 0154 Square Root Calculator these answers are not exact. The two numbers total 10 but the product is not exactly 20. There is no precise answer but you can get closer and closer. A spreadsheet or graphic calculator will help.

0159 Angles of a Triangle

1. The angles should fit together to make a straight line with any triangle.

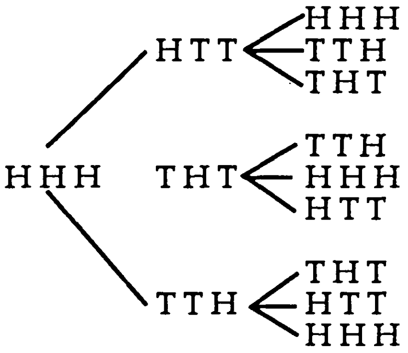
2. a) The 3 angles of any triangle fit together to make a straight line.

   b) The 3 angles of a triangle add together to make 180 degrees and this is the same as 2 right angles.

0161 The Three Coin Problem

At each stage there are 3 possible moves:

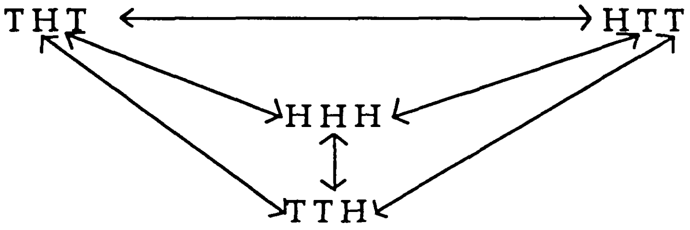
- i) leave the 1st coin, turn 2nd and 3rd;
- ii) leave the 2nd coin, turn 1st and 3rd;
- iii) leave the 3rd coin, turn 1st and 2nd.



You do not need to go on forever.

You only need to go as far as this diagram shows because that is when the combinations begin to repeat.

The diagram below shows how the combinations repeat.



0162 2, 3, 4, 5

These answers only show one way for each number. It is likely that you found different ways. Get someone to check your solutions.

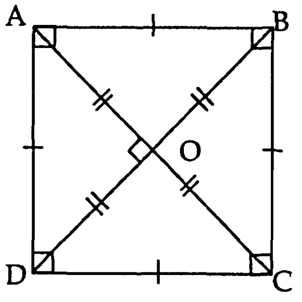
1 = $\frac{5+2}{4+3}$
2 = (4 x 3) – (5 x 2)
3 = 5 + 4 – (2 x 3)
4 = $\frac{4 \times 5}{2+3}$
5 = (4 x 3) – (2 + 5)
6 = 2 + 3 + 5 – 4
7 = (5 x 3) – (4 x 2)
8 = 2 + 4 + 5 – 3
9 = (2 x 5) + 3 – 4
10 = 3 + 4 + 5 – 2
11 = 4 x 5 – 3^2
12 = 2 + 3 + √4 + 5
13 = (3 x 5) + 2 – 4
14 = 2 + 3 + 4 + 5
15 = (4 x 5) – 2 – 3
16 = (3 + 5) x (4 – 2)
17 = (3 x 5) + 4 – 2
18 = 4 + 5 + 3^2
19 = (4 x 5) + 2 – 3
20 = 4 x 5 x (3 – 2)
21 = [(2 + 3) x 5] – 4
22 = (2 x 5) + (3 x 4)
23 = (3 x 5) + (2 x 4)
24 = 2^4 + 3 + 5
25 = 5^2 x (4 – 3)

0164 Patterns with 11 and 13

$\begin{array}{r}1\text{ r }9\\11\overline{)20}\end{array}$	1 + 9 = 10	$\begin{array}{r}1\text{ r }7\\13\overline{)20}\end{array}$	3 x 1 + 7 = 10
$\begin{array}{r}2\text{ r }8\\11\overline{)30}\end{array}$	2 + 8 = 10	$\begin{array}{r}2\text{ r }4\\13\overline{)30}\end{array}$	3 x 2 + 4 = 10
$\begin{array}{r}3\text{ r }7\\11\overline{)40}\end{array}$	3 + 7 = 10	$\begin{array}{r}3\text{ r }1\\13\overline{)40}\end{array}$	3 x 3 + 1 = 10
$\begin{array}{r}4\text{ r }6\\11\overline{)50}\end{array}$	4 + 6 = 10	$\begin{array}{r}3\text{ r }11\\13\overline{)50}\end{array}$	3 x 3 + 11 = 20
$\begin{array}{r}5\text{ r }5\\11\overline{)60}\end{array}$	5 + 5 = 10	$\begin{array}{r}4\text{ r }8\\13\overline{)60}\end{array}$	3 x 4 + 8 = 20
$\begin{array}{r}6\text{ r }4\\11\overline{)70}\end{array}$	6 + 4 = 10	$\begin{array}{r}5\text{ r }5\\13\overline{)70}\end{array}$	3 x 5 + 5 = 20
$\begin{array}{r}7\text{ r }3\\11\overline{)80}\end{array}$	7 + 3 = 10	$\begin{array}{r}6\text{ r }2\\13\overline{)80}\end{array}$	3 x 6 + 2 = 20
$\begin{array}{r}8\text{ r }2\\11\overline{)90}\end{array}$	8 + 2 = 10	$\begin{array}{r}6\text{ r }12\\13\overline{)90}\end{array}$	3 x 6 + 12 = 30
$\begin{array}{r}9\text{ r }1\\11\overline{)100}\end{array}$	9 + 1 = 10	$\begin{array}{r}7\text{ r }9\\13\overline{)100}\end{array}$	3 x 7 + 9 = 30

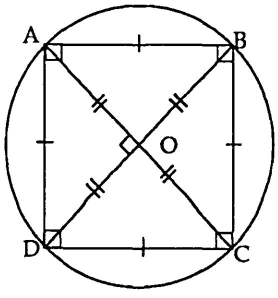
0165 Cyclic Quadrilateral

All squares are cyclic quadritaterals. To justify this you need to think about the definition of a square. A square is a shape with:



- four sides of equal length
  AB = BC = CD = DA.
- four angles of equal size (90°)
  ∠ DAB = ∠ABC = ∠ BCD = ∠ CDA = 90°
- diagonals crossing at right-angles
  ∠ DOA = ∠AOB = ∠ BOC = ∠COD = 90°
- diagonals of equal length and which bisect each other
  AO = OC = BO = OD

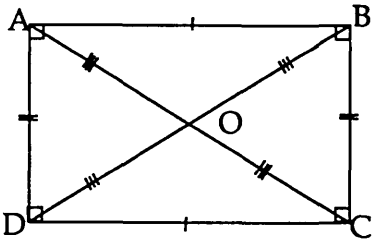
In any square it is possible to draw a circle through the vertices with AO, BO, CO and DO being radii and O as the centre of the circle.



continued/

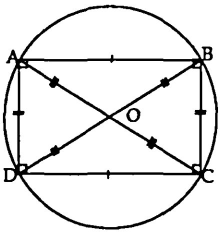
0165 Cyclic Quadrilateral (cont)

All rectangles are cyclic quadrilaterals. To justify this you need to think about the definition of a rectangle. A rectangle is a shape with:



- four sides
- opposite sides are equal
  AB = CD and AD = BC
- four angles of equal size
  ∠ DAB = ∠ABC = ∠ BCD = ∠ CDA = 90°
- diagonals which are of equal length which bisect each other
  AO = OC = BO = OD

In any rectangle it is always possible to draw a circle through the vertices with AO, BO, CO and DO being radii and O as the centre of the circle.



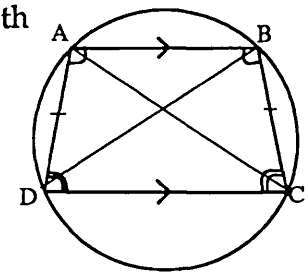
Only special cases of rhombi will be cyclic quadrilaterals. For a rhombus to be a cyclic quadrilateral it must have:

- all angles equal to 90° (normally called a square).

Only special cases of parallelograms will be cyclic quadrilaterals. For a parallelogram to be a cyclic quadrilateral it must have:

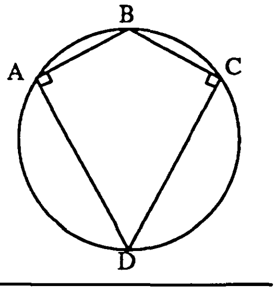
- diagonals of equal length
- four angles which are equal to 90° (normally called a rectangle).

Only special cases of trapezia will be cyclic quadrilaterals. For a trapezium to be a cyclic quadrilateral it must have:



- the 2 non-parallel sides of the trapezium equal in length
  AD = BC
- diagonals of equal length
  AC = BD
- 2 pairs of equal angles
  ∠DAB = ∠ABC and ∠ADC =∠BCD

Only special cases of kites will be cyclic quadrilaterals. For a kite to be a cyclic quadrilateral it must have:



- a pair of angles which are right-angles
  ∠BAD = ∠BCD = 90°

You may like to justify why these special cases are cyclic quadrilaterals.

0166 Area of a Triangle

1. 3 units 2. 2 units 3. 3 square units

4. Many possible answers. Your table should show that

The area of a triangle is always HALF of the base x height.

1. Base = 5; height = 3; area = $\frac{1}{2}(5 \times 3) = 7\frac{1}{2}$ square units.
2. Base = 6; height = 4; area = $\frac{1}{2}(6 \times 4) = 12$ square units.
3. Base = 4; height = 4; area = $\frac{1}{2}(4 \times 4) = 8$ square units.
4. Base = 2; height = 4; area = $\frac{1}{2}(2 \times 4) = 4$ square units.
5. Base = 3; height = 7; area = $\frac{1}{2}(3 \times 7) = 10\frac{1}{2}$ square units.
6. Base = 5; height = 4; area = $\frac{1}{2}(5 \times 4) = 10$ square units.

0167 x for Breakfast

x	x + 4
7	11
19	23
5	9
13	17
32	36
87	91

a) x → x + 2

x	x + 2
10	12
6	8
14	16
4	6
13	15
10	12
1	3
$6\frac{1}{2}$	$8\frac{1}{2}$
$2\frac{1}{2}$	$4\frac{1}{2}$

b) x → x − 6

x	x − 6
10	4
6	0
14	8
4	-2
13	7
18	12
9	3
$6\frac{1}{2}$	$\frac{1}{2}$
$10\frac{1}{2}$	$4\frac{1}{2}$

c) x → 2x

x	2x
10	20
6	12
14	28
4	8
13	26
6	12
$1\frac{1}{2}$	3
$6\frac{1}{2}$	13
$2\frac{1}{4}$	$4\frac{1}{2}$

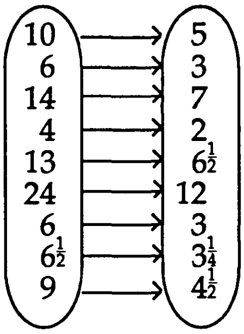
d) x → 3x

x	3x
10	30
6	18
14	42
4	12
13	39
4	12
1	3
$6\frac{1}{2}$	$19\frac{1}{2}$
$1\frac{1}{2}$	$4\frac{1}{2}$

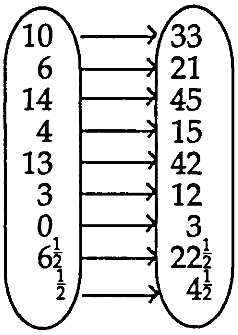
continued/

0167 x for Breakfast (cont)

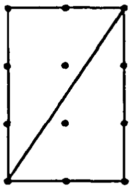
e) $x \longrightarrow x \div 2$



f) $x \longrightarrow 3x + 3$

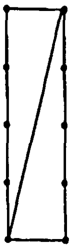


0168 Right-angled Triangles



The area of the rectangle is 6 squares so the area of each triangle is 3 squares.

1.



The area of the rectangle is 4 squares so the area of each triangle is 2 squares.

2. 4 squares
3. 7½ squares
4. 5 squares
5. 6 squares
6. 2 squares
7. 8 squares
8. 10 squares

0169 Half a Rectangle

1. 3 squares
2. 2 squares
3. 10 squares
4. 4 squares
5. 4½ squares
6. 7 squares
7. 3 squares
8. 2½ squares
9. 6 squares
10. 18 squares
11. 22½ squares
12. 12 squares
13. 6 squares
14. 10 squares
15. 8 squares

If you used centimetre squared dotty paper, the unit of area is cm^2.

0170 Hex

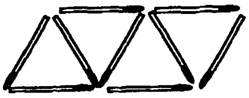
If she plays correctly, the person who goes first will always win. Try it on a smaller board if you are not convinced.

0171 TV Drinks

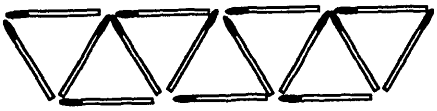
1. Coffee is Davindra's drink.
2. John's three drinks are coffee, cider and lemonade.
3. John drank cider.
4. Bill and John drank lemonade.
5. Ann did not have a drink.
6. Coffee was the most popular drink.
7. Milk was the least popular drink.
8. Bill and Helen had two drinks.
9. John had the most drinks.
10. John and Bill both drank lemonade.

Show your own arrow diagram to your teacher.

0172 A Match for Anyone



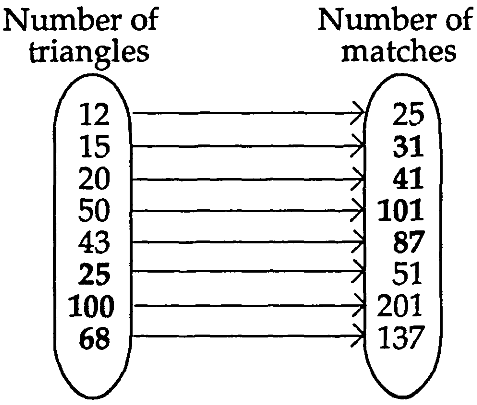
4 triangles and 9 matches



7 triangles and 15 matches

You should have spotted the rule 'double the number of triangles and add one'.

- Show your own patterns to someone else and check that they follow the rule 'double the number of triangles and add one'.
- Check that your mapping diagram also follows the rule 'double the number of triangles and add one'.



Number of triangles	Number of matches
12	25
15	**31**
20	**41**
50	**101**
43	**87**
25	51
100	201
68	137

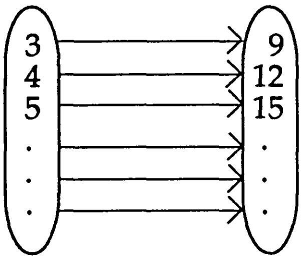
0173 Mapping Machines

1. When 4 goes in, 8 comes out.

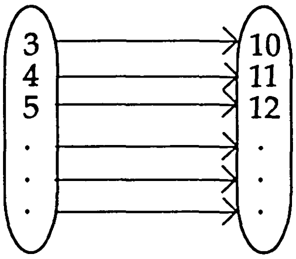
2. If 20 comes out, 10 went in.

Show your diagram to some else to check that your 'out' numbers are double your 'in' numbers.

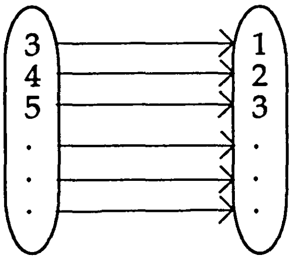
a) treble



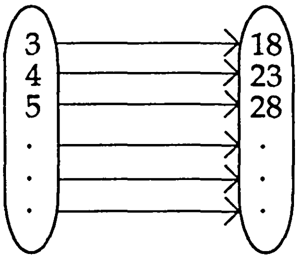
b) add seven



c) subtract two

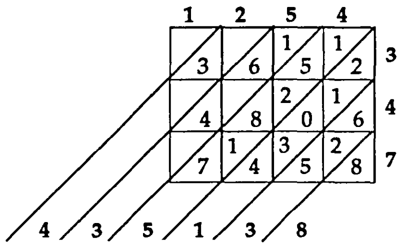


d) multiply by five and then add three

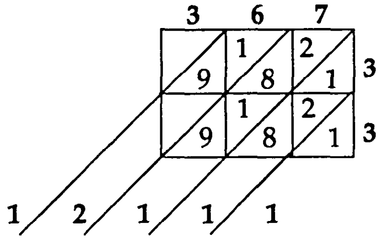


Show your diagrams to someone else to check that your 'out' numbers follow the rules of the mapping machines for your 'in' numbers.

0174 Gelosia

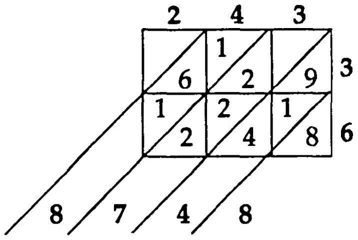


1.



367 x 33 = 12111

2.

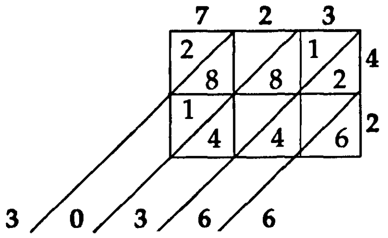


243 x 36 = 8748

continued/

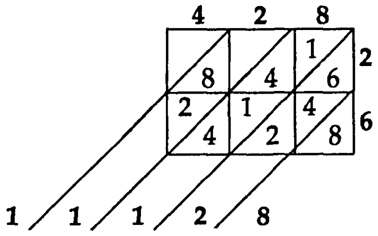
0174 Gelosia (cont)

3.



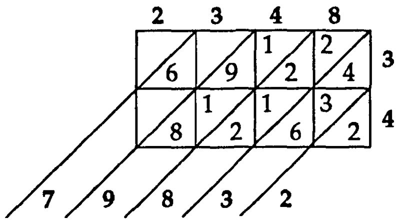
723 x 42 = 30366

4.



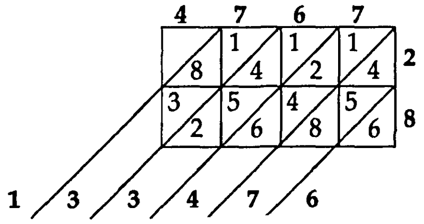
428 x 26 = 11128

5.



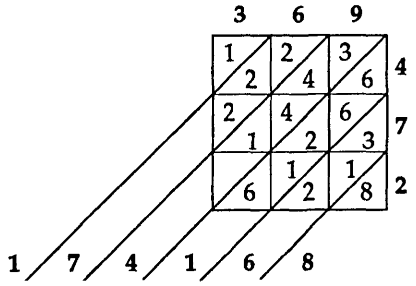
2348 x 34 = 79832

6.



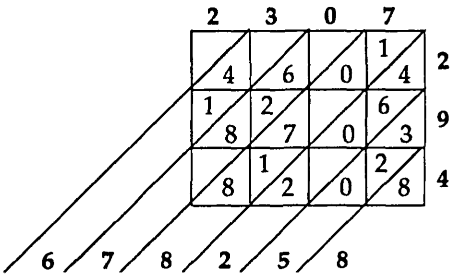
4767 x 28 = 133476

7.



369 x 472 = 174168

8.



2307 x 294 = 678258

0177 Shearing a Triangle

The area of a triangle is equal to $\frac{1}{2}$ x base x height.

So if two triangles have the same base and the same height, their areas must also be the same.

0178 Rectangles Worksheet

	length cm	width cm	area cm^2
A	3	2	6
1	2	1	2
2	3	1	3
3	4	2	8
4	3	3	9
5	5	1	5

- Area of a rectangle = length x width
- You can check each row of your table to make sure that this formula works.

0179 Four 4's

Useful hints

- $4 \div \cdot 4 = 10$
- $4! = (4 \times 3 \times 2 \times 1) = 24$
- $4 \div \cdot\dot{4} \approx 9$

Here is one answer for each number from 1 to 20. Yours are likely to be different.

$(4 - 4) + (4 \div 4) = 1$

$(\cdot 4 \times \sqrt{4} \times \sqrt{4}) + \cdot 4 = 2$

$\sqrt{4} + \sqrt{4} - (4 \div 4) = 3$

$\sqrt{(4 \times 4 \times 4 \div 4)} = 4$

$\sqrt{(4 \times 4)} + \frac{4}{4} = 5$

$(4 \times 4 \div \sqrt{4}) - \sqrt{4} = 6$

$4 + 4 - \frac{4}{4} = 7$

$(4 \times 4) - (4 + 4) = 8$

$4 + 4 + \frac{4}{4} = 9$

$(4 \times 4) - (4 + \sqrt{4}) = 10$

$\frac{4}{\cdot 4} + \frac{4}{4} = 11$

$(4 \times 4) - (\sqrt{4} + \sqrt{4}) = 12$

$(4! \div \sqrt{4}) + \frac{4}{4} = 13$

$4 + 4 + 4 + \sqrt{4} = 14$

$(4 \times 4) - \frac{4}{4} = 15$

$\frac{4 \times 4 \times 4}{4} = 16$

$(4 \times 4) + \frac{4}{4} = 17$

$(4 \times 4) + (4 - \sqrt{4}) = 18$

$4! - 4 - \frac{4}{4} = 19$

$(4 \times 4) + (\sqrt{4} + \sqrt{4}) = 20$

0181 Alf, Mike or Leena

Mike was right because the rule 'add three' **always** works:

$3 \xrightarrow{+3} 6$
$7 \xrightarrow{+3} 10$
$1 \xrightarrow{+3} 4$
$4 \xrightarrow{+3} 7$
$5 \xrightarrow{+3} 8$
$2\frac{1}{2} \xrightarrow{+3} 5\frac{1}{2}$
$26 \xrightarrow{+3} 29$
$0 \xrightarrow{+3} 3$

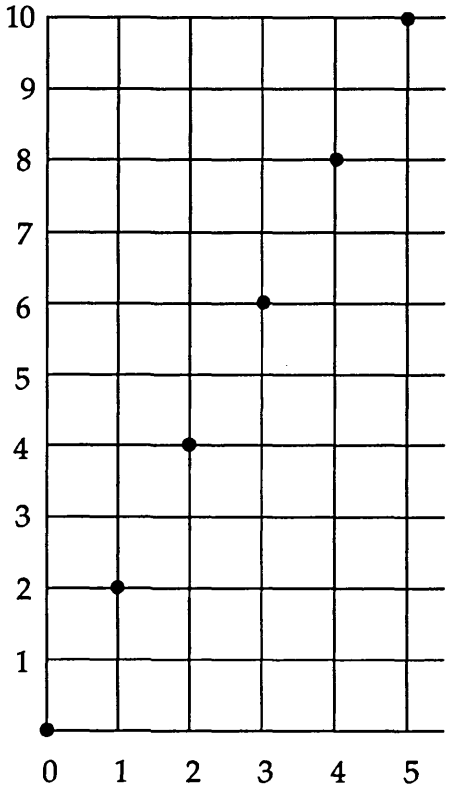
1. add ten
2. multiply by six
3. square (or multiply by itself)
4. subtract three
5. double and then add three
6. divide by five and find the remainder

0182 Mappings to Graphs

'Add two'

$0 \longrightarrow 2$ (0, 2)
$1 \longrightarrow 3$ (1, 3)
$2 \longrightarrow 4$ (2, 4)
$3 \longrightarrow 5$ (3, 5)
$4 \longrightarrow 6$ (4, 6)
$5 \longrightarrow 7$ (5, 7)

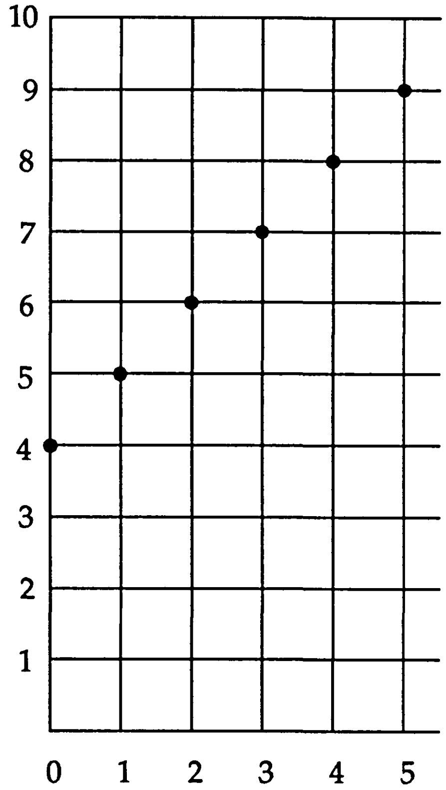
1. 'Double'



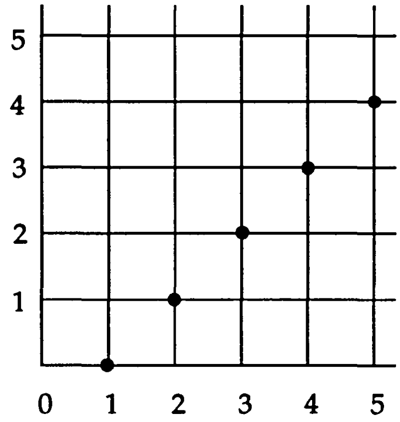
continued/

0182 Mappings to Graphs (cont)

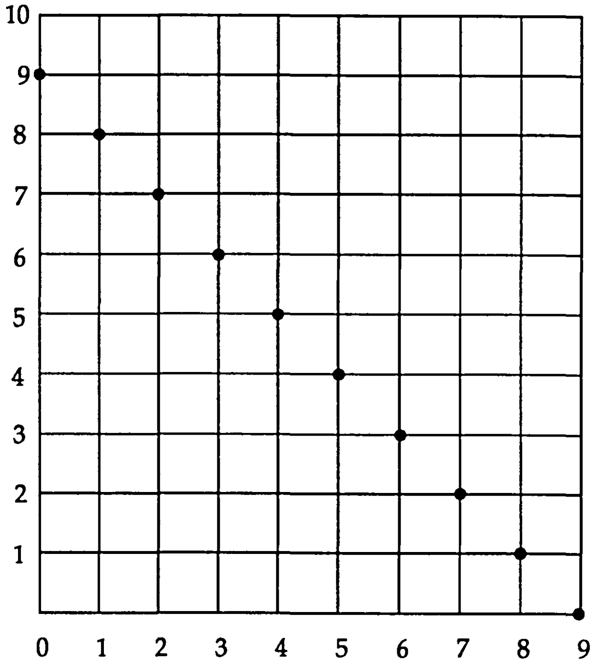
2. 'Add four'



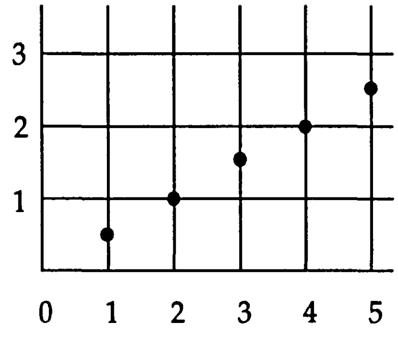
3. 'Subtract 1'



4. 'Subtract from 9'



5. 'Divide by 2'



0183 Graphs to Mappings

(2, 0)	2	→	0
(3, 1)	3	→	1
(4, 2)	4	→	2
(5, 3)	5	→	3
(6, 4)	6	→	4
(7, 5)	7	→	5
(8, 6)	8	→	6

1.

(0, 1)	0	→	1
(1, 2)	1	→	2
(2, 3)	2	→	3
(3, 4)	3	→	4
(4, 5)	4	→	5
(5, 6)	5	→	6
(6, 7)	6	→	7
(7, 8)	7	→	8
(8, 9)	8	→	9
(9, 10)	9	→	10

The rule is
'add one'

2.

(1, 2)	1	→	2
(2, 4)	2	→	4
(3, 6)	3	→	6
(4, 8)	4	→	8
(5, 10)	5	→	10

The rule is
'double'

3.

(1, 1)	1	→	1
(2, 2)	2	→	2
(3, 3)	3	→	3
(4, 4)	4	→	4
(5, 5)	5	→	5
(6, 6)	6	→	6
(7, 7)	7	→	7
(8, 8)	8	→	8
(9, 9)	9	→	9
(10, 10)	10	→	10

The rule is
'leave unchanged'

4

(2, 1)	2	→	1
(3, 3)	3	→	3
(4, 5)	4	→	5
(5, 7)	5	→	7
(6, 9)	6	→	9

The rule is
'double and subtract three'

5.

(0, 4)	0	→	4
(2, 5)	2	→	5
(4, 6)	4	→	6
(6, 7)	6	→	7
(8, 8)	8	→	8
(10, 9)	10	→	9

The rule is
'divide by two and add four'

6.

(0, 7)	0	→	7
(1, 6)	1	→	6
(2, 5)	2	→	5
(3, 4)	3	→	4
(4, 3)	4	→	3
(5, 2)	5	→	2
(6, 1)	6	→	1
(7, 0)	7	→	0

The rule is
'subtract from seven'

- Did all your graphs make straight lines?
- Which rules made 'steeper' lines?
- If there were some graphs that were too difficult to find the rule, show them to your teacher.

0184 Number Puzzle

1. The rule is 'add the number at the side to the number at the top'.

+	7	2	5	8	4
2	9	4	7	10	6
3	10	5	8	11	7
5	12	7	10	13	9
9	16	11	14	17	13
6	13	8	11	14	10

2. The rule is 'multiply the number at the top by the number at the side'.

x	9	0	7	6	2
3	27	0	21	18	6
8	72	0	56	48	16
8	72	0	56	48	16
3	27	0	21	18	6
4	36	0	28	24	8

3. The rule is 'subtract the number at the top from the number at the side.

–	5	3	5	6	4
11	6	8	6	5	7
12	7	9	7	6	8
6	1	3	1	0	2
7	2	4	2	1	3
16	11	13	11	10	12

0185 Which is larger?

- Jamaica is larger.
- Which other two countries did you choose?

0187 x for Tea

Add five

$3 \longrightarrow 8$

$7 \longrightarrow 12$

$1 \longrightarrow 6$

$11 \longrightarrow 16$

$2\frac{1}{2} \longrightarrow 7\frac{1}{2}$

$14\frac{1}{4} \longrightarrow 19\frac{1}{4}$

$x \longrightarrow x + 5$

1. $x \longrightarrow x + 7$
2. $x \longrightarrow 4x$
3. $x \longrightarrow \frac{x}{9}$
4. $x \longrightarrow 6 - x$
5. $x \longrightarrow 3x - 4$

continued/

0187 x for Tea (cont)

6. Subtract seven
$x \longrightarrow x - 7$

7. Multiply by five
$x \longrightarrow 5x$

8. Double and subtract one
$x \longrightarrow 2x - 1$

9. Divide by three
$x \longrightarrow \frac{x}{3}$

10. Subtract from thirteen
$x \longrightarrow 13 - x$

11. Square (multiply by itself)
$x \longrightarrow x^2$

0188 Checking Pythagoras

1. a) 81cm^2
b) 144cm^2
c) 225cm^2
d) Yes

2. The hypotenuse should measure 13cm.
a) 169cm^2
b) The squares on the other two sides are 25cm^2 and 144cm^2.
Added together they make 169cm^2.
c) Yes

3. The hypotenuse should measure 10cm.
a) 100cm^2
b) The squares on the other two sides are 36cm^2 and 64cm^2.
Added together they make 100cm^2.
c) Yes

4. a) 9cm^2 and 16cm^2
b) 25cm^2 (9cm^2 +16cm^2 = 25cm^2)
c) 5cm (25cm^2 = 5cm x 5 cm)
d) The hypotenuse should measure 5cm if you have drawn it accurately.

5. a) The hypotenuse is 20cm because: 12 x 12 = 144
16 x 16 = 256
144 + 256 = 400 = 20 x 20
b) The hypotenuse should measure 20cm if you have drawn it accurately.

6. a) 676cm^2 (26cm x 26cm)
b) 576cm^2 (24cm x 24cm)
c) 100cm^2 (676cm^2 – 576cm^2)
d) 10cm (100cm^2 = 10cm x 10cm)

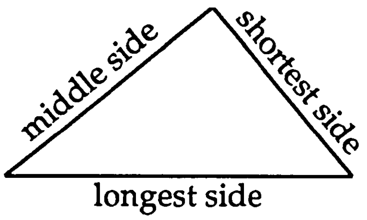
0189 Looking for Right Angles

1. Triangle a is a right-angled triangle.
Triangle b is a right-angled triangle.
Triangle c is not a right-angled triangle.
Triangle d is a right-angled triangle.
Triangle e is a right-angled triangle.
Triangle f is not a right-angled triangle.
Triangle g is a right-angled triangle.
Triangle h is not a right-angled triangle.

continued/

0189 Looking for Right Angles (cont)

2.



Using Pythagoras' Theorem to check whether the triangle is a right-angled triangle, the square of the hypotenuse (longest side) **must** be equal to the sum of the squares on the other two sides.

	Square on longest side	Sum of squares on other two sides	Right-angled?
Triangle **a**	25 x 25 = 625	(20 x 20) + (15 x 15) = 400 + 225 = 625	yes 625 = 625
Triangle **b**	15 x 15 = 225	(12 x 12) + (9 x 9) = 144 + 81 = 225	yes 225 = 225
Triangle **c**	12 x 12 = 144	(9 x 9) + (7 x 7) = 81 + 49 = 130	no 144 ≠ 130
Triangle **d**	10 x 10 = 100	(8 x 8) + (6 x 6) = 64 + 36 = 100	yes 100 = 100
Triangle **e**	13 x 13 = 169	(12 x 12) + (5 x 5) = 144 + 25 = 169	yes 169 = 169
Triangle **f**	7 x 7 = 49	(5 x 5) + (3 x 3) = 25 + 9 = 34	no 49 ≠ 34
Triangle **g**	5 x 5 = 25	(4 x 4) + (3 x 3) = 16 + 9 = 25	yes 25 = 25
Triangle **h**	3 x 3 = 9	(2 x 2) + (2 x 2) = 4 + 4 = 8	no 9 ≠ 8

- The square on the longest side is equal to the sum of the squares on the other two sides for triangles a, b, d, e and g. This checks that these triangles are right-angled and the others are not.

0190 Using Pythagoras

1.
   - The sizes of the squares which could be drawn on the two shorter sides are 36cm^2 and 64cm^2.
   - Added together they make 100cm^2.
   - So the square on the hypotenuse must be 100cm^2.
   - The hypotenuse must be 10cm.

2. The hypotenuse should measure 10cm if your triangle is drawn accurately.

3. a)
   - $(5\text{cm})^2 + (12\text{cm})^2 =$
   - $25\text{cm}^2 + 144\text{cm}^2 = 169\text{cm}^2$
   - So the square on the hypotenuse must be 169cm^2.
   - The hypotenuse is the square root of 169 ($\sqrt{169}$), which is 13cm.

continued/

0190 Using Pythagoras (cont)

3. b)
- $(9\text{cm})^2 + (12\text{cm})^2 =$
- $81\text{cm}^2 + 144\text{cm}^2 = 225\text{cm}^2$
- So the square on the hypotenuse must be 225cm^2.
- The hypotenuse must be 15cm.

c)
- $(30\text{cm})^2 + (40\text{cm})^2 =$
- $900\text{cm}^2 + 1600\text{cm}^2 = 2500\text{cm}^2$
- So the square on the hypotenuse must be 2500cm^2.
- The hypotenuse must be 50cm.

Telegraph pole
- $(3\text{m})^2 + (4\text{m})^2 =$
- $9\text{m}^2 + 16\text{m}^2 = 25\text{m}^2$
- So the square on the hypotenuse must be 25m^2.
- The hypotenuse must be 5m.

Tree
- $(10\text{m})^2 + (24\text{m})^2 =$
- $100\text{m}^2 + 576\text{cm}^2 = 676\text{m}^2$
- So the square on the hypotenuse must be 676m^2.
- The hypotenuse must be 26m.

0191 Pythagoras Problems

1. Pythagoras' Theorem can be used with right-angled triangles.

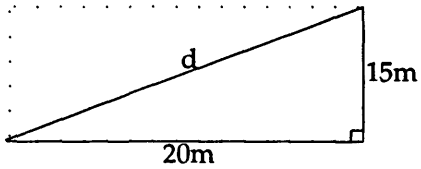
2. $12^2 + 16^2 =$
$144 + 256 = 400$
The hypotenuse is 20 because $\sqrt{400} = 20$.

3. $x^2 = 12^2 + 9^2$
$x^2 = 144 + 81$
$x^2 = 225$
$x = 15$

$y^2 = 12^2 + 5^2$
$y^2 = 144 + 25$
$y^2 = 169$
$y = 13$

$50^2 = 40^2 + z^2$
$2500 = 1600 + z^2$
$2500 - 1600 = z^2$
$900 = z^2$
$30 = z$

4. Let the distance from one corner of the hall to the other corner be d.
So d is the hypotenuse of a right-angled triangle.



$d^2 = 225+400$
$d^2 = 625$
$d = 25\text{m}$

The distance of walking around the edge of the hall is 20m + 15m = 35m.
So you save 10m by walking along the diagonal (hypotenuse) of the hall.

5. The following are perfect combinations, i.e. are right-angled triangles.

Combination a	6, 8, 10.	because	$6^2 + 8^2 = 10^2$ $36 + 64 = 100$
Combination b	10, 24, 26	because	$10^2 + 24^2 = 26^2$ $100 + 576 = 676$
Combination d	18, 24, 30	because	$18^2 + 24^2 = 30^2$ $324 + 576 = 900$
Combination e	15, 36, 39	because	$15^2 + 36^2 = 39^2$ $225 + 1296 = 1521$

continued/

0191 Pythagoras Problems (cont)

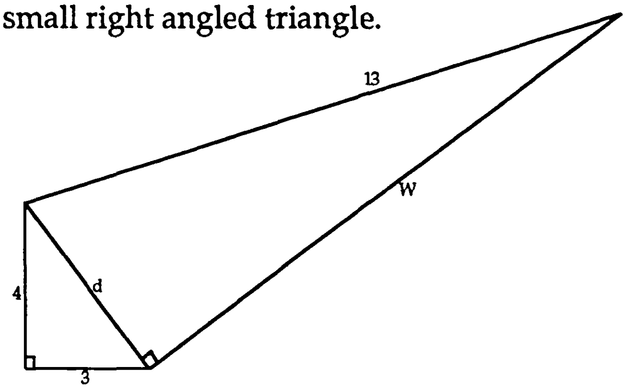
Combination f	7, 24, 25	because	$7^2 + 24^2 = 25^2$ $49 + 576 = 625$
Combination g	15, 20, 25	because	$15^2 + 20^2 = 25^2$ $225 + 400 = 625$
Combination i	21, 28, 35	because	$21^2 + 28^2 = 35^2$ $441 + 784 = 1225$

6. - Let d be the hypotenuse of the small right angled triangle.

$d^2 = 4^2 + 3^2$
$d^2 = 16 + 9$
$d = 5$

- In the larger triangle.

$d^2 + w^2 = 13^2$
$25 + w^2 = 169$
$w^2 = 169 - 25$
$w^2 = 144$
$w = 12$ because $\sqrt{144} = 12$.



0211 Perpendicular Bisectors

8. The perpendicular bisectors of the 3 sides of a triangle should meet at a point every time.

0212 Bisecting an Angle

8. The bisectors of the 3 angles of each triangle should meet at a point each time.

0213 The Circumcircle

- The centre of the circumcircle will be inside an acute-angled triangle.
- The centre of the circumcircle will be outside an obtuse-angled triangle.
- The centre of the circumcircle will be on the hypotenuse of a right-angled triangle. Therefore the angle at any point of the circumference standing on the diameter will always be 90°.

0214 Using a Ruler

1. The biro is 13cm long.
2. The nail is 5cm long.
3. The match is 4cm long.
4. The top of the card is 21cm long.

continued/

0214 Using a Ruler (cont)

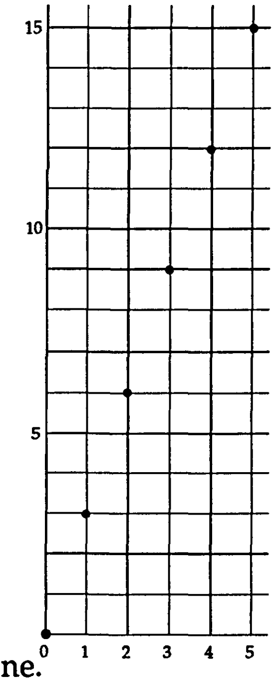
5. The side of the card is 29½cm long.

6. Show your measurements to your teacher.

0215 Drawing the Line

'multiply by three'

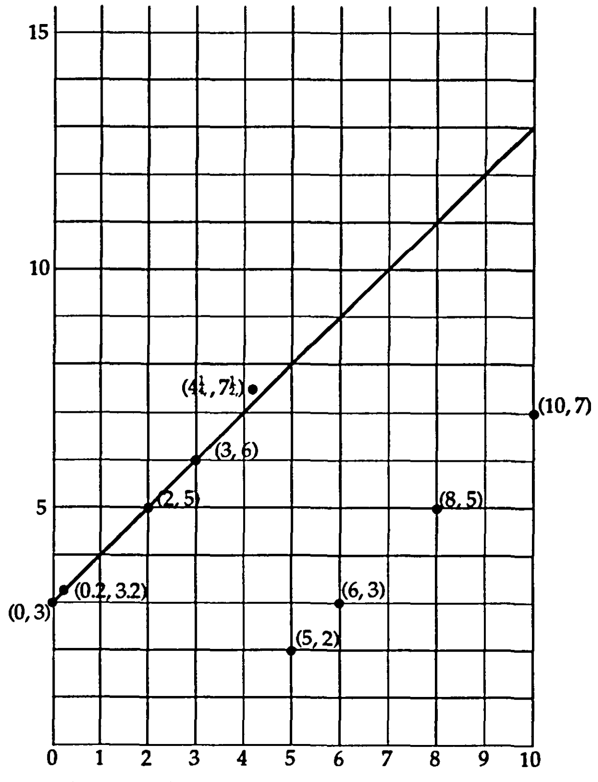
0	⟶	0	(0, 0)
1	⟶	3	(1, 3)
2	⟶	6	(2, 6)
3	⟶	9	(3, 9)
4	⟶	12	(4, 12)
5	⟶	15	(4, 15)



(½, 1½) (1¼, 3¾) (4½, 13½)

(2½, 7½) (¼, ¾) (4¼, 12¾)

- The points of these co-ordinates lie on the same straight line.
- Your own co-ordinates should all come from the rule 'multiply by three'.



- These points lie on the graph: (3, 6) (2, 5) (0, 3) (0.2, 3.2)
- The rule 'add three' works for these points.

0220 Triangle Numbers 1

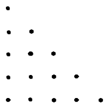
1\. a) 1, 3, 6, 10

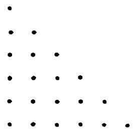
b) The triangle numbers are made like this:

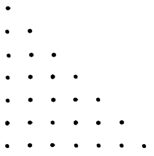
$$1 \xrightarrow{+2} 3 \xrightarrow{+3} 6 \xrightarrow{+4} 10$$

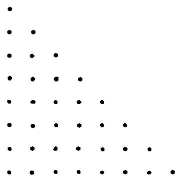
c) 15, 21, 28

2\. These are the next 4 patterns:

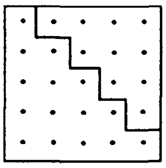




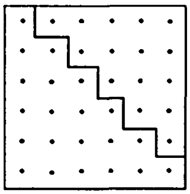




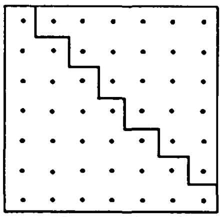
3\. These are the next 3 patterns:



4th and 5th



5th and 6th



6th and 7th

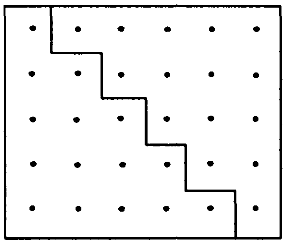
4\. a) Sum of 1st and 2nd triangle numbers = $4 = 2 \times 2 = 2^2$
b) Sum of 2nd and 3rd triangle numbers = $9 = 3 \times 3 = 3^2$
c) Sum of 3rd and 4th triangle numbers = $16 = 4 \times 4 = 4^2$
d) Sum of 9th and 10th triangle numbers = $100 = 10 \times 10 = 10^2$

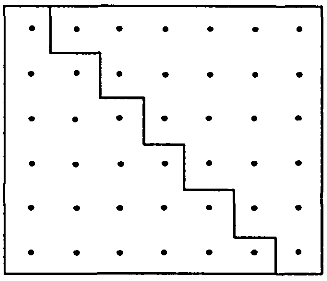
5\. If you add any triangle number to the next one you make a square number.
1st triangle number + 2nd triangle number = 2nd square number
2nd triangle number + 3rd triangle number = 3rd square number
3rd triangle number + 4th triangle number = 4th square number

---

0221 Triangle Numbers 2

1\. The next 2 patterns are:



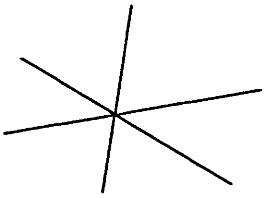


2\. a) 1st triangle number is $\frac{1}{2}(1 \times 2) = 1$
b) 2nd triangle number is $\frac{1}{2}(2 \times 3) = 3$
c) 3rd triangle number is $\frac{1}{2}(3 \times 4) = 6$
d) 4th triangle number is $\frac{1}{2}(4 \times 5) = 10$
e) 5th triangle number is $\frac{1}{2}(5 \times 6) = 15$
f) 10th triangle number is $\frac{1}{2}(10 \times 11) = 55$

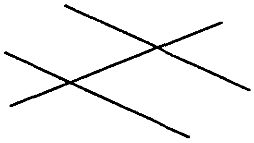
The answers certainly should agree with those you obtained for Triangle Numbers 1.

0222 Triangle Numbers 3

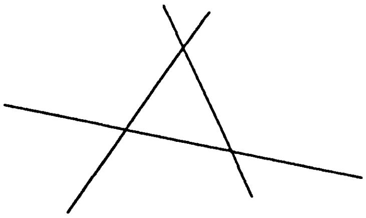
1. 3 strips could have 1, 2, or 3 intersections:



1 intersection



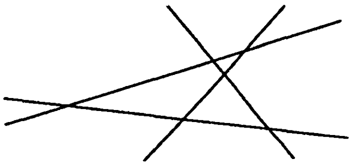
2 intersections



3 intersections

- The greatest number of intersections for 3 strips is 3.

2. The greatest number of intersections you can get from 4 strips is 6.



3.

Number of strips		Greatest number of intersections
1	→	0
2	→	1
3	→	3
4	→	6
5	→	10
6	→	15
7	→	21

4.
- The numbers in the 'Greatest number of intersections' column are the triangle numbers.
- The number in the 'Number of strips' column goes up by 1 each time.

0224 Area of a Parallelogram

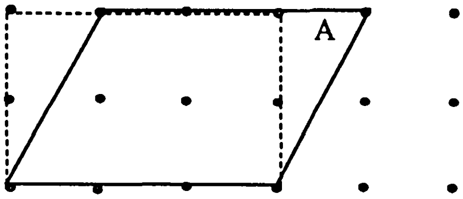
The area of the parallelogram drawn is 6 square units.

Check the results in your table using the formula

area of parallelogram = base x perpendicular height.

1. 24cm^2
2. 14cm^2
3. 45cm^2
4. 24cm^2
5. 35cm^2

0226 Shearing Parallelograms



The area of the rectangle is the same as the area of the parallelogram because when we make the rectangle we cut off triangle A and add on an identical triangle to the opposite side.

1. If we make a rectangle which has the same **base** and also same **height** as a parallelogram, then the 2 shapes will cover the same **area.**
2. To find the area of a rectangle we work out **base x height.**
3. So to find the area of a parallelogram we work out **base x height.**

	Base cm	Height cm	Area cm^2
a	3	6	18
b	7	2	14
c	2	4	8
d	6	4.5	27
e	2	5	10

0227 Parallelogram Problems

Your answers for each question should be approximately the same but are not likely to be exactly the same. This is because measurement is always approximate, not exact. How much did your two answers vary for each question?

1. approximately $9cm^2$
2. approximately $16cm^2$
3. approximately $25cm^2$

0228 From Parallelogram to Rectangle

7 A Height = 5cm
Base = 4cm
Area = $20cm^2$

8. B Height = 5cm
Base = 4cm
Area = $20cm^2$

9. A Height = 3cm
Base = 5cm
Area = $15cm^2$

B Height = 3cm
Base = 5cm
Area = $15cm^2$

10. If a parallelogram and a rectangle have the same height and base, they have the same area.

0229 Shearing a Rectangle

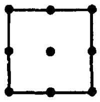
- The height of the books will stay the same.
- The area of the books will stay the same.

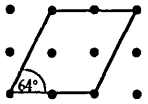
This is true for all shears of rectangles because: area of rectangle = base x height
area of parallelogram = base x height.

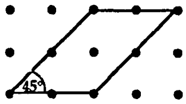
Since the base does not move and the height remains the same, the area must remain the same.

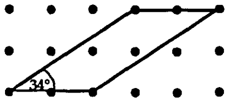
- It is interesting to study how the angle changes with the amount of shear and to show the information on a graph.

e.g. with base = 2cm and height = 2cm.

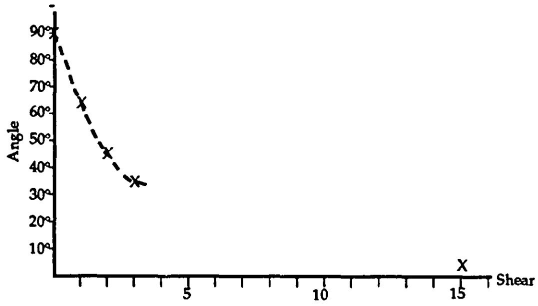








Shear	0	1	2	3	... 15
Height	2	2	2	2	... 2
Angle	90	64	45	34	... 4.2

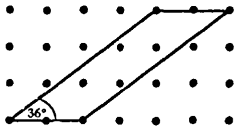


What would happen if you graphed the results for a different rectangle? Would the graph look the same?

- It is also interesting to look at the ratio of height ÷ shear for different rectangles and to record the results in a table.

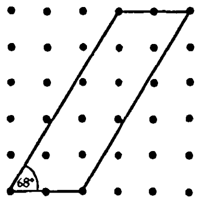
e.g. Height = 3cm
Shear = 4cm

$\frac{\text{Height}}{\text{Shear}} = \frac{3}{4} = 0.75$



Height = 5cm
Shear = 3cm

$\frac{\text{Height}}{\text{Shear}} = \frac{5}{3} = 1.67$



	Shear							
Height	1	2	3	4	5	6	7	8
1								
2								
3				0.75				
4								
5			1.67					
6								
7								
8								

Can you see any patterns in your table?

The ratio $\frac{\text{height}}{\text{shear}}$ gives the tangent of the angle of shear.

Compare your results by using the 'tan' button on your calculator.

0230 Squares Pegs in Round Holes

You will have different arrow diagrams depending on which squares you made.

- Your answer might be:

3 ⟶ 9
6 ⟶ 36
2 ⟶ 4
4 ⟶ 16
5 ⟶ 25
7 ⟶ 49

The rule is 'multiply by itself' (or square). If you multiply the first number by itself you **always** get the second number.

- The new arrow diagram might be:

4 ⟶ 12
3 ⟶ 8
5 ⟶ 16
6 ⟶ 20

The rule is 'subtract 1 and then multiply by 4'.

0232 Inscribed Circle

5. You should find that angle a = angle b
angle c = angle d
angle e = angle f

6. The straight lines are **angle bisectors.**
Angle bisectors are lines which split the angle into 2 equal halves.

8. You should find that the angle bisectors of a triangle meet at the centre of the inscribed circle.

0233 Rectangle Numbers

1. 2 x 3 = **6.**

2. 2 x 6 = **12.**

3. **3 x 4 = 12.**

4. 3 x 5 = 15
● ● ● ● ●
● ● ● ● ●
● ● ● ● ●

5. 7 x 4 = 28
● ● ● ●
● ● ● ●
● ● ● ●
● ● ● ●
● ● ● ●
● ● ● ●
● ● ● ●

6. 4 x 6 = 24
● ● ● ● ● ●
● ● ● ● ● ●
● ● ● ● ● ●
● ● ● ● ● ●

7. 2 x 4 = 8
● ● ● ●
● ● ● ●

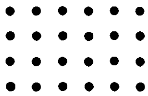
8. 5 x 4 = 20
● ● ● ●
● ● ● ●
● ● ● ●
● ● ● ●
● ● ● ●

9. 3 x 7 = 21
● ● ● ● ● ● ●
● ● ● ● ● ● ●
● ● ● ● ● ● ●

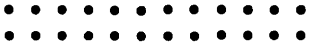
continued/

0233 Rectangle Numbers (cont)

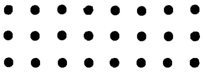
10. 4 x 6 = 24



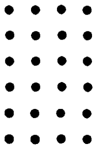
2 x 12 = 24



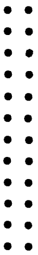
3 x 8 = 24



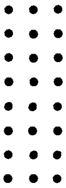
6 x 4 = 24



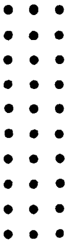
12 x 2 = 24



8 x 3 = 24

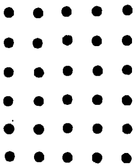


11. 10 x 3 = 30

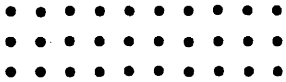


15 x 2 = 30

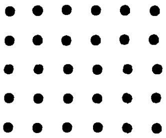
6 x 5 = 30



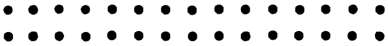
3 x 10 = 30



5 x 6 = 30



2 x 15 = 30



0235 Finding the Angles of a Triangle

You probably did not get **exactly** 180° for each triangle but you should have been somewhere between 178° and 182°.

It is difficult to be accurate to one degree unless you are using a very sharp pencil and a good angle indicator or protractor.

1. 44° + 36° + **100°** = 180°
2. 60° + 40° + **80°** = 180°
3. 57° + 35° + **88°** = 180°
4. 60° + 50° + **70°** =180°
5. 73° + 28° + **79°** = 180°
6. 36° + 72° + **72°** = 180°
7. 108° + 15° + **57°** = 180°
8. 26° + 90° + **64°** = 180°
9. 124° + 55° + **1°** = 180°

0236 Triangle Problems

Whichever way you work out the area it should come to the same number. Your answers may not be **exactly** the same but they should be fairly close. Your ruler usually measures accurately to the nearest millimetre but that may not be good enough. If you used a blunt pencil, that would also make it more difficult to measure accurately.

1. a) Area = $\frac{1}{2}$ x 3 x 1.9 = 2.85cm^2
   b) Area = $\frac{1}{2}$ x 4 x 3.9 = 7.8cm^2
   c) Area = $\frac{1}{2}$ x 4 x 2 = 4cm^2
   d) Area = $\frac{1}{2}$ x 4 x 1.9 = 3.8cm^2
   e) Area = $\frac{1}{2}$ x 2 x 2 = 2cm^2
   f) Area = $\frac{1}{2}$ x 4.9 x 1.5 = 3.675cm^2
   g) Area = $\frac{1}{2}$ x 5.6 x 2 = 5.6cm^2
   h) Area = $\frac{1}{2}$ x 4.8 x 1.9 = 4.56cm^2

   Your measurements may differ if you used a different side as the base.

2. Area = $\frac{1}{2}$ x 3.9 x 5 = 9.75cm^2 or Area = $\frac{1}{2}$ x 5.8 x 3.3 = 9.57cm^2 or Area = $\frac{1}{2}$ x 5 x 3.9 = 9.75cm^2

3. Area = $\frac{1}{2}$ x 7 x 4 =14cm^2 or Area = $\frac{1}{2}$x 4.9 x 5.7 = 13.965cm^2 or Area = $\frac{1}{2}$ x 5.7 x 4.9 = 13.965cm^2

0238 Anytown City Centre

1.

Start	Route	End
Garage	ES	**Church**
Cinema	WWN	**Market**
Station	ESW	**School**
Market	EESS	**Baths**
School	ENWNW	**Market**

2. There are many possible routes each time. Make sure that a friend has checked your answers.

3.

Start	Route	Finish
Pub	E	Hospital
Church	W	School
Garage	EW	Garage
Hospital	NE	Station
Hospital	EES	Church

0239 5 x 5 Square

- The number in the corner opposite the star shows the area of the rectangle.
- 9 should appear in this position.
- The pattern is a times table square.

1	2	3	4	5
2	4	6	8	10
3	6	9	12	15
4	8	12	16	20
5	10	15	20	25

0240 Odds and Evens Tables

1. a) An even number added to an even number **always** makes an **even** number (never makes an odd number).

   b) An odd number added to an odd number **always** makes an **even** number (never makes an odd number).

   c) An odd number added to an even number **always** makes an **odd** number (never makes an even number).

   d) An even number added to an odd number **always** makes an **odd** number (never makes to even number).

2. Second number

First number

+	ODD	EVEN
ODD	Even	Odd
EVEN	**Odd**	**Even**

3. Second number

First number

X	ODD	EVEN
ODD	**Odd**	**Even**
EVEN	**Even**	**Even**

Second number

First number

–	ODD	EVEN
ODD	**Even**	**Odd**
EVEN	**Odd**	**Even**

- It is not possible to make an operations table for division because the answers are not always integers (whole numbers).

  For instance, 6 ÷ 4 = 1.5 and 1.5 is neither odd nor even.

- In the subtraction table above it has been assumed that -5 is an odd number and that -6 is even. Also, 0 is even.

0241 A Secret Code

1. MEET ME TODAY.
2. CALL THE POLICE.
3. THE GOLD IS BY THE TREE.
4. GO TO THE HUT AT TEN.
5. I AM NOW A CODE BREAKER GRADE ONE.

0242 Cracking the Code

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	1

1. WAIT FOR ME.
2. MEET YOU AFTER DARK.
3. 9 10 5 6   7 19 16 14   21 9 6   19 16 3 3 6 19 20
4. 4 16 14 6   21 16   21 9 6   1 16 16

0244 More Sorting

1. B
2. A
3. outside
4. outside
5. D
6. C
7. outside
8. D

0245 Venn Diagrams

1. 5
2. 6
3. 2
4. 4

continued/

0245 Venn Diagrams (cont)

5. 6

6. Your drawing must be any **black triangle.**

7. Your drawing must be any **triangle** which is **not black.**

8. Your drawing must be any **black** shape which is **not a triangle**.

9. Your drawing must be any shape which is **not black** and **not a triangle.**

0248 Making Ten

1. 10 + 0 = 10
9 + 1 = 10
8 + 2 = 10
7 + 3 = 10
6 + 4 = 10
5 + 5 = 10
4 + 6 = 10
3 + 7 = 10
2 + 8 = 10
1 + 9 = 10
0 + 10 = 10
Eleven different ways

2. 7 + 0 = 7
6 + 1 = 7
5 + 2 = 7
4 + 3 = 7
3 + 4 = 7
2 + 5 = 7
1 + 6 = 7
0 + 7 = 7
Eight different ways

3. Show your work to your teacher.

0249 How many ways?

1. 0 + 7 = 7
1 + 6 = 7
2 + 5 = 7
3 + 4 = 7
4 + 3 = 7
5 + 2 = 7
6 + 1 = 7
7 + 0 = 7
8 ways

2. 0 + 9 = 9
1 + 8 = 9
2 + 7 = 9
3 + 6 = 9
4 + 5 = 9
5 + 4 = 9
6 + 3 = 9
7 + 2 = 9
8 + 1 = 9
9 + 0 = 9
10 ways

3. 0 + 15 = 15
1 + 14 = 15
2 + 13 = 15
3 + 12 = 15
4 + 11 = 15
5 + 10 = 15
6 + 9 = 15
7 + 8 = 15
8 + 7 = 15
9 + 6 = 15
10 + 5 = 15
11 + 4 = 15
12 + 3 = 15
13 + 2 = 15
14 + 1 = 15
15 + 0 = 15
16 ways

- Did you notice that: i) 3 + 4 = 7
and 4 + 3 = 7
ii) 4 + 11 = 15
and 11 + 4 = 15
so it does not matter which way you add two numbers?
If you had to do the sum 3 + 63, it might be easier to do 63 + 3.

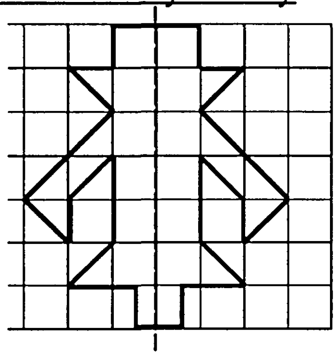
- The number of ways is one more than the number you are trying to add up to, so if you are trying to add up to 23 there are 24 ways.

0250 Less Than, More Than

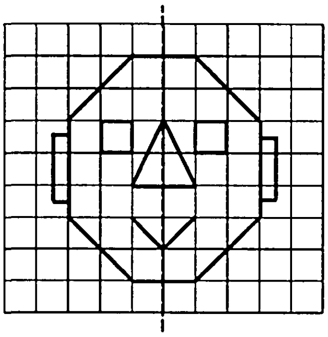
1. 5 more than 4 is **9.**
2. 3 more than 2 is **5.**
3. 4 more than **3** is 7.
4. **5** more than 5 is 10.
5. 7 more than 1 is **8.**
6. 5 less than 9 is **4.**
7. 3 less than 7 is **4.**
8. 4 less than **8** is 4.
9. **3** less than 10 is 7.
10. 6 less than 9 is **3.**

0251 Mirror Symmetry

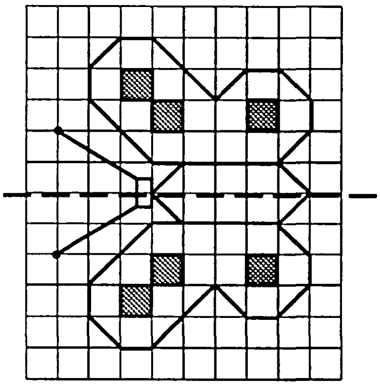
1.



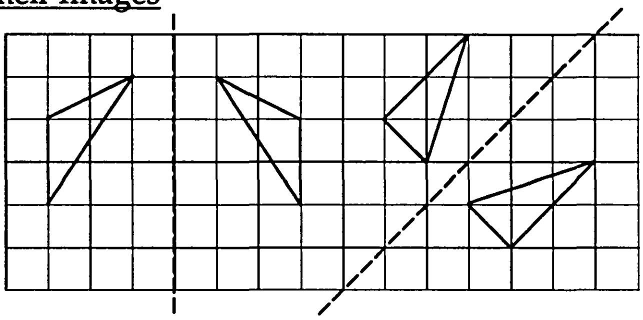
2.



3.

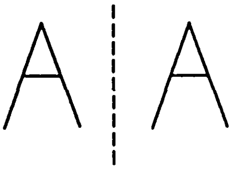


0255 Points and their Images

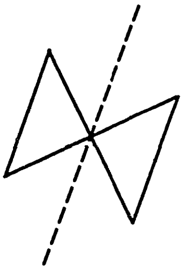


The dotted lines drawn are called **mirror lines or axes of symmetry.**

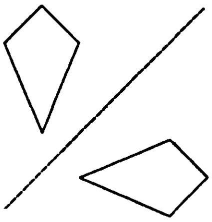
1.



2.



3.

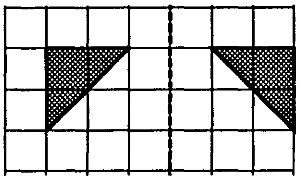


4.

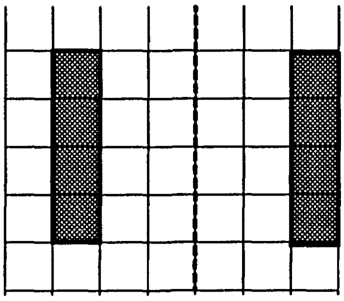
continued/

0255 Points and their Images (cont)

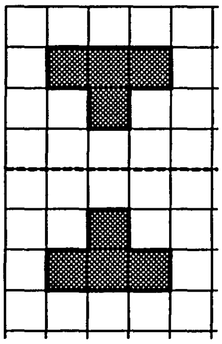
1.



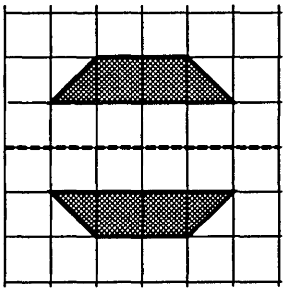
2



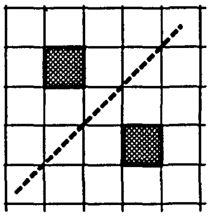
3.



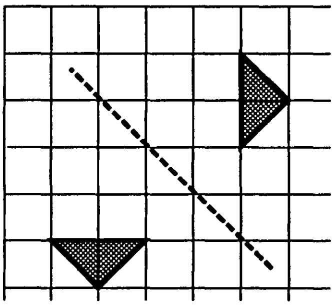
4.



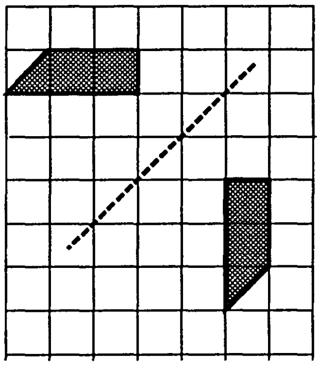
5.



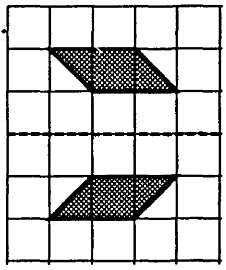
6.



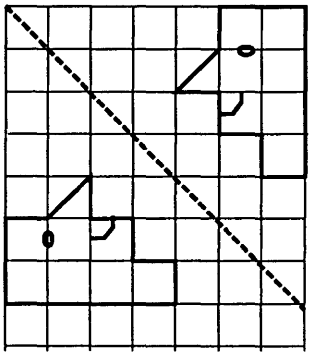
7.



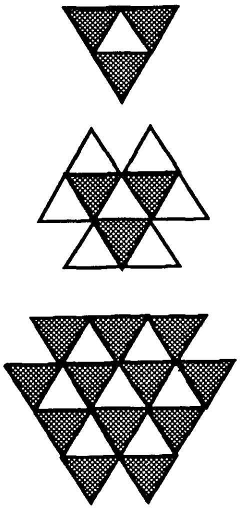
8.



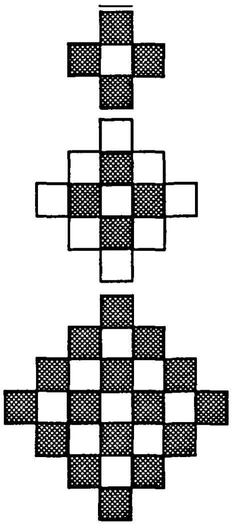
9.



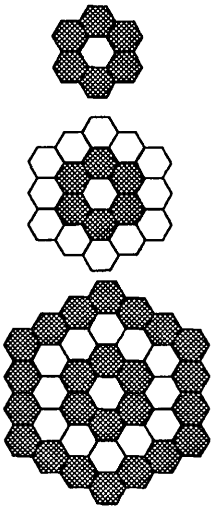
0256 Shapes and Numbers



3
↓
6
↓
9
↓
12
↓
15
↓
add on 3



4
↓
8
↓
12
↓
16
↓
20
↓
add on 4



6
↓
12
↓
18
↓
24
↓
30
↓
add on 6

0257 Squidge

*	1	2	3	4	5	6	7	8	9	10
1	3	5	7	9	11	13	15	17	19	21
2	5	8	11	14	17	20	23	26	29	32
3	7	11	15	19	23	27	31	35	39	43
4	9	14	19	24	29	34	39	44	49	54
5	11	17	23	29	35	41	47	53	59	65
6	13	20	27	34	41	48	55	62	69	76
7	15	23	31	39	47	55	63	71	79	87
8	17	26	35	44	53	62	71	80	89	98
9	19	29	39	49	59	69	79	89	99	109
10	21	32	43	54	65	76	87	98	109	120

The next 3 numbers in each row are:

11	12	13
23	25	27
35	38	41
47	51	55
59	64	69
71	77	83
83	90	97
95	103	111
107	116	125
119	129	139
131	142	153

The numbers in each row and column increase by the same amount.
It is symmetrical about the leading diagonal.

- In row 1 each number increases by 2
  In row 2 each number increases by 3
  In row 3 each number increases by 4 . . .
- In column 1 each number increases by 2
  In column 2 each number increases by 3 . . .

0258 Squidgeree

⊛	1	2	3	4	5	6
1	2	6	12	20	30	42
2	6	16	30	48	70	96
3	12	30	54	84	120	162
4	20	48	84	128	180	240
5	30	70	120	180	250	330
6	42	96	162	240	330	432

Row 1 increases by 4, 6, 8, 10, 12
Row 2 increases by 10, 14, 18, 22, 26
Row 3 increases by 18, 24, 30, 36, 42
Row 4 increases by 28, 36, 44, 52, 60
Row 5 increases by 40, 50, 60, 70, 80
Row 6 increases by 54, 66, 78, 90, 102

There are the same patterns in the columns.
e.g. column 1 increases by 4, 6, 8, 10, 12

- The table is symmetrical about the leading diagonal.

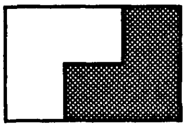
⧆	1	2	3	4	5	6
1	1	1	1	1	1	1
2	1	0	1	2	3	4
3	1	1	3	5	7	9
4	1	2	5	8	11	14
5	1	3	7	11	15	19
6	1	4	9	14	19	24

You may have -1 in the first row and column.
This is also correct.

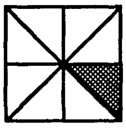
0259 Shading Fractions

The **number** of shapes that you shade is important - **not** which ones.

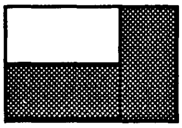
$\frac{1}{2}$



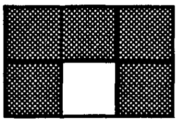
$\frac{1}{8}$



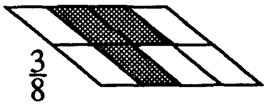
$\frac{2}{3}$



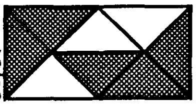
$\frac{5}{6}$



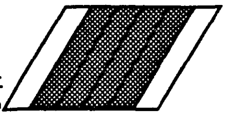
$\frac{3}{8}$



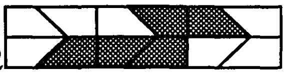
$\frac{5}{8}$



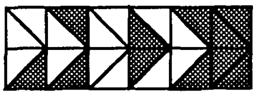
$\frac{4}{6}$



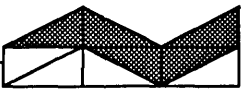
$\frac{5}{12}$



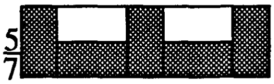
$\frac{11}{24}$



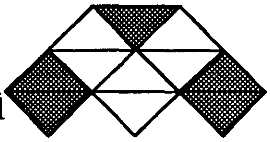
$\frac{5}{9}$



$\frac{5}{7}$



$\frac{5}{11}$



0261 Co-ordinates 1

1.
   - The rock is at (4, 1).
   - The wreck is at (4, 4).
   - The treasure is at (1, 2).

2. a) The sandbank
   b) The cave
   c) The treasure
   d) The lookout

3. a) The swamp
   b) The lake
   c) The castle is at ($2\frac{1}{2}$, $1\frac{1}{2}$).

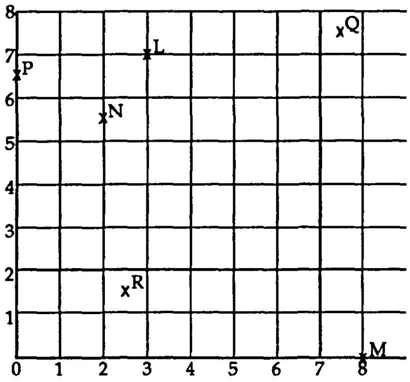
4. Ask a friend to check your map and the positions of the places you have marked.

0262 Co-ordinates 2

3. (3, 1) is not the same as (1,3). (1, 3) are the co-ordinates of A.
   To get to (3, 1), go across 3, then up 1. You will not arrive at A.

4. B is at (4, 4) C is at (6, 4) D is at (5, 8) E is at (7, 3) F is at (6, 0)
   G is at (0, 0) H is at (0, 4) I is at (3, $2\frac{1}{2}$) J is at ($4\frac{1}{2}$, 6) K is ($4\frac{1}{2}$, $1\frac{1}{2}$)

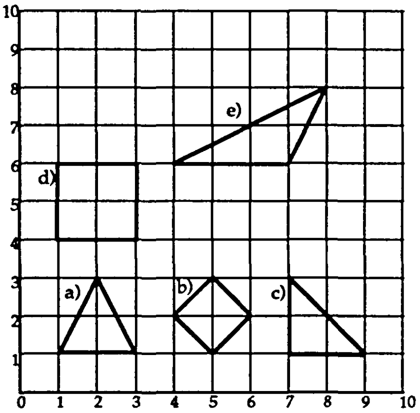
5.



0263 Co-ordinates 3

1. a) Sammy's nose is at (1, 4)
   b) The end of his tail is at (14, 3)
   c) His eye is at (2, 4½)
   d) The bottom of his ear is at (2½, 4)

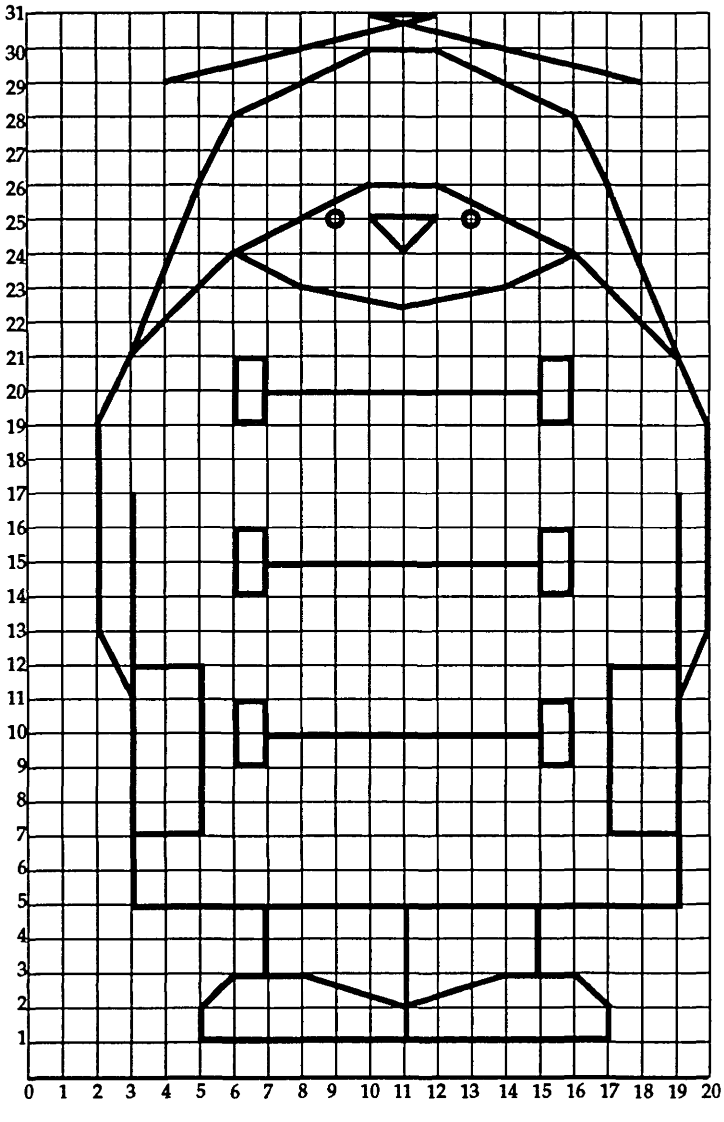
2.



a. The shape is an isosceles triangle.
b. The shape is a square.
c. The shape is a isosceles right-angled triangle.
d. The shape is a square.
e. The shape is a triangle.

0264 Cartoon Co-ordinates

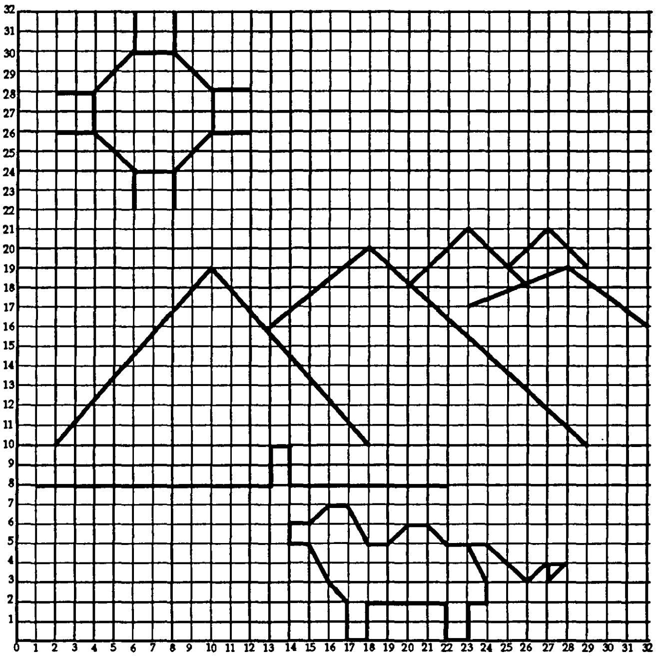
Bear



continued/

0264 Cartoon Co-ordinates (cont)

Landscape



This was designed
by a girl in a
London school.

You might like to design a cartoon of your own.

0265 Odd and Even

a) 2 pairs so 4 is even.
b) 3 pairs and one counter left over so 7 is odd.

c) 11 is odd.
d) 23 is odd.
e) 10 is even.
f) 5 is odd.
g) 17 is odd.

h) 18 is even.
i) 25 is odd.
j) 14 is even.
k) 3 is odd.
l) 1 is odd.

0267 Angles of a Polygon

1. 180° x 2 = 360° The angles of a quadrilateral add up to 360°.

2. 180° x 3 = 540° The angles of a pentagon add up to 540°.

3. 180° x 4 = 720° The angles of a hexagon add up to 720°.
180° x 5 = 900° The angles of a heptagon add up to 900°.
180° x 6 = 1080° The angles of an octagon add up to 1080°.

4.

Shape	No. of sides	No. of triangles	Angle sum
Triangle	3	1	180°
Quadrilateral	4	2	360°
Pentagon	5	3	540°
Hexagon	6	4	720°
Heptagon	7	5	900°
Octagon	8	6	1080°
Decagon	10	8	1440°

5. To find the angle sum of any polygon,
'find the number of sides, subtract 2 then multiply by 180°'.

0268 Exterior Angles of Polygons

For all your polygons the angles should fit together to make one whole turn or 360°. If you walk around the perimeter of a polygon you will turn through all its exterior angles. In doing this, you will make one complete turn, i.e. 360°.

0269 Finding Exterior Angles

a = 107°
b = 88°
c = 78°
d = 87°
sum = 360°

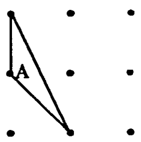
- In each of your polygons the sum should always be 360°.

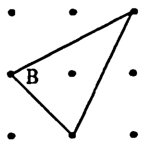
1. a = 124°
2. b = 110°
3. c = 130°
4. d = 60°
5. e = 95°
6. f = 105°

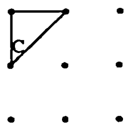
0271 Pins and Polygons

Here are **some** of the answers.

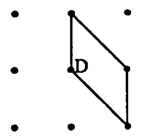
3 pins

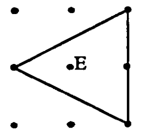


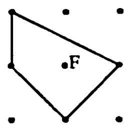




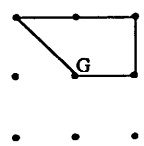
4 pins

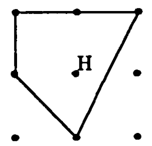


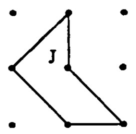




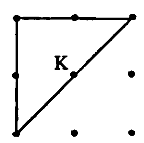
5 pins

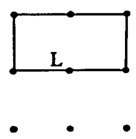


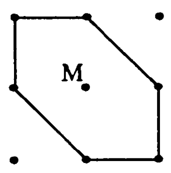




6 pins







Sorting by shapes

Triangles = A, B, C, E, K
Quadrilaterals = D, F, G, H, L
Pentagons = J
Hexagons = M

Other ways of sorting your shapes could be:

- regular and irregular shapes,
- shapes containing right-angles and not containing right angles.

You may have sorted them in a different way, if so, show your work to your teacher.

0272 Vehicle Survey

Which type of vehicle did you see most often? If the type of vehicle you saw most was cars, then cars are the **mode**. Show your worksheet and bar chart to your teacher.

0273 How much longer?

1. DG = 4cm
   CE = 3.5cm
   DG is 0.5cm longer than **CE**.
   DG is 5mm longer than **CE**.

2. EF = 1.5cm
   FG = 1cm
   EF is 0.5cm longer than FG.
   EF is 5mm longer than FG.

3. CF = 5cm
   FH = 4cm
   CF is 1cm longer than FH.
   CF is 10mm longer than FH.

4. AE = 7.5cm
   EH = 5.5cm
   AE is 2cm longer than EH.
   AE is 20mm longer than EH.

5. BF = 6cm
   CF = 5cm
   BF is 1cm longer than CF.
   BF is 10 mm longer than CF.

6. BG = 7cm
   GE = 2.5cm
   BG is 4.5cm longer than GE.
   BG is 45mm longer than GE.

7. FB = 6cm
   CE = 3.5cm
   FB is 2.5cm longer than CE.
   FB is 25mm longer than CE.

8. AB = 3cm
   GH = 3cm
   Neither AB nor GH is longer. They are both the same length.

9. DH = 7cm
   BE = 4.5cm
   DH is 2.5cm longer than BE.
   DH is 25mm longer than BE.

10. FH = 4cm
    CA = 4cm
    FH and CA are both the same length.

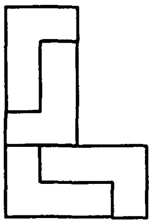
11. X——————————Y

12. ——————————

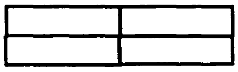
13. ————————

0275 Tetromino 2

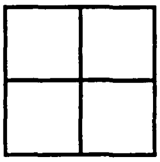
1. 'L' tetromino



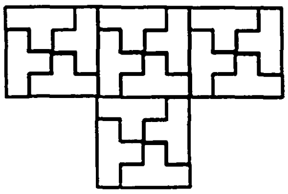
Long tetromino



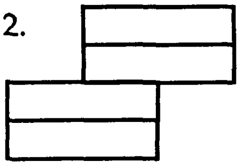
Square tetromino



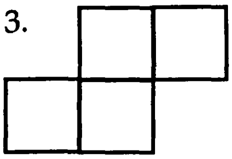
'T' tetromino.



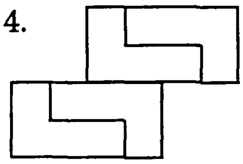
2.



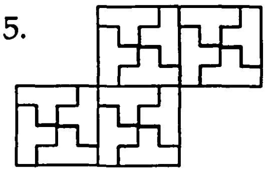
3.



4.



5.



0276 The Tiger Game

Make a sketch of a game where the sheep block the tigers from winning.

0278 Five Field Kono

Did it matter which player started?

0279 High Jump Game

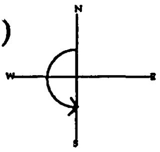
Did it matter which player started?

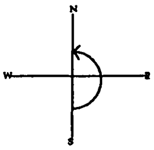
0281 Angles: The Compass

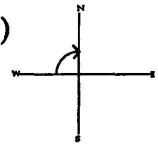
	Start	Turn		End
		which way	how much	
a)	North	left	½ turn	South
b)	South	left	½ turn	North
c)	West	right	¼ turn	North
d)	North	right	¾ turn	West
e)	East	left	2 turns	East
f)	North	right	1½ turns	South
g)	West	right	¾ turn	South
h)	North	left	¼ turn	West
i)	East	left	½ turn	West
j)	West	right	1 turn	West
k)	South	right	¾ turn	East
l)	West	right	¾ turn	South

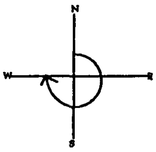
continued/

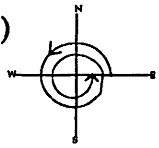
0281 Angles: The Compass (cont)

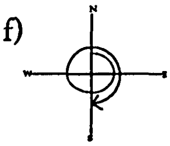
a)


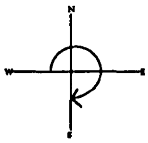
b)


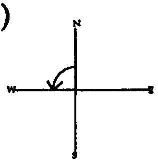
c)


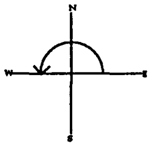
d)


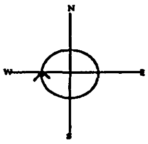
e)


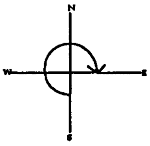
f)


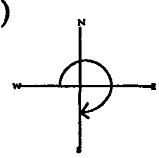
g)


h)


i)


j)


k)


l)


0284 Angles from Tessellations

1. There are 4 angles at A.
2. $\frac{1}{4}$ turn. A complete turn is 360°, so $\frac{1}{4}$ turn is 90°.
3. The interior angle at each corner of a square is 90°, or a right-angle.
4. There are 6 angles at B.
5. The angle at each corner of an equilateral triangle is 60°.
6. The angle at each corner of a regular hexagon is 120°.
7. There are 3 angles at D.
   One is 90° (an angle of a square) so the angle of a regular octagon is 135°.

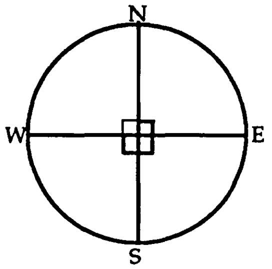
0285 The Clock

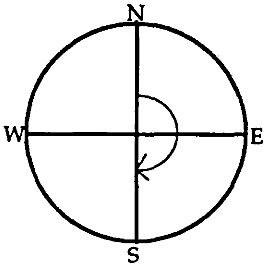
The second hand moves fastest

- The minute hand takes 1 hour to make 1 whole turn.
- The hour hand takes 12 hours to make 1 whole turn.

1.	a)	12 hours	b)	6 hours	c)	3 hours	d)	18 hours
2.	a)	60 minutes	b)	15 minutes	c)	45 minutes	d)	1 minute
3.	a)	60 seconds	b)	30 seconds	c)	15 seconds		
4.	a)	1 turn	b)	$\frac{1}{2}$ turn	c)	$\frac{1}{4}$ turn	d)	$\frac{1}{60}$ turn
	e)	$\frac{1}{6}$ turn	f)	$\frac{1}{12}$ turn				
5.	a)	$1\frac{1}{2}$ turns	b)	2 turns	c)	$\frac{1}{12}$ turn.		

0286 Right Angles





- There are 4 right-angles.
- If you rotate from North clockwise to South, you turn through 2 right-angles.
- If the hand of a clock turns from the 3 down to the 6, it turns through 1 right-angle.

	Start	Which way	End	Number of Right-angles
a)	South	Anticlockwise	East	1
b)	East	Anticlockwise	South	3
c)	South	Clockwise	West	1
d)	North	Clockwise	East	1
e)	West	Anticlockwise	North	3
f)	South	Anticlockwise	North	2
g)	12	Clockwise	6	2
h)	3	Clockwise	12	3
i)	6	Clockwise	9	1
j)	9	Clockwise	3	2
k)	12	Anticlockwise	9	1
l)	12	Anticlockwise	3	3

0288 Rolling Two Dice

- The highest possible score is **12**. (6 + 6)
- The lowest possible score is **2**. (1 + 1)
- It is very likely that column 7 was the first one to fill up. If not, it was probably column 6 or column 8.
- The columns with the least squares shaded are probably columns 2 and 12. Columns 3 and 11 are not likely to be very full either.
- The reasons for this are to do with the different ways 2 dice can land. There is only one possible way to score 12, (6 + 6) but there are several ways to score 7.
  Can you find how many ways?

0290 Experiments

1. If you throw a dice 60 times, you will probably get about 10 fours. This is because there are 6 numbers on a dice. The dice is equally likely to land on any of them. Did your experiment match your prediction?

2. If you toss a coin 50 times, you will probably get about 25 heads. This is because the coin is equally likely to land heads or tails. Did your experiment match your prediction?

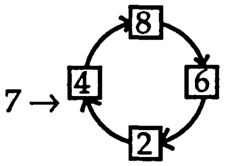
3. With the spinner, you will probably get 2 about 10 times. There are 5 numbers and they are all equally likely. Did your experiment match your prediction?

0291 Which Set?

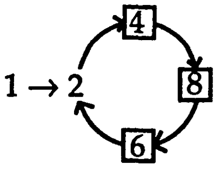
1. 3, 6, 9 and 12 are inside triangle A
2. 2, 4, 6, 8, 10 and 12 are inside square B.
3. 1, 2, 3, 4, 5, 6 and 7 are inside circle C.
4. 9 is inside triangle A, but not inside square B or circle C.
5. 3 lies inside triangle A and circle C.
6. 1, 2, 4, 5, 7, 8, 10, 11 and 13 are not inside triangle A.
7. 2 lies inside square B and circle C.
8. 3 and 6 are both in triangle A and circle C.
9. 6 is inside all three shapes.
10. 12 is inside triangle A and square B but not in circle C.
11. Triangle A contains multiples of 3.
    Square B contains multiples of 2.
    Circle C contains numbers less than 8.
    11 and 13 do not belong in any of these shapes, so they have been left outside.

0292 Doubling Patterns

1. 7→14 → 28 → 56 → 112 → 224 → 448 → 896 → 1792 → 3584 → 7168
   7 → 4 → 8 → 6 → 2 → 4 → 8 → 6 → 2 → 4 → 8



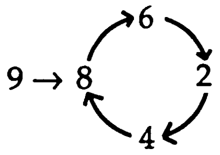
2. 1 → 2 → 4 → 8 → 16 → 32 → 64 →128 → 256 → 512 → 1024
   1 → 2 → 4 → 8 → 6 → 2 → 4 → 8 → 6 → 2 → 4



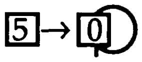
continued/

0292 Doubling Patterns (cont)

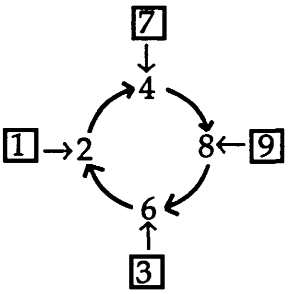
3. 9 →18 → 36 → 72 → 144 → 288 → 576 → 1152 → 2304 → 4608 → 9216
9 → 8 → 6 → 2 → 4 → 8 → 6 → 2 → 4 → 8 → 6

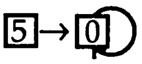


4. 5 → 10 → 20 → 40 → 80 → 160 → 320 → 640 → 1280 → 2560 → 6120
5 → 0 → 0 → 0 → 0 → 0 → 0 → 0 → 0 → 0 → 0



5.





0294 Measuring Lengths

Your measurements may vary slightly from these.

1. 1.9cm

2. AB = 6.0cm
CD = 5.6cm
EF = 2.8cm
GH = 8.1cm
JK = 13.9cm

3. In order of size, shortest first, the lengths are, EF, CD, AB, GH, JK.

5. 5.5cm
4.8cm
2.8cm
4.3cm
17.4cm

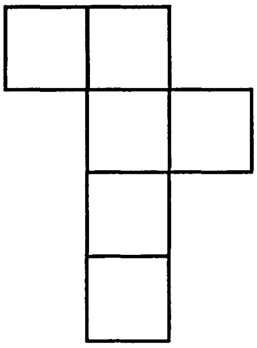
6. 6.8cm
4.8cm
2.8cm
7.6cm
22.0cm

7. 2.2cm
3.4cm
6.4cm
5.1cm
4.0cm
5.5cm
2.5cm
6.1cm
5.5cm
40.7cm

0295 Nets of a Cube

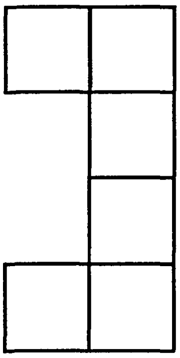
This shape will fold up to make a cube.

This is a net of a cube.



This shape will **not** fold up to make a cube.

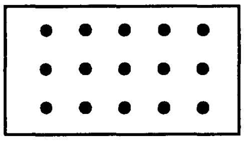
This is **not** a net of a cube.



If you found some more nets of a cube, check them yourself by making sure they fold into cubes.

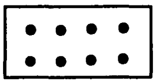
There are 36 arrangements of 6 squares joined edge to edge of which 6 form a net of a cube. Did you find them all?

0297 More Rectangle Numbers

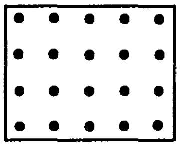


This rectangle of dots shows that 15 is a **rectangle** number.

1. Here are 2 more **rectangle** numbers:



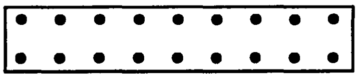
8

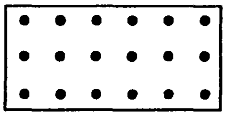


20

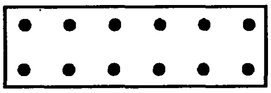
There are many more. Show your rectangle numbers to your teacher.

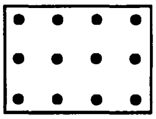
2. a) There are 2 different patterns for 18.



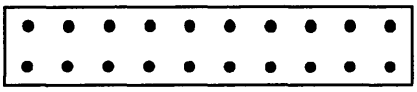


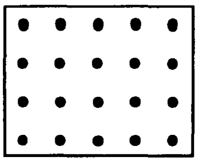
b) There are 2 different patterns for 12.





c) There are 2 different patterns for 20.

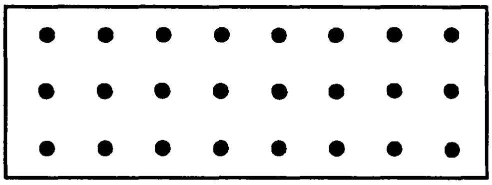


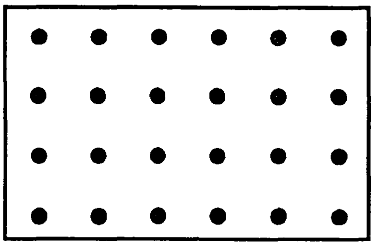


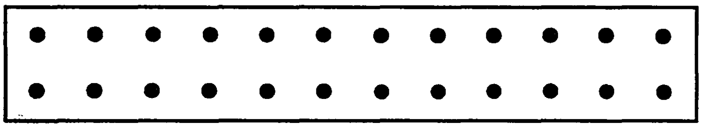
continued/

0297 More Rectangle Numbers (cont)

3. Many possible answers, for instance, 24 has 3 different patterns.



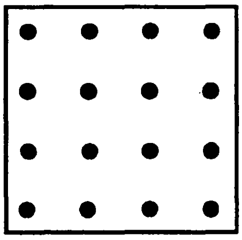




36 has 4 different patterns and 60 has five patterns. Can you see why?

0298 Square Numbers

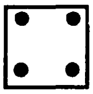
1. The last rectangle is a square.



16 = 4 x 4

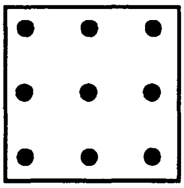
A rectangle with all its sides equal is a square so 16 is a special rectangle number called a square number.

2. a)



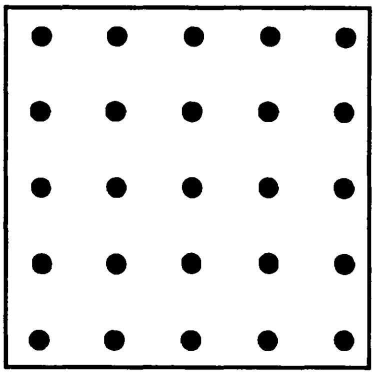
4 = 2 x 2

b)



9 = 3 x 3

c)



25 = 5 x 5

3. Many possible answers, you may have found the next five square numbers which are 36, 49, 64, 81 and 100.

4. 49 and 64 are square numbers; 28, 62 and 78 are not.

0299 Three Squared

a) $4^2 = 16$ (4 x 4)
b) $5^2 = 25$ (5 x 5)
c) $7^2 = 49$ (7 x 7)
d) $12^2 = 144$ (12 x 12)
e) $100 = 10^2$ (10 x 10)
f) $81 = 9^2$ (9 x 9)

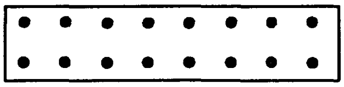
$1^2 = 1 \times 1 = 1$
$2^2 = 2 \times 2 = 4$
$3^2 = 3 \times 3 = 9$
$4^2 = 4 \times 4 = 16$
$5^2 = 5 \times 5 = 25$
$6^2 = 6 \times 6 = 36$
$7^2 = 7 \times 7 = 49$
$8^2 = 8 \times 8 = 64$
$9^2 = 9 \times 9 = 81$
$10^2 = 10 \times 10 = 100$
$11^2 = 11 \times 11 = 121$
$12^2 = 12 \times 12 = 144$

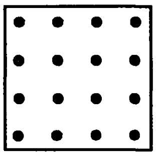
- There are 10 square numbers between 1 and 100, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.
- 1 000 000 = 1000^2, so there are 1000 square numbers between 1 and 1 000 000.

0307 Factors

2. a) 5 is a factor of 15
3 is a factor of 15
b) 2 is a factor of 12
6 is a factor of 12
c) 1 is a factor of 7
7 is a factor of 7

3. There are 3 patterns for 16.





{factors of 16} = {1, 2, 4, 8, 16}

4. a) {factors of 12} = {1, 2, 3, 4, 6, 12}
b) {factors of 20} = {1, 2, 4, 5, 10, 20}
c) {factors of 21} = {1, 3, 7, 21}
d) {factors of 9} = {1, 3, 9}

5. a) {factors of 5} = {1, 5}
b) {factors of 30} = {1, 2, 3, 5, 6, 10, 15, 30}
c) {factors of 23} = {1, 23}
d) {factors of 24} = {1, 2, 3, 4, 6, 8, 12, 24}
Get your teacher to check your own numbers.

6. The factors of a number are the numbers which can be divided into it without a remainder.

0308 Prime Numbers

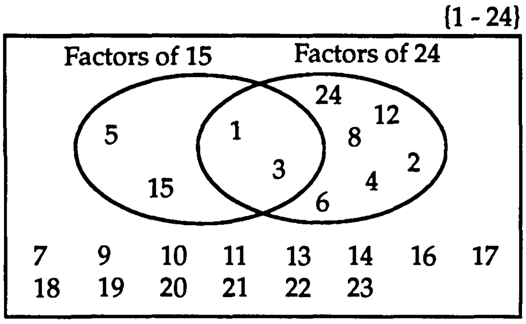
1. 7 has only a straight line pattern and therefore only 2 factors, so 7 is a prime number.

2. The prime numbers under 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

0310 Common Factors

1. {factors of 15} = (1, 3, 5, 15}
{factors of 24} = {1, 2, 3, 4, 6, 8, 12, 24}

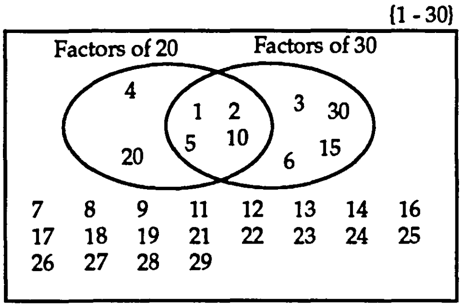
2.



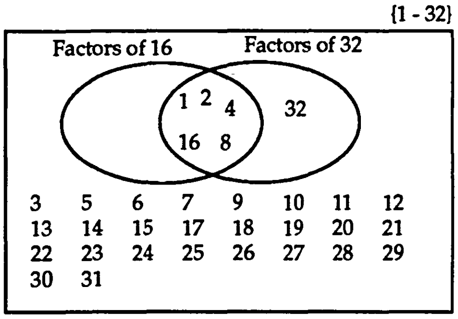
3. 1 and 3 are factors of both 15 and 24.
{common factors of 15 and 24} = {1, 3}

4 {factors of 20} = {1, 2, 4, 5, 10, 20}
{factors of 30} = {1, 2, 3, 5, 6, 10, 15, 30}
{factors of 16} = {1, 2, 4, 8, 16}
{factors of 32} = {1, 2, 4, 8, 16, 32}
{factors of 10} = {1, 2, 5, 10}
{factors of 15} = {1, 3, 5, 15}
{factors of 18} = {1, 2, 3, 6, 9, 18}

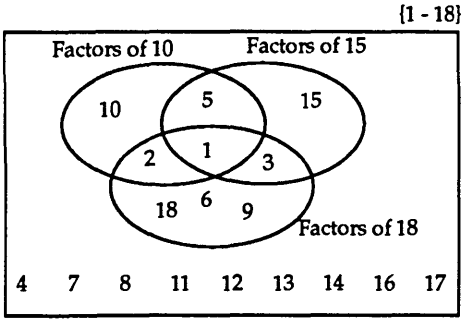
5. a)



b)



c)



{common factors of 20 and 30} = {1, 2, 5, 10}
{common factors of 16 and 32} = {1, 2, 4, 8, 16}
{common factors of 10, 15 and 18} = {1}

0311 Factor Finder

25																									25
24																								24	
23																							23		
22																						22			
21																					21				
20																				20					
19																			19						
18																		18							
17																	17								
16																16									
15															15										
14														14											
13													13												
12												12												12	
11											11											11			
10										10										10					
9									9									9							
8								8								8								8	
7							7							7							7				
6						6						6						6						6	
5					5					5					5					5					5
4				4				4				4				4				4				4	
3			3			3			3			3			3			3			3			3	
2		2		2		2		2		2		2		2		2		2		2		2		2	
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

1. The number in column 6 are 1, 2, 3 and 6.
   All these numbers are factors of 6.

2. All the factors of 8 are in column 8.

3. All the factors of 24 are in column 24.

4. The common factors of 15 and 20 are the factors which are in both columns.
   The common factors of 15 and 20 are 1 and 5.

5. The numbers that have exactly two factors are 2, 3, 5, 7, 11, 13, 17, 19, 23.

6. The factors of these numbers are the number itself and 1.
   A definition of a prime number is a number with only 2 factors, 1 and the number itself.

- Show your descriptions of the other patterns that you found to your teacher.

0313 Spots in Sequence

- 1, 5, 9, 13, 17, 21, 25, 29, 33
- Each number is 4 more than the one before.

a) 3, 6, 9, 12, 15, 18, 21, 24
b) 1, 3, 6, 10, 15, 21, 28, 36
c) 1, 4, 9, 16, 25, 36, 49, 64
d) 2, 6, 12, 20, 30, 42, 56, 78
e) 1, 8, 16, 24, 32, 40, 48, 56

0314 Dots in Sequence

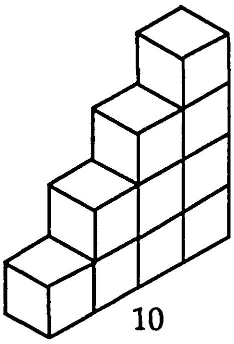
a) The sequence of dots on the perimeters is 4, 8, 12, 16, 20 . . .
The numbers are the multiples of 4.
Each successive square has another dot on each side so there are 4 new dots each time.

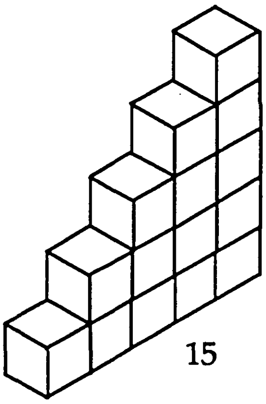
b) The sequence of dots inside the square is 1, 5, 13, 25, 41 . . .
The numbers increase by 4 more each time.
A difference table shows how the sequence works.

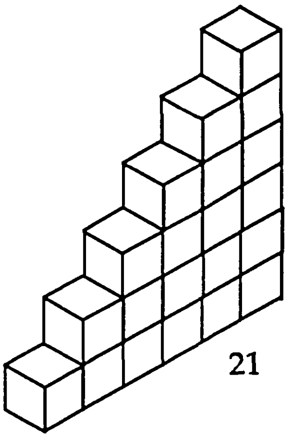
	1		5		13		25		41
1st difference		4		8		12		16	
2nd difference			4		4		4		

0315 Staircases

1. The next 3 staircases are:







1, 3, 6, 10, 15, 21, 28, 36 . . .
These are the triangle numbers.
The sequence is made by adding on 2, 3, 4, 5, 6 . . .

2. 1, 4, 9, 16, 25, 36 . . .
These are the square numbers.
The sequence can be made by adding on 3, 5, 7, 9, 11 . . .

3. 1, 6, 15, 28, 45, 66, 91, 120 . . .
The best way to see how the sequence works is to look at a difference table.

	1		6		15		28		45		66		91		120
1st differences:		5		9		13		17		21		25		29	
2nd differences:			4		4		4		4		4		4		

Try to describe the sequence in words, using the difference table to help.

0316 Counting On

1.	2→ 5→ 8→ 11→ 14 → 17	Count on 3
2.	4→ 8→ 12→ 16→ 20 → 24	Count on 4
3.	1→ 3→ 5→ 7→ 9 → 11	Count on 2
4.	2→ 6→ 10→ 14→ 18→ 22	Count on 4
5.	1→ 6→ 11→ 16→ 21→ 26	Count on 5
6.	15→ 13→ 11→ 9→ 7→ 5	Count back 2
7.	17→ 14→ 11→ 8→ 5 → 2	Count back 3
8.	20→ 16→ 12→ 8→ 4 → 0	Count back 4
9.	18→ 16→ 14 → 12→ 10→ 8	Count back 2
10.	19 → 16 → 13 →10 → 7	Count back 3

0317 Sequences of Numbers

1.	2, 5, 8, 11, 14, **17, 20, 23**	Add 3
2.	3, 7, 11, 15, 19, **23, 27, 31**	Add 4
3.	50, 47, 44, 41, 38, **35, 32, 29**	Subtract 3
4.	9, $10\frac{1}{2}$, 12, $13\frac{1}{2}$, 15, **$16\frac{1}{2}$, 18, $19\frac{1}{2}$**	Add $1\frac{1}{2}$
5.	1, 2, 4, 8, 16, **32, 64, 128**	Double
6.	1, 10, 100, 1000, **10000, 100000, 1000000**	Multiply by 10
7.	1, 3, 7, 15, **31, 63, 127**	Double and add 1
8.	32, 16, 8, 4, 2, **1, $\frac{1}{2}$, $\frac{1}{4}$**	Divide by 2
9.	2, 6, 18, 54, **162, 486, 1458**	Multiply by 3
10.	1, 1, 2, 3, 5, 8, 13, **21, 34, 55**	Add together last 2 numbers

---

0320 Turning Patterns

Did you make at least two patterns?
If you enjoyed this activity, you may like to make a pattern using two different shapes.

0321 Blocking Game

Did it matter who started?

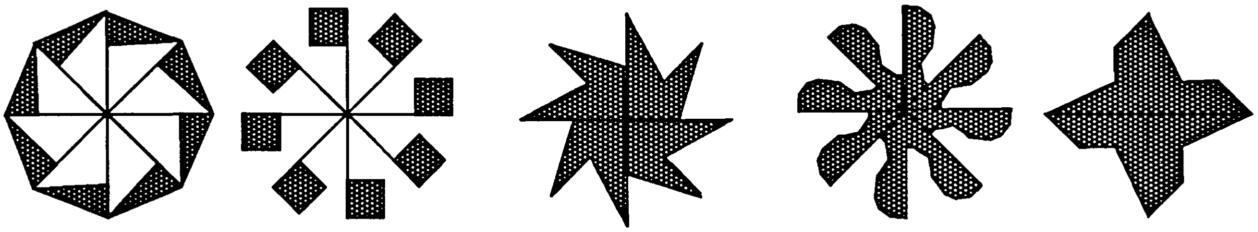
0322 Cutting Up Rectangles

Learn the names of the shapes you have made.
Could you make any different shapes with the triangles?

0323 Metre and Centimetre

When you have found 5 things in each list, show you list to your teacher.

0324 Rotations



Make a display of your own rotation patterns.

0326 Tessellations of Quadrilaterals

Make sure each time that:
- a) all the quadrilaterals are the same.
- b) there are no gaps in between the quadrilaterals.

Is it always possible to make a tessellation from a quadrilateral?

0327 Centres of Rotation

Get someone else to check your answers by using tracing paper.

0328 Tessellating Pentominoes

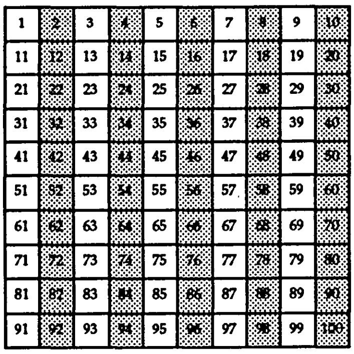
Make sure each time that:
- a) All the pentominoes are the same,
- b) There are no gaps in between the pentominoes.

Do all the pentominoes tessellate?

Which pentomino tessellates in the largest number of different ways? Why?

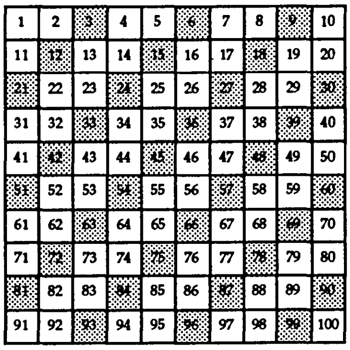
0330 Multiple Patterns

1.



1	**2**	3	**4**	5	**6**	7	**8**	9	**10**
11	**12**	13	**14**	15	**16**	17	**18**	19	**20**
21	**22**	23	**24**	25	**26**	27	**28**	29	**30**
31	**32**	33	**34**	35	**36**	37	**38**	39	**40**
41	**42**	43	**44**	45	**46**	47	**48**	49	**50**
51	**52**	53	**54**	55	**56**	57	**58**	59	**60**
61	**62**	63	**64**	65	**66**	67	**68**	69	**70**
71	**72**	73	**74**	75	**76**	77	**78**	79	**80**
81	**82**	83	**84**	85	**86**	87	**88**	89	**90**
91	**92**	93	**94**	95	**96**	97	**98**	99	**100**

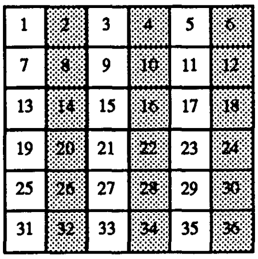
2. Multiples of 3 do not make a column pattern.



1	2	**3**	4	5	**6**	7	8	**9**	10
11	**12**	13	14	**15**	16	17	**18**	19	20
21	22	23	**24**	25	26	**27**	28	29	**30**
31	32	**33**	34	35	**36**	37	38	**39**	40
41	**42**	43	44	**45**	46	47	**48**	49	50
51	52	53	**54**	55	56	**57**	58	59	**60**
61	62	**63**	64	65	**66**	67	68	**69**	70
71	**72**	73	74	**75**	76	77	**78**	79	80
81	82	83	**84**	85	86	**87**	88	89	**90**
91	92	**93**	94	95	**96**	97	98	**99**	100

3. Multiples of 5 and 10 make column patterns.

5.



1	**2**	3	**4**	5	**6**
7	**8**	9	**10**	11	**12**
13	**14**	15	**16**	17	**18**
19	**20**	21	**22**	23	**24**
25	**26**	27	**28**	29	**30**
31	**32**	33	**34**	35	**36**

Multiples of 2 make column patterns.

6. Multiples of 3 and 6 also make column patterns.

7. Multiples of 7 make a column pattern on a number square with 7 columns.

8. On a 10 square, multiples of 2, 5 and 10 give column patterns.
On a 6 square, multiples of 2, 3 and 6 give column patterns.
On a 7 square, multiples of 7 give column patterns.

On a 12 square, multiples of 2, 3, 4, 6 and 12 will give column patterns.
The multiples which give column patterns are the **factors** of the square size.

0331 Prime Factors

- 2 x 3 x 3 x 3 = 54
- 2 and 3 are the prime factors of 54.

- 2 x 2 x 2 x 3 x 3 = 72
- 2 and 3 are the prime factors of 72.

- 3 x 3 x 7 = 63
- 3 and 7 are the prime factors of 63.

- 2 x 2 x 5 x 5 = 100
- 2 and 5 are the prime factors of 100.

0333 Equivalent Fractions

These are the equivalent fractions you can find using the diagrams.

1. There are 8 triangles. 4 are shaded.
   There are 4 rectangles. 2 are shaded.
   There are 2 squares. 1 is shaded.

   $$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$$

2. There are 6 small rectangles. 3 are shaded.
   There are 2 large rectangles. 1 is shaded.

   $$\frac{3}{6} = \frac{1}{2}$$

3. There are 6 triangles. 4 are shaded.
   There are 3 rhombuses. 2 are shaded.

   $$\frac{4}{6} = \frac{2}{3}$$

4. There are 9 squares. 3 are shaded.
   There are 3 rectangles. 1 is shaded.

   $$\frac{3}{9} = \frac{1}{3}$$

5. There are 12 triangles. 4 are shaded.
   There are 6 rectangles. 2 are shaded.

   $$\frac{4}{12} = \frac{2}{6}$$

6. There are 12 triangles. 4 are shaded.
   There are 3 arrow shapes (hexagons). 1 is shaded.

   $$\frac{4}{12} = \frac{1}{3}$$

7. There are 9 rectangles. 3 are shaded.
   There are 3 'T' shapes (octagons). 1 is shaded.

   $$\frac{3}{9} = \frac{1}{3}$$

8. There are 8 trapeziums. 2 are shaded.
   There are 4 hexagons. 1 is shaded.

   $$\frac{2}{8} = \frac{1}{4}$$

9. There are small 16 triangles. 4 is shaded.
   There are 4 large triangles. 1 is shaded.

   $$\frac{4}{16} = \frac{1}{4}$$

10. There are 18 squares. 3 are shaded.
    There are 6 'L' shapes (hexagons). 1 is shaded.

    $$\frac{3}{18} = \frac{1}{6}$$

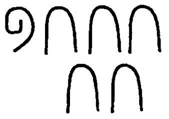
There are other fractions equivalent to each of these. If you have different answers show them to your teacher.

0334 Egyptian Numbers

1. 223
2. 335
3. 376
4. 629
5. 651
6. 676

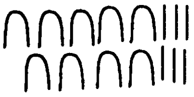
7.

8.

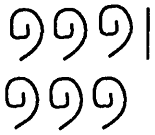


9.

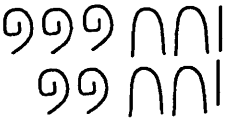
10.



11.



12.



0338 Summing the Odds

Using this square number pattern these results were found.

$1 = 1 = 1^2$
$1 + 3 = 4 = 2^2$
$1 + 3 + 5 = 9 = 3^2$
$1 + 3 + 5 + 7 = 16 = 4^2$
$1 + 3 + 5 + 7 + 9 = 25 = 5^2$
$1 + 3 + 5 + 7 + 9 + 11 = 36 = 6^2$
$1 + 3 + 5 + 7 + 9 + 11 + 13 = 49 = 7^2$
$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64 = 8^2$
$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 = 81 = 9^2$
$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100 = 10^2$
$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 = 121 = 11^2$
$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 = 144 = 12^2$

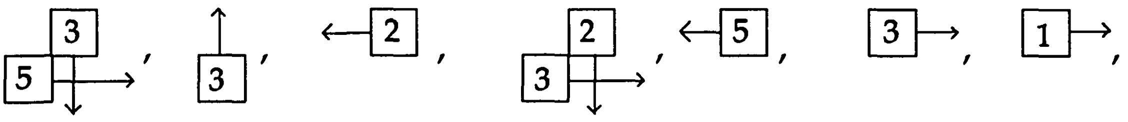
A quick way of finding the sum of the first 12 odd numbers is to calculate 12^2 which is 144.
So the sum of the first 25 odd numbers will be 25^2 which is 625.
The general rule for the sum of the first 'n' odd numbers = n^2.

You may like to extend this activity. For instance, the same diagram could be used to show this square number pattern.

$2^2 = 1^2 + 3$
$3^2 = 2^2 + 5$
$4^2 = 3^2 + 7$
. .
. .
. .
$n^2 =$.

0339 Vector Messages

1. IT MAKES YOU SMILE
2.



3. Did your friend understand your message?

0340 Is it Rigid?

1. The triangle is rigid. All triangles are rigid.

2. The diagonal divides the quadrilateral into 2 triangles which are rigid.

3. For a pentagon you need 2 diagonals.
For a hexagon you need 3 diagonals.

a 4-sided figure needs 1 diagonal
a 5-sided figure needs 2 diagonals
a 6-sided figure needs 3 diagonals
.   .
.   .
.   .

The rule is 'subtract 3 from the number of sides to get the number of diagonals'.

0341 Nodes

1. One goes to C, two go to B.

2. There are 3 paths from B.

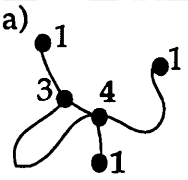
3. 1 goes to C and 2 go to A.
B is a node of order **3**.

4. C is a **4**-node.
D is a **1**-node.

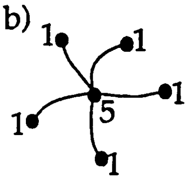
5.

Number of 1-Nodes	Number of 3-Nodes	Number of 4-Nodes	Number of 5-Nodes	Total Number of Nodes
1	1	2	0	4

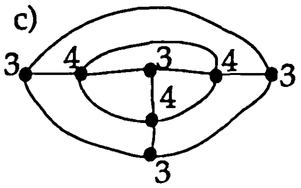
a)



b)

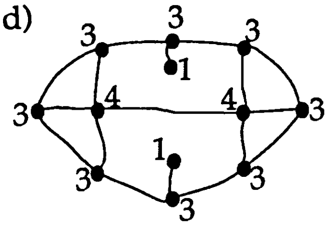


c)

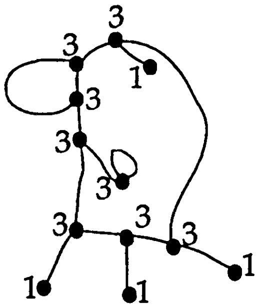


	Number of 1-Nodes	Number of 3-Nodes	Number of 4-Nodes	Number of 5-Nodes	Total Number of Nodes
a)	3	1	1	0	5
b)	5	0	0	1	6
c)	0	4	3	0	7

d)



e)



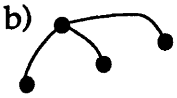
	Number of 1-Nodes	Number of 3-Nodes	Number of 4-Nodes	Number of 5-Nodes	Total Number of Nodes
d)	2	8	2	0	12
e)	4	8	0	0	12

0342 About Nodes

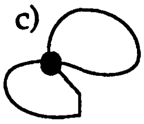
It is impossible to draw a network for d and g. There are many answers for some of the others but here are some suggestions.

a)

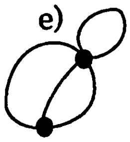
b)



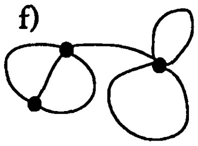
c)



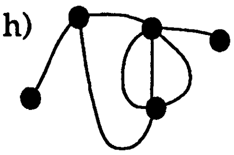
e)



f)



h)



- A network cannot be drawn if it has an **odd** number of **odd** nodes. It is impossible to draw a network for d and g because d has 1 odd node and g has 3 odd nodes.
- Test some networks to see if the rule works. Can you see why the rule works?

0344 Counter Hopping Puzzle

Here is one answer. Perhaps you can find some more.

Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ Ⓙ

1. F ⇒ I, D ⇒ A, H ⇒ C, B ⇒ E, J ⇒ G.

0345 Nim

Did you find that the first player always lost?

0346 Sequences in Squares

1.

+ 2 →

+ 3 ↓

7	9	11	13	15
10	12	14	16	18
13	15	17	19	21
16	18	20	22	24
19	21	23	25	27

2.

+ 2 →

+ 3 ↓

0	2	4	6	8
3	5	7	9	11
6	8	10	12	14
9	11	13	15	17
12	14	16	18	20

3.

+ 2 →

− 3 ↓

11	13	15	17	19
8	10	12	14	16
5	7	9	11	13
2	4	6	8	10
-1	1	3	5	7

4.

− 2 →

+ 3 ↓

9	7	5	3	1
12	10	8	6	4
15	13	11	9	7
18	16	14	12	10
21	19	17	15	13

5.

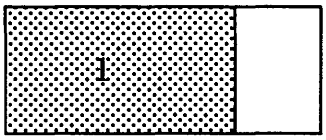
− 2 →

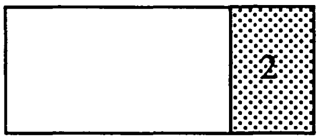
− 3 ↓

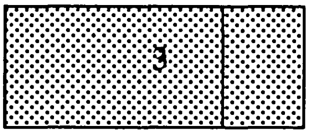
21	19	17	15	13
18	16	14	12	10
15	13	11	9	7
12	10	8	6	4
9	7	5	3	1

0347 How Many Rectangles?

With 1 dividing line there are 3 rectangles.







With 2 dividing lines there are 6 rectangles.
With 3 dividing lines there are 10 rectangles.
4 15
8 45

A mapping diagram with the first few results shows the pattern:

Dividing lines Rectangles
1 ⟶ 3
2 ⟶ 6
3 ⟶ 10
4 ⟶ 15
5 ⟶ 21
6 ⟶ 28

- The number of rectangles is always a **triangle** number.
  You can see the reason for this if you count systematically,
  e.g. for 4 dividing lines:
  Number of single rectangles = 5
  Number of double rectangles = 4
  Number of treble rectangles = 3
  Number of 4-rectangles = 2
  Number of 5-rectangles = 1
  The total number of rectangles is 15 (1 + 2 + 3 + 4 + 5).

With 1 horizontal dividing line:

Vertical dividing lines Rectangles
1 ⟶ 9
2 ⟶ 18
3 ⟶ 30
4 ⟶ 45
5 ⟶ 63

- The number of rectangles is always 3 times a triangle number.

With 2 horizontal dividing lines:

Vertical dividing lines Rectangles
1 ⟶ 18
2 ⟶ 36
3 ⟶ 60
4 ⟶ 90

- The number of rectangles is always 6 times a triangle number.

- In general the numer of rectangles is always the product of 2 triangle number.

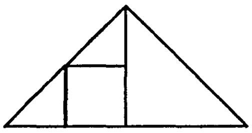
  Which two?

0348 Tangram Teasers

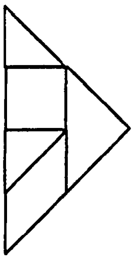
How many other ways did you find of making a triangle using 3 pieces?

Here is one way of making a triangle with:

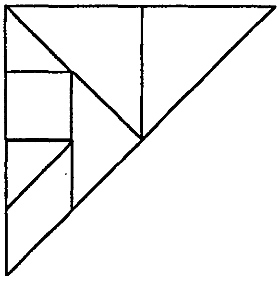
4 pieces



5 pieces



7 pieces.



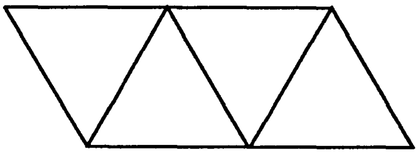
You may have found different ones.

We could not find a triangle using 6 pieces. Could you?

Show your solutions for making a square from tangram pieces to your teacher.

0349 Tetrahedron Nets

1. A tetrahedron has 4 faces.
2. Each face is a triangle.
3. a) 4 triangles. b) 3 triangles. c) 5 triangles
   d) 4 triangles. e) 4 triangles.
4. The only other net of a tetrahedron is (a):



0352 Table Squares

1.

x2 →

x2 ↓

1	2	4	8	16
2	4	8	16	32
4	8	16	32	64
8	16	32	64	128
16	32	64	128	256

2.

x2 →

x2 ↓

3	6	12	24	48
6	12	24	48	96
12	24	48	96	192
24	48	96	192	384
48	96	192	384	768

continued/

0352 Table Squares (cont)

3\. x3 →

÷2 ↓

16	48	244	432	1296
8	24	72	216	648
4	12	36	108	324
2	6	18	54	162
1	3	9	27	81

4\. ÷2 →

÷2 ↓

32	16	8	4	2
16	8	4	2	1
8	4	2	1	$\frac{1}{2}$
4	2	1	$\frac{1}{2}$	$\frac{1}{4}$
2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

5\. x2 →

÷3 ↓

81	162	324	648	1296
27	54	108	216	432
9	18	36	72	144
3	6	12	24	48
1	2	4	8	16

0353 Bowling Tom

1\. 13

2\. 28

3\. 28

4\. 35

5\. 43

6\. 48

7\. 58

8\. 73

0354 Tom the Bowling Champ

Here are all the possible answers. You should have drawn one answer only.

1\. ○○○ or ○○○
⊗○ ○⊗
⊗ ⊗

2\. ○⊗○ or ○⊗○ or ⊗○⊗
⊗○ ○⊗ ○○
⊗ ⊗ ⊗

3\. ⊗○○ or ⊗○○ or ○○⊗ or ○○⊗
⊗○ ○⊗ ⊗○ ○⊗
⊗ ⊗ ⊗ ⊗

4\. ⊗○○ ⊗○○ ○○⊗ ○○⊗
⊗○ or ○⊗ or ⊗○ or ○⊗
○ ○ ○ ○

5\. ⊗⊗○ ⊗⊗○ ○⊗⊗ ○⊗⊗
⊗○ or ○⊗ or ⊗○ or ○⊗
⊗ ⊗ ⊗ ⊗

6\. ○⊗○ ⊗○⊗ ⊗○⊗
⊗⊗ or ⊗○ or ○⊗
⊗ ⊗ ⊗

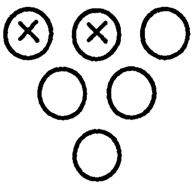
7\. ○⊗○ or ○○○
○○ ⊗⊗
⊗ ⊗

8\. ⊗⊗⊗
⊗⊗
⊗

0355 Bowling Tom's Problem

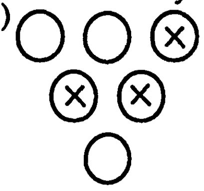
1. You can have these 4 answers in any order:

a)



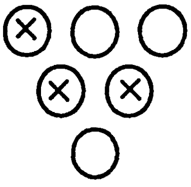
score 35

b)



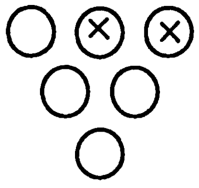
score 35

c)



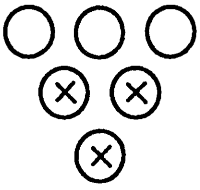
score 35

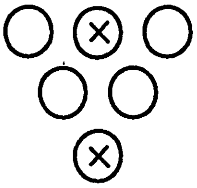
d)



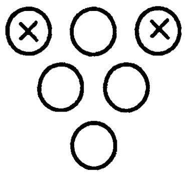
score 35

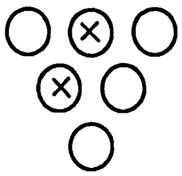
2. There are 2 ways of scoring 23.

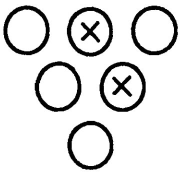




3. There are 3 ways of scoring 30.







0359 How many Colours?

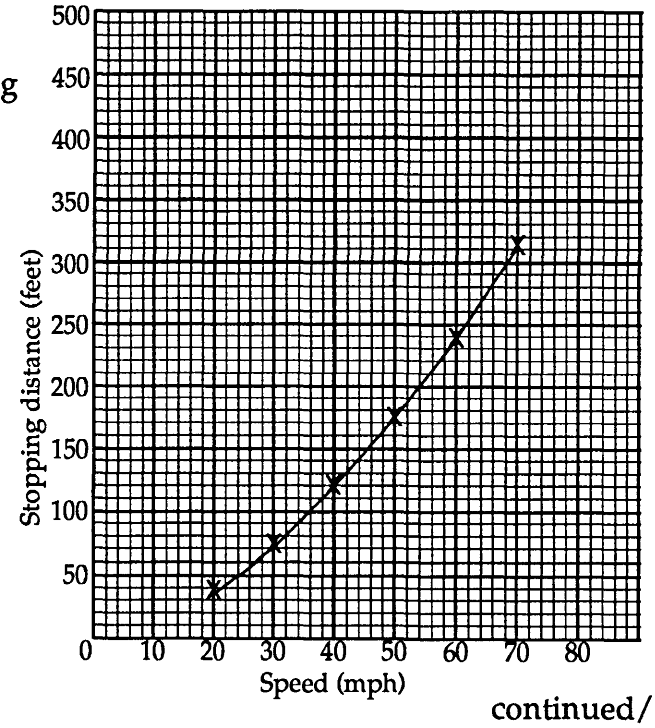
These answers show the **least** number of colours you need for each drawing.

1. 2 colours
2. 2 colours
3. 3 colours
4. 3 colours
5. 4 colours
6. 3 colours
7. 3 colours

0362 No Brakes Bruce

1. A good estimate for Bruce's stopping distance should be between 130 - 165 feet.

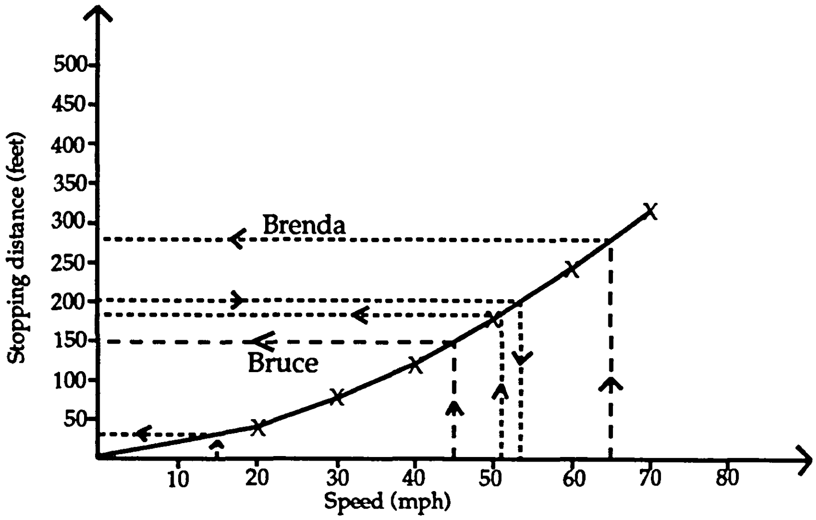
   A good estimate for Brenda's stopping distance should be between 250 - 305 feet.



continued/

0362 No Brakes Bruce (cont)

2.



Bruce's stopping distance is nearly 150 feet.
Brenda's stopping distance is about 280 feet.

3. About 187 feet.

4. About 26 feet.

5. About 54mph.

6. The curve should go through (0, 0) because a car which is standing still (0mph) does not require any stopping distance (0 feet).

0363 Painting Cubes

In the 4 x 4 x 4 cube there are 64 small cubes.

- 8 small cubes have 3 red faces.
  24 small cubes have 2 red faces.
  24 small cubes have 1 red face.
  8 small cubes have 0 red face.

Your table should look something like this:

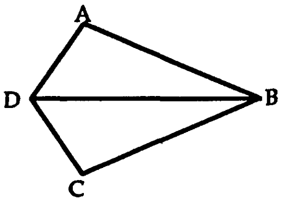
cube	3 red faces	2 red faces	1 red face	0 red face	total
1 x 1 x 1	0	0	0	0	1
2 x 2 x 2	8	0	0	0	8
3 x 3 x 3	8	12	6	1	27
4 x 4 x 4	8	24	24	8	64
⋮	⋮	⋮	⋮	⋮	⋮
n x n x n					

- If you didn't find many patterns, try looking for square numbers and cube numbers.

0364 Using a Triangle

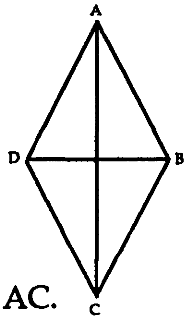
The right-angled triangles in 1772 are congruent.
i.e. corresponding sides are the same length and corresponding angles are the same size.

Kite



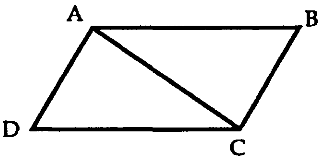
• angles	$\angle A = \angle C$.
• length of sides	Adjacent sides are equal AB = BC, AD=DC.
• diagonals	Diagonals meet at right angles.
• symmetry	A kite has 1 line of symmetry along the diagonal BD

Rhombus



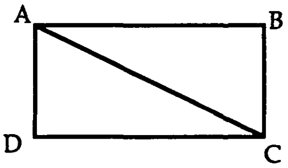
• angles	Opposite angles are equal. $\angle B = \angle D, \angle A = \angle C$.
• parallel lines	Opposite sides are parallel.
• length of sides	All 4 sides are equal.
• diagonals	The diagonals cross at 90° and bisect each other.
• symmetry	There are 2 lines of symmetry BD and AC.

Parallelogram



• angles	Opposite angles are equal. $\angle A = \angle C, \angle B = \angle D$.
• parallel lines	Opposite sides are parallel.
• length of sides	Opposite sides are equal.
• symmetry	There are no lines of symmetry but there is rotational symmetry, order 2.

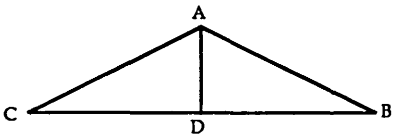
Rectangle

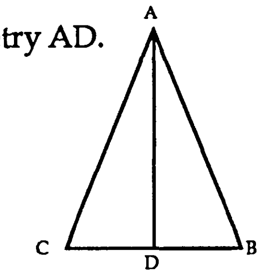


• angles	All angles are 90°.
• parallel lines	Opposite sides are parallel.
• length of sides	Opposite sides are equal.
• diagonals	The diagonals are equal.
• symmetry :	There are 2 lines of symmetry and a rectangle also has rotational symmetry order 2.

Isosceles Triangles

• angles	Two angles are equal. $\angle B = \angle C$.
• length of sides	Two sides are equal. AB = AC.
• symmetry	There is 1 line of symmetry AD.





0365 A Million

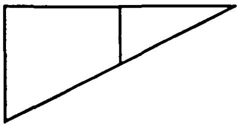
1. You have not lived a million days and it is impossible to know anyone who has. Work out 1 000 000 ÷ 365 and see for yourself.

2. A big book might have 500 words on every page and 1 000 pages. That would be 500 000 words which is only half a million.

3. Your answer will depend upon the size of your desk.
   Find how many cubes you need to cover the desk one layer thick. Then find how many layers by dividing into a million.

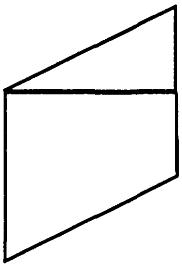
0366 2 Piece Square

- The 2 pieces can be used to make this triangle.

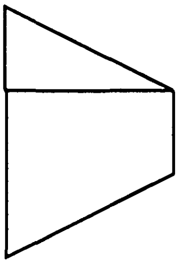


- The 2 pieces can also be used to make:

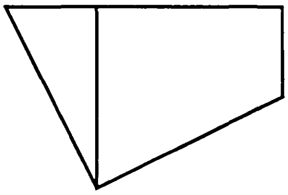
a parallelogram



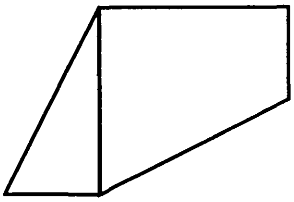
a trapezium



an irregular quadrilateral



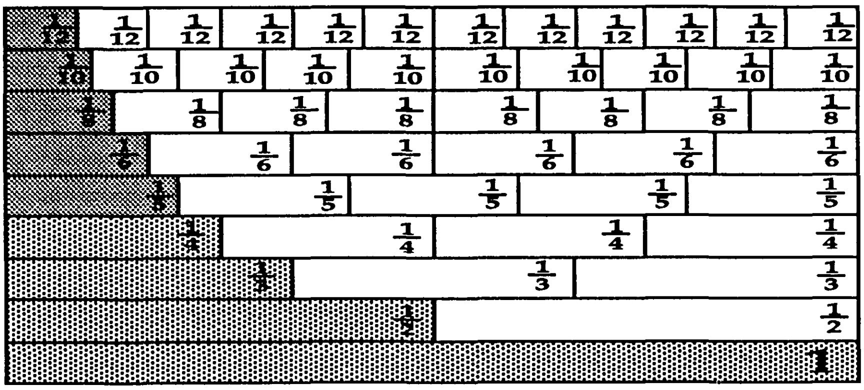
- Here is one way to make a pentagon, you may have found different ones.



0367 Fraction Wall

1. There are four $\frac{1}{4}$ bricks are the same as the whole brick.

2.



continued/

0367 Fraction Wall (cont)

3\. a) $\frac{1}{10}$ is smaller than $\frac{1}{3}$

$\frac{1}{10} < \frac{1}{3}$

b) $\frac{3}{8}$ is smaller than $\frac{3}{4}$

$\frac{3}{8} < \frac{3}{4}$

c) $\frac{2}{3}$ is smaller than $\frac{7}{10}$ is smaller than $\frac{3}{4}$

$\frac{2}{3} < \frac{7}{10} < \frac{3}{4}$

4\. Smallest first $\frac{7}{12}, \frac{3}{5}, \frac{5}{8}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$

5\. Here are some equivalent fractions. If your answers are different, check them with your teacher.

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$

$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$

$\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$

0371 Rotate a Shape

Patterns drawn by rotating a shape can also be created using LOGO.

0376 A Hundred

Show your 6 sentences to your teacher.

These are some ideas for what you might have written about 100.

- 100 is even.
- 100 is in the 2, 4, 5, 10, . . . times table.
- 100 is a square number.
- The square root of a 100 is 10.

Show your answers to your teacher.

0377 Vector Sea

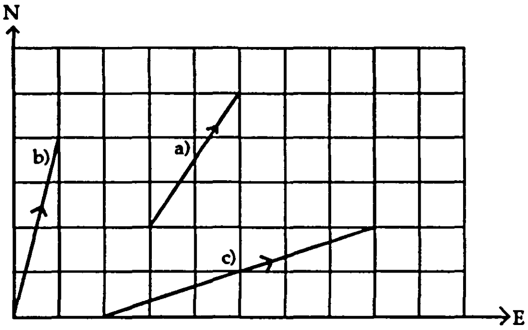
1\. $\begin{pmatrix}1\\3\end{pmatrix}$

2\. a) $\begin{pmatrix}2\\2\end{pmatrix}$ b) $\begin{pmatrix}2\\1\end{pmatrix}$ c) $\begin{pmatrix}3\\0\end{pmatrix}$

continued/

0377 Vector Sea (cont)

3.



4. The captain wrote the vector $\begin{pmatrix}2\\6\end{pmatrix}$.

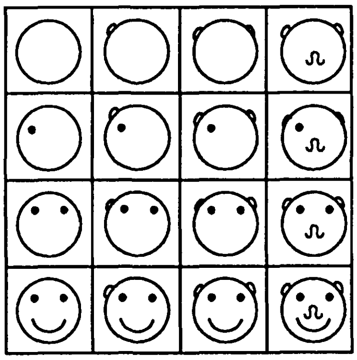
0379 Threeline Victory

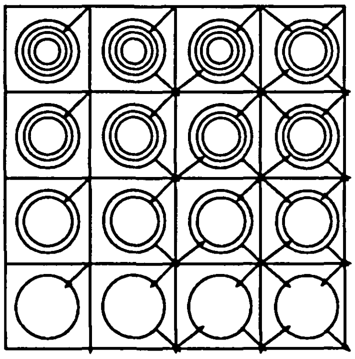
If you enjoyed playing this game you may like to use MicroSMILE program 'Lines'.

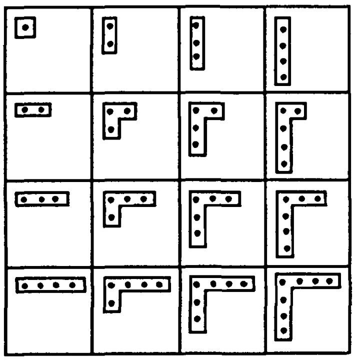
0381 Cuboids

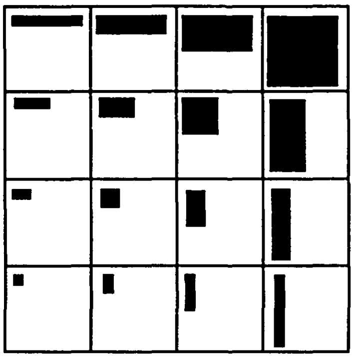
2. You should have found the cuboid with measurements 2l, w and h.
3. There are 6 different arrangements of 4 cuboids.
4. There are 3 different arrangements of 5 cuboids.
   There are 9 different arrangements of 6 cuboids.
5. There are 9 different arrangements of 15 cuboids.
   Check that you can find all of them.

0383 Worksheet



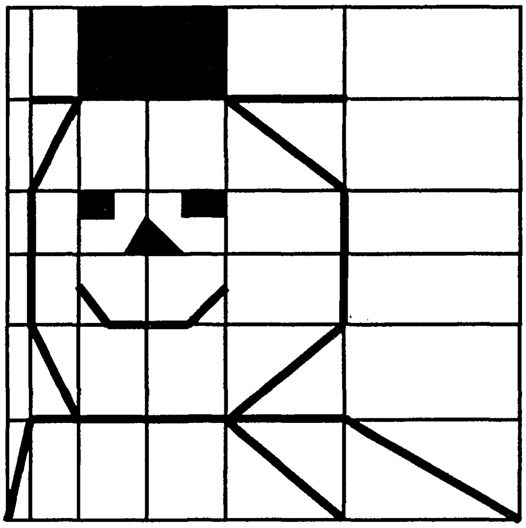






0384 Worksheets

One of your answers should look like this.

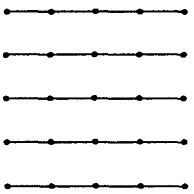


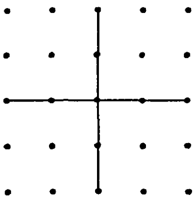
0386 Think of a Number

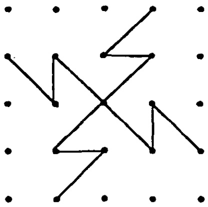
In the first game the answer is always 1.
In the new game, it is always 0.

0387 Quarters

There are many possible answers and many tessellations that can be made from them.
Here are 3 possible answers.







Make a display of your tessellation.

0388 Power

1. $2^1 = 2$
   $2^2 = 2 \times 2 = 4$
   $2^3 = 2 \times 2 \times 2 = 8$
   $2^4 = 2 \times 2 \times 2 \times 2 = 16$
   $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$
   $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$
   $2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$
   $2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$
   $2^9 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$
   $2^{10} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$
   $2^{11} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2048$
   $2^{12} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 4096$

continued/

0388 Power (cont)

2. a) 2^{10} = **1024**

b) 8 x 128
2^3 x 2^7
2^{10} = **1024**

c) 32 x 32
2^5 x 2^5
2^{10} = **1024**

d) 4 x 256
2^2 x 2^8
2^{10} = **1024**

e) 16 x 256
2^4 x 2^8
2^{12} = **4096**

f) 8 x 32 x 16
2^3 x 2^5 x 2^4
2^{12} = **4096**

3. a) 2^3 = **8**

b) 4096 ÷ 256
2^{12} ÷ 2^8
2^4 = **16**

c) 2048 ÷ 16
2^{11} ÷ 2^4
2^7 = **128**

d) 1024 ÷ 512
2^{10} ÷ 2^9
2^1 = **2**

e) 512 ÷ 32
2^9 ÷ 2^5
2^4 = **16**

4. 3^1 = 3
3^2 = 3 x 3 = 9
3^3 = 3 x 3 x 3 = 27
3^4 = 3 x 3 x 3 x 3 = 81
3^5 = 3 x 3 x 3 x 3 x 3 = 243
3^6 = 3 x 3 x 3 x 3 x 3 x 3 = 729
3^7 = 3 x 3 x 3 x 3 x 3 x 3 x 3 = 2187
3^8 = 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 = 6561
3^9 = 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 = 19683
3^{10} = 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 = 59049
3^{11} = 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 = 177147
3^{12} = 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 = 531441

a) 243 x 27
3^5 x 3^3
3^8 = **6561**

b) 81 x 81
3^4 x 3^4
3^8 = **6561**

c) 729 ÷ 81
3^6 ÷ 3^4
3^2 = **9**

d) 2187 ÷ 243
3^7 ÷ 3^5
3^2 = **9**

0390 Surfaces

These answers are probably correct but you might disagree.

Seat of chair	**Kneecap**	**Football**	**Coke can**	**Match**
flat	curved	curved	curved	flat
smooth	smooth	smooth	smooth	rough
hard	hard	hard	hard	hard

New pencil	**Balloon**	**Sellotape**	**Door mat**	**Dice**
flat	curved	curved	flat	flat
smooth	smooth	smooth	rough	smooth
hard	soft	hard	soft	hard

0392 Circumference

We measured the diameter of some round objects and found these results.

Objects	Diameter	Circumference
Roll of Sellotape	9.3cm	28cm
Tin of baked beans	7.4cm	22cm
Top of waste paper bin	19cm	57cm
Bottom of waste paper bin	16.5cm	50cm

By multiplying the diameter by 3 you get a good approximation to the circumference.

3 x diameter is approximately equal to the circumference

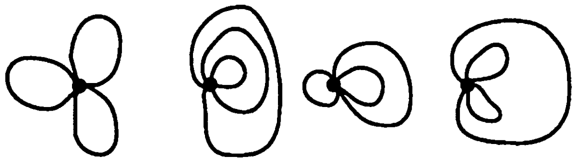
- Check your own results.

For each object, multiply the diameter measurement by 3. Your answer should be close to the measurement for the circumference.

If you are unsure about your answers, check them with your teacher.

0393 Loops

Here are all the diagrams for 1 node and 3 arcs:



For 1 node and 4 arcs, there are 9 different diagrams
For 1 node and 5 arcs, we found 20 different diagrams, but we weren't sure we had found all the answers.
Try to find a systematic method so that you can be sure.

0394 Concentric Circles

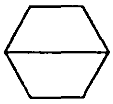
A dartboard is another example of a pattern made from concentric circles. Can you draw it?

0396 Hexagons

Have you divided the hexagon into 2 equal parts (in 2 different ways), 3 equal parts, 4 equal parts, 6 equal parts and 12 equal parts?

- What shapes did you get by dividing the hexagon into equal parts?

  e.g. one way of dividing a hexagon into two equal parts gives 2 trapezia.



0397 Operations

Clock Arithmetic

3. $4 \oplus 10 = 2$

4. a) $6 \oplus 7 = 1$ c) $2 \oplus 6 = 8$ e) $7 \oplus 7 = 2$

   b) $11 \oplus 4 = 3$ d) $8 \oplus 5 = 1$ f) $12 \oplus 12 = 12$

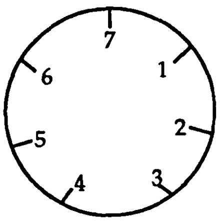
6. Second number

First number

⊕	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	1
2	3	4	5	6	7	8	9	10	11	12	1	2
3	4	5	6	7	8	9	10	11	12	1	2	3
4	5	6	7	8	9	10	11	12	1	2	3	4
5	6	7	8	9	10	11	12	1	2	3	4	5
6	7	8	9	10	11	12	1	2	3	4	5	6
7	8	9	10	11	12	1	2	3	4	5	6	7
8	9	10	11	12	1	2	3	4	5	6	7	8
9	10	11	12	1	2	3	4	5	6	7	8	9
10	11	12	1	2	3	4	5	6	7	8	9	10
11	12	1	2	3	4	5	6	7	8	9	10	11
12	1	2	3	4	5	6	7	8	9	10	11	12

7. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

9.



Second number

First number

⊕	1	2	3	4	5	6	7
1	2	3	4	5	6	7	1
2	3	4	5	6	7	1	2
3	4	5	6	7	1	2	3
4	5	6	7	1	2	3	4
5	6	7	1	2	3	4	5
6	7	1	2	3	4	5	6
7	1	2	3	4	5	6	7

- Yes, {1, 2, 3, 4, 5, 6, 7} is closed under ⊕ 'addition'.

continued/

Using Cards

2. Card B

3. Second card

First card

On top of	A	B	C	D
A	A	D	A	D
B	D	B	B	D
C	A	B	C	D
D	D	D	D	D

4. Yes, {A, B, C, D} is closed under the operation 'on top of'.

5. C on top of A = **A**
C on top of B = **B**
C on top of C = **C**
C on top of D = **D**

A on top of C = **A**
B on top of C = **B**
C on top of C = **C**
D on top of C = **D**

6. C is the identity.

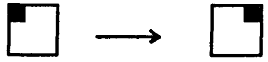
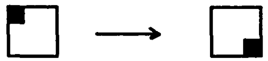
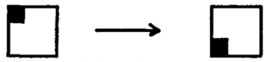
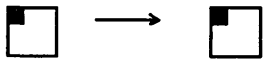
7. C is the identity of {A, B, C, D } under the operation 'on top of' because it causes no change.

8. The row belonging to the identity is the same as the top of the table.
The column belonging to the identity is the same as the side of the table.

9. The identity is 12.

10. The identity for Martian clock numbers under 'addition' is 7.

Rotation

A	Rotate through 90°	
B	Rotate through 180°	
C	Rotate through 270°	
D	Rotate through 360°	

4. A followed by B gives the same result as C.

5. A

6. Second Instruction

First Instruction

	A	B	C	D
A	B	C	D	A
B	C	D	A	B
C	D	A	B	C
D	A	B	C	D

7. Yes, all the letters in the table belong to {A, B, C, D}.

8. D is the identity because it causes no change.

continued/

0397 Operations (cont)

Ordinary addition

1.

+	1	3	5	7
1	2	4	6	8
3	4	6	8	10
5	6	8	10	12
7	8	10	12	14

+	0	2	4	6
0	0	2	4	6
2	0	4	6	8
4	4	6	8	10
6	6	8	10	12

2. No

3. No

4. No

5. Yes. The identity is 0.

6.

X	1	3	5	7
1	1	3	5	7
3	3	9	15	21
5	5	15	25	35
7	7	21	35	49

a) No, the set is not closed.
b) 1 is the identity.

X	0	2	4	6
0	0	0	0	0
2	0	4	8	12
4	0	8	16	24
6	0	12	24	36

No, the set is not closed.
There is no identity.

Rotating Blackboard

2. {A, B, C} is closed.

3. C is the identity as it causes no change.

5. A ✻ B = C

   B ✻ A = C

- A and B combine in either order to give C, the identity. So A and B are the inverses of each other.

6. C is the inverse of C.

7. C is the inverse of A.

8. The inverse is 5.

continued/

Remainders

1. Second number

First number

⊗	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

2. Yes, {1, 2, 3, 4, 5, 6} is closed.

3. The identity is 1.

4. The inverse of 1 is 1.
   The inverse of 2 is 4.
   The inverse of 3 is 5.
   The inverse of 4 is 2.
   The inverse of 4 is 3.
   The inverse of 6 is 6.

5.

$\boxed{X}$	1	2	3
1	1	2	3
2	2	0	2
3	3	2	1

6. a) {1, 2, 3} is not closed under the operation $\boxed{X}$.
   b) 1 is the identity.
   c) $\left.\begin{array}{l}\text{1 is the inverse of 1.}\\ \text{3 is the inverse of 3.}\end{array}\right\}$ i.e. they are self inverses.
   2 has no inverse.

An Exercise

1. a) Yes
   b) Y
   c) The inverse of X is Z.
   Y is a self inverse.
   The inverse Z is X.

2. a) Yes
   b) No
   c) Therefore, no inverses.

3. a) No
   b) 7
   c) They are all self inverses.

4. a) Yes
   b) A
   c) A and D are self inverse
   The inverse of B is C.
   The inverse of C is B.

continued/

0397 Operations (cont)

5.

⊕	K	L	M
K	L	M	K
L	M	K	L
M	K	L	M

Permutations

2.

	E	F	G	H	J	K
E	K	H	J	F	G	E
F	J	K	H	G	E	F
G	H	J	K	E	F	G
H	G	E	F	J	K	H
J	F	G	E	K	H	J
K	E	F	G	H	J	K

a) It is closed.
b) K is the identity
c) E, F, G and K are self inverses
J is the inverse of H
H is the inverse of J

0398 4 + 3 x 2

1. (2 + 3) x 4 = 20
or 2 + (3 x 4) = 14

2. 10 – (2 + 3) = 5
or (10 – 2) + 3 = 11

3. (4 x 5) + (6 – 5) = 21
or 4 x (5 + 6 – 5) = 24
or 4 x (5 + 6) – 5 = 39

4. 16 – (8 – 4 – 2 – 1) = 15
or 16 – 8 – (4 – 2 – 1) = 7
or 16 – 8 – 4 – (2 – 1) = 3
or 16 – (8 – 4 – 2) – 1 = 13
or 16 – (8 – 4) – (2 – 1) = 11
or (16 – 8 – 4 – 2) – 1 = 1
or 16 – 8 – (4 – 2) – 1 = 5
or 16 – (8 – 4) – 2 – 1 = 9

5. 35

6. 9

7. 24

8. 2

9. 10

10. 18

11. 1

12. ((3 + 5) x 2) + 7
= (8 x 2) + 7
= 16 + 7
= 23

13. 2 x (13 – (15 ÷ 3))
= 2 x (13 – 5)
= 2 x 8
= 16

14. 16 ÷ (16 ÷ (16 ÷ 8))
= 16 ÷ (16 ÷ 2)
= 16 ÷ 8
= 2

15. (5 x 4) – 1 = 19

16. 2 + (3 x 10) = 32

17. 24 ÷ (6 + 2) = 3

18. (10 – 5) x (2 + 7) = 45

0399 Cubes

There are 6 different cuboids you can make with 24 cubes:

	Total surface area in sq. units.	Total edge length in units
24 x 1 x 1	2[(24 x 1) + (24 x 1) + (1 x 1)] = 98	4(24 + 1 + 1)= 104
12 x 2 x 1	2[12 x 2) + (12 x 1) + (2 x 1)] = 76	4(12 + 2 + 1) = 60
8 x 3 x 1	2[(8 x 3) + (8 x 1) + (3 x 1)] = 70	4(8 + 3 + 1) = 48
6 x 4 x 1	2[(6 x 4) + (6 x 1) + (4 x 1)] = 68	4(6 + 4 + 1) = 44
4 x 3 x 2	2[4 x 3) + (4 x 2) + (3 x 2)] = 52	4(4 + 3 + 2) = 36
6 x 2 x 2	2[(6 x 2) + (6 x 2) + (2 x 2)] = 56	4(6 + 2 + 2) = 40

- The cuboid with the largest total surface area is the 24 x 1 x 1 cuboid.
- The cuboid with the smallest total surface area is the 4 x 3 x 2 cuboid.
- The cuboid with the longest total edge length is the 24 x 1 x 1 cuboid.
- The cuboid with the shortest total edge length is the 4 x 3 x 2 cuboid.

With 36 cubes there are 8 different cuboids:

	Total surface area in sq. units.	Total edge length in units
36 x 1 x 1	2[(36 x 1) + (36 x 1) + (1 x 1)] = 146	4(36 + 1 + 1) = 152
18 x 2 x 1	2[(18 x 2) + (18 x 1) + (2 x 1)] = 112	4(18 + 2 + 1) = 84
12 x 3 x 1	2[(12 x 3) + (12 x 1) + (3 x 1)] = 102	4(12 + 3 + 1) = 64
9 x 4 x 1	2[(9 x 4) + (9 x 1) + (4 x 1)] = 98	4(9 + 4 + 1) = 56
9 x 2 x 2	2[(9 x 2) + (9 x 2) + (2 x 2)] = 80	4(9 + 2 + 2) = 52
6 x 6 x 1	2[(6 x 6) + (6 x 1) + (6 x 1)] = 96	4(6 + 6 + 1) = 52
6 x 3 x 2	2[(6 x 3) + (6 x 2) + (3 x 2)] = 72	4(6 + 3 + 2) = 44
4 x 3 x 3	2[(4 x 3) + (4 x 3) + (3 x 3)] = 66	4(4 + 3 + 3) = 40

- The cuboid with the largest total surface area is the 36 x 1 x 1 cuboid.
- The cuboid with the smallest total surface area is the 4 x 3 x 3 cuboid.
- The cuboid with the longest total edge length is the 36 x 1 x 1 cuboid.
- The cuboid with the shortest total edge length is the 4 x 3 x 3 cuboid.

With 48 cubes there are 9 different cuboids:

48 x 1 x 1
16 x 3 x 1
24 x 2 x 1
6 x 4 x 2
8 x 6 x 1
12 x 4 x 1
8 x 3 x 2
4 x 4 x 3
12 x 2 x 2

- 48 x 1 x 1 has the largest surface area (194 sq. units) and the greatest total edge length (200 units).
- 4 x 4 x 3 has the smallest surfaces area (80 sq. units) and the shortest total edge length (44 units).

0400 Folding Symmetry

Draw the line of symmetry on each of your shapes.

0401 Add 'em

What methods did you use to add decimals in your head?

0402 Adding Fractions

1. $\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \frac{9}{10}$

2. $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

3. $\frac{1}{2} + \frac{3}{5} = \frac{5}{10} + \frac{6}{10} = \frac{11}{10} = 1\frac{1}{10}$

4. $\frac{4}{7} + \frac{1}{3} = \frac{12}{21} + \frac{7}{21} = \frac{19}{21}$

5. $\frac{1}{6} + \frac{2}{3} = \frac{1}{6} + \frac{4}{6} = \frac{5}{6}$

6. $\frac{3}{4} + \frac{4}{5} = \frac{15}{20} + \frac{16}{20} = \frac{31}{20} = 1\frac{11}{20}$

7. $\frac{3}{8} + \frac{7}{12} = \frac{9}{24} + \frac{14}{24} = \frac{23}{24}$

8. $\frac{5}{6} + \frac{8}{9} = \frac{15}{18} + \frac{16}{18} = \frac{31}{18} = 1\frac{13}{18}$

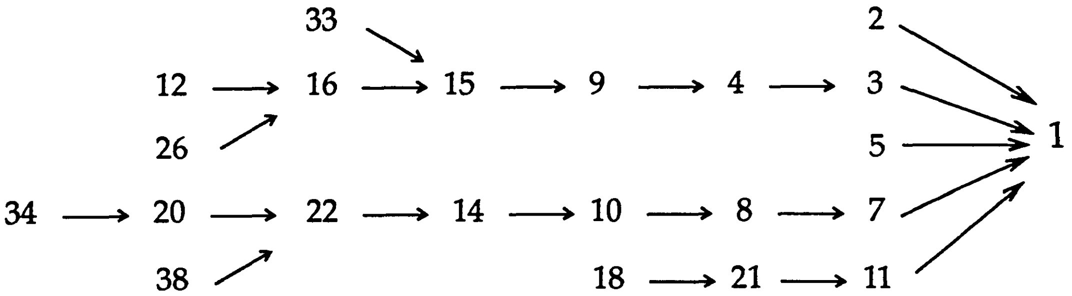
9. $\frac{5}{7} + \frac{3}{5} = \frac{25}{35} + \frac{21}{35} = \frac{46}{35} = 1\frac{11}{35}$

10. $\frac{1}{7} + \frac{1}{8} = \frac{8}{56} + \frac{7}{56} = \frac{15}{56}$

Your answers may differ if you chose different equivalent fractions to add, but each of your answers should be an equivalent fraction to the answer given here. If you are not sure, check with your teacher.

0403 Factor Chains

Here is part of a factor chain diagram.



continued/

0403 Factor Chains (cont)

- What happens if you start a factor chain with 6 or 28?
  The Greeks thought that numbers like 6 and 28 were special and they called them perfect numbers.

  To find out more you may like to read pages 144 - 151 in "Mathematics on Vacation" written by Joseph S. Madachy.

0404 Solids

Cube:	8 vertices 6 surfaces 12 edges	Cylinder:	0 vertices 3 surfaces 2 edges
Triangular Prism:	6 vertices 5 surfaces 9 edges	Tetrahedron:	4 vertices 4 surfaces 6 edges
Square-based pyramid:	5 vertices 5 surfaces 8 edges	Cuboid:	8 vertices 6 surfaces 12 edges

If you have drawn other solids and counted their vertices, surfaces and edges, show you answers to your teacher.

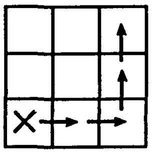
0406 Two Folds

Draw the two lines of symmetry on your shape.

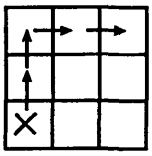
0408 Cube Moving

There are several different paths the X-cube can take. Some of them are reflections of each other.

This

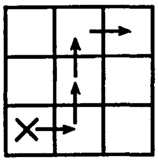


is a reflection of

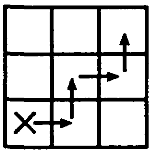


and needs 17 moves.

This route



takes 15 moves but the 'best' route is the staircase which takes 13 moves.



Looking carefully at the best route, you might have noticed that the first step of the X-cube took 4 moves and every other step took 3.

Is the staircase route the best on a 4 x 4 board?
5 x 5 board?
6 x 6 board?
.
.
.

continued/

0408 Cube Moving (cont)

Can you predict the minimum number of moves for a n x n board?

Investigate for rectangle boards 2 squares wide:

$3 \times 2 \longrightarrow \quad 9$
$4 \times 2 \longrightarrow$
$5 \times 2 \longrightarrow$
.
.
.
$n \times 2 \longrightarrow$

Investigate for rectangle boards 3 squares wide, 4 squares wide, . . .
Show all your results in a table.

	2	3	4	5	6
2	5	9			
3	9	13			
4					
5					
6					

Describe and explain the patterns in the table.

0409 Pins

- The band is touching 9 pins on the perimeter.
- There are 2 pins inside.
- The area of the shape is $5\frac{1}{2}$ squares.

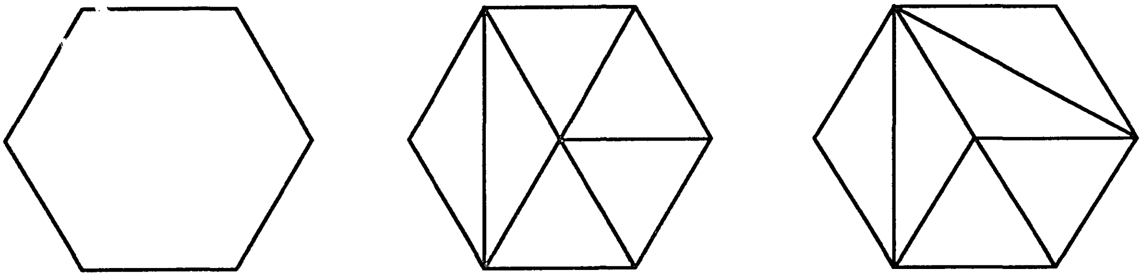
There are three variables in this investigation, the pins on the perimeter (p), the pins inside (n) and the area (A). Try several shapes with 2 pins inside and record your results in a table, trying to organise them in order of size.

Pins inside(n)	Pins on perimeter (p)	Area (A)
2	6	4
2	7	$4\frac{1}{2}$
2	8	5
2	9	$5\frac{1}{2}$
2	.	.
2	.	.
2	.	.
2	p	.

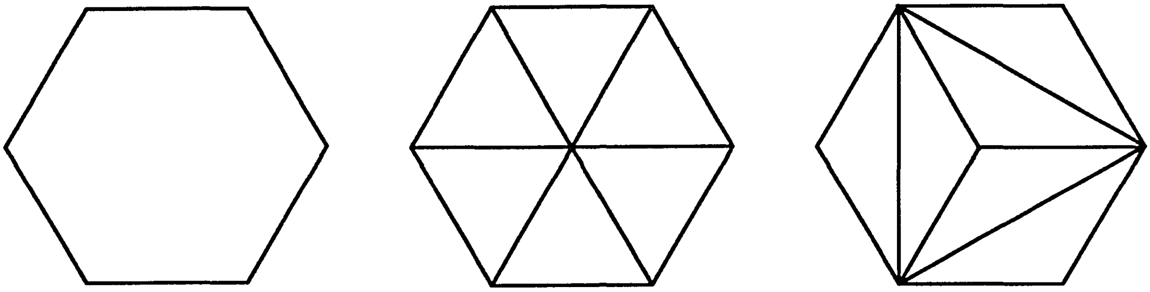
Describe what happens for 0 pins inside, 1 pin inside, 3 pins inside, . . . n pins inside.

0411 Hexagon Dissection

Here is one way to use the 13 pieces to make 3 separate hexagons.



Here is another way to use the 13 pieces to make 3 separate hexagons.



0412 Bracelets

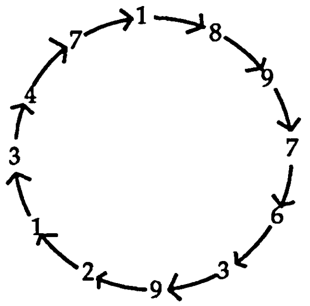
- Each number in the bracelet is obtained by adding the two previous number, dividing the answer by 10, and writing down the remainder.
  This is called modulo arithmetic.
- The series does not go on for ever.
- Altogether there are 6 bracelets (including 0 ⟶ 0).

This is a bracelet with 12 members:

What is the sum of diametrically opposite numbers?

What is the sum of all the digits in this bracelet?

Answer these questions for the other bracelets?



In investigations like this it is important to set out your work in an ordered way to avoid repetition. One way would be to make a table to contain all the possible combinations of numbers. Different colours could be used to indicate different bracelets.

0414 Bi-Fractions

2	=	10 (base two)	1/2	$=^1/_{10}$ (base two)
4	=	100 (base two)	1/4	$=^1/_{100}$ (base two)
3	=	11 (base two)	1/3	$=^1/_{11}$ (base two)

continued/

0414 Bi-Fractions (cont)

To change a fraction to a decimal, divide the numerator by the denominator.

$$\begin{array}{rl} & \underline{0.0101\ldots} \\ 11\,) & 1.0000\ldots \\ & 11 \\ & 100 \\ & \;\;11 \\ & \;\;\;1 \text{ etc. so } {}^{1}/_{3} = 0.01\dot{0}\dot{1} \text{ (base two)} \end{array}$$

The investigation will be easier if you look for patterns.

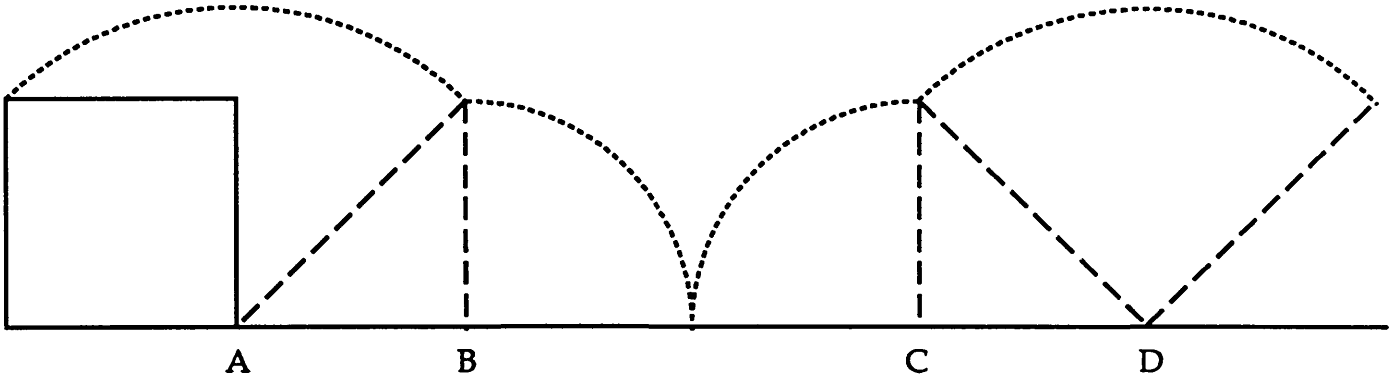
- Base ten fractions can be set up in an array like this:

$$\begin{array}{ccccc} {}^{1}/_{2} & {}^{1}/_{3} & {}^{1}/_{4} & {}^{1}/_{5} & {}^{1}/_{6}\ldots \\ & {}^{2}/_{3} & {}^{2}/_{4} & {}^{2}/_{5} & {}^{2}/_{6}\ldots \\ & & {}^{3}/_{4} & {}^{3}/_{5} & {}^{3}/_{6}\ldots \\ & & & {}^{4}/_{5} & {}^{4}/_{6}\ldots \end{array}$$

You might use this to help you discover patterns in bi-fractions.

0415 Follow the Path

- This shows the path (locus) of the corner of a 3cm square, rotated along a straight line.



- The path could be described as:
  a quarter circle with radius 4.2cm, centre A, followed by
  a quarter circle with radius 3cm, centre B, followed by
  a quarter circle with radius 3cm, centre C, followed by
  a quarter circle with radius 4.2cm, centre D . . .
- How would you describe the path of the centre of the square?
- You could describe the path of your own shapes in a similar way.

0416 Eggs

These are some things to consider.

- The boxes should stack, but not in a complicated way.
- The boxes should fit together neatly side by side, perhaps they should tessellate.
- What shape would be best for the packing case?
  The boxes should fit inside it.
- The boxes should not be awkward in a shopping basket.
- The boxes should be stable so that they don't fall off the shelf.

What shape did you choose?

Recently some boxes are designed to hold 10 eggs. Compare your design to these.

0417 String

The area of the shape could range from 0cm^2 - 103cm^2 to the nearest cm^2.

Here are some things to consider when investigating the possible areas.

- Try looking at one type of shape only, e.g. rectangle.
  Which is the largest rectangle?
  Which is the smallest rectangle?
- Try other shapes, e.g. triangles, hexagons, . . .
  Which triangle has the largest area?
  Which hexagon has the largest area?
- What is the largest shape you can make?
- Try looking at regular polygons in order of number of sides.

You may like to use a spreadsheet to calculate results more quickly. Here is part of a spreadsheet to find all possible rectangles with sides of integer lengths.

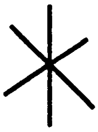
Height	Width	Area
1	17	17
2	16	32
3	15	45
4	14	56
5	13	65
6	12	72
7	11	77
8	10	80
9	9	81
10	8	80

A graph of these results will allow you to see the rectangle with the largest area more easily.

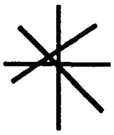
0421 Cross Dots

Hint: Before you start your investigation you will need to decide how you are going to define an intersection with more than two lines.

Is this just one intersection?



How many intersections is this?



- With 2 crosses and 3 dots, you get 3 intersections.
  This table shows what happens with 2 crosses and different numbers of dots.

No. of Crosses	No. of Dots	No. of Intersections
2	3	3
2	4	6
2	5	.
2	6	.
2	7	.
2	8	.

Look for and describe any patterns in the number of intersections.
Can you find the general rule?

If you start with 3 crosses there are 2 different sorts of intersections:

- points at which 2 lines cross
- points at which 3 lines cross

No. of Crosses	No. of Dots	No. of Points at which 2 lines cross	No. of Points at which 3 lines cross
3	3	6	1
3	4	.	.
3	5	.	.
3	6	.	.
3	7	.	.

Investigate what happens

- if you start with 4 crosses?
- if you start with dots and crosses not evenly spaced?
- if you start with dots and crosses on perpendicular lines?

0422 Some Sums

45 can be made in 5 different ways:
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9
5 + 6 + 7 + 8 + 9 + 10
7 + 8 + 9 + 10 + 11
14 + 15 + 16
22 + 23

There are also 5 ways of expressing 63, 75, 81, 90, 99, . . .as the sum of consecutive numbers.

Looking at 2, 3, 4, 5, 6, 7 and 8 consecutive numbers it is possible to find consecutive sums for the first 100 numbers except 88 and powers of 2.

It is important to look for rules and patterns and these are found when you work systematically. e.g. Which numbers can be expressed as the sum of

• **two** consecutive whole numbers	3,	5,	7,	9,	11,	. . .
• **three** consecutive whole numbers	6,	9,	12,	15,	18,	. . .
• **four** consecutive whole numbers	10,	14,	18,	22,	26,	. . .
• five consecutive whole numbers	15,	20,	25,	30,	35,	. . .

⋮

Describe any patterns you notice.

- How do the numbers increase in each sequence?
- What is the first number in each pattern?
- Can you write a rule for each pattern?
- Can you write a general rule which determines how many consecutive sums each number has?

0423 Clock Arithmetic

1. 9
2. 1
3. 2
4. 6
5. 9
6. 8
7. 7
8. 7
9. 1
10. 11
11. 5
12. 1
13. 11
14. 12
15. 12
16. 2
17. 7
18. 12

0424 How Many Routes?

1.

	A	B	C	D
A	0	0	1	2
B	0	0	1	0
C	1	1	0	1
D	2	0	1	0

2.

	A	B	C	D
A	0	1	1	1
B	1	0	1	1
C	1	1	0	1
D	1	1	1	0

continued/

0424 How Many Routes? (cont)

3.

	A	B	C	D	E
A	0	1	0	2	0
B	1	0	2	0	0
C	0	2	0	1	0
D	2	0	1	0	1
E	0	0	0	1	0

4.

	A	B	C	D
A	2	1	0	0
B	1	0	1	2
C	0	1	0	0
D	0	2	0	2

5.

	W	X	Y	Z
W	2	1	0	0
X	1	0	1	2
Y	0	1	0	0
Z	0	2	0	2

6. The last two tables are the same. This is because the networks are topologically equivalent.

0425 Colour Competition

If you think you have found a network which needs 5 colours show it to your teacher.

0426 Traversable?

1.	Yes	2.	Yes	3.	Yes	4.	Yes
5.	No	6.	No	7.	Yes	8.	No

9.

	Number of odd nodes	Number of even nodes	Is it traversable? Yes or No?
1.	2	4	Yes
2.	0	4	Yes
3.	0	5	Yes
4.	2	1	Yes
5.	4	1	No
6.	8	0	No
7.	2	3	Yes
8.	4	2	No

10. If the number of odd nodes is 2 or less, the network is traversable.

11. Did the rule work for the networks that you have drawn? If not, get someone to check your results.

0427 Cut Half Shuffle

10 shuffles are needed to get 10 cards back into the original order:

1	2	4	8	5	10	9	7	3	6	1
2	4	8	5	10	9	7	3	6	1	2
3	6	1	2	4	8	5	10	9	7	3
4	8	5	10	9	7	3	6	1	2	4
5	10	9	7	3	6	1	2	4	8	5
6	1	2	4	8	5	10	9	7	3	6
7	3	6	1	2	4	8	5	10	9	7
8	5	10	9	7	3	6	1	2	4	8
9	7	3	6	1	2	4	8	5	10	9
10	9	7	3	6	1	2	4	8	5	10

The piles of cards are shown by the **columns.**
If you look at the rows you will see this cycle in each row.

$$\begin{tikzcd}
 & 1 \arrow[r] & 2 \arrow[r] & 4 \arrow[r] & 8 \arrow[dr] & \\
6 \arrow[ur] & & & & & 5 \arrow[dl] \\
 & 3 \arrow[ul] & 7 \arrow[l] & 9 \arrow[l] & 10 \arrow[l] &
\end{tikzcd}$$

Only 6 shuffles are needed for 8 cards:

1	2	4	8	7	5	1
2	4	8	7	5	1	2
3	6	3	6	3	6	3
4	8	7	5	1	2	4
5	1	2	4	8	7	5
6	3	6	3	6	3	6
7	5	1	2	4	8	7
8	7	5	1	2	4	8

There are 2 distinct cycles:

$$\begin{tikzcd}
 & 1 \arrow[r] & 2 \arrow[r] & 4 \arrow[dr] & \\
5 \arrow[ur] & & & & 8 \arrow[dl] \\
 & {} \arrow[ul] & 7 \arrow[l] & {} \arrow[l] &
\end{tikzcd}
\qquad
\begin{tikzcd}
 & {} \arrow[r] & {} \arrow[dr] & \\
3 \arrow[ur] & & & 6 \arrow[dl] \\
 & {} \arrow[ul] & {} \arrow[l] &
\end{tikzcd}$$

Look at the top cards for each set of shuffles, (the numbers at the top of each column), they all seem to start with 1, 2, 4, 8, . . .

0428 One Difference Logichain

Did you manage to make a one-difference logichain which used all 64 logiblocks?

0429 Squaring

1. $3^2 < 3.8^2 < 4^2$
   $9 < 3.8^2 < 16$
   3.8^2 is approximately 14

2. $4^2 < 4.3^2 < 5^2$
   $16 < 4.3^2 < 25$
   4.3^2 is approximately 19.

continued/

0429 Squaring (cont)

3. $6^2 < 6.4^2 < 7^2$
$36 < 6.4^2 < 49$
6.4^2 is approximately 41.

4. $7^2 < 7.5^2 < 8^2$
$49 < 7.5^2 < 64$
7.5^2 is approximately 56.

5. $1^2 < 1.2^2 < 2^2$
$1 < 1.2^2 < 4$
1.2^2 is approximately 1.5.

6. $14^2 < 14.1^2 < 15^2$
$196 < 14.1^2 < 225$
14.1^2 is approximately 199.

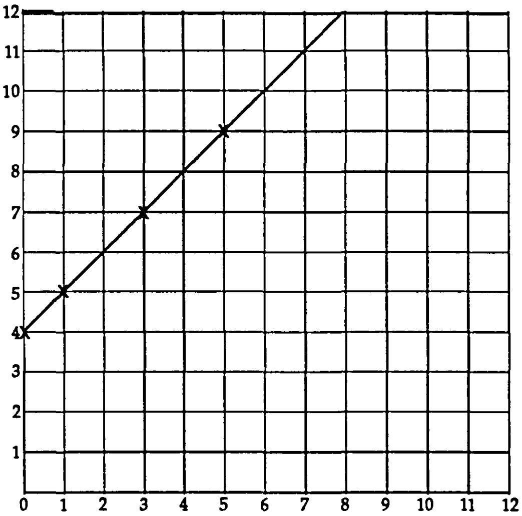
7. $11^2 < 11.7^2 < 12^2$
$121 < 11.7^2 < 144$
11.7^2 is approximately 136.

Your approximations will probably not be exactly the same. You can check how accurate you were by using a calculator.

0430 Parallel Lines

1.

x	→	x + 4	
0	→	4	(0, 4)
1	→	5	(1, 5)
3	→	7	(3, 7)
5	→	9	(5, 9)



3. Yes all the points on the line follow the rule x → x + 4.

4.

x	→	x + 5	
0	→	5	(0, 5)
1	→	6	(1, 6)
2	→	7	(2, 7)
3	→	8	(3, 8)
4	→	9	(4, 9)
5	→	10	(5, 10)

x	→	x + 6	
0	→	6	(0, 6)
1	→	7	(1, 7)
2	→	8	(2, 8)
3	→	9	(3, 9)
4	→	10	(4, 10)
5	→	11	(5, 11)

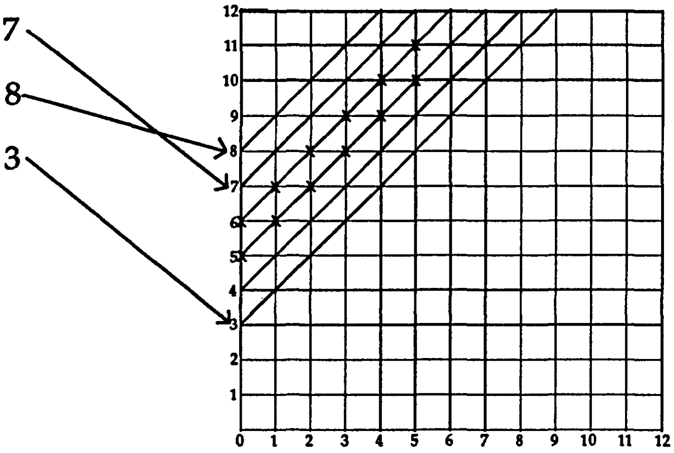
continued/

0430 Parallel Lines (cont)

5. x → x + 7

x → x + 8

x → x + 3



6. They are all parallel.

7. a) The line x → x + 4 crosses the y-axis at (0, 4)
b) The line x → x + 7 crosses the y-axis at (0, 7)
The line x → x + 8 crosses the y-axis at (0, 8)
The line x → x + 3 crosses the y-axis at (0, 3)

c) The line x → x + 12 crosses the y-axis at (0, 12)

Any mapping of the form x → mx + c will cross the y axis at c.

0431 Tower of Hanoi

It is easier to spot the pattern if you start with 3 discs. For 3 discs the least number of moves is 7.

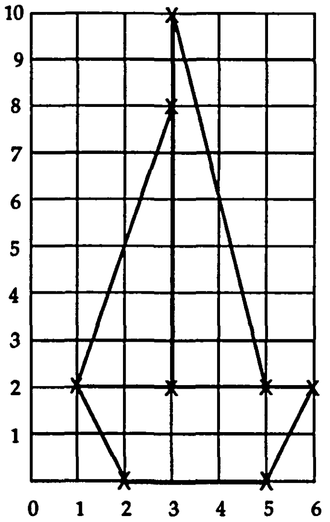
3 discs	→	7 moves
4 discs	→	15 moves
5 discs	→	31 moves

Can you spot a rule for *any* number?

0432 Moving Pictures

1. (3, 5)
(3, 4)
(1, 1)
(3, 1)
(5, 1)
(6, 1)
(2, 0)
(5, 0)

2. (3, 10)
(3, 8)
(1, 2)
(3, 2)
(5, 2)
(6, 2)
(2, 0)
(5, 0)

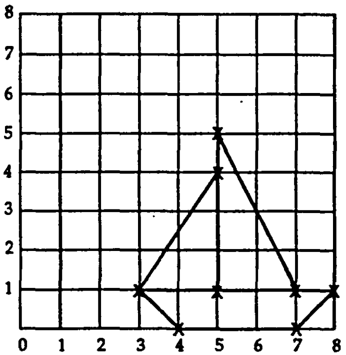


The picture has doubled in height.

continued/

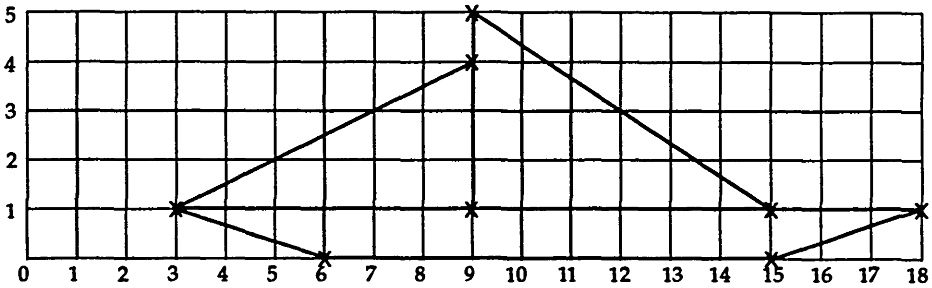
0432 Moving Pictures (cont)

3. (5, 5)
(5, 4)
(3, 1)
(5, 1)
(7, 1)
(8, 1)
(4, 0)
(7, 0)



The picture has moved 2 squares to the right.

4. (9, 5)
(9, 4)
(3, 1)
(9, 1)
(15, 1)
(18, 1)
(6, 0)
(15, 0)



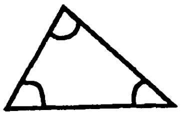
The picture has trebled in width.

5. The picture will treble in height.

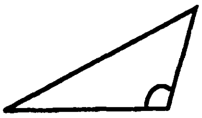
6. The picture will move up 2 squares.

0433 Acute/Obtuse

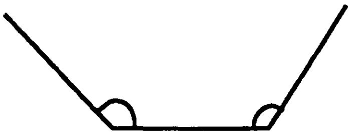
1.



2.

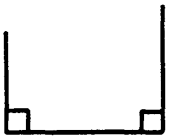


3.



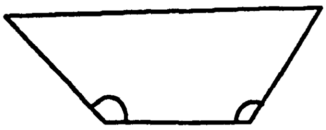
Impossible

4.

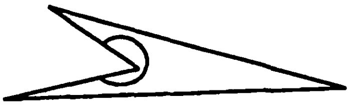


Impossible

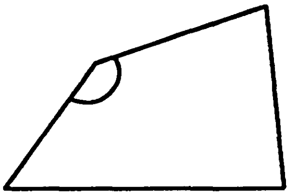
5.



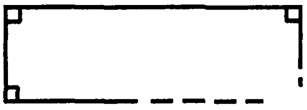
6.



7.



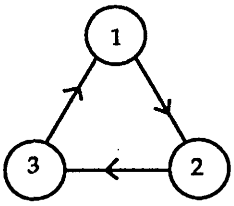
8.



If there are 3 right angles,
the other angle must also be a right angle.

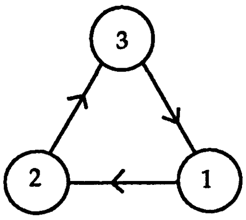
0435 Cycles

If you start with 3 numbers



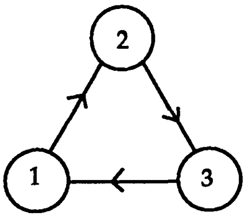
gives 1 → 2 → 1 → 2 . . .
and 3 → 3 → 3 → 3 . . .

or



which gives 3 → 3 → 3 → . . .
and 1 → 2 → 1 → 2 → . . .

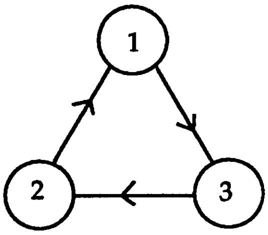
or



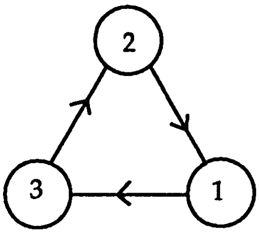
which gives 2 → 1 → 2 → 1 → . . .
and 3 → 3 → 3 → 3 →. . .

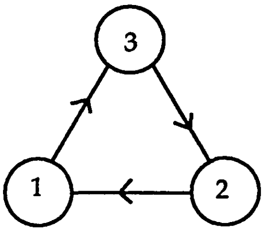
Can you see why these three give the same result?

Three other possibilities would be



and





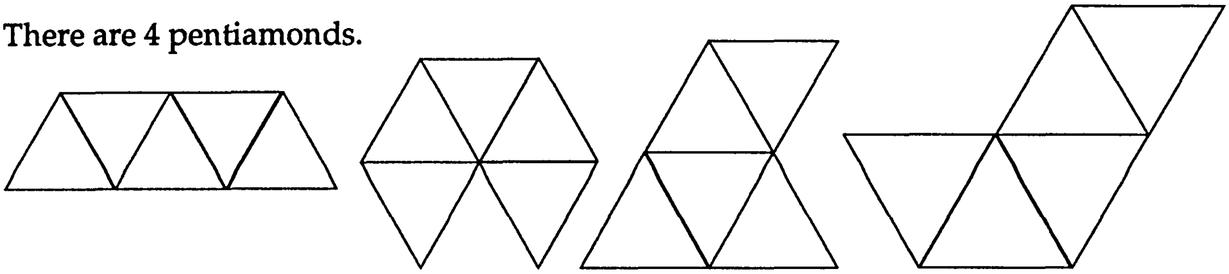
which all give 1 → 3 → 3 → . . .
2 → 3 → 3 → . . .
3 → 3 → 3 → . . .

So there are only 2 different possibilities with 3 numbers. One way of writing the results could be (1, 2, 3) → (1, 2), (3)
(1, 3, 2) → (3)
Can you see why?

With 4 numbers there are 6 different cycles. Can you find them all? How many of the cycles let you reach every position?

0436 Polyiamonds

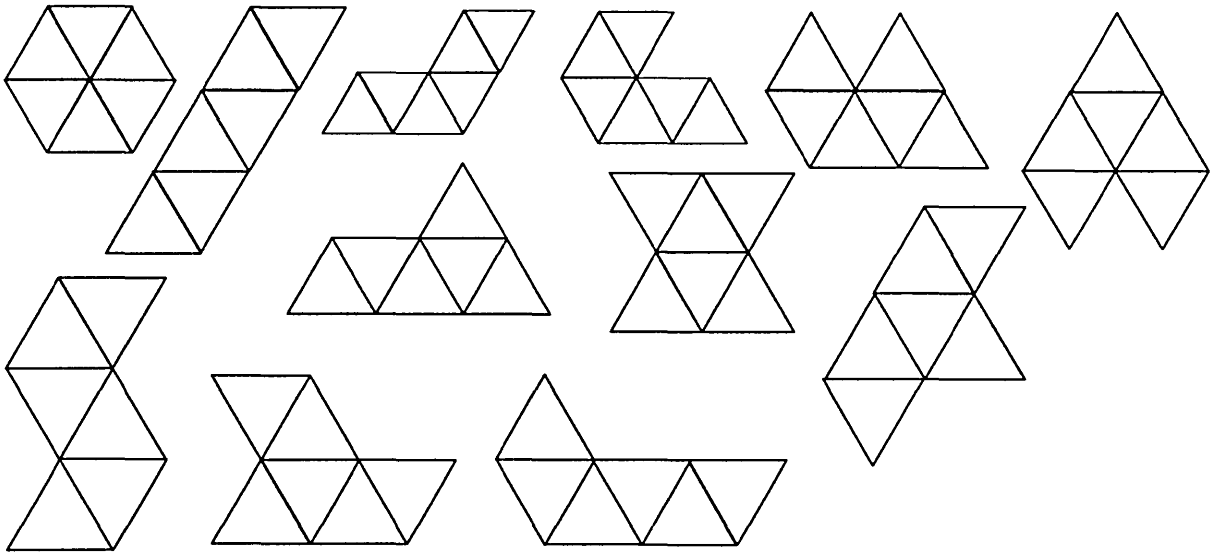
There are 4 pentiamonds.



continued/

0436 Polyiamonds (cont)

There are 12 hexiamonds.



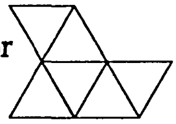
If you get more than this check that the shapes really are different and not just rotated or reflected.

Number of equilateral triangles		Number of different polyiamonds
1	⟶	1
2	⟶	1
3	⟶	1
4	⟶	3
5	⟶	4
6	⟶	12
.		.
.		.
.		.

Is there a pattern?

Tessellation of polyiamonds.

Three of the four pentiamonds will tessellate, based on 180° rotations and translations. Can you find them? The fourth pentiamond does not tessellate but makes an interesting pattern when repeatedly rotated through 60°.

The hexiamonds will all tessellate except for 

Do any of the hexiamonds produce interesting rotation patterns?

Nets of solids of polyiamonds.

The sensible way to approach this question is to ask which solids could be made. There are no solids with 2, 3 or 5 triangular faces and so diamonds, triamonds and pentiamonds cannot be nets.

Two of the three tetriamonds are nets of the tetrahedron. Which two?

Can you find the five hexiamonds which are nets? Will any heptiamonds be nets? What about octiamonds?

0437 Chess

5 players: (call them A, B, C, D and E)

Total number of games.

A plays	B C D E	4
B plays	C D E	3
C plays	D E	2
D plays	E	1

$4 + 3 + 2 + 1 = 10$ games

Note: $\frac{5 \times 4}{2} = 10$

4 players:	$3 + 2 + 1$	= 6 games	$\frac{4 \times 3}{2} = 6$
7 players:	$6 + 5 + 4 + 3 + 2 + 1$	= 21 games	

.
.
.

n players: . . .

0439 Rectangle Diagonal

There is an easy rule for squares. Did you spot it?
Here is a table for rectangles where one side is 2.

Rectangles	2 x 1	2 x 2	2 x 3	2 x 4	2 x 5	2 x 6
Squares on Diagonals	2	2	4	4	6	. . .

Can you see how the table continues?
How many squares for a 2 x 16 rectangle, 2 x 17 rectangle, . . . 2 x n rectangle?

Investigate rectangles where one side is 3: 3 x 1, 3 x 2, 3 x 3, 3 x 4, . . . 3 x n

It will help to draw a table to combine your results for all the rectangles you have drawn.

	1	2	3	4	5	6	7
1							
2	2	2	4	4	6	6	8
3							
4							
5							
6							
7							

Number of squares which diagonals pass through.

Describe any patterns in the table. Give reasons if you can.

0443 Who Won?

The girls won. Here is **one** way to work it out:

240 is a multiple of both 60 **and** 80.

$$\frac{47}{60} \xrightarrow[\times 4]{\times 4} = \frac{188}{240} \quad \text{and} \quad \frac{61}{80} \xrightarrow[\times 3]{\times 3} = \frac{183}{240}$$

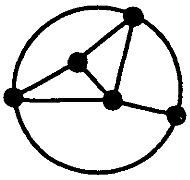
$$\frac{188}{240} > \frac{183}{240}$$

$$\text{so} \quad \frac{47}{60} > \frac{61}{80}$$

Alternatively you may have changed the fractions into decimals or percentages . . .

0445 Points and Lines

For 5 points each point **cannot** be joined to every other point.



9 lines can be drawn.

This table may help to study your results.

Points	Lines	Regions
3	3	2
4		
5		

Look for patterns in the table and reasons for those patterns.

Hint: How many points are there on the boundary?

0448 Favourite Colours

1. 2
2. 5
3. 4
4. 4
5. 3
6. 18

- Show the pie-chart you have drawn of your class' favouite colour to your teacher.

0450 Trick or Treat

If Kitty has 2 sweets left after sharing a box of sweets between 4 monsters, the number of sweets must have been

4 + 2 or
4 + 4 + 2 or
4 + 4 + 4 + 2 . . .
i.e. **6,** 10, 14, 18, 22, **26,** 30, . . .

If Kitty had 1 sweet left after sharing a box of sweets between 5 ghosts, the number of sweets must have been

5 + 1 or
5 + 5 + 1 or
5 + 5 + 5 + 1 . . .
i.e. **6,** 11, 16, 21, **26,** 31, . . .

So the box of sweets could have contained: 6, 26, 46, . . .
What do you notice about these numbers?

You may like to use a spreadsheet to solve the skeleton and Frankenstein problem and some similar problems of your own.

0452 Inside or Outside?

By looking at the shape it is very hard to determine whether A or B is inside or out.

1. Each line from A crosses the curve 15, 25, 19, 23 and 23 times.

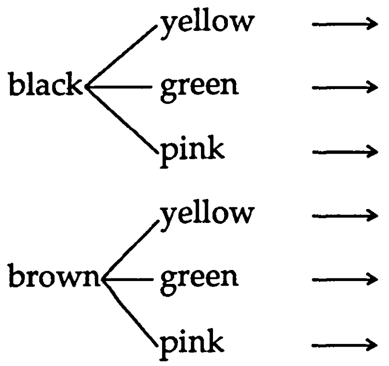
2. The lines will all cross the curve an even number of times.

3. If the point is **inside** the curve, the lines cross it an **odd** number of times.
If the point is **outside** the curve, the lines cross it **even** number of times.

4. C is outside the curve.
D is inside the curve.
E is outside the curve.
F is outside the curve.
G is outside the curve.

0453 What Can I Wear?

1.

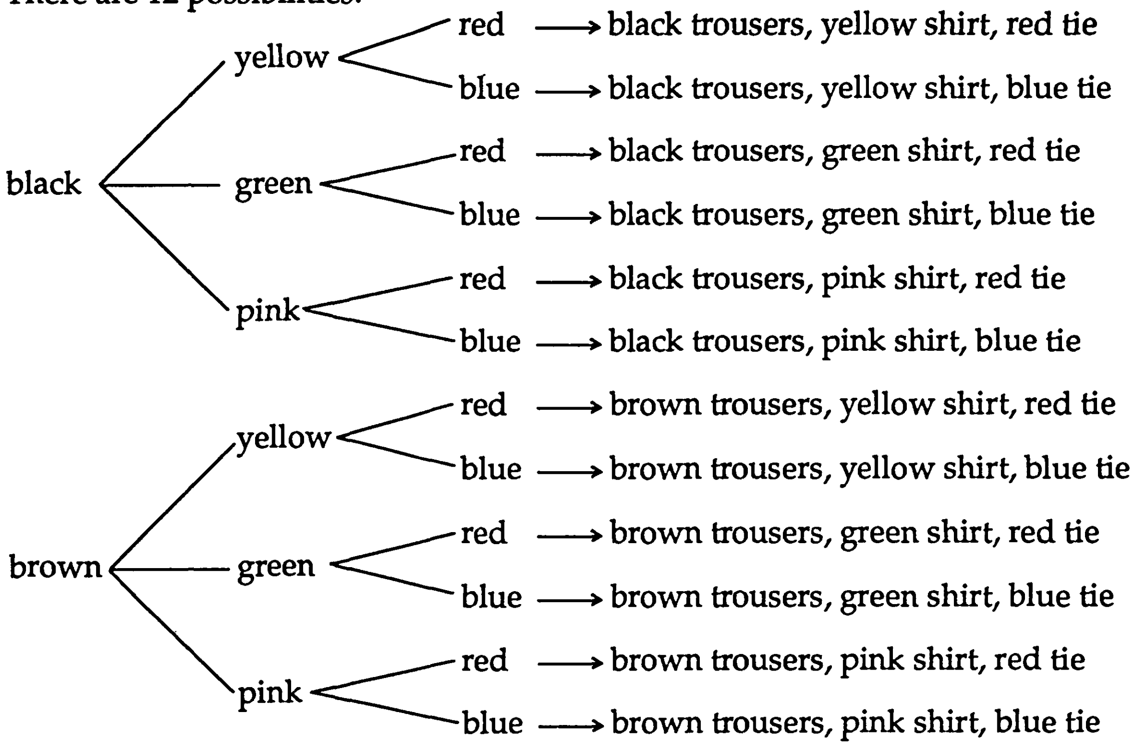


black trousers, yellow shirt
black trousers, green shirt
black trousers, pink shirt
brown trousers, yellow shirt
brown trousers, green shirt
brown trousers, pink shirt

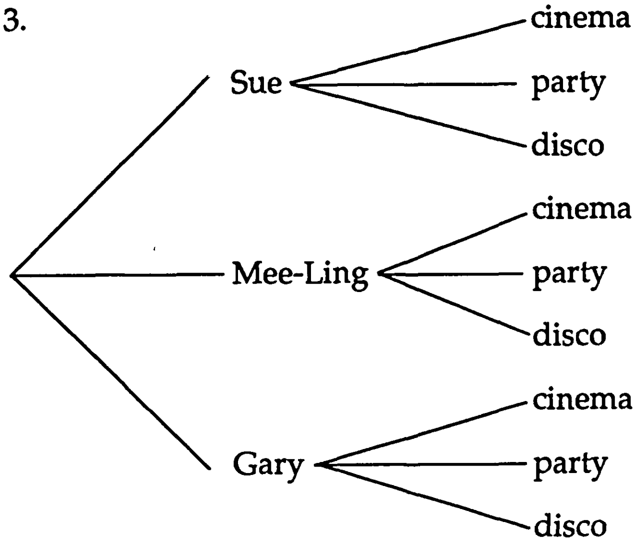
continued/

0453 What Can I Wear? (cont)

2. There are 12 possibilities.

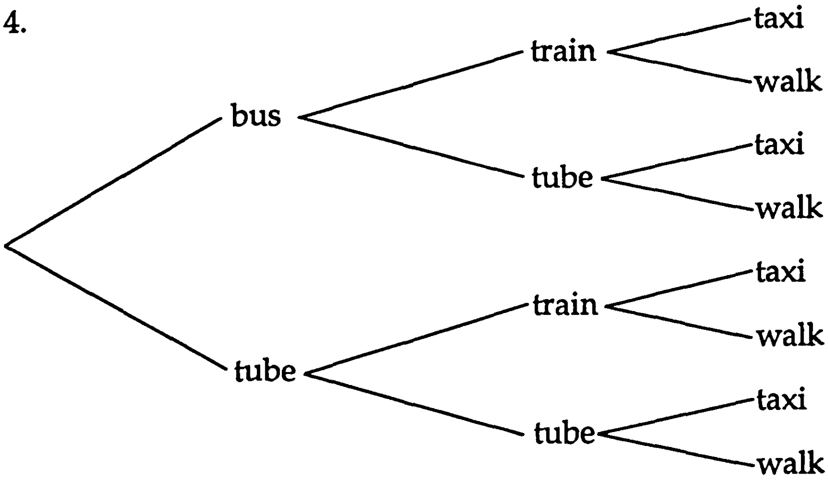


3.



9 possibilities

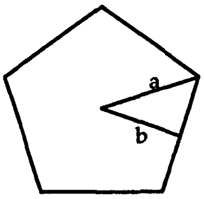
4.



8 possibilities

0454 Post Box

Probably the best approach is to start with the difficult shape, the regular pentagon.

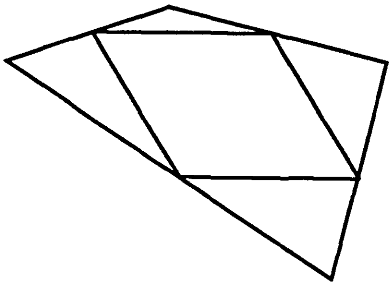


Then overlap this with a circle whose radius r is between "a and b".
Then choose one definite value for the radius r.
This will only give one answer - there could be others.

Can you justify that each of your shapes cannot be posted through the wrong hole?

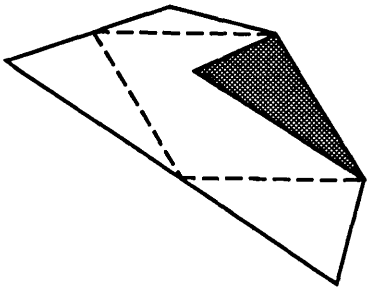
0455 Mid Points

3. You should have found that the new quadrilateral is a parallelogram.



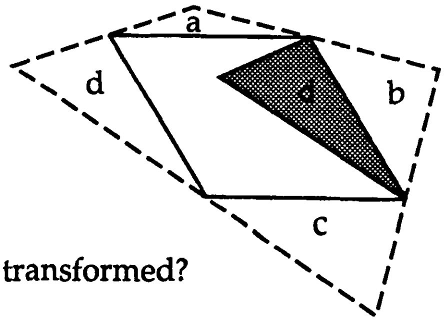
What shape did you get when the original quadrilateral was a square, a rhombus, a rectangle, a kite, etc?

4. When the triangles are folded over, they do **not always** fit into the central shape but they **do sometimes**.



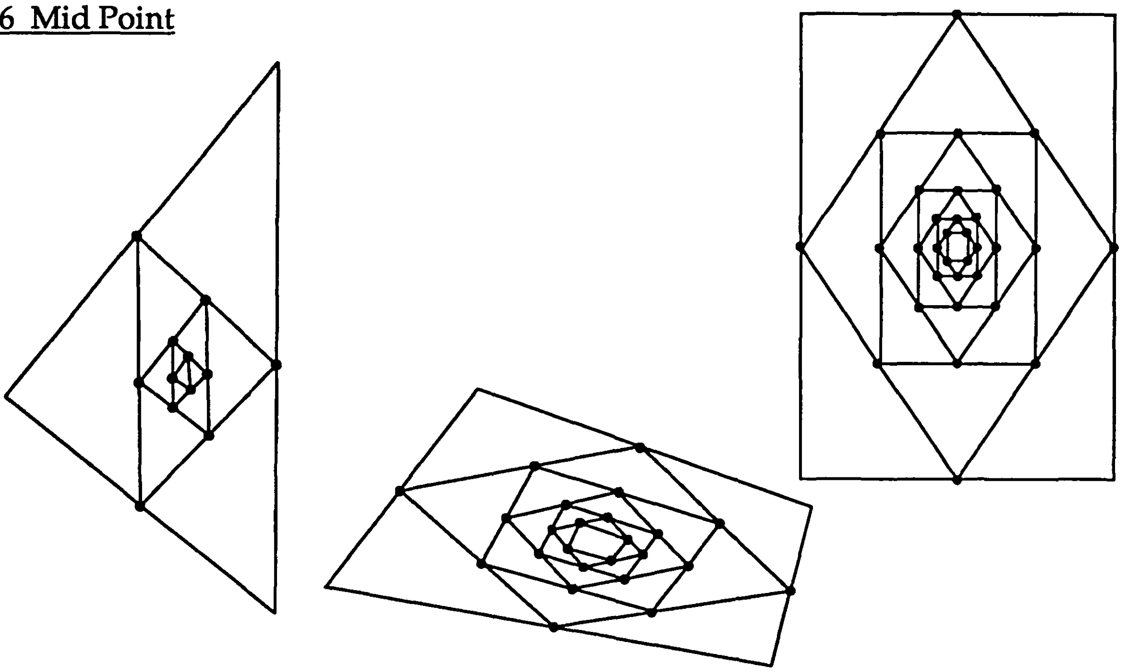
Which shapes could you start with to make them fit?

5. If you cut off the 4 triangles they **will always** fit on to the quadrilateral. In this case, triangle **d** has been translated.



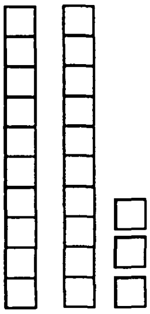
How have the other triangles been transformed?
Try to find a rule.

0456 Mid Point



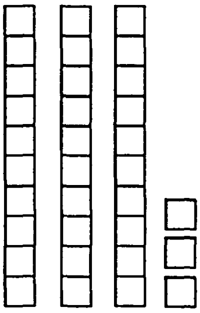
0457 Number Pictures

1.



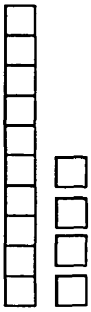
23

2.



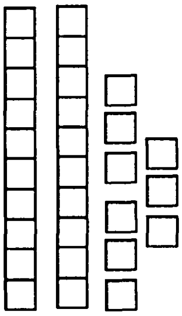
33

3.



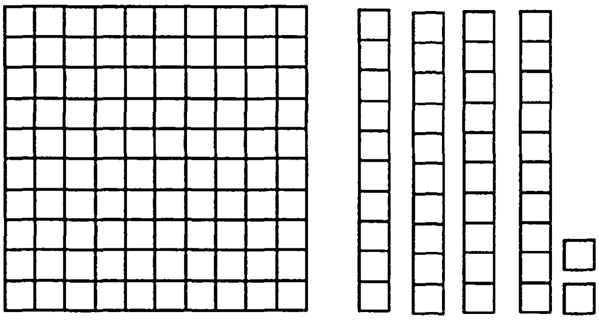
14

4.



29

5.



142

6. 35

7. 43

8. 14

9. 101

10. 20

11. 127

12. 65

13. 10

14. 6

15. 11

0458 Adding Numbers

1. 38 2. 34 3. 58 4. 45 5. 89
6. 77 7. 55 8. 147 9. 138 10. 379

0459 Adding Shapes

1. 52 2. 71 3. 85 4. 70 5. 30
6. 60 7. 110 8. 117 9. 60 10. 163

0460 Carry on Adding

1. 42 2. 73 3. 77 4. 106
5. 95 6. 88 7. 219 8. 521
9. 258 10. 424 11. 531 12. 458
13. 515 14. 937 15. 890 16. 419
17. 373 18. 730 19. 933 20. 1332

0461 Venus Clock

1. 1 6. 1 11. 1
2. 3 7. 3 12. 2
3. 0 8. 0 13. 0
4. 3 9. 2 14. 3
5. 2 10. 3 15. 0

16.

2nd Number

1st Number

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

2nd Number

1st Number

−	0	1	2	3
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

N.B. The patterns in the table are symmetrical about the diagonals shown.
Each row and each column contains 0, 1, 2 and 3.

0463 Paper Power

- 1 cut → 2 pieces
- 2 cuts → 4 pieces
- 3 cuts → 8 pieces

No. of Cuts	No. of Pieces
0	$1 = 2^0$
1	$2 = 2^1$
2	$4 = 2^2$
3	$8 = 2^3$
4	$16 = 2^4$
5	$32 = 2^5$
6	$64 = 2^6$

- After 10 there will be $2^{10} = 1024$ pieces

No. of Cuts	No. of Pieces
0	$1 = 3^0$
2	$3 = 3^1$
4	$9 = 3^2$
6	$27 = 3^3$
8	$81 = 3^4$
10	$243 = 3^5$
12	$729 = 3^6$

- Were you able to convince yourself that $2^0 = 1$ and $3^0 = 1$?

0464 Subtracting

1. 13
2. 21
3. 11
4. 5
5. 33
6. 10
7. 24
8. 103
9. 321
10. 235

0465 Subtraction

1. 8
2. 18
3. 7
4. 19
5. 27
6. 27
7. 17
8. 4
9. 87

0467 Subtract

1.

8	5	12	16
H	E	L	P

2.

1	18	19	5	14	1	12
A	R	S	E	N	A	L

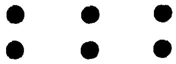
3. Show the one you made up to your teacher.

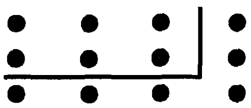
0468 Watch Out

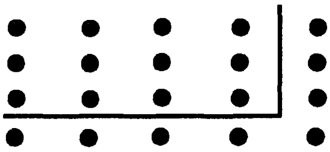
Try blocks in **one row only**.

- How many more policemen are needed for each block you add?
- How many policemen are needed if the blocks are put in a square?
  e.g. 1 block, 4 blocks, 9 blocks etc.

If you start with 2 blocks then add one row and one column every time, do you find a pattern?







Try arranging the blocks into other shapes.
You could put your results in a table like this:

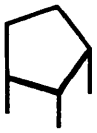
		No. of blocks along			
		1	2	3	4
No. of blocks down	1	2	3		
	2			6	
	3				
	4				

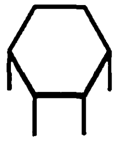
No. of police

What patterns do you notice?

0469 World without Rectangles

Write down a list of objects made of rectangles. Make the list as long as possible.
Which of the objects could you make into a different shape?
e.g.





Do you think you are allowed to use squares? Why not? Explain.

0470 The Nephroid

- An envelope is a wrapper or a covering
- To envelop means to wrap up, cover or surround.

Curves made from straight lines are called envelopes because the straight lines surround the curve. In fact these straight lines are tangents to the curve.

x 3 → nephroid (2 cusps)
x 2 → cardioid (1 cusp)
x 4 → like the shapes above but with 3 cusps
+ 15 → circle

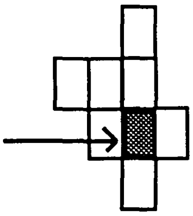
- Nephroid means kidney-shaped (nephro comes form the Greek word for kidney).
- Cardioid means heart shaped (cardio comes from the Greek work for heart).

0471 Border Patterns

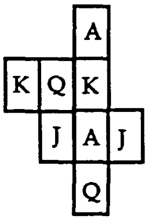
You may like to make a display of your border patterns.

0472 Sort the Cards

This card governs all the rest.

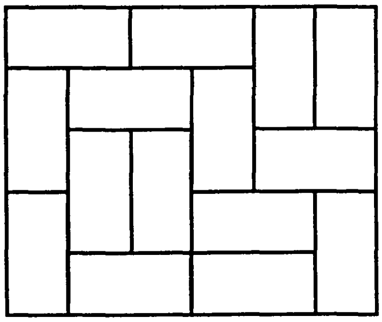


Here is one solution:

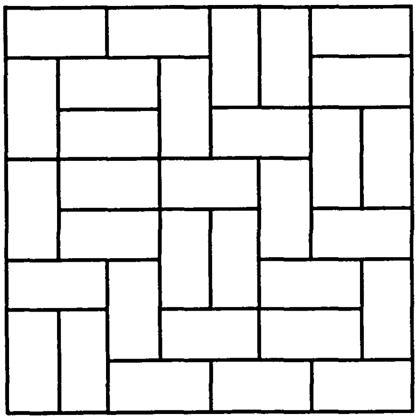


0473 Fault - Lines

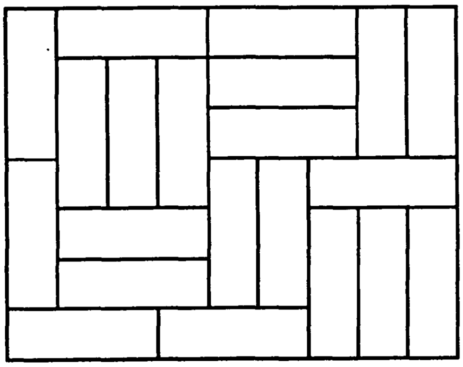
1. The smallest fault-free rectangle is 5 x 6 (apart from 1 x 2).



2. a) The smallest square is 8 x 8.



b) The smallest rectangle from straight trominoes is 7 x 9.



0474 Triominoes

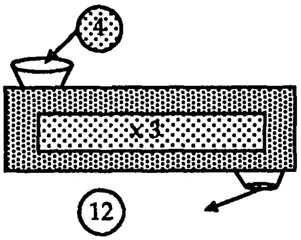
Learn any of the times tables you do not know.

0475 All Change

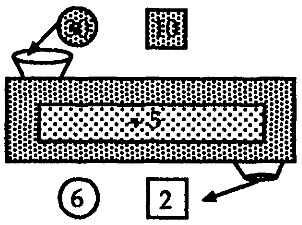
Show your completed worksheet to your teacher.

0476 Mapping Worksheet

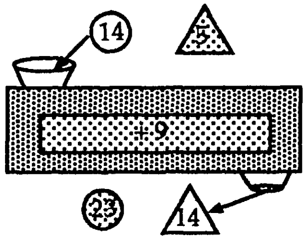
1.



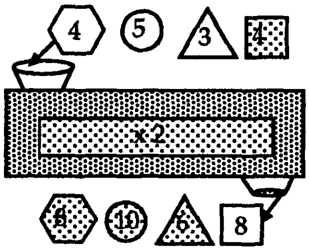
2.



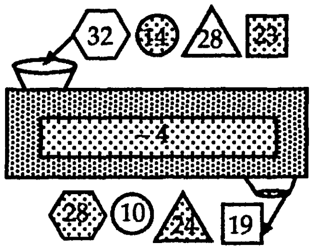
3.



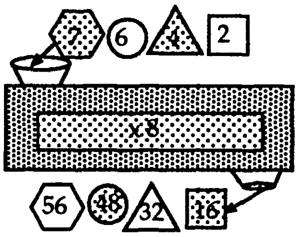
4.



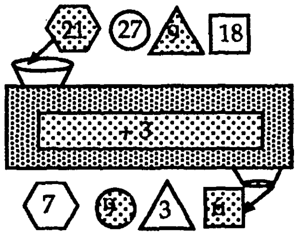
5.



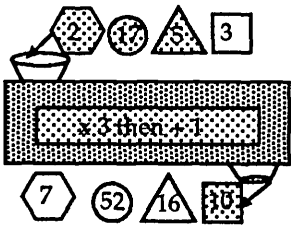
6.



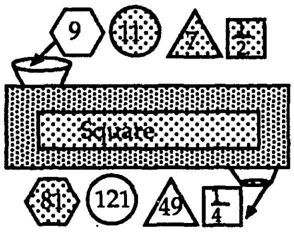
7.



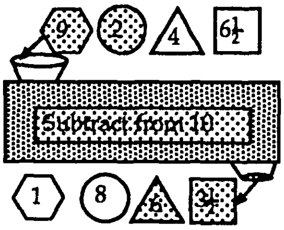
8.



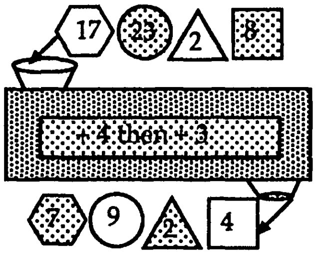
9.



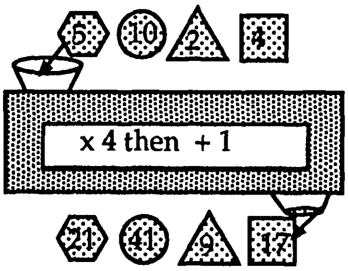
10.



11.

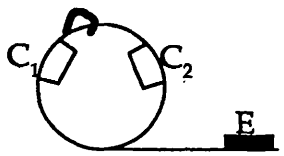


12.



0477 Shunting

This solution requires 9 moves. Your solution may be different.



1st move: Engine collects C_2 and leaves it in the side track.

2nd move: Engine goes under the bridge to collect C_1 and moves it to the side track.

3rd move: Engine reverses both carriages to the right of the bridge.

4th move: Engine leaves C_2 and moves C_1 into the side track.

5th move: Engine leaves C_1 and moves to C_2.

6th move: Engine takes C_2 to the left of the bridge.

7th move: Engine deposits C_2 and goes under the bridge.

8th move: Engine picks up C_1 and deposits it to the right of the bridge.

9th move: Engine returns to the side track.

0478 Patterns with Squares

Show your own patterns to your teacher.

0481 Where's That Town?

1. Wells is at $(5\frac{1}{2}, 3\frac{1}{2})$
   Cowes is at (8, 1)
   London is at (10, 3)
   Oxford is at (8, 4)
   Huntingdon is at $(10, 5\frac{1}{2})$
   Welshpool is at (5, 7).
   Manchester is at (7, 9)
   Sheffield is at $(8, 8\frac{1}{2})$
   York is at (9, 10)
2. Dublin is at (2, 9)
3. Limerick is at (-3, 8)
   Tralee is at $(-3\frac{1}{2}, 6\frac{1}{2})$
   Cork is at $(-1\frac{1}{2}, 6)$
4. Nantes is at $(6, -7\frac{1}{2})$
   Poitiers is at $(8\frac{1}{2}, -9)$
   Rouen is at (11, -2)
   Le Mans is at (9, -6)
   Brest is at $(2, -3\frac{1}{4})$

0483 Star Puzzle

5. The puzzle with 5 counters is impossible.

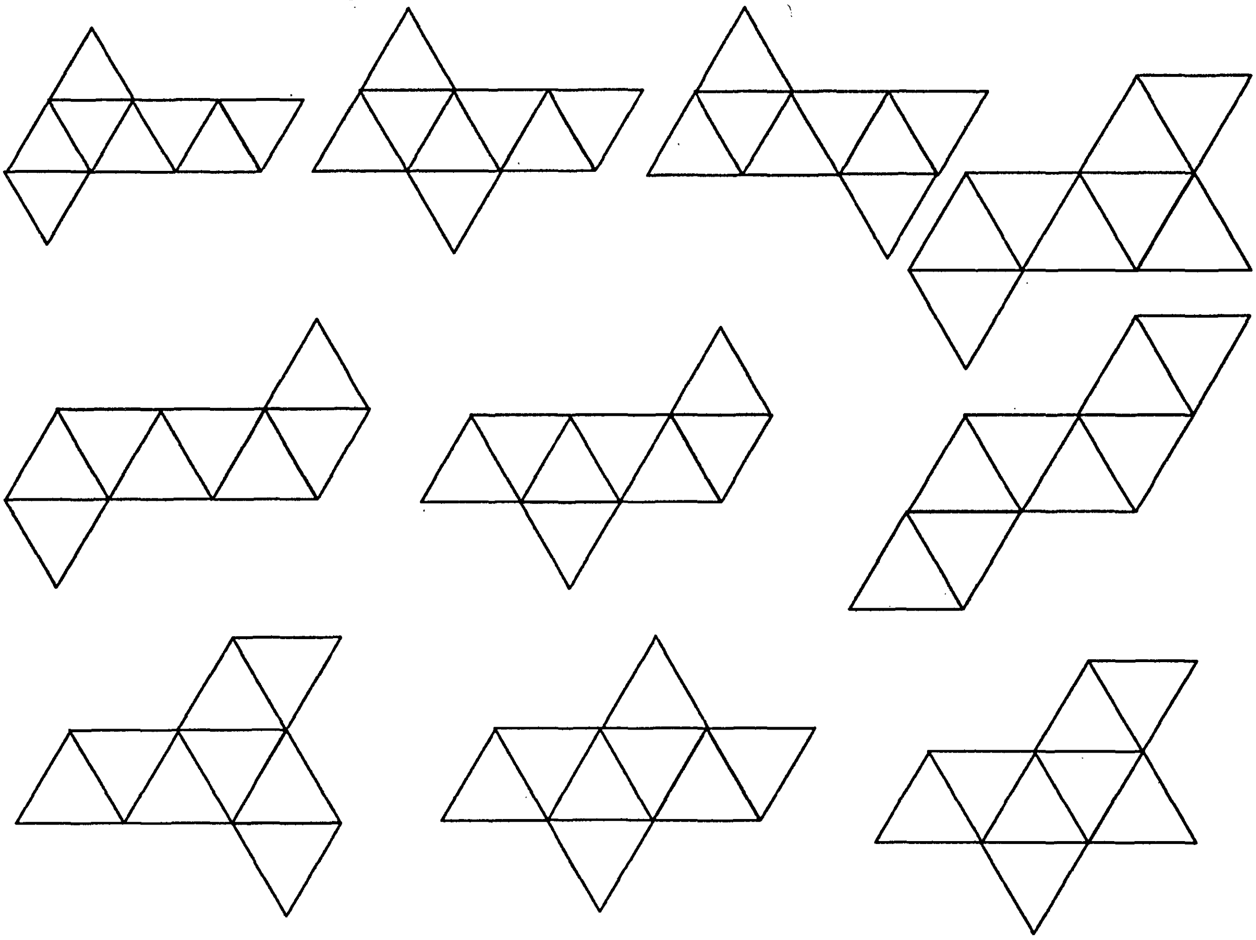
6. **7 counters**
   If you start at one circle and move along the straight lines from one circle to another you will return to the circle you started from, having visited every circle; i.e. the path is **continuous.**

   5 counters
   If you start at one circle and move along the straight lines from one circle to another you will return to the circle you started from, but you will not have visited every circle;
   i.e. the path is **not continuous.**

0484 Octahedron Nets

1. It is **regular** because all the faces are the same.

6. There are 10 other possible nets. How many were you able to find?



0485 Pamphlets

One way to solve this problem is by trial and improvement. Using a spreadsheet will allow you to refine your answers.

Type A - 26 sold,
Type B - 14 sold,
Type C - 60 sold.

0488 Happy Numbers

Can you see why the card says that you already know 16 unhappy numbers?

The happy numbers between 1 and 100 are:

1, 7, 10, 13, 19, 23, 28, 31, 32, 44, 49, 68, 70, 79, 82, 86, 91, 94, 97 and 100.

0489 The Underground

The journey time depends on the route you choose. Here are the shortest times.

2. 14 minutes.

3. 18 minutes.

4. 22 minutes.

5. 22 minutes.

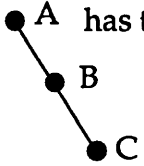
6. Many possible answers, convince someone else that you have found the quickest time.

7. Many possible answers, convince someone else that you have found the quickest time.

8. The best route is using the Victoria Line, because there are only 9 stations, and there is no changing lines.

9. Here are some of the places:
   - Monument,
   - High St. Kensington,
   - Earls Court . . .

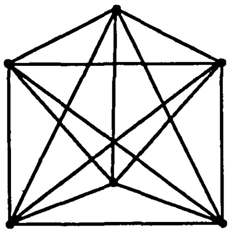
0490 Dots and Lines

The answers to the investigation assume that when dots are in line, the line joining A to C is not an "extra" line, so that  has two lines only.

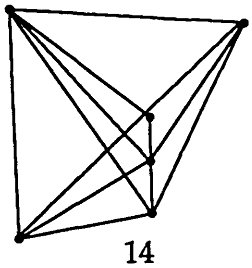
If you decide that this is really three lines you will always get the maximum total of lines for each diagram.

For 6 dots

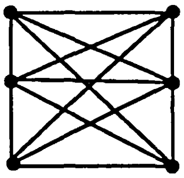
A The different totals are:



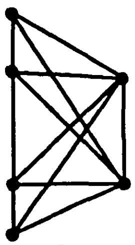
15



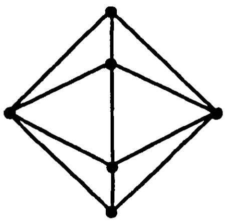
14



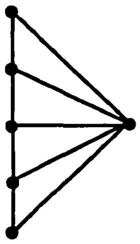
13



12



11



9

5

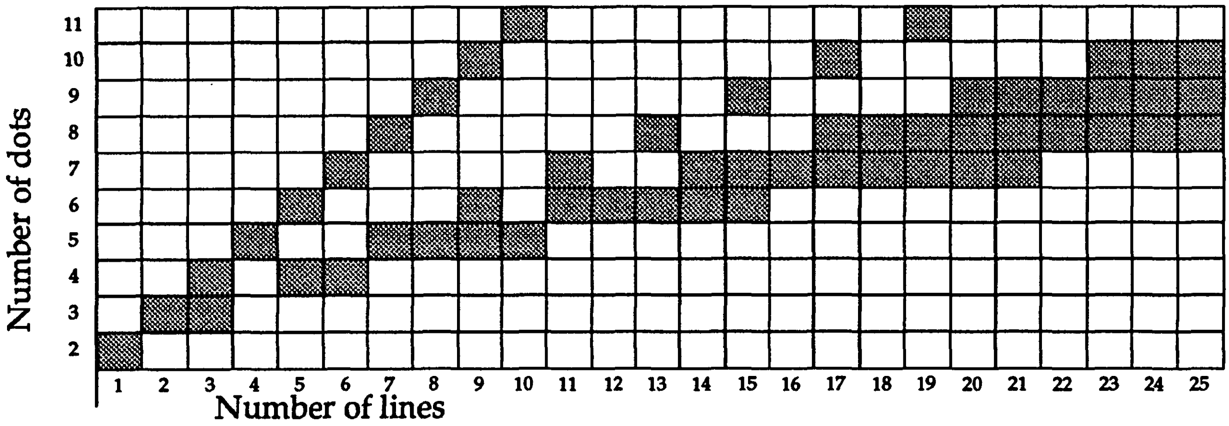
continued/

0490 Dots and Lines (cont)

B The maximum number of lines is 15.
It is useful to organise your results in a table.

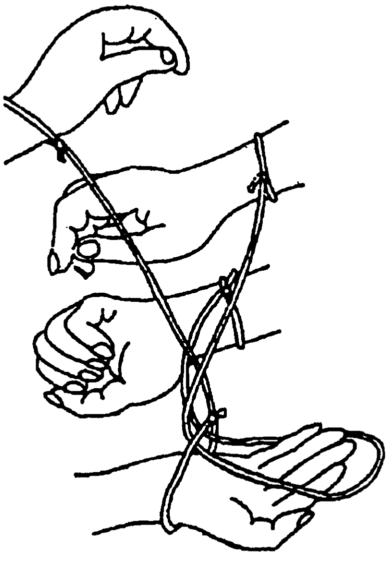
No. of Dots		Maximum total no. of lines
1	⟶	0
2	⟶	1
3	⟶	3
4	⟶	6
5	⟶	10
6	⟶	15
7	⟶	21
.		
.		
.		
n	⟶	

This chart records the number of lines possible for each number of dots.



The pattern shows that for n dots the minimum total of lines is . . .
The maximum total of lines is...

0492 The Inseparables



0493 Sam Shape

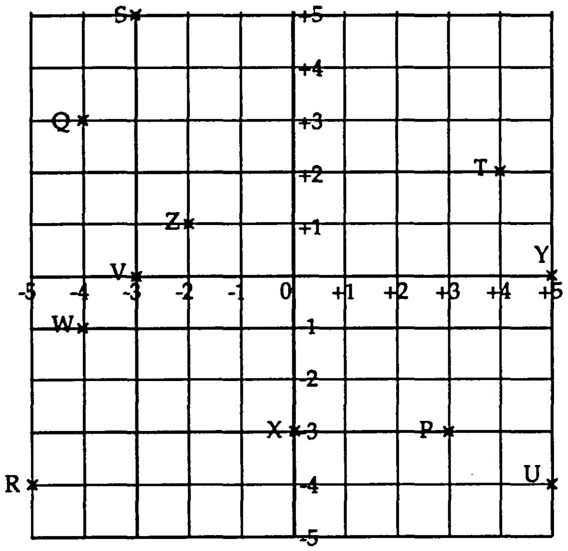
1. A square has 4 sides.
2. A triangle has 3 sides.
3. A rectangle has 4 sides.
4. This shape ○ is a circle.
5. This shape ▭ is a rectangle.
6. This shape △ is a triangle.
7. This shape □ is a square and also a rectangle.

0494 All Co-ordinates

1. B is at (1, 4) C is at (4, 3) D is at (3, 5) E is at (4, 0)
   F is at (2, -1) G is at (-2, 4)

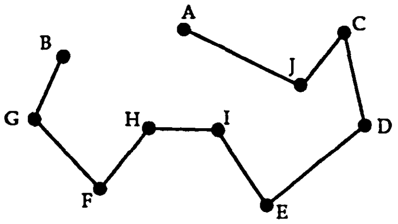
2. H is at (+4, -2) I is at (-1, +2) J is at (-5, -2) K is at (+5, +4)
   L is at (-4, +2) M is at (-2, -3) N is at (0, -4)

3.



0495 Routey

Here is one possible answer:



Did you find another route?

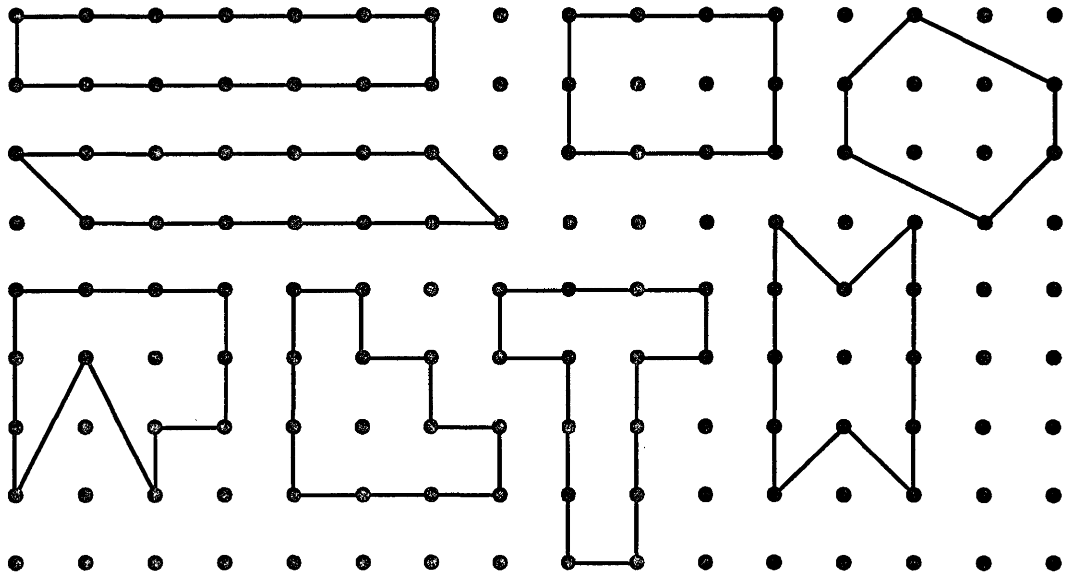
0496 Junior Contig

Who won?
What was the winner's score?
What was the other person's score?

Which number was the easiest to make? Can you explain why?

0498 Area

Here are some possible answers:



0500 Hey Mr Porter!

1. - It is not possible to get from Paddington to Tottenham Court Road, without changing trains. (The no. 7 bus goes direct).

   - These are the 2 most sensible routes:

     { Paddington - Notting Hill Gate (District or Circle)
     Notting Hill Gate - Tottenham Court Road, (Central)

     { Paddington - Oxford Circus (Bakerloo)
     Oxford Circus - Tottenham Court Road (Central)

   - Of the 2 routes shown above, the route via Oxford Circus is the shorter. . .

   - . . . the route via Notting Hill Gate goes near Hyde Park.

   - There are 8 stations on the 1st route and 6 stations on the 2nd route.
     (On the 2nd route the Circle or Hammersmith and City line could be used for the first part of the journey from Paddington to Baker Street. This would mean changing once more but there would only be 5 stations on the route).

continued/

2. The Underground map is more suitable for finding routes, the number of stations, where you can change, etc.

   The Visitors' London map is more suitable for finding distances, noting the position of Hyde Park, etc.

   More of Pat's questions would require the use of the Underground map than the Visitors' London map.

3. The Underground map is not drawn to scale. The direction of the lines is not necessarily the true direction of the Underground route.

   The Visitors' London map is drawn to scale. It shows more information about the central area, e.g. public buildings. The central Underground routes and connections and the network relationship of the stations are the same on both maps.

Answers • Answers • Answers

0001
to
0500

Answers