Answers • Answers • Answers

0501 to 1300

SMILE MATHEMATICS

SMILE

ANSWERS 0501-1300

Answers

0501 to 1300

SMILE MATHEMATICS

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This book contains answers to all the SMILE activities between 0501 - 1300, in numerical order.

As well as giving the answers there are also:

- explanations about how solutions have been arrived at,
- hints or prompts if you get stuck,
- ideas for extending some activities.

Use this book after you have completed each activity, so that you have immediate feedback on your work. You will remember the work more clearly and be able to identify any difficulties or misconceptions more easily. If you have made errors, look through your work again to see if you can spot where you have made an error. If you then do not understand why your answer is incorrect always seek help from your teacher so that she can help you to clarify any mis-understandings.

You can also use this book while you are working on an activity as it contains hints if you get stuck, or want to know how continue.

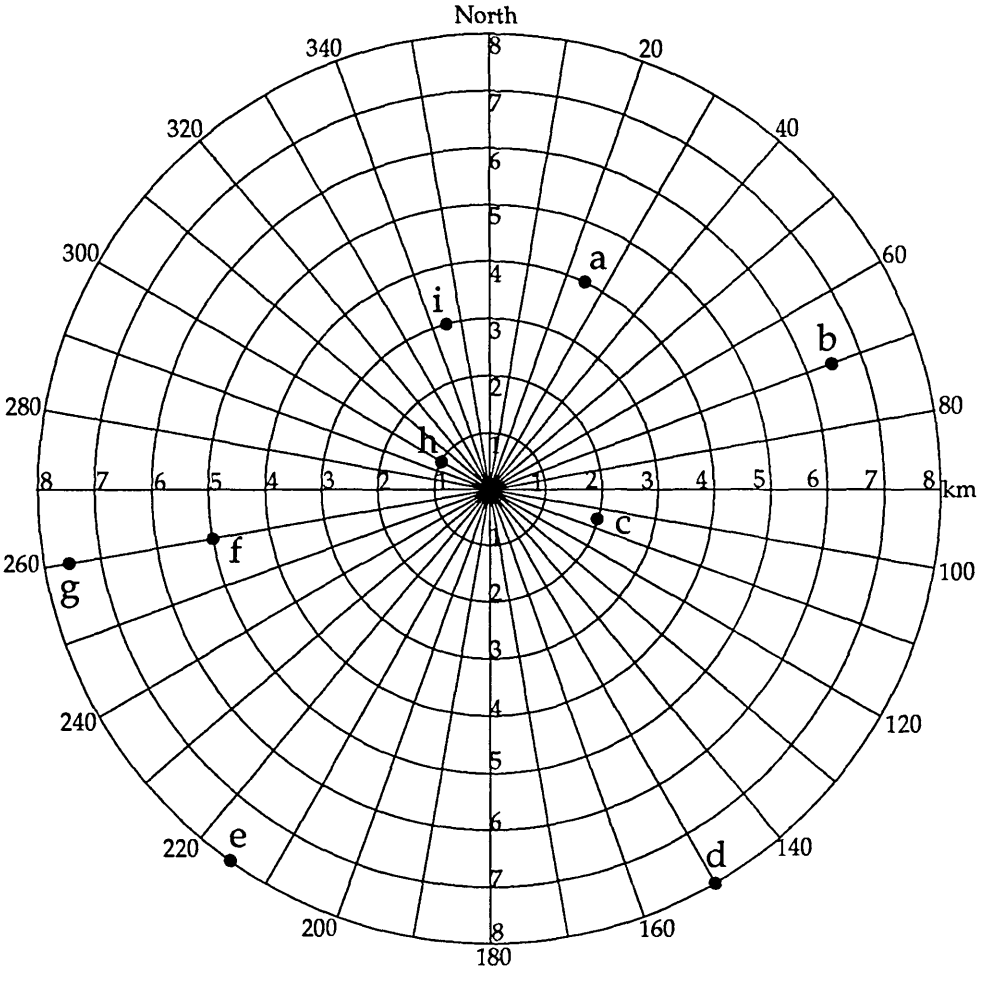
Remember, using the answer book to check your work or to help you if you are stuck is not cheating.

0510 Radar

1.

Object	Polar co-ordinates
A	(6, 030)
B	**(3.5 060)**
C	**(7.5, 105)**
D	**(4, 140)**
E	**(7, 140)**
F	**(3.5, 218)**
G	**(6, 250)**
H	**(8, 290)**
I	**(4, 300)**
J	**(6, 345)**

2.



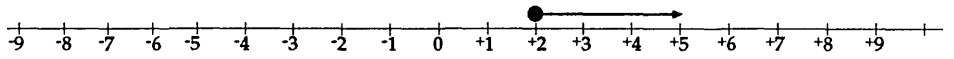
0516 Adding Directed Numbers

a) $^{+}7 + {}^{-}4 = \boxed{^{+}3}$

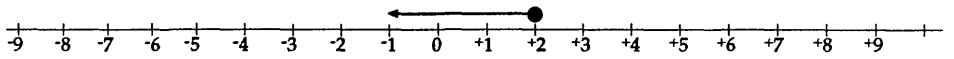
b) $^{-}3 + {}^{+}10 = \boxed{^{+}7}$

c) $^{-}5 + {}^{+}3 = \boxed{^{-}2}$

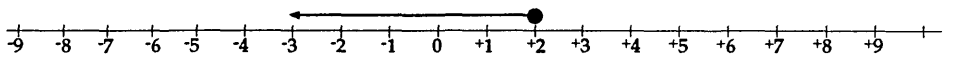
1. a) $^{+}5$



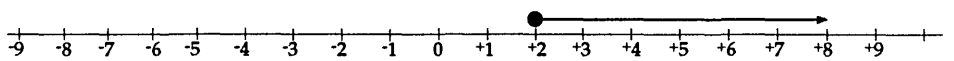
b) $^{-}1$



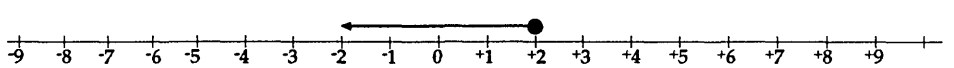
c) $^{-}3$



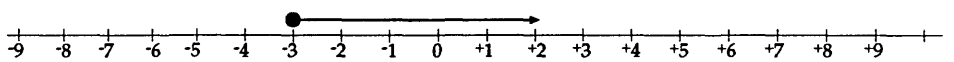
d) $^{+}8$



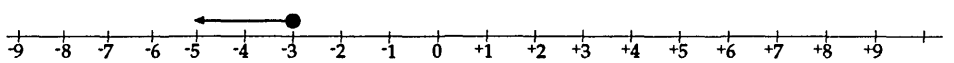
e) $^{-}2$



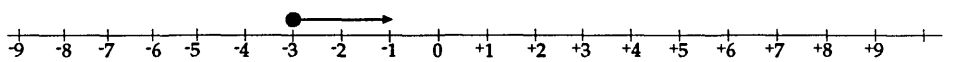
2. a) $^{+}2$



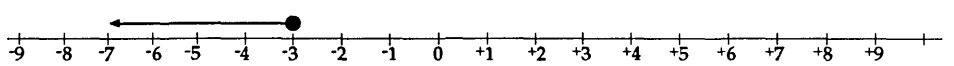
b) $^{-}5$



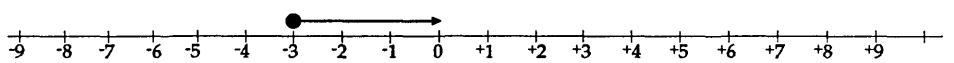
c) $^{-}1$



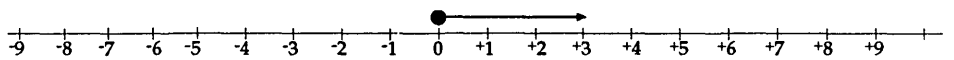
d) $^{-}7$



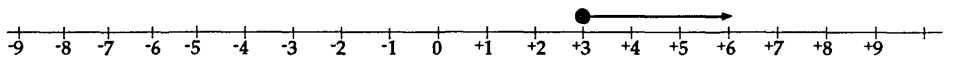
e) 0



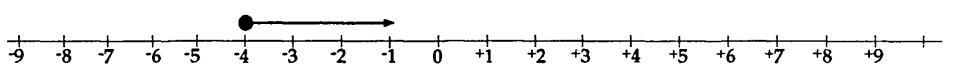
3. a) $^{+}3$



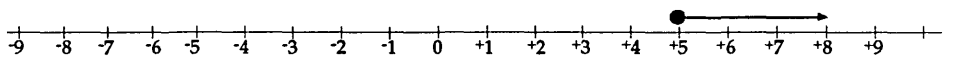
b) $^{+}6$



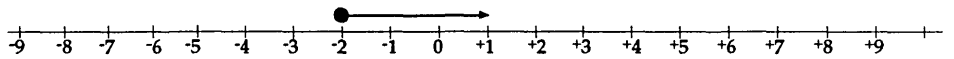
c) $^{-}1$



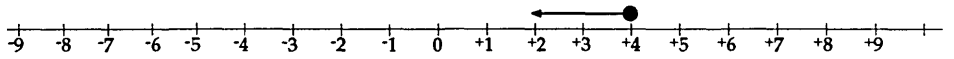
d) $^{+}8$



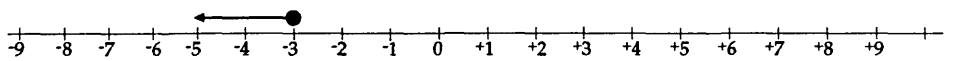
e) $^{+}1$



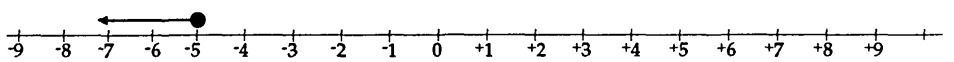
4. a) $^{+}2$



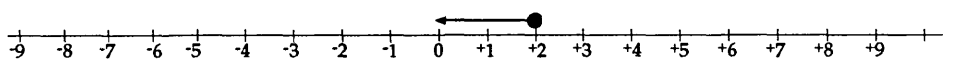
b) $^{-}5$



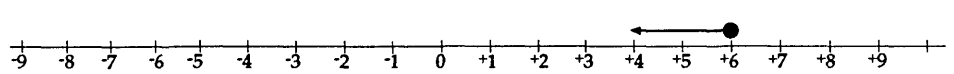
c) $^{-}7$



d) 0



e) $^{+}4$



continued/

0516 Adding Directed Numbers (cont)

5. a) $^{+}2 + {}^{+}3 = \boxed{{}^{+}5}$ b) $^{+}3 + \boxed{{}^{+}4} = {}^{+}7$ c) $^{+}5 + {}^{-}3 = \boxed{{}^{+}2}$ d) $^{+}4 + {}^{-}5 = \boxed{{}^{-}1}$

e) $^{+}3 + \boxed{{}^{-}3} = 0$ f) $^{-}4 + \boxed{{}^{+}6} = {}^{+}2$ g) $\boxed{{}^{+}6} + {}^{-}4 = {}^{+}2$ h) $\boxed{{}^{-}7} + {}^{+}2 = {}^{-}5$

i) $\boxed{{}^{+}2} + {}^{-}7 = {}^{-}5$ j) $^{-}4 + \boxed{{}^{+}4} = 0$ k) $^{-}3 + \boxed{{}^{-}4} = {}^{-}7$

l) $^{+}5 + {}^{+}2 + {}^{-}6 = \boxed{{}^{+}1}$ m) $\boxed{{}^{-}4} + {}^{+}2 + {}^{-}5 = {}^{-}7$

0517 Subtracting Directed Numbers

Pattern 1 Adding to $^{+}4$

$^{+}4$	+	$^{-}4$	=	0
$^{+}4$	+	$^{-}3$	=	$^{+}1$
$^{+}4$	+	$^{-}2$	=	$\boxed{{}^{+}2}$
$^{+}4$	+	$^{-}1$	=	$\boxed{{}^{+}3}$
$^{+}4$	+	0	=	$^{+}4$
$^{+}4$	+	$^{+}1$	=	$\boxed{{}^{+}5}$
$^{+}4$	+	$^{+}2$	=	$^{+}6$
$^{+}4$	+	$^{+}3$	=	$\boxed{{}^{+}7}$

Pattern 2 Subtracting from $^{+}4$

$^{+}4$	–	$^{+}4$	=	0
$^{+}4$	–	$^{+}3$	=	$^{+}1$
$^{+}4$	–	$^{+}2$	=	$\boxed{{}^{+}2}$
$^{+}4$	–	$^{+}1$	=	$\boxed{{}^{+}3}$
$^{+}4$	–	0	=	$^{+}4$
$^{+}4$	–	$^{-}1$	=	$\boxed{{}^{+}5}$
$^{+}4$	–	$^{-}2$	=	$^{+}6$
$^{+}4$	–	$^{-}3$	=	$\boxed{{}^{+}7}$

- One example to show that adding $^{-}4$ is the same as subtracting $^{+}4$ is:
  From Pattern 1 $^{+}4 + {}^{-}4 = 0$
  From Pattern 2 $^{+}4 - {}^{+}4 = 0$
- One example to show that adding $^{+}3$ is the same as subtracting $^{-}3$ is:
  From Pattern 1 $^{+}4 + {}^{+}3 = {}^{+}7$
  From Pattern 2 $^{+}4 - {}^{-}3 = {}^{+}7$

1. a) $^{+}6 - {}^{+}4 = \boxed{{}^{+}2}$ b) $^{+}5 - {}^{-}7 = \boxed{{}^{+}12}$ c) $^{+}8 - {}^{+}4 = \boxed{{}^{+}4}$ d) $^{+}3 - {}^{+}8 = \boxed{{}^{-}5}$

e) $^{-}5 - {}^{+}4 = \boxed{{}^{-}9}$ f) $^{-}6 - {}^{-}5 = \boxed{{}^{-}1}$ g) $^{-}3 - {}^{-}9 = \boxed{{}^{+}6}$ h) $^{-}5 - {}^{+}5 = \boxed{{}^{-}10}$

i) $^{+}8 - {}^{+}8 = \boxed{0}$ j) $^{+}5 - {}^{+}3 = \boxed{{}^{+}2}$ k) $^{-}2 - {}^{-}4 = \boxed{{}^{+}2}$ l) $^{+}3 - {}^{-}5 = \boxed{{}^{+}8}$

m) $^{-}4 - {}^{+}1 = \boxed{{}^{-}5}$ n) $^{-}12 - {}^{-}20 = \boxed{{}^{+}8}$

continued/

0517 Subtracting Directed Numbers (cont)

2. a) $\boxed{+2} - {}^{-}3 = {}^{+}5$ $\quad -{}^{-}3$ is the same as $+{}^{+}3$ so $^{+}2 - {}^{-}3 = {}^{+}5$

   $\boxed{+2} + {}^{+}2 = {}^{+}5$ $\quad {}^{+}2 + {}^{+}3 = {}^{+}5$

   b) $\boxed{+2} - {}^{-}5 = {}^{+}7$ c) $\boxed{+7} - {}^{+}5 = {}^{+}2$ d) $\boxed{-4} - {}^{-}2 = {}^{-}2$ e) $\boxed{+4} - {}^{+}3 = {}^{+}1$

   f) $\boxed{-1} - {}^{-}4 = {}^{+}3$ g) $\boxed{-3} - {}^{+}1 = {}^{-}4$ h) $\boxed{+4} - {}^{-}2 = {}^{+}6$ i) $\boxed{0} - {}^{+}2 = {}^{-}2$

   j) $\boxed{+4} - {}^{+}5 = {}^{-}1$ c) $\boxed{-7} - {}^{-}5 = {}^{-}2$

3. a) ${}^{+}5 + {}^{+}8 = \boxed{+13}$ b) ${}^{+}3 + \boxed{+7} = {}^{+}10$ c) ${}^{+}2 - \boxed{-7} = {}^{+}9$ d) $\boxed{-3} + {}^{+}3 = 0$

   e) $\boxed{-5} - {}^{-}2 = {}^{-}3$ f) ${}^{-}8 + {}^{-}6 = \boxed{-14}$ g) ${}^{-}13 - {}^{-}4 + {}^{-}1 = \boxed{-10}$

4. Here are examples of three pairs of numbers that make the expressions true. If you are unsure whether your answers are correct, show them to your teacher, or check them, using the number line.

$^{+}\triangle 7 + {}^{+}(5) = {}^{+}12$	$^{-}\triangle 10 - {}^{+}(7) = {}^{-}17$	$^{+}\triangle 20 - {}^{-}(1) = {}^{+}21$	$^{-}\triangle 20 - {}^{-}(27) = {}^{+}7$
$^{+}\triangle 6 + {}^{+}(6) = {}^{+}12$	$^{-}\triangle 1 - {}^{+}(16) = {}^{-}17$	$^{+}\triangle 14 - {}^{-}(7) = {}^{+}21$	$^{-}\triangle 5 - {}^{-}(12) = {}^{+}7$
$^{+}\triangle 3 + {}^{+}(9) = {}^{+}12$	$^{-}\triangle 13 - {}^{+}(4) = {}^{-}17$	$^{+}\triangle 10 - {}^{-}(11) = {}^{+}21$	$^{-}\triangle 7 - {}^{-}(14) = {}^{+}7$

0518 (Do It First)

1. a)
   $(5 \times 4) + 7$
   $20 + 7 = \mathbf{27}$
   $5 \times (4 + 7)$
   $5 \times 11 = \mathbf{55}$

   b)
   $(6 \times 11) - 3$
   $66 - 3 = \mathbf{63}$
   $6 \times (11 - 3)$
   $6 \times 8 = \mathbf{48}$

   c)
   $(18 \div 2) + 4$
   $9 + 4 = \mathbf{13}$
   $18 \div (2 + 4)$
   $18 \div 6 = \mathbf{3}$

   d)
   $(24 \div 6) - 3$
   $4 - 3 = \mathbf{1}$
   $24 \div (6 - 3)$
   $24 \div 3 = \mathbf{8}$

   e)
   $(7 + 3) \times 5$
   $10 \times 5 = \mathbf{50}$
   $7 + (3 \times 5)$
   $7 + 15 = \mathbf{22}$

   f)
   $(8 - 2) \times 3$
   $6 \times 3 = \mathbf{18}$
   $8 - (2 \times 3)$
   $8 - 6 = \mathbf{2}$

   g)
   $(4 + 8) \div 2$
   $12 \div 2 = \mathbf{6}$
   $4 + (8 \div 2)$
   $4 + 4 = \mathbf{8}$

   h)
   $(12 - 9) \div 3$
   $3 \div 3 = \mathbf{1}$
   $12 - (9 \div 3)$
   $12 - 3 = \mathbf{9}$

   i)
   $(5 \times 6) \div 3$
   $30 \div 3 = \mathbf{10}$
   $5 \times (6 \div 3)$
   $5 \times 2 = \mathbf{10}$

   j)
   $(12 \div 3) \times 2$
   $4 \times 2 = \mathbf{8}$
   $12 \div (3 \times 2)$
   $12 \div 6 = \mathbf{2}$

All are different except for i), so brackets are nearly always important.

continued/

0518 (Do It First) (cont)

2. Here are two pairs of expressions which give the same answer.

$(18 + 8) + 3$		$(5 \times 2) \times 4$	
$26 + 3$	$= 29$	10×4	$= 40$
$18 + (8 + 3)$		$5 \times (2 \times 4)$	
$18 + 11$	$= 29$	5×8	$= 40$

In each case, the same numbers and operations are used, but the brackets are in different positions. If you are unsure about your answers show them to your teacher.

3.

a)	$3 \times (5 + 7)$	$= 36$	b)	$(3 \times 5) + 7$	$= 22$
c)	$(1 + 4) \times 8$	$= 40$	d)	$1 + (4 \times 8)$	$= 33$
e)	$6 - (2 \times 2)$	$= 2$	f)	$(6 - 2) \times 2$	$= 8$
g)	$11 + (9 \times 3)$	$= 38$	h)	$(11 + 9) \times 3$	$= 60$
i)	$15 - (2 \times 7)$	$= 1$	j)	$16 \div (4 + 4)$	$= 2$
k)	$(18 - 6) \div 3$	$= 4$	l)	$17 - (7 + 1)$	$= 9$
m)	$(5 - 3) - 2$	$= 0$	n)	$(20 \div 2) \times 5$	$= 50$
o)	$(14 + 8) \div 2$	$= 11$	p)	$16 - (2 \times 6)$	$= 4$

0528 Multiplying

1.

a)	42	b)	72	c)	36	d)	48
e)	63	f)	63	g)	56	h)	56

2.

17	x	5		
10	x	5	→	50
7	x	5	→	35
	Add		→	85

3.

a) 13×7
$10 \times 7 \rightarrow 70$
$3 \times 7 \rightarrow 21$
Add $\rightarrow$ **91**

b) 15×6
$10 \times 6 \rightarrow 60$
$5 \times 6 \rightarrow 30$
Add $\rightarrow$ **90**

c) 18×4
$10 \times 4 \rightarrow 40$
$8 \times 4 \rightarrow 32$
Add $\rightarrow$ **72**

d) 19×5
$10 \times 5 \rightarrow 50$
$9 \times 5 \rightarrow 45$
Add $\rightarrow$ **95**

e) 14×7
$10 \times 7 \rightarrow 70$
$4 \times 7 \rightarrow 28$
Add $\rightarrow$ **98**

f) 15×5
$10 \times 5 \rightarrow 50$
$5 \times 5 \rightarrow 25$
Add $\rightarrow$ **75**

g) 17×8
$10 \times 8 \rightarrow 80$
$7 \times 8 \rightarrow 56$
Add $\rightarrow$ **136**

h) 18×9
$10 \times 9 \rightarrow 90$
$8 \times 9 \rightarrow 72$
Add $\rightarrow$ **162**

i) 19×7
$10 \times 7 \rightarrow 70$
$9 \times 7 \rightarrow 63$
Add $\rightarrow$ **133**

continued/

0528 Multiplying (cont)

4. a) 76 One possible short-cut for 19 x 4 could be: Do 20 x 4 → 80
   Then take 4 away → **76**

   b) 80 One possible short-cut for 16 x 5 could be: Do 8 x 5 → 40
   Then double it → **80**

   c) 70 One possible short-cut for 5 x 14 could be: Do 10 x 14 → 140
   Then halve it → **70**

   d) 48 One possible short-cut for 16 x 3 could be: Do 8 x 3 → 24
   Then double it → **48**

   e) 180 One possible short-cut for 6 x 30 could be: Do 6 x 10 → 60
   Then treble it → **180**

   f) 360 One possible short-cut for 9 x 40 could be: Do 10 x 40 → 400
   Then take 40 away → **360**

   You may have found different short-cuts from these, but they should always give you the same final answer.

5. a) 60 b) 160 c) 170 d) 180
   e) 210 f) 300 g) 100 h) 990

6. a) 80 b) 320 c) 120 d) 250
   e) 270 f) 420 g) 700 h) 640

7. The short-cut is to '*add a nought*'.

0549 Marbles

Number won	6	2	3	7	2	4	4	8	0	18	21	10	12	4
Number lost	2	6	7	3	4	2	4	3	5	20	25	15	7	19
Change in number	**4** more	**4** less	**4** less	**4** more	**2** less	**2** more	**0**	**5** more	**5** less	**2** less	**4** less	**5** less	**5** more	**15** less

1. -2 + -4 = -6
2. 3 + -2 = **+1**
3. 7 + -5 = **+2**
4. 6 + -5 = **+1**
5. -4 + 3 = **-1**
6. 3 + -4 = **-1**
7. -2 + -7 = **-9**
8. -7 + -2 = **-9**
9. -6 + 5 = **-1**
10. 5 + -6 = **-1**
11. -6 + -7 = **-13**
12. 2 + -2 = **0**
13. -2 + 2 = **0**
14. -5 + -5 = **-10**

0550 Adding Shifts

2. $^{+}5 + {}^{-}2 = \boxed{^{+}3}$

3. $\boxed{^{+}4} + \boxed{^{-}7} = \boxed{^{-}3}$

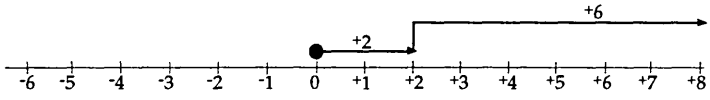
4. $\boxed{^{+}3} + \boxed{^{-}1} = \boxed{^{+}2}$

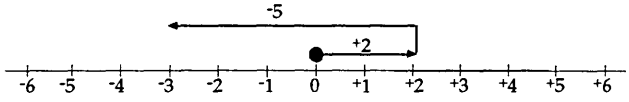
5. $\boxed{^{-}6} + \boxed{^{+}8} = \boxed{^{+}2}$

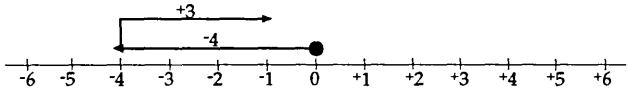
6. $\boxed{^{+}4} + \boxed{^{-}5} = \boxed{^{-}1}$

7. $\boxed{^{-}5} + \boxed{^{+}5} = \boxed{0}$

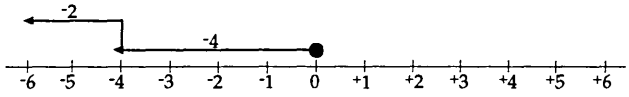
8. $\boxed{^{-}2} + \boxed{^{-}5} = \boxed{^{-}7}$

9. 

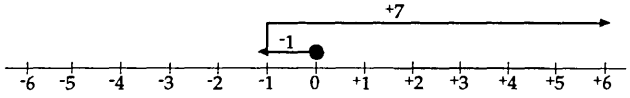
10. 

11. 

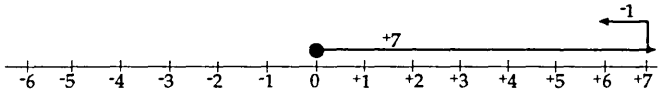
$^{-}4 + {}^{+}3 = \boxed{^{-}1}$

12. 

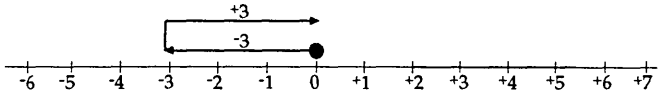
$^{-}4 + {}^{-}2 = \boxed{^{-}6}$

13. 

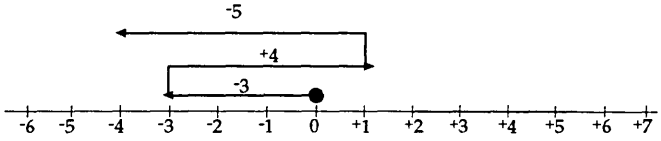
$^{-}1 + {}^{+}7 = \boxed{^{+}6}$

14. 

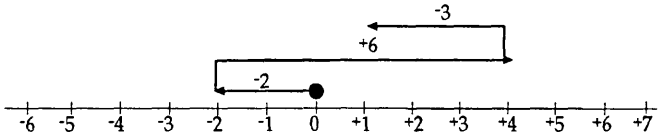
$^{+}7 + {}^{-}1 = \boxed{^{+}6}$

15. 

$^{-}3 + {}^{+}3 = \boxed{0}$

16. 

$^{-}3 + {}^{+}4 + {}^{-}5 = \boxed{^{-}4}$

17. 

$^{-}2 + {}^{+}6 + {}^{-}3 = \boxed{^{+}1}$

0557 A Special Number

One possible answer is: 6 2 1 0 0 0 1 0 0 0
How many different solutions are there?

0560 Symmetrical Cross Cut

- Solids 2, 3, 4 and 6 will give symmetrical halves.
- Solids 8, 9 and 10 can all be cut with a plane of symmetry.

Solid 11 has **seven** planes of symmetry.

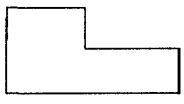
Solid 12 has **one** plane of symmetry.

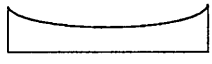
Solid 13 has **one** plane of symmetry.

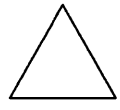
Solid 14 has **four** planes of symmetry.

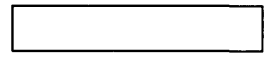
Solid 15 has **nine** planes of symmetry.

16. Any cross-cut through the centre of each of the three solids would give a plane of symmetry. Show your list of objects with the number of planes of symmetry to your teacher.

17. 

18. 

19. 

20. 

0563 Digit Sum

Here is one method for solving this problem

- Start by looking at the sum of all the digits from 1 - 100.

1st row		1	2	3	4	5	6	7	8	9	sum of units = 45
2nd row	10	11	12	13	14	15	16	17	18	19	sum of tens = 10
3rd row	20	21	22	.	.	.					
4th row	30	31	.	.	.						
5th row	40	.	.	.							
.	.	.									
.	.	.									
.	.	.									

The sum of the unit digits in the first row is 45.
What is the sum of the unit digits in the second row?
What is the sum of all the unit digits?

continued/

0563 Digit Sum (cont)

The sum of the tens digits in the second row is 10.
What is the sum of the tens digits in the third row?
What is the sum of all the tens digits?

Total of all the digits = sum of unit digits + sum of tens digits + 1
Total of all the digits from 1 - 100 is 901. Make sure you understand why.

- Then look at the sum of all the digits from 1 - 1000.
  Total of all the digits from 1 - 1000 is 17501. Make sure you understand why.

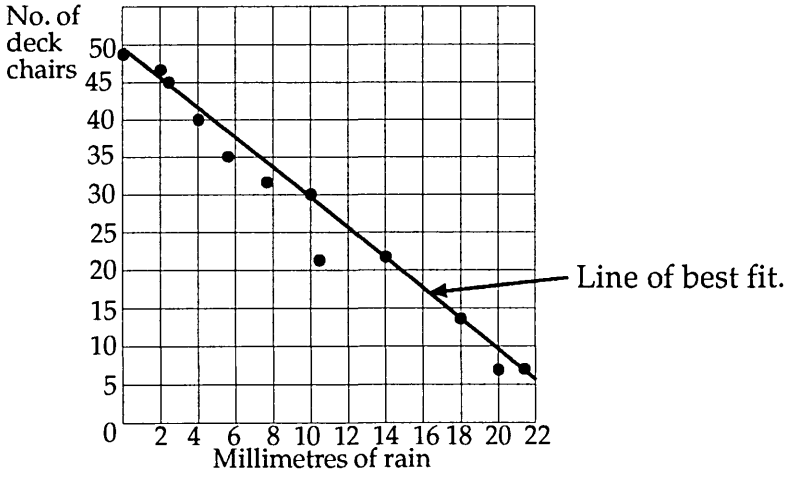
- Finally look at the sum of all the digits from 1 - 10000.
  Convince someone else that your solution is right.

0574 Line of Best Fit

1. It is the *line that fits the general position of the points best.* The points lie in one direction, i.e. from the bottom left hand corner to the top right hand corner. The line of best fit goes in the same direction. The graph shows that generally the taller the person is, the bigger their feet. This is an example of **positive correlation.**

2. About 180cm.

3. About size 39 - 40.

4. 

5. About 30 chairs.

6. About 13mm.

7. No.

8. The graph shows that the more it rains, the less people use deckchairs. The points lie in one direction, i.e. from the top left hand corner to the bottom right hand corner. This is an example of **negative correlation.**

continued/

0574 Line of Best Fit (cont)

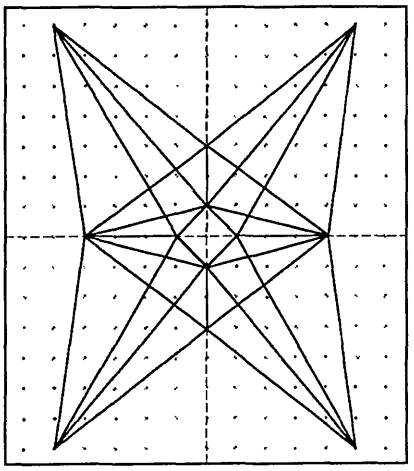
- A matches scattergram 3 or 1. This shows **negative correlation.**
- B matches scattergram 6 or 4. This shows **positive correlation.**
- C matches scattergram 2 or 5. This shows no correlation.
- D matches scattergram 4 or 6. This shows **positive correlation.**
- E matches scattergram 2 or 5. This shows no correlation.

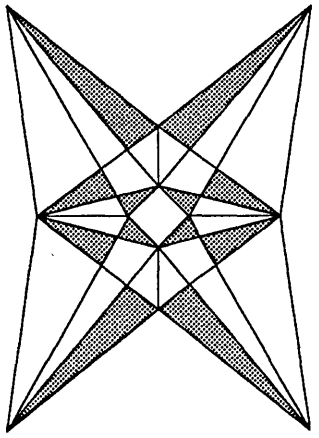
0576 Block and Back

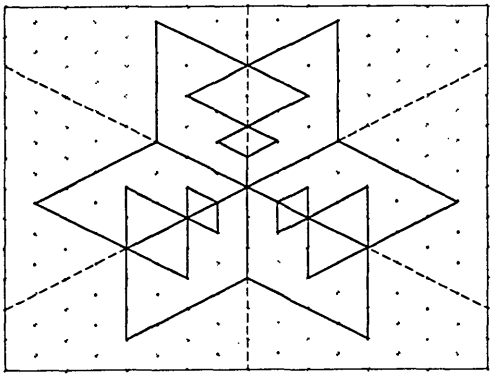
You might find it easier to group the base 10 apparatus in 10's.

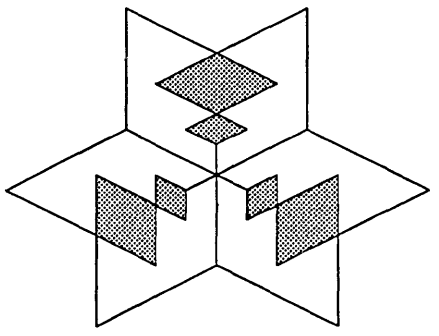
0577 Reflect

These show the designs reflected in the mirror lines and one way of shading them.









If you are unsure whether you have shaded your designs correctly, check your completed design with a mirror.

Did you reflect your own design in 2, 3 or 4 mirror lines? Show your own design to your teacher. You may like to make a display of your work.

0579 Two Loops

Who won? Did you play more than one game? Did it get easier the more you played?

0580 2-D Domino

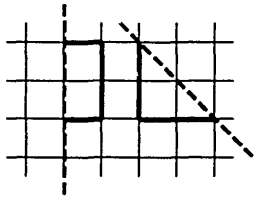
Who won? There are enough squares to fit in all the logiblocks, but you are unlikely to have placed them all. Explain why.

0581 Using a Mirror

Before you start

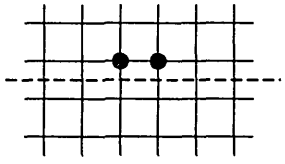
- By putting a mirror along the line and looking at the reflection you should see:
  $3 \times 3 = 8 + 1$ The sum is correct.
- The number is 83. $8 \times 10 + 3 = 83$

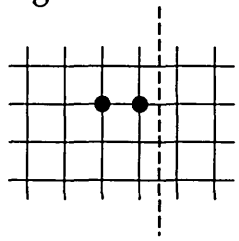
- Place a mirror along the dotted line to see a square.

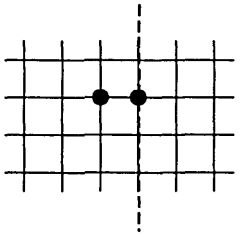


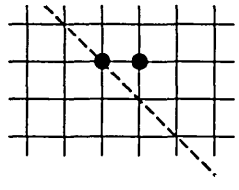
Page 1 - Dots

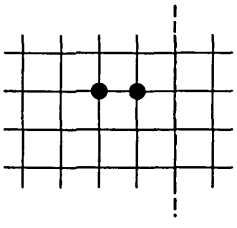
These diagrams show where the mirror must be placed to make the picture. To create some of the pictures you will have to rotate the image.

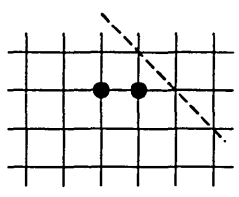
1. 

2. 

3. 

4. 

5. 

6. 

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Page 2 - Shapes

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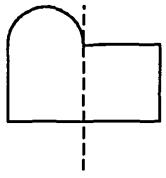
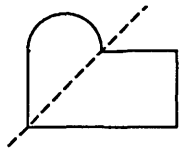
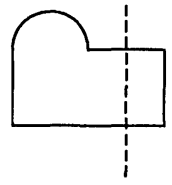
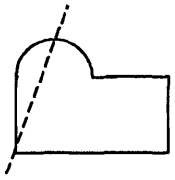
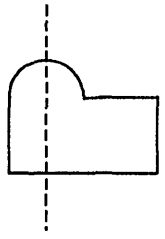
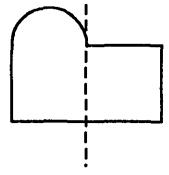
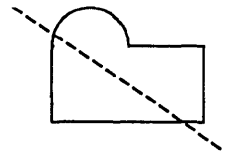
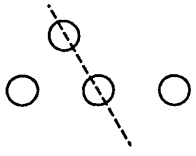
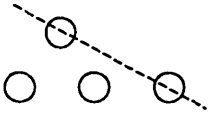
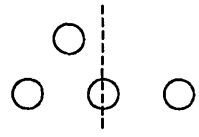
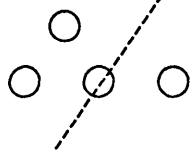
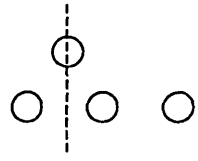
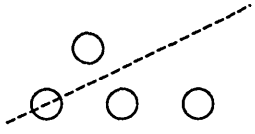
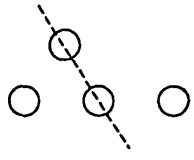
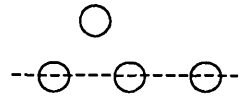
1. 
2. 
3. 
4. 
5. 
6. 
7. 

Page 3 - White Dots

These diagrams show where the mirror must be placed to make the picture. To create some of the pictures you will have to rotate the image.

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 

Page 4 - Dots and Squares

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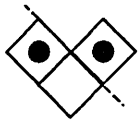
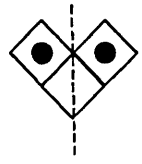
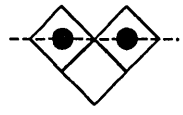
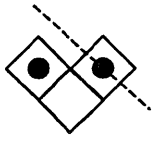
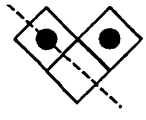
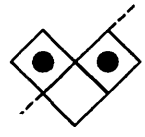
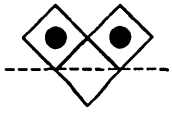
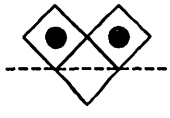
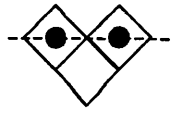
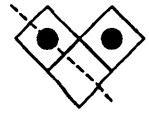
1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 

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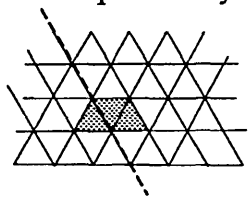
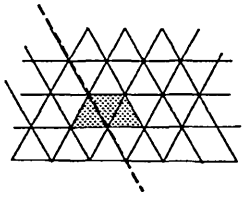
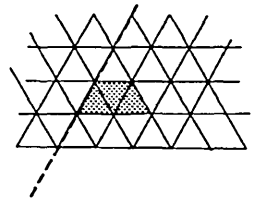
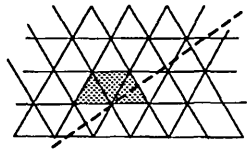
Page 5 - Triangle Shapes

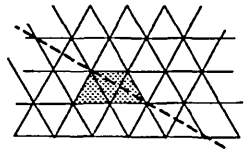
These diagrams show where the mirror must be placed to make the picture. To create some of the pictures you will have to rotate the image.

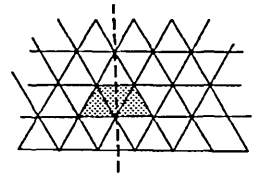
1. 

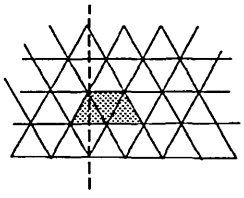
2. 

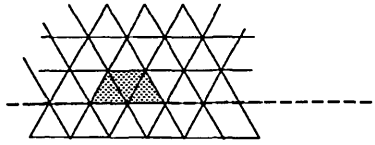
3. 

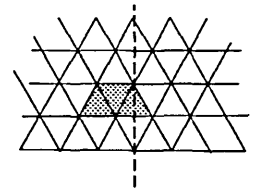
4. 

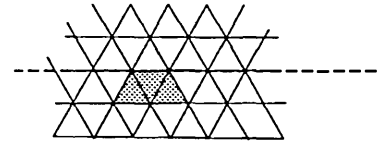
5. 

6. 

7. 

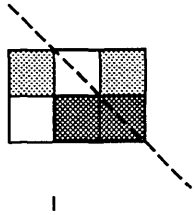
8. 

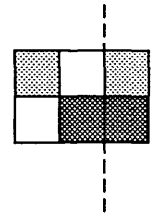
9. 

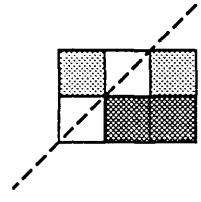
10. 

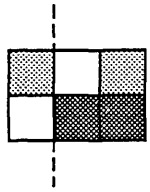
Page 6 - Shaded Squares

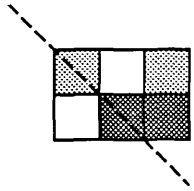
These diagrams show where the mirror must be placed to make the picture. To create some of the pictures you will have to rotate the image.

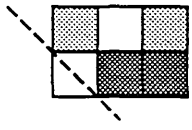
1. 

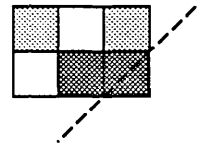
2. 

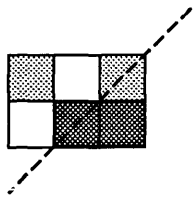
3. 

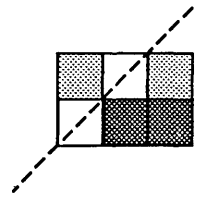
4. 

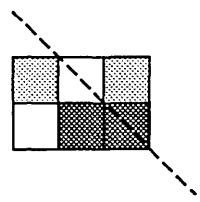
5. 

6. 

7. 

8. 

9. 

10. 

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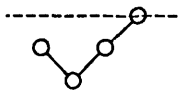
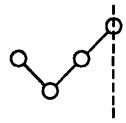
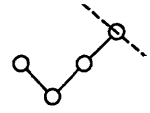
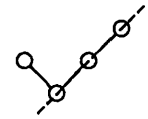
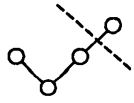
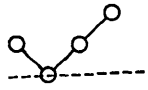
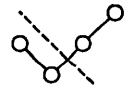
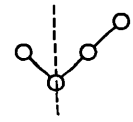
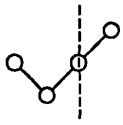
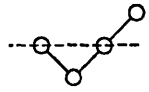
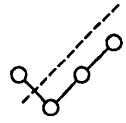
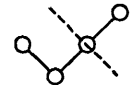
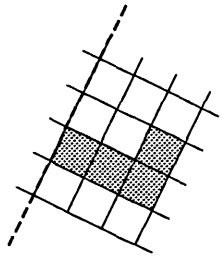
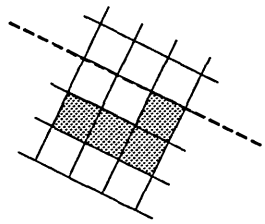
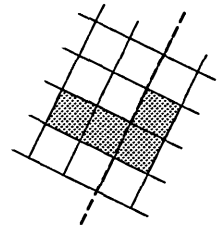
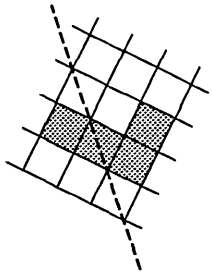
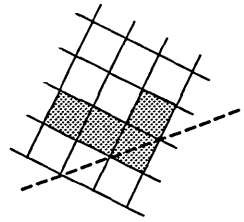
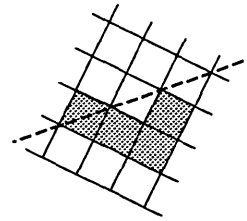
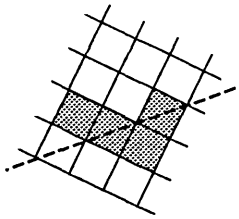
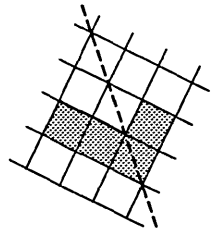
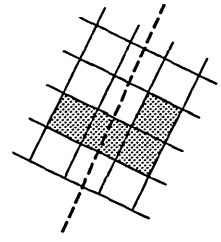
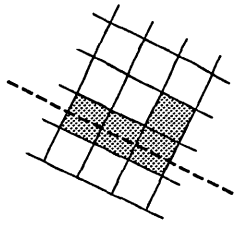
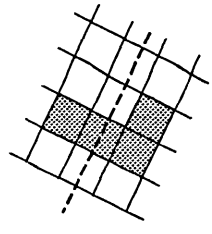
Page 7 - Links

These diagrams show where the mirror must be placed to make the picture. To create some of the pictures you will have to rotate the image.

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 
12. 

Page 8 - Square Shapes

These diagrams show where the mirror must be placed to make the picture. To create some of the pictures you will have to rotate the image.

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 

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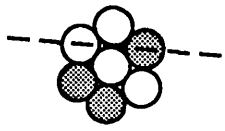
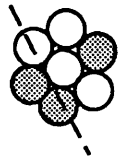
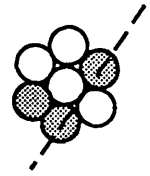
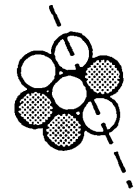
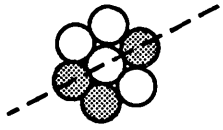
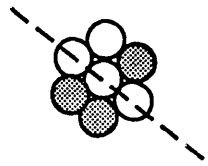
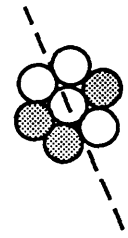
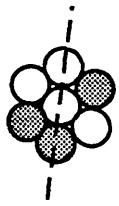
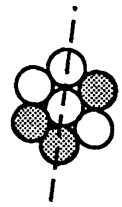
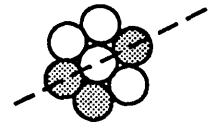
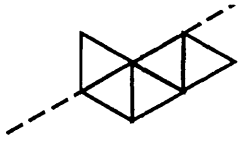
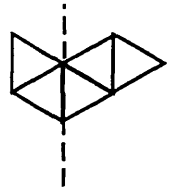
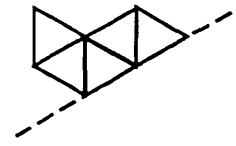
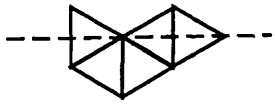
Page 9 - Flowers

These diagrams show where the mirror must be placed to make the picture. To create some of the pictures you will have to rotate the image.

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 

Page 10 - More Triangles

These diagrams show where the mirror must be placed to make the picture. To create some of the pictures you will have to rotate the image.

1. 
2. 
3. 
4. 
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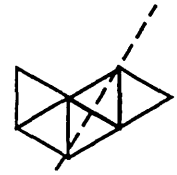
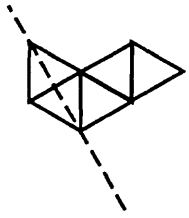
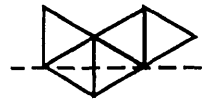
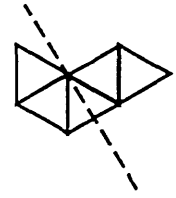
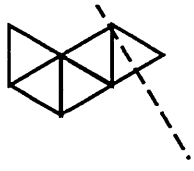
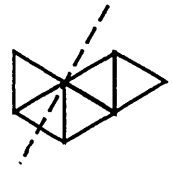
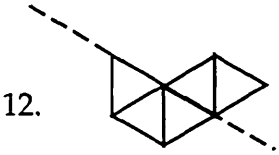
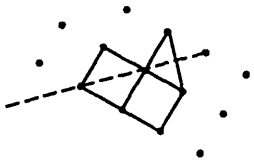
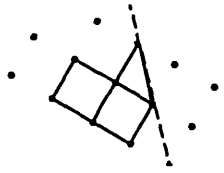
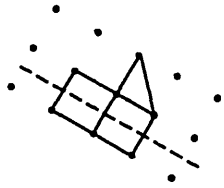
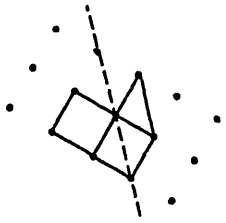
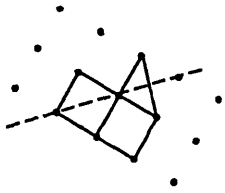
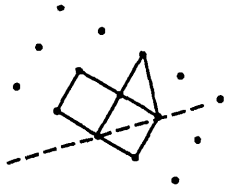
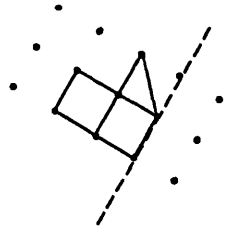
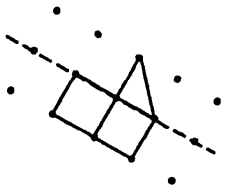
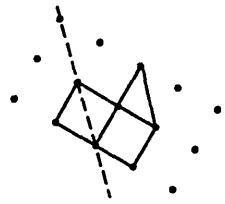
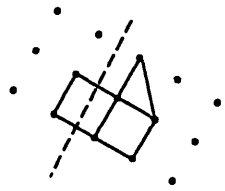
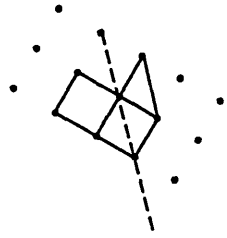
6. 
7. 
8. 
9. 
10. 
11. 
12. 

continued/

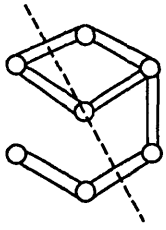
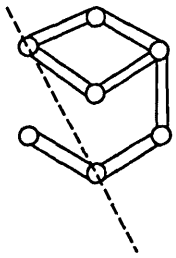
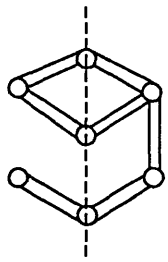
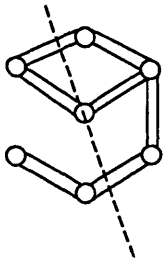
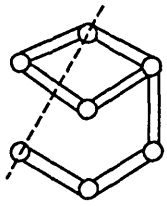
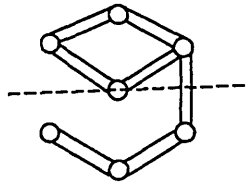
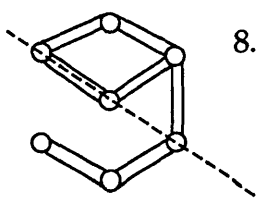
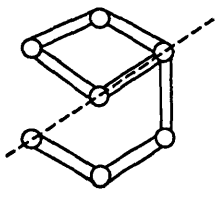
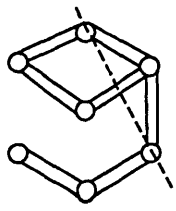
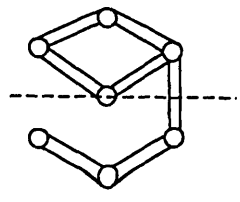
Page 11 - Joined Dots

These diagrams show where the mirror must be placed to make the picture. To create some of the pictures you will have to rotate the image.

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 

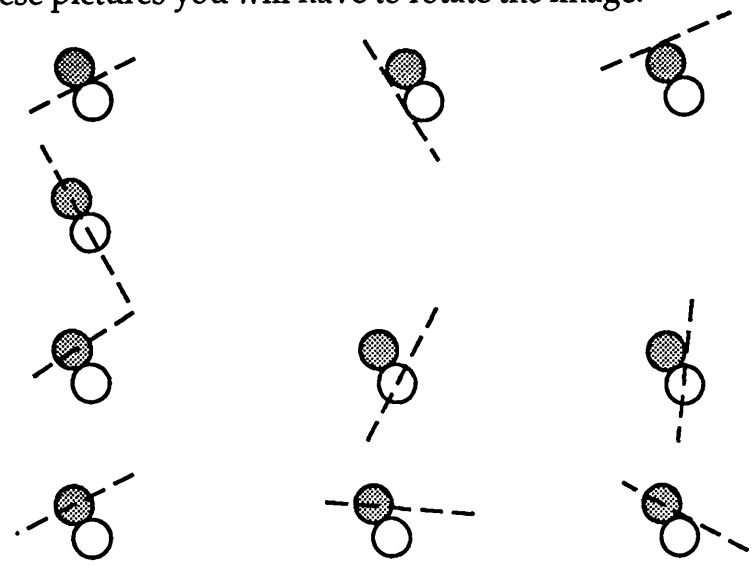
Page 12 - The 9 Puzzle

These diagrams show where the mirror must be placed to make the picture. To create some of the pictures you will have to rotate the image.

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 

continued/

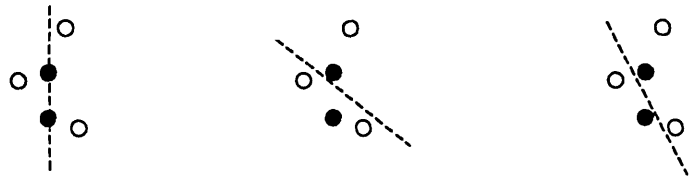
Target Test
Part 1
These diagrams show where the mirror must be placed to make the picture. To create all of these pictures you will have to rotate the image.



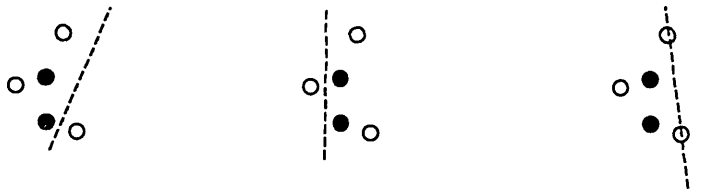
The D shape can be used to create the other four shapes.

Target Test
Part 2
These diagrams show where the mirror must be placed to make the picture. To create all of these pictures you will have to rotate the image.

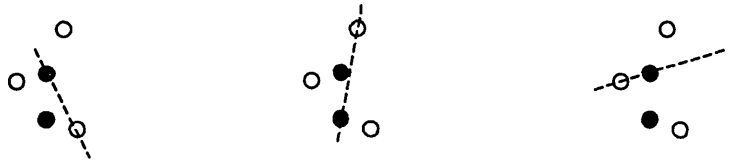
a)



b)



c)



0583 Animations

Were you able to make a 'flick book' or create a 'tumbling shape'? Make a display of your work and show it to your teacher.

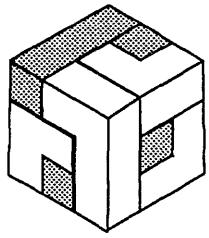
0585 Three Loops

Who won? Did you play more than one game? Did it get easier the more you played?

0589 The Soma Cube

There are many possible solutions.

Here is one:



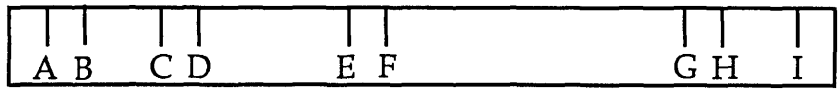
How many different ways did you find? You might like to draw one of your solutions on isometric paper to show how you did it.

0590 Less Marks are Best

- Here is one way to measure from 1cm to 10cm.
  For 1cm you can use AB.
  For 2cm you can use BC.
  For 3cm you can use EF.
  For 4cm you can use AD.
  For 5cm you can use EG.
  For 6cm you can use DG.
  For 7cm you can use CG.
  For 8cm you can use AF.
  For 9cm you can use BG.
  For 10cm you can use AG.
  If your answers are different, check them with your teacher.

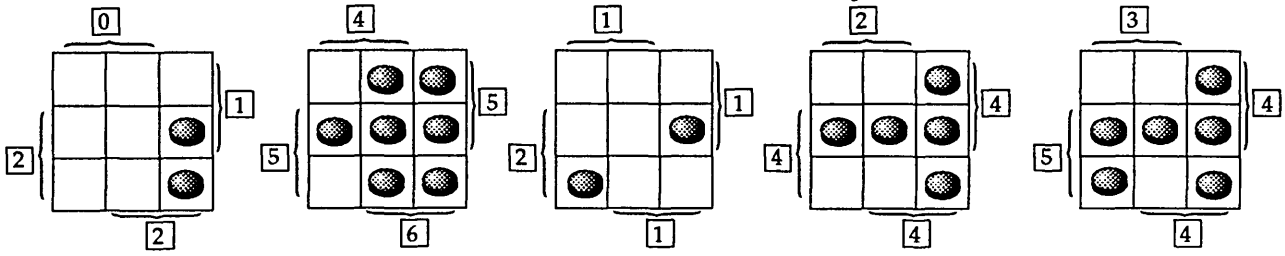
- Here is one possible way to make all the measurements using only 6 marks.

- To measure up to 10cm in half centimetres you will need 9 marks.

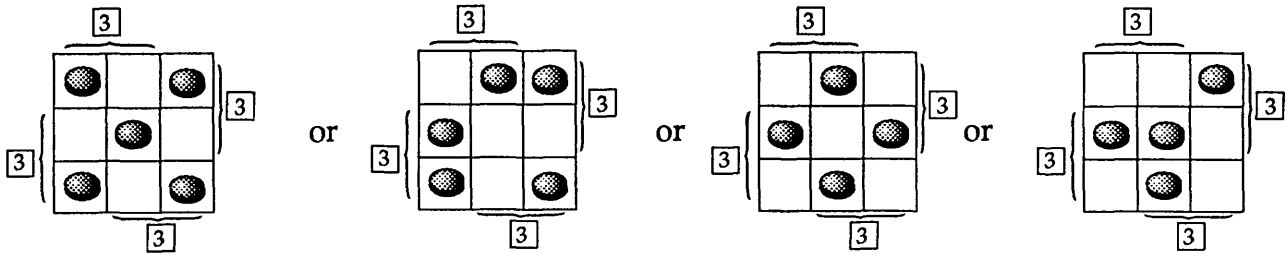


0591 Counter Placing

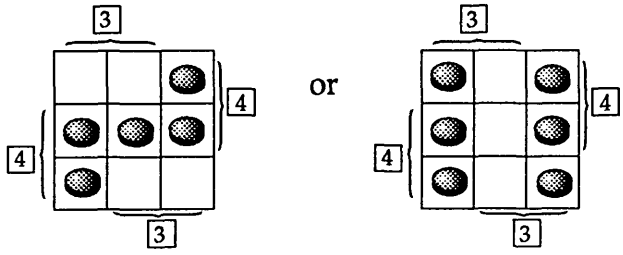
You may have more results than these if you have counted reflections and rotations as different.



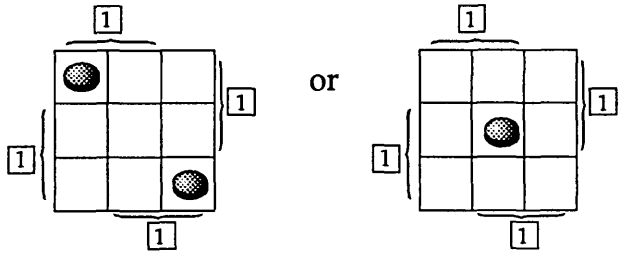
Here are four different solutions, you were asked to find two.



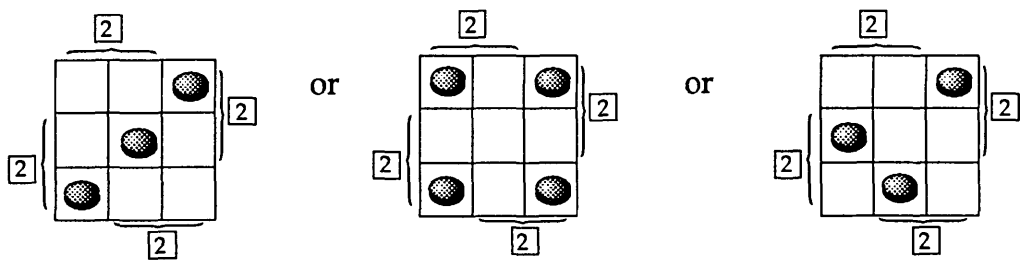
Here are the two different solutions.



Here are two different solutions. You may have found three if you allowed rotations.



Here are three different solutions. You may have found more if you allowed reflections and rotations.



0592 Powerful Rules

Powers of Two and Three

1.

POWERS OF TWO		
2	2	2^1
4	2 x 2	2^2
8	2 x 2 x 2	2^3
16	2 x 2 x 2 x 2	2^4
32	**2 x 2 x 2 x 2 x 2**	**2^5**
64	**2 x 2 x 2 x 2 x 2 x 2**	**2^6**
128	**2 x 2 x 2 x 2 x 2 x 2 x 2**	2^7
256	**2 x 2 x 2 x 2 x 2 x 2 x 2 x 2**	**2^8**
512	2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2	**2^9**
1024	**2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2**	**2^{10}**
2048	**2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2**	**2^{11}**
4096	**2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2**	**2^{12}**
8192	**2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2**	**2^{13}**
16384	**2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2**	**2^{14}**
32768	2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2	**2^{15}**

POWERS OF THREE		
3^1	3	3
3^2	3 x 3	9
3^3	3 x 3 x 3	27
3^4	3 x 3 x 3 x 3	81
3^5	**3 x 3 x 3 x 3 x 3**	**243**
3^6	**3 x 3 x 3 x 3 x 3 x 3**	**729**
3^7	**3 x 3 x 3 x 3 x 3 x 3 x 3**	2187
3^8	**3 x 3 x 3 x 3 x 3 x 3 x 3 x 3**	**6561**
3^9	3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3	**19683**
3^{10}	**3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3**	59049
3^{11}	**3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3**	**177147**
3^{12}	3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3	**531441**
3^{13}	**3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3**	1594323
3^{14}	**3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3**	**4782969**
3^{15}	3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3	14348907

continued/

2. a) 2^6 means $2 \times 2 \times 2 \times 2 \times 2 \times 2$
   b) 2^3 means $2 \times 2 \times 2$
   c) 3^1 means 3
   d) 3^6 means $3 \times 3 \times 3 \times 3 \times 3 \times 3$
   e) k^7 means $k \times k \times k \times k \times k \times k \times k$
   f) a^6 means $a \times a \times a \times a \times a \times a$

3. a) $7^3 = 343$ c) $9^3 = 729$ e) $14^2 = 196$
   b) $10^6 = 1000000$ d) $6^4 = 1296$ f) $1^5 = 1$

Multiplying Powers

1. $5^3 = 5 \times 5 \times 5$
   $5^4 = 5 \times 5 \times 5 \times 5$
   so $5^3 \times 5^4 = (5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5)$
   $= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
   $= 5^7$

2. a) $6^2 = 6 \times 6$
   $6^3 = 6 \times 6 \times 6$
   so $6^2 \times 6^3 = (6 \times 6) \times (6 \times 6 \times 6)$
   $= 6 \times 6 \times 6 \times 6 \times 6$
   $= 6^5$

   b) $4^3 = 4 \times 4 \times 4$
   $4^2 = 4 \times 4$
   so $4^3 \times 4^2 = (4 \times 4 \times 4) \times (4 \times 4)$
   $= 4 \times 4 \times 4 \times 4 \times 4$
   $= 4^5$

   c) $7^6 \times 7^2 = (7 \times 7 \times 7 \times 7 \times 7 \times 7) \times (7 \times 7)$
   $= 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$
   $= 7^8$

   d) $2^4 \times 2^3 = (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2)$
   $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
   $= 2^7$

   e) $12^5 \times 12 = (12 \times 12 \times 12 \times 12 \times 12) \times 12$
   $= 12^6$

   f) $5^5 \times 5^2 = (5 \times 5 \times 5 \times 5 \times 5) \times (5 \times 5)$
   $= 5^7$

3. a) $6^2 \times 6^3 = 6^{2+3} = 6^5$
   b) $4^3 \times 4^2 = 4^{3+2} = 4^5$
   c) $7^6 \times 7^2 = 7^{6+2} = 7^8$
   d) $2^4 \times 2^3 = 2^{4+3} = 2^7$
   e) $12^5 \times 12 = 12^{5+1} = 12^6$
   f) $5^5 \times 5^2 = 5^{5+2} = 5^7$

continued/

0592 Powerful Rules (cont)

4. a) $2^5 \times 2^2 = 2^7$ e) $5^{63} \times 5^2 = 5^{65}$
   b) $4^2 \times 4^3 = 4^5$ f) $a^5 \times a^3 = a^8$
   c) $3^4 \times 3^6 = 3^{10}$ g) $n^2 \times n^7 = n^9$
   d) $15^{10} \times 15^3 = 15^{13}$ h) $a^m \times a^n = a^{m+n}$

Using Powers to Multiply

1. $2^3 \times 2^5 = 2^8 = 256$
2. $2^4 \times 2^6 = 2^{10} = 1024$
3. $2^2 \times 2^7 = 2^9 = 512$
4. $2^1 \times 2^{13} = 2^{14} = 16384$
5. $2^4 \times 2^{10} = 2^{14} = 16384$
6. $2^5 \times 2^5 = 2^{10} = 1024$
7. $2^2 \times 2^3 \times 2^4 = 2^9 = 512$
8. $2^1 \times 2^4 \times 2^9 = 2^{14} = 16384$
9. $2^5 \times 2^5 \times 2^5 = 2^{15} = 32768$
10. $3^2 \times 3^3 = 3^5 = 243$
11. $3^4 \times 3^1 = 3^5 = 243$
12. $3^3 \times 3^5 = 3^8 = 6561$
13. $3^{10} \times 3^3 = 3^{13} = 1594323$
14. $3^1 \times 3^2 \times 3^3 = 3^6 = 729$
15. $3^2 \times 3^2 \times 3^2 = 3^6 = 729$
16. $3^4 \times 3^3 \times 3^2 \times 3^1 = 3^{10} = 59049$
17. $3^4 \times 3^4 \times 3^1 \times 3^1 = 3^{10} = 59049$
18. $3^3 \times 3^3 \times 3^3 \times 3^3 = 3^{12} = 531441$

Dividing Powers

1. a) $\frac{2^7}{2^3} = \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2 \times 2 \times 2}{\cancel{2} \times \cancel{2} \times \cancel{2}} = 2^4$

   b) $\frac{3^7}{3^3} = \frac{\cancel{3} \times \cancel{3} \times \cancel{3} \times 3 \times 3 \times 3 \times 3}{\cancel{3} \times \cancel{3} \times \cancel{3}} = 3^4$

   c) $\frac{2^9}{2^4} = \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2 \times 2 \times 2 \times 2}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}} = 2^5$

   d) $\frac{2^7}{2^4} = \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2 \times 2}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}} = 2^3$

   e) $\frac{5^{10}}{5} = \frac{\cancel{5} \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{\cancel{5}} = 5^9$

   f) $\frac{10^5}{10^2} = \frac{\cancel{10} \times \cancel{10} \times 10 \times 10 \times 10}{\cancel{10} \times \cancel{10}} = 10^3$

   g) $\frac{7^9}{7^8} = \frac{\cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times 7}{\cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7}} = 7^1 = 7$

   h) $\frac{6^5}{6^2} = \frac{\cancel{6} \times \cancel{6} \times 6 \times 6 \times 6}{\cancel{6} \times \cancel{6}} = 6^3$

   i) $\frac{18^5}{18^2} = \frac{\cancel{18} \times \cancel{18} \times 18 \times 18 \times 18}{\cancel{18} \times \cancel{18}} = 18^3$

   j) $\frac{q^6}{q^2} = \frac{\cancel{q} \times \cancel{q} \times q \times q \times q \times q}{\cancel{q} \times \cancel{q}} = q^4$

   k) $\frac{m^8}{m^7} = \frac{\cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m} \times m}{\cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m}} = m^1 = m$

continued/

2. a) $2^7 \div 2^3 = 2^{7-3} = 2^4$
   b) $3^7 \div 3^3 = 3^{7-3} = 3^4$
   c) $2^9 \div 2^4 = 2^{9-4} = 2^5$
   d) $2^7 \div 2^4 = 2^{7-4} = 2^3$
   e) $5^{10} \div 5^1 = 5^{10-1} = 5^9$
   f) $10^5 \div 10^2 = 10^{5-2} = 10^3$
   g) $7^9 \div 7^8 = 7^{9-8} = 7^1$
   h) $6^5 \div 6^2 = 6^{5-2} = 6^3$
   i) $18^5 \div 18^2 = 18^{5-2} = 18^3$
   j) $q^6 \div q^2 = q^{6-2} = q^4$
   k) $m^8 \div m^7 = m^{8-7} = m^1$

3. a) 3^4 b) 13^8 c) 9^4 d) 17^1 e) 2^1 f) a^{11}

Powers of Ten

1.

10	10	10^1
100	10 x 10	10^2
1000	10 x 10 x 10	10^3
10000	10 x 10 x 10 x 10	10^4
100000	10 x 10 x 10 x 10 x 10	10^5
1000000	10 x 10 x 10 x 10 x 10 x 10	10^6
10000000	10 x 10 x 10 x 10 x 10 x 10 x 10	10^7
100000000	10 x 10 x 10 x 10 x 10 x 10 x 10 x 10	10^8
1000000000	10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10	10^9
10000000000	10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10	10^{10}

2. Your answer may be different, but a number such as 10^{10} would be used to describe a very large amount such as the population of the world.

3. a) $10^3 \div 10^2 = 10^1$
   b) $10^5 \div 10^2 = 10^3$
   c) $10^6 \div 10^3 = 10^3$
   d) $10^5 \div 10^4 = 10^1$
   e) $10^{10} \div 10^1 = 10^9$
   f) $10^3 \div 10^3 = 10^0$ 10^0 is not in your table. Using your table, what do you think 10^0 is?

4. Any number divided by itself will give the answer 1.
   Any number to the power of 0 = 1.
   If you are unsure whether your answers are correct, check them with your teacher.

Mixed Bag

1. a) 27
   b) 343
   c) 81
   d) 1
   e) 0
   f) 1296
   g) $6^4 = 1296$
   h) $6^4 = 1296$
   i) $11^2 = 121$
   j) $8 \times 25 = 200$
   k) $2^{10} = 1024$
   l) $2^4 = 16$

continued/

0592 Powerful Rules (cont)

2. a) $125 = 5^3$
   b) $7776 = 6^5$
   c) $14641 = 11^4$
   d) $169 = 13^2$

3. a) 9^4
   b) 6^{34}
   c) y^3
   d) $2^7 \times 3^9$
   e) 3^6
   f) 8^{16}
   g) $5^{10} \div 5^2 = 5^8$
   h) $5^4 \times 5^4 = 5^8$
   i) $2^2 \times 5$
   j) $7^0 = 1$

4. When you multiply powers of the same number, you **add** the powers.
   When you divide powers of the same number, you **subtract** the powers.
   If you are unsure about your answer, show your work to your teacher.

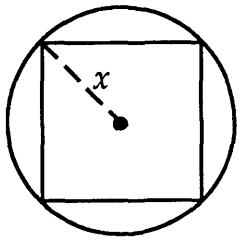
0595 Best Fitting Peg

Here is one explanation.

You may have decided to look at the problem in a different way. If your results are significantly different, explain your work to your teacher.

Let: x = radius of the circle
y = length of the side of square

- **A square peg in a round hole.**



$y = \sqrt{2}x$

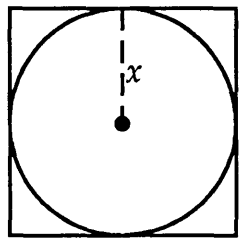
Area of square peg $= 2x^2$
Area of round hole $= \pi x^2$

$$\frac{\text{Area of space}}{\text{Area of round hole}} = \frac{(\pi - 2)x^2}{\pi x^2} = \frac{\pi - 2}{\pi}$$

$= 0.363$

Percentage of space $= 36.3\%$

- **A round peg in a square hole.**



$y = 2x$

Area of round peg $= \pi x^2$
Area of square hole $= 4x^2$

$$\frac{\text{Area of space}}{\text{Area of square hole}} = \frac{(4 - \pi)x^2}{4x^2} = \frac{4 - \pi}{4}$$

$= 0.215$

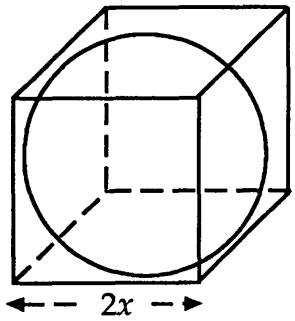
Percentage of space $= 21.5\%$

A round peg in a square hole fits best because $21.5\% < 36.3\%$.
The amount of space is smaller.

continued/

Let: x = the radius of the sphere
y = the length of the side of the cube.

- **A sphere inside a cube**



$y = 2x$

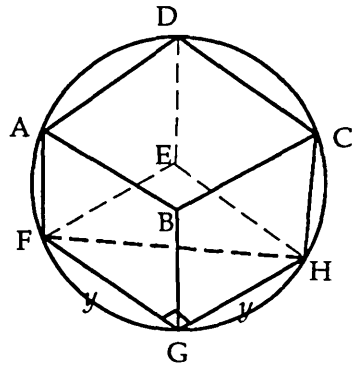
Volume of sphere $= \frac{4}{3}\pi x^3$
Volume of cube $= 8x^3$

$$\frac{\text{Volume of space}}{\text{Volume of cube}} = \frac{(8-\frac{4}{3}\pi)x^3}{8x^3}$$

$$= 0.476$$

Percentage of space = 47.6%

- **A cube inside a sphere**

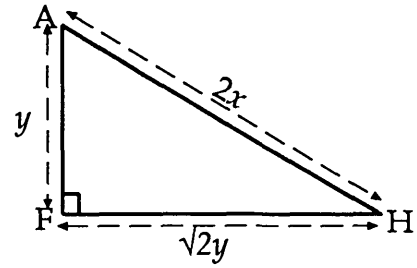


Each vertex of the cube, A, B, C, D, E, F, G and H touches the edge of the sphere.

In triangle FGH

$FH^2 = y^2 + y^2$
$FH = \sqrt{2}y$

In triangle AFH



AH = $2x$ (diameter of the sphere)
$y^2 + (\sqrt{2}y)^2 = (2x)^2$

$3y^2 = 4x^2$

$y = \frac{2x}{\sqrt{3}}$

Volume of cube $= \left(\frac{2x}{\sqrt{3}}\right)^3$
Volume of sphere $= \frac{4}{3}\pi x^3$

$$\frac{\text{Volume of space}}{\text{Volume of sphere}} = \frac{(\frac{4}{3}\pi - \frac{8}{3\sqrt{3}})x^3}{\frac{4}{3}\pi x^3}$$

$$= 0.632$$

Percentage of space = 63.2%

A sphere inside a cube fits best because 47.6% < 63.2%.

0596 Undercover Agent

1. W O R D A S P I D I S T R A

23, 15, 18, 4 **1, 19, 16, 9, 4, 9, 19, 20, 18, 1**

$$\begin{pmatrix} \mathbf{23} & \mathbf{15} \\ \mathbf{18} & \mathbf{4} \end{pmatrix} \qquad \begin{pmatrix} \mathbf{1} & \mathbf{19} & \mathbf{16} & \mathbf{9} & \mathbf{4} \\ \mathbf{9} & \mathbf{19} & \mathbf{20} & \mathbf{18} & \mathbf{1} \end{pmatrix}$$

Coder

$$\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} \mathbf{23} & \mathbf{15} \\ \mathbf{18} & \mathbf{4} \end{pmatrix} = \begin{pmatrix} \mathbf{41} & \mathbf{19} \\ \mathbf{100} & \mathbf{42} \end{pmatrix} \qquad \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} \mathbf{1} & \mathbf{19} & \mathbf{16} & \mathbf{9} & \mathbf{4} \\ \mathbf{9} & \mathbf{19} & \mathbf{20} & \mathbf{18} & \mathbf{1} \end{pmatrix} = \begin{pmatrix} \mathbf{10} & \mathbf{38} & \mathbf{36} & \mathbf{27} & \mathbf{5} \\ \mathbf{29} & \mathbf{95} & \mathbf{92} & \mathbf{72} & \mathbf{11} \end{pmatrix}$$

41, 19, 100, 42 10, 38, 36, 27, 5, 29, 95, 92, 72, 11

2. M E E T A T A N I M A L F A R M

13, 5, 5, 20, 1, 20 1, 14, 9, 13, 1, 12 6, 1, 18, 13

Coder

$$\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 13 & 5 \\ 5 & 20 \end{pmatrix} = \begin{pmatrix} 18 & 25 \\ 41 & 70 \end{pmatrix} \qquad \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 20 \end{pmatrix} = \begin{pmatrix} 21 \\ 62 \end{pmatrix} \qquad \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 14 & 9 \\ 13 & 1 & 12 \end{pmatrix} = \begin{pmatrix} 14 & 15 & 21 \\ 41 & 31 & 54 \end{pmatrix} \qquad \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 6 & 1 \\ 18 & 13 \end{pmatrix} = \begin{pmatrix} 24 & 14 \\ 66 & 41 \end{pmatrix}$$

18, 25, 41, 70 21, 62 14, 15, 21, 41, 31, 54 24, 14, 66, 41

3. Decoder

$$\begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 41 & 19 \\ 100 & 42 \end{pmatrix} = \begin{pmatrix} 23 & 15 \\ 18 & 4 \end{pmatrix} \qquad \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 10 & 38 & 36 & 27 & 5 \\ 29 & 95 & 92 & 72 & 11 \end{pmatrix} = \begin{pmatrix} 1 & 19 & 16 & 9 & 4 \\ 9 & 19 & 20 & 18 & 1 \end{pmatrix}$$

23, 15, 18, 4 1, 19, 16, 9, 4, 9, 19, 20, 18, 1

CODE **W O R D A S P I D I S T R A**

4. 26, 23, 53, 64 34, 14, 19, 19, 88, 33, 43, 52 13, 29, 32, 34, 78, 89 27, 33, 75, 84

$$\begin{pmatrix} 26 & 23 \\ 53 & 64 \end{pmatrix} \qquad \begin{pmatrix} 34 & 14 & 19 & 19 \\ 88 & 33 & 43 & 52 \end{pmatrix} \qquad \begin{pmatrix} 13 & 29 & 32 \\ 34 & 78 & 89 \end{pmatrix} \qquad \begin{pmatrix} 27 & 33 \\ 75 & 84 \end{pmatrix}$$

Decoder

$$\begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 26 & 23 \\ 53 & 64 \end{pmatrix} = \begin{pmatrix} 25 & 5 \\ 1 & 18 \end{pmatrix} \qquad \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 34 & 14 & 19 & 19 \\ 88 & 33 & 43 & 52 \end{pmatrix} = \begin{pmatrix} 14 & 9 & 14 & 5 \\ 20 & 5 & 5 & 14 \end{pmatrix} \qquad \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 13 & 29 & 32 \\ 34 & 78 & 89 \end{pmatrix} = \begin{pmatrix} 5 & 9 & 7 \\ 8 & 20 & 25 \end{pmatrix} \qquad \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 27 & 33 \\ 75 & 84 \end{pmatrix} = \begin{pmatrix} 6 & 15 \\ 21 & 18 \end{pmatrix}$$

25, 5, 1, 18 14, 9, 14, 5, 20, 5, 5, 14 5, 9, 7, 8, 20, 25 6, 15, 21, 18

Y E A R N I N E T E E N E I G H T Y F O U R

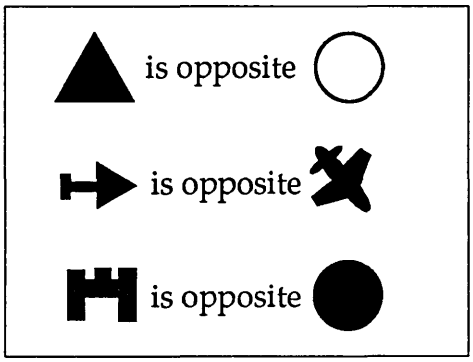
0597 Sunita's Day

1. a) 1 hour and 45 minutes.
   b) 1 hour.
   c) $\frac{1}{2}$ hour.
   d) 5 hours and 45 minutes.

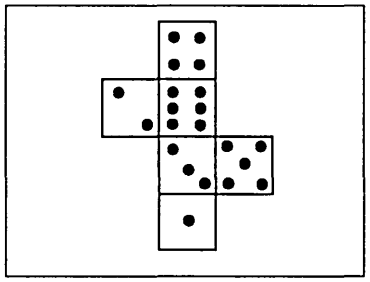
2. 15 hours.

3. 9 hours.

0600 In your Mind



These are not the same dice, because the top left dice and the bottom dice imply that the ✚ symbol and the ∧ symbol occupy the same face.



0601 Anti-magic Squares

Here are some possible answers.

9	5	7	→ 21
1	2	3	→ 6
4	8	6	→ 18
↙	↓	↓	↓ ↘
13	14	15	16 17

9	8	7	→ 24
2	1	6	→ 9
3	4	5	→ 12
↙	↓	↓	↓ ↘
11	14	13	18 15

1	9	8	→ 18
3	2	7	→ 12
4	5	6	→ 15
↙	↓	↓	↓ ↘
14	8	16	21 9

0603 Numbering the Pages

There are 32 pages.

You should have found that with 6 sheets:

- on any one sheet, the four numbers always add up to 50.
- on any one side the two numbers add up to 25.

Here are the results of looking at the page number patterns for other sizes of newspaper.

Number of sheets	Number of pages	Total of numbers on any one sheet.	Total of numbers on any one side.
1	4	10	5
2	8	18	9
3	12	26	13
4	16	34	17
5	20	42	21
6	24	50	25
.	.	.	.
.	.	.	.
.	.	.	.
n	?	?	?

0611 Hop or Slide

No answer to the puzzle but the minimum number of moves needed, that we found, was 35. Did you manage to do the puzzle in fewer moves?

0614 Powers of Tens

10^6	1000000	10 x 10 x 10 x 10 x 10 x 10
10^5	100000	10 x 10 x 10 x 10 x 10
10^4	10000	10 x 10 x 10 x 10
10^3	1000	10 x 10 x 10
10^2	100	10 x 10
10^1	10	10
10^0	1	1
10^{-1}	$\frac{1}{10}$	$\frac{1}{10}$
10^{-2}	$\frac{1}{100}$	$\frac{1}{10} \times \frac{1}{10}$
10^{-3}	$\frac{1}{1000}$	$\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$
10^{-4}	$\frac{1}{10000}$	$\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$
10^{-5}	$\frac{1}{100000}$	$\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$
10^{-6}	$\frac{1}{1000000}$	$\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$

0616 The Unknown Square

$4^2 = 3^2 + (2 \times 3) + 1$
$16 = 9 + 6 + 1$

1. Let $(x + 1) = 8$, so $x = 7$
$8^2 = (7 + 1)^2$
$64 = 7^2 + (2 \times 7) + 1^2$
$64 = 49 + 14 + 1$

2.

4^2	$(3 + 1)^2$	$3^2 + (2 \times 3) + 1^2$	$9 + 6 + 1$	16
5^2	$(4 + 1)^2$	$\mathbf{4^2 + (2 \times 4) + 1^2}$	**16 + 8 + 1**	**25**
6^2	$(5 + 1)^2$	$\mathbf{5^2 + (2 \times 5) + 1^2}$	**25 + 10 + 1**	**36**
$\mathbf{7^2}$	$(6 + 1)^2$	$\mathbf{6^2 + (2 \times 6) + 1^2}$	**36 + 12 + 1**	**49**
$\mathbf{8^2}$	$(7 + 1)^2$	$\mathbf{7^2 + (2 \times 7) + 1^2}$	**49 + 14 + 1**	**64**
$\mathbf{9^2}$	$(8 + 1)^2$	$\mathbf{8^2 + (2 \times 8) + 1^2}$	**64 + 16 + 1**	**81**

3. You should have found that the numbers in the last column are the squares of the numbers in the first column.

4. $101^2 = (100 + 1)^2$
$= 100^2 + (2 \times \mathbf{100}) + \mathbf{1^2}$
$= 10000 + \mathbf{200} + \mathbf{1}$
$= \mathbf{10201}$

5. a) $21^2 = (20 + 1)^2$
$= 20^2 + (2 \times 20) + 1^2$
$= 400 + 40 + 1$
$= 441$

b) $51^2 = (50 + 1)^2$
$= 2500 + (2 \times 50) + 1^2$
$= 2601$

c) $301^2 = (300 + 1)^2$
$= 90000 + (2 \times 300) + 1^2$
$= 90601$

6. $1\frac{1}{2}^2 = (\frac{1}{2} + 1)^2$
$= \frac{1}{2}^2 + (2 \times \frac{1}{2}) + 1^2$
$= \frac{1}{4} + 1 + 1$
$= 2\frac{1}{4}$

0617 Looking Around

Your answers will vary according to what is in your classroom. If you are unsure whether you answers are correct, show your work to your teacher. You probably found that cubes and cuboids were the solids that had the most examples.

0618 Action Decimals

Using Squares

1. a) $3\frac{2}{10} \times 4 = 12\frac{8}{10}$
   b) $3.2 \times 4 = 12.8$

2. a) $2\frac{6}{10} \times 3 = 7\frac{8}{10}$
   b) $2.6 \times 3 = 7.8$

3. a) $17\frac{6}{10}$
   b) 17.6

4. a) $8\frac{6}{10}$
   b) 8.6

5. a) $2\frac{1}{10}$
   b) 2.1

6. a) 2
   b) 2.0

7. a) $23\frac{1}{10}$
   b) 23.1

8. a) $16\frac{8}{10}$
   b) 16.8

9. a) $4\frac{74}{100}$
   b) 4.74

10. a) $4\frac{89}{100}$
    b) 4.89

11. a) $7\frac{26}{100}$
    b) 7.26

12. a) $13\frac{17}{100}$
    b) 13.17

13. a) $2\frac{84}{100}$
    b) 2.84

14. a) $3\frac{12}{100}$
    b) 3.12

15. a) $6\frac{25}{100}$
    b) 6.25

16. a) $6\frac{75}{100}$
    b) 6.75

17. a) $22\frac{75}{100}$
    b) 22.75

18. a) $2\frac{75}{100}$
    b) 2.75

19. a) $\frac{25}{100}$
    b) 0.25

20. 8.68

21. 5.74

22. 11.56

23. 2.52

24. 11.97

25. 0.06

Using Dots

1. 4.83

2. 7.98

3. 2.58

4. 1.62

5. 0.01

6. 5.89

0629 Time Tiles

Show your finished arrangement of Time Tiles to your teacher.

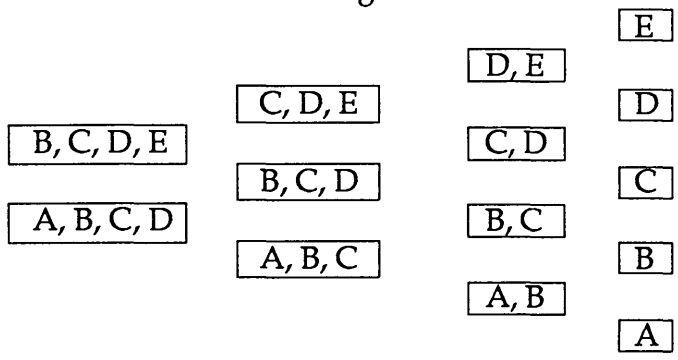
0634 Sidings

Did it make a difference whether you went first or second? Discuss your strategies with your teacher.

1. The orange train could get to four finishing points, B, C, D or E.

2. The grey train could get to three finishing points, B, C or D.

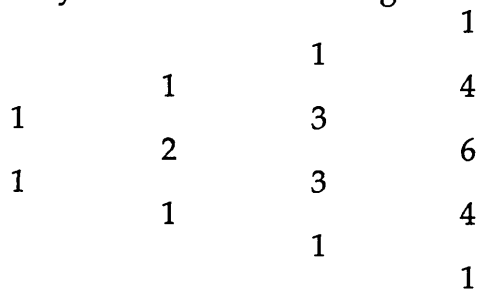
3. The answers for all the sidings are:



4. There are three ways to reach the position of the grey train.

5. There is one way to reach the position of the orange train.

6. The numbers of ways to reach all the sidings are:



Have you seen this arrangement of numbers before?

Total number of routes to A, B, C, D and E = 1 + 4 + 6 + 4 + 1 = 16

7. A = 1 1 1 1 This is the only route that finishes at A.

8. a) B
   b) There are three more routes that finish at B, 0 1 1 1, 1 1 0 1 and 1 1 1 0.

9.

Finish	Routes
A	1 1 1 1
B	1 1 1 0 1 1 0 1 1 0 1 1 0 1 1 1 0
C	1 1 0 0 1 0 1 0 1 0 0 1 0 1 1 0 0 1 0 1 0 0 1 1
D	1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1
E	0 0 0 0

0660 Palindromes

There have been many palindromes made up in every major language, but they are not easy to invent. Here are two famous examples:

- A man, a plan, a canal, Panama
- Was it a car or a cat I saw?

The palindromic number investigation is a famous unsolved problem. It has been suggested that, no matter what number you start with, you will arrive at a palindrome after a finite number of steps. No one knows whether this conjecture is true or false. The smallest number which may be a counter example is 196. Computers have carried it to hundreds of thousands of steps without obtaining a palindrome (but nobody had yet proved that it will never produce one).

Here are some questions which may help you investigate palindromic numbers further.

- What are the prime factors of 343?

- 777, 434 and 252 are also palindromic numbers with three digits that are multiples of 7. Find all the remaining numbers of this kind, and prove that the sum of the first pair of digits must be 7 or 14.

- How many of these palindromic multiples of 7 are exactly divisible by 3?

- If the three digit palindromic number 100a+10b+a is a multiple of 11, prove that $2a = b$ *or* $2a = b + 11$. Hence find all such numbers, and verify that when each of them is divided by 11, the quotient is either a multiple of 11, or one more than a multiple of 11.

- Find four three-digit palindromic numbers that are multiples of 19.

- Show that all the three-digit palindromic numbers that are multiples of 29 are also multiples of 8. Verify that the sum of the first pair of digits is always a multiple of 5.

- Show that all the three-digit palindromic numbers that are multiples of 31 are also multiples of 7, and that the sums of their three digits are exactly divisible by 11.

0667 Codes

You will know if you have used the code correctly because your answer will be a sentence.

- Key word: 'ANSWERS' (see page 15)
  If you can read this you must be an expert decoder. Well done!

0669 Pegboards

When you set out your results, make it clear what rules you used.

0674 A Hungry Death?

- Open the box labelled bomb first. Why?
  If the sandwiches are not in this box, open the box labelled secret.
  Why should you not open the box labelled lunch?

0675 Cube Cuts

The minimum number of cuts for a 3 x 3 x 3 cube is 6. It is not possible to do it with fewer cuts even if you arrange the pieces after each cut.

- What was the minimum number of cuts for larger cubes and cuboids?

0676 Regular Tetrahedron Study Group

0676j

1. a) They are all examples of the same solid.
   b) The size is different. The materials each one is made from, are different.
   c) Yes
   d) Triangular, each face is an equilateral triangle.
   e) 4 faces, 6 edges, 4 vertices.

2. a) Because these solids have four plane triangular faces.
   b) A triangle which has all sides equal in length.

3. a) Many possible answers.
   b) They do not pack together very easily.
   c) They preserve things better i.e. they keep the razor blades sharp and free from rust. Pyramids are good for preserving things. Apparently one of the reasons why Ancient Egyptian mummies, etc. have lasted so long is because they were placed inside the pyramids.

4. Your commentary should contain all the facts mentioned above.
   It may, of course, contain more.

5. Show your display to your teacher.

0677 Logic Maps

If you played the game using the same board and attributes as on the card, which logiblocks was it best to have?

0678 7 Card Add

How did you decide when the game was finished? Who won?
What was the highest score?

0679 7 Card Minus

How did you decide when the game was finished? Who won?
What was the highest score?

0680 Intercepted Messages

1. Coder A $= \begin{pmatrix} 50 & 66 & 81 \\ 85 & 114 & 139 \end{pmatrix}$ Decoder A $= \begin{pmatrix} 15 & 18 & 23 \\ 5 & 12 & 12 \end{pmatrix}$

   $=$ **50 66 81 85 114 139** $=$ **15 18 23 5 12 12**

2. The patterns in the coders and decoders suggest that

   for Coder $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the Decoder would be $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

   So, if the Coder is $\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ then the Decoder is $\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$

   Did your decoder work?

To get the decoder
1.&2. You should have found that the rule works for all the other coders.

3. Decoder $= \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$

4. $\begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 54 & 8 & 63 & 59 \\ 40 & 7 & 47 & 44 \end{pmatrix}$

   $= \begin{pmatrix} 14 & 1 & 16 & 15 \\ 12 & 5 & 15 & 14 \end{pmatrix}$

   $=$ 14 1 16 15 12 5 15 14
   N A P O L E O N

5. A $\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$ B $\begin{pmatrix} -2 & 1 \\ -5 & 2 \end{pmatrix}$ C $\begin{pmatrix} -2 & 7 \\ -1 & 3 \end{pmatrix}$

   D $\begin{pmatrix} -2 & 3 \\ -3 & 4 \end{pmatrix}$ E $\begin{pmatrix} 5 & -3 \\ -3 & 2 \end{pmatrix}$

0683 Fraction Sort

Here are the fifteen matching goups.

Fraction		Decimal		Percentage
$\frac{1}{100}$	=	0.01	=	1%
$\frac{1}{50}$	=	0.02	=	2%
$\frac{1}{20}$	=	0.05	=	5%
$\frac{1}{10}$	=	0.1	=	10%
$\frac{1}{5}$	=	0.2	=	20%
$\frac{1}{4}$	=	0.25	=	25%
$\frac{3}{10}$	=	0.3	=	30%
$\frac{2}{5}$	=	0.4	=	40%
$\frac{1}{2}$	=	0.5	=	50%
$\frac{3}{5}$	=	0.6	=	60%
$\frac{7}{10}$	=	0.7	=	70%
$\frac{3}{4}$	=	0.75	=	75%
$\frac{4}{5}$	=	0.8	=	80%
$\frac{9}{10}$	=	0.9	=	90%
$\frac{10}{10}$	=	1	=	100%

0684 Forty Towers

The following 4 - towers are all different:

6 towers with green at the bottom

6 towers with green second from the bottom

6 towers with green third from the bottom

6 towers with green at the top

There are 24 different 4 - towers.

continued/

0684 Forty Towers (cont)

Number of colours	Number of different towers	
1	1	1 which can be written as 1!
2	2	2 x 1 which can be written as 2!
3	6	3 x 2 x 1 which can be written as 3!
4	24	4 x 3 x 2 x 1 which can be written as 4!
10	3628800	10x9x8x7x6x5x4x3x2x1 which can be written as 10!
40	*	40 x 39 x 38 . . . 3x2x1 which can be written as 40!
n	*	n(n – 1)(n – 2) . . . 3 x 2 x 1 which can be written as n!

* Why do you think these answers have not been given?
• ! is a symbol meaning 'factorial'.

0685 Brick Walls

Make a display of your work and show it to your teacher.

0689 Random Code

b = 8	f = 19	h = 7	k = 10	m = 2
n = 26	q = 4	t = 24	w = 1	x = 6

The message is: Random means not regular, by chance, without order.

0691 . . . and now Swahili

1 → moja	2 → mbili,	3 → tatu,	4 → nne,	5 → tano,
6 → sita,	7 → saba,	8 → nane,	9 → tisa,	10 → kumi.

0694 Which Switches?

1. a) A and B
   b) A or B (or both)
   c) A or B or C
   d) A and C or B and C
   e) A and B or C and D
   f) A and C or A and D or B and C or B and D

0695 Locate the Error

In the 4th row down, the 3rd and 4th arrows from the left are interchanged.

0696 Number Codex

The sentence is This code is easy.

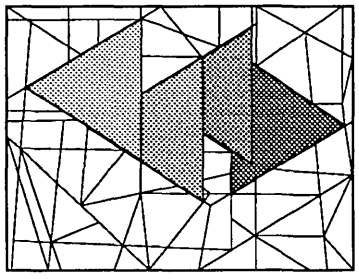
a) This message uses code three.
b) I am now using code five.
c) This is code two.
d) And this is code four.

1.	Code 1	2.	Code 2	3.	Code 5
4.	Code 2	5.	Code 3	6.	Code 2
7.	Code 1	8.	Code 4	9.	Code 3
10.	Code 3	11.	Code 5	12.	Code 5
13.	Code 1 and 5	14.	Code 1 and 5		

15. Ask a friend to try out your answer.

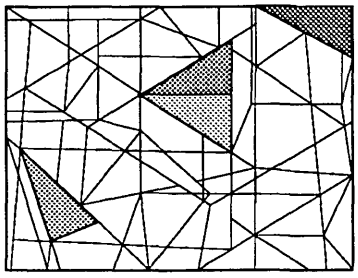
0697 Hidden Shapes

- Four hidden equilateral triangles.



The equilateral triangles are identical in size and shape.
They are **congruent**.

- Four hidden right-angled triangles.

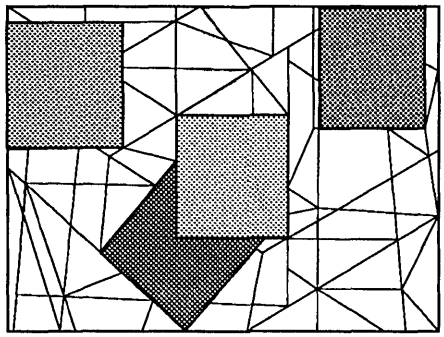


The right-angled triangles are identical in size and shape.
They are **congruent**.

continued/

0697 Hidden Shapes (cont)

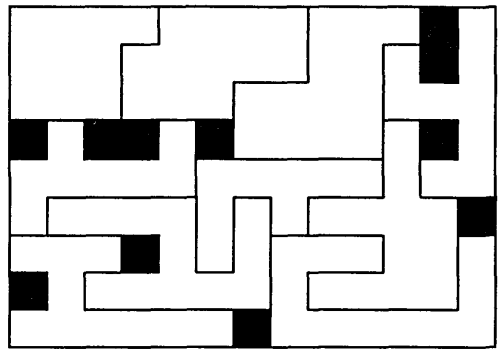
- Four hidden squares.



The squares are identical in size and shape.
They are **congruent**.

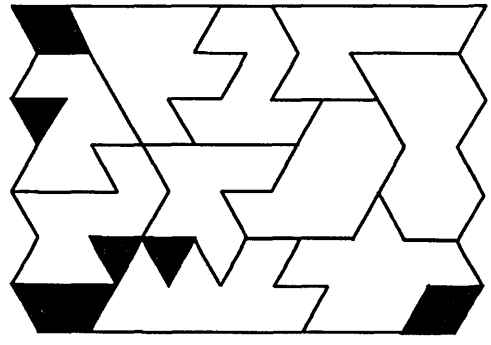
0701 Fitting the Tiles A

With this kind of work, it does not help much to see other people's answers. You will know if you have done the puzzle or not. If you find the work difficult, discuss it with someone.
Here is the answer to workpage 8 which is very hard.



0702 Fitting the Tiles B

With this kind of work, it does not help much to see other people's answers. You will know if you have done the puzzle or not. If you find the work difficult, discuss it with someone.
Here is the answer to workpage 8 which is very hard.



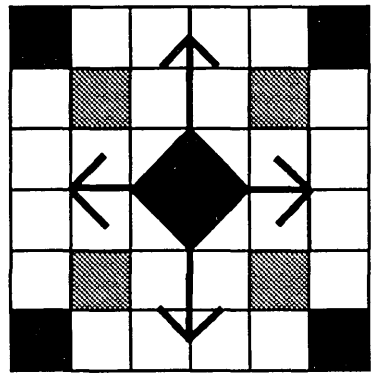
0705 Crosswords

1. 1	7	■	2. 2	3. 5
2	■	4. 3	■	0
■	5. 2	6	4	■
6. 1	■	6	■	7. 4
8. 6	9	■	9. 9	9

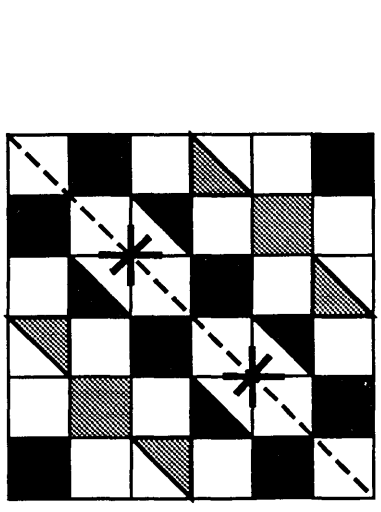
■	1. 1	1	2. 1	■
3. 2	4	■	4. 3	5. 2
1	■	6. 5	■	9
7. 6	0	■	8. 2	7
■	9. 1	0	5	■

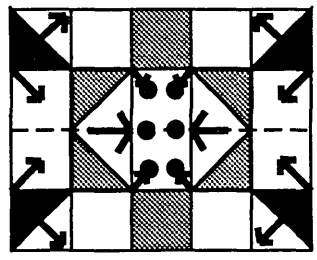
0709 Reflection

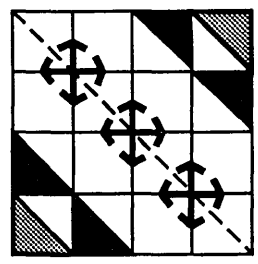
This is the completed reflected pattern.

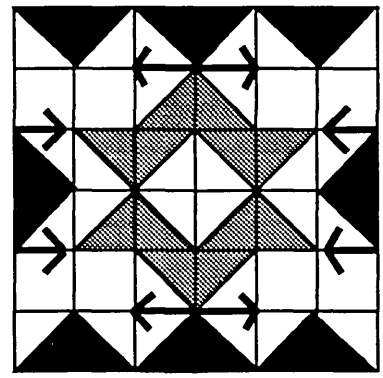


You should have drawn at least two of these patterns.









Show your own completed design to your teacher. You may like to make a display of your work.

0713 Jumping Jack

3 spaces.
He would land at 11.
He would land at 14.
The sequence is 2, 5, 8, **11**, **14**, ...

1. 1, 6, 11, **16**, **21**, ...
2. 12, 15, 18, **21**, **24**, ...
3. 2, 6, 10, **14**, **18**, ...
4. 7, 11, 15, **19**, **23**, ...
5. 9, 13, 17, **21**, **25**, ...
6. 1, 7, 13, **19**, **25**, ...
7. 3, 8, 13, **18**, **23**, ...
8. 4, 7, 10, **13**, **16**, ...
9. 5, 8, 11, **14**, **17**, ...
10. 6, 9, 12, **15**, **18**, ...

0716 Relations

1. a) It is shaded because Jothi is the daughter of Laxmi.
   b) It is not shaded because Minesh is not the daughter of Parvati.

2. **B** "is the sister of"

	G	P	S	D	L	J	M
G	Δ	Δ	Δ	Δ	Δ	Δ	Δ
P	Δ	Δ	Δ	Δ	Δ	Δ	Δ
S	Δ	Δ	Δ	▲	Δ	Δ	Δ
D	Δ	Δ	Δ	Δ	Δ	Δ	Δ
L	Δ	Δ	Δ	Δ	Δ	Δ	Δ
J	Δ	Δ	Δ	Δ	Δ	Δ	▲
M	Δ	Δ	Δ	Δ	Δ	Δ	Δ

3. **C** "is the parent of"

	G	P	S	D	L	J	M
G	Δ	Δ	▲	▲	Δ	Δ	Δ
P	Δ	Δ	▲	▲	Δ	Δ	Δ
S	Δ	Δ	Δ	Δ	Δ	Δ	Δ
D	Δ	Δ	Δ	Δ	Δ	▲	▲
L	Δ	Δ	Δ	Δ	Δ	▲	▲
J	Δ	Δ	Δ	Δ	Δ	Δ	Δ
M	Δ	Δ	Δ	Δ	Δ	Δ	Δ

continued/

4. **D** "is the child of"

	G	P	S	D	L	J	M
G	Δ	Δ	Δ	Δ	Δ	Δ	Δ
P	Δ	Δ	Δ	Δ	Δ	Δ	Δ
S	▲	▲	Δ	Δ	Δ	Δ	Δ
D	▲	▲	Δ	Δ	Δ	Δ	Δ
L	Δ	Δ	Δ	Δ	Δ	Δ	Δ
J	Δ	Δ	Δ	▲	▲	Δ	Δ
M	Δ	Δ	Δ	▲	▲	Δ	Δ

5. **E** Transpose of C

	G	P	S	D	L	J	M
G	Δ	Δ	Δ	Δ	Δ	Δ	Δ
P	Δ	Δ	Δ	Δ	Δ	Δ	Δ
S	▲	▲	Δ	Δ	Δ	Δ	Δ
D	▲	▲	Δ	Δ	Δ	Δ	Δ
L	Δ	Δ	Δ	Δ	Δ	Δ	Δ
J	Δ	Δ	▲	▲	Δ	Δ	Δ
M	Δ	Δ	▲	▲	Δ	Δ	Δ

They are the same matrix.

6. **F** "is the grandchild of"

	G	P	S	D	L	J	M
G	Δ	Δ	Δ	Δ	Δ	Δ	Δ
P	Δ	Δ	Δ	Δ	Δ	Δ	Δ
S	Δ	Δ	Δ	Δ	Δ	Δ	Δ
D	Δ	Δ	Δ	Δ	Δ	Δ	Δ
L	Δ	Δ	Δ	Δ	Δ	Δ	Δ
J	▲	▲	Δ	Δ	Δ	Δ	Δ
M	▲	▲	Δ	Δ	Δ	Δ	Δ

G Transpose of F

	G	P	S	D	L	J	M
G	Δ	Δ	Δ	Δ	Δ	▲	▲
P	Δ	Δ	Δ	Δ	Δ	▲	▲
S	Δ	Δ	Δ	Δ	Δ	Δ	Δ
D	Δ	Δ	Δ	Δ	Δ	Δ	Δ
L	Δ	Δ	Δ	Δ	Δ	Δ	Δ
J	Δ	Δ	Δ	Δ	Δ	Δ	Δ
M	Δ	Δ	Δ	Δ	Δ	Δ	Δ

Transpose of matrix F shows the relationship "is the grandparent of".

7. **H** "is the brother or sister of"

	G	P	S	D	L	J	M
G	Δ	Δ	Δ	Δ	Δ	Δ	Δ
P	Δ	Δ	Δ	Δ	Δ	Δ	Δ
S	Δ	Δ	Δ	▲	Δ	Δ	Δ
D	Δ	Δ	▲	Δ	Δ	Δ	Δ
L	Δ	Δ	Δ	Δ	Δ	Δ	Δ
J	Δ	Δ	Δ	Δ	Δ	Δ	▲
M	Δ	Δ	Δ	Δ	Δ	▲	Δ

I Transpose of H

	G	P	S	D	L	J	M
G	Δ	Δ	Δ	Δ	Δ	Δ	Δ
P	Δ	Δ	Δ	Δ	Δ	Δ	Δ
S	Δ	Δ	Δ	▲	Δ	Δ	Δ
D	Δ	Δ	▲	Δ	Δ	Δ	Δ
L	Δ	Δ	Δ	Δ	Δ	Δ	Δ
J	Δ	Δ	Δ	Δ	Δ	Δ	▲
M	Δ	Δ	Δ	Δ	Δ	▲	Δ

Matrix I shows the same relationship as matrix H, "is the brother or sister of".

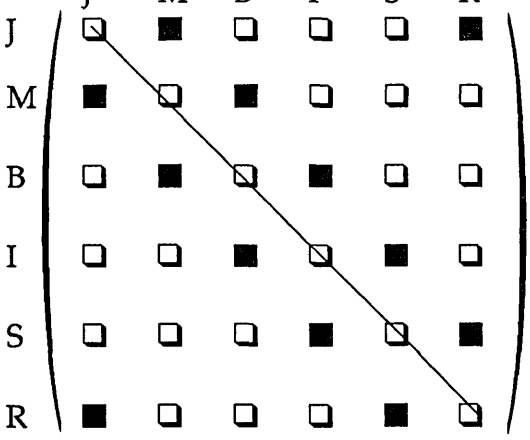
continued/

8. The leading diagonal goes from the top left hand corner to the bottom right hand corner of the matrix.

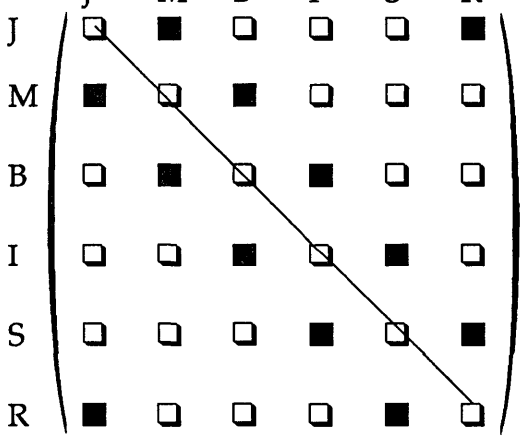
   Matrices H and I are symmetrical about the leading diagonal.

   "Is the brother or sister of" is a reciprocal relationship as Jothi cannot be Minesh's sister without Minesh being Jothi's brother.

9. **J** "is next to"

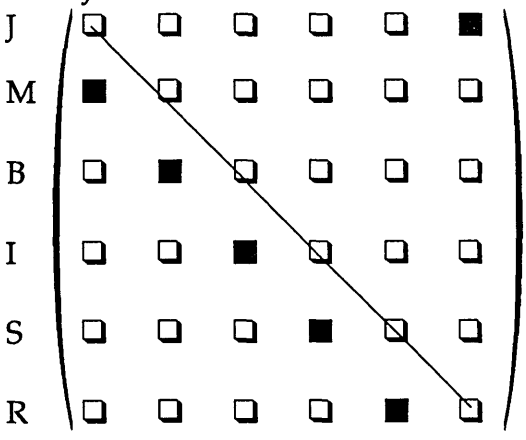


K Transpose of J

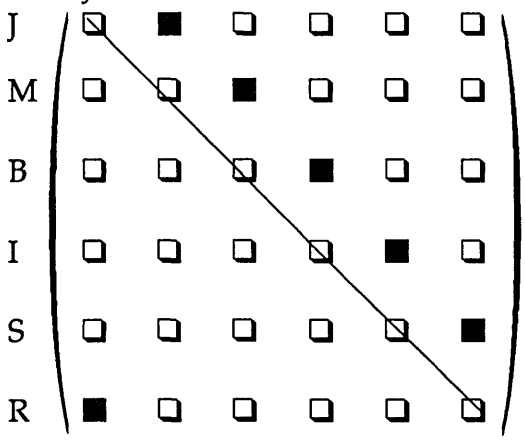


Matrix K shows the same relationship as matrix J "is next to".

10. **L** "is on the left of"

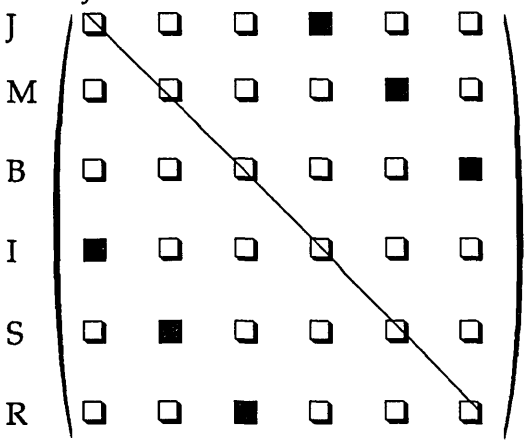


M Transpose of L

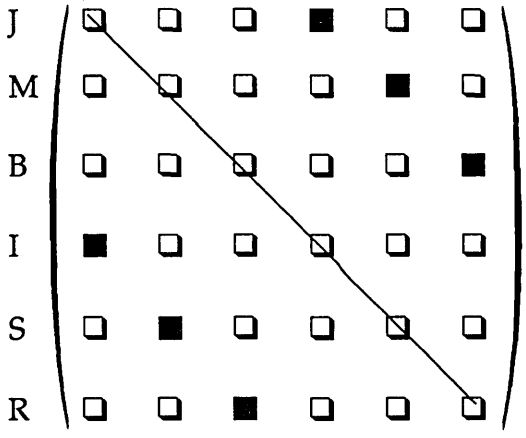


Matrix M shows the relationship "is on the right of".

11. **N** "is opposite to"



O Transpose of N



Matrix O shows the same relationship as matrix M "is opposite to"

continued/

0716 Relations (cont)

12. Matrices J, K, N and O are each symmetrical about the leading diagonal.

Matrices J and K are self-inverses because Briony cannot be next to Irving without Irving being next to Briony.

Matrices N and O are also self-inverses.

0718 Secret Identities

1. G E O R G E

$$\begin{pmatrix} 7 & 5 & 15 \\ 18 & 7 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 7 & 5 & 15 \\ 18 & 7 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 64 & 34 & 70 \\ 25 & 12 & 20 \end{pmatrix}$$

64 34 70 25 12 20

2. 64 34 70 25 12 20

$$\begin{pmatrix} 64 & 34 & 70 \\ 25 & 12 & 20 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 64 & 34 & 70 \\ 25 & 12 & 20 \end{pmatrix}$$

$$\begin{pmatrix} 14 & 10 & 30 \\ 36 & 14 & 10 \end{pmatrix}$$

14 10 30 36 14 10

N J ? ? N J

This does **not** make sense.

3. Coder Word Message

CODING $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 11 \\ 20 \end{pmatrix}$

Decoder Message Word

DECODING $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 11 \\ 20 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

4. $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

continued/

5. B: $\begin{pmatrix}0 & -1\\ 1 & 2\end{pmatrix}\begin{pmatrix}2 & 1\\ -1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

C: $\begin{pmatrix}5 & -2\\ 3 & -1\end{pmatrix}\begin{pmatrix}-1 & 2\\ -3 & 5\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

D: $\begin{pmatrix}-1 & 3\\ -1 & 2\end{pmatrix}\begin{pmatrix}2 & -3\\ 1 & -1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

Yes, the rule works.

6. $\begin{pmatrix}1 & -2\\ -1 & 4\end{pmatrix}\begin{pmatrix}4 & 2\\ 1 & 1\end{pmatrix} = \begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}$

It is double the identity matrix.

7. $\begin{pmatrix}\frac{1}{2} & -1\\ -\frac{1}{2} & 2\end{pmatrix}\begin{pmatrix}4 & 2\\ 1 & 1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

It works.

8. $\begin{pmatrix}\frac{1}{2} & -1\\ -\frac{1}{2} & 2\end{pmatrix}\begin{pmatrix}64 & 34 & 70\\ 25 & 12 & 20\end{pmatrix} = \begin{pmatrix}7 & 5 & 15\\ 18 & 7 & 5\end{pmatrix}$

7 5 15 18 7 5

G E O R G E

9. $\begin{pmatrix}1 & 0\\ -2 & 3\end{pmatrix}\begin{pmatrix}3 & 0\\ 2 & 1\end{pmatrix} = \begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}$ You need to divide by 3.

So the decoder is $\frac{1}{3}\begin{pmatrix}1 & 0\\ -2 & 3\end{pmatrix}$ or $\begin{pmatrix}\frac{1}{3} & 0\\ -\frac{2}{3} & 1\end{pmatrix}$

because $\frac{1}{3}\begin{pmatrix}1 & 0\\ -2 & 3\end{pmatrix}\begin{pmatrix}3 & 0\\ 2 & 1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

10. $\begin{pmatrix}2 & -1\\ -1 & 4\end{pmatrix}\begin{pmatrix}4 & 1\\ 1 & 2\end{pmatrix} = \begin{pmatrix}7 & 0\\ 0 & 7\end{pmatrix}$ You need to divide by 7.

So the decoder is $\frac{1}{7}\begin{pmatrix}2 & -1\\ -1 & 4\end{pmatrix}$ or $\begin{pmatrix}\frac{2}{7} & -\frac{1}{7}\\ -\frac{1}{7} & \frac{4}{7}\end{pmatrix}$

11. $\begin{pmatrix}1 & -1\\ -4 & 5\end{pmatrix}\begin{pmatrix}5 & 1\\ 4 & 1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

So $\begin{pmatrix}1 & -1\\ -4 & 5\end{pmatrix}$ is the decoder.

continued/

0718 Secret Identities (cont)

12. $\begin{pmatrix}2 & -4\\ 1 & 3\end{pmatrix}\begin{pmatrix}3 & 4\\ -1 & 2\end{pmatrix} = \begin{pmatrix}10 & 0\\ 0 & 10\end{pmatrix}$ You need to divide by 10.

So the decoder is $\frac{1}{10}\begin{pmatrix}2 & -4\\ 1 & 3\end{pmatrix}$ or $\begin{pmatrix}\frac{2}{10} & -\frac{4}{10}\\ \frac{1}{10} & \frac{3}{10}\end{pmatrix}$

13. $\begin{pmatrix}2 & 3\\ -1 & -1\end{pmatrix}\begin{pmatrix}-1 & -3\\ 1 & 2\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

So $\begin{pmatrix}2 & 3\\ -1 & -1\end{pmatrix}$ is the decoder.

14. $\begin{pmatrix}3 & -2\\ -2 & 2\end{pmatrix}\begin{pmatrix}2 & 2\\ 2 & 3\end{pmatrix} = \begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}$ You need to divide by 2.

So the decoder is $\frac{1}{2}\begin{pmatrix}3 & -2\\ -2 & 2\end{pmatrix}$ or $\begin{pmatrix}\frac{3}{2} & -1\\ -1 & 1\end{pmatrix}$

15. $\begin{pmatrix}\frac{1}{3} & 0\\ -\frac{2}{3} & 1\end{pmatrix}$ $\begin{pmatrix}\frac{2}{7} & -\frac{1}{7}\\ -\frac{1}{7} & \frac{4}{7}\end{pmatrix}$ $\begin{pmatrix}1 & -1\\ -4 & 5\end{pmatrix}$ $\begin{pmatrix}\frac{2}{10} & -\frac{4}{10}\\ \frac{1}{10} & \frac{3}{10}\end{pmatrix}$ $\begin{pmatrix}2 & 3\\ -1 & -1\end{pmatrix}$ $\begin{pmatrix}\frac{3}{2} & -1\\ -1 & 1\end{pmatrix}$

0719 Cuboid Nets

The cuboid has 6 faces.
The faces are all rectangles.

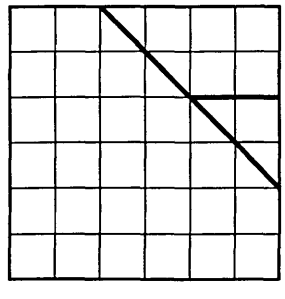
0720 Nets of Pyramids

Show your pyramids to your teacher. Did you check the nets of the pyramids which you made?

5. The sides of the triangles are not long enough and do not meet.

0721 Square Tangram

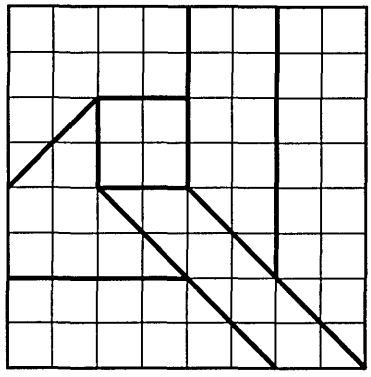
4. One solution is:



continued/

0721 Square Tangram (cont)

5. A possible solution is:



6. 100 square centimetres

7. 36 square centimetres and 64 square centimetres

8. $10^2 = \boxed{6}^2 + \boxed{8}^2$
   $100 = \boxed{36} + \boxed{64}$

0722 Prove It

Two results may be noticed:

1. If the original number is even, the result is a whole number.
   If the original number is odd, the result is an even number ending with a quarter.

2. The result is always the square of half the original number.

Proof

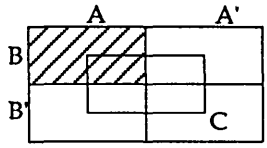
- Start with an even number: $2n$
  square it: $(2n)^2 = 4n^2$
  divide by 4: n^2

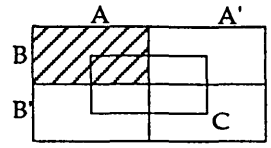
  Since n is a whole number, n^2 is a whole number.
  n^2 is also the square of half the original number.

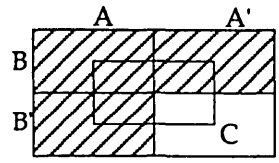
- Start with an odd number: $2n - 1$
  square it: $(2n - 1)^2 = 4n^2 - 4n + 1$
  divide by 4: $n^2 - n + \frac{1}{4} = n(n - 1) + \frac{1}{4}$.

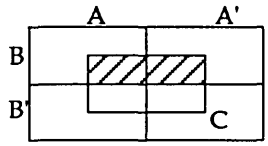
  Since n is a whole number, so is $(n - 1)$. If one is odd, the other is even.
  Hence $n(n - 1)$ is always even. So, the result is an even number $+ \frac{1}{4}$.
  Also, $n^2 - n + \frac{1}{4} = (n - \frac{1}{2})^2 = \frac{(2n - 1)^2}{2}$
  So $n^2 - n + \frac{1}{4}$ is the square of half the original number.

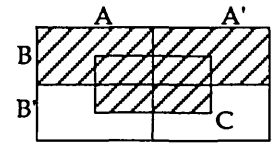
0723 3 Set Carroll Diagrams

3. i) 

ii) 

iii) 

iv) 

v) 

i) and ii) are the same.

4. The same area is shaded on each diagram so:
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is true.

5. i) false ii) true iii) false iv) false
v) true vi) true

6. Identities are equations which are true for all sets A, B and C.
As a double-check make sure your identities are true for the following sets:
A = {1, 2, 3, 4}
B = {1, 2, 3}
C = {3, 4, 5}

0725 Race Track

Write down the last four vectors you used and show your teacher.

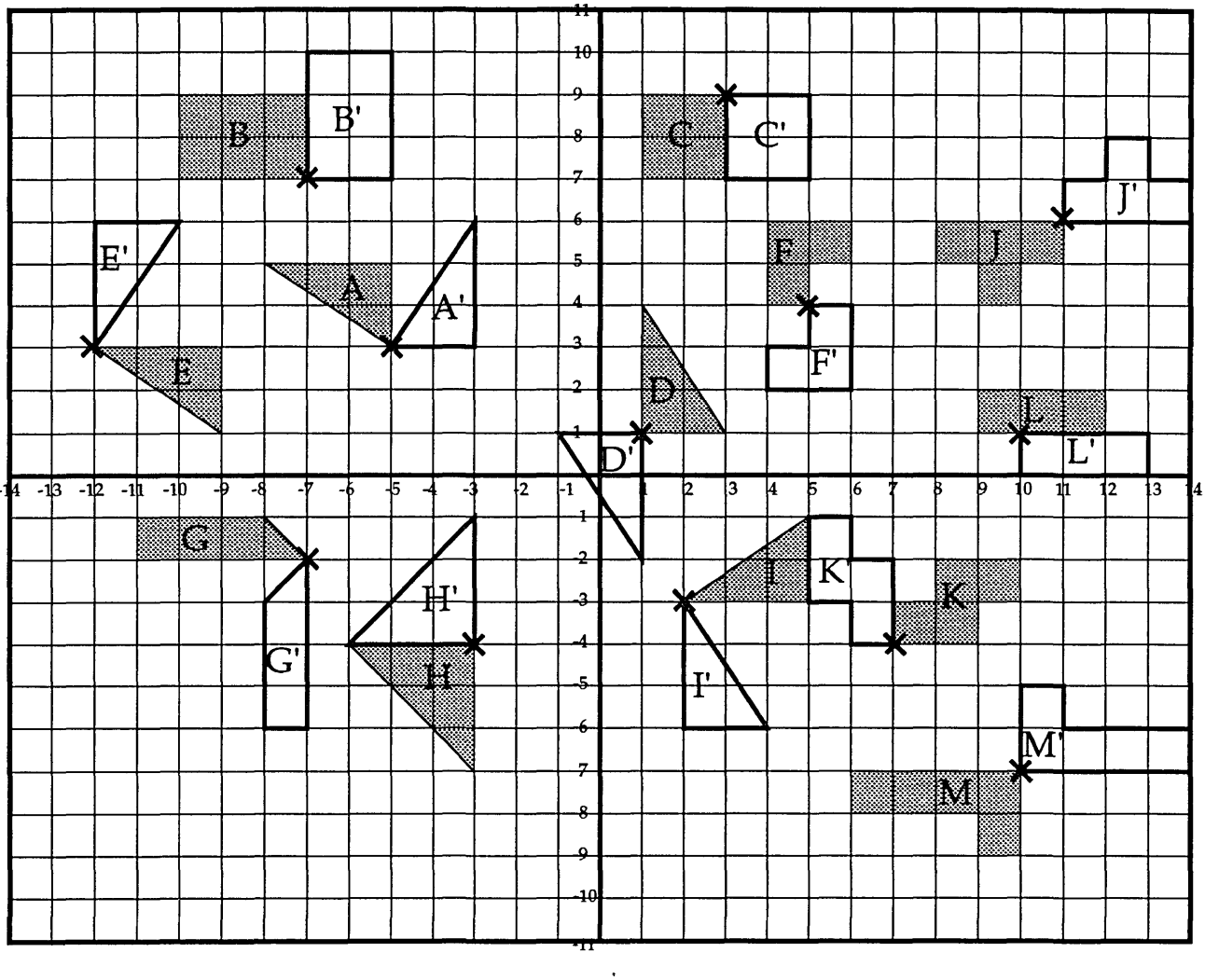
0727 Who's Who?

Bill always tells the truth so he is not in the middle. Since Bill is not in the middle the person on the left hand side is lying so this person is not Bill. Therefore, Bill is on the right hand side.
Bill always tells the truth. Therefore Will is in the middle and Phil must be on the left hand side.

0728 Spiral Patterns

- When the numbers are in a different order, the pattern is a reflection of the original one.
- If one of the numbers is negative the pattern does not join up to the start.
- Four-number spirals are different. They do not join up to the start whereas five-number spirals do join up.
- (1, 3) was used to make the pattern on the isometric grid. Triangular patterns tend to dominate an isometric grid.

0730 Rotation Worksheet



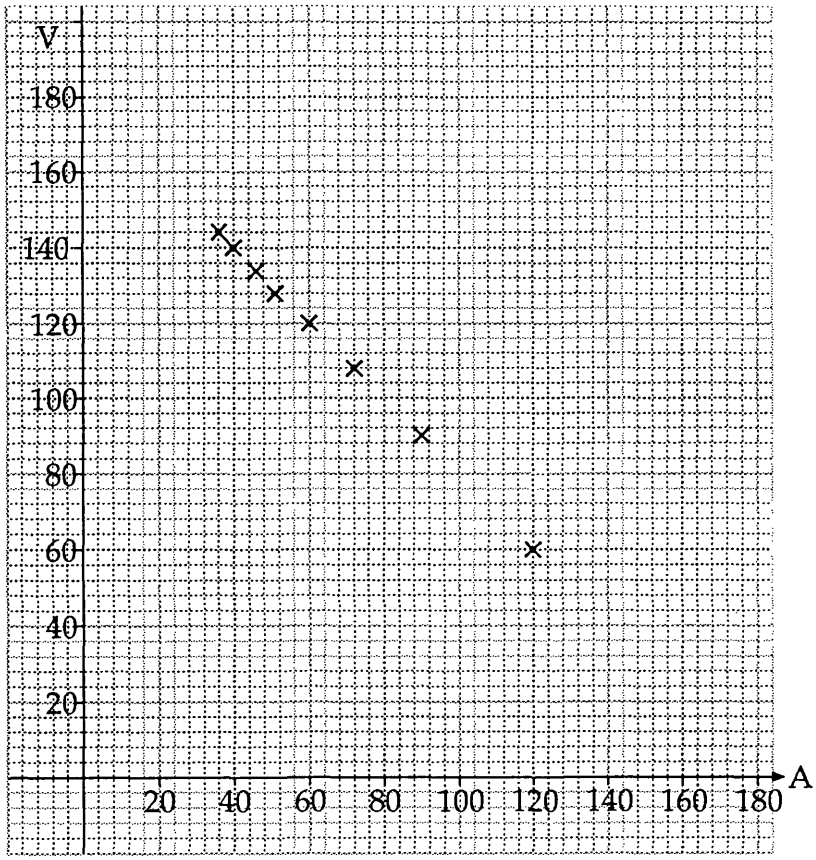
0731 Regular Polygons

1.& 2.

Polygon	n	V	A
Triangle	3	60°	120°
Square	4	90°	90°
Regular Pentagon	5	108°	72°
Regular Hexagon	6	120°	60°
Regular Heptagon	7	128.5°	51.5°
Regular Octagon	8	135°	45°
Regular Nonagon	9	140°	40°
Regular Decagon	10	144°	36°

continued/

3.



The points should lie on a straight line.

4. You must not join the points up as there are no possible values for V between say 60° and 90°. You cannot have a polygon with 'half sides'!

5. $V + A = 180°$

6. OPQ is an isosceles triangle.

7. OP bisects the angle V.

8. $\angle OPQ = \frac{1}{2}V$

9. $\angle PQO = \frac{1}{2}V$

10. The sum of angles of a triangle = 180°
In OPQ, $\frac{1}{2}V + \frac{1}{2}V + A = 180°$
$V + A = 180°$

11. The first method uses patterns spotted from specific examples to help reach a generalisation between V and A. This is called an inductive approach.

The second uses a method of proof for a polygon with any number of sides. This is a deductive method.

You may wish to discuss the two different approaches with your teacher.

0732 Ruler, Pencil, Compass

- Triangles 4. and 6. can be drawn.
- Triangles 5. and 7. cannot be drawn.

8. By construction you can see that the two arcs you draw never cut each other so it is not possible to obtain the third point of the triangle. You should try to work out a relationship between the longest length and the two shorter ones.

9. Triangle 4 and 1 are isosceles triangles.
Triangle 6 is an equilateral triangle.

0734 Start with a^2

1. Using the diagram you can see that square (a + b) is bigger by
   - two rectangles a x b and
   - a square b^2

2. $(a + b)^2 = a^2 + \boxed{\mathbf{2ab}} + \boxed{\mathbf{b^2}}$

3. $(5 + 3)^2 = 5^2 + (2 \times 5 \times 3) + 3^2$
   $8^2 = 25 + 30 + 9$
   $8^2 = 64$
   The identity works.

4. $(102)^2 = (100 + 2)^2$
   $= 100^2 + (2 \times 100 \times 2) + 2^2$
   $= 10000 + 400 + 4$
   $(102)^2 = 10404$

5. $(2a)^2 = a^2 + 2a^2 + a^2$
   $= 4a^2$

6. $(a + b)^2 = a^2 + 2ab + b^2$
   Difference is 2ab

7. $96^2 = (100 - 4)^2$
   $= 100^2 - (2 \times 100 \times 4) + 4^2$
   $= 10000 - 800 + 16$
   $= 9216$

8. $(a - c)^2 = \boxed{\mathbf{a^2 - 2ac + c^2}}$

9. $(a - b)^2 = a^2 - (2ab - b^2)$
   $= a^2 - \boxed{\mathbf{2ab}} + \boxed{\mathbf{b^2}}$

continued/

10. $(a-b)^2 = a^2 - 2ab + b^2$
$(7-4)^2 = 49 - (2 \times 7 \times 4) + 4^2$
$3^2 = 49 - 56 + 16$
$3^2 = 9$

11. $99^2 = a^2 - 2ab + b^2$
$(100-1)^2 = 100^2 - (2 \times 100 \times 1) + 1^2$
$= 10000 - 200 + 1$
$= 9801$

12. $(a-b)^2 = a^2 - 2ab + b^2$
Difference $= (a^2 - 2ab + b^2) - (a^2 - b^2)$
$= 2b^2 - 2ab$

13. $(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
Difference $= 4ab$

14. length $= a + b$
width $= a - b$

15. $(a+b)(a-b) = \boxed{a^2} - \boxed{b^2}$

16. $(10+1)(10-1) = 10^2 - 1^2$
$11 \times 9 = 100 - 1$
$99 = 99$

17. $103 \times 97 = (100+3)(100-3)$
$= 100^2 - 3^2$
$= 10000 - 9$
$= 9991$

18. a) $x^2 + 2xy + y^2 = (a+b)^2 + 2(a+b)(a-b) + (a-b)^2$
$= a^2 + 2ab + b^2 + 2(a^2 - b^2) + a^2 - 2ab + b^2$
$= a^2 + 2ab + b^2 + 2a^2 - 2b^2 + a^2 - 2ab + b^2$
$= 4a^2$

b) $(x+y)^2 = (a+b+a-b)^2$
$= (2a)^2$
$= 4a^2$

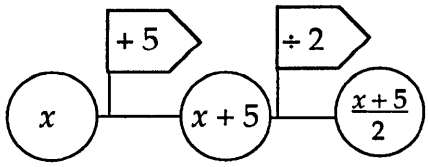
therefore $x^2 + 2xy + y^2 = (x+y)^2$

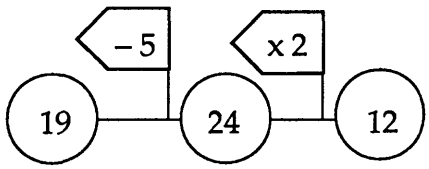
0735 Knots Worksheet

1.	12 knots	2.	20 knots		
3.	4 knots	4.	6 knots	5.	1 knot
6.	25 knots	7.	24 kots	8.	24 knots
9.	35 knots	10.	48 knots	11.	40 knots
12.	36 knots	13.	35 knots	14.	64 knots

0736 Solving Equations

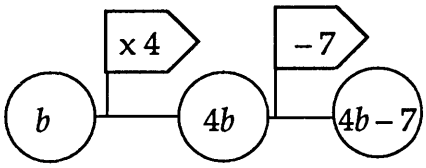
1.

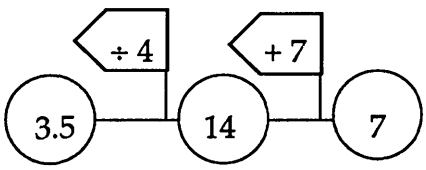




$x = 19$

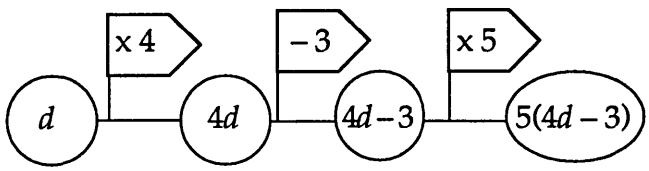
2.

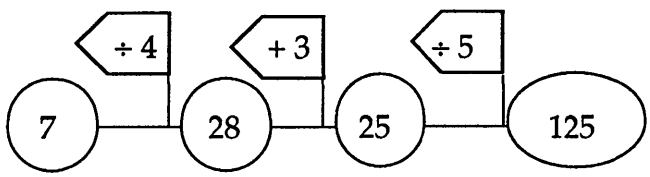




$b = 3.5$

3.

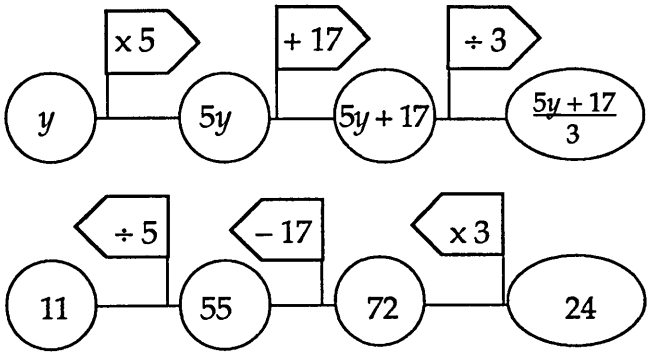




$d = 7$

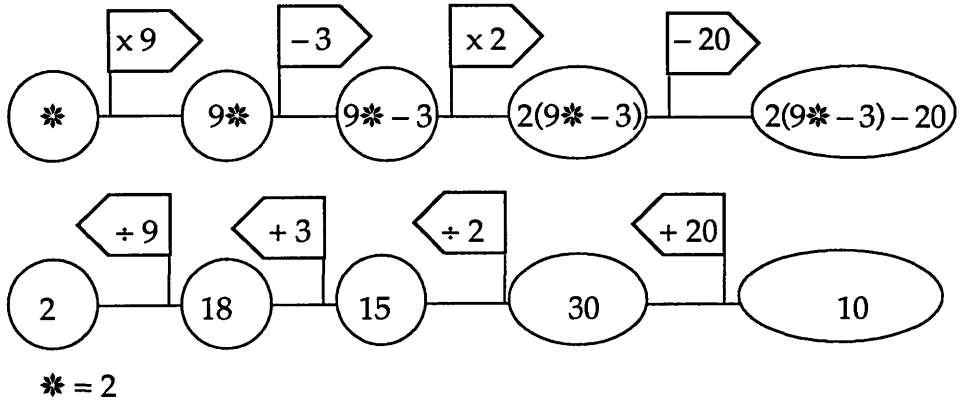
continued/

4.



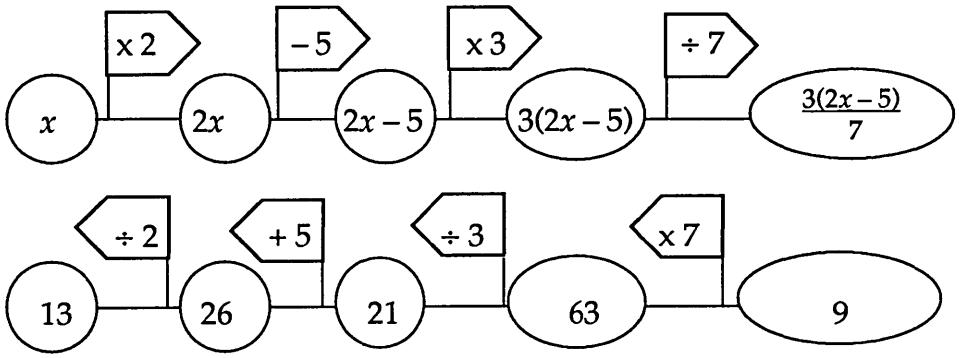
$y = 11$

5.



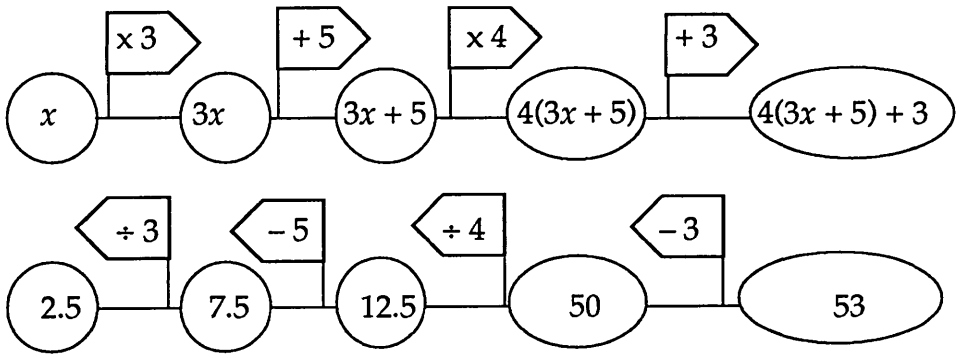
✲ = 2

6.



$x = 13$

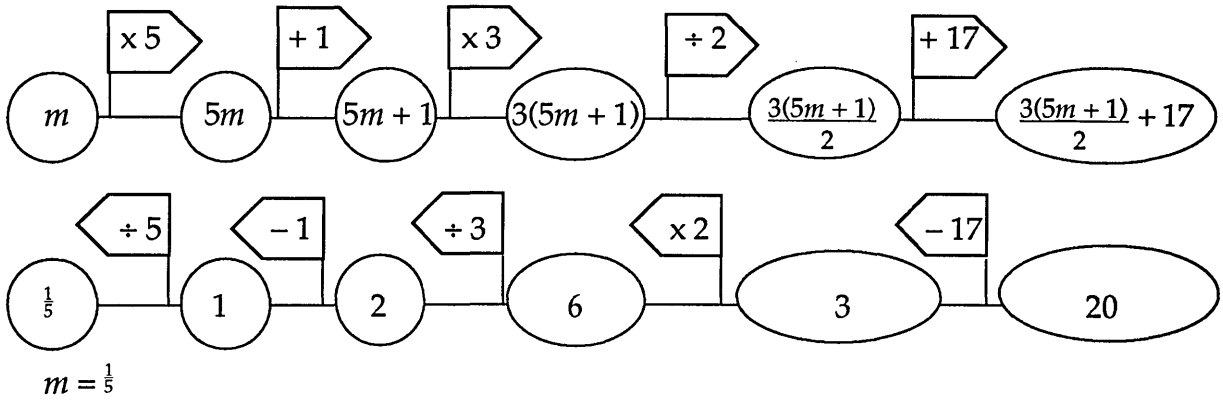
7.



$x = 2.5$

continued/

8.



$m = \frac{1}{5}$

0737 What Chance?

1. a) 5
   b) 0

2. a) You probably found that the '1' column won. If your result was different, discuss your graph with your teacher.
   b) There are more ways to score a difference of 1 than any other number.
   c) You probably had to throw your dice more than 40 times.

3.

Second dice

First dice	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

a) Here are some of the things you may have noticed.

- The first row is the 6th row reversed.
  The second row is the 5th row reversed etc.
- The first column is the 6th column reversed etc.
- The leading diagonal from the top left hand corner to the bottom right contains only 0's.
- Lines parallel to the leading diagonal contain the same number.
- The numbers in the secondary diagonal, from the bottom left hand corner to the top right, are symmetrical. Numbers in lines parallel to the secondary diagonal are also symmetrical.
- In any horizontal or vertical line the numbers only go up or down one at a time.

continued /

3. b) 36

4. a)

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

There are 6 squares shaded.

The probability of scoring 0 is $\frac{6}{36}$ $(\frac{1}{6})$ because, of the 36 possible answers, 6 are zero.

b)

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

There are 10 squares shaded. $\frac{10}{36}$

c)

Difference	0	1	2	3	4	5
Probability	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$

d) The probability fractions add up to $\frac{36}{36} = 1$
since 0, 1, 2, 3, 4, 5 are the only possible results.

5. You are likely to have less money.
Of the 36 possible results:
- 28 would give 1p more.
- 8 would each give 4p less.

So, in 36 goes, you would expect a loss of 4p.

0738 The Family of Quadrilaterals

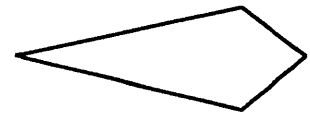
1. The definition of a kite in Geometry Facts SMILE 2163 is:
"A kite is a quadrilateral and has:
- two pairs of adjacent equal sides
- one pair of equal angles
- one line of symmetry."

continued/

2. The definition of a rhombus in Geometry Facts SMILE 2163 is:
"A rhombus is a quadritateral and has:
- four equal sides
- opposite sides equal
- opposite sides parallel
- two lines of symmetry."

3. a) It is possible to draw a kite that does not fit the definition of a rhombus.
This kite does not fit the definition of a rhombus because:

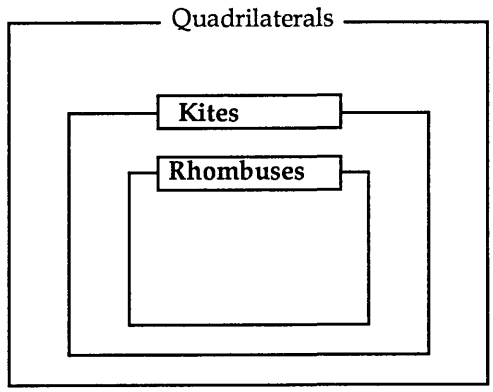
- all four sides are not equal
- it only has one line of symmetry
- opposite sides are not parallel.



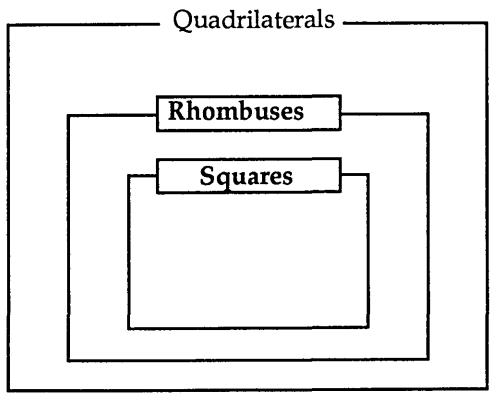
b) It is not possible to draw a rhombus that does not fit the definition of a kite.
Any rhombus will fit the definition of a kite:

- two pairs of adjacent equal sides (all four sides are equal)
- one pair of equal angles (there are two pairs of equal angles)
- one line of symmetry (there are two lines of symmetry)

4. "All **rhombuses** are **kites**".
"**Rhombuses** are special cases of **kites**".



5. a) **Squares** are special cases of **rhombuses.**



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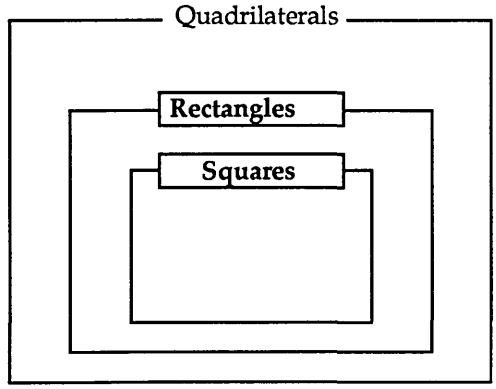
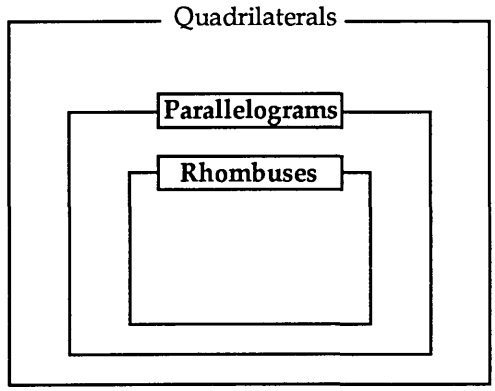
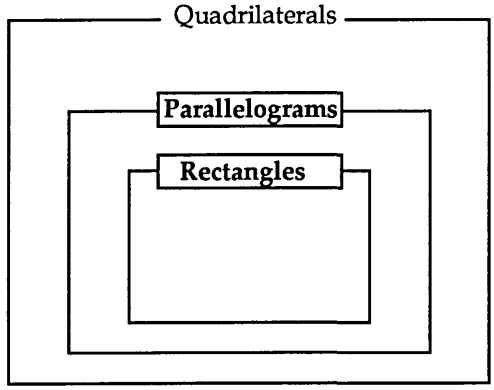
5. b) **Squares** are special cases of **rectangles.**



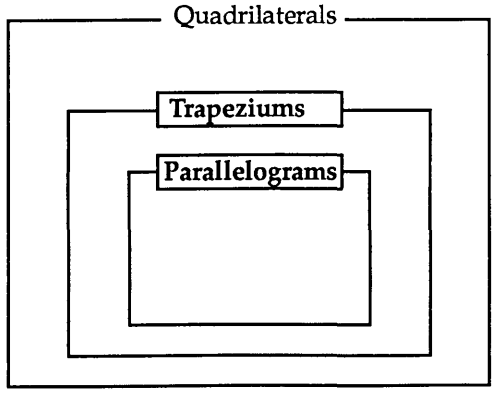
c) **Rhombuses** are special cases of **parallelograms.**



d) **Rectangles** are special cases of **trapeziums.**



f) **Parallelograms** are special cases of **trapeziums.**



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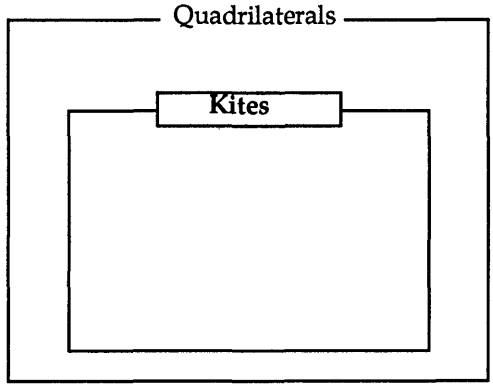
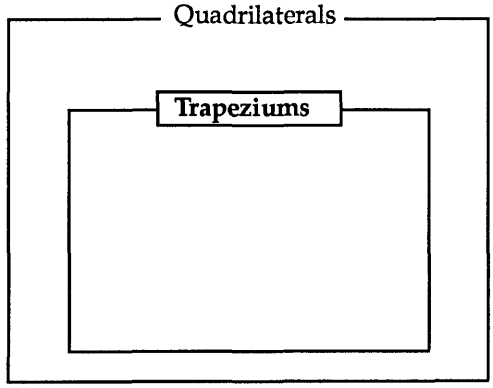
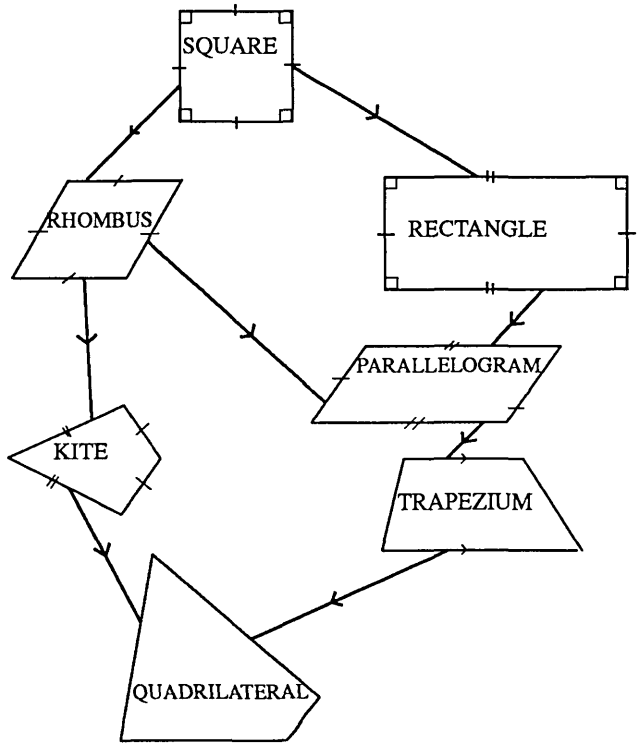
5. i) **Kites** are special cases of **quadrilaterals.**



j) **Trapeziums** are special cases of **quadrilaterals.**



6.



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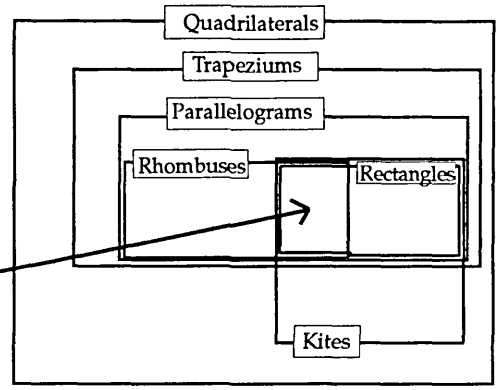
0738 The Family of Quadrilaterals (cont)

Challenge

What shape belongs in here?



0740 Solve It

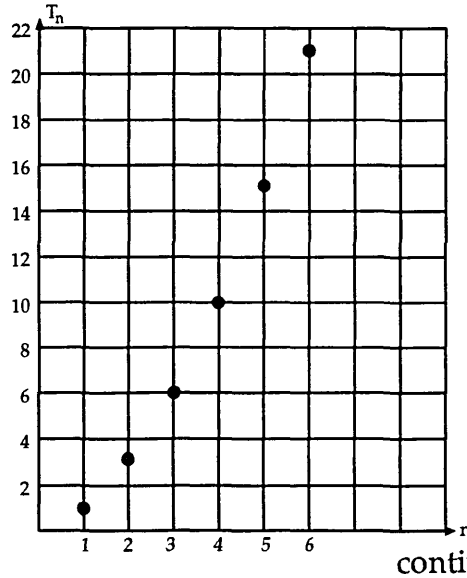
1. $x = 18$
2. $x = 71$
3. $x = 0.2$
4. $x = 7.25$
5. $p = 10$
6. $x = 9$
7. $y = 46$
8. $t = 5$
9. $a = 0.2$
10. $p = -1$
11. $\phi = 14$
12. $x = -5$
13. $\theta = 9$
14. $\beta = -3$
15. $\Omega = 7$

0741 The Thirty-Eighth Triangle Number

1.

n	1	2	3	4	5	6
T_n	1	3	6	10	15	21

n must be a whole number in order to calculate T_n, so joining the points would have no meaning.



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2. $T_1 + T_2 = 2^2$
$T_2 + T_3 = 3^2$
$T_3 + T_4 = 4^2$
. . . . .
. . . . .
. . . . .
$T_{n-1} + T_n = \mathbf{n^2}$

3. $T_1 + 2 = T_2$
$T_2 + 3 = T_3$
$T_3 + 4 = T_4$
. . . . .
. . . . .
. . . . .
$T_{n-1} + \mathbf{n} = T_n$

4. $T_{n-1} = \mathbf{n^2 - T_n}$

$T_{n-1} = \mathbf{T_n - n}$

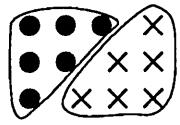
$\mathbf{n^2 - T_n} = \mathbf{T_n - n}$

$2T_n = n^2 + n$

$T_n = \frac{1}{2}(n^2 + n)$

$= \frac{n}{2}(n + 1)$

You should have found that your formula worked for any example.
The number pattern does confirm the formula.
e.g.



$2T_3 = 3^2 + 3$

The shape of the graph is a curve. This agrees with the quadratic expression for T_n.

a) $T_{100} = \frac{100}{2} \times 101$
$= 5050$

b) $T_{38} = \frac{38}{2} \times 39$
$= 741$

0742 Make Up a Flag-Chart

Make up a flag chart for your teacher to solve.

0743 Solving by Graphs

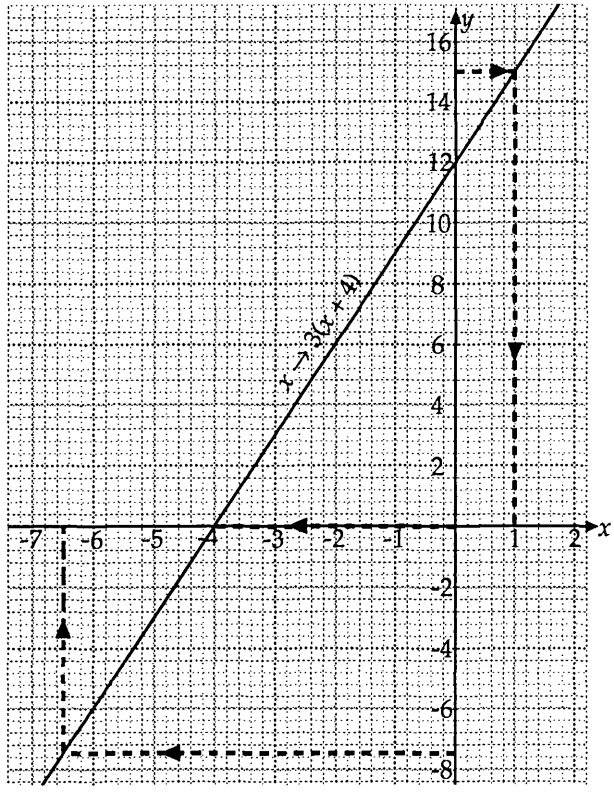
1.

x	$\rightarrow$	$2x - 3$
-2	$\rightarrow$	-7
-1	$\rightarrow$	**-5**
0	$\rightarrow$	**-3**
4	$\rightarrow$	5
6	$\rightarrow$	**9**

2. (-2, -7)
(-1, **-5**)
(0, **-3**)
(4, 5)
(6, **9**)

6. a) $x = 3$ Check $2 \times \mathbf{3} - 3 = 3$ ✓
b) $x = -1$ Check $2 \times \mathbf{-1} - 3 = -5$ ✓
c) $x = 5$ Check $2 \times \mathbf{5} - 3 = 7$ ✓
d) $x = 1.5$ Check $2 \times \mathbf{1.5} - 3 = 0$ ✓

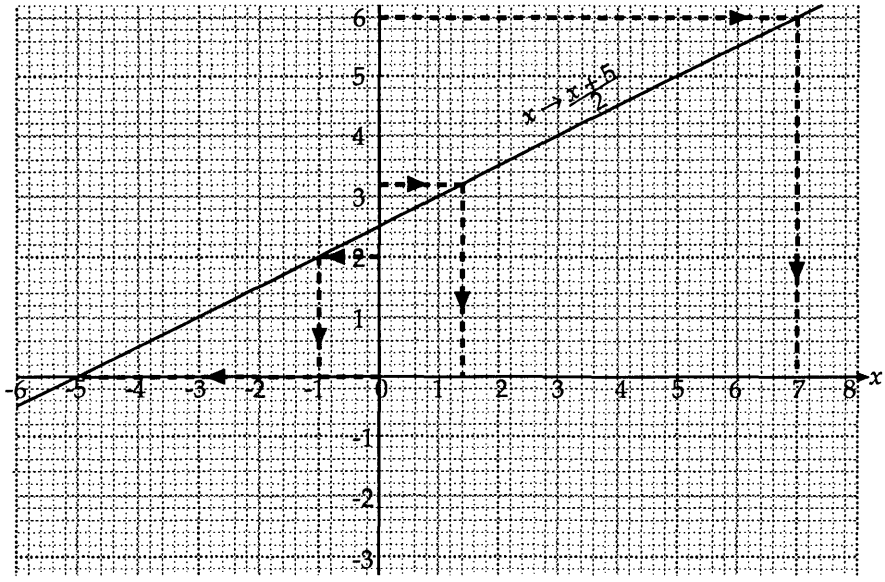
7.



a) $x = 1$ Check $3(1 + 4) = 15$ ✓
b) $x = -6.5$ Check $3(\mathbf{-6.5} + 4) = -7.5$ ✓
c) $x = -4$ Check $3(\mathbf{-4} + 4) = 0$ ✓

continued/

8.



a) $x = -1$
b) $x = 7$
c) $x = 1.4$
d) $x = -5$

0744 Equations and Graphs

1. You might have guessed the correct answer - but you won't always be so lucky!

2. $x = -3$ gives the same result for both mappings, but you will not necessarily have chosen this value for x.

3. If you draw the graphs accurately the lines cross where $x = -3$.

4. (-3, 2)

5. -3 maps to the same number.

$$x \to \frac{x+7}{2} \qquad\qquad x \to 2(x+4)$$

$$-3 \to \frac{-3+7}{2} = 2 \qquad\qquad -3 \to 2(-3+4) = 2$$

6.

$$-3$$

$$\frac{-3+7}{2} = 2(-3+4)$$

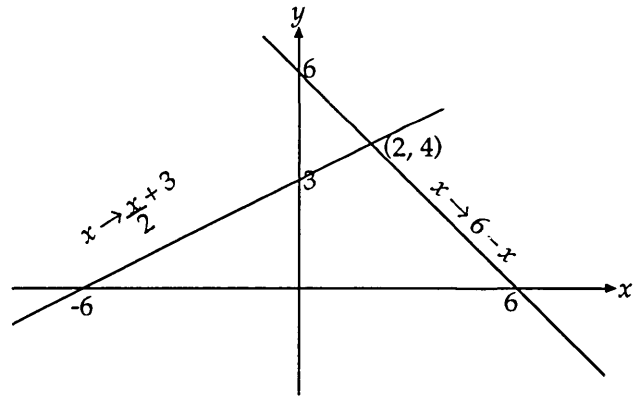
$$\frac{4}{2} = 2 \times 1$$

$$2 = 2$$

continued/

7. a) $x \to 6-x$ $\quad x \to \frac{x}{2}+3$

*6	→	0	*6	→	6
2	→	4	2	→	4
0	→	6	0	→	3
-2	→	8	-2	→	1
-6	→	12	-6	→	-3



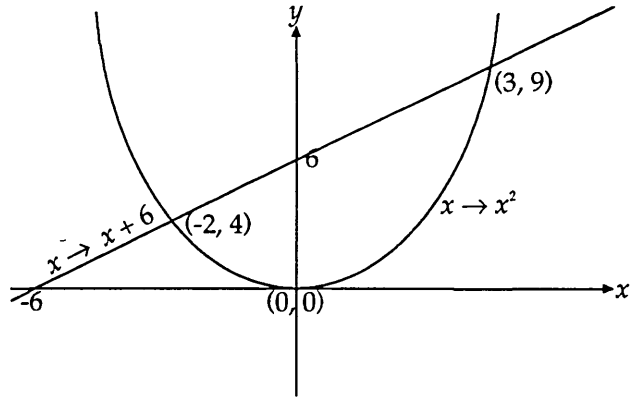
The solution to $6-x = \frac{x}{2}+3$ is (2, 4)

$6-\mathbf{2} = \frac{\mathbf{2}}{2}+3$

$4 = 4$ ✓

b) $x \to x^2$ $\quad x \to x+6$

*3	→	9	*3	→	9
2	→	4	2	→	8
0	→	0	0	→	6
-2	→	4	-2	→	4
-3	→	9	-6	→	0



The solutions to $x^2 = x+6$ are (3, 9) and (-2, 4).

Check for (3, 9) → $3^2 = 3+6$

$9 = 9$ ✓

Check for (-2, 4) → $x^2 = x+6$

$(\mathbf{-2})^2 = \mathbf{-2}+6$

$4 = 4$ ✓

* You may have chosen different values for x.

---

1. These mapping diagrams show that $x \to 2x$ and $x \to \frac{1}{2}x$ are inverses of each other.

x	→	$2x$	x	→	$\frac{1}{2}x$
4	→	8	8	→	4
10	→	20	20	→	10
2	→	4	4	→	2
15	→	30	30	→	15

x	→	$\frac{1}{2}x$	x	→	$2x$
4	→	2	2	→	4
10	→	5	5	→	10
2	→	1	1	→	2
15	→	$7\frac{1}{2}$	$7\frac{1}{2}$	→	15

continued/

2. a)

x	$\rightarrow$	$x+7$	x	$\rightarrow$	$x-7$
4	$\rightarrow$	11	11	$\rightarrow$	4
10	$\rightarrow$	17	17	$\rightarrow$	10
2	$\rightarrow$	9	9	$\rightarrow$	2
15	$\rightarrow$	30	30	$\rightarrow$	15

The inverse is $x \rightarrow x-7$

b)

x	$\rightarrow$	x	x	$\rightarrow$	x
4	$\rightarrow$	4	4	$\rightarrow$	4
10	$\rightarrow$	10	10	$\rightarrow$	10
2	$\rightarrow$	2	2	$\rightarrow$	2
15	$\rightarrow$	15	15	$\rightarrow$	15

The inverse is $x \rightarrow x$

c)

x	$\rightarrow$	$\frac{x}{6}$	x	$\rightarrow$	$6x$
6	$\rightarrow$	1	1	$\rightarrow$	6
3	$\rightarrow$	$\frac{1}{2}$	$\frac{1}{2}$	$\rightarrow$	3
12	$\rightarrow$	2	2	$\rightarrow$	12
9	$\rightarrow$	$1\frac{1}{2}$	$1\frac{1}{2}$	$\rightarrow$	9

The inverse is $x \rightarrow 6x$

d)

x	$\rightarrow$	$x-2$	x	$\rightarrow$	$x+2$
6	$\rightarrow$	4	4	$\rightarrow$	6
3	$\rightarrow$	1	1	$\rightarrow$	3
12	$\rightarrow$	2	2	$\rightarrow$	12
9	$\rightarrow$	7	7	$\rightarrow$	9

The inverse is $x + 2$

3. $x \rightarrow x$ is a self inverse mapping.

4. a)

x	$\rightarrow$	$3-x$	x	$\rightarrow$	$3-x$
6	$\rightarrow$	-3	-3	$\rightarrow$	6
3	$\rightarrow$	0	0	$\rightarrow$	3
2	$\rightarrow$	1	1	$\rightarrow$	2
1	$\rightarrow$	2	2	$\rightarrow$	1

b)

x	$\rightarrow$	$\frac{4}{x}$	x	$\rightarrow$	$\frac{4}{x}$
4	$\rightarrow$	1	1	$\rightarrow$	4
8	$\rightarrow$	$\frac{1}{2}$	$\frac{1}{2}$	$\rightarrow$	8
2	$\rightarrow$	2	2	$\rightarrow$	2
1	$\rightarrow$	4	4	$\rightarrow$	1

c)

x	$\rightarrow$	$10-x$	x	$\rightarrow$	$10-x$
6	$\rightarrow$	4	4	$\rightarrow$	6
3	$\rightarrow$	7	7	$\rightarrow$	3
12	$\rightarrow$	-2	-2	$\rightarrow$	12
9	$\rightarrow$	1	1	$\rightarrow$	9

d)

x	$\rightarrow$	$\frac{10}{x}$	x	$\rightarrow$	$\frac{10}{x}$
1	$\rightarrow$	10	10	$\rightarrow$	1
2	$\rightarrow$	5	5	$\rightarrow$	2
5	$\rightarrow$	2	2	$\rightarrow$	5
10	$\rightarrow$	1	1	$\rightarrow$	10

These mappings are all self inverses because the inverse **is the same as** the original mapping.

5. Any mapping of the form $x \rightarrow k - x$

e.g. $x \rightarrow 10 - x$

$x \rightarrow 3 - x$

$x \rightarrow 43 - x$

or $x \rightarrow \frac{k}{x}$

e.g. $x \rightarrow \frac{4}{x}$

$x \rightarrow \frac{10}{x}$

$x \rightarrow \frac{43}{x}$ are self inverses.

If you are unsure about your mappings, show your work to your teacher.

0746 Pascal's Triangle

0746a Sorting Yard

1. LRR

2. 8

3. Yes, RLL

Combinations	Path	Total number of paths
Three L's	LLL	1
Two L's and one R	LLR,LRL,RLL	3
Two R's and one L	RRL,RLR,LRR	3
Three R's	RRR	1

Investigation hints:

- How many different codes are there with 4 levels?
- How many combinations are there with
  - i) 4L's
  - ii) 3L's, 1R
  - iii) 2L's, 2R's
  - iv) 1L, 3R's
  - v) 4R's?
- What about with 5 levels, 6 levels . . .?

0746b Flipping Coins

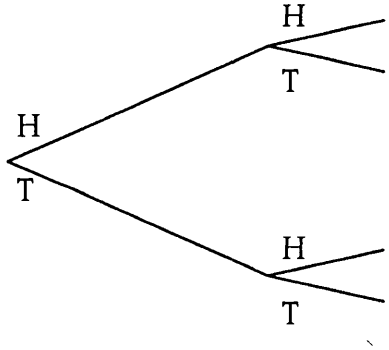
1. 8

2. 3 (HHT, HTH, THH)

3.

Combinations	Paths	Total number of paths	Probability
3 Heads	HHH	1	$\frac{1}{8}$
2 Heads and 1 Tail	HHT,HTH,THH	3	$\frac{3}{8}$
2 Tails and 1 Head	TTH,THT,HTT	3	$\frac{3}{8}$
3 Tails	TTT	1	$\frac{1}{8}$
		Total of probabilities	$\frac{8}{8} = 1$

4. First Throw / Second Throw



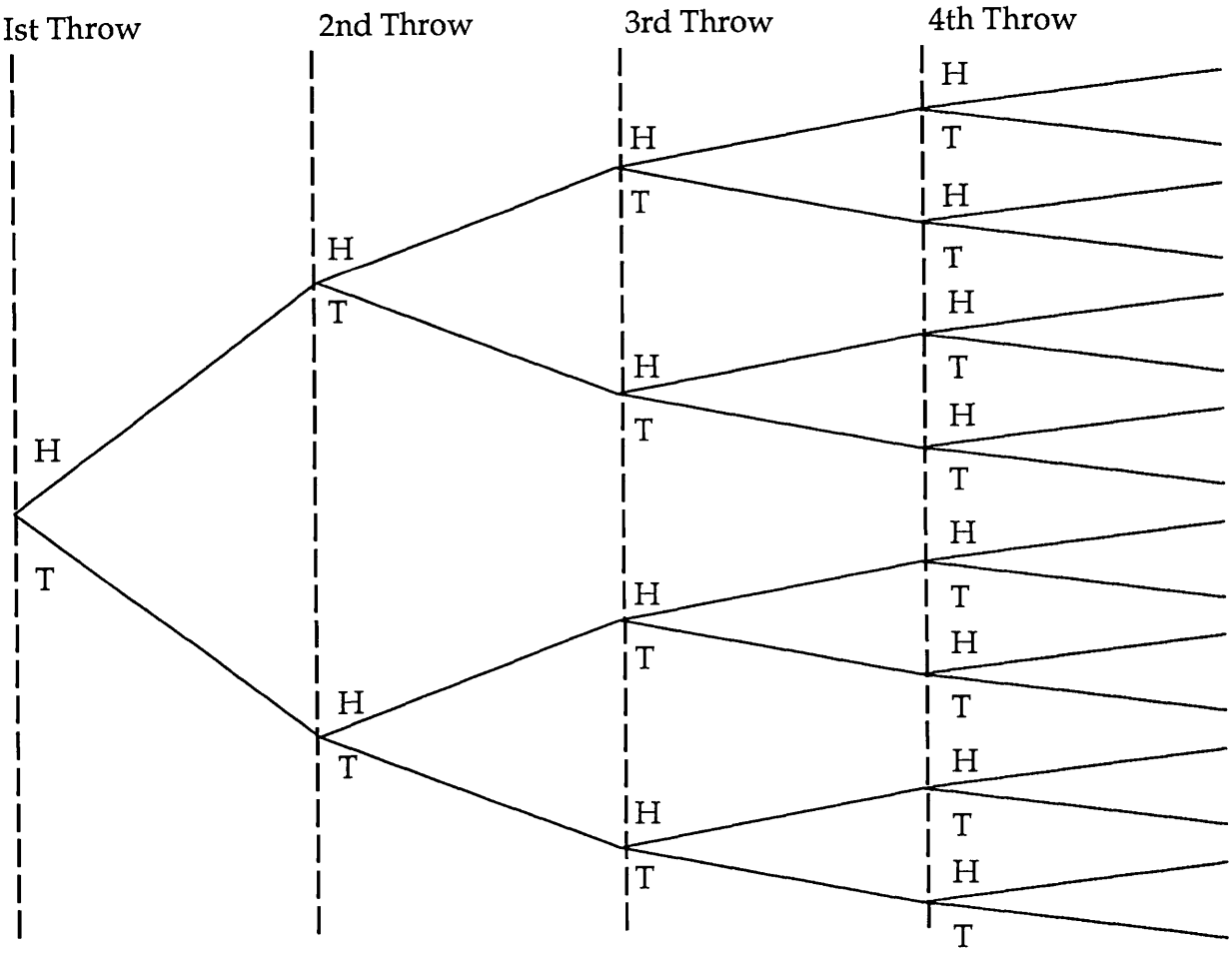
1 way of throwing 2 heads; Probability = $\frac{1}{4}$
1 way of throwing 2 tails; Probability = $\frac{1}{4}$
2 ways of throwing 1 head, 1 tail; Probability = $\frac{2}{4}$
Total probabilities = $\frac{4}{4}$
= 1

continued/

0746b Flipping Coins (cont)

5. It is not a fair game.
There are 4 different ways the two coins can fall.
In three cases, a head is showing, so Fred pays John 1p.
In one case, no head is showing, so John pays Fred 2p.
So, after every 4 throws, John would expect to pay Fred 2p and Fred would expect to pay John 3p.

6.



Combinations	Total number of paths	Probability
4 heads	1	$\frac{1}{16}$
3 heads, 1 tail	4	$\frac{4}{16}$
2 heads, 2 tails	6	$\frac{6}{16}$
1 head, 3 tails	4	$\frac{4}{16}$
3 tails	1	$\frac{1}{16}$
	Total of probabilities =	$\frac{16}{16}$
	=	1

continued/

0746c Coin Game

- You should have found that most counters finished at C and D and that least finished at A and F.

- There is 1 route which finishes at A
  There are 5 routes which finishes at B
  There are 10 routes which finishes at C
  There are 10 routes which finishes at D
  There are 5 routes which finishes at E
  There is 1 route which finishes at F.

1. A is 5 squares to the left of the start square, so the only way to get there is to toss 5 successive heads.

2. The 5 paths show all the possible different combinations of 4 heads and 1 tail.

3.

Home Square	Routes	Total
A	HHHHH	1
B	HHHHT HHHTH HHTHH HTHHH THHHH	5
C	HHHTT HHTHT HTHHT THHHT HHTTH HTHTH THHTH HTTHH THTHH TTHHH	10
D	TTTHH TTHTH THTTH HTTTH TTHHT THTHT HTTHT THHTT HTHTT HHTTT	10
E	TTTTH TTTHT TTHTT THTTT HTTTT	5
F	TTTTT	1

continued/

4. For player 1, the table would be reversed. A's routes become F's, B's become E's, etc.

5. Possible routes for 5 x 5 board are:
A → 1 route
B → 4 routes
C → 6 routes
D → 4 routes
E → 1 route

For a 4 x 4 board:
A → 1 route
B → 3 routes
C → 3 routes
D → 1 route

For a 3 x 3 board:
A → 1 route
B → 2 routes
C → 1 route

There is a pattern. Look at Pascal's triangle.

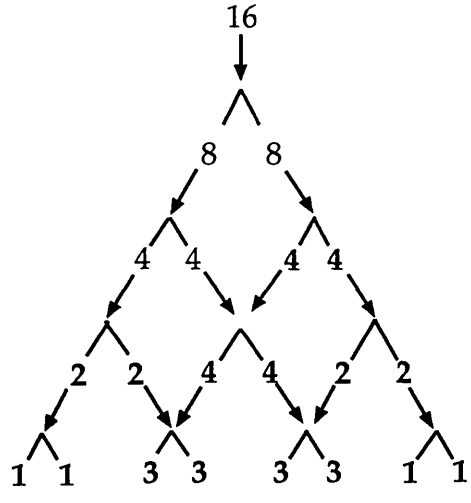
```
                1
              1   1
            1   2   1                →  A 3 x 3 board gives the 3rd line of
          1   3   3   1                  Pascal's triangle
        1   4   6   4   1            →  A 5 x 5 board gives the 5th line of
      1   5   10  10  5   1              Pascal's triangle
    1   6   15  20  15  6   1
  1   7   21  35  35  21  7   1
```

So you can use Pascal's triangle to find the number of routes to each home square on 6 x 6 board and an 8 x 8 board.

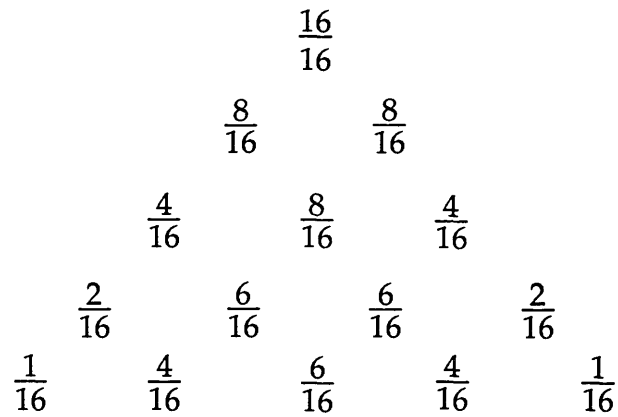
0746d Probability Maze

Just guessing rarely gives an accurate answer. However if you are using prior knowledge from other experiments, your guess might be quite accurate.
The more times an experiment is carried out, the more likely it is that the experimental probability will become closer to the theoretical probability.

a)



b)



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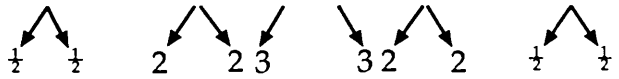
0746d Probability Maze (cont)

With one more row, you obtain:

a) 

b) $\frac{1}{32}$ $\frac{5}{32}$ $\frac{10}{32}$ $\frac{10}{32}$ $\frac{5}{32}$ $\frac{1}{32}$

- The results give a form of Pascal's Triangle

0746e Three Men in a Boat

1. 4 ways. Davy Hilary Simon Tim

2. 6 ways: Davy and Hilary, Davy and Simon, Davy and Tim, Hilary and Simon, Hilary and Tim, Simon and Tim

3. 4 ways: Davy, Hilary, Simon; Davy, Hilary, Tim; Davy, Simon, Tim; Hilary, Simon, Tim

4. 5 ways.

5. 10 ways: Davy, Hilary; Davy, Simon; Davy, Tim; Davy, Ronnie; Hilary, Tim; Hilary, Ronnie; Hilary, Simon; Simon, Tim; Simon, Ronnie; Tim, Ronnie

6. 10 ways: Davy, Hilary, Simon; Davy, Hilary, Tim; Davy, Hilary, Ronnie; Davy, Simon, Ronnie; Davy, Simon, Tim; Davy, Tim, Ronnie; Hilary, Simon, Tim; Hilary, Simon, Ronnie; Hilary, Tim, Ronnie; Simon, Tim, Ronnie

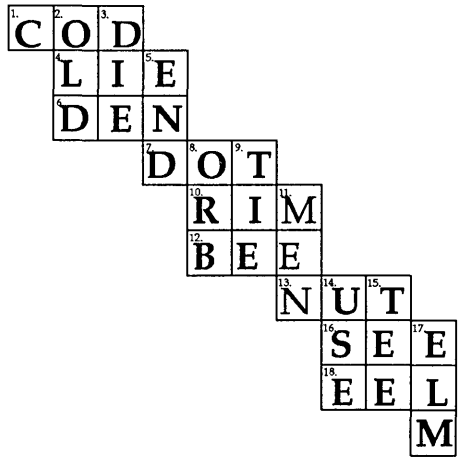
7. 5 ways.

All these activities relate to Pascal's triangle.

```
          1
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
```

If you cannot see the connection, look back at your work once more.

0748 The Times Crossword



0749 Three Numbers

8	6	5	2	3	6
1	4	6	8	7	2
7	1	3	5	5	5
9	4	8	9	3	7
6	4	2	7	2	1
2	2	7	5	1	8

1. There are probably several answers. These are possible ones:
   a) 22 = (6 x 4) – 2
   b) 16 = 8 + 7 + 1
   c) 26 = (9 x 2) + 8
   d) 21 = 7 x (5 – 2)
   e) 3 = (7 – 1) – 3

2. Were there any 'answers' which were impossible to find?

0750 Monopoly

1. Discuss your reasons with your teacher.

2.

Score	2	3	4	5	6	7	8	9	10	11	12
Ways of getting the score	1 + 1	1 + 2 2 + 1	1 + 3 3 + 1 2 + 2	1 + 4 4 + 1 2 + 3 3 + 2	1 + 5 5 + 1 2 + 4 4 + 2 3 + 3	1 + 6 6 + 1 2 + 5 5 + 2 3 + 4 4 + 3	2 + 6 6 + 2 3 + 5 5 + 3 4 + 4	3 + 6 6 + 3 4 + 5 5 + 4	4 + 6 6 + 4 5 + 5	5 + 6 6 + 5	6 + 6
Total	1	2	3	4	5	6	5	4	3	2	1

continued/

3. 5 ways

4. 3 ways

5. You are more likely to score a 6 because there are more ways of getting this score.

6. Equally likely - there are 4 ways of getting a 5 and 4 ways of getting a 9.

7. 36 ways altogether.

8. It is dangerous because there are 6 different ways you can get a score that will land you on Mayfair.

9.

	1	2	3	4	5	6
1	2	3	4	5	6	
2	3	4	5	6		8
3	4	5	6		8	9
4	5	6		8	9	10
5	6		8	9	10	11
6		8	9	10	11	12

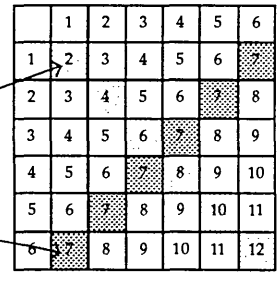
11.

Score	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

12. Assume a run of 36 goes:
The boys will take 36p.
They are likely to pay out

a) 6 lots of 3p = 18p

b) 6 lots of 2p = 12p



Therefore expected profit is (36p – 30p) = 6p

13. There is a high probability of getting a 7 so it might be wiser to wait until after you throw before developing your property.

0752 Repeating Digits

The answer is always the three digit number you started with. By repeating the three digits, to make a six digit number, you have multiplied the original three digit number by 1001 which is 7 x 11 x 13.

Challenge
When you have made up your flow chart, test it.
You should find your answer is always the four digit number you started with.

0753 Cafe Prices

1. e means eggs, b means bacon

2. $2e + b = 31$
   $3e + 2b = 52$

3. $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}e\\b\end{pmatrix} = \begin{pmatrix}\mathbf{10}\\\mathbf{11}\end{pmatrix}$

4. $\begin{pmatrix}e\\b\end{pmatrix} = \begin{pmatrix}10\\11\end{pmatrix}$

   so e = **10** and b = **11**

5. Yes

6. $\mathbf{c + 2t = 28}$

7. $3c + 5t = 74$ $\quad \begin{pmatrix}3 & 5\\\mathbf{1} & \mathbf{2}\end{pmatrix}\begin{pmatrix}c\\t\end{pmatrix} = \begin{pmatrix}74\\\mathbf{28}\end{pmatrix}$
   $1c + 2t = 28$

8. Inverse of $\begin{pmatrix}3 & 5\\1 & 2\end{pmatrix}$ is $\begin{pmatrix}2 & -5\\-1 & 3\end{pmatrix}$

   $$\begin{pmatrix}2 & -5\\-1 & 3\end{pmatrix}\begin{pmatrix}3 & 5\\1 & 2\end{pmatrix}\begin{pmatrix}c\\t\end{pmatrix} = \begin{pmatrix}2 & -5\\-1 & 3\end{pmatrix}\begin{pmatrix}74\\28\end{pmatrix}$$

   Inverse Inverse

   so $\begin{pmatrix}c\\t\end{pmatrix} = \begin{pmatrix}8\\10\end{pmatrix}$

   so 1 choc ice costs 8p
   1 tub costs 10p

9. 1 tea costs 8p, 1 coffee costs 11p.

10. 1 bun costs 5p, 1 cake costs 8p.

0755 Rectangles to Regions

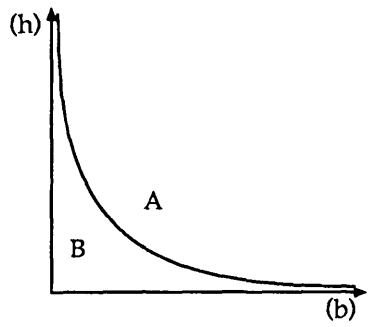
1. Show your results to your teacher.

2. The points on your graph should make a smooth curve. If they do not, check your work with you teacher.

3. For each point, the values for b and h, should give $bh = 36$.

4. For each point, the values for b and h, should give $bh < 36$.

5. For each point, the values for b and h, should give $bh > 36$.
Points in region S represent $bh > 36$.

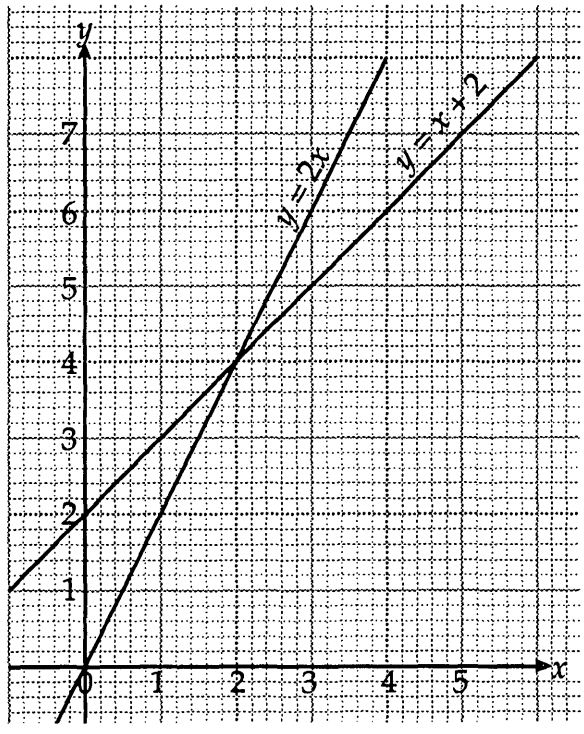
6. 

Any point in region B (below the curve) will give $bh < 24$.

Any point in region A (above the curve) will give $bh > 24$.

7. a) h $\approx$ 2.82cm
b) Any value of h where h > 2.82cm.
c) Any value of h where h < 2.82cm.

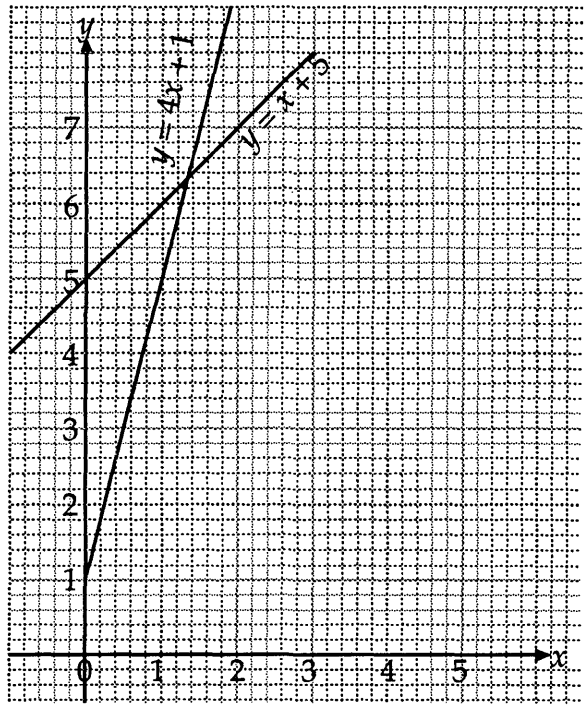
0756 Points of Intersection

1. 

Point of intersection is (2, 4)

continued/

2.



Point of intersection is approximately (1.3, 6.3).

3.

x	y
2	
	7
1.5	
	6.5
1.375	
	6.375
1.344	
	6.344
1.336	
	6.336

Point of intersection is (1.34, 6.34) to two decimal places.

4. $x = \frac{y}{5}$

x	y
1	
	4
0.8	
	3.8
0.76	
	3.76
0.752	
	3.752
0.7504	
	3.7504
0.75	
	3.75

Point of intersection is (0.75, 3.75) to two decimal places.

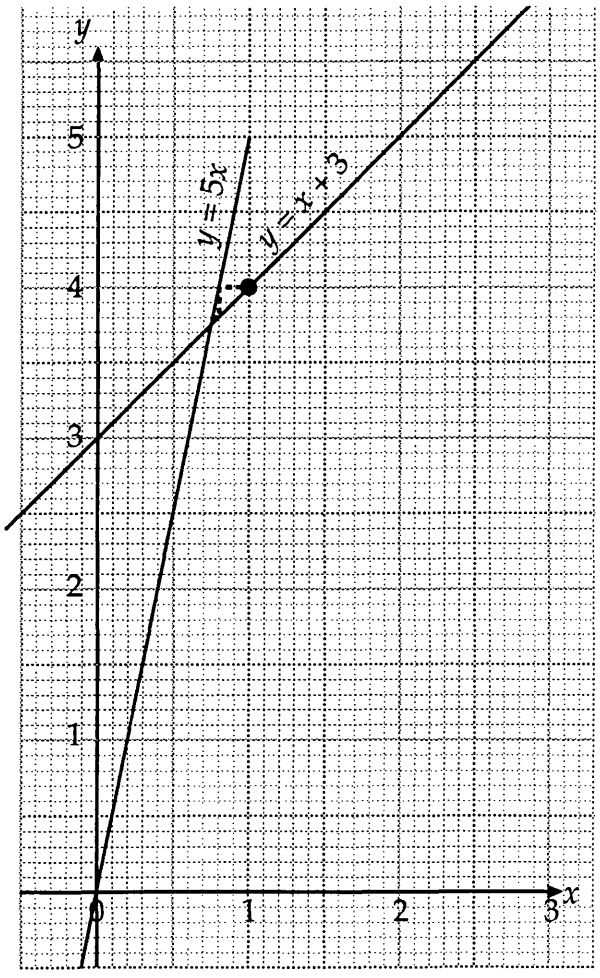
continued/

5. $x = \frac{y-3}{7}$ start with $x = 1$. You may have chosen a different starting number.

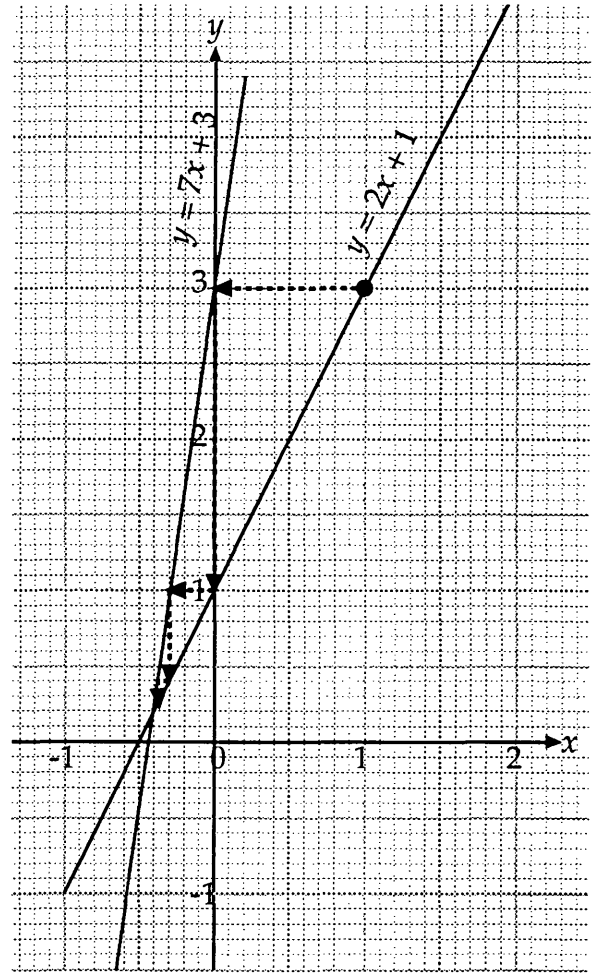
x	y
1	
	3
0	
	1
-0.286	
	0.429
-0.367	
	0.265
-0.391	
	0.219
-0.397	
	0.205
-0.399	
	0.202
-0.40	
	0.20

Point of intersection is (-0.40, 0.20) to two decimal places.

6. Line graphs for question 4



Line graphs for question 5



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7. For question 4 $x = y - 3, y = 5x$

x	y
1	
	5
2	
	10
7	
	35

For question 5 $x = \frac{y-1}{2}, y = 7x + 3$

x	y
1	
	10
4.5	
	34.5
16.75	
	120.25

In each case, the iterative method does not work. All the results **diverge** away from the answer. Your first guess should always be used in the equation having the shallower gradient.

8. Rewrite the equation $y = 5x + 1$.
The equation $y = 2x$, having the shallower gradient, must be used to calculate y using your initial guess for x.

0757 Centigrade and Fahrenheit

0757a Substituting for C

F = 104.

Yes, C = 40. Use this substituting method to check your other values of C.

0757b Substituting for F

$$C = \frac{5(36)}{9} = 20$$

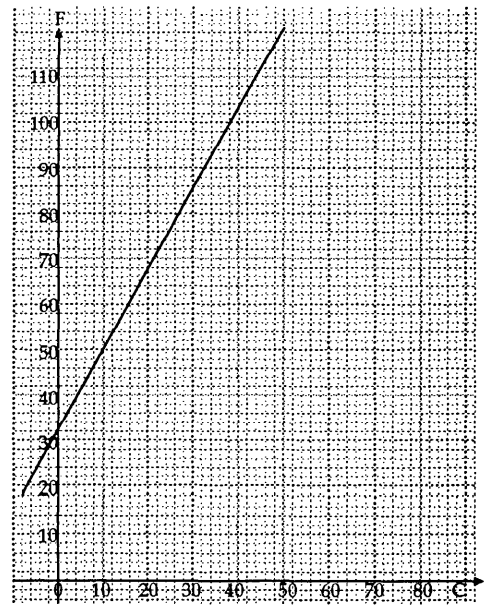
$$F = \frac{9 \times 20}{5} + 32 = \frac{180}{5} + 32 = 68$$

Again you can check your values of F using this substituting method.

0757c Plotting Points

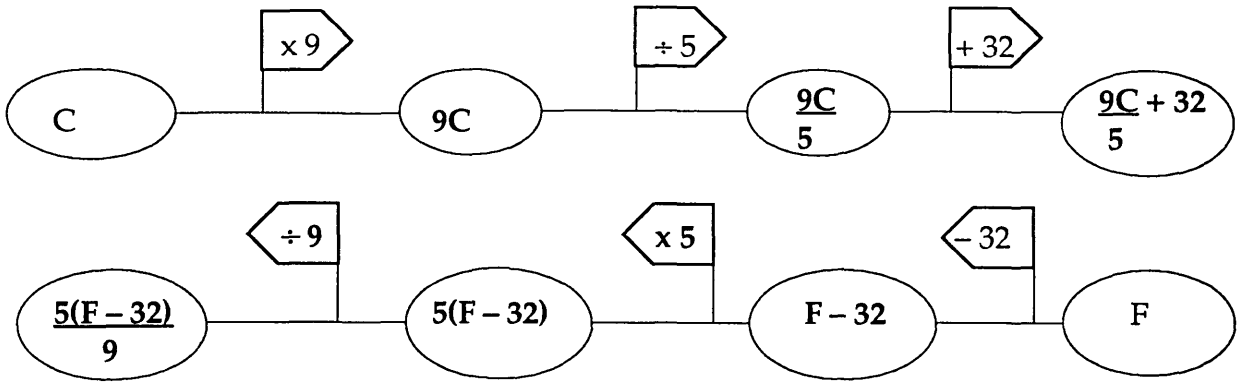
If you have any points which are not in a straight line, check your calculation.

Once you have drawn your conversion graph, you can use it to convert Centigrade temperatures to Fahrenheit, and Fahrenheit to Centigrade.



continued/

0757d Inverse Flags



Try out the flow diagram for different values of C and F.

0757e Arrangement 1

$5(F-32) = 9C$

$\frac{5(F-32)}{9} = C$

0757f Rearrangement 2

Divide both sides by **5**

$\frac{9C}{5} = F-32$

Add **32** to both sides

$\frac{9C}{5} + 32 = F$

0758 Odd One Out

1.

Rule	Number
Odd numbers	158
Divide by 7 and the remainder is 4	143
3 digit numbers	53

2. Here are some rules, yours may be different. If you are unsure about your rules, check them with your teacher.

Rule	Number
2 digit numbers	9
Divide by 5 and the remainder is 4	25
Odd numbers	64
Square numbers	79

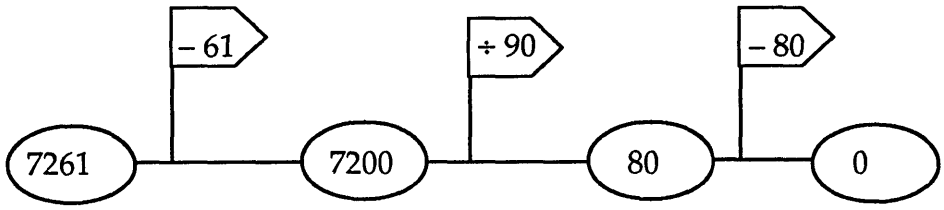
3. Ask someone else to check your rules for your set of numbers.

0760 Quickly to Zero

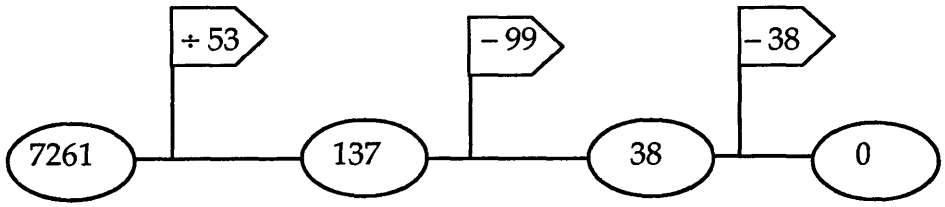
A number with 4 digits can **always** be reduced to zero using 2 digits at a time in a **maximum** of 3 steps. You could add, subtract, multiply or divide using any 2 digits but as you are reducing the number to zero it is better to divide and subtract.

- Subtraction of a positive 2 digit number will always reduce the number.
- Division by a positive 2 digit number greater than one will always reduce the number.

Here is one way of reducing 7261 to 0.

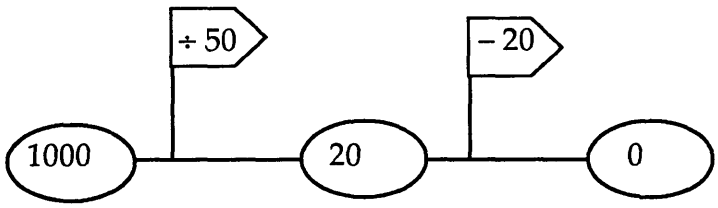


Here is another way.



We were unable to reduce it in fewer than 3 steps.

It is possible to reduce some 4 digit numbers in fewer than 3 steps.
e.g.



Other examples we found were:

1500	→	divide by 30	then	subtract 50
2225	→	divide by 25	then	subtract 89
2500	→	divide by 50	then	subtract 50
4032	→	divide by 64	then	subtract 63
9801	→	divide by 99	then	subtract 99

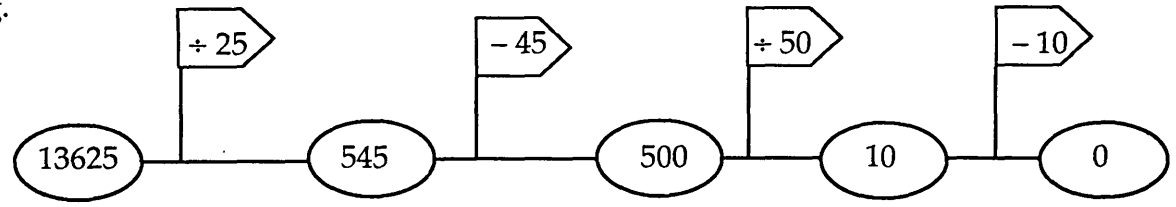
- Any 4 digit number which is the product of two 2 digit numbers can be reduced in 2 steps by dividing by the largest factor with 2 digits then subtracting the result.

- Any 4 digit number which is not a product of two 2 digit numbers, needs to have a 2 digit number subtracted first before dividing by the largest factor and subtracting the result.

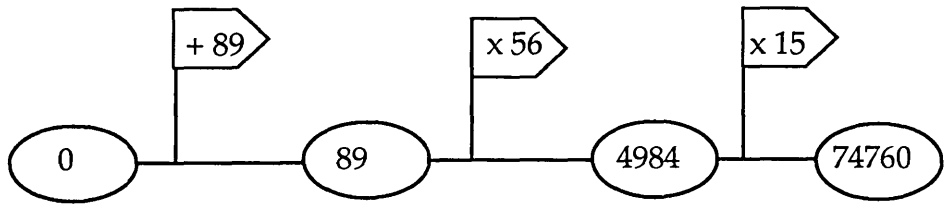
continued/

0760 Quickly to Zero (cont)

Any 5 digit number can be reduced to zero in a maximum of 4 steps.
e.g.



Can you find a 5 digit number which can be reduced in fewer steps? You might want to work backwards to find some examples.
e.g. Start with 0, add a 2 digit number and multiply by two 2 digit numbers.



So it is possible to reduce 74760 in 3 steps. (divide by 15, divide by 56 then subtract 89).
What sort of 5 digit numbers can be reduced in fewer than 4 steps?

- What about 6 digit numbers?

0761 Orbits

Each answer is obtained by going back to the original calculation and only rounding the answer in the final calculation. If you used answers to previous calculations your answers will be less accurate.

1. 40000km

2. Using π button — 12732km to nearest whole kilometre.
   12732.395km to the nearest metre
   Using π = 3.14 — 12739km to nearest whole kilometre.
   12738.854km to the nearest metre.

3. Using π button — 6366km to nearest whole kilometre.
   6366.198km to nearest metre.
   Using π = 3.14 — 6369km to nearest whole kilometre.
   6369.427km to nearest metre

4. Using the π button or π as 3.14 the orbit of the satellite is 40050km to the nearest whole kilometre.

5. Approximately 1 hour and 20 minutes.

6. Length of rope = 40000km
   Rope required using the π button or π = 3.14 is 0.006km or 6 metres.
   In fact 2π metres extra is needed. Can you see why?

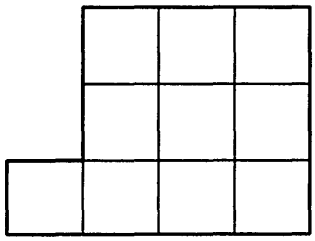
0772 Angle Estimation

Mark all your angles with the accurate measurement.

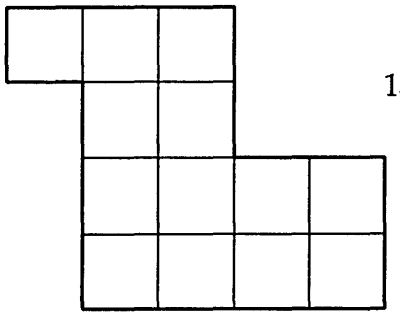
0773 Matching the Tiles A

Page 1 - Using dotty paper
You should have drawn six of the tiles.

Page 2 - How many squares?

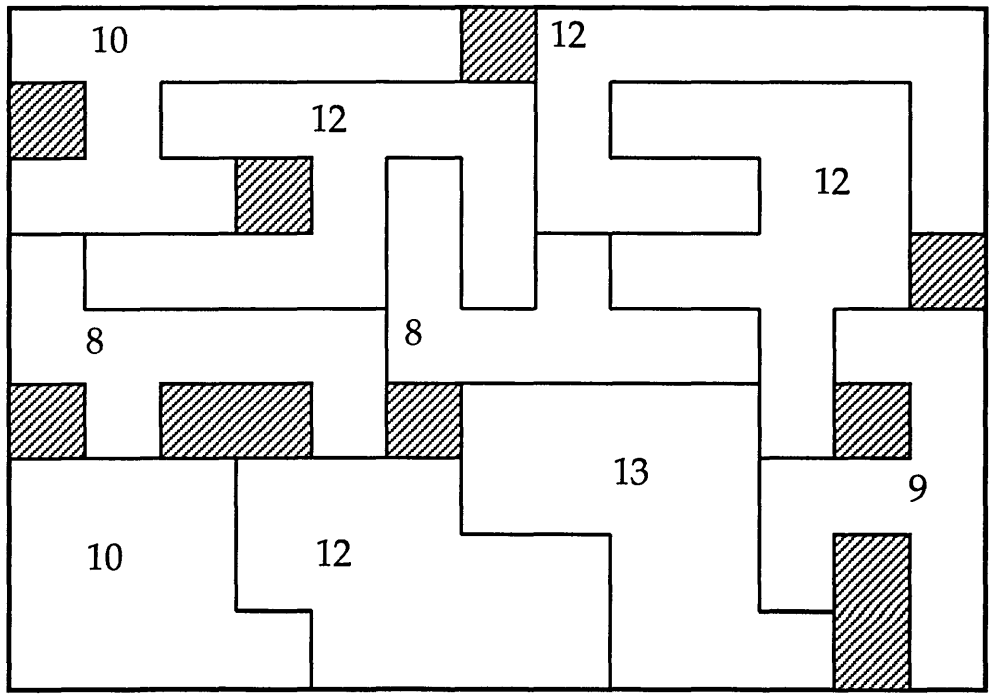


10 squares



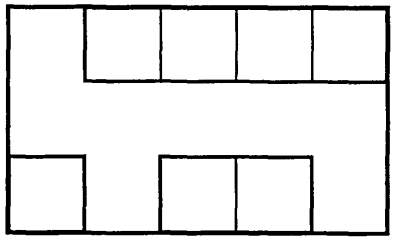
13 squares

Worksheet 2

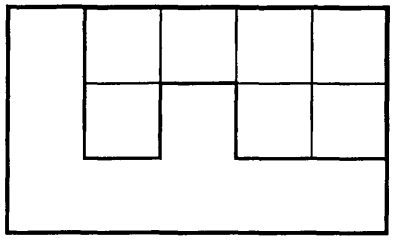


continued/

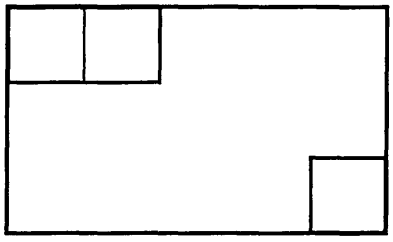
Page 3 - Tiles in a rectangle.
Worksheet 3

1. 

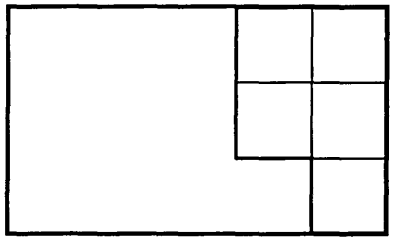
Area of rectangle = 15
Number of extra squares = 7
so area of tile = 15 – 7
= 8 squares

2. 

Number of extra squares = 7
so area of tile = 8

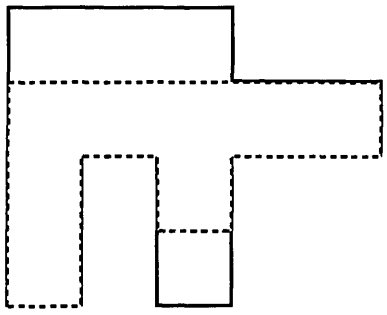
3. 

Number of extra squares = 3
so area of tile = 12

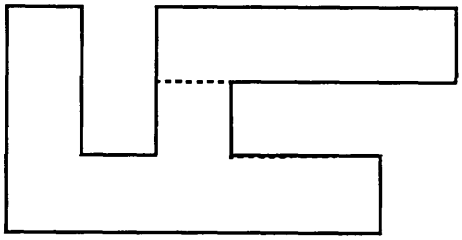
4. 

Number of extra squares = 5
so area of tile = 10

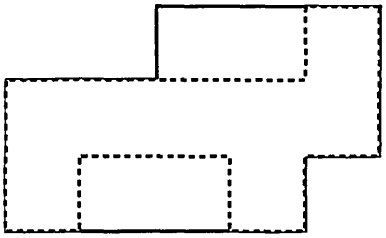
Page 4 - Shapes within tiles
You should have drawn two of these.



or



or



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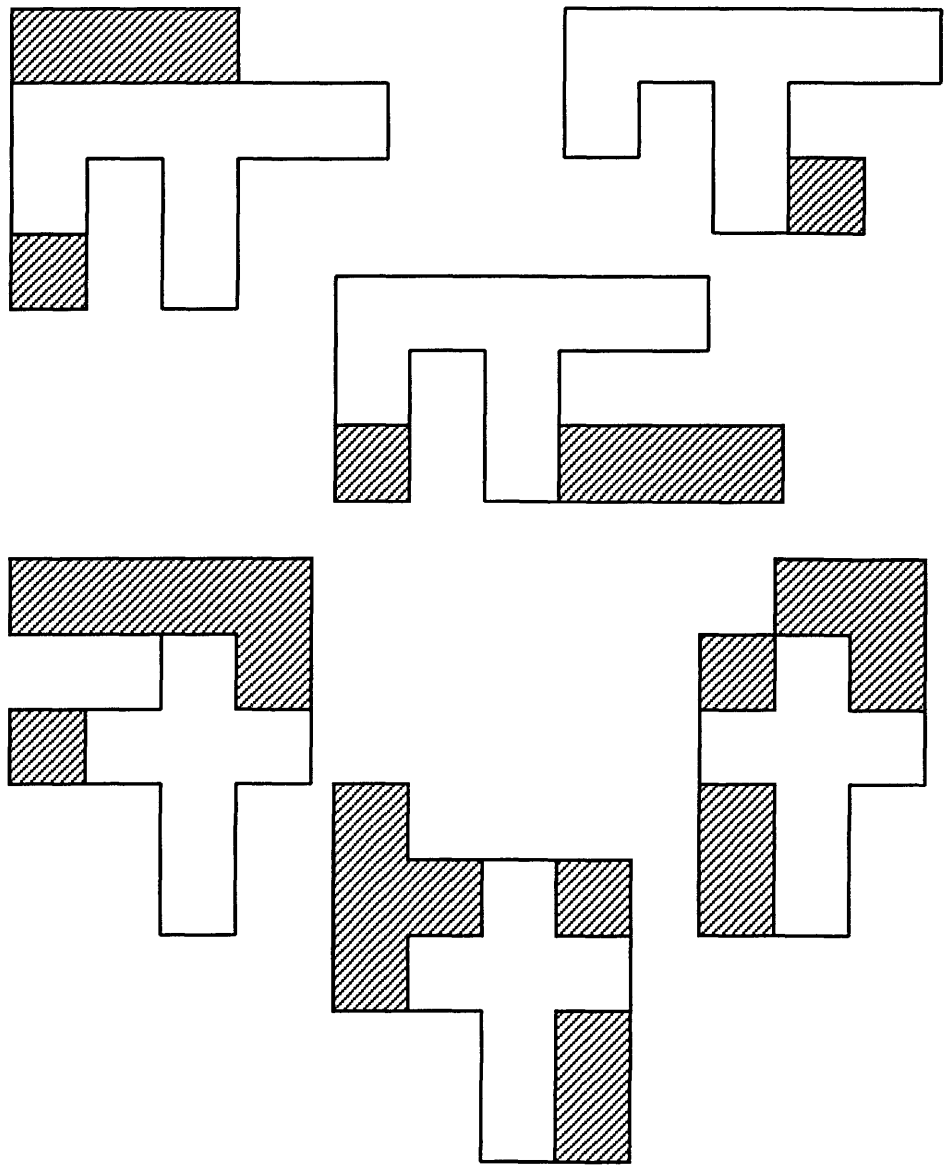
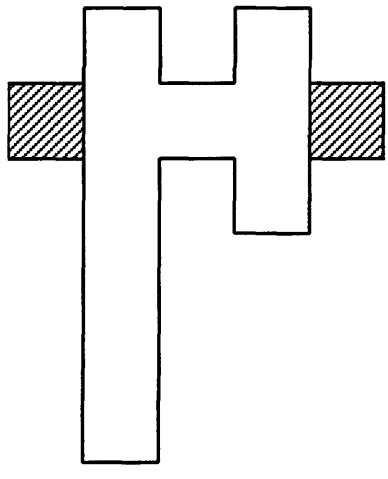
Worksheet 4



Page 5 - Two extra squares



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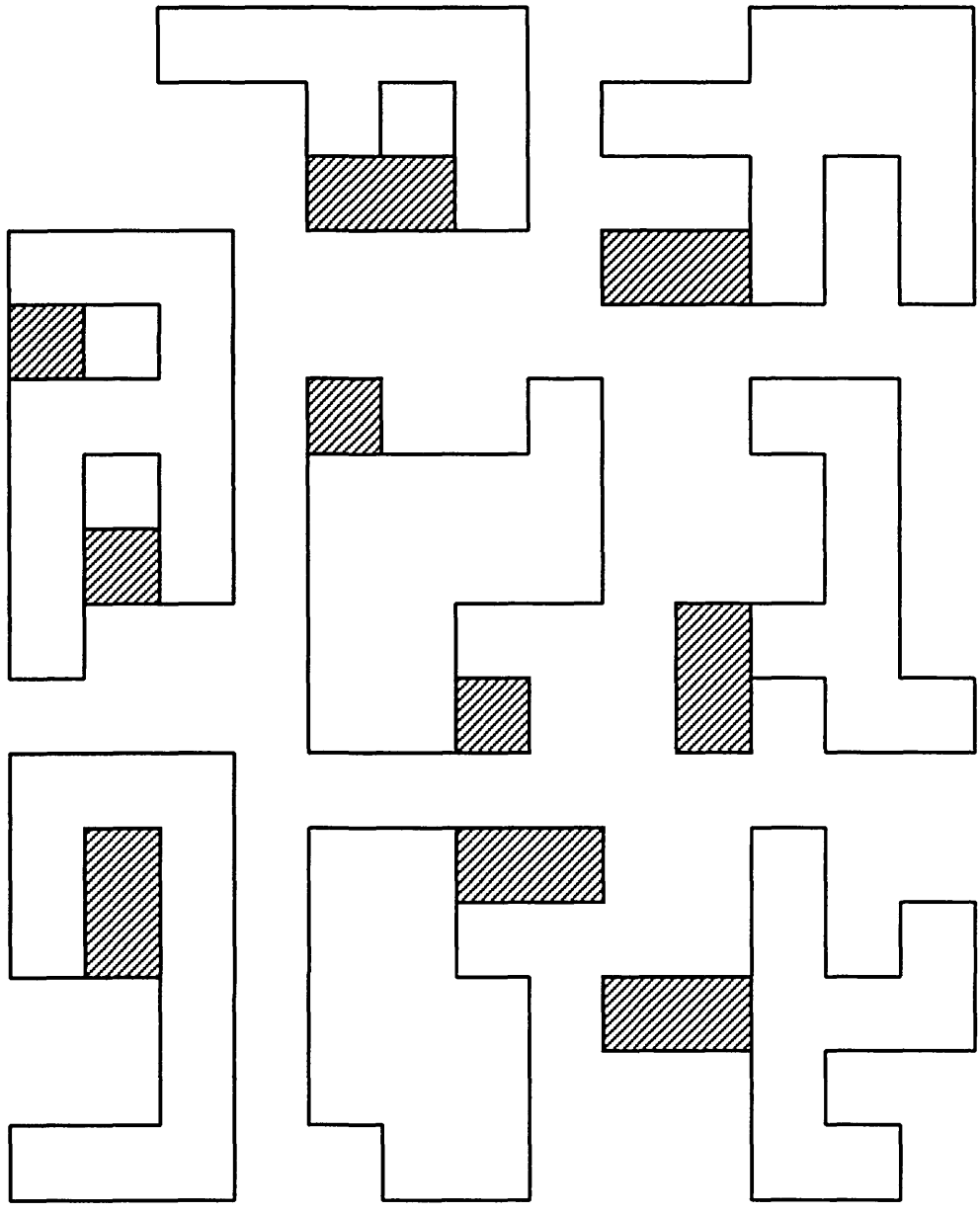
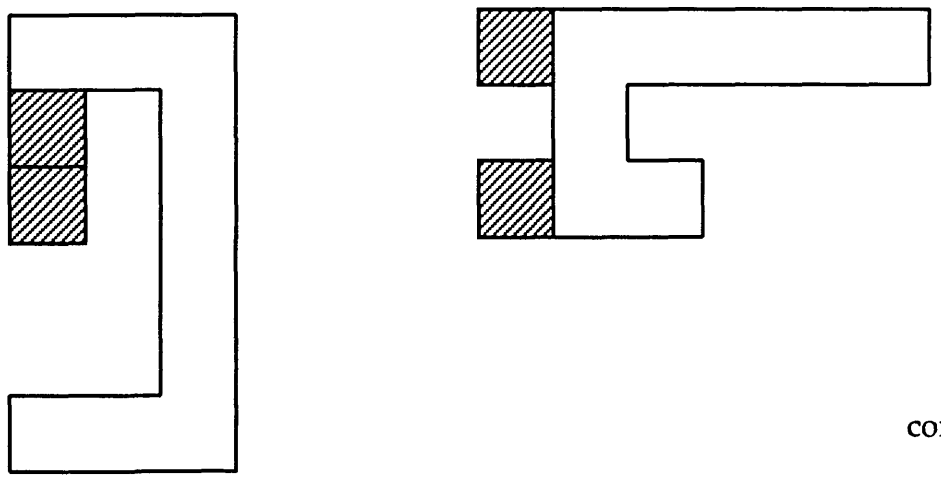
Worksheet 5

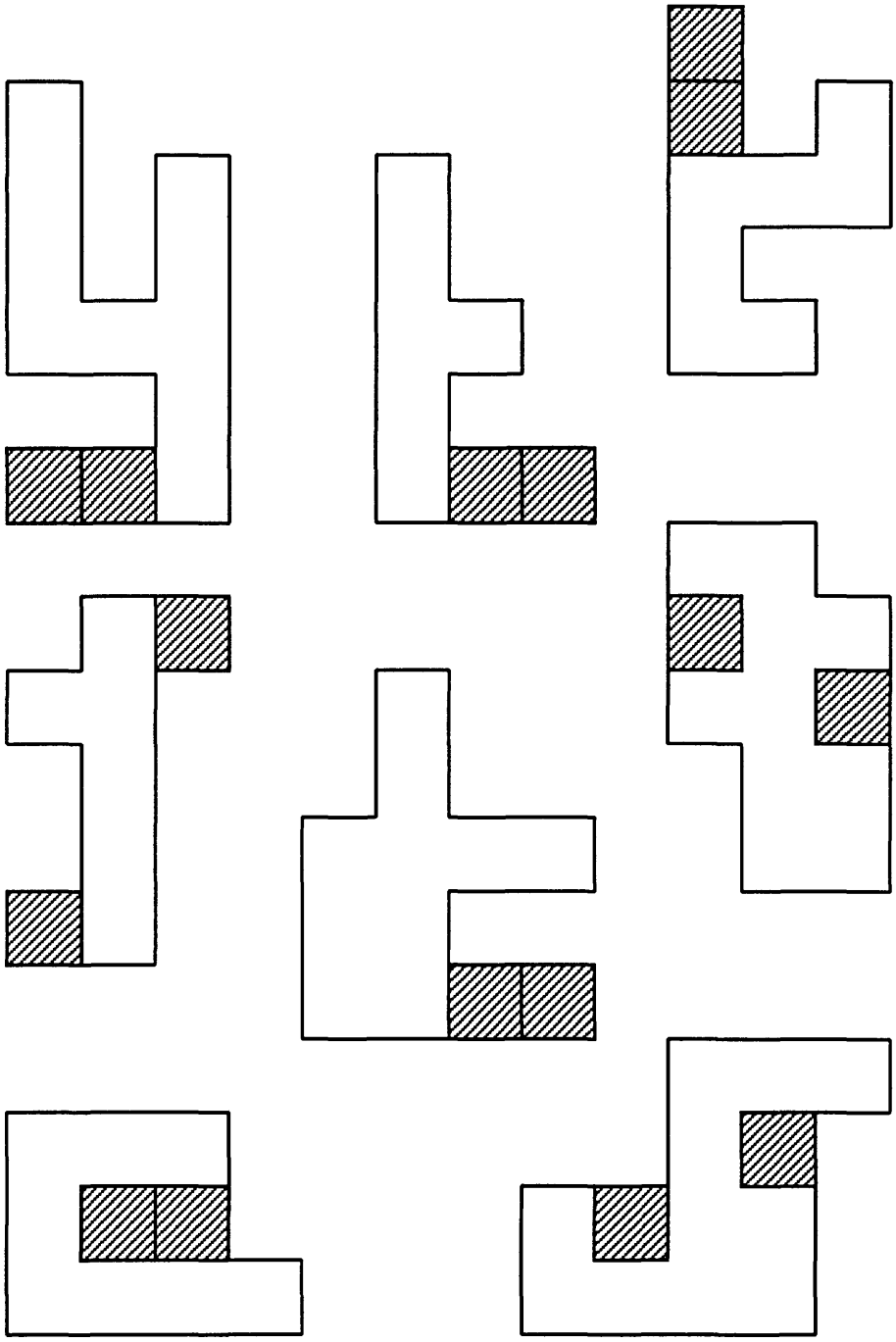


Page 6 - Two missing squares



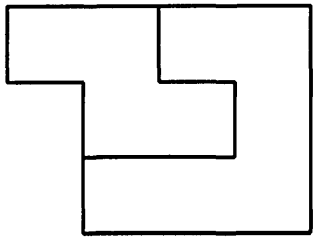
continued/

Worksheet 6

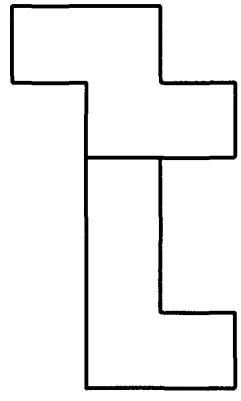


Page 7 - Parts of tiles
Worksheet 7

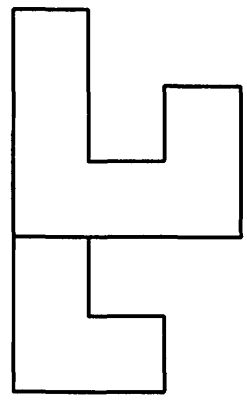
2.



3.



4.



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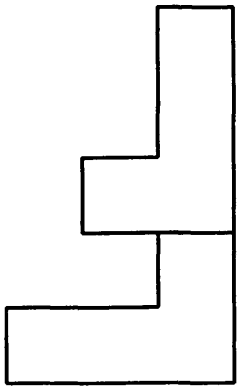
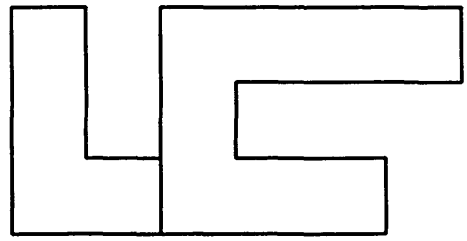
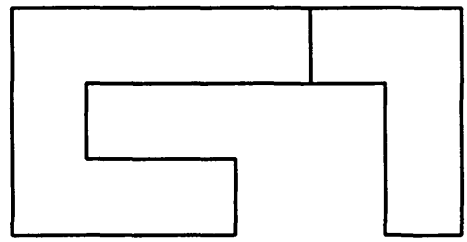
Worksheet 7(cont)

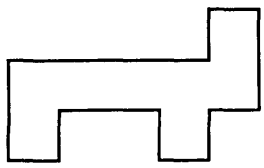
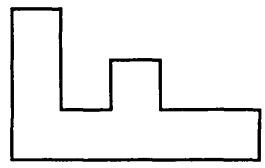
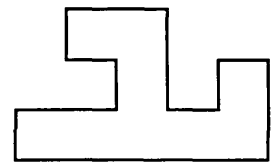
5. 

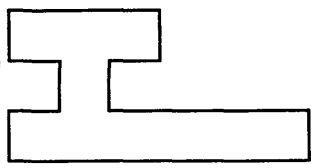
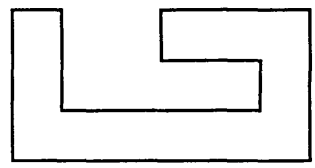
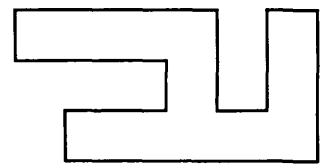
6. 

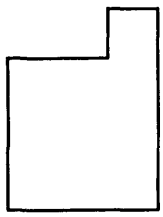
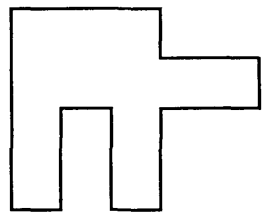
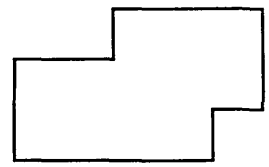
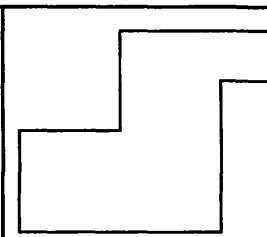
and



Page 8 - How many matches?
Worksheet 8

			
Area	8	8	9
Perimeter	18	18	20

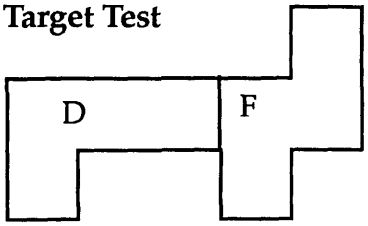
			
Area	10	12	12
Perimeter	22	26	26

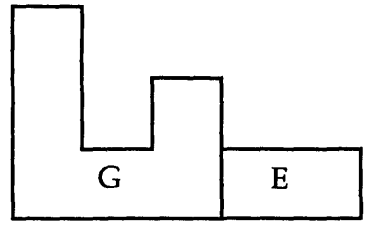
				
Area	10	12	12	13
Perimeter	14	22	16	18

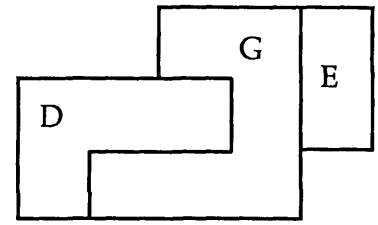
The largest tile with the largest area does not have the longest perimeter.
The nearer to a square each tile is, the shorter the perimeter.

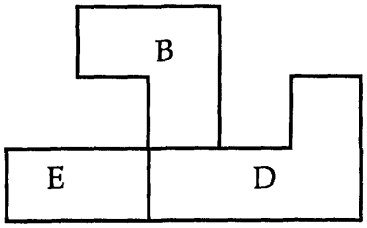
continued/

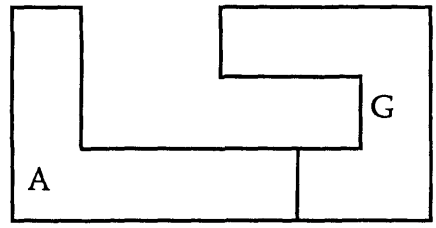
Target Test

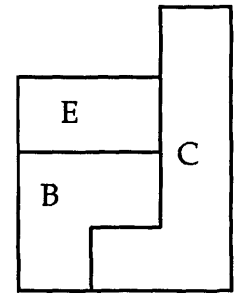


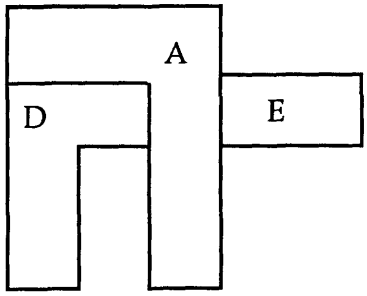


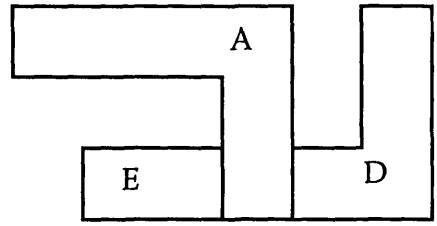


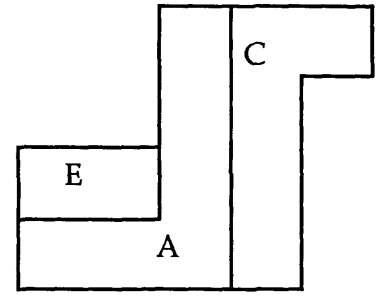




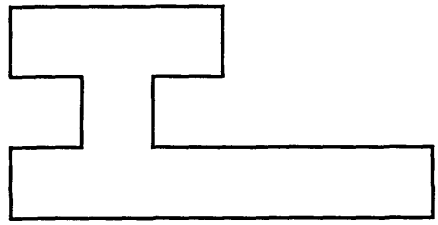






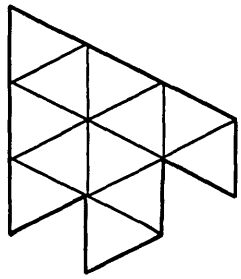


This tile could not be made from the shapes.

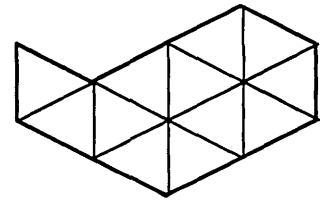


Page 1 - Using dotty paper
You should have drawn six tiles.

Page 2 - How many triangles?



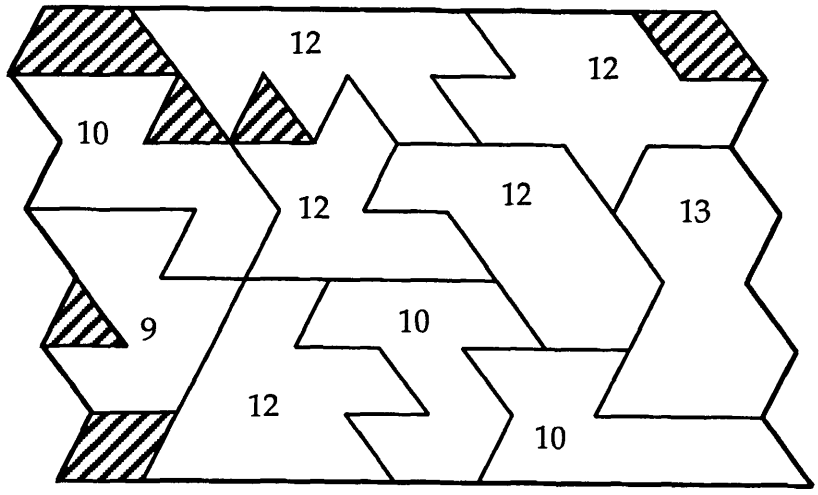
12 triangles



12 triangles

continued/

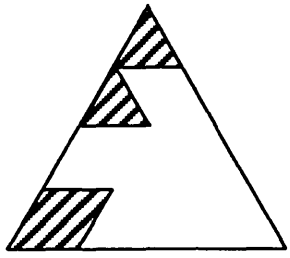
Worksheet 2



Page 3 - Tiles in a parallelogram

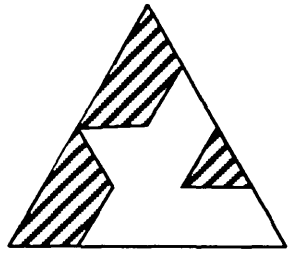
Worksheet 3

1.



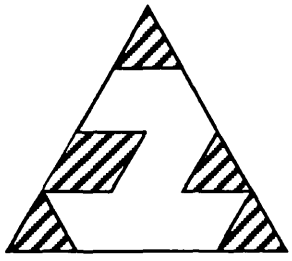
Area of big triangle = 16
Number of extra triangles = 4
so area of tile = 12

2.



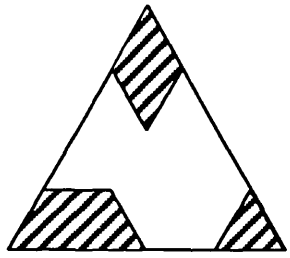
Number of extra triangles = 7
so area of tile = 9

3.



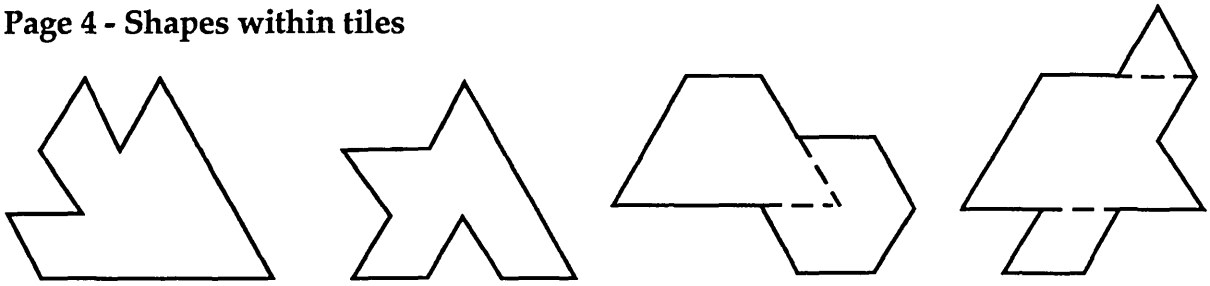
Number of extra triangles = 6
so area of tile = 10

4.



Number of extra triangles = 6
so area of tile = 10

Page 4 - Shapes within tiles



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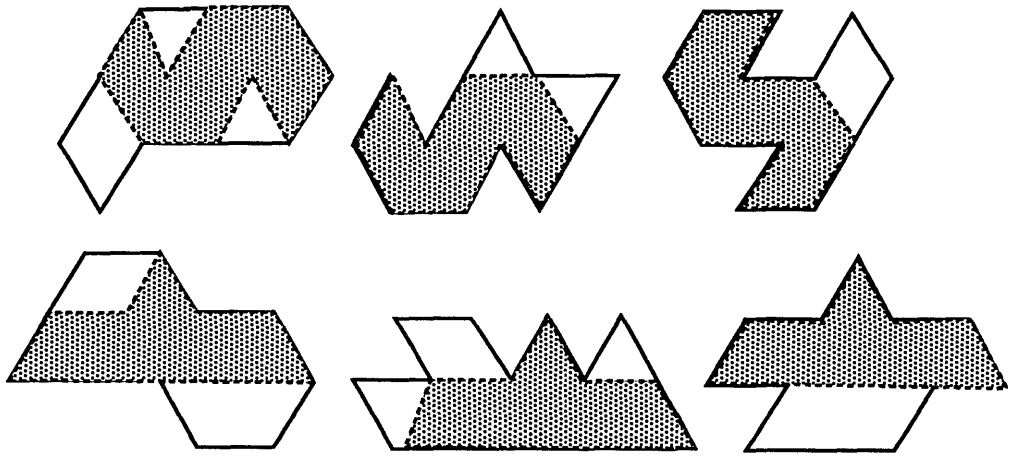
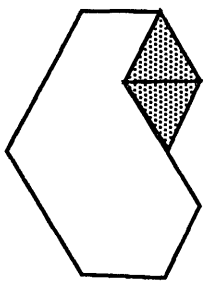
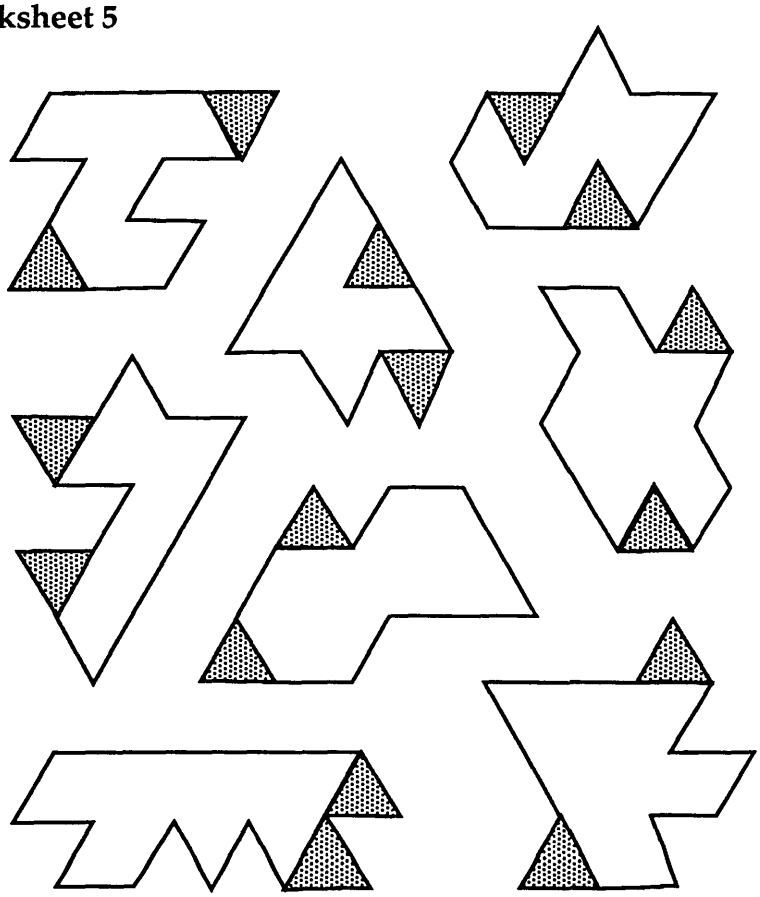
Worksheet 4



Page 5 - Two extra triangles



Worksheet 5



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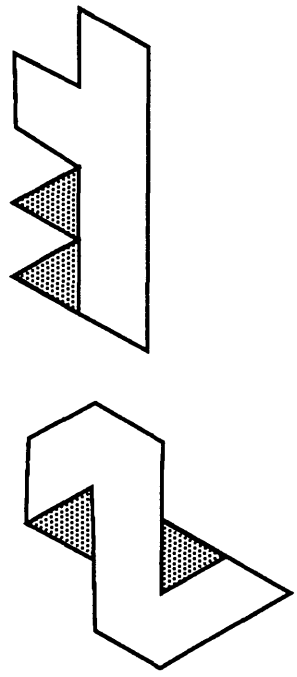
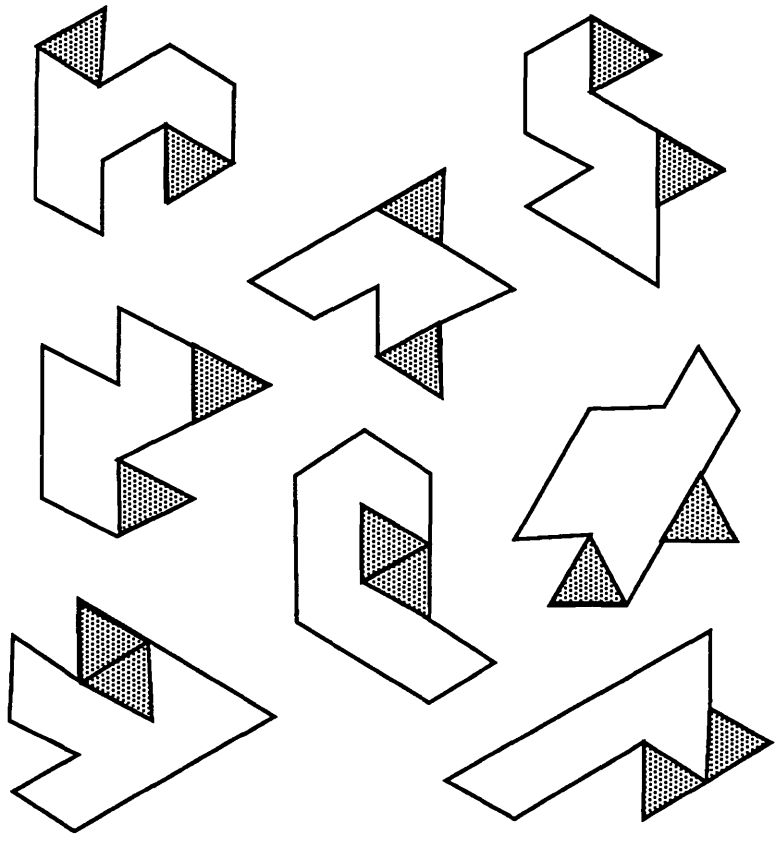
Page 6 - Two missing triangles

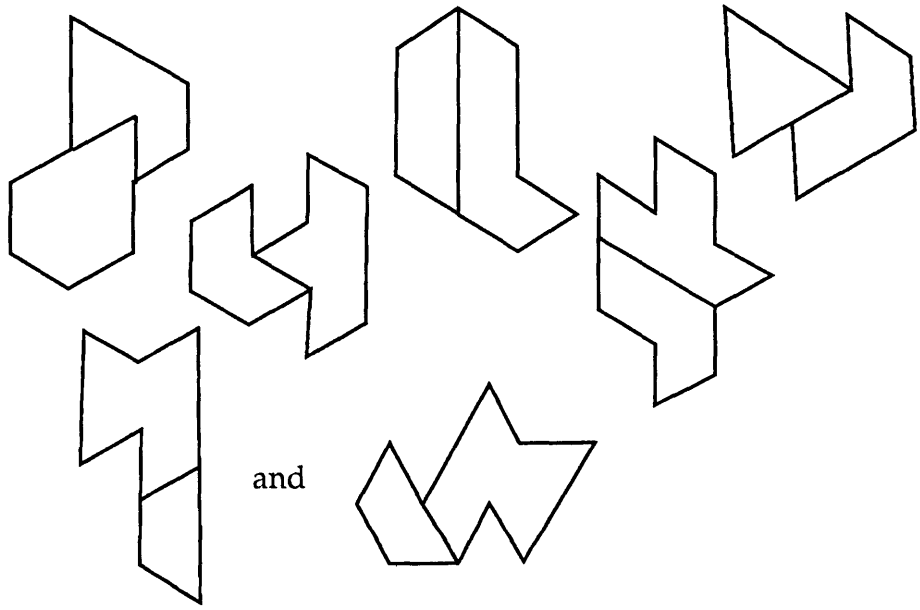


Worksheet 6

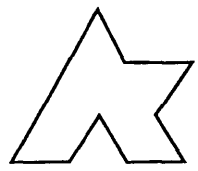
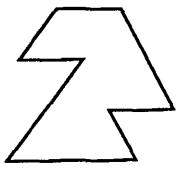
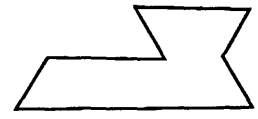
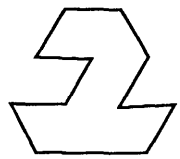
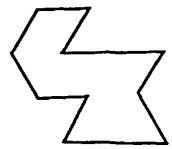
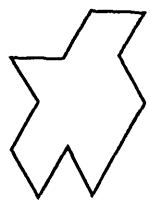
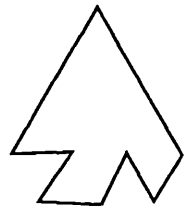
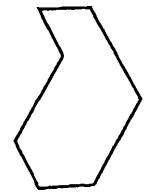
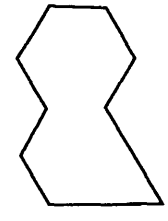


continued/

Page 7 - Parts of tiles



Page 8 - How many matches?

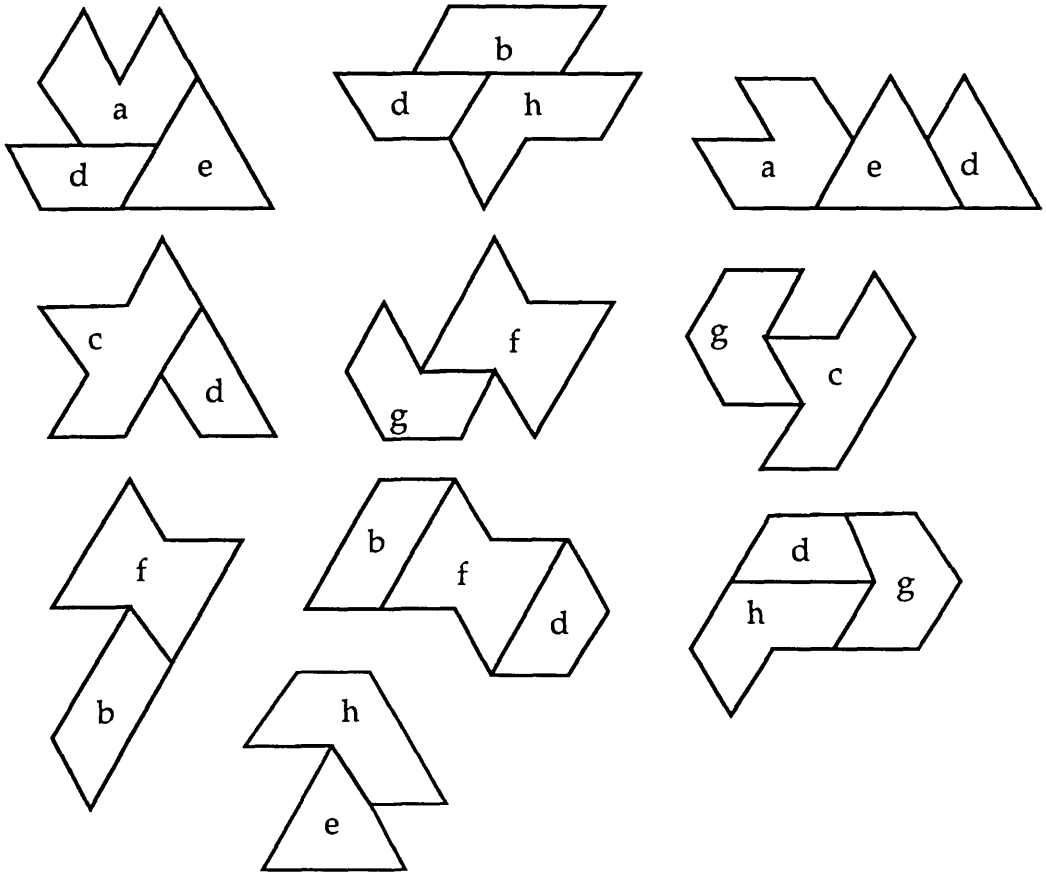
			
Area	9	9	10
Perimeter	11	11	12
			
Area	10	10	12
Perimeter	12	12	14
				
Area	12	12	12	13
Perimeter	12	12	10	11

The tile with the largest area does not have the longest perimeter.
Thin tiles have longer perimeters than tiles which are shaped more like 'hexagons'.

continued/

0774 Matching the Tiles B (cont)

Target Test



0775 Measuring Angles

1. 360°

2. 90°

3.

Angle	Size (in turns)	Size (in degrees)	Size (in right angles)
	1 turn	360°	4 right angles
	$\frac{1}{2}$ turn	180°	2 right angles
	$\frac{1}{4}$ turn	90°	1 right angle
	$\frac{3}{4}$ turn	270°	3 right angles
	$\frac{1}{8}$ turn	45°	$\frac{1}{2}$ right angle

d) The size of the angle is 57°.

continued/

0775 Measuring Angles (cont)

4. If your answers are close, mark them as correct.
If your answers are very different show them to your teacher.

c) 68° d) 11°
e) 243° f) 117° Your answers to e) and f) should add up to 360°. Why?
g) 159° h) 21° Your answers to g) and h) should add up to 180°. Why?
i) 45° j) 45° Your answers to i) and j) should be the same because they are vertically opposite angles.
k) 90° l) 270° Your answers to k) and l) should add up to 360°. Why?
m) 90° n) 90°
o) 45° p) 135° q) 180°
Your answers to o) and p) and q) should add up to 360°. Why?

0776 Drawing Angles

1. Get someone else to check your angles.

2.&3. Each pair of angles adds up to 360°.

0777 Satellite Signals

Your estimate of each angle should be within ±5° of the actual measured angle. If you require further practice you may like to use MicroSMILE Program Angle 90.

Turn	Estimate	Clockwise or anticlockwise	Measure
A to B		Anticlockwise	33°
B to C		Clockwise	69°
C to D		Clockwise	43°
D to E		Anticlockwise	137°
E to F		Anticlockwise	56°
F to G		Anticlockwise	100°
G to H		Anticlockwise	97°

0778 Tangrams

You may like to make a display of your own shapes that you have created using the tangram pieces.

0779 Greatest Product

1. $3 \times 3 \times 3 \times 3 = 81$

2. The greatest product for any number can be made by following this rule:
   - If the number is a multiple of 3, split it into 3's.
     e.g. 18 $3 \times 3 \times 3 \times 3 \times 3 \times 3$
   - If the number is 1 less than a multiple of 3, split it into 3's and one 2.
     e.g. 17 $3 \times 3 \times 3 \times 3 \times 3 \times 2$
   - If the number is 1 more than a multiple of 3, split it into 3's and two 2's.
     e.g. 16 $3 \times 3 \times 3 \times 3 \times 2 \times 2$

0780 Long Multiplication Revision

	423		
	x 26		
First step	8460	=	423 x 20
Second step	2538	=	423 x 6
Third step	10998	=	(423 x 20) + (423 x 6)
	641		
	x 83		
First step	51280	=	641 x 80
Second step	1923	=	641 x 3
Third step	53203	=	(641 x 80) + (641 x 3)
	374		
	x 29		
First step	7480	=	374 x 20
Second step	3366	=	374 x 9
Third step	10846	=	(374 x 20) + (374 x 9)
	544		
	x 47		
First step	21760	=	544 x 40
Second step	3808	=	544 x 7
Third step	25568	=	(544 x 40) + (544 x 7)
	208		
	x 36		
First step	6240	=	208 x 30
Second step	1248	=	208 x 6
Third step	7488	=	(208 x 30) + (208 x 6)

0781 The Inverse

1. Returning to the city
2. Pay back £1
3. No inverse, you cannot put an egg back together.
4. Letting air out of a tyre.
5. Taking 6 away from a number.
6. No inverse.
7. Turning anticlockwise through an angle of 60°.
8. No inverse
9. Multiplying a number by 2.
10. Turning the empty mug upright.
11. No inverse.
12. No inverse.
13. Show your list of operations and inverses to your teacher.

0782 Number Pattern Proof

1. Yes, the equations are true, both sides of the equation are: $\frac{1}{2}$, $\frac{4}{3}$ or $1\frac{1}{3}$, $\frac{9}{4}$ or $2\frac{1}{4}$

2. The next three are all true:

$4 \times \frac{4}{5} = 4 - \frac{4}{5}$ $\frac{16}{5}$ or $3\frac{1}{5}$

$5 \times \frac{5}{6} = 5 - \frac{5}{6}$ $\frac{25}{6}$ or $4\frac{1}{6}$

$6 \times \frac{6}{7} = 6 - \frac{6}{7}$ $\frac{36}{7}$ or $5\frac{1}{7}$

3. Yes, the 20th equation is: $20 \times \frac{20}{21} = 20 - \frac{20}{21}$ $\frac{400}{21}$ or $19\frac{1}{21}$

continued/

4. $n \times \frac{n}{n+1} = n - \frac{n}{n+1}$

$$\frac{n^2}{n+1} = \frac{n(n+1)-n}{n+1}$$

$$= \frac{n^2+n-n}{n+1}$$

$$= \frac{n^2}{n+1}$$

5. a) You have already verified that the equation is true when n = 4, 5, 6, and 20.

If n = 200, $n \times \frac{n}{n+1} = \frac{400}{201}$ and $n - \frac{n}{n+1} = \frac{400}{201}$

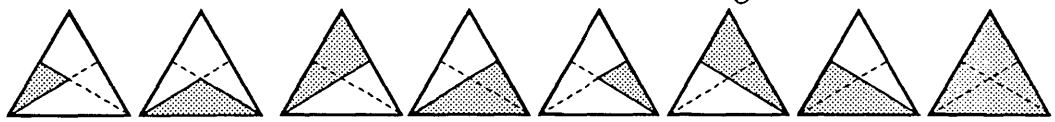
If n = 0, $n \times \frac{n}{n+1} = 0$ and $n - \frac{n}{n+1} = 0$

b) If $n = \frac{1}{2}$, $n \times \frac{n}{n+1} = \frac{1}{6}$ and $n - \frac{n}{n+1} = \frac{1}{6}$

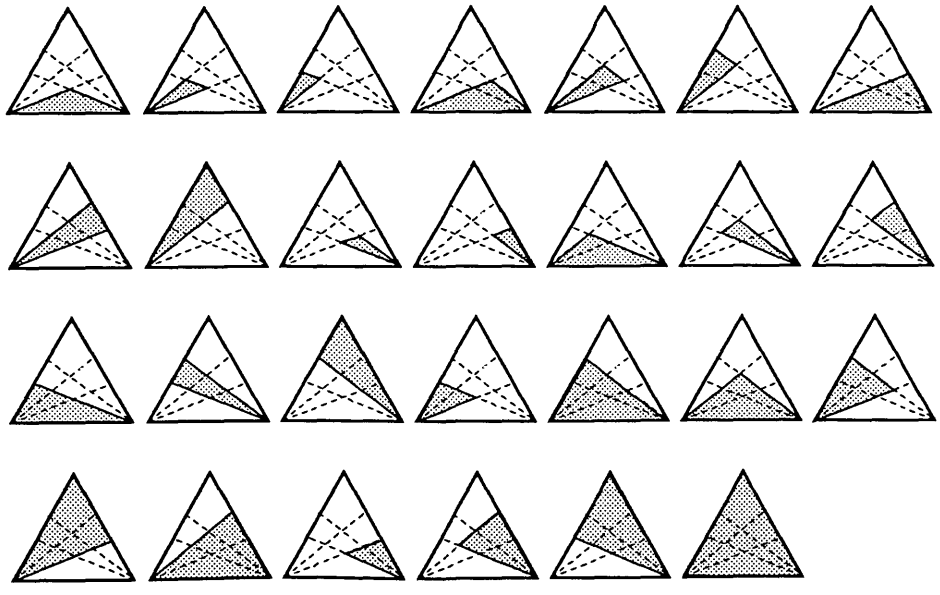
c) If n = -3, $n \times \frac{n}{n+1} = -4.5$ and $n - \frac{n}{n+1} = -4.5$

When n = -1 the equations are not true.
Can you see why?

With 1 line drawn from two vertices there are 8 triangles.



With 2 lines drawn from two vertices there are 27 triangles.



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0783 Cubes from Triangles (cont)

- The number of triangles are cube numbers: $1 \times 1 \times 1 = 1$
  $2 \times 2 \times 2 = 8$
  $3 \times 3 \times 3 = 27$
  This suggests the next diagram will have $4 \times 4 \times 4 = 64$ triangles

  If you counted the number of triangles methodically you should have found that there were 64 triangles.

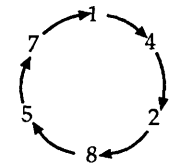
Number of lines drawn from two vertices	Number of triangles
0	$1 = 1^3$
1	$8 = 2^3$
2	$27 = 3^3$
3	$64 = 4^3$
n	$= (n + 1)^3$

0784 142 857 Times Tables

1.

x	142857
1	142857
2	285714
3	428571
4	571428
5	714285
6	857142
7	999999

2. For the first 6 rows the digits (1, 4, 2, 8, 5, 7) repeat in cycles. In the 7th row the pattern breaks down.



3.

8	1142856
9	1285713

4. In the 8th row one is added to the digits from the 1st row and the final digit reduces by one. The same is true of the connection between the 2nd and the 9th row. From here you can fill in the next five rows.

10	1428570
11	1571427
12	1714284
13	1857141
14	1999998

5. The best way to check will be to extend your spreadsheet.

continued/

0784 142 857 Times Tables (cont)

6. $\frac{1}{7} = 0.\dot{1}42857\dot{}$

$\frac{2}{7} = 0.\dot{2}85714\dot{}$

$\frac{3}{7} = 0.\dot{4}28571\dot{}$

The digits are the same as in the 142857 times tables. The 'sevenths' family is unique. Any number of sevenths, (except multiples of 7) when written in decimal form, always have the same recurring pattern.
The pattern is a six-figure cycle of 142857.

Using the pattern in the 142857 times tables, $\frac{7}{7} = 0.9999999$. Can you explain why?

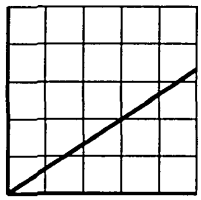
0788 Free Hand Angles

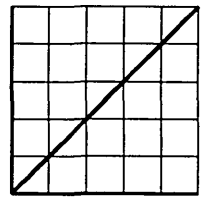
1. $\angle b = 22\frac{1}{2}°$
$\angle c = 45°$
$\angle d = 67\frac{1}{2}°$
$\angle e = 90°$

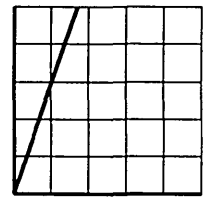
2. If your angles were very inaccurate, get your teacher to check your work.

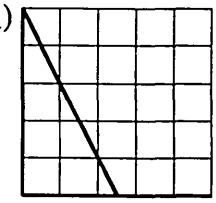
0789 Gradient

1. Yes, a straight line always has the same gradient no matter where you measure it.

2. a)  Gradient = $\frac{2}{3}$

b)  Gradient = 1

c)  Gradient = 3

d)  Gradient = -2

3. The gradient is the same as the coefficient of x
e.g if $x \rightarrow 5x + 3$ the gradient is 5

0790 A London Panorama

2. Marble Arch 283°
Houses of Parliament 170°
St Pauls Cathedral 068°

continued/

0790 A London Panorama (cont)

3. If your bearings were significantly different, check your work with your teacher.

4. Ask someone else to check your bearings.

5. Generally these panoramas give accurate directions and some idea of what type of building to look for. They do not give distances (as a rule) and they don't allow for obstacles like rivers or buildings.

0791 A Millionaire

These answers can only be a guide because the rate of exchange varies frequently, as you will see from the two examples given on the activity.

- On Friday April 1st, The Guardian shows 4.11Dm for German currency.
- A month later on Tuesday May 3rd, The Daily Mail shows a drop to 4.00Dm.

These answers are based on the rate of exchange of Friday 17th May 1996

1. USA £1 = $1.4775
   No. A dollar is worth $\frac{1}{1.4775}$ or £0.68.
   1000000US$ = £680000
   1477500US$ = £1000000

2. Italy £1 = 2306L
   A million lire are worth £433.65.

3. France £1 = 7.59F
   A million francs are worth £131752.31.
   A French millionaire would need another £868247.69, which is about another 7 million francs.

4. Germany £1 = 2.25Dm
   3 million marks will be £1333333.33.

0792 Wage Bargaining

		Managing Director	Foreman	Skilled Worker	Unskilled Worker
	Original Wage	6000	4000	3000	2000
1.	With 10% added	6600	4400	3300	2200
2.	Shared £2100 equally	6300	4300	3300	2300

continued/

0792 Wage Bargaining (cont)

3. a) The managing director would get more by voting for the 10% increase.
   b) The skilled worker would get the same either way.

4. If the skilled workers supported the unskilled workers (by voting with them or by abstaining) the equal share would be most popular.

0793 Approximation & π

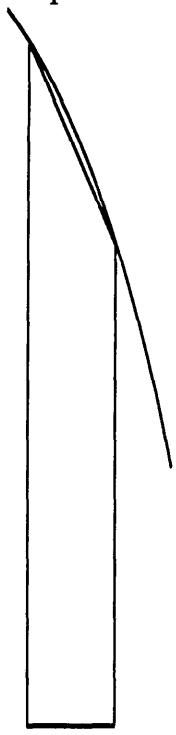
1. Area = AB x BC

2.- 4. You should find that as you increase the number of sectors they get closer and closer to forming a rectangle.

5. As the sectors get closer to forming a rectangle, AB gets closer to being the same as half the circumference.

The circumference	=	$2\pi r$
So AB	=	πr (approx)
BC	=	r because it is formed by a straight side of a circle sector. This is a radius.

6. Area of trapezium = $\frac{1}{2}$(Sum of parallel lengths) x (distance between)



Area of 1st strip	=	$\frac{1}{2}(20) \times 1 = 10$
2nd strip	=	$\frac{1}{2}(19.8) \times 1 = 9.9$
3rd strip	=	$\frac{1}{2}(19.3) \times 1 = 9.65$
.		. .
.		. .
.		. .
10th strip	=	$\frac{1}{2}(4.4) \times 1 = 2.2$
		$78.5cm^2$
Area of the circle	=	$4 \times 78.5cm^2$
	=	$314cm^2$

Area of circle		=	πr^2
So	314	=	$\pi 10^2$
So	$\frac{314}{100}$	=	π
	3.14	=	π

7. You should find π will have the same value whatever the size of the circle.

0794 The Trapezium

2. a = 1 unit b = 4 units h = 2 units

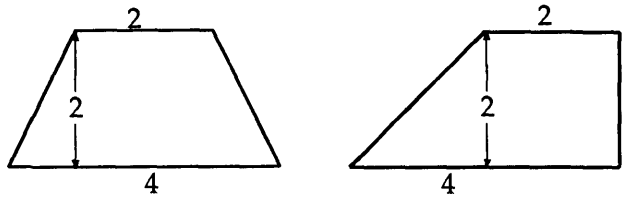
3. 5 square units

4.&5. If you cannot see a pattern in your table, show your results to your teacher.

6. To find the area of each trapezium:
   - add the lengths of the two parallel sides (a + b)
   - multiply by the distance between them (h)
   - halve.

7. a) 6 square units
   b) 8 square units
   c) $10\frac{1}{2}$ square units

8. Get someone to check your trapezia. Here are two possible answers for a).



9. If a = 0 the trapezium becomes a triangle.
   $A = \frac{(0 + b)h}{2} = \frac{bh}{2}$ = area of a triangle.

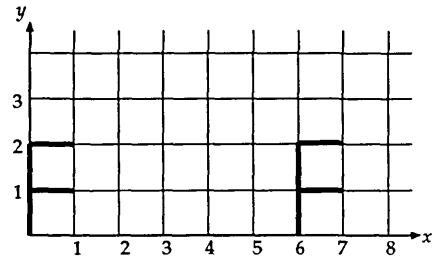
10. a) If b = 0 the shape becomes a triangle. Area = $\frac{ah}{2}$
    b) If h = 0 the shape becomes a straight line. Area = 0
    c) If a = b the shape becomes a parallelogram. Area = bh
    d) If a = b = h the shape becomes either a square or a parallelogram. Area = h^2

0795 The Transformation Game

Here is one possible start to the game.

Translate $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$

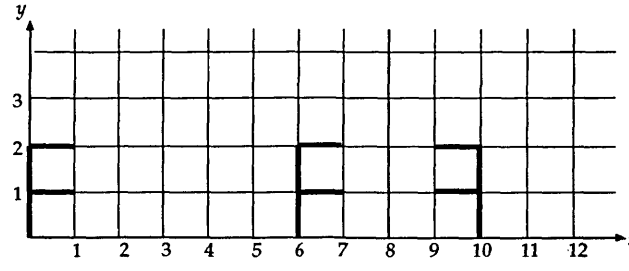
Moves the F at (0, 0) to (6, 0).



then

Reflect in $x = 8$

Translate the F at (6, 0) to this position.



0796 Darts Probability

1. Radius of the board = 30cm
   Area of the board = πr^2
   = $\pi 30^2$
   = $2827cm^2$

   Area of double ring = $\pi R^2 - \pi r^2$ where R = 24 and r = 23
   = $\pi(R + r)(R - r)$
   = $\pi \times 47 \times 1$
   = $148cm^2$

   Probability that John scores a double with his first throw = $\frac{148}{2827}$ = 0.05

2. Area of 50 circle = πr^2 where r is 1cm
   Probability of scoring a 50 = πcm^2
   = $\frac{\pi}{2827}$ = 0.001

3. Area of non-scoring area = $\pi R^2 - \pi r^2$ where R = 30 and r = 24
   = $\pi(R + r)(R - r)$
   $\pi \times 54 \times 6$
   = $1018cm^2$

   Probability of missing the scoring area = $\frac{1018}{2827}$ = 0.36

   John would be expected to miss the scoring area at least once in every three throws.

4. Score 60 $\Rightarrow$ treble 20
   Area of treble ring = $\pi R^2 - \pi r^2$
   = $\pi(R + r)(R - r)$
   = $\pi \times 25 \times 1$
   = $78.5cm^2$
   Area of treble 20 = $\frac{78.5cm^2}{20}$
   = $3.9cm^2$

   Probability of scoring 60 with one dart = $\frac{3.9}{2827}$ = 0.001

5. a) John can beat a 45 score in 6 different ways:
   - 50
   - treble 20
   - treble 19
   - treble 18
   - treble 17
   - treble 16.

continued/

5. b)

Area of the 50 circle	=	πcm^2		
Area of the 5 treble areas giving a score greater than 45	=	5 x 3.9	=	$19.6cm^2$
Total area where John can beat a 45 score	=	$\pi + 19.6cm^2$		
	=	$22.8cm^2$		
Probability that John's team starts first	=	$\frac{22.8}{2827}$	=	0.008

0797 Matrices and Transformations

1.

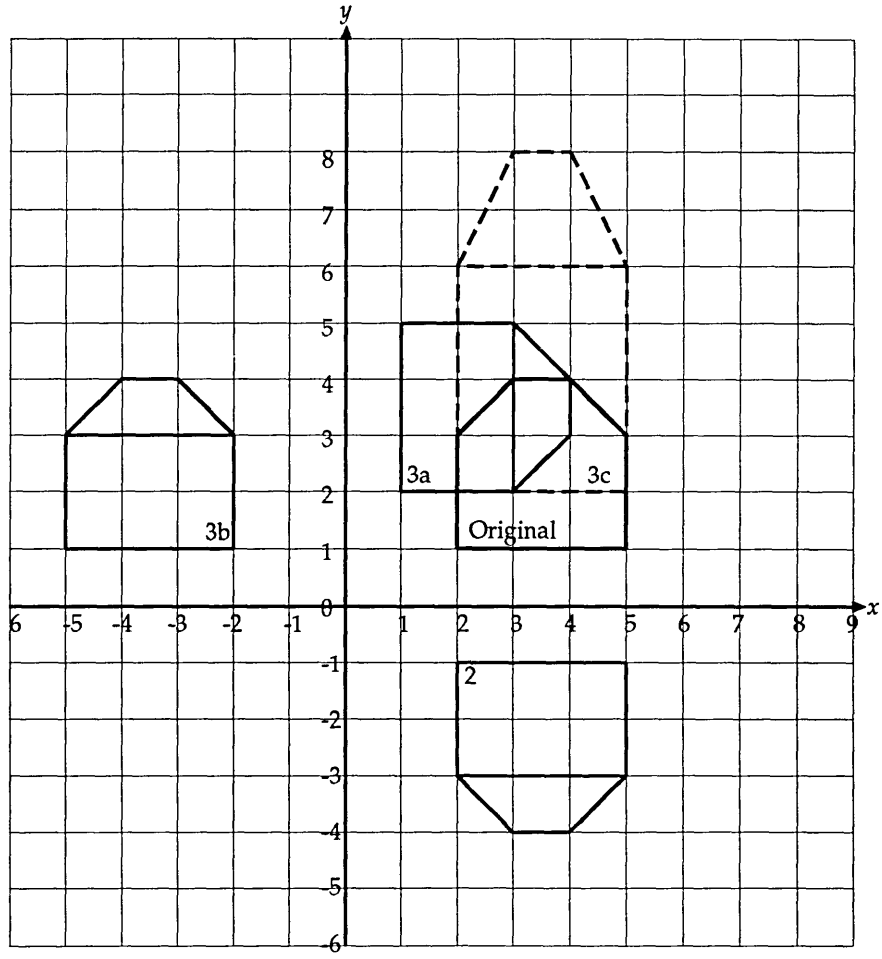
(2, 3)	→	(2, -3)	(5, 3)	→	(5, -3)
(3, 4)	→	(3, -4)	(5, 1)	→	(5, -1)
(4, 4)	→	(4, -4)	(2, 1)	→	(2, -1)

2. Reflection in the x - axis

3. a) (2, 3) →(3, 2)
(3, 4) →(4, 3)
(4, 4) →(4, 4)
(5, 3) →(5, 3)
(5, 1) →(1, 5)
(2, 1) →(1, 2)
Reflection in the line $y = x$

b) → (-2, 3)
→ (-3, 4)
→ (-4, 4)
→ (-5, 3)
→ (-5, 1)
→ (-2, 1)
Reflection in the y - axis

c) → (2, 6)
→ (3, 8)
→ (4, 8)
→ (5, 6)
→ (5, 2)
→ (2, 2)



Enlargement of the height of the house, scale factor +2

continued/

4. $(2, 3) \rightarrow (2, -3) \rightarrow (4, -3)$
$(3, 4) \rightarrow (3, -4) \rightarrow (6, -4)$
$(4, 4) \rightarrow (4, -4) \rightarrow (8, -4)$
$(5, 3) \rightarrow (5, -3) \rightarrow (10, -3)$
$(5, 1) \rightarrow (5, -1) \rightarrow (10, -1)$
$(2, 1) \rightarrow (2, -1) \rightarrow (4, -1)$
Reflection in y - axis and enlargement of width by scale factor +2.

5. $\rightarrow (-2, 3) \rightarrow (3, -2)$
$\rightarrow (-3, 4) \rightarrow (4, -3)$
$\rightarrow (-4, 4) \rightarrow (4, -4)$
$\rightarrow (-5, 3) \rightarrow (3, -5)$
$\rightarrow (-5, 1) \rightarrow (1, -5)$
$\rightarrow (-2, 1) \rightarrow (1, -2)$
Rotation of 90° clockwise.

6. "Change the sign of the x co-ordinate" and
"double the y co-ordinate".

7. a) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

b) You should have found that $\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

8. $\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

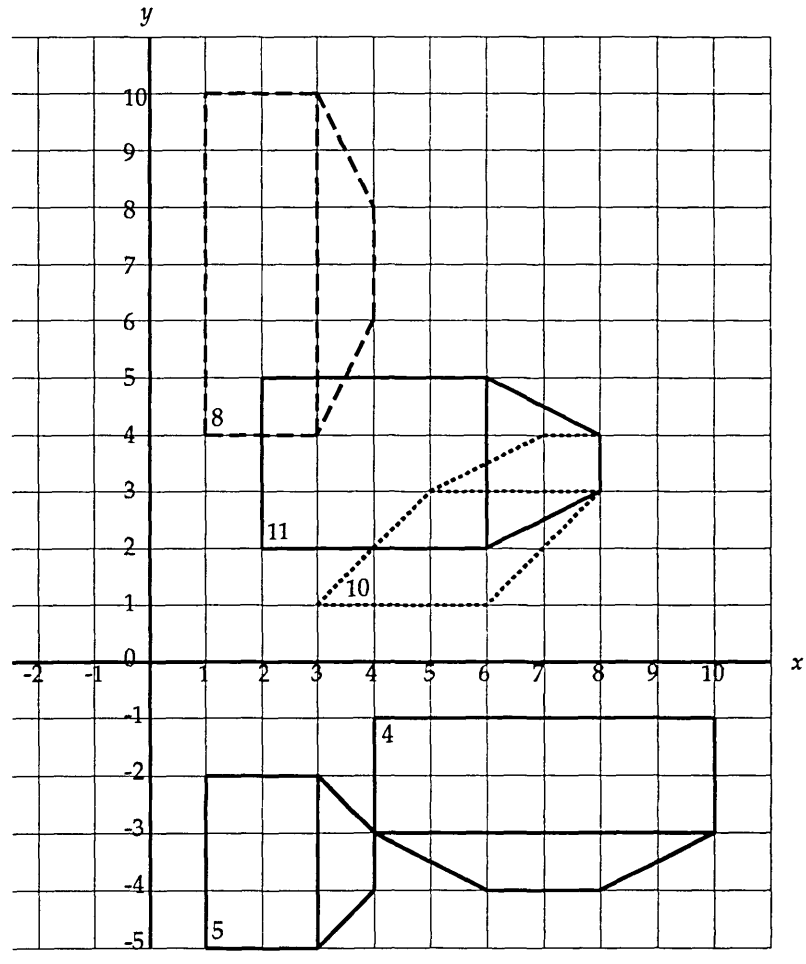
$\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 10 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

Enlargement of width scale factor 2 and reflection in $y = x$.



continued/

9. a) $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ b) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

10. a)
$(2, 3) \rightarrow (5, 3)$
$(3, 4) \rightarrow (7, 4)$
$(4, 4) \rightarrow (8, 4)$
$(5, 3) \rightarrow (8, 3)$
$(5, 1) \rightarrow (6, 1)$
$(2, 1) \rightarrow (3, 1)$

11. a)
$(2, 3) \rightarrow (6, 2)$
$(3, 4) \rightarrow (8, 3)$
$(4, 4) \rightarrow (8, 4)$
$(5, 3) \rightarrow (6, 5)$
$(5, 1) \rightarrow (2, 5)$
$(2, 1) \rightarrow (2, 2)$

b) $\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$

---

1. - 2. What you see will depend on your classroom.

3. 4 corners assuming your classroom is rectangular.

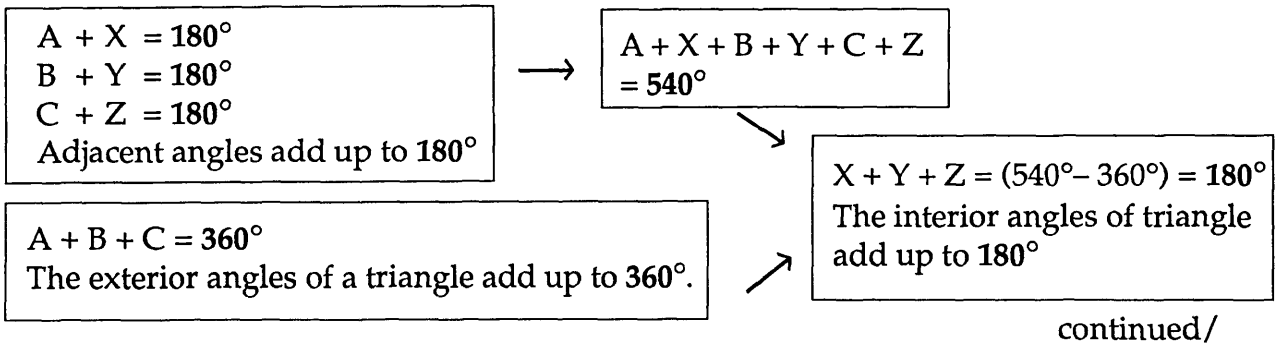
4. No

5. The four exterior angles of a quadrilateral add up to 360 degrees.

6. The three exterior angles of a triangle add up to 360 degrees.

7. The exterior angles of *any* polygon always add up to 360 degrees.

8. No



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0800 Polygons' Interior Angles (cont)

Cutting and sticking the 3 corners makes an approximate straight line.
Measuring the 3 angles gives X + Y + Z = 180° (approx.)

These are both approximate answers for the triangles concerned but the first method is more general.

1. a) A + B + C + D = **360°**

   b)
   $$\begin{array}{rcl} A + W &=& 180° \\ B + X &=& 180° \\ C + Y &=& 180° \\ D + Z &=& 180° \\ \hline A + B + C + D + W + X + Y + Z &=& 720° \end{array}$$

   c) So W + X + Y + Z = **360°**

   d) The interior angles of a quadrilateral total **360°**

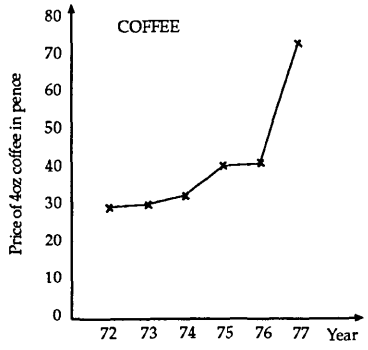
2. a) 540°
   b) 1080°

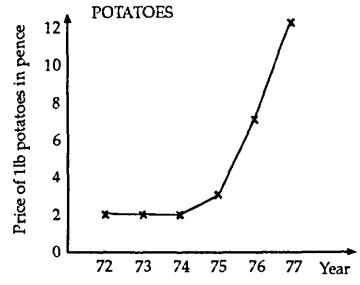
3.

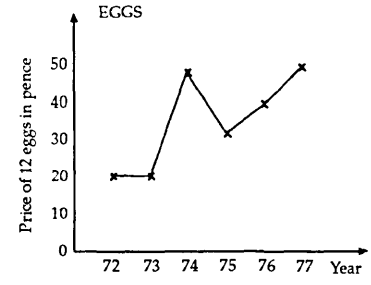
Polygon	Number of Sides	Sum of interior angles (in degrees)
Triangle	3	(3 x 180) – 360 = 180
Quadrilateral	4	(4 x 180) – 360 = 360
Pentagon	5	(5 x 180) – 360 = 540
Hexagon	6	(6 x 180) – 360 = 720
Octagon	8	(8 x 180) – 360 = 1080
Decagon	10	(10 x 180) – 360 = 1440
-	22	(22 x 180) – 360 = 3600
-	n	(n x 180) – 360 = 180n – 360

0804 Inflation

1. Your graphs may look slightly different to these, depending upon what scale you used, they should show the same rises and falls.
   It is always best to choose a scale appropriate to the range in prices.







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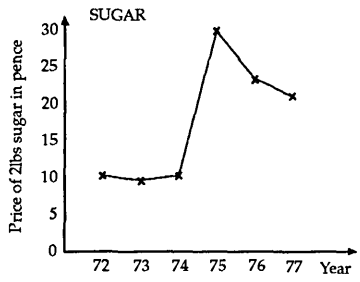
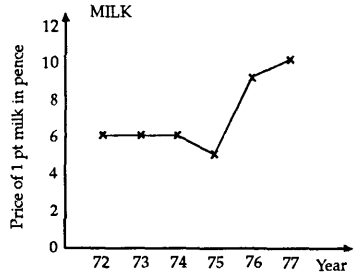
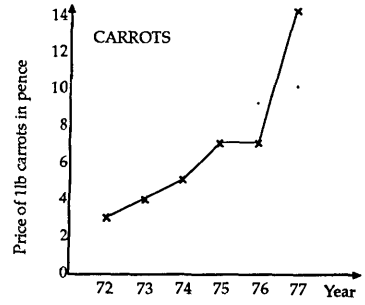
1.







2. a) carrots and sugar
   b) sugar
   milk and sugar
   Prices depend on other factors as well as inflation, for instance availability.

3. a) Although the prices of sausages and sugar increased by different amounts both increases represent just over 100% from 1972 to 1977.
   b)

4oz coffee	43p
1lb potatoes	10p
12 eggs	28p
1pt milk	4p
1lb carrots	11p

4. Coffee increased by the greatest amount between 1972 and 1977. (43p)
   Milk increased by the least amount. (4p)

5.

Food	1972 price	1977 price	price increase	price increase ÷ 1972 price	percentage increase
1lb sausages	21p	44p	23p	$\frac{23}{21} = 1.095$	1.095 x 100% = 109.5%
4oz coffee	29p	72p	43p	$\frac{43}{29} = 1.483$	1.483 x 100% = 148.3%
1lb potatoes	2p	12p	10p	$\frac{10}{2} = 5.000$	5.000 x 100% = 500.0%
12 eggs	20p	48p	28p	$\frac{28}{20} = 1.400$	1.400 x 100% = 140.0%
2lbs sugar	10p	21p	11p	$\frac{11}{10} = 1.100$	1.100 x 100% = 110.0%
1pt milk	6p	10p	4p	$\frac{4}{6} = 0.666$	0.666 x 100% = 66.6%
1lb carrots	3p	14p	11p	$\frac{11}{3} = 3.667$	3.667 x 100% = 366.7%

6. Potatoes have the greatest percentage price increase. (500%)
   Milk has the lowest percentage price increase. (66.6%)

7. The answers for question 4. are *actual* price increases.
   The answers for question 6. are *percentage* price increases.

8.

	a) 1972 prices	b) 1977 prices
1lb sausages	.21	.44
8oz coffee	.58	1.44
5lbs potatoes	.10	.60
6 eggs	.10	.24
3 pts milk	.18	.30
1lb carrots	.03	.14
Total	**£1.20**	**£3.16**

continued/

0804 Inflation (cont)

8. c) $\frac{1.96}{1.20} \times 100\% = 163.3\%$

   d) This does not give an accurate value of inflation as only a few items are considered.

9. Your graph is likely to be steeper if the rate of inflation is high, and less steep if the rate of inflation is low. However, this will also depend on the scale you chose to use for the vertical axis.

0805 An Average Pack of Workcards

B Average shoe size

Asma's size is probably the mode, the most common size.

4. The mean is 6.32. You can't get shoes in decimal sizes.

5. Mode = 7
   Median = 6

6. The mode is now 6.
   The median is now 7 because more large sizes have been added to the class.

C Average ability

Asma's geography mark was near the middle so the median is a good guide.
The median mark is 52. You may also have used the mean. The mean is 58% to the nearest whole number.

D Average mark

The mean is 53%. The median is 48%.

7. In C there are two modes (45 and 76) one at each end of the group. Neither of them is average for the group. In D, there is no mode as all the scores are different.

E Average breakfast cereal

Cornflakes is the mode.

8. It is impossible to work out the median and the mean if the data is not numerical.

F Average speed

1. The average speed going is 50mph.

2. The average speed returning is 33mph.

3. The average speed for the whole trip is 200 miles ÷ 5 hours = 40mph.

9. Average speed doesn't indicate very much in this type of problem. It is not necessarily very representative because the driver may have travelled at 70mph for most of the journey and stopped (speed = 0mph) for a long time.

continued/

G Average weekend

9. An 'average weekend' involves no calculation. You use your judgement about what you do most often, but no two weekends are ever *exactly* the same.

H Average weight

There are 25 people in Asma's class, but you can't tell exactly what they weigh. For instance, the lowest weight is between 38kg and 39kg to the nearest kg, but that is as accurate as you can be.

10. The modal group is 54-55kg. A good estimate for the mode is 54.5kg because that is the middle of the modal group.

    The median is in the 52-53kg group.

    The mean is impossible to find unless you decide to use the mid-point of each group. Then there's 1 at 38.5
    1 at 42.5
    1 at 44.5
    1 at 48.5
    5 at 50.5
    4 at 52.5 and so on . . .
    The total of these weights is 1318.5kg and so the mean is 52.74kg

I Average wage

The true average of all the wages (the mean) is £5000.

11. The mean average suits the boss best because £5000 average is high compared with the wages that 25 of the workers get.

12. The workers prefer to use the mode £3500, because this is what the biggest group of workers (the unskilled) get.

13. The majority of workers don't like to include the Director's salary because it is so high compared with the others. Only one person gets such high pay and this gives bias to the calculation. Excluding the Director, the mean of all the other workers is £4300. The majority of the workers earn £4800 or £3500.
    The mean average of these 24 people is £4095.83.

    Perhaps you can see why managers and workers often argue about wages! Which average gives a true picture depends upon your point of view.

0806 Trapezium to Parallelogram

3. Area = 11cm^2

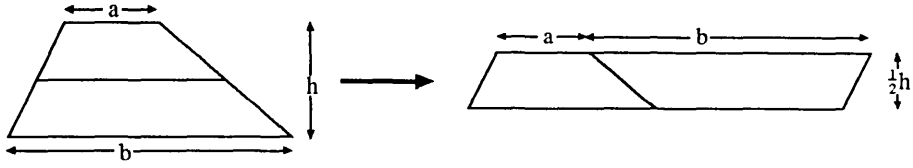
4. The two trapezia shown have areas:

    $^1/_2 \times 2 \times (2 + 4) = 6\text{cm}^2$ $^1/_2 \times 2 \times (2 + 5.8) = 7.8\text{cm}^2$

continued/

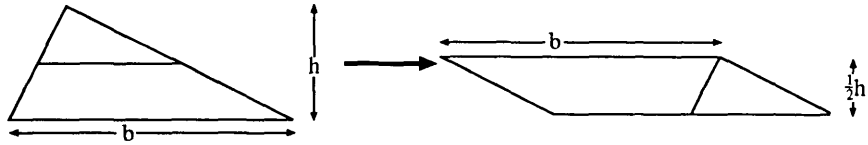
0806 Trapezium to Parallelogram (cont)

5.



Area of any trapezium = $^1/_2$ x h x (a + b)
because the area of the parallelogram = height x base

6.



Area of any triangle = $^1/_2$h x b

0808 Code Breaking

⌞•	𝍸 𝍸 //	12	T
⌋	𝍸 ///	8	H
□	𝍸 //	7	I or S
⌞•⌟	𝍸 //	7	I or S
⌃	𝍸 /	6	C
‹•	𝍸	5	
›	𝍸	5	
⌟	𝍸 𝍸 𝍸	15	E
⌄	𝍸 ////	9	A
•⌟	////	4	
⌞•⌟	//	2	
‹	𝍸	5	
⌄•	////	4	
⌞•⌟	/	1	
⌜	//	2	
⌜⌝	////	4	
⌞⌟	///	3	
⌃•	//	2	
⌜•⌟	//	2	

The secret message is: "This code is a direct substitution code which means that each letter of the alphabet has been replaced by a different symbol"

continued/

0808 Code Breaking (cont)

The second message is: "This is a bit more difficult"

The symbols were chosen by using the four parts of ✕ i.e. ＞ ∨ ＜ ∧

and the nine parts of ⧺ i.e. ⌋ ⌊ ⌊ ⌋ □ ⊏ ⌉ ⊓ ⌈

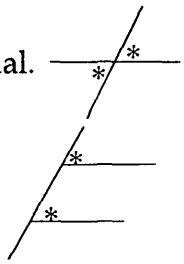
This makes 13 different symbols; by placing at dot inside each of these we have 13 more - one for each letter of the alphabet.

0809 Fold It

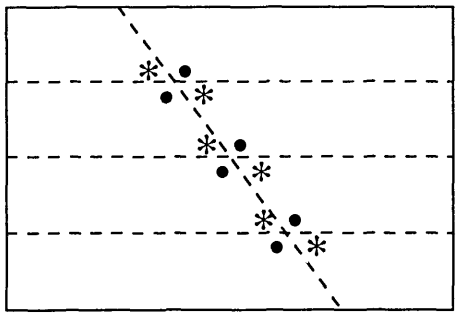
5. **a = e = i = l = h = d**
   b = c = f = g = j = k

6. Vertically opposite angles are equal.

   Corresponding angles are equal.



8. This shows the angles you would expect to be equal.



9. **a + b** = 180°
   b + d = 180°
   d + c = 180°
   c + a = 180°

a + b = 180° and
b + d = 180°
so **a = d**

a + b = 180° and
c + a = 180°
so **b = c**

Parallel lines are always the same distance apart, they never meet.

When a line intersects parallel lines the corresponding angles are equal.
a and **e** are corresponding angles, so are **b** and **f**.

0812 Irregular Areas

1. A good estimate would be 27 squares so that the area would be 270000m^2.

2. a) Measuring the length of the rectangles to the nearest mm gives
$(100 \times 600) + (100 \times 580) + (100 \times 530) + (100 \times 450) + (100 \times 340) + (100 \times 250) + (100 \times 160)$
$= 100 \times (600 + 580 + 530 + 450 + 340 + 250 + 160)$
$= 291000\text{m}^2$

   b) The answer will be inaccurate because each rectangle will include other parts of the land. e.g. Rectangle A will include parts of the River Severn.

3. $^1/_2 \times 100(600 + 580) + {}^1/_2 \times 100(580 + 530) + {}^1/_2 \times 100(530 + 450) +$
$^1/_2 \times 100(450 + 340) + {}^1/_2 \times 100(340 + 250) + {}^1/_2 \times 100(250 + 160) +$
$^1/_2 \times 100(160 + 120)$
$= {}^1/_2 \times 100(1180 + 1110 + 980 + 790 + 590 + 410 + 280)$
$= {}^1/_2 \times 100(5340)$
$= 267000\text{m}^2$

4. Low Field can be split into five trapezia and one triangle. Your trapezia may be slightly different to these but the final answer should be very close.
$^1/_2 \times 100(330 + 590) + {}^1/_2 \times 100(590 + 700) + {}^1/_2 \times 100(700 + 660) +$
$^1/_2 \times 100(660 + 540) + {}^1/_2 \times 100(540 + 330) + {}^1/_2 (330 \times 120)$
$= {}^1/_2 \times 100(920 + 1290 + 1360 + 1200 + 870) + {}^1/_2 \times 39600$
$= {}^1/_2 \times 100(5640) + 19800$
$= 282000 + 19800$
$= 301800\text{m}^2$

0813 Sectors of Circles

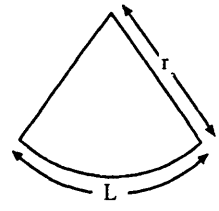
Area of sector $= \frac{x}{360}$ of area of circle

$\therefore A = \frac{x}{360} \times \pi r^2$

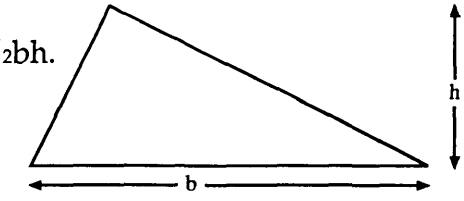
$\therefore A = \frac{L}{2\pi r} \times \pi r^2$ because $\frac{x}{360} = \frac{L}{2\pi r}$

$\therefore A = \frac{Lr}{2}$

1. Formula for area of sector = $^1/_2$Lr



which is like the formula for the area of triangle = $^1/_2$bh.



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0813 Sectors of Circles (cont)

2. There are four large sectors and four small sectors so the total area is:

$4(\frac{1}{2} \times 3.7 \times 3.5) + 4(\frac{1}{2} \times 1.05 \times 2)$

$= (2 \times 3.7 \times 3.5) + (2 \times 1.05 \times 2)$

$= (7 \times 3.7) + (4 \times 1.05)$

$= 25.9 + 4.2$

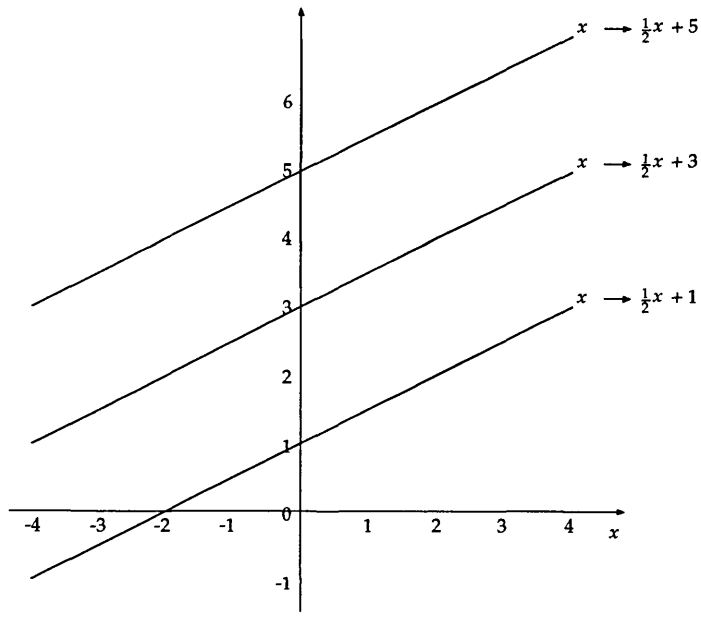
Area $= 30.1\ cm^2$

3. Show your design and your calculations to find the area of metal required to your teacher.

0817 Straight Line Graphs

1.

x	$\frac{1}{2}x + 1$
-3	$-\frac{1}{2}$
-2	0
-1	$\frac{1}{2}$
0	1
1	$1\frac{1}{2}$
2	2
3	$2\frac{1}{2}$



2.

x	$\frac{1}{2}x + 3$
-3	$1\frac{1}{2}$
-2	2
-1	$2\frac{1}{2}$
0	3
1	$3\frac{1}{2}$
2	4
3	$4\frac{1}{2}$

x	$\frac{1}{2}x + 5$
-3	$3\frac{1}{2}$
-2	4
-1	$4\frac{1}{2}$
0	5
1	$5\frac{1}{2}$
2	6
3	$6\frac{1}{2}$

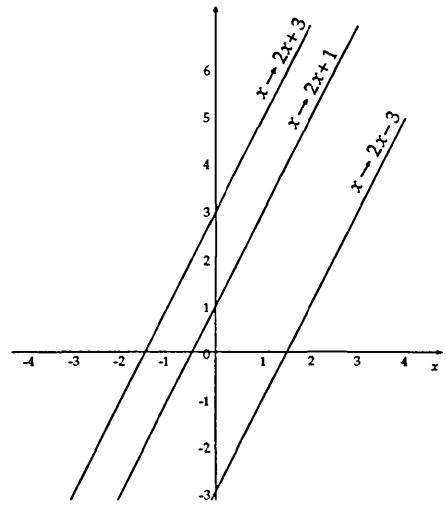
continued/

0817 Straight Line Graphs (cont)

3. The constant number in the mapping (i.e. **+3** in $x \rightarrow \frac{1}{2}x + 3$) gives the point of intersection on the y axis, the **intercept**.

4. The three lines are all parallel to one another.
Their gradients (steepness) are the same.
The x coefficient (i.e. $\frac{1}{2}$ in $x \rightarrow \frac{1}{2}x + 3$) gives the **gradient** of each line.

5. The three lines are parallel to one another, so their gradients are the same.
They have a different gradient from the lines on page 1.
The gradient is 2.
The constants give the intercepts.



6. These lines will not be parallel.
The gradients are all different.
The values of the gradients are 1, 2 and 3 respectively.
The lines will cross the vertical axis at the same point, they have the same intercept, +1.

The graph of $x \rightarrow \frac{5}{2}x + 1$ will also pass through +1 on the vertical axis.
The gradient will be $\frac{5}{2}$.

7. The gradient will be 5.
The line will pass through +3 on the vertical axis.

0818 Differences Between Squares

Page 2

$4^2 - 1 = (4 + 1)(4 - 1)$
$16 - 1 = 5 \times 3$
$15 = 15$
So $4^2 - 1$ and $(4 + 1)(4 - 1)$ are the same.

You should have found that:

$5^2 - 1$	=	$(5 + 1)(5 - 1)$	and	$6^2 - 1$	=	$(6 + 1)(6 - 1)$
$25 - 1$	=	6×4		$36 - 1$	=	7×5
24	=	24		35	=	35

continued/

Page 3

$a^2 - 1 = (a + 1)(a - 1)$
$10^2 - 1 = (10 + 1)(10 - 1)$
$100 - 1 = 11 \times 9$
$99 = 99$
So the identity works for a = 10.
Using the identity $21 \times 19 = (20 + 1)(20 - 1)$.
$= 20^2 - 1$
$= 400 - 1$
$= 399$

Page 4

$6^2 - 2^2 = (6 + 2)(6 - 2)$
$36 - 4 = 8 \times 4$
$32 = 32$
So $6^2 - 2^2$ and $(6 + 2)(6 - 2)$ are the same.
Whatever size you chose for a, you should have found $a^2 - 2^2 = (a + 2)(a - 2)$

Page 5

$a^2 - 2^2 = (a + 2)(a - 2)$
If $a = 2^1/_2$ $2^1/_2{}^2 - 2^2 = (2^1/_2 + 2)(2^1/_2 - 2)$
$6^1/_4 - 4 = 4^1/_2 \times {}^1/_2$
$2^1/_4 = 2^1/_4$
So the identity works for $a = 2^1/_2$.
Using the identity $32 \times 28 = (30 + 2)(30 - 2)$
$= 30^2 - 2^2$
$= 900 - 4$
$= 896$

Page 6

$5^2 - 3^2 = (5 + 3)(5 - 3)$
$25 - 9 = 8 \times 2$
$16 = 16$
So $5^2 - 3^2$ and $(5 + 3)(5 - 3)$ are the same.
Whatever size you chose for a, you should have found $a^2 - 3^2 = (a + 3)(a - 3)$

Page 7

The identity is $a^2 - 3^2 = (a + 3)(a - 3)$.
$a^2 - 3^2$ should be equal to $(a + 3)(a - 3)$ for any integer or fraction you tried.
If a = 3 $3^2 - 3^2 = (3 + 3)(3 - 3)$
$9 - 9 = 6 \times 0$
$0 = 0$
So the identity works for a = 3 because both sides reduce to zero.

Page 8

The identity $a^2 - b^2 = (a + b)(a - b)$ should always work.
It can be written as 'The difference between the squares of two numbers ($a^2 - b^2$) is equal to the sum of the two numbers (a + b) multiplied by the difference of the two numbers (a – b)'.

0819 Prove Your Identity

1. $1^2 = 0^2 + 1$
   $2^2 = 1^2 + 3$
   $3^2 = 2^2 + 5$
   $4^2 = 3^2 + 7$
   $10^2 = 9^2 + 19$
   $20^2 = 19^2 + 39$

   $n^2 = (n-1)^2 + (2n-1)$

   proof: $RHS = (n-1)^2 + (2n-1)$
   $= (n-1)(n-1) + (2n-1)$
   $= n(n-1) - 1(n-1) + 2n - 1$
   $= n^2 - 2n + 1 + 2n - 1$
   $= n^2$
   $= LHS$

2. $1^2 = 0^2 + 1$
   $2^2 = 1^2 + 2 + 1$
   $3^2 = 2^2 + 3 + 2$
   $4^2 = 3^2 + 4 + 3$
   $10^2 = 9^2 + 10 + 9$
   $20^2 = 19^2 + 20 + 19$

   $n^2 = (n-1)^2 + n + (n-1)$

   proof: $RHS = (n-1)^2 + n + (n-1)$
   $= (n-1)(n-1) + n + (n-1)$
   $= n^2 - 2n + 1 + n + n - 1$
   $= n^2$
   $= LHS$

3. $2^2 = 0^2 + 4 \times 1$
   $3^2 = 1^2 + 4 \times 2$
   $4^2 = 2^2 + 4 \times 3$
   $5^2 = 3^2 + 4 \times 4$
   $10^2 = 8^2 + 4 \times 9$
   $21^2 = 19^2 + 4 \times 20$

   $n^2 = (n-2)^2 + 4 \times (n-1)$

   proof: $RHS = (n-2)^2 + 4 \times (n-1)$
   $= (n-2)(n-2) + 4n - 4$
   $= n^2 - 4n + 4 + 4n - 4$
   $= n^2$
   $= LHS$

continued/

0819 Prove Your Identity (cont)

4. $2^2 = 0^2 + 2 \times 2 + 2 \times 0$
   $3^2 = 1^2 + 2 \times 3 + 2 \times 1$
   $4^2 = \mathbf{2^2} + 2 \times 4 + 2 \times 2$
   $\mathbf{5^2} = 3^2 + \mathbf{2 \times 5 + 2 \times 3}$
   $10^2 = \mathbf{8^2 + 2 \times 10 + 2 \times 8}$

   $n^2 = \mathbf{(n-2)^2 + (2n) + 2(n-2)}$

   proof: RHS $= (n-2)^2 + 2n + 2(n-2)$
   $= (n-2)(n-2) + 2n + 2n - 4$
   $= n^2 - 4n + 4 + 2n + 2n - 4$
   $= n^2$
   $=$ LHS

5. 6. and 7. If you managed to find another series, write the equation and prove the identity, show it to your teacher to check.

0820 Equations from Squares

Page 3

1.& 2.a) $3 \times 3 = (\mathbf{2 \times 4}) + 1$ b) $8 \times 8 = (\mathbf{7 \times 9}) + 1$ c) $6 \times 6 = (5 \times 7) + 1$
$9 = 8 + 1$ $64 = 63 + 1$ $36 = 35 + 1$

3. $20 \times 20 = (\mathbf{19 \times 21}) + 1$

4. $n \times n = [(n-1) \times (n+1)] + 1$

5. You should find the equation $n^2 = (n+1)(n-1) + 1$ is true for any positive integer, n.

6. a) $n = 1^1/_2$ LHS $= 1^1/_2{}^2$ RHS $= (1^1/_2 + 1)(1^1/_2 - 1) + 1$
$= 2^1/_4$ $= 2^1/_2 \times {}^1/_2 + 1$
$= 1^1/_4 + 1$
$= 2^1/_4$

b) $n = 0$ LHS $= 0^2$ RHS $= (0+1)(0-1) + 1$
$= 0$ $= 1 \times -1 + 1$
$= -1 + 1$
$= 0$

c) $n = 8.7$ LHS $= 8.7^2$ RHS $= (8.7 + 1)(8.7 - 1) + 1$
$= 75.69$ $= 9.7 \times 7.7 + 1$
$= 74.69 + 1$
$= 75.69$

d) $n = -4$ LHS $= -4^2$ RHS $= (-4 + 1)(-4 - 1) + 1$
$= 16$ $= (-3 \times -5) + 1$
$= 15 + 1$
$= 16$

In each case the LHS = RHS so the equation is true.

continued/

Page 5

1.& 2. a) $5^2 + 5 = \mathbf{5 \times 6}$ b) $3^2 + 3 = \mathbf{3 \times 4}$ c) $7^2 + 7 = 7 \times 8$

3. $20^2 + 20 = \mathbf{20 \times 21}$

4. $n^2 + n = n \times (n + 1)$

5. You should have found that equation $n^2 + n = n(n + 1)$ was true for all values of n that you tried. But is it true for any value?
It is not possible to be sure by just trying more and more values.
To answer the question you would need to prove it.

Page 7

1. $7^2 - 3^2 = \mathbf{10 \times 4}$

2. a) $5^2 - 3^2 = \mathbf{8 \times 2}$ b) $7^2 - 2^2 = \mathbf{9 \times 5}$
c) $6^2 - 2^2 = \mathbf{8 \times 4}$ d) $8^2 - 5^2 = \mathbf{13 \times 3}$

3. $20^2 - 3^2 = \mathbf{23 \times 17}$

4. $x^2 - y^2 = (x + y) \times (x - y)$

5. RHS $= (x + y)(x - y)$
$= x(x - y) + y(x - y)$
$= x^2 - xy + yx - y^2$
$= x^2 - y^2$
= LHS

6. a) $51^2 - 49^2 = (51 + 49)(51 - 49)$
$= 100 \times 2$
$= 200$

b) $77^2 - 67^2 = (77 + 67)(77 - 67)$
$= 144 \times 10$
$= 1440$

Page 8

$4^2 = \mathbf{1 + 3 + 5 + 7}$

1. i) $5^2 = \mathbf{1 + 3 + 5 + 7 + 9}$
ii) $10^2 = \mathbf{1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19}$

2. $20^2 = 1 + 3 + \ldots + \mathbf{39}$

3. $n^2 = 1 + 3 + \ldots + \mathbf{(2n - 1)}$

continued/

0820 Equations from Squares (cont)

4. i) n = 6 LHS = 6^2 = 36 RHS = 1 + 3 + 5 + 7 + 9 + 11 = 36

ii) n = 15 LHS = 15^2 = 225 RHS = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29 = 225

iii) n = 1/2 LHS = $^1/_2{}^2$ = $^1/_4$ RHS does not have a meaning for this value.

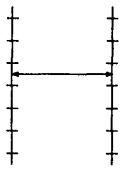
iv) n = -2 LHS = -2^2 = 4 RHS does not have a meaning for this value.

The equation is true for only positive integer values of x.

0822 Reducing to a Point

If you want to explore what happens with different mappings there is one important thing to remember:

Make sure the distance between the lines stays the same.



With many investigations it is a good idea to begin by making the problem as simple as possible.

In this investigation it is sensible to study groups of mappings such as;

$x \rightarrow 2x$	or	$x \rightarrow 2x$
$x \rightarrow 2x + 3$		$x \rightarrow 3x$
$x \rightarrow 2x - 5$		$x \rightarrow 4x$
$x \rightarrow 2x + 10$		$x \rightarrow 5x$
. . . etc.		. . . etc.

Recording the meeting points on one diagram will help to find patterns.

Then, other groups of mappings can be studied such as;

$x \rightarrow {}^1/_2x$	or	$x \rightarrow -2x$
$x \rightarrow {}^2/_3x$		$x \rightarrow -3x$
$x \rightarrow {}^3/_5x$		$x \rightarrow -4x$
. . . etc.		. . . etc.

What happens with $x \rightarrow x$?

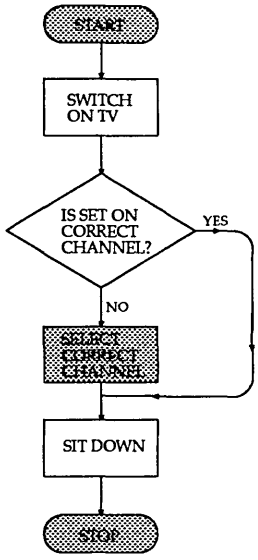
Once patterns for these groups of mappings have been found, it will help you to make predictions for other mappings such as;

$x \rightarrow 2x + 1$
$x \rightarrow 7x - 3$
$x \rightarrow {}^1/_2x + 10$
$x \rightarrow -3x + 4$
. . . etc.

. . . and generalise the meeting point for any mapping.

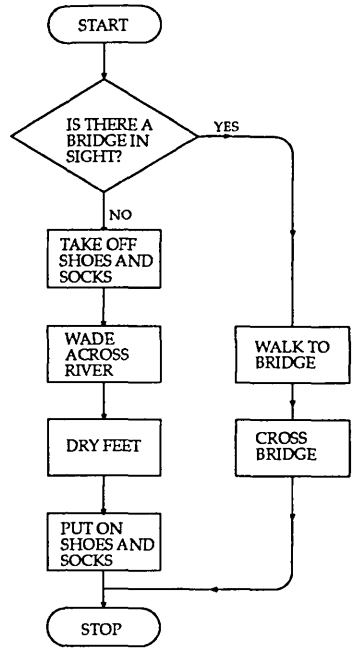
0823 Filling Flowcharts

1. Watching TV

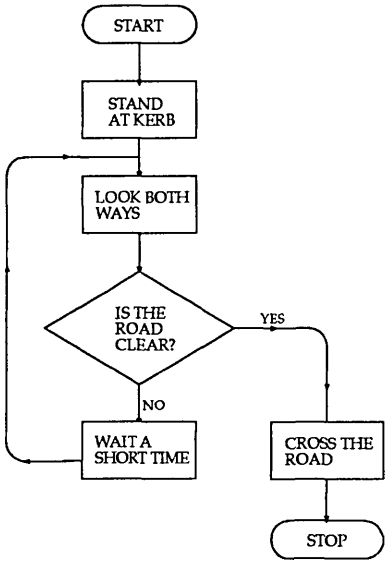


If you have a remote control the order you do this may be different. If so redraw it and show it to your teacher.

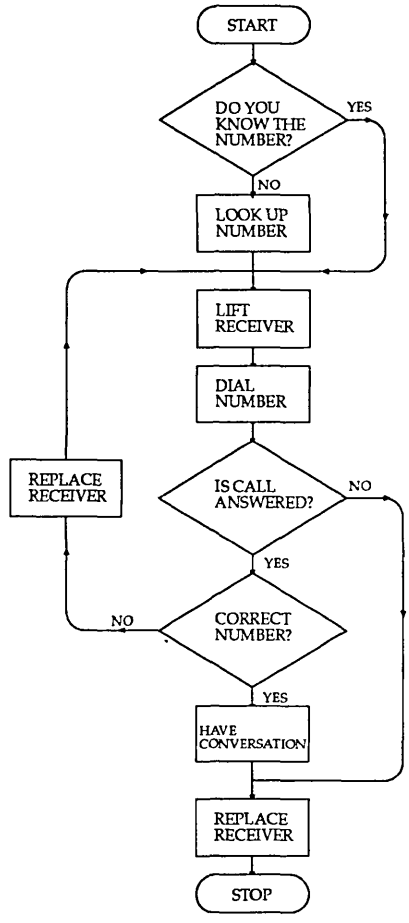
2. Crossing a river



3. Crossing the road



4. Phoning a friend



5. Ask your teacher to check your flowchart.

0824 The Golden Rectangle

The work with the Golden Rectangle is intricately involved with Fibonacci numbers.

This is explained by the following.

For a Golden Rectangle $\frac{b}{a} = \frac{a+b}{b}$ (see 0824a).

For a Fibonacci sequence any four consecutive numbers in the sequence can be described by a, b, a + b, a + 2b (see 0824e). The first two terms in this sequence are in the ratio $\frac{b}{a}$, the second and third terms are in the ratio $\frac{a+b}{b}$.

In a Fibonacci sequence, the first few ratios of pairs are not equal, but if you take later pairs, you will find they get closer and closer to 1.618 . . .

$\frac{3}{2} = 1.5$ $\qquad\qquad$ $\frac{34}{21} = 1.61905$

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . .

$\frac{5}{3} = 1.667$ $\qquad\qquad$ $\frac{89}{55} = 1.61818$

If, for example, accuracy is to just one decimal place $\frac{8}{5}$ and $\frac{13}{8}$ are both approximated to 1.6. For accuracy of two decimal places it is necessary to continue the sequence, until $\frac{21}{13}$ and $\frac{34}{21}$ which are both approximated to 1.62.

Agreement to four decimal places is not reached until fifteen terms in the sequence are calculated. To quickly generate the terms of the Fibonacci sequence and the ratios of terms a spreadsheet can be used.

Formula used here → = A2 + A3

	A	B	
1	Fibonacci sequence	Ratio of terms	
2	1		
3	1	1	← Formula used here = A3/A2
4	2	2	
5	3	1.5	
6	5	1.66666667	
7	8	1.6	
8	13	1.625	
9	21	1.61538462	
10	34	1.61904762	
11	55	1.61764706	
12	89	1.61818182	
13	144	1.61797753	
14	233	1.61805556	
15	377	1.61802575	← These terms are
16	610	1.61803714	← both 1.6180 correct to 4 decimal places.

continued/

If the sequence is extended far enough it is possible to find consecutive pairs of number for which $\frac{b}{a}=\frac{a+b}{b}$ to whatever accuracy required.

Notes on specific cards

0824a The Golden Rectangle

1. a = 16.2cm approx. b = 27cm approx. so $\frac{b}{a}$ =1.6

2. $\frac{b}{a}-\frac{a}{b}$ =1.66 – 0.6 = 1 approx.

3. If you have an 8 digit display calculator your value could be as accurate as 1.6180339.
   If you use a spreadsheet, accuracy to fifteen significant figures is possible

	A	B
4 1	102334155	1.61803398874989
4 2	165580141	1.61803398874990
4 3	267914296	1.61803398874990
4 4	433494437	1.61803398874990

4. Choose any value for a.
   Then b = golden ratio x a.

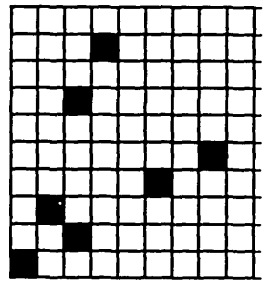
0824b Two Rectangle Surveys

We do not know of any research previously done on this, so the SMILE Centre will welcome any reports and conclusions.

0824c Cornering the Queen

You probably found that the Fibonacci numbers turned up in your co-ordinates of the 'safe squares' and 'winning squares'.

You could use the MicroSMILE program *Queens* to help you explore this problem.



0824d Is it Golden?

1.

	Rectangle A	Rectangle B
a)	$\frac{16.9}{9.3}$ = 1.817	$\frac{17.8}{11}$ = 1.618
b)	square has side 9.3cm	square has side 11cm
c)	$\frac{9.3}{7.6}$ = 1.224	$\frac{11}{6.8}$ = 1.618

continued/

2. In rectangle B the ratio remains the same.

3. As rectangle B is a golden rectangle, the ratio of the small rectangle will remain the same.
As rectangle A is not a golden rectangle, the ratio of the small rectangle will not remain the same.

0824e Fibonacci Sequence

1. $1 \times 1,\ 1 \times 1,\ 2 \times 2,\ 3 \times 3,\ 5 \times 5,\ 8 \times 8,\ 13 \times 13,\ 21 \times 21.$

2. You should find the ratio of the sides your final rectangle is approximately equal to 1.618.

3. Each term of the Fibonacci sequence is the sum of the two previous terms.

4. 21, 34, 55.

5. The ratios are given correct to 3 decimal places.
$\frac{1}{1} = 1,\ \frac{2}{1} = 2,\ \frac{3}{2} = 1.5,\ \frac{5}{3} = 1.667,\ \frac{8}{5} = 1.6,\ \frac{13}{8} = 1.625,\ \frac{21}{13} = 1.615,\ \frac{34}{21} = 1.619.$

6. The sides of the squares in question (1) are the numbers in the Fibonacci sequence.

7. The ratio of the sides of the final rectangle in question (2) is in the sequence of ratios of successive terms of the Fibonacci sequence. These are getting nearer to the golden ratio.

8. 1, 3, 4, 7, 11, 18, 29, 47, . . .

$\frac{3}{1} = 3,\ \frac{4}{3} = 1.333,\ \frac{7}{4} = 1.75,\ \frac{11}{7} = 1.571,\ \frac{18}{11} = 1.636,\ \frac{29}{18} = 1.611,\ \frac{47}{29} = 1.621 \ldots$

9. The ratio of successive terms of the rectangle approaches the golden ratio.

10. The ratios of successive terms of any rectangle formed from a Fibonacci-type sequence will eventually approach the golden ratio.

$\frac{b}{a} = \frac{c}{b}$

$c = a + b$

$\frac{b}{a} = \frac{a+b}{b}$

continued/

0824f A Dissection Problem

It is the angle that is wrong. In question 1, for example, the slope of the bottom is $\frac{5}{13}$ (= 0.385) and the slope of the top diagonal is $\frac{3}{8}$ (= 0.375). By the time you have drawn the diagram and cut it out you will not be able to detect such a small error. After all, in terms of the area there is only an error of 1 in 169 (0.6%).

0824g The Pentagram

2. $x + y = z$
   $2x = y$
   $3x = z$

3. $5x$

4. $180°$

5. $5x = 180°$
   $x = 36°$

6. $y = 72°$
   $z = 144°$

9. $a + b = c$ $a + 2b = d$
   $d - c = b$ $2c - d = a$

11. a = **3.5**cm
    b = **5.6**cm
    c = **9.0**cm
    d = **14.7**cm

12. $\frac{d}{c} = \frac{14.7}{9.0} = 1.633$
    $\frac{c}{b} = \frac{9.0}{5.5} = 1.636$
    $\frac{b}{a} = \frac{5.6}{3.5} = 1.6$

13. and 14. You probably guessed the answers had something to do with the golden ratio.
    The answers to the following questions help to explain why.

15. The three angles of each isosceles triangle are 36°, 72° and 72°.

16. The three triangles are similar because they have the same angles.

17. Because the triangles are similar,the ratio of the sides are equal.

continued/

0824 The Golden Rectangle (cont)

18. $\frac{c}{b}=\frac{b}{a}$

$c = a + b$

$\frac{a+b}{b}=\frac{b}{a}$

19. Rearranging the equation above gives $\frac{b}{a}-\frac{a}{b}=1$.

0827 Clover Leaf

1. a) A to E = $200 + {}^1/_4(2\pi .50) + {}^1/_4(2\pi.100) + {}^1/_4(2\pi.50) + 100$
   = 614m
   b) F to C = 100 + 50 + 175 + 50 + 175 + 50 + 150
   = 750m
   c) A to G = $200 + 50 + 175 + 50 + 75 + {}^3/_4(2\pi.75) + 75 + 50 + 175 + 50 + 75$
   = 1.328km
   d) B to E = FC – 75m
   = 675m
   e) D to C = AG – 25m
   = 1.303km

2. 1.957km { = $200 + 50 + 175 + 50 + 75 + {}^3/_4(2\pi.75) + 75 + 25$ all doubled!}

0830 Regrouping

4 triangles, **12** squares and **8** circles are needed for 4(T + 3S + 2C).

1. 4S + 8C = 4(**S** + **2C**)

2. 12S + 18C = 3(**4S** + **6C**)
   12S + 18C = 6(**2S** + **3C**)
   12S + 18C = 2(**6S** + **9C**)

3. **Two** because 8T + 12C = 2(4T + 6C)
   and 8T + 12C = 4(2T + 3C)

4. **Eight** because 32C + 8T + 24S = **8**(4C + T + 3S)

5. a) 4C + 12T = 4(C + 3T)
   b) 6S + 12T + 6C = 6(S + 2T + C)
   c) 7F + 14L = 7(F + 2L)
   d) 10x + 15y = 5(2x + 3y)
   e) 16a + 4b + 8c + 8d = 4(4a + b + 2c + 2d)

6. The maximum number of groups is the *highest common factor* of all the numbers.

0831 Primes and Proof

0831A Squares and Primes

A good way to tackle this without omitting any possibilities is to investigate the sums of pairs of squares. You will have soon realised that, with the exception of $1^2 + 1^2 = 2$, one of the squares must be odd and one must be even. *Why?*

Here are the pairs of odd and even squares whose sums are less than 100:

$1^2 + 2^2 = 5$	$3^2 + 2^2 = 13$	$5^2 + 2^2 = 29$	$7^2 + 2^2 = 53$	$9^2 + 2^2 = 85$
$1^2 + 4^2 = 17$	$3^2 + 4^2 = 25$	$5^2 + 4^2 = 41$	$7^2 + 4^2 = 65$	$9^2 + 4^2 = 97$
$1^2 + 6^2 = 37$	$3^2 + 6^2 = 45$	$5^2 + 6^2 = 61$	$7^2 + 6^2 = 85$	
$1^2 + 8^2 = 65$	$3^2 + 8^2 = 73$	$5^2 + 8^2 = 89$		

The *primes* which are the sum of 2 different squares are: 5, 13, 17, 29, 37, 41, 53, 61, 73, 89, 97.

Each of these primes is one more than a multiple of 4, i.e. of the form $x = 4n + 1$ when n is an integer. What values can n take for x to be prime?

If $a^2 + b^2$ is prime, then a^2 is odd and b^2 is even.

Firstly, a^2 is odd $\Rightarrow$ a is odd

$\Rightarrow$ $a = 2m + 1$ *what values can m take?*

so that $a^2 = (2m + 1)^2$

$= 4m^2 + 4m + 1$

Secondly, b^2 even $\Rightarrow$ b is even

$\Rightarrow$ $b^2 = 2k$ *what values can k take?*

so that $b^2 = 4k^2$

Therefore if $a^2 + b^2 = (4m^2 + 4m + 1) + 4k^2$

$= 4(k^2 + m^2 + m) + 1$

It is therefore a *necessary* condition that $a^2 + b^2 = 4n + 1$ ($n = 1, 2, 3 \ldots$) for $a^2 + b^2$ to be prime, but it is *not* a *sufficient* condition. *Why?*

0831B Proof about Primes

The flowchart always gives a whole number if you start with a prime number.

The flowchart performs the mapping $p \rightarrow \frac{p^2 - 1}{24}$ and so $\frac{p^2 - 1}{24}$ is a whole number ($p \neq 2, 3$).

Therefore $(p^2 - 1)$ must be a multiple of 24.

But $(p^2 - 1) = (p - 1)(p + 1)$ and so $(p - 1)(p + 1)$ must be a multiple of 24.

continued/

Proof: Firstly p is prime $\Rightarrow$ p is odd
$\Rightarrow$ (p – 1) and (p + 1) are both even numbers.

(p – 1) and (p + 1) are consecutive even numbers *Why?*

One number is a multiple of 4
+
the other number is a multiple of 2

Secondly, (p – 1), p, (p + 1) are three consecutive numbers so that one of them must be a multiple of 3.

p is not, so either (p – 1) or (p + 1) is a multiple of 3.

One number is a multiple of 4
+
the other number is a multiple of 2

↖ One of these is a multiple of 3
↙

These three conditions can be combined into two possibilities:

One number is a multiple of 12
+
the other number is a multiple of 2

or

One number is a multiple of 4
+
the other is a multiple of 6

The result of multiplying either pair will be a number which is a multiple of 24.

0831C Primes and Factorials

For n = 2, 3, 4, 5, 6, 7, 8, 9, 10 . . .

A = {2, 3, 5, 7. . .}
B = {4, 6, 8, 9, 10 . . .}
A is the set of prime numbers.
B contains only composite numbers.

A counter example to this is n = 1,

$$(n-1)! = 0! = 1, \qquad \frac{(n-1)!+1}{n} = 2$$

so 1 should go in list A, but 1 is not a prime number.

The flowchart performs the mapping $k \rightarrow \frac{(k-1)!+1}{k}$

For composite numbers $\frac{(n-1)!+1}{n}$ is not a whole number

and so (n – 1)! + 1 cannot be a multiple of n.

continued/

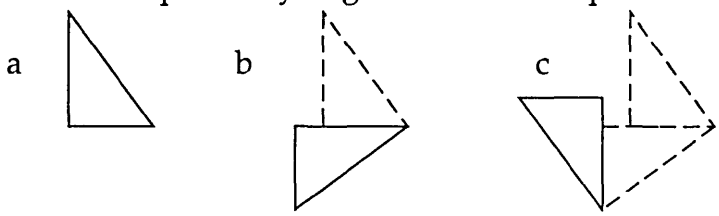
Proof: Consider the composite number n which is factorised into its factors (a x b x c x . . .).
a, b, c . . . are all less than n and will therefore also be factors in (n – 1)!
$\frac{(n-1)!}{n}$ is the same as $\frac{(n-1)!}{a,b,c...}$ and will therefore be a whole number.

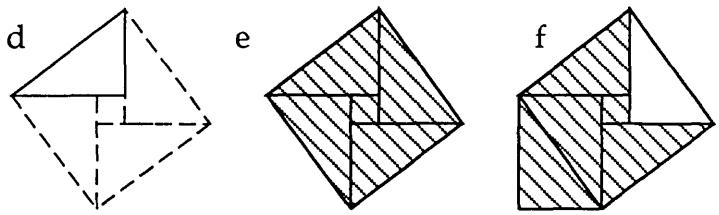
For example, 12 can be factorised into (6 x 2).
11! = 11 x 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1

$$\frac{11!}{12} = \frac{11!}{6x2} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times \cancel{6} \times 5 \times 4 \times 3 \times \cancel{2} \times 1}{\cancel{6} \times \cancel{2}} = 3\,326\,400$$

Since $\frac{(n-1)!}{n}$ is a whole number, $\frac{(n-1)!+1}{n}$ cannot be.

0831 D Euclid
Pythagoras' Theorem:
The most simple proof is given by Dr Jacob Bronowski in his book "The Ascent of Man". See p. 159. Pythagoras could have proved his theorem this way in about 500 BC.





a, b, c, d build up the squares on the hypotenuse (e)

f, g transform the pattern to an 'L'-shape

Bronowski shows (h) that the 'L'-shape is the same as two squares - what size are the squares?

You will find other geometric and algebraic proofs in some mathematics text-books.

Consecutive Composites:
21! + 2, 21! + 3, . . . and so on . . .up to 21! + 21 are respectively multiples of 2, 3 . . .21.
Therefore none of these twenty numbers is prime.

0832 Short Division

1. 42
2. 11
3. 21
4. 21
5. 10
6. 143
7. 210
8. 321
9. 101

0833 Short Division - Carrying

1. 24
2. 13
3. 14
4. 28
5. 311 rem 1
6. 203
7. 151
8. 36 rem 1
9. 121 rem 2
10. 26 rem 2
11. 72 rem 1

0834 Dividing Strips

1. 24cm ÷ 4 = 6cm
2. 16cm ÷ 8 = 2cm
3. **18 ÷ 3 = 6**
4. 5
5. 5
6. 4
7. $2\frac{1}{2}$
8. $2\frac{1}{2}$
9. 13
10. 4

0835 Sharing a Block

Problem 1
Each person could have 1 flat or 3 longs or 9 units.

Problem 2
Each person could have 1 flat, 1 long and 1 unit. There is 1 unit left over.

Problem 3

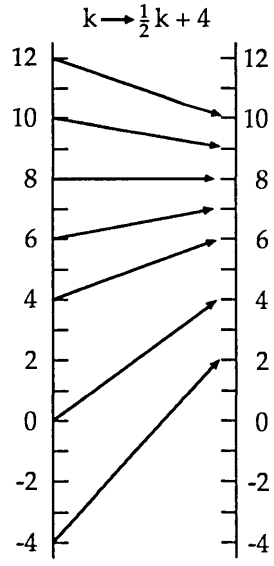
a) Each person could have 2 flats or 8 longs or 16 units each
b) Each person could have 1 flat, 1 long and 1 unit. There is 1 unit left over.

Problem 4

a) Each person could have 2 flats or 20 longs or 200 units.
b) Each person could have 2 flats and 5 longs.
c) Each person could have 1 flat, 6 longs and 6 units. There are 4 units left over.

0837 Inverse Mappings

1.



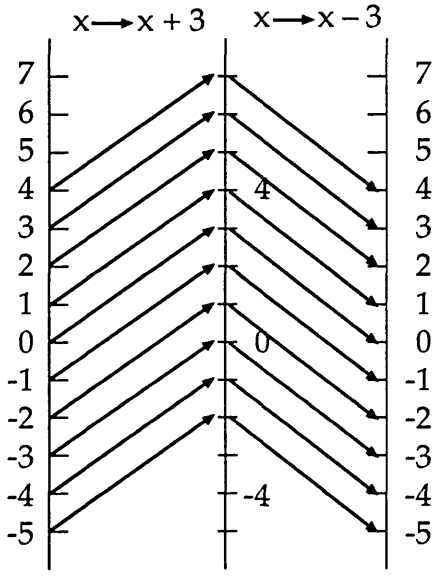
2. Show your mapping diagram to your teacher.

3. The central line is used to show numbers which come out of the first mapping machine and then go into the second mapping machine.

4. The number from the first mapping (2x + 1) is then put through the second mapping $x \rightarrow x - 4$, which can be written $(2x + 1) \rightarrow (2x + 1) - 4$

   So the combined mapping can be described by $x \rightarrow (2x + 1) - 4$
   or, more simply $x \rightarrow 2x - 3$.

5.



6. The single mapping is $x \rightarrow x$.
   Show your mapping diagram to your teacher.

7. The result of combining $x \rightarrow \frac{x+7}{2}$ and $x \rightarrow 2x - 7$ is $x \rightarrow x$.

8. a) and c) are inverse pairs.

continued/

0837 Inverse Mappings

9. The inverse of b) $x \to 7 + x$ is $x \to x - 7$

The inverse of d) $x \to \frac{x}{2} + 3$ is $x \to 2(x - 3)$.

10. $x \to \frac{x+7}{2}$

0838 Scale Factor

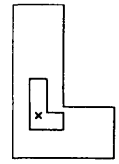
A Enlargement by scale factor 3 makes each side of the letter three times bigger.
Enlargement by scale factor 2 makes each side of the letter two times bigger.
Enlargement by scale factor $\frac{1}{2}$ makes each side of the letter half as big.

Enlargement by scale factors between 0 and 1 make shapes smaller, but these are still called enlargements.
These rules apply whatever the starting shape you choose.

B The position of the enlarged shape depends on the position of the centre of enlargement. Each of these shows an enlargement of the L shape by scale factor 3.

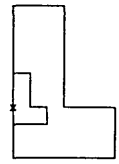
1. Centre of enlargement inside the shape,

   enlarged shape surrounds the starting shape.



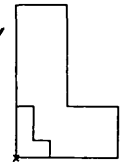
2. Centre of enlargement on the edge of the shape,

   enlarged shape shares that edge with the starting shape.



3. Centre of enlargement on the corner of the shape,

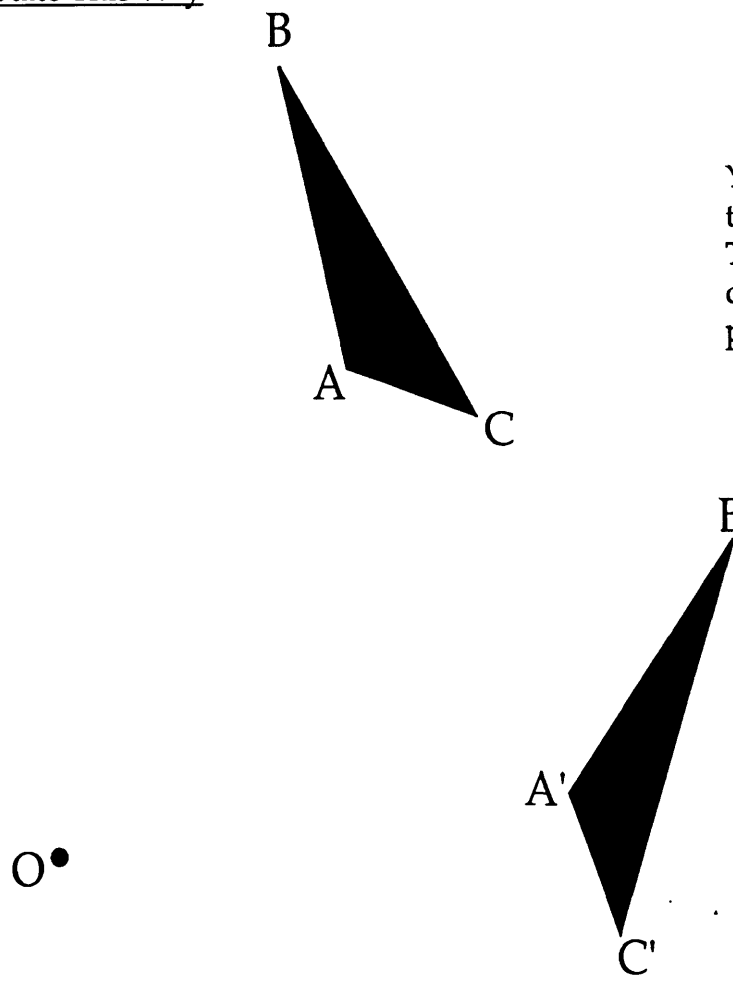
   enlarged shape shares two edges with the starting shape.



C The area of the shape will change according to the square of the scale factor.
Enlargement by scale factor 2, the area is increased 4 times;
Enlargement by scale factor 3, the area is increased 9 times;

Enlargement by scale factor $\frac{1}{4}$, the area is reduced to $\frac{1}{16}$ of the original.

0839 Rotate This Way

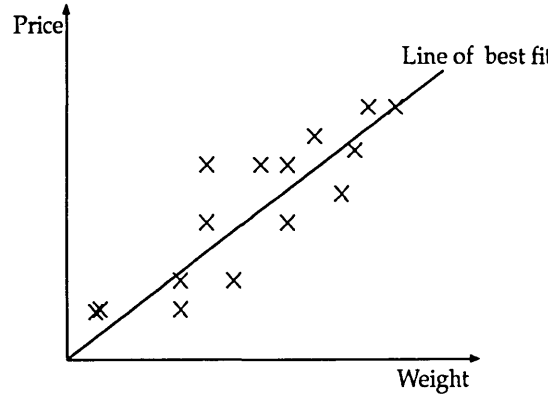


Your other diagrams will need to be measured for accuracy. The position of the image depends where you put the point O, the centre of rotation.

0840 Best Buy

Each of your scattergrams will show a *positive correllation,* this is because larger packages of any food cost more than smaller ones.

If you put weight on the horizontal axis and price on the vertical axis of your scattergram, any product that is lower than the line of best fit will represent good value for money.



If you entered your data into a database you will be able to quickly produce different types of graphs to represent your data. Write some comments on each graph to explain what they show. Which type of graph shows the most information and most clearly?

0843 Very Large Numbers

1. $1000 = 10 \times 10 \times 10 = 10^3$

2. $10000000 = 10^7$

3. $10^6 = 1000000 = 1$ million

4.

a)	300000	$= 3 \times 100000$	$= 3 \times 10^5$
b)	70000	$= 7 \times 10000$	$= \mathbf{7 \times 10^4}$
c)	80000000	$= 8 \times \mathbf{10000000}$	$= \mathbf{8 \times 10^7}$
d)	2000	$= 2 \times 1000$	$= 2 \times 10^3$
e)	90000000000	$= 9 \times 10000000000$	$= 9 \times 10^{10}$

5. 10^5 or 1×10^5.

6. Mercury is 6×10^7 km
Venus is 10^8 km
Mars is 2×10^8 km

7. At the time the Leapfrogs LINKS BOOK was produced, the figures were as follows:
a) The edge of our galaxy is 10^{20} miles away.
b) The edge of the known universe is 10^{24} miles away.
However, these distances are a matter of disagreement between astronomers. Estimates for the distance to the edge of the known universe vary between 12000000000 or 1.2×10^{10} light years and 20000000000 or 2×10^{10} light years.

8. $3.64 \times 10^1 = 36.4$
$3.64 \times 10^2 = 364$
$3.64 \times 10^3 = 3640$
$3.64 \times 10^4 = 36400$
$3.64 \times 10^5 = 364000$

9. The power tells you the number of places the decimal point moves to the right. (Mathematicians say that the decimal point stays still and it's the figures that move to the left! Why do they argue this?)

10.

256.3	$= 2.563 \times 100$	$= 2.563 \times 10^2$
137.6	$= 1.376 \times \mathbf{100}$	$= \mathbf{1.376 \times 10^2}$
9200	$= 9.2 \times \mathbf{1000}$	$= \mathbf{9.2 \times 10^3}$
72310	$= \mathbf{7.231 \times 10000}$	$= \mathbf{7.231 \times 10^4}$
22	$= \mathbf{2.2 \times 10}$	$= \mathbf{2.2 \times 10^1}$

11. The Moon is 3.82171×10^5km from the Earth.
The edge of the Solar System is 3.573×10^9 miles from the Earth.

continued/

0843 Very Large Numbers (cont)

12. and 13. Distances from the Sun (smallest first) in km:

Mercury	5.8×10^7
Venus	1.08×10^8
Earth	1.49×10^8
Saturn	1.43×10^9
Pluto	5.898×10^9
Alpha Centauri	4.035×10^{13}
Vega	2.53368×10^{14}
Alpha Cygni	1.53×10^{16}

14. Saturn is about 10 times further from the Sun than the Earth.

15. Alpha Centauri is about 10^5 times further from the Sun than the Earth - more accurately it is about 3×10^5 times further, roughly 300000 times the distance from the Sun!

16. a) 3.8×10^9
b) 1.9×10^6
The bigger power is always the bigger number (for positive numbers in standard form) - write out the numbers to check.

17. Show your answers to your teacher.

0844 Very Small Numbers

$0.0000006 = 6 \times 10^{-7}$

1. $0.0000001 = 10^{-7}$

2. a) 0.6 $= 6 \times 0.1$ $= 6 \times 10^{-1}$
b) 0.0006 $= 6 \times 0.0001$ $= \mathbf{6 \times 10^{-4}}$
c) 0.00062 $= 6.2 \times 0.0001$ $= \mathbf{6.2 \times 10^{-4}}$
d) 0.00129 $= 1.29 \times \mathbf{0.001}$ $= \mathbf{1.29 \times 10^{-3}}$
e) 0.0075 $= 7.5 \times 0.001$ $= 7.5 \times 10^{-3}$

3. The power of 10 indicates how many places the decimal point has moved. (Mathematicians say that the decimal point stays still and it is the figures that move to the left! Why do they argue this?)

4. Bacteria 2.5×10^{-5}cm diameter.
Blood cells 7.5×10^{-4}cm diameter

continued/

0844 Very Small Numbers (cont)

5. Smallest first:

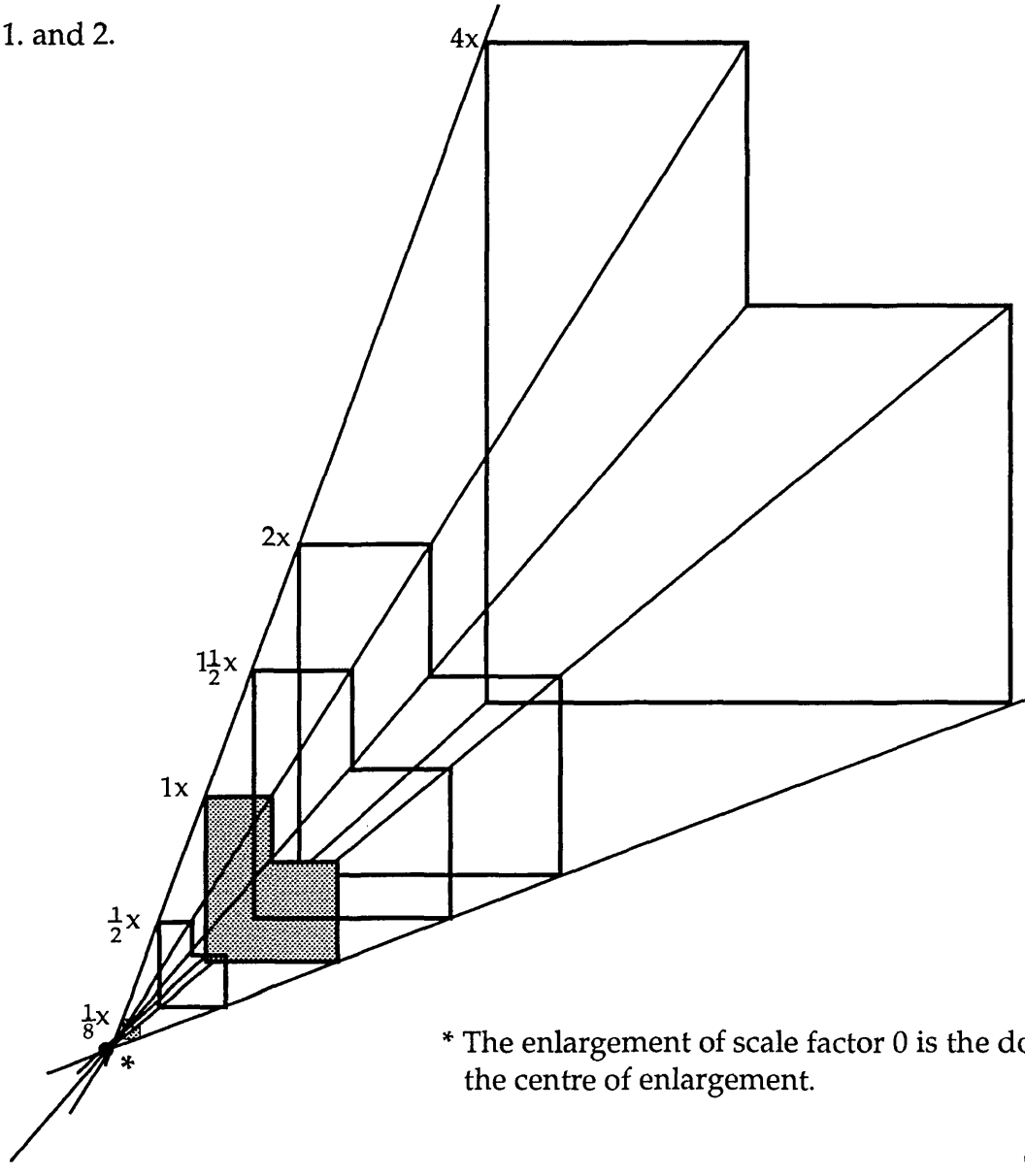
Hydrogen atom	2×10^{-7} mm
Pneumonia bacteria	10^{-6} mm
Molecule (egg white protein)	10^{-5} mm
Flu virus	5×10^{-5} mm
Mumps virus	2.25×10^{-4} mm
Pin prick	10^{-1} mm
Paramecium Protozoa	2×10^{-1} mm

6. a) 9.3×10^{9}
   b) 4.6×10^{-3}
   The number with the biggest power of 10 is always biggest.
   In a) 10^{9} is bigger than 10^{8}. In b) 10^{-3} is bigger than 10^{-4}.

7. Show your answers to your teacher.

0845 Negative Scale Factor

1. and 2.



* The enlargement of scale factor 0 is the dot, the centre of enlargement.

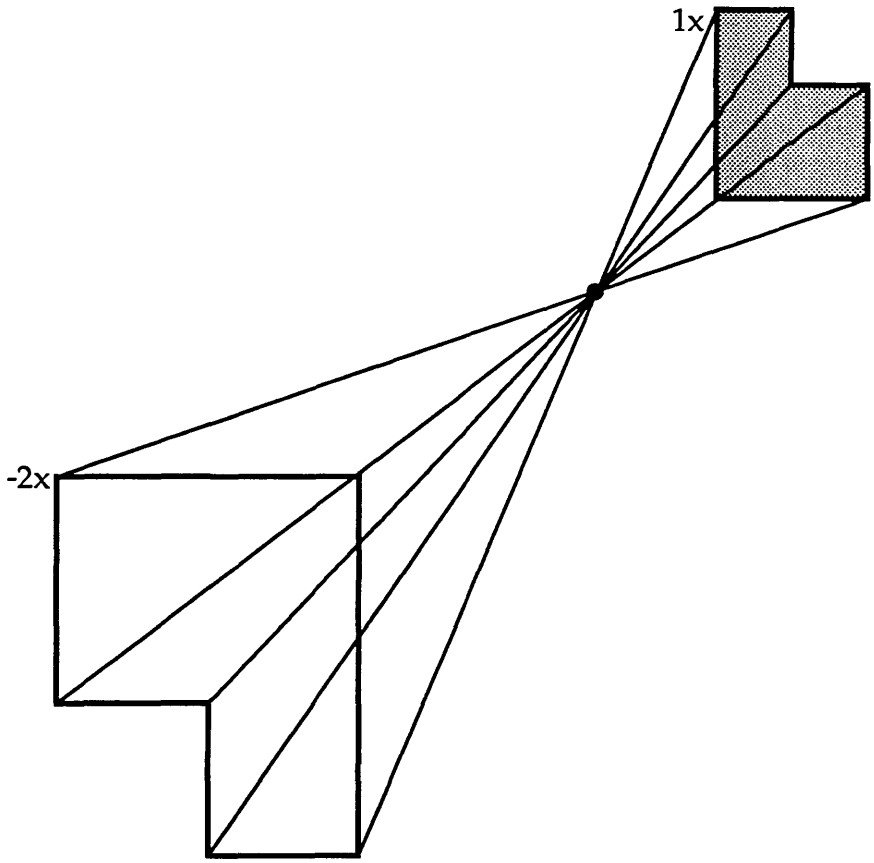
continued/

0845 Negative Scale Factor

3. As the scale factor diminishes the shape diminishes.

4. With a scale factor of -2, the shape is twice as far away, but in the opposite direction from O.

5.



6. The scale factor -1 rotates the shape through 180°.

7. The position of the dot does not change the effect of the negative scale factor, only the position of the image.

8. $^{+}3 \times {}^{+}2 = {}^{+}\mathbf{6}$

9. $^{+}3 \times {}^{-}2 = {}^{-}\mathbf{6}$

10. $^{-}5 \times {}^{+}2 = {}^{-}\mathbf{10}$

11. $^{-}5 \times {}^{-}2 = {}^{+}\mathbf{10}$

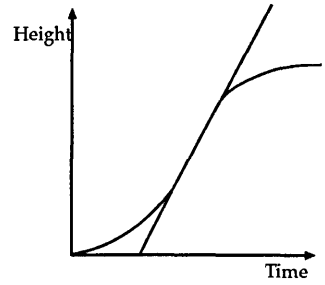
12. a) On positive arrows, a positive scale factor gives a positive arrow.
a negative scale factor gives a negative arrow.

b) On negative arrows, a positive scale factor gives a negative arrow.
a negative scale factor gives a positive arrow.

0847 Hot Mustard

A linear equation will describe the main growth fairly accurately. However, an equation of the form $y = mx - c$ will not pass through the origin (0, 0). This is because of the slow start while the seeds germinate.

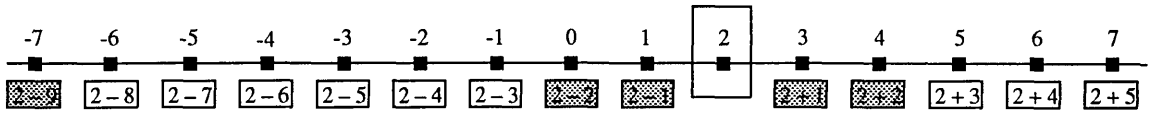
If you continue to record the growth of the seeds, the growth curve will eventually tail off. Why?



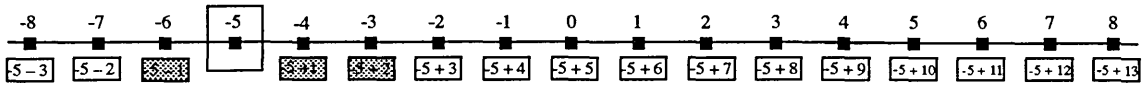
For further information, see 'Life Size'
by Gibbons & Blofield, chapter 6.

0849 Anywhere on the Number Line

1.

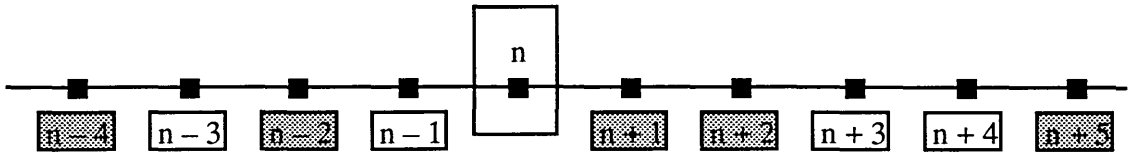


2.



3. The labels will depend on the position you choose for the start.

4.

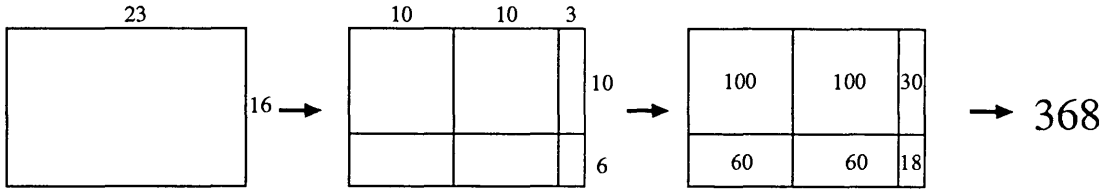


5. If n = 3, $(n + 1) = 4$
$(n - 3) = 0$
$(n + 3) = 6$

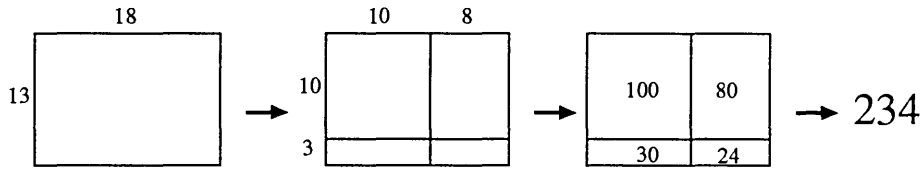
6. If n = 7, $(n - 1) = 6$ $3(n + 1) = 24$ $(n - 1)(n - 4) = 18$
$n(n + 3) = 70$ $n(n - 4)(n + 3) = 210$

0850 Multiplication Problem

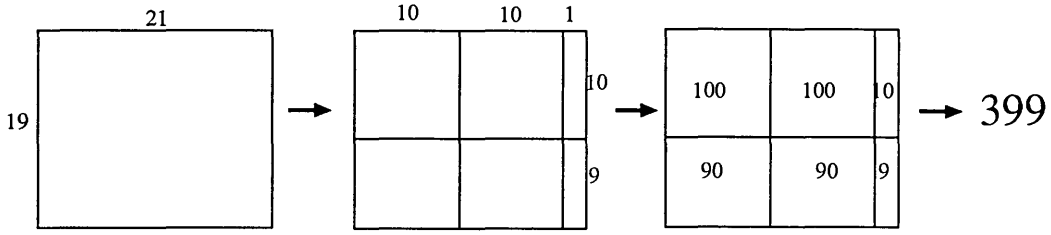
23 x 16



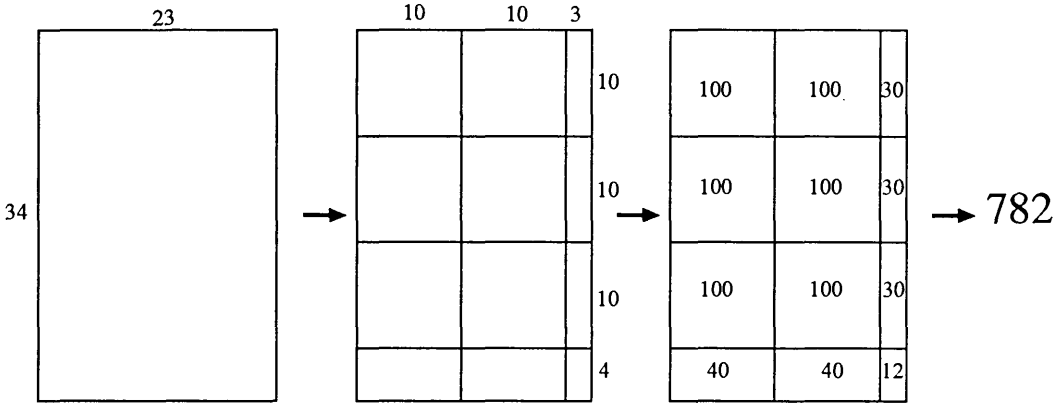
1. a) 18 x 13 = 234



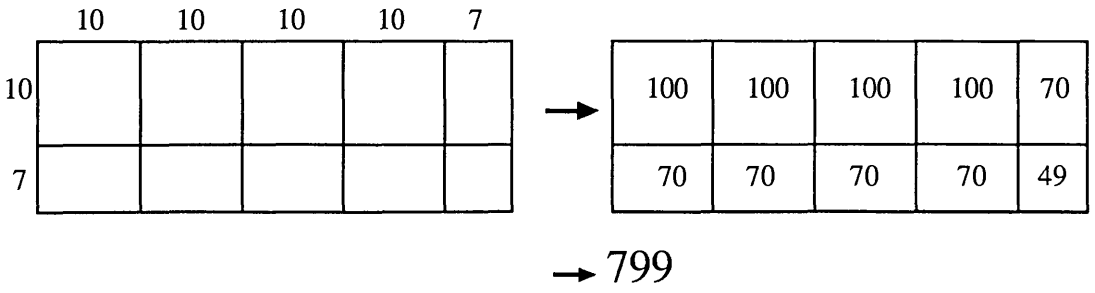
b) 21 x 19 = 399



c) 23 x 34 = 782



d) 47 x 17 = 799



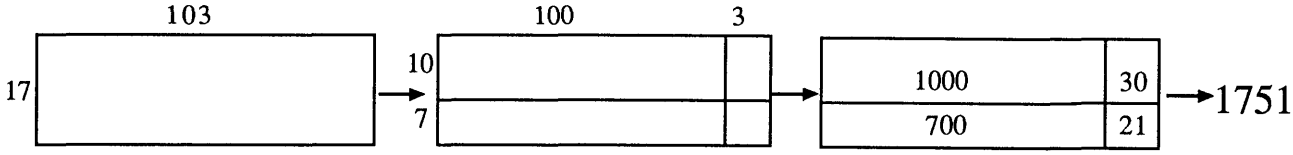
continued/

0850 Multiplication Problem (cont)

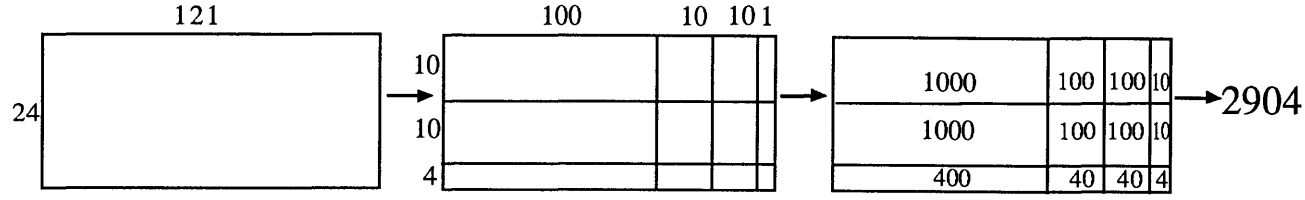
2. The method is not so good for numbers between 50 and 100 because you need to draw so many squares. It still works but it is not very quick.

   For numbers between 100 and 150 it is quicker because you can draw the 100 first:

   103 x 17 = 1751



   121 x 24 = 2904

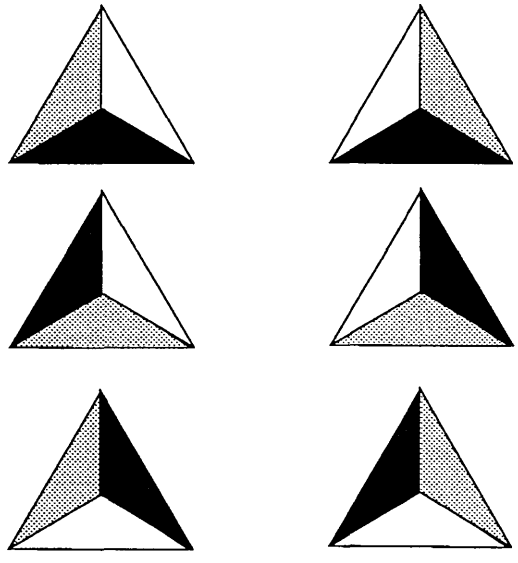


0851 Tile Patterns

Make a drawing or take a photo of one of your tile patterns.

0852 Colouring Triangles

Colour the triangles first, before cutting them out.
Here is one way of colouring the triangles so that each one *looks* different.



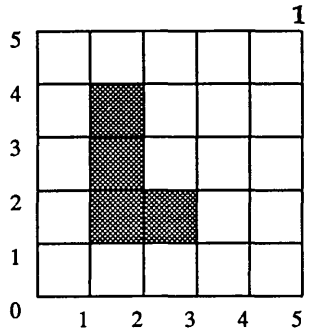
Are you pleased with your design when you fitted the 6 triangles together so that each colour touches a different colour?

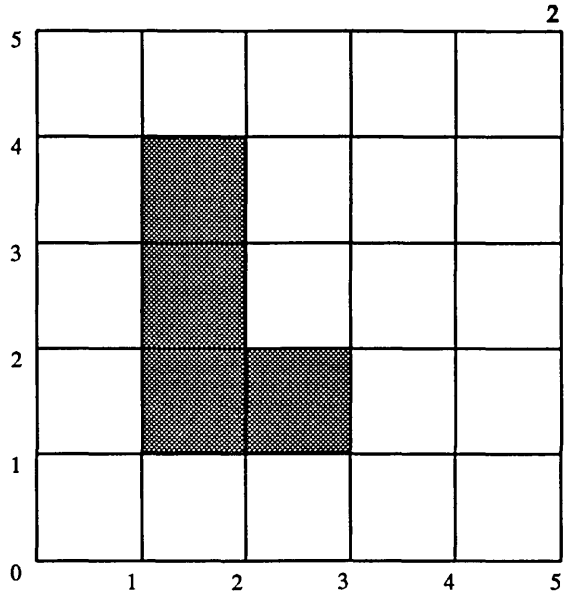
Stick your design into your book or make a display for the wall.

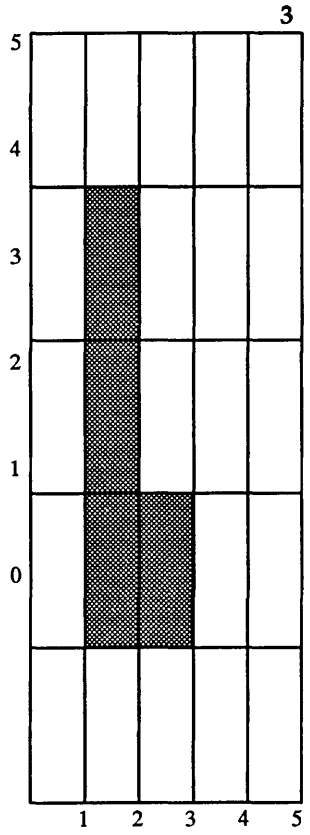
0853 Grids

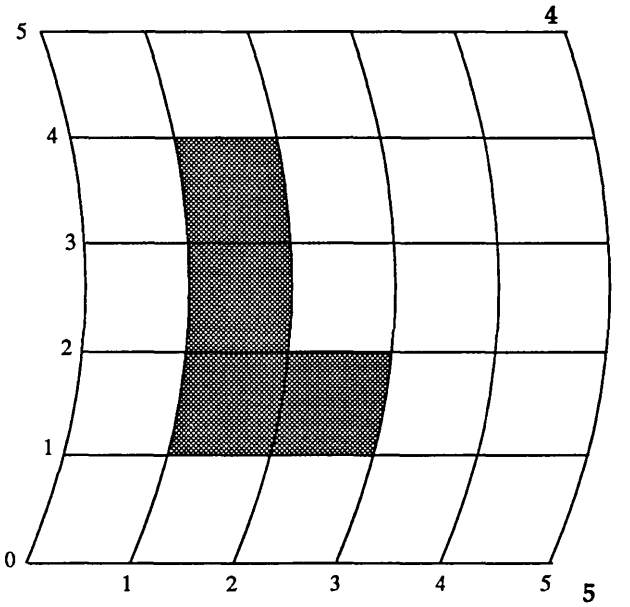
1. A is at (1, 1)
   B is at (3, 1)
   C is at (3, 2)
   D is at (2, 2)
   E is at (2, 4)
   F is at (1, 4)

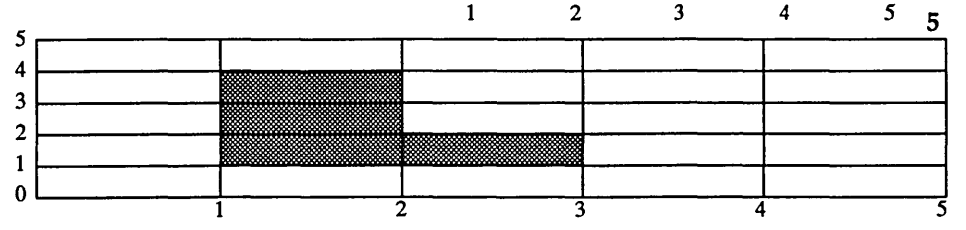
2.











3. Show your own shape on the worksheet to your teacher.
   Discuss how the shape has changed on each grid.

0854 Perimeter

1. Triangle

2. AB = 7cm

3. BC = 6cm

4. CA = 7cm

5. The perimeter is 20cm. (7cm + 6cm + 7cm = 20cm)

6. Each side of the square is 3cm. The perimeter is 12cm.

7. a) Rectangle: length = 7cm
   width = $3\frac{1}{2}$cm (or 3.5cm)
   perimeter = 21cm

   b) Trapezium: base = 8cm
   top = $2\frac{1}{2}$cm (or 2.5cm)
   right side = $3\frac{1}{2}$cm (or 3.5cm)
   left side = 3cm
   perimeter = 17cm

   c) Parallelogram top and bottom = 4cm
   left and right = 3cm
   perimeter = 14cm

   d) Hexagon starting at the top, the sides are:
   $2\frac{1}{2}$cm + $3\frac{1}{2}$cm + 4cm + 4cm + 3cm + 3cm
   perimeter = 20cm

0855 How Long?

1. AB = 2cm
2. AC = 6cm
3. AE = 12cm
4. BC = 4cm
5. BE = 10cm
6. AF = 15cm
7. CF = 9cm
8. CE = 6cm
9. DF = 5cm

10. and 11. Did you make good estimates?
If not, practise with some other objects - your shoe, your finger, . . .

continued/

0855 How Long? (cont)

12. GH = 2cm

    JK = $7\frac{1}{2}$cm (or 7.5cm)

    LM = $7\frac{1}{2}$cm (or 7.5cm)

    NP = $10\frac{1}{2}$cm (or 10.5cm)

    TS = $2\frac{1}{4}$cm (or 2.2cm)

    UV = $4\frac{3}{4}$cm (or 4.8cm)

    QR = $4\frac{1}{4}$cm (or 4.2cm)

If your measurements are very different show your work to your teacher.

0856 How Hot?

1.

Tuesday	22°C
Wednesday	25°C
Thursday	24°C
Friday	35°C

2. Friday

3. Monday

4. 15°C

- Show your measurements of the temperature in your classroom to your teacher.
- You probably decided that the temperature inside your classroom was higher than the temperature outside.

0857 It's Raining

1. 115mm
2. 70mm
3. 45mm
4. 25mm
5. 210mm
6. 40mm
7. 100mm
8. 60mm
9. 190mm

continued/

0857 It's Raining (cont)

10. January

11. March

You would probably choose July for your holiday as it normally has least rain and is warm. However, there are other factors to consider, like the fact that you have to be at school during July!

0858 Round Objects

Your teacher will need to check your measurements.

0859 Triangle Pairs

2. **a** and **b** are the same, they are congruent triangles.
c and **d** are the same, they are congruent triangles.
e and **f** are the same, they are congruent triangles.
g and **h** are the same, they are congruent triangles.

Triangles are **congruent** if they have the same lengths and the same angles.

3. **a**, **b**, **c** and **d** have one square corner.

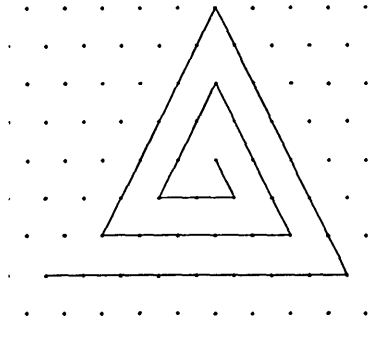
4. **a** and **b** fit together to make a square.
c and **d** fit together to make a rectangle.

5. There are many possible shapes - a rhombus and a parallelogram are just two possibilities.

0860 The Same Area

C, F and G have the same area (16 squares)
D, E and H have the same area (18 squares)

0861 Triangle Spirals

1. 

On the first spiral the numbers go up one each time.

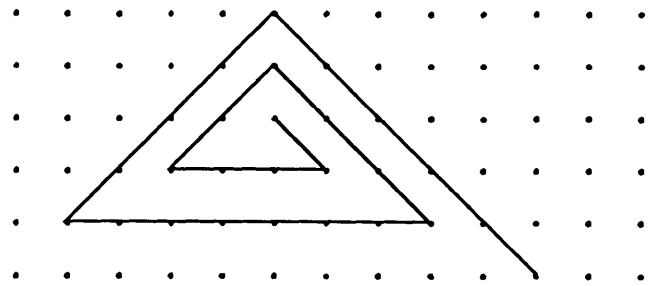
2	3	4	5	6	7	8	9	...

continued/

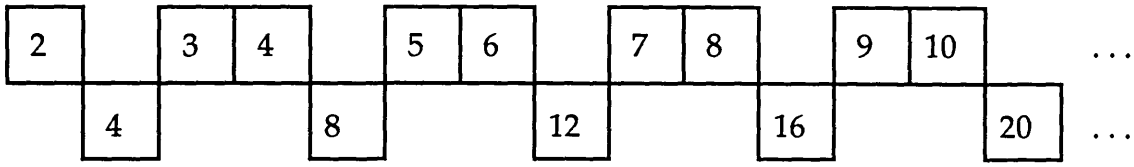
0861 Triangle Spirals (cont)

2. The pattern in the second spiral is more difficult to see.

It can be split into two patterns.



First pattern

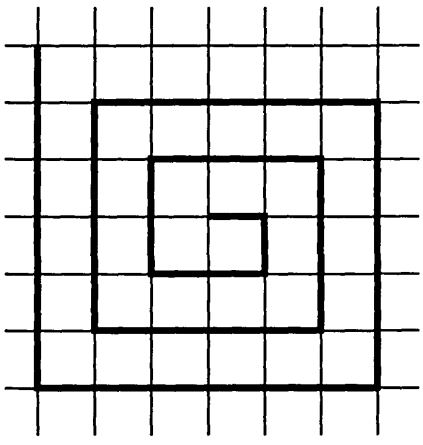


Second pattern

The first pattern goes up by one each time.
The second pattern goes up by four each time.
After the first two numbers, there are two numbers from the first pattern, then one number from the second pattern and so on.

0862 Square Spirals

1.



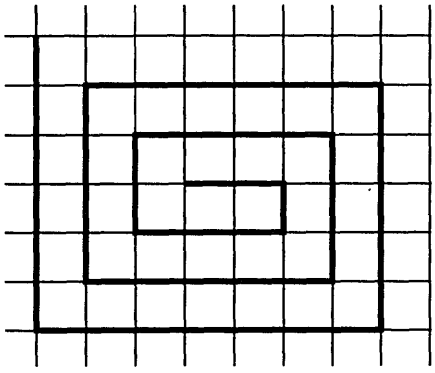
2. The sequence for this spiral goes 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7 . . .

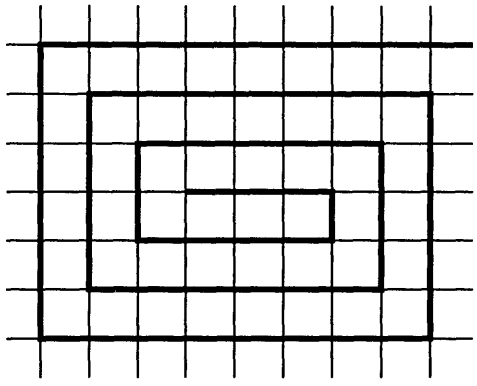
The pattern is: add 0, add 1, add 0, add 1, . . .

continued/

0862 Square Spirals (cont)

3.





The number sequences for thes spirals are:

2, 1, 3, 2, 4, 3, 5, 4, 6, 5, 7, 6 . . . 3, 1, 4, 2, 5, 3, 6, 4, 7, 5, 8, 6 . . .

The patterns are:
subtract 1, add 2, subtract 1, add 2, . . . subtract 2, add 3, subtract 2, add 3, . . .

Ask your teacher to check the sequences and patterns for your spirals.

0863 Dealing the Cards

1. $52 \div 2 = \mathbf{26}$

2. $52 \div 4 = \mathbf{13}$ so **13** cards each

3. $48 \div 4 = \mathbf{12}$
   $48 \div 8 = \mathbf{6}$
   $48 \div 6 = \mathbf{8}$
   $\mathbf{48 \div 3 = 16}$

4. $30 \div 3 = 10$ so **10** cards each

5. $52 \div \mathbf{4} = 13$ so **4** players are needed

6. a) 3 players have 7 cards each $3 \times 7 = 21$ so that is 21 cards
      $52 - 21 = 31$ so **31** cards remain.
   b) $52 \div 7 = 7$, remainder 3
      So 7 players could play, but it would not be a good game with only three cards over.
      6 players is really the largest group because there would be 11 cards over.

7. $52 \div 9 = 5$, remainder 7 so **5** people could play.
   There would be a remainder of **7** cards.

0864 People in Villages

1.  stands for five people

2. a) Eastway 70 b) Weston 105 c) Garfang 120 d) Hilton 95 e) Mossley 140

3.

Dunston														
Melway														
Galton														
Hibberton														

Pictograms are a good way of displaying data.

0866 Sharing Counters

1. $12 \div 2 = \mathbf{6}$

2. $12 \div 3 = \mathbf{4}$

3. $18 \div 2 = \mathbf{9}$
   $18 \div 6 = \mathbf{3}$
   $18 \div 9 = \mathbf{2}$

4. $24 \div 2 = \mathbf{12}$
   $24 \div 3 = \mathbf{8}$
   $27 \div 3 = \mathbf{9}$
   $28 \div 7 = \mathbf{4}$
   $30 \div 5 = \mathbf{6}$
   $32 \div 8 = \mathbf{4}$
   $36 \div 9 = \mathbf{4}$
   $39 \div 3 = \mathbf{13}$

0867 Dividing Counters

1. $15 \div 3 = \mathbf{5}$

2. $23 \div 5 = 4$ remainder **3**

3. a) $18 \div 6 = \mathbf{3}$
   b) $18 \div 5 = \mathbf{3}$ remainder **3**

4. a) $17 \div 3 = \mathbf{5}$ **remainder 2**
   b) $25 \div 5 = \mathbf{5}$
   c) $21 \div 4 = \mathbf{5}$ **remainder 1**
   d) $7 \div 2 = \mathbf{3}$ **remainder 1**
   e) $35 \div 7 = \mathbf{5}$
   f) $35 \div 6 = \mathbf{5}$ **remainder 5**
   g) $43 \div 7 = \mathbf{6}$ **remainder 1**
   h) $31 \div 9 = \mathbf{3}$ **remainder 4**

0868 Evens Worksheet

Twos

1 x 2	2 x 2	3 x 2	4 x 2	5 x 2	6 x 2	7 x 2	8 x 2	9 x 2	10 x 2	11 x 2
2	4	**6**	**8**	**10**	**12**	**14**	**16**	**18**	**20**	**22**

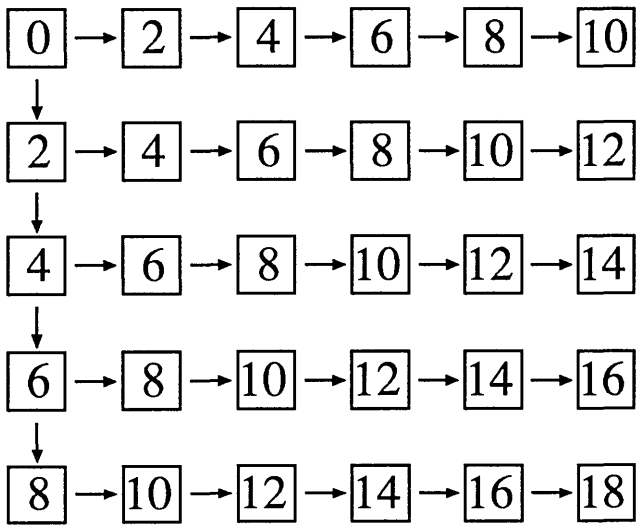
Pairs

1 pair	2 pairs	3 pairs	4 pairs	5 pairs	6 pairs	7 pairs
2	4	**6**	**8**	**10**	**12**	**14**

Pages

Left hand page numbers in a book are usually even numbers. Can you explain why?

Add a Couple



Even

18, 26, 42, 10, 4

An even number always ends in 0, 2, 4, 6 or 8

0869 Puzzle Worksheet

How many scramble?

1000		100
10	18	2
1	20	16

Rifle range

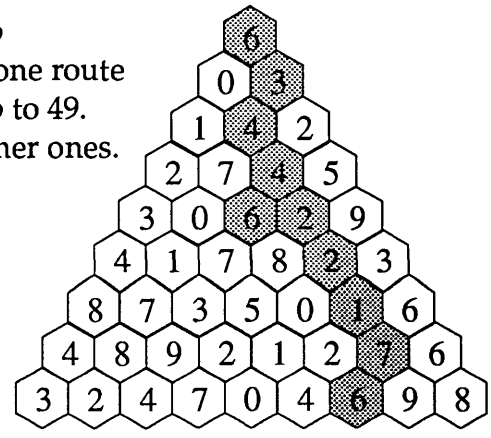
Sue	330
Doug	320
Karin	310
Leroy	290
Pat	350

Hidden picture

The multiples of 4 shaded in give a horse.

Route climb

This shows one route that adds up to 49. There are other ones.



0870 Find the Stranger

'e' is not a capital or upper case letter.

There are several ways to answer each question. You should have one of these answers.

	Odd one out		Reason
1.	17		not a multiple of 10
		or	doesn't end in 0
2.	hen		does not have four legs
	or rat		not a farm animal
	or horse		more than 3 letters
3.	3		not an even number
	or 12		a two digit number
4.	cabbage		not grown under ground
5.	3 x 7		does not equal 24

For these questions you should have two different answers:

6.	banana	not round
or	tomato	not a sweet fruit
or	apple	not six letters
7.	2 x 1	not a division
or	15 ÷ 3	does not equal 2
8.	four	not three letters
or	one	not an even number
or	ten	not a single digit number

You should have found some of these answers:

9.	(120 ÷ 10) ÷ 2	contains brackets *or* has three numbers *or* has two division signs
	72 ÷ 2	does not equal 6
	18 ÷ 3	contains an odd number
10.	6	single digit number
	26	not a multiple of 6 or not in 6 times table
11.	9	single digit number
	or	odd number
	or	not a multiple of 2 or not in 2 times table
	or	not a multiple of 4 or not in 4 times table
	or	not a multiple of 8 or not in 8 times table

12. Did you write down the reason why 8 is the odd one out for your set of numbers?

0871 Heavier/Lighter

Show your work to your teacher to check.

0872 How Heavy?

Show your work to your teacher to check.

0873 New Clothes

Show your work to your teacher to check.

0876 Identities

1. $a = 12, b = 3$

$$\begin{aligned} a(a + b) &= a^2 + ab \\ 12 \times (12 + 3) &= 12^2 + (12 \times 3) \\ 12 \times 15 &= 144 + 36 \\ 180 &= 180 \end{aligned}$$

2. $(a + b)(a + c) = \mathbf{a^2 + ab + ac + bc}$

3. To check your identity, you should substitute some values for a, b and c and check that both sides of the identity are equal.

e.g. $a = 3, b = 5, c = 12$

$$\begin{aligned} (a + b)(a + c) &= a^2 + ab + ac + bc \\ (3 + 5)(3 + 12) &= 3^2 + (3 \times 5) + (3 \times 12) + (5 \times 12) \\ 8 \times 15 &= 9 + 15 + 36 + 60 \\ 120 &= 120 \end{aligned}$$

4. $(m + n)^2 = (m + n)(m + n)$
$= \mathbf{m^2 + 2mn + n^2}$

5. Substitute some values for m and n and check that both sides of the identity are equal.

e.g. $m = 4, n = 7$

$$\begin{aligned} (m + n)^2 &= m^2 + 2mn + n^2 \\ (4 + 7)^2 &= 4^2 + (2 \times 4 \times 7) + 7^2 \\ 11^2 &= 16 + 56 + 49 \\ 121 &= 121 \end{aligned}$$

6. $(x + y + z)z = \mathbf{xz + yz + z^2}$

7. $(a + 2b)(2a + b) = \mathbf{2a^2 + 5ab + 2b^2}$

continued/

0876 Identities (cont)

8. a) $(x + y + z)z = xz + yz + z^2$

e.g. $x = 0.2, y = 0.6, z = 1.3$

$(0.2 + 0.6 + 1.3)1.3 = (0.2 \times 1.3) + (0.6 \times 1.3) + 1.3^2$

$2.1 \times 1.3 = 0.26 + 0.78 + 1.69$

$2.73 = 2.73$

$(m + n)^2 = m^2 + 2mn + n^2$

e.g. $m = {}^1/_2, n = {}^1/_4$

$({}^1/_2 + {}^1/_4)^2 = ({}^1/_2)^2 + (2 \times {}^1/_2 \times {}^1/_4) + ({}^1/_4)^2$

$({}^3/_4)^2 = {}^1/_4 + {}^1/_4 + {}^1/_{16}$

${}^9/_{16} = {}^9/_{16}$

b) $(x + y + z)z = xz + yz + z^2$

e.g. $x = -7, y = -2, z = -4$

$(-7 + -2 + -4)-4 = (-7 \times -4) + (-2 \times -4) + -4^2$

$-13 \times -4 = 28 + 8 + 16$

$52 = 52$

$(m + n)^2 = m^2 + 2mn + n^2$

e.g. $m = -9, n = -1$

$(-9 + -1)^2 = -9^2 + (2 \times -9 \times -1) + -1^2$

$-10^2 = 81 + 18 + 1$

$100 = 100$

0877 Angle 4 Review

1. $x = \mathbf{40°}$

2. $y = \mathbf{70°}$

3. In an equilateral triangle each angle is **60°**

4. $a = \mathbf{40°}, b = \mathbf{75°}$

6. $c = \mathbf{120°}, d = \mathbf{37°}$

7. $g = \mathbf{130°}, e = f = \mathbf{50°}$

5. $100° + 40° + 60° + 75° + 45° + 40° = \mathbf{360°}$

8. $h = \mathbf{68°}, j = \mathbf{105°}$
The angle sum of a pentagon is **540°**.

9. A triangle has angle sum = **180°**. There are **three** triangles. So the angle sum $= 3 \times 180° = \mathbf{540°}$.

10. There are five triangles. So the angle sum $= 5 \times 180° = \mathbf{900°}$.

11. Angles **r** and **s** are the same.

12. $w = \mathbf{25°}$
$x = \mathbf{60°}$
$x + y + z = \mathbf{360°}$

continued/

0877 Angle 4 Review (cont)

13. $k = 30°$
$m + n + o + p + q = 360°$

14. **1, 3** and **5** are the same. **4** and **2** are the same.

15. *You may have slightly different answers to these.*
A **right angle** is a square corner.
An **acute angle** is less than a right angle.
An **obtuse angle** is more than more than one right angle, but less than two.

16. An **equilateral** triangle has all **three sides** and **three angles equal.**
An **isosceles** triangle has **two sides** and **two angles equal.**

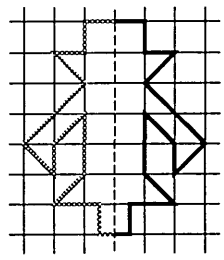
0880 Symmetry Pack

0880B and **0880G**

Shape	lines of symmetry
Rectangle	2
Square	4
Kite	1
Isosceles Triangle	1
Regular Pentagon	5
Parallelogram	0
Scalene Triangle	0
Circle	infinite - an unlimited number

0880E
The message says 'If you use a mirror you will be able to read what this says.'

0880F 1.



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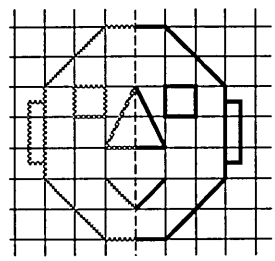

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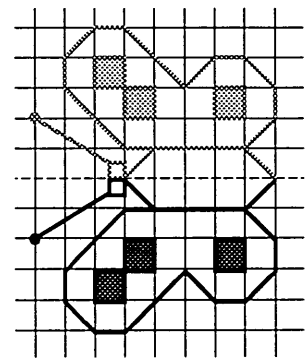
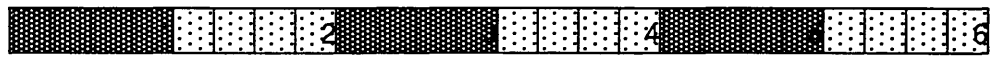


Your work from these and the other cards in the pack will make a good display for your classroom.

0881 24 Squares

1. $24 \div 2 = 12$ 

2. $24 \div 3 = 8$ 

3. $24 \div 4 = 6$ 

4. $24 \div 6 = 4$ 

5. $24 \div 8 = 3$ 6. $24 \div 12 = 2$ 7. $24 \div 24 = 1$ 8. $24 \div 1 = 24$

9. $36 \div 2 = 18$ 10. $36 \div 3 = 12$ 11. $36 \div 4 = 9$ 12. $36 \div 6 = 6$

13. $36 \div 9 = 4$ 14. $36 \div 12 = 3$ 15. $36 \div 18 = 2$ 16. $36 \div 36 = 1$

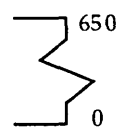
0882 "Lies, Damned Lies and Statistics"

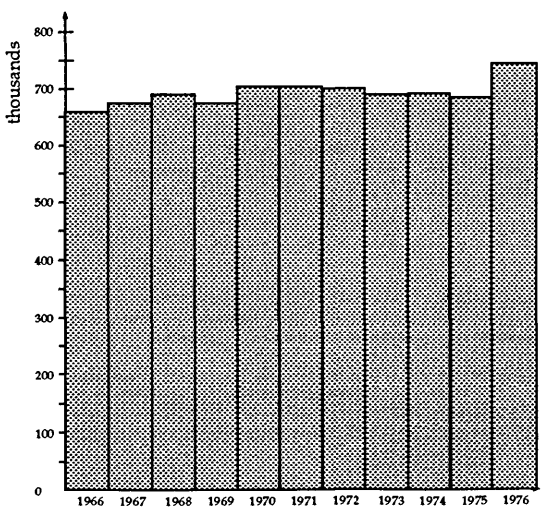
1. No!

2. 1966 Approximately 660 thousand
   1976 Approximately 745 thousand

3. Approximately 85 thousand

4. $\frac{85}{660}$ is 0.128787 so the increase on the 1966 figures is about 13% or $\frac{1}{8}$.
   Definitely not 400%!

5. The graph gives this impression because only the top part of the graph is shown, notice the scale is broken.
   The portion from 0 to 650 is squashed, this is indicated by 

6. 

continued/

7. The newspaper wanted to influence the way people think about things, particularly politics and politically related topics. They wanted to give the impression that the Civil Service had grown more than it actually had.

In the following examples some questions to consider have been posed. It is not likely that you will be able to answer all of them, but bear in mind that the people who published the graphs probably couldn't answer them either.

'A' level results slump
Why was only the top of the graph shown? *In order to exaggerate the decline, to make comprehensive schools look bad.*

The implication is that comprehensive reorganisation was the reason for the decline. *However, there was a decline throughout the period from 1964 to 1976.*

How did these results compare with national percentages? *It is possible that the Manchester results reflect a general decline. Perhaps 'A' levels have become more difficult. Perhaps there are fewer other options for 16 year olds (like jobs) and many more are staying on and taking 'A' levels, including some less able students.*

Were students taking 'A' level courses at school or FE Colleges? *It isn't clear whether the results included all schools and colleges or just comprehensive schools.*

Sleep as a series of curves
What has been measured? And who has measured it? Is it meant to represent normal sleep patterns or those after having a bed-time drink?

The volume of passenger traffic at Heathrow
Predictions are difficult because until 1970 the increase was fairly linear. It was really only 1973 that gave the graph such an upturn. Was this a freak year? Or are the planners right to think that this is the start of an ever increasing slope?

If their slope is correct, what would they have expected for the volume of passengers to be for 1985? . . . for 2005? Does this imply London needs a new airport by 2005?

The way the grapefruit slices
The diagrams give the impression that the Angus Steak House makes much more profit than the Crown Inn. Infact the Angus Steak House makes a net profit of 17.25p, whilst the Crown Inn's net profit is 19p.

When you calculate the actual costs you get the following:

	Crown Inn	Angus Steak House
Cost to customer	50p	32p
Cost to restaurant	6p	5p
Profit	44p	27p
Overheads (wages, running costs, etc.)	25p	9.75p
Net profit (after overheads)	19p	17.25p

continued/

0882 "Lies, Damned Lies and Statistics" (cont)

The questions to ask are:
Why should you pay so much more for your grapefruit at the Crown Inn?

Why do the diagrams suggest that the Crown Inn is making less profit than most of the other restaurants?

Why are the overheads so much less at the Angus Steak House?

How much did the grapefruit cost each hotel?

The falling pupil roll in a London school
Why is the gradient of the graph steeper in July/September 1975 than in July/September 1974, although the percentage decline is smaller?

How many pupils were in the school in (a) July 1975, (b) September 1975 and (c) October 1975? *Does the graph show that the roll fell by* $\frac{1}{3}$*?*

When you first looked at the graph did you think that there might be no pupils in the school in October 1975? *Why might the newspaper want to give this impression?*

Tea cups
The statistics indicated are really the heights of the cups. Tom drinks twice as much as Dick, but the diagrams exaggerate this.

Your eye is influenced by the area of the cups drawn, or by the volume of the cups (because you *know* that a cup is a 3-D object). Tom's cup would only $\frac{1}{2}$ fill Harry's cup.

Tom's cup is 8 times bigger (in volume) than Dick's cup, so the graph suggests he drank 8 times more tea than Dick. Harry's cup is 15 times bigger (in volume) than Dick's!

Finally, keep an eye open for biased reporting. Statistics are usually accurate, but they can be presented to the public in a misleading way.

0883 Short-Cuts

There are very many short cuts that can help people to multiply and divide mentally. Even if you don't know all your multiplication tables by heart, you can use the ones you do know to work out the ones you don't. It is important to build up your list of short cuts and not to always rely on a calculator!

Here are some examples that you may have included. Can you see why they work?

Short-cuts for multiplication

To multiply by 10	if a whole number, add a nought.
	if a decimal number, move the numbers one place to the left.

continued/

0883 Short-Cuts (cont)

To multiply by 100		if a whole number, add two noughts. if a decimal number, move the numbers two places to the left.
To multiply by 20	*or*	multiply by ten, then double. double, then multiply by ten.
To multiply by 5	*or*	multiply by ten, then halve. halve, then multiply by ten.
To multiply by 4		double and double again.
To multiply by 9		multiply by ten, then take away the number you are multiplying.
To multiply by 11		multiply by ten, then add on the number you are multiplying.
To multiply by 15		multiply by ten, then halve and add the two answers.

Short-cuts for division

To divide by 10	move the numbers one place to the right.
To divide by 100	move the numbers two places to the right.
To divide by 20	divide by 10, then divide by 2.
To divide by 5	divide by 10, then double.
To divide by 4	halve, then halve again.
To divide by 8	halve, then halve, then halve again.

0884 Positive or Negative

The examples and problems in this booklet demonstrate the general rules:

Positive x Positive = Positive
Negative x Negative = Positive
Negative x Positive = Negative
Positive x Negative = Negative

They can best be summarised by the table:

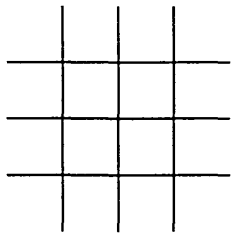
X	+	−
+	+	−
−	−	+

However, you may not agree that Friends and Enemies always fits this pattern. Sometimes just because someone is a friend of your friend it doesn't necessarily mean they will be your friend too! And your enemy's enemy, need not be your friend!

0885 Number Noughts and Crosses

You may like to think of all the possible ways you can make a winning line of 10.

You could extend the game to a board like this and use the numbers 0 to 15, to make a winning line of 20.



0886 Two Sets

One way you could sort the shapes is:

Quadrilaterals (Four sided shapes) A, D, G, H, I, J	Triangles (Three sided shapes) B, C, E, F, K

How many *different* ways did you find to sort the shapes?

0888 SMILER'S Flowchart

Your answers will depend on how your teacher organises your lessons.

0889 Old Oak

1. 30m
   Tell your teacher which building you think is about 30m high.

2. 10m

3. Between 2 and 3 metres

4. 40 years

5. 50 years

6. After 100 years
   The oak tree changes by different amounts in 10 years depending on which period of 10 years you are talking about. The 10 years in which it changes the most is between 20 and 30 years, when it grows at the fastest rate. Between 110 and 120 years, it doesn't change at all.

0892 Swing

The time for one complete swing is the time for ten complete swings divided by 10.

In such an experiment, variations in time for small swings which could be the result of changing the size of the swing or changing the mass cannot be detected.

The length of the string does affect the time of the swing.

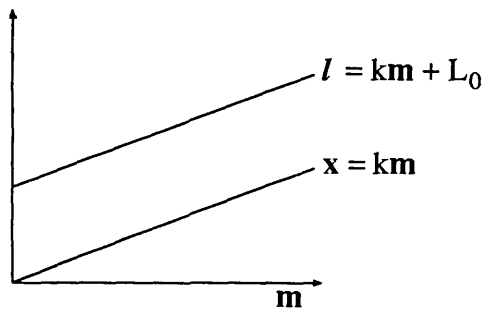
A pedulum of length about 25cm will give a period of swing of 1 second. (Theoretically, it is 24.84cm.)

0893 Stretch

Your results will depend upon the spring that you use. However, in general, you should find that the extension of the spring **x** (the amount of stretch) is **directly proportional** to the mass **m** which causes it, $x \propto m$, or $x = km$. As long as you do not overstretch the spring, you should get a straight line graph.

These equations describe the relationships between l l, **m**, **x** and L_0 (the length of the unstretched spring):

$$x = l - L_0$$
$$x = km$$
$$l = km + L_0$$



What values did you find for the number k and L_0?

0894 Force Meet

Did you always manage to force the second player to meet your position?

What other rules did you use?

0895 Jumps

1. Twos and fives first meet at [10]: 5 jumps of 2 = [10] and 2 jumps of 5 = [10]
   5 x 2 = 2 x [5]

2. Twos and eights first meet at [8]: 1 jump of 8 = [8] and [4] jumps of 2 = [8]
   1 x [8] = [4] x 2

3. [9] jumps of [2] = [18] and [2] jumps of 9 = 18
   9 x 2 = [2] x [9]

continued/

0895 Jumps (cont)

4. Fives and threes first meet at [15]: [3] jumps of 5 = [15] and [5] jumps of 3 = [15]
[3] x [5] = [5] x [3]

5. [3] jumps of [7] = [21] and [7] jumps of [3] = [21]
[3] x [7] = [7] x [3]

6. [3] jumps of [6] = [18] and [6] jumps of [3] = [18]
[3] x [6] = [6] x [3]

Or you might have found that threes and sixes first meet at 6.
[1] jump of 6 = [6] and [2] jumps of [3] = [6]
[1] x [6] = [2] x [3]

7. and 8. Show your own jumps to your teacher.

0896 How Thick?

- Ten pennies are approximately 14mm, so one penny is approximately $14 \div 10 = 1.4$mm thick.
- One hundred cards are approximately 33mm, so one card is approximately $33 \div 100 = 0.33$mm or $\frac{1}{3}$mm thick.
- Five hundred sheets of graph paper are approximately 5cm, so one sheet of graph paper is approximately $5 \div 500 = 0.01$cm or 0.1mm thick.
- Two exercise books are approximately 5mm (depending on the type of book), so if 100 pages are 5mm, one page is approximately $5 \div 100 = 0.05$mm thick.
- Different threads have different thicknesses. The thickness is measured in Denier, because millimetres are too big! 15 denier tights are made with thinner thread than 30 denier tights. How many denier would be equal to 1mm?

0897 Statistics Three Review

1. 2 votes

2.

Death's Door	0 votes
Loud Hailers	3 votes
Sadie	4 votes
The Stretch	3 votes

You may not have included the votes for Death's Door in your answer, as they are not shown on the pie chart.

3. Each vote is represented by 30°.

continued/

4. $\frac{3}{12}$ or $\frac{1}{4}$.

5. $\frac{4}{12}$ of the votes went to "Sadie", so their part of the pie chart is 120°.

6. The histogram shows that "Death's Door" got no votes.

7. 5000 records

8. 7500 records

9. "Look Back" by Sadie. *But not if everyone has already bought it!*

10.

"See You"	35000
"Rat a tat'	7500
"Come on over"	30000
"Look back"	50000
"Love me too"	27500

11. The total number of records sold is 150000

12. $\frac{50000}{150000} = \frac{5}{15}$ or $\frac{1}{3}$

13.

Diagram	**Mode**
Piechart	Sadie
Histogram	Sadie
Pictogram	"Look Back" by Sadie

14. The friends preferred "The Stretch" and "Loud Hailers" to "Smilers", but the "Smilers" record sales were second only to Sadie.

15. The record sales probably give a better guide as the sample size is larger, i.e. more people's opinions are included.

16. It is reasonable to expect 20 votes for the "Loud Hailers".
$\frac{1}{3}$ of the record sales were for "Look Back" and $\frac{1}{3}$ of the votes were for "Sadie".
$\frac{1}{5}$ of the record sales were for "Come on over" so it might be expected that $\frac{1}{5}$ of the votes to go to "Loud Hailers".

0898 Keys

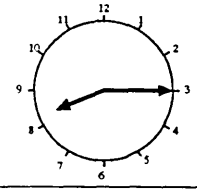
Ask some people to try out your 'key', to check that it works and that you have included all possibilities.

Did you find out how Linnaeus classified living things? This is an excerpt from Children's Brittanica;

> **Linnaeus, Carolus** (Carl Von Linne)
> "Linnaeus established what is now called the *binomial* (meaning 'having two names') naming of plants and animals because he gave each two Latin names, the first being the name of its genus and the second that of its species. For example, he called the common daisy *Bellis perennis. Bellis* is the name of its genus *perennis* of the species.
>
> In the case of plants, Linnaeus based this system mainly on the number of stamens and pistils the different flowers possess. Although different methods are now used, the work that Linnaeus did on the genera and species of plants has been the inspiration of later botanists and also the foundation of their work."

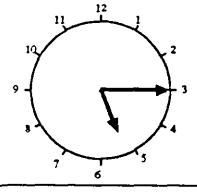
0899 Time Bingo

Once you have played "Time Bingo" a few times, you should be able to recognise the time whether it is:

	written in words	*or*	written in numbers	*or*	shown on a clock face.
e.g.	Eight fifteen		8.15		

0900 24 Hour Bingo

Once you have played "24 Hour Bingo" a few times, you should be able to recognise the time whether it is:

	written in words	*or*	written in numbers	*or*	shown on a clock face.
e.g.	Homework Quarter past five		17.15		

0903 Millions

0903A A Long Walk
A million paces is 800km.
She can't do it in England, but she can just about in Great Britain, if it were possible to walk from John O'Groats to Lands End in a straight line.
There are several countries in Europe, and elsewhere, where it is possible, e.g. France, Spain, Italy.

0903B A Million Strokes
A million strokes would take 11 days, 13 hours, 46 minutes, 40 seconds (non-stop).
In working hours (8 hours per day for 5 days a week) it would take 6 weeks, 4 days, 5 hours, 46 minutes, 40 seconds.

0903C Piles of Paper
500 sheets of paper are about 5cm high, so 1 million sheets would be 100m high.

0903D All the Tea in China
A million cups would need 2500 pounds (lb.) of tea, this would cost £875.

0903E The Size of a Book
An average novel of 200 pages, with each page having about 500 words, contains approximately 100000 words. If a book contained a million words, it would need to have about 2000 pages.

0903F Hours and Hours
You haven't lived a million hours.
A person who has lived a million hours would be 114 years old. There are a few people in the world who are 114 years old, many of these are living in Ecuador in South America.

0903G Ages and Ages
Cleopatra was born less than a million days ago. A million days is about 2738 years.
In 1996, a million days ago was 742 B.C. Cleopatra was born in 69 B.C.

In 1996, another million days will be 4734 A.D.

0904 Carry on Subtracting

	66			91	
−	18		−	72	
	48			**19**	
	192			125	
−	127		−	53	
	65			**72**	
	461			703	
−	295		−	25	
	166			**678**	

0905 Domino Puzzles

Here is a solution to Puzzle 4.

9	3	3	3	3	0	5
9	1	1	1	1	0	5
9	4	4	4	4	0	5
6	2	2	2	2	0	5
6	1	1	1	1	3	4
6	5	5	5	5	3	4
6	2	2	2	2	3	4
6	0	0	0	0	3	4

Did you record your results for any of the puzzles?

0906 Tak Tiles A

Show your teacher how some of your shapes fitted.

0907 Tak Tiles B

Show your teacher how some of your shapes fitted.

0908 Tak Tiles C

Show your teacher how some of your shapes fitted.

0909 Tak Tiles D

Show your teacher how some of your shapes fitted.

0982 Letters for Lengths

			1.	2.	3.
A	a)	the **area**	30	$3x^2$	$8x^2$
	b)	the **perimeter**	22	$8x$	$12x$

B
1. $x = 2$
2. $x = 3$
3. $x = 5$
4. $x = 1$

C
1. $x = 2$
2. $x = 3$
3. $x = 2.5$
4. $x = 5$
5. $x = 0.5$

			1.	2.	3.
D	a)	the **area**	xy	$6ab$	$4p^2$
	b)	the **perimeter**	$2(x + y)$	$2(2a + 3b)$	$8p$

E *Some possible values for x and y are:*

x	1	2	3	4	6	8	12	24
y	24	12	8	6	4	3	2	1

or others if x and y are not integers.

F *Some possible values for a and b are:*

a	3	6	4.5	1.5
b	3	1	2	4

or others.

1007 Cumulative Frequency and Quartiles

1. 160.5cm

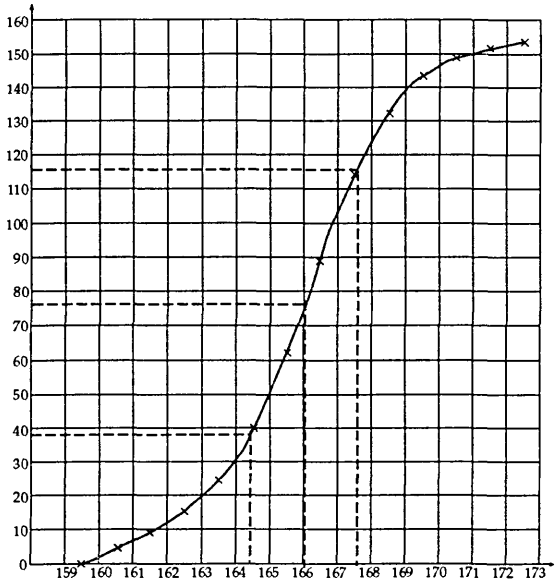
2.

Height (cm)	Frequency	Cumulative frequency	Heights of students represented by cumulative frequency
160	4	4	< 160.5cm
161	5	9	< 161.5cm
162	6	15	< 162.5cm
163	9	24	< 163.5cm
164	16	40	< 164.5cm
165	22	62	< 165.5cm
166	27	89	< 166.5cm
167	25	114	< 167.5cm
168	18	132	< 168.5cm
169	11	143	< 169.5cm
170	6	149	< 170.5cm
171	3	152	< 171.5cm
172	2	154	< 172.5cm

The last entry in the cumulative frequency column should be 154, this is the total number of students.

continued/

3. The point (159.5, 0) belongs to the curve because there are no students who have a height of 159.5 or less.

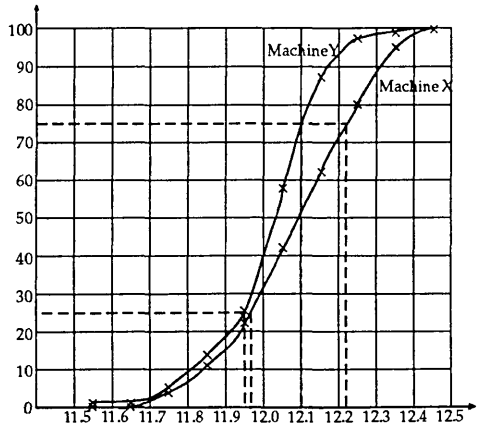
4. 

5. a) 166.1cm
   b) 164.5cm
   c) 167.6cm
   d) 3.1cm

Your answers may differ slightly from these, as they are dependent on the accuracy of the graph.

6. a)

Diameter (mm)	11.5	11.6	11.7	11.8	11.9	12.0	12.1	12.2	12.3	12.4
X Frequency	0	0	3	8	11	20	20	18	15	5
X Cum. Freq.	0	0	3	11	22	42	62	80	95	100
Y Frequency	1	0	4	9	13	31	30	9	2	1
Y Cum. Freq.	1	1	5	14	27	58	88	97	99	100



b) Inter-quartile range:
Machine X = 12.22 – 11.97 = 0.25mm
Machine Y = 12.1 – 11.95 = 0.15mm

Your answers may differ slightly from these, as they are dependent on the accuracy of the graph.

c) Machine Y produces more consistent results and so is more reliable.

1011 Dividing in a given ratio

A 1. $\mathbf{b}-\mathbf{a}$ 2. $\frac{3}{5}\mathbf{b}-\frac{3}{5}\mathbf{a}$ 3. $\frac{2}{5}\mathbf{b}-\frac{2}{5}\mathbf{a}$

B 1. $\overrightarrow{OP}=\overrightarrow{OA}+\overrightarrow{AP}$

2. $\mathbf{r} = \mathbf{a}+\frac{3}{5}\mathbf{b}-\frac{3}{5}\mathbf{a} = \frac{2}{5}\mathbf{a}+\frac{3}{5}\mathbf{b}$

C $\mathbf{r} = \frac{2}{5}\begin{pmatrix}2\\6\end{pmatrix}+\frac{3}{5}\begin{pmatrix}8\\3\end{pmatrix} = \begin{pmatrix}\frac{4}{5}\\\frac{12}{5}\end{pmatrix}+\begin{pmatrix}\frac{24}{5}\\\frac{9}{5}\end{pmatrix} = \begin{pmatrix}\frac{28}{5}\\\frac{21}{5}\end{pmatrix} = \begin{pmatrix}5\frac{3}{5}\\4\frac{1}{5}\end{pmatrix}$

The co-ordinates of P are $(5\frac{3}{5}, 4\frac{1}{5})$.

D 1. $\overrightarrow{OQ}=\overrightarrow{OA}+\overrightarrow{AQ}$

$\mathbf{s} = \mathbf{a}+\frac{3}{10}\mathbf{b}-\frac{3}{10}\mathbf{a} = \frac{7}{10}\mathbf{a}+\frac{3}{10}\mathbf{b} = \frac{1}{10}(7\mathbf{a}+3\mathbf{b})$

2. $\mathbf{s} = \frac{7}{10}\begin{pmatrix}2\\6\end{pmatrix}+\frac{3}{10}\begin{pmatrix}8\\3\end{pmatrix} = \begin{pmatrix}\frac{14}{10}\\\frac{42}{10}\end{pmatrix}+\begin{pmatrix}\frac{24}{10}\\\frac{9}{10}\end{pmatrix} = \begin{pmatrix}\frac{38}{10}\\\frac{51}{10}\end{pmatrix} = \begin{pmatrix}3\frac{8}{10}\\5\frac{1}{10}\end{pmatrix}$

Co-ordinates of Q are (3.8, 5.1)

E 1. $\mathbf{t}=\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b}$

2. T = (4, 5)

3. If 2AT ≠ TB show your diagram to your teacher.

F 1. $\overrightarrow{AP} = \frac{m}{m+n}\overrightarrow{AB} = \frac{m}{m+n}(\mathbf{b}-\mathbf{a}) = \frac{m}{m+n}\mathbf{b}-\frac{m}{m+n}\mathbf{a}$

2. $\overrightarrow{OP} = \overrightarrow{OA}+\overrightarrow{AP} = \mathbf{a}+\frac{m}{m+n}\mathbf{b}-\frac{m}{m+n}\mathbf{a}$

$= \frac{m+n-m}{m+n}\mathbf{a}+\frac{m}{m+n}\mathbf{b}$

$= \frac{n}{m+n}\mathbf{a}+\frac{m}{m+n}\mathbf{b} = \frac{1}{m+n}(n\mathbf{a}+m\mathbf{b})$

G 1. $\frac{1}{3}(2\mathbf{a}+\mathbf{b})$

2. $\frac{1}{5}(3\mathbf{a}+2\mathbf{b})$

3. $\frac{1}{7}(2\mathbf{a}+5\mathbf{b})$

continued/

1011 Dividing in a given ratio (cont)

H 1. $\overrightarrow{OE} = \frac{1}{4}(3\overrightarrow{OC} + \overrightarrow{OD})$

2. $E = (4\frac{1}{4}, 5)$

1013 Vector Magnitude

A 1. 2 2. 3 3. $\sqrt{(2^2 + 3^2)} = \sqrt{13}$

B 1. $\sqrt{13}$ 2. 13 3. $\sqrt{53}$

C 1. $\sqrt{17}$ 2. $\sqrt{18}$ 3. $\sqrt{68}$ 4. 3

D 1. 4 2. a_1 3. a_2 4. 0

E 1. a_1 2. a_2 magnitude of $\overrightarrow{PQ} = \sqrt{(a_1^2 + a_2^2)}$

F $a = 5$ or -5, $b = 4$ or -4, $c = 3$ or -3, $d = 0$

G 1. $\sqrt{13}$

2. 7

3. $\sqrt{40}$

4. Yes $7^2 = (\sqrt{40})^2 + 3^2$

5. 7

6. Yes

H 1. a_3 2. a_1 3. a_2 4. a_3 5. $\sqrt{(a_1^2 + a_3^2)}$

$OR^2 = a_1^2 + a_2^2 + a_3^2$

J 1. $PQ^2 = 5$ 2. $HK^2 = 149$ 3. $PQ = \sqrt{5}$, $HK = \sqrt{149}$

K 1. $\sqrt{13}$ 2. $\sqrt{41}$ 3. $\sqrt{48}$

1028 Isometries

A 1. Yes
2. Yes
3. Yes

continued/

1028 Isometries (cont)

B The mirror is in a position described by:
'm_3 translated towards m_2 by a distance equivalent to the distance between m_2 and m_1'.

C 1. M_3R_{180}, where R_{180} is a 'rotation of 180°, centre intersection of m_2 and m_1'.

or M_1 followed by a translation twice the distance between m_2 and m_1 and at right angles to these mirror lines.

2. As q. 1 above.

A Reflection and rotation are also isometries.

B

	T	M	R	G
T	T	G or M	R	G
M	G or M	R or T	M or G	T or R
R	R	M or G	R or T	M or G
G	G	T or R	M or G	R or T

C 1. Rotation, the invariant point is the centre of rotation.
2. Reflection, the invariant line is the mirror line.

D Glide reflection in y = 0 line.

E $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ 5 \end{pmatrix}$

1081 Puzzles

Puzzles A

1. $3n + 6 = 33$ $n = 9$
2. $7n + 5 = 61$ $n = 8$
3. $8n + 3 = 35$ $n = 4$
4. $6n + 9 = 51$ $n = 7$
5. $7n + 10 = 59$ $n = 7$

continued/

1081 Puzzles (cont)

Puzzles B

1.	$6r + 5 = 23$	$r = \mathbf{3}$	One bag of rice weighs **3kg**.
2.	$8b + 60 = 300$	$b = \mathbf{30}$	One book costs **30p**.
3.	$24 + 26 + 4s = 60$	$s = \mathbf{2.5}$	One pair of socks costs **£2.50**.
4.	$6s + 3 = 12$	$s = \mathbf{1.5}$	He uses **1.5m** of material for one shirt.
5.	$2c + 3c = 6$	$c = \mathbf{1.2}$	The cost for one child was **£1.20**.

1094 Volume of Prisms

1. $5 \times 5 \times 5 = 125\text{cm}^3$

2. $\pi \times 2^2 \times 6 = 75.40\text{cm}^3$
   or $3.14 \times 2^2 \times 6 = 75.36\text{cm}^3$

3. $\frac{1}{2}(4 \times 3) \times 6 = 36\text{cm}^3$

4. $3 \times 7 \times 4 = 84\text{cm}^3$

5. $\pi \times 3^2 \times 7 = 197.92\text{cm}^3$
   or $3.14 \times 3^2 \times 7 = 197.82\text{cm}^3$

6. $2 \times 7 \times 5 = 70\text{cm}^3$

7. $\frac{1}{2}(\pi \times 2^2 \times 8) = 50.27\text{cm}^3$
   or $\frac{1}{2}(3.14 \times 2^2 \times 8) = 50.24\text{cm}^3$

8. $\frac{1}{2}(1 + 3) \times 3 \times 8 = 48\text{cm}^3$

9. $[(3 \times 3) + (3 \times 2)] \times 8 = 120\text{cm}^3$

10. $\frac{1}{4}(\pi \times 4^2 \times 5.25) = 65.97\text{cm}^3$
    or $\frac{1}{4}(3.14 \times 4^2 \times 5.25) = 65.94\text{cm}^3$

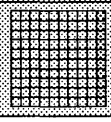
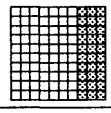
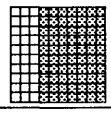
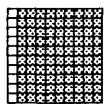
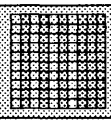
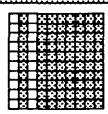
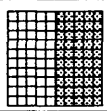
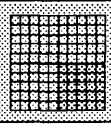
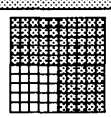
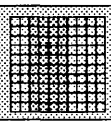
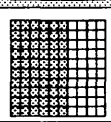
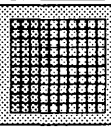
1095 Percentages

A

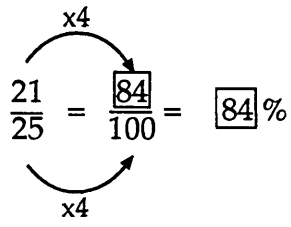
1. 9%
2. 13%
3. 19%
4. 53%
5. 61%
6. $\frac{50}{100} = 50\%$
7. $\frac{25}{100} = 25\%$
8. 100%

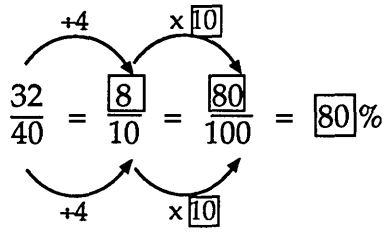
continued/

B In each of these it doesn't matter *which* squares you shaded, but you should have shaded the *same number* of squares.

	Shading	Fraction	Decimal		%
1.		$\frac{1}{10}$	0.1	$\frac{10}{100}$	10%
2.		$\frac{3}{10}$	0.3	$\frac{30}{100}$	30%
3.		$\frac{7}{10}$	0.7	$\frac{70}{100}$	70%
4.		$\frac{9}{10}$	0.9	$\frac{90}{100}$	90%
5.		$\frac{6}{10}$	0.6	$\frac{60}{100}$	60%
6.		$\frac{8}{10}$	0.8	$\frac{80}{100}$	80%
7.		$\frac{1}{2}$	0.5	$\frac{50}{100}$	50%
8.		$\frac{1}{4}$	0.25	$\frac{25}{100}$	25%
9.		$\frac{3}{4}$	0.75	$\frac{75}{100}$	75%
10.		$\frac{2}{5}$	0.4	$\frac{40}{100}$	40%
11.		$\frac{3}{5}$	0.6	$\frac{60}{100}$	60%
12.		$\frac{7}{20}$	0.35	$\frac{35}{100}$	35%

1096 Marks to percentages





Autumn term	
Physical Education	76%
English	68%
French	70%
History	68%
Geography	50%
Mathematics	82%
Science	70%
Art	70%
Music	50%
Technology	90%

Summer term	
Physical Education	85%
English	80%
French	84%
History	80%
Geography	65%
Mathematics	90%
Science	70%
Art	68%
Music	55%
Technology	90%

1. Summer term

2. They were the same.

3. Art

4. Physical Education, English, French, History, Geography, Mathematics, Music

5. Geography

1097 Fractions to Percentages

A 1. $\frac{19}{20} = \frac{95}{100} = 0.95 = 95\%$

2. $\frac{13}{20} = \frac{65}{100} = 0.65 = 65\%$

3. $\frac{3}{5} = \frac{60}{100} = 0.6 = 60\%$

continued/

1097 Fractions to Percentages (cont)

4. $\frac{18}{25} = \frac{72}{100} = 0.72 = 72\%$

5. $\frac{23}{25} = \frac{92}{100} = 0.92 = 92\%$

6. $\frac{36}{60} = \frac{60}{100} = 0.6 = 60\%$

7. $\frac{69}{75} = \frac{23}{25} = \frac{92}{100} = 0.92 = 92\%$

8. $\frac{26}{40} = \frac{13}{20} = \frac{65}{100} = 0.65 = 65\%$

9. $\frac{102}{120} = \frac{17}{20} = \frac{85}{100} = 0.85 = 85\%$

10. $\frac{26}{40} = \frac{13}{20} = \frac{65}{100} = 0.65 = 65\%$

B

1. 0.75 = 75%
2. 0.375 = 37.5%
3. 0.625 = 62.5%
4. 0.875 = 87.5%
5. 0.0625 = 6.25%

C

A $\frac{42}{150} = 0.28 = 28\%$

B $\frac{128}{320} = 0.40 = 40\%$

C $\frac{54}{180} = 0.30 = 30\%$

D $\frac{84}{480} = 0.175 = 17.5\%$

E $\frac{27}{72} = 0.375 = 37.5\%$

continued/

F $\frac{64}{256} = 0.25 = 25\%$

If you wanted a good chance of passing your MOT, you would probably choose garage D. However, the low percentage of failures at this garage may indicate that they are not so strict as the others and allow through unsafe cars. In this case you would probably choose garage B, which may have the highest standards.

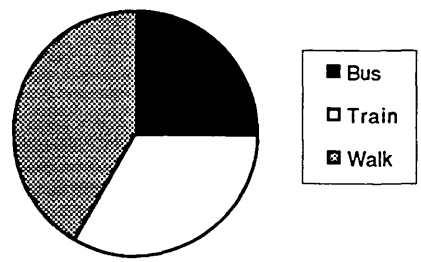
D

Gloucester	$\frac{6400}{8000}$	= 80%
Harrogate	$\frac{2220}{6000}$	= 37%
Jarrow	$\frac{2100}{7500}$	= 28%
Keswick	$\frac{2550}{15000}$	= 17%
Leicester	$\frac{11970}{21000}$	= 57%
Margate	$\frac{1140}{3800}$	= 30%

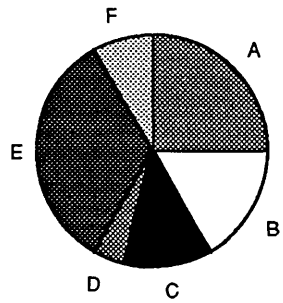
1101 Pie Charts

- $\frac{5}{8}$ of the children have school dinners.
  75 children have school dinners.

- $\frac{1}{4}$ of 60 = **15** children travel by bus.
- $\frac{1}{3}$ of 60 = **20** children travel by train.
- $\frac{5}{12}$ of 60 = **25** children walk.



1. a)



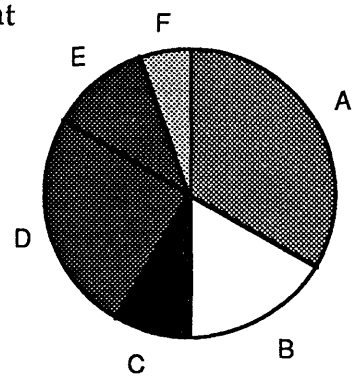
b)

A	60 students	D	**10** students
B	**40** students	E	**80** students
C	**30** students	F	**20** students

continued/

1101 Pie Charts (cont)

2. In the college, there were 360 students, so each student is represented by 1° on the pie chart.



Sector A is 120°
Sector B is 60°
Sector C is 30°
Sector D is 90°
Sector E is 40°
Sector F is 20°

3. In the primary school, there were 180 children, so each child is represented by 2° on the pie chart.

Sector	Angle	Votes
A	60°	30
B	120°	60
C	90°	45
D	30°	15
E	40°	20
F	20°	10

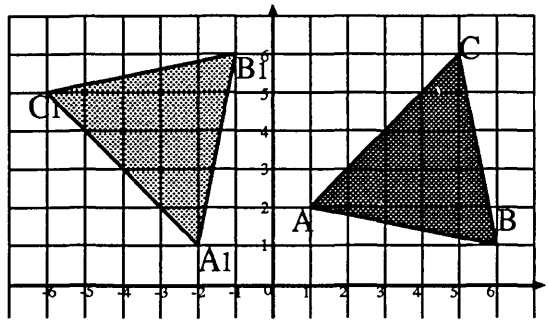
4.

	A	B	C	D	E	F
Secondary school	60	40	30	10	80	20
College	120	60	30	90	40	20
Primary school	30	60	45	15	20	10
Total	**210**	**160**	**105**	**115**	**140**	**50**

The mode chocolate is type **A**.

1112 Rotation

1. a)



b) A (1, 2) → A_1 (-2, 1)
B (6, 1) → B_1 (-1, 6)
C (5, 6) → C_1 (-6, 5)

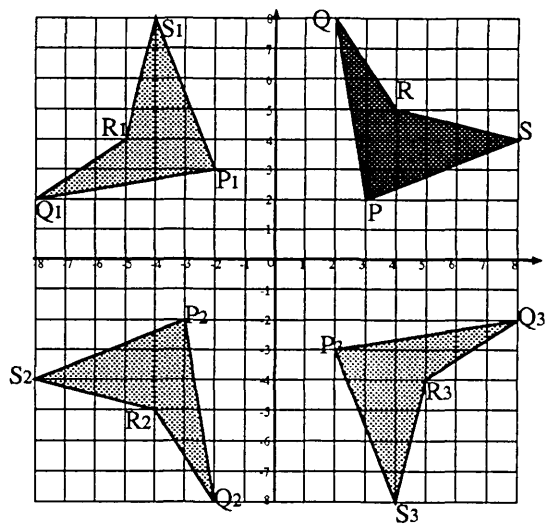
continued/

2. b) P (3, 2) → P_1 **(-2, 3)**
Q (2, 8) → Q_1 **(-8, 2)**
R (4, 5) → R_1 **(-5, 4)**
S (8, 4) → S_1 **(-4, 8)**

3. b) P (3, 2) → P_2 (-3, -2)
Q (2, 8) → Q_2 (-2, -8)
R (4, 5) → R_2 (-4, -5)
S (8, 4) → S_2 (-8, -4)

4. b) P (3, 2) → P_3 (2, -3)
Q (2, 8) → Q_3 (8, -2)
R (4, 5) → R_3 (5, -4)
S (8, 4) → S_3 (4, -8)

2, 3, 4, a)



A On your graph 1. 36°F should be equal to 2°C and 2. -17°C should be equal to 2°F, if they are not equal you should redraw your graph as it is not accurate.

B
1. 158°F
2. 36°F
3. 58°F
4. 200°F
5. 100°F
6. 164°F

C
1. 16°C
2. 48°C
3. 96°C
4. 74°C
5. 28°C
6. 66°C

- Newspapers give up to date information on foreign currency.
Here is the information for converting miles to kilometres and pounds to kilograms. The figures in the centre column can be used for both conversions.
e.g. 1 mile = 1.61km
1km = 0.62 miles

miles		km
0.62	1	1.61
1.24	2	3.22
1.86	3	4.83
2.49	4	6.44
3.11	5	8.05
.	.	.
.	.	.
.	.	.

lb		kg
2.20	1	0.45
4.41	2	0.91
6.61	3	1.36
8.82	4	1.81
11.02	5	2.27
.	.	.
.	.	.
.	.	.

1123 Translation

Translation -5 parallel to x-axis

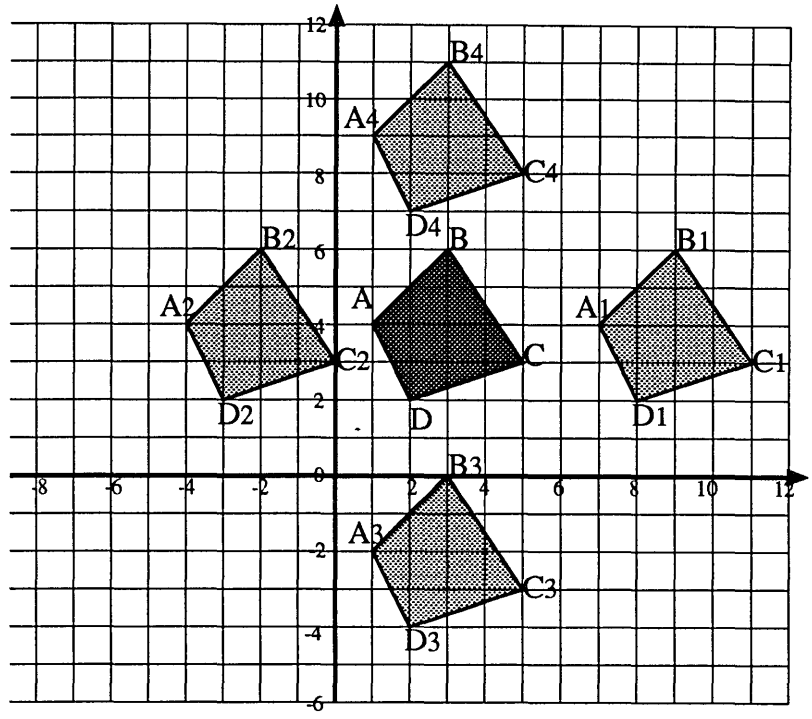
A (1, 4) → A_2 (-4, 4)
B (3, 6) → B_2 (-2, 6)
C (5, 3) → C_2 (0, 3)
D (2, 2) → D_2 (-3, 2)

Translation -6 parallel to y-axis

A (1, 4) → A_3 (1, -2)
B (3, 6) → B_3 (3, 0)
C (5, 3) → C_3 (5, -3)
D (2, 2) → D_3 (2, -4)

Translation +5 parallel to y-axis

A (1, 4) → A_4 (1, 9)
B (3, 6) → B_4 (3, 11)
C (5, 3) → C_4 (5, 8)
D (2, 2) → D_4 (2, 7)

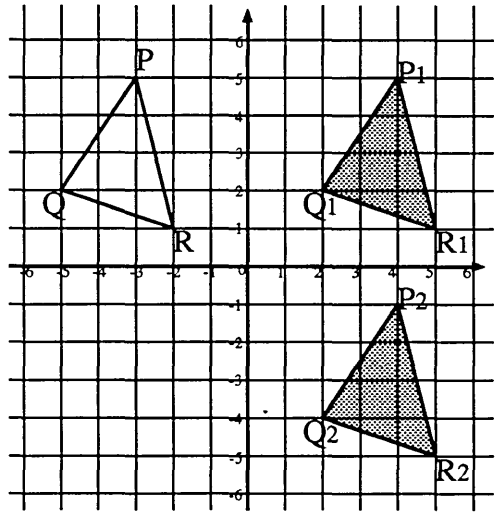


Translation 1, +7 parallel to x-axis

P (-3, 5) → P_1 (4, 5)
Q (-5, 2) → Q_1 (2, 2)
R (-2, 1) → R_1 (5, 1)

Translation 2, -6 parallel to y-axis

P_1 (4, 5) → P_2 (4, -1)
Q_1 (2, 2) → Q_2 (2, -4)
R_1 (5, 1) → R_2 (5, -5)



Combined translations, +7 parallel to x-axis followed by -6 parallel to y-axis.

P (-3, 5) → P_2 (4, -1)
Q (-5, 2) → Q_2 (2, -4)
R (-2, 1) → R_2 (5, -5)

1127 Time-Distance Graphs

Reasons for it being impossible to travel at a constant speed include:

- *when you start cycling it is harder than when you are travelling along, you slowly build up speed.*
- *it is necessary to slow down for traffic lights, pedestrian crossings, traffic jams, etc.*

1. The cyclist reached her destination at **12 noon**, she was **30km** from home.

2. She took her first rest at **10.30 a.m.** She rested for **30 minutes**.

continued/

3. She was **17km** from home.

4. She travelled **13km** between 11 a.m. and 12 noon.

5. Her average speed
   a) between 9 a.m. and 10 a.m. was **10 kilometres per hour (km/hr)**
   b) between 10 a.m. and 10.30 a.m. was **14km/hr**

6. She took a rest between **12 noon** and **1 p.m.**

7. She had **30km** to travel home.

8. Her average speed on her return journey was **15km/hr.**

9. a) At 11.30 a.m. she was **23.5km** from home.
   a) At 1.30 p.m. she was **22.5km** from home.

10. a) She was 13km from home at **10.12 a.m.**
    b) She was 22km from home at **11.24 a.m.**

A 1. The destination was **240km** from the motorist's home. She arrived at **2.30 p.m.**

2. a) She was probably travelling on the motorway between **10.30a.m.** and **11.00 a.m.** and between **1.30 p.m.** and **2.00 p.m.** (When the gradient of the graph is the steepest.)
   b) She was probably involved in a traffic delay **1.00 p.m.** and **1.30 p.m.** (When the gradient of the graph is the least steep.)

3. She took her first stop at **10 a.m.** She was **96km** from home.

4. The motorist's distance from home:
   a) at 9.30 a.m. was **68km.**
   b) at 1.30 p.m. was **184km.**
   c) at 11.30 a.m. was **156km.**
   d) at 1.45 p.m. was **200km.**
   e) at 12.30 p.m. was **172km.**

5. The motorist's distance from home:
   a) was 200km at **1.45 p.m.**
   b) was 96km at **10 a.m. - 10.30 a.m.**
   c) was 140km at **11 a.m.**
   d) was 224km at **2.06 p.m.**
   e) was 36km at **8.54 a.m.**

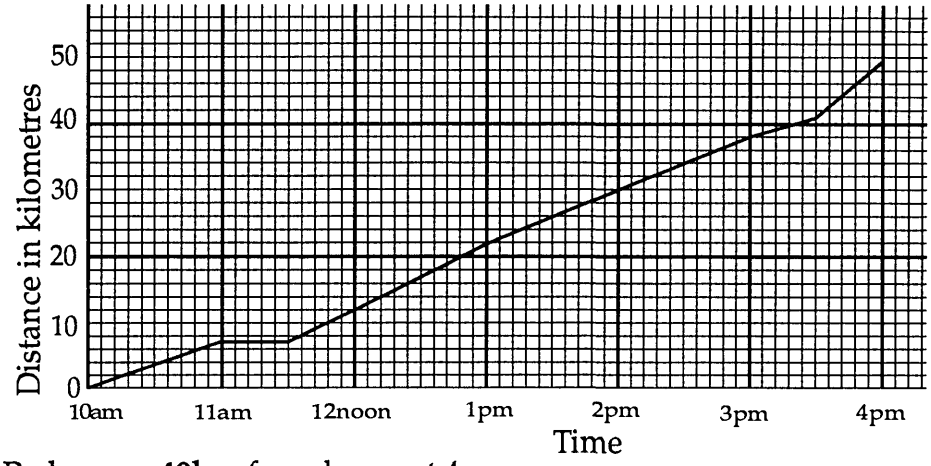
6. The motorist's average speed between:
   a) 9 a.m. and 10 a.m. was **56 km/hr.**
   b) 10.30 a.m. and 11 a.m. was **88 km/hr.**
   c) 1 p.m. and 1.30 p.m. was **24 km/hr.**
   d) 1.30 p.m. and 2 p.m. was **72 km/hr.**
   e) 2 p.m. and 2.30 p.m. was **40 km/hr.**

7. The motorist's average speed for the whole journey was **48 km/hr.**

continued/

1127 Time-Distance Graphs (cont)

B Time-distance graph for Pedro's journey



Pedro was **49km** from home at 4 p.m.

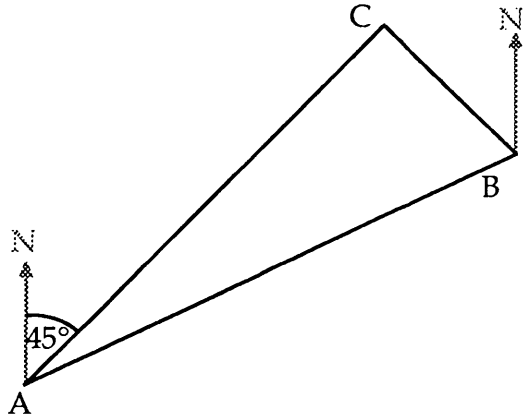
1130 Journeys

Different rulers and angle indicators or protractors will give slightly different answers. It is impossible to be exactly accurate when measuring, there is always some degree of error. If your answers are within 3 degrees above or below the answers given and the distances are within 2mm (converted using the appropriate scale) mark your answers as correct.

1. 910km, bearing 090°.

2. 910km, bearing 280°.

3. a)



b) 14km, bearing 045°

	A to B		B to C		A to C	
	Distance	Bearing	Distance	Bearing	Distance	Bearing
4.	700km	175°	**1400km**	**052°**	1200km	082°
5.	500km	090°	375km	250°	**200km**	**131°**
6.	185km	110°	260km	270°	**110km**	235°
7.	1000km	220°	1640km	020°	780km	**354°**

8. 66km, bearing 338°

1132 What's the Probability?

1. a) $\frac{1}{6}$ b) $\frac{1}{6}$ c) $\frac{3}{6} = \frac{1}{2}$ d) $\frac{2}{6} = \frac{1}{3}$

2. a) $\frac{4}{52} = \frac{1}{13}$ b) $\frac{4}{52} = \frac{1}{13}$ c) $\frac{13}{52} = \frac{1}{4}$ d) $\frac{26}{52} = \frac{1}{2}$

3. a) $\frac{1}{3}$ b) $\frac{1}{3}$ c) $\frac{2}{3}$

4. a) 1 b) 0

5. a) $\frac{1}{10}$ b) $\frac{5}{10} = \frac{1}{2}$ c) $\frac{4}{10} = \frac{2}{5}$ d) $\frac{4}{10} = \frac{2}{5}$

6. It is not sensible, as although there are three possible outcomes; home win, draw, or away win, they are not all equally likely.

1136 Solving Equations

A	1.	$x = 1.5$	2.	$x = 5$	3.	$x = 4$	4.	$x = 1.5$	5.	$x = -1$
B	1.	$x = 5$	2.	$x = 7$	3.	$x = -2$	4.	$x = 6$	5.	$x = 15$
C	1.	$x = 3$	2.	$x = 2$	3.	$x = 2.5$	4.	$x = 11$	5.	$x = -2$
D	1.	$x = 4.5$	2.	$x = -1.5$	3.	$x = 3$	4.	$x = 6.5$	5.	$x = 11.5$
	6.	$x = 13$	7.	$x = 8$	8.	$x = -12$	9.	$x = 4$	10.	$x = 17$

1137 Solving Harder Equations

A	1.	$x = 6$	2.	$x = -6$	3.	$x = 1.5$	4.	$x = -2$	5.	$x = 2.25$
B	1.	$x = 3$	2.	$x = 3$	3.	$x = 2.4$	4.	$x = 2.4$	5.	$x = -5$
	6.	$x = -6$	7.	$x = -5$	8.	$x = -2$	9.	$x = 1.5$	10.	$x = 1.\dot{3}$
C	1.	$x = 2$	2.	$x = 5$	3.	$x = 1.5$	4.	$x = 4$	5.	$x = 2$
	6.	$x = 3$	7.	$x = 2$	8.	$x = 0.5$	9.	$x = -2$	10.	$x = -13$
D	1.	$x = 2$	2.	$x = -4$	3.	$x = 1.5$	4.	$x = 1$	5.	$x = -1.5$
	6.	$x = -4$	7.	$x = 10$	8.	$x = 2.\dot{3}$	9.	$x = 2$	10.	$x = -1$

1156 Transformations

1. Reflection, rotation and translation.

2. i) a) Rotation of 180° about O (0, 0).
   b) Reflection in x axis followed by reflection in y axis
   or Reflection in y axis followed by reflection in x axis.

   ii) Rotation of 180° about O (0, 0).

3. i) Translation -7 parallel to x axis followed by translation -5 parallel to y axis.
   ii) Yes, the order of the two translations doesn't make any difference.

4. i) Reflection in x axis followed by rotation through 90° anti-clockwise about O.
   or Reflection in y axis followed by rotation through 90° clockwise about O.
   or Rotation through 90° clockwise about O followed by reflection in y - axis.
   or Rotation through 90° anti-clockwise about O followed by reflection in x - axis.

   ii) Reflection in x axis followed by rotation through 90° anti-clockwise about O.
   or Reflection in y axis followed by rotation through 90° clockwise about O.
   or Rotation through 90° clockwise about O followed by reflection in y - axis.
   or Rotation through 90° anti-clockwise about O followed by reflection in x - axis.

5. i) Reflection in x axis followed by rotation through 90° clockwise about O.
   or Reflection in y axis followed by rotation through 90° anti-clockwise about O.
   or Rotation through 90° clockwise about O followed by reflection in x - axis.
   or Rotation through 90° anti-clockwise about O followed by reflection in y - axis.

   ii) Reflection in x axis followed by rotation through 90° clockwise about O.
   or Reflection in y axis followed by rotation through 90° anti-clockwise about O.
   or Rotation through 90° clockwise about O followed by reflection in x - axis.
   or Rotation through 90° anti-clockwise about O followed by reflection in y - axis.

6. i) Rotation through 90° clockwise about O followed by translation 5 units parallel to x axis.
   or Translation 5 units parallel to y axis followed by rotation through 90° clockwise about O.

   ii) Rotation through 90° anti-clockwise about O followed by translation -5 units parallel to y axis.
   or Translation -5 units parallel to x axis followed by rotation through 90° anti-clockwise about O.

Here are some facts you may have included in your summary:

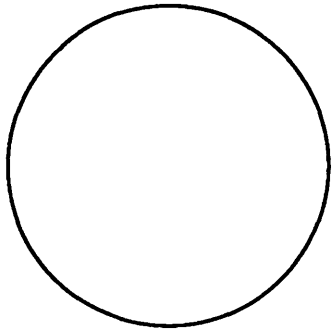
- 'A reflection followed by a rotation' is restored to original position by repeating the same transformations.
- 'A rotation followed by a reflection' is restored to original position by repeating the same transformations.
- 'A translation followed by another translation' will have the same effect whatever order.
- 'A translation' is restored to original position by an equal and opposite translation.

1170 Compass Constructions

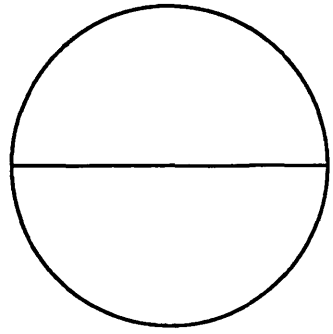
Here are some hints to help you start to reconstruct the designs using a **ruler** and **compass**.

Top design

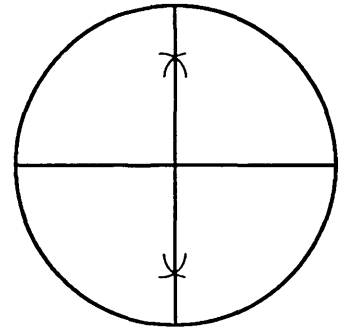
Start with a circle . . .



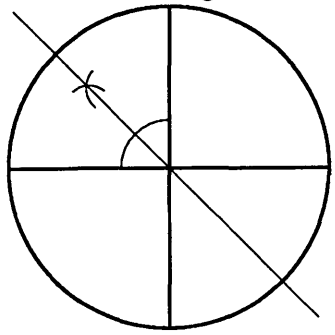
. . . draw in a diameter . . .



. . . construct the perpendicular bisector *(see Smile 0211)*. . .



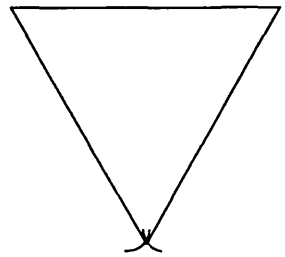
. . . bisect the angle *(see Smile 0212)* . . .



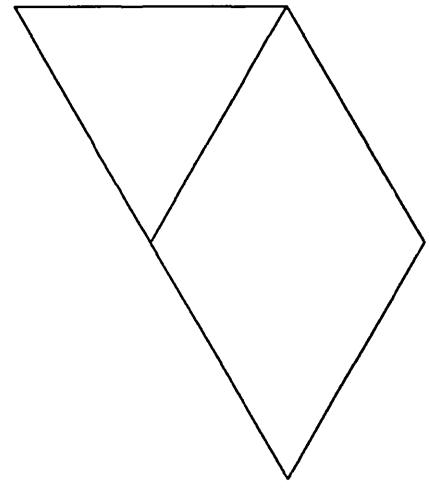
Bottom design

Start with a straight line . . .

. . . construct an equilateral triangle *(see Smile 1287)* . . .



. . . adapt this method to construct a rhombus. . . .



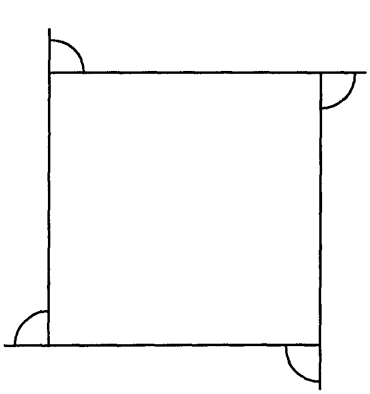
continued/

1170 Compass Constructions (cont)

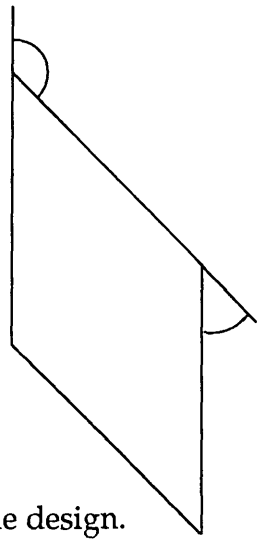
If you are creating the designs in Logo a different approach is needed.

Top design

Create a procedure to draw a square . . .

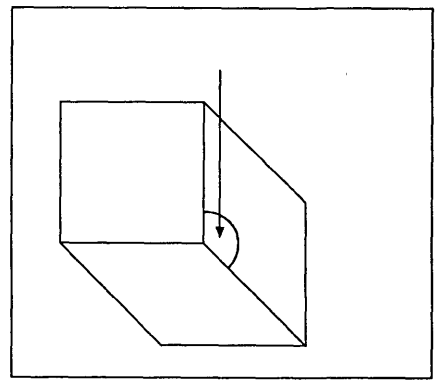


. . . and one to draw a rhombus with sides the same length . . .



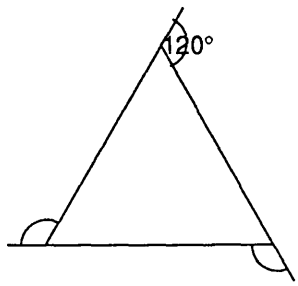
. . . combine them to create the design.

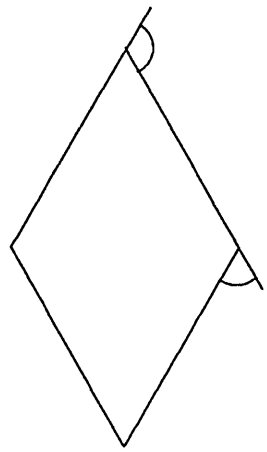
What is the size of this angle?



Bottom design

Create procedures to draw an equilateral triangle and a rhombus . . .





1177 Vectors

1. i) $\overrightarrow{NM} = \mathbf{a}$ $\overrightarrow{LM} = \mathbf{b}$

   ii) $\mathbf{a} + \mathbf{b}$

2. $\mathbf{a} + \mathbf{b}$

3. $\overrightarrow{LM} = \mathbf{a}$ $\overrightarrow{KM} = 2\mathbf{a}$

continued/

1177 Vectors (cont)

4. $\overrightarrow{PR} = \frac{1}{2}\mathbf{b}$ $\overrightarrow{RQ} = \frac{1}{2}\mathbf{b}$

5. i) $\overrightarrow{TV} = \mathbf{a} + \mathbf{b}$ ii) $\overrightarrow{VT} = -(\mathbf{a} + \mathbf{b})$

1. $\mathbf{a} - \mathbf{b}$ 2. $\mathbf{b} + \mathbf{a}$

3. $\mathbf{b} - \mathbf{a}$ 4. $\mathbf{a} + \mathbf{b}$

5. i) $\overrightarrow{BC} = \mathbf{a} + \mathbf{b}$ ii) D is the midpoint of BC. Can you see why?
$\overrightarrow{BD} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$

1178 More Vectors

a
- $\overrightarrow{OP} = \mathbf{a} + \mathbf{c}$
- $\overrightarrow{OP} = \mathbf{b} - \mathbf{c}$

Adding
- $2\overrightarrow{OP} = \mathbf{a} + \mathbf{b}$

so $\overrightarrow{OP} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$

b
- $\overrightarrow{XP} = \frac{1}{2}\mathbf{z} - \frac{1}{2}\mathbf{y}$
- $\overrightarrow{ZR} = \frac{1}{2}\mathbf{y} - \frac{1}{2}\mathbf{x}$
- $\overrightarrow{YQ} = \frac{1}{2}\mathbf{x} - \frac{1}{2}\mathbf{z}$

Adding
- $\overrightarrow{XP} + \overrightarrow{ZR} + \overrightarrow{YQ} = 0$

c
- $\overrightarrow{OW} = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

1179 Column Vectors

1. a) i) $\overrightarrow{BA} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

ii) $\overrightarrow{BM} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

b) (5, 3)

c) $\sqrt{34}$

d) $\frac{3}{5}$

2. a) i) $\mathbf{b} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$ ii) $\mathbf{c} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ iii) $\mathbf{d} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ iv) $\mathbf{a} + \mathbf{c} = \begin{pmatrix} 2 \\ 9 \end{pmatrix}$

b) i) $\overrightarrow{OB} = \mathbf{m} + \mathbf{n}$ ii) $\overrightarrow{AC} = \mathbf{n} + \mathbf{s}$

iii) $\overrightarrow{AD} = \mathbf{n} + \frac{1}{3}\mathbf{s}$

c) $\tan^{-1} \frac{7}{2} = 74°$

1202 Significant Figures

A

1.	a)	$50cm^2$	6.	a)	8.9kg	11.	a)	41kg
	b)	$49.7cm^2$		b)	8.94kg		b)	41.0kg
2.	a)	£280 000	7.	a)	11m	12.	a)	21 litres
	b)	£284 000		b)	10.8m		b)	20.8 litres
3.	a)	7.8cm	8.	a)	£38 000	13.	a)	0.90km
	b)	7.82cm		b)	£37 700		b)	0.901km
4.	a)	0.067m	9.	a)	$40cm^3$	14.	a)	5.9m
	b)	0.674m		b)	$40.0cm^3$		b)	5.94m
5.	a)	0.00048	10.	a)	$0.71m^2$	15.	a)	$11cm^2$
	b)	0.000484		b)	$0.707m^2$		b)	$10.9cm^2$

continued/

1202 Significant Figures (cont)

B

1.	0.667	6.	0.714
2.	0.833	7.	0.917
3.	0.455	8.	0.538
4.	0.0875	9.	0.385
5.	0.417	10.	0.143

1208 Percentage Sales

	Profit	Selling Price
1.	20p	£1.00
2.	17p	£1.87
3.	£1.62	£7.02
4.	75p	£5.75
5.	£1.94	£11.64

	Reduction	Sale Price
1.	7p	63p
2.	£4.50	£13.50
3.	£3.10	£12.40
4.	£2.90	£5.80
5.	66p	£3.74

1233 Frequency Graphs

- The mean of £25 is misleading as only eight employees earn this amount or more, while 15 earn less than £25.

1.

No. of absentees x Frequency			Total
11	x	1	11
9	x	2	18
8	x	1	8
6	x	7	42
4	x	10	40
3	x	5	15
1	x	1	1
		Total	**135**

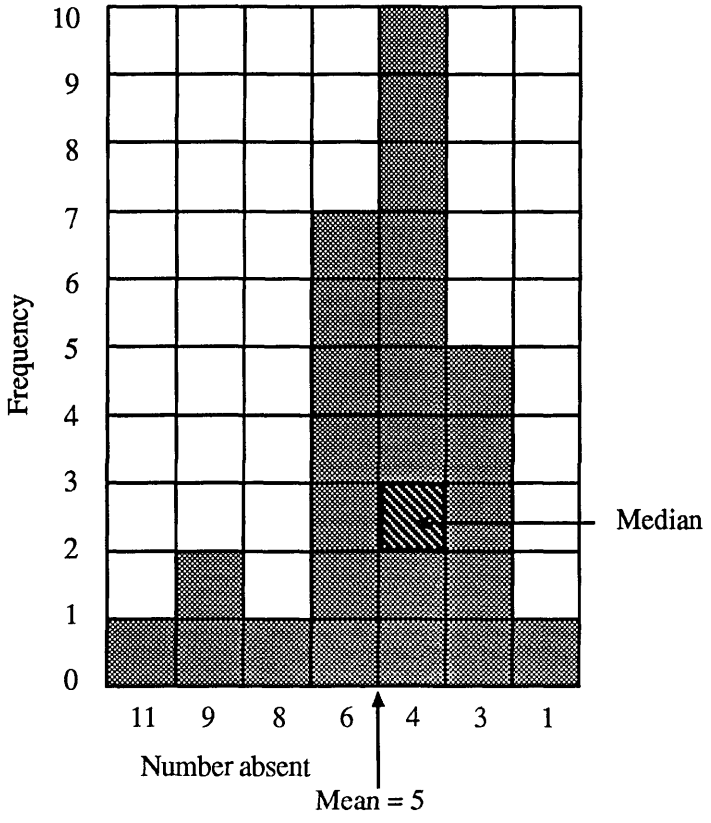
Total number of tutor groups. **27** absentees

continued/

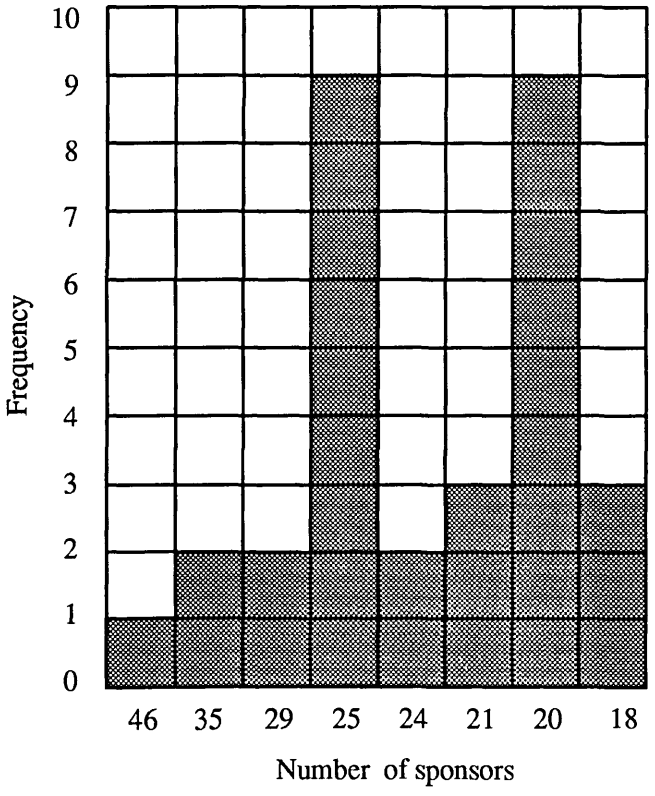
2. Mean number of absentees per tutor group = 135 ÷ 27 = **5**

3. Median number of absentees per tutor group = **4**

4.& 5.



1.



continued/

1233 Frequency Graphs (cont)

2.

No.of sponsors	x	Frequency	Total
46	x	1	46
35	x	2	70
29	x	2	58
25	x	9	225
24	x	2	48
21	x	3	63
20	x	9	180
18	x	3	54
		Total	**744**

Total number of walkers: **31**

sponsors

Mean number of sponsors = 744 ÷ 31 = **24**

3. Median number of sponsors = **24**

4. The mean and the median are both 24.

1257 Volume of cuboids

1. a) $180cm^3$ b) $44mm^3$
   c) $62.5cm^3$ d) $52.08m^3$

2. $540cm^3$

3. $10m^3$

4. Although the volumes are both $60cm^3$, the density of sugar is greater than that of tea, so you would expect the sugar to weigh more.

5. To calculate the volume the measurements must all be in the same units:
   either 5m by 1m by 0.5m, giving a volume of $2.5m^3$
   or 500cm by 100cm by 50cm, giving a volume of $2500000cm^3$

6. $1cm^3$ = 2cm x 2cm x $x$cm, where x is the distance between graduation marks.
   x = **0.25cm**

7. $50cm^3$ or $50000mm^3$

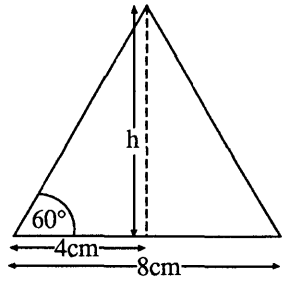
8. 60 packs would fit, they would weigh 15kg

9. 80 matchboxes

10. 1 litre = $1000cm^3$
    $10m^2 = 100000cm^2$
    Thickness = 1000 ÷ 100000 = 0.01cm

1258 The Biggest Vase

Equilateral triangle base: each side is 8cm



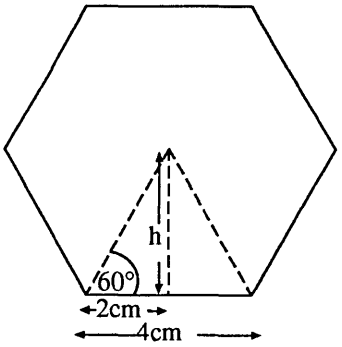
$h = 4 \tan 60°$
$= 6.928$

Area of triangle $= 4 \times 6.928$
$= \mathbf{27.71}\text{cm}^2$ to 2 d.p.

Square base: each side is 6cm

Area of square $= 6 \times 6$
$= \mathbf{36}\text{cm}^2$

Regular hexagon base: each side is 4cm

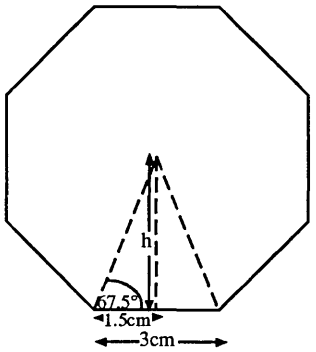


$h = 2 \tan 60°$
$= 3.464$

Area of triangle $= 2 \times 3.464$
$= 6.928\text{cm}^2$

Area of hexagon $= 6 \times 6.928$
$= \mathbf{41.57}\text{cm}^2$ to 2 d.p.

Regular octagon base: each side is 3cm



$h = 1.5 \tan 67.5°$
$= 3.621$

Area of triangle $= 1.5 \times 3.621$
$= 5.432\text{cm}^2$

Area of octagon $= 8 \times 5.432$
$= \mathbf{43.46}\text{cm}^2$ to 2 d.p.

- The four vases all have the same height, so the vase which will hold the most water is the one with the largest base area, the one with the regular octagon base.
- As the number of sides increases, the area of the base increases, so you would expect a vase with a circular base (with an infinite number of sides) to hold even more water than the regular octagon base vase.
- Perimeter of circle is 24cm and $C = 2\pi r$.

$2 \times \pi \times r = 24$

$r = \dfrac{24}{2\pi}$

$r = 3.820$

Area of circle $= \pi r^2$
$= \pi \times 3.820^2$
$= 45.84\text{cm}^2$

This confirms the prediction that the cylindrical vase would hold the most water.

1259 Lengths of Similar Objects

1. 390cm = 3.9m

2. 2000cm = 20m

3.

real object	scale model
320m	1.6m
35m	**0.175m or 17.5cm**
10m or 1000cm	5cm

4. 3 : 360 = 1 : 120 Yes, the model is similar to the real bus.

5. Model height = 4.5cm, model length = 6cm

6. a) 6cm by 5cm by 3.5cm
   b) 18cm by 8cm
   c) 12cm by 12.5cm by 5cm

7. a) 4cm
   b) 2.5cm

8. a) 1cm by 1.2cm
   b) 6.6cm by 2cm
   c) 12cm by 10cm

9. a) i) 1cm represents 50 000cm = 0.5km
      ii) 5cm represents 250 000cm = 2.5km

   b) 8cm represents 160 000cm = 1.6km

10. 50°, 40°, 90°

1261 Volumes of Similar Objects

1.

	Side length	Surface Area	Volume
A	1cm	**6cm²**	**1cm³**
B	3cm	**54cm²**	**27cm³**
C	5cm	**150cm²**	**125cm³**

2.

	Side length	Surface Area	Volume
A to B	1 : 3	**1 : 9**	**1 : 27**
A to C	**1 : 5**	1 : 25	**1 : 125**
C to B	**5 : 3**	**25 : 9**	125 : 27

continued/

1261 Volumes of Similar Objects (cont)

3.

	Side length	Surface Area	Volume
D to E	1 : 3	**1 : 9**	**1 : 27**

	Side length	Surface Area	Volume
D	**2cm**	**24cm²**	**8cm³**
E	**6cm**	**216cm²**	**216cm³**

These values agree with the ratios above.

- The similar solid with enlargement scale factor 2, uses 32 cubes.

- The ratio of corresponding surface areas is 1 : 4.
- The ratio of corresponding volumes is 1 : 8.

- The ratio of corresponding side lengths is 1 : 3.
- The ratio of corresponding surface areas is 1 : 9.
- The ratio of corresponding volumes is 1 : 27.

In general, if you enlarge a solid by scale factor x.

- The ratio of corresponding side lengths is $1 : x$.
- The ratio of corresponding surface areas is $1 : x^2$.
- The ratio of corresponding volumes is $1 : x^3$.

1267 Cumulative Frequency from Grouped Data

- From the cumulative frequency graph, the median mark gained in the exam is **45.5** marks.
- From the cumulative frequency graph, **27** students gained a mark of 75 or less.

- The actual median was **45.9** marks.
- **27** students actually gained a mark of 75 or less.

The estimates from the cumulative frequency graph, were very close to the actual results.

- A child who cycled 40.499km would be placed in the 21 - 40 class.
- A child who cycled 40.503km would be placed in the 41 - 60 class.

- The median distance cycled was approximately **46**km.
- Approximately **15** children cycled 45km or less.

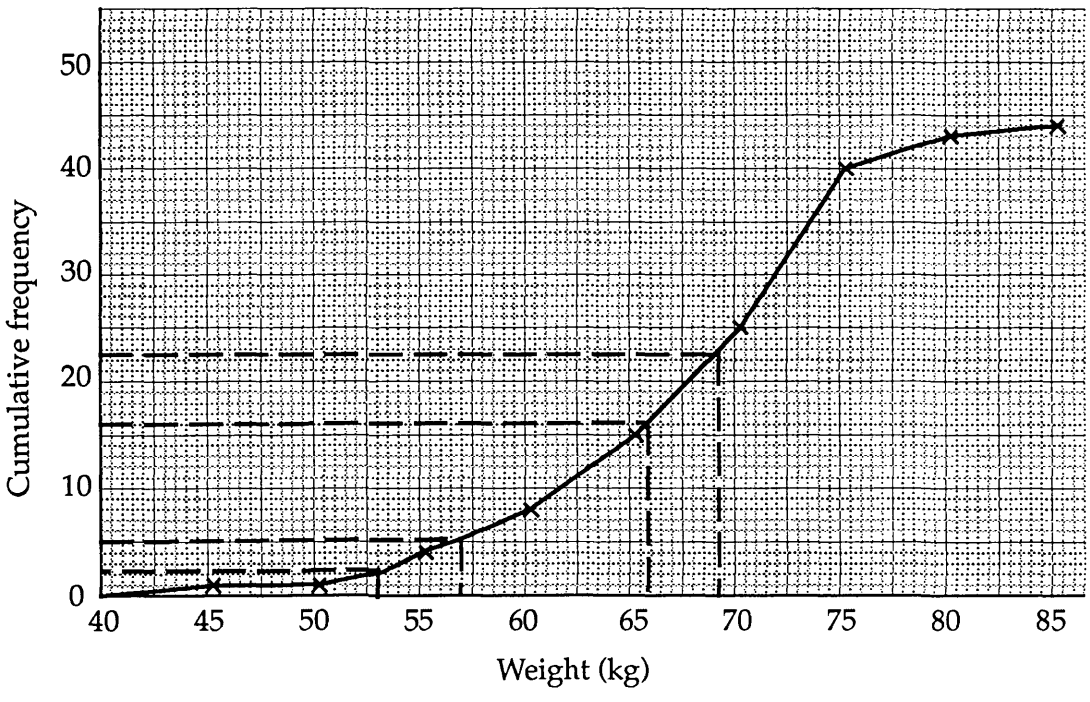
continued/

1267 Cumulative Frequency from Grouped Data (cont)

1. ▩ Weight is an example of continuous data.

▩

Weight of students (kg)	Cumulative Frequency	Plot
less than 45.5	1	(45.5, 1)
less than 50.5	1	(50.5, 1)
less than 55.5	4	(55.5, 4)
less than 60.5	8	(60.5, 8)
less than 65.5	15	(65.5, 15)
less than 70.5	25	(70.5, 25)
less than 75.5	40	(75.5, 40)
less than 80.5	43	(80.5, 43)
less than 85.5	44	(85.5, 44)



▩ The median weight of the students is 69.5kg to the nearest kg.

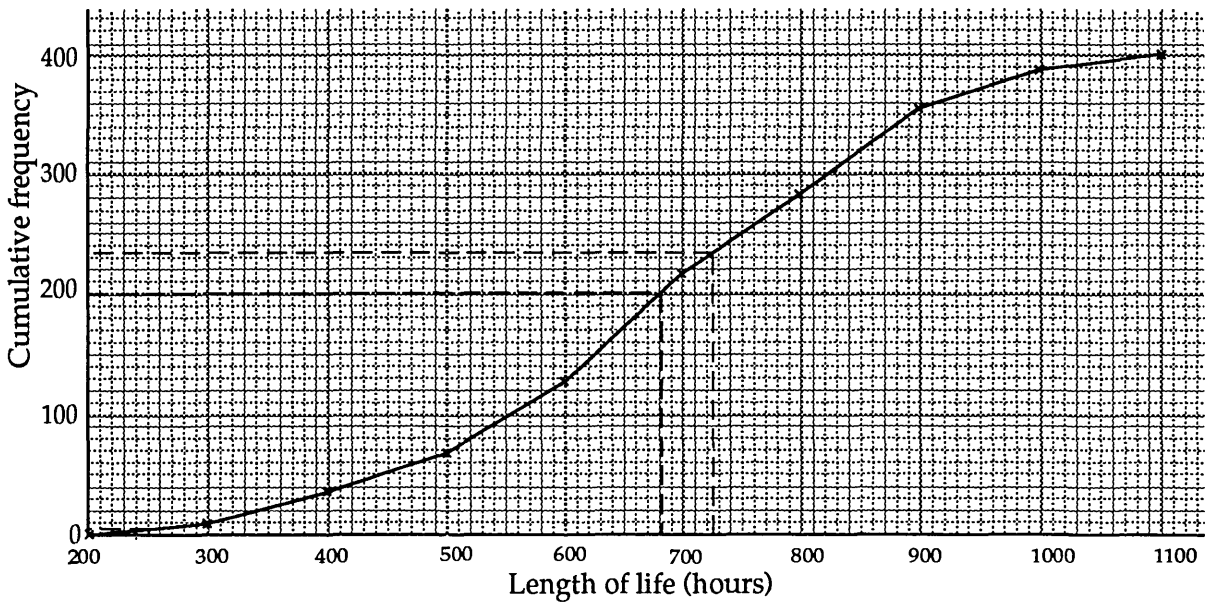
▩ *These answers are given to the nearest whole student.*

a) 2 students b) 5 students c) 16 students

continued/

2. Length of life of light bulbs is an example of continuous data.

Length of life (hrs)	Cumulative Frequency
less than 199.5	0
less than 299.5	10
less than 399.5	36
less than 499.5	68
less than 599.5	128
less than 699.5	216
less than 799.5	292
less than 899.5	354
less than 999.5	388
less than 1099.5	400



a) 5 bulbs b) 235 bulbs

The median life of the bulbs is 682 hours.

a) Examples of discrete data are; number of brothers and sisters, shoe size, age next birthday, monthly salary, amount of money in pocket, etc.

b) Examples of continuous data are; height, distance lived from school, length of foot, time taken to travel to work, etc.

1269 Probability

Picking a card from a full pack of playing cards.

1. a) $\frac{1}{52}$

   b) $\frac{51}{52}$ $\qquad \frac{1}{52} + \frac{51}{52} = 1$

2. a) $\frac{4}{52} = \frac{1}{13}$

   b) $\frac{48}{52} = \frac{12}{13}$

3. $\frac{13}{52} = \frac{1}{4}$

4. $\frac{39}{52} = \frac{3}{4}$

The sum of these answers ($\frac{1}{4} + \frac{3}{4}$) is 1, since the events are mutually exclusive and between them account for all the possible outcomes.

Picking a card from a pack of playing cards with one card missing.

5. a) $\frac{13}{51}$

   b) $\frac{38}{51}$ $\qquad \frac{13}{51} + \frac{38}{51} = 1$

6. a) $\frac{12}{51}$

   b) $\frac{39}{51}$ $\qquad \frac{12}{51} + \frac{39}{51} = 1$

7. a) $\frac{3}{51} = \frac{1}{17}$

   b) $\frac{48}{51} = \frac{16}{17}$ $\qquad \frac{3}{51} + \frac{48}{51} = 1$

One dice is rolled.

8. $\frac{2}{6} = \frac{1}{3}$

9. 0

10. 0

11. $\frac{5}{6}$

12. $\frac{2}{6} = \frac{1}{3}$

continued/

Two dice are rolled.

- Table of all possible scores when two dice are rolled:

Dice 2 \ Dice 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

13. $\frac{3}{36} = \frac{1}{12}$

14. $\frac{33}{36} = \frac{11}{12}$

Three coins are tossed.

- Possible outcomes when three coins are tossed are:
  HHH, HHT, HTH, THH, TTH, THT, HTT, TTT

15. $\frac{3}{8}$

16. $\frac{4}{8} = \frac{1}{2}$

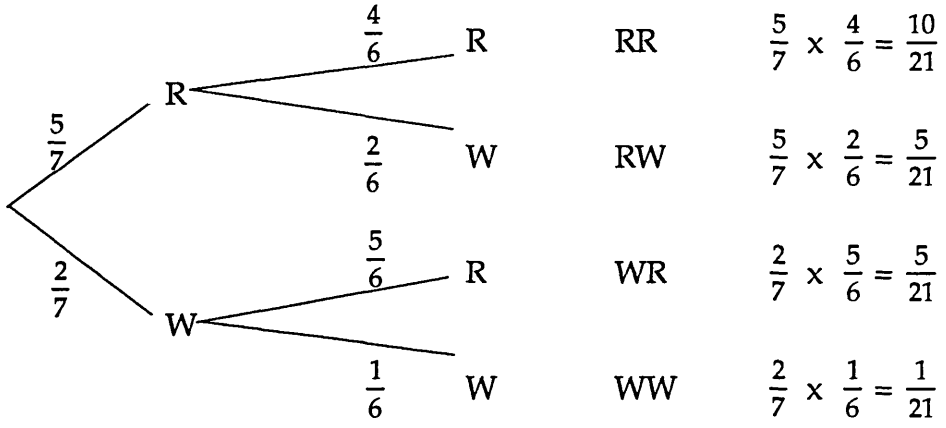
17. 0 is the smallest probability of an event.
    1 is the largest probability of an event.

18. $1 - \frac{1}{20} = \frac{19}{20}$

19. No, she is not correct, the probability of a tail is $\frac{1}{2}$.
    A fairly convincing argument is to say that the coin has no memory; at each separate throw, a tail and a head are equally likely events no matter how many times one result has occured in succession.

20. 400. The probability of throwing a number greater than 2 is $\frac{2}{3}$, $600 \times \frac{2}{3} = 400$.

1272 Combined Probabilities from Tree Diagrams

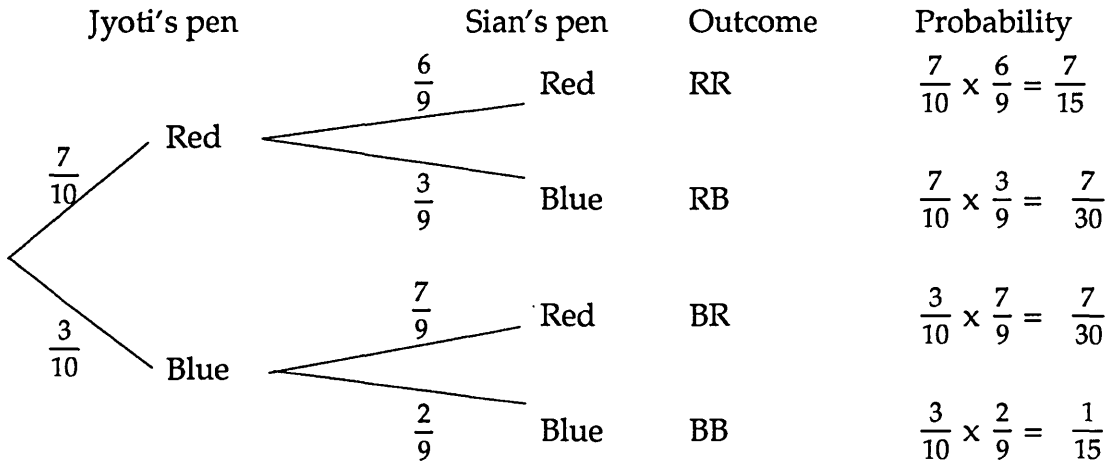


The probability both balls are the same colour is $\frac{10}{21} + \frac{1}{21} = \frac{11}{21}$.

■ The probability both balls are different colours is $\frac{5}{21} + \frac{5}{21} = \frac{10}{21}$.

■ The sum of the probability that both balls are the same colour and the probability that both balls are different colours equals 1, because these are the only possibilities, either one or the other will definitely happen, the two events are *mutually exclusive.*

1.



$\frac{7}{15} + \frac{7}{30} + \frac{7}{30} + \frac{1}{15} = 1$

a) $\frac{7}{10} \times \frac{6}{9} = \frac{7}{15}$

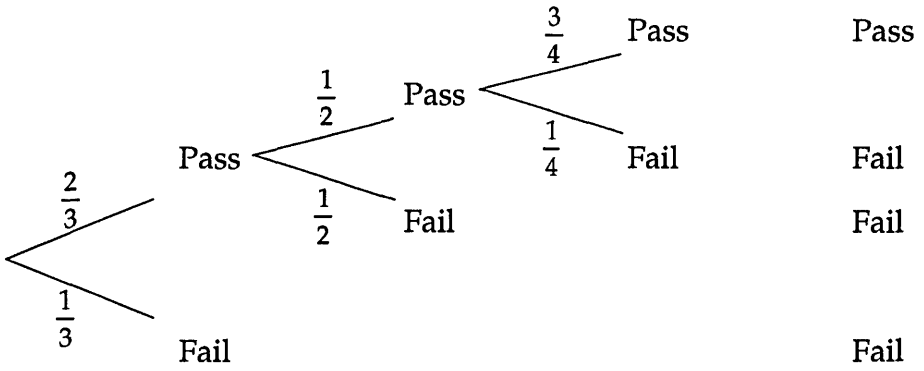
b) $\frac{3}{10} \times \frac{2}{9} = \frac{1}{15}$

c) $\frac{7}{15} + \frac{1}{15} = \frac{8}{15}$

d) $\frac{7}{30} + \frac{7}{30} = \frac{7}{15}$

continued/

2.



It is unnecessary to draw all the tree diagram since if a car fails on one aspect, the overall outcome is a fail.

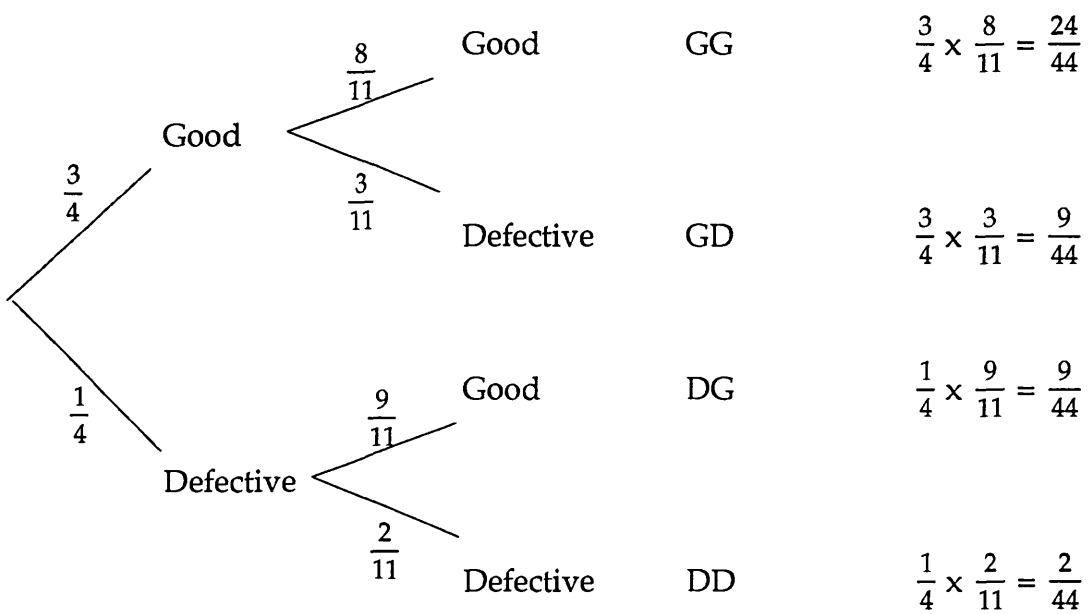
The probability that the car will pass its test is $\frac{2}{3} \times \frac{1}{2} \times \frac{3}{4} = \frac{1}{4}$.

3. a) i) $\frac{3}{4}$ ii) $\frac{1}{4}$

b) i) $\frac{9}{11}$ ii) $\frac{2}{11}$

c) i) $\frac{8}{11}$ ii) $\frac{3}{11}$

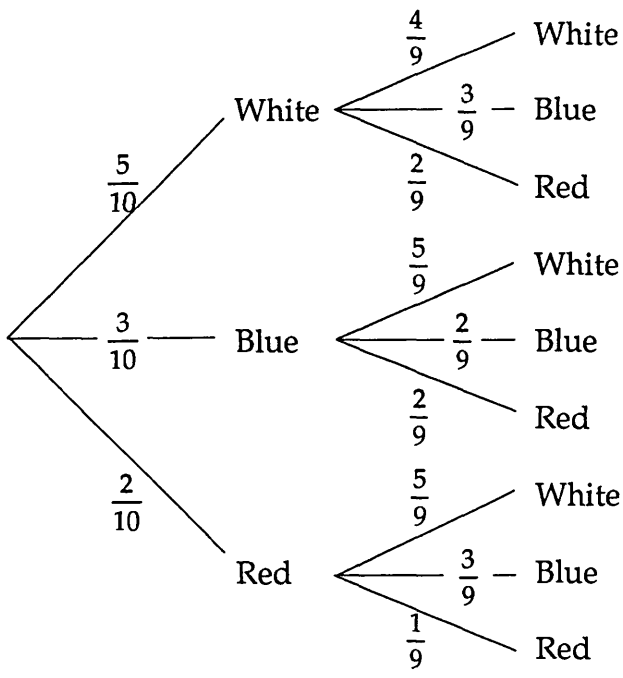
d)



i) $\frac{3}{4} \times \frac{8}{11} = \frac{24}{44} = \frac{6}{11}$ ii) $\frac{1}{4} \times \frac{2}{11} = \frac{2}{44} = \frac{1}{22}$ iii) $\frac{9}{44} + \frac{9}{44} = \frac{18}{44} = \frac{9}{22}$

continued/

4.



Outcome	Probability
WW	$\frac{5}{10} \times \frac{4}{9} = \frac{20}{90}$
WB	$\frac{5}{10} \times \frac{3}{9} = \frac{15}{90}$
WR	$\frac{5}{10} \times \frac{2}{9} = \frac{10}{90}$
BW	$\frac{3}{10} \times \frac{5}{9} = \frac{15}{90}$
BB	$\frac{3}{10} \times \frac{2}{9} = \frac{6}{90}$
BR	$\frac{3}{10} \times \frac{2}{9} = \frac{6}{90}$
RW	$\frac{2}{10} \times \frac{5}{9} = \frac{10}{90}$
RB	$\frac{2}{10} \times \frac{3}{9} = \frac{6}{90}$
RR	$\frac{2}{10} \times \frac{1}{9} = \frac{2}{90}$

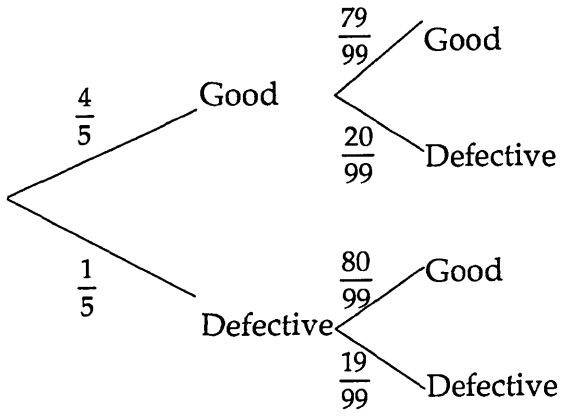
a) $\frac{20}{90} + \frac{6}{90} + \frac{2}{90} = \frac{28}{90} = \frac{14}{45}$

b) $1 - \frac{14}{45} = \frac{31}{45}$

5. a) $\frac{80}{100} = \frac{4}{5}$

b) i) $\frac{79}{99}$ ii) $\frac{20}{99}$

c)



Outcome	Probability
GG	$\frac{4}{5} \times \frac{79}{99} = \frac{316}{495}$
GD	$\frac{4}{5} \times \frac{20}{99} = \frac{80}{495}$
DG	$\frac{1}{5} \times \frac{80}{99} = \frac{80}{495}$
DD	$\frac{1}{5} \times \frac{19}{99} = \frac{19}{495}$

i) $\frac{316}{495}$ ii) $\frac{19}{495}$ iii) $\frac{80}{495} + \frac{80}{495} = \frac{160}{495} = \frac{32}{99}$

1275 Volumes and Surface Areas of Cylinders

All answers are given to appropriate degrees of accuracy. Where 2 answers are given, the first answer is the one you would get if using the π button, the answer in the brackets is the one you would get if using the approximation π = 3.14.

A Volumes

1. a) $48cm^3$ b) $924cm^3$ ($923cm^3$) c) $723cm^3$ ($722cm^3$)

2. First cylinder volume = $151cm^3$ ($151cm^3$) Second cylinder volume = $113cm^3$ ($113cm^3$)
   You may have expected the volume of the two cylinders to be equal, however, when the volume is calculated the radius of the cylinder is *squared*.
   First cylinder volume = $4^2 \times \pi \times 3cm^3$ Second cylinder volume = $3^2 \times \pi \times 4cm^3$

3. 5cm

4. 31.8cm (31.8cm)

B Surface Areas

1. a) $3519cm^2$ ($3517cm^2$) b) $1188cm^2$ ($1187cm^2$) c) $3848cm^2$ ($3847cm^2$)

2. a) $75.40cm^2$ ($75.36cm^2$) b) $75.40cm^2$ ($75.36cm^2$)
   The surface areas are equal. You may not have expected this because of the volumes, but when calculating surface areas the dimensions are *not* squared.

3. Area of curved surface = $8.47cm^2$ ($8.46cm^2$)

4. Total surface area = $100cm^2$ ($100cm^2$)

5. Curved surface area = $1.7m^2$ ($1.7m^2$). There should be enough paint in the tin.

1278 Multiplying Directed Numbers

	-4	-3	-2	-1	0	+1	+2	+3	+4
+4	-16	-12	-8	-4	0	+4	+8	+12	+16
+3	-12	-9	-6	-3	0	+3	+6	+9	+12
+2	-8	-6	-4	-2	0	+2	+4	+6	+8
+1	-4	-3	-2	-1	0	+1	+2	+3	+4
0	0	0	0	0	0	0	0	0	0
-1	+4	+3	+2	+1	0	-1	-2	-3	-4
-2	+8	+6	+4	+2	0	-2	-4	-6	-8
-3	+12	+9	+6	+3	0	-3	-6	-9	-12
-4	+16	+12	+8	+4	0	-4	-8	-12	-16

continued/

1278 Multiplying Directed Numbers (cont)

1. a) $^{-}8$ b) $^{+}9$ c) $^{-}6$
   d) $^{+}2$ e) $^{-}1$ f) $^{-}3$
   g) $^{-}16$ h) $^{+}16$ i) $^{-}16$

2. The numbers coloured red are positive whole numbers, they should be in the top right corner and the bottom left corner. The numbers coloured green are negative whole numbers, they should be in the top left corner and the bottom right corner.

3.

a positive number	x	a positive number	=	a positive number
a positive number	x	a negative number	=	**a negative number**
a negative number	x	a positive number	=	**a negative number**
a negative number	x	a negative number	=	**a positive number**
zero	x	a positive number	=	zero
zero	x	a negative number	=	**zero**

4. a) $^{-}6$ b) $^{-}18$ c) $^{-}50$
   d) $^{+}32$ e) 0 f) $^{+}40$
   g) 0 h) $^{-}42$ i) $^{+}24$
   j) $^{+}24$ k) 0 l) $^{-}36$
   m) $^{+}63$ n) $^{+}63$ o) $^{-}66$
   p) 0 q) $^{+}48$ r) $^{+}100$
   s) $^{+}16$ t) $^{-}144$ u) $^{+}56$

1279 Dividing by Directed Numbers

1. a)
   i) $\boxed{^{+}2} \times {}^{-}3 = {}^{-}6$ $\quad {}^{-}6 \div {}^{-}3 = \boxed{^{+}2}$
   ii) $\boxed{^{-}4} \times {}^{-}4 = {}^{+}16$ $\quad {}^{+}16 \div {}^{-}4 = \boxed{^{-}4}$
   iii) $\boxed{^{-}3} \times {}^{+}3 = {}^{-}9$ $\quad {}^{-}9 \div {}^{+}3 = \boxed{^{-}3}$
   iv) $\boxed{^{+}3} \times {}^{-}4 = {}^{-}12$ $\quad {}^{-}12 \div {}^{-}4 = \boxed{^{+}3}$
   v) $\boxed{0} \times {}^{+}3 = 0$ $\quad 0 \div {}^{+}3 = \boxed{0}$
   vi) $\boxed{^{+}4} \times {}^{+}2 = {}^{+}8$ $\quad {}^{+}8 \div {}^{+}2 = \boxed{^{+}4}$
   vii) $\boxed{^{+}1} \times {}^{-}2 = {}^{-}2$ $\quad {}^{-}2 \div {}^{-}2 = \boxed{^{+}1}$
   viii) $\boxed{^{-}1} \times {}^{+}3 = {}^{-}3$ $\quad {}^{-}3 \div {}^{+}3 = \boxed{^{-}1}$

   b) Any positive or negative number when divided by zero has no solution.

2. a) $^{+}4$ b) $^{+}4$ c) $^{-}4$ d) $^{-}4$

3. a) $^{+}4$ b) $^{+}3$ c) $^{+}3$ d) $^{+}3$

continued/

1278 Multiplying Directed Numbers (cont)

1. a) $^{-}8$ b) $^{+}9$ c) $^{-}6$
   d) $^{+}2$ e) $^{-}1$ f) $^{-}3$
   g) $^{-}16$ h) $^{+}16$ i) $^{-}16$

2. The numbers coloured red are positive whole numbers, they should be in the top right corner and the bottom left corner. The numbers coloured green are negative whole numbers, they should be in the top left corner and the bottom right corner.

3.

a positive number	x	a positive number	=	a positive number
a positive number	x	a negative number	=	**a negative number**
a negative number	x	a positive number	=	**a negative number**
a negative number	x	a negative number	=	**a positive number**
zero	x	a positive number	=	zero
zero	x	a negative number	=	**zero**

4. a) $^{-}6$ b) $^{-}18$ c) $^{-}50$
   d) $^{+}32$ e) 0 f) $^{+}40$
   g) 0 h) $^{-}42$ i) $^{+}24$
   j) $^{+}24$ k) 0 l) $^{-}36$
   m) $^{+}63$ n) $^{+}63$ o) $^{-}66$
   p) 0 q) $^{+}48$ r) $^{+}100$
   s) $^{+}16$ t) $^{-}144$ u) $^{+}56$

1279 Dividing by Directed Numbers

1. a)

i)	$\boxed{^{+}2} \times {}^{-}3 = {}^{-}6$	${}^{-}6 \div {}^{-}3 = \boxed{^{+}2}$
ii)	$\boxed{^{-}4} \times {}^{-}4 = {}^{+}16$	${}^{+}16 \div {}^{-}4 = \boxed{^{-}4}$
iii)	$\boxed{^{-}3} \times {}^{+}3 = {}^{-}9$	${}^{-}9 \div {}^{+}3 = \boxed{^{-}3}$
iv)	$\boxed{^{+}3} \times {}^{-}4 = {}^{-}12$	${}^{-}12 \div {}^{-}4 = \boxed{^{+}3}$
v)	$\boxed{0} \times {}^{+}3 = 0$	$0 \div {}^{+}3 = \boxed{0}$
vi)	$\boxed{^{+}4} \times {}^{+}2 = {}^{+}8$	${}^{+}8 \div {}^{+}2 = \boxed{^{+}4}$
vii)	$\boxed{^{+}1} \times {}^{-}2 = {}^{-}2$	${}^{-}2 \div {}^{-}2 = \boxed{^{+}1}$
viii)	$\boxed{^{-}1} \times {}^{+}3 = {}^{-}3$	${}^{-}3 \div {}^{+}3 = \boxed{^{-}1}$

   b) Any positive or negative number when divided by zero has no solution.

2. a) $^{+}4$ b) $^{+}4$ c) $^{-}4$ d) $^{-}4$

3. a) $^{+}4$ b) $^{+}3$ c) $^{+}3$ d) $^{+}3$

continued/

1279 Dividing by Directed Numbers (cont)

4.

a positive number	÷	a positive number	=	a positive number
a positive number	÷	a negative number	=	**a negative number**
a negative number	÷	a positive number	=	**a negative number**
a negative number	÷	a negative number	=	**a positive number**
zero	÷	a positive number	=	zero
zero	÷	a negative number	=	**zero**
a positive number	÷	zero	=	no solution
a negative number	÷	zero	=	**no solution**

5.

a)	$^{-}2$	b)	$^{+}2$	c)	$^{-}2$	d)	$^{+}2.5$
e)	$^{-}2$	f)	$^{+}4.5$	g)	$^{+}9$	h)	$^{-}2.75$
i)	$^{-}7$	j)	$^{-}3$	k)	$^{+}1.\dot{6}$	l)	0
m)	$^{-}2.5$	n)	$^{-}1.5$	o)	$^{-}10$		

1281 Using Gradients

1. a) The graph shows the cost of up to 10 litres of petrol.
The gradient shows the cost is 50p per litre.

b) The graph shows the distance travelled for up to 2.5 litres of petrol.
The gradient shows that the petrol is used at a rate of 12 km per litre.

c) The graph shows the cost of up to 25 centimetres of ribbon.
The gradient shows that the cost is 5p per 10 centimetres.

d) The graph shows a car travelling at a steady speed for three hours.
The gradient shows that this speed is 40km/h.

e) The graph shows water being emptied from a bath.
The gradient shows that it is being emptied at a rate of 15 litres per minute.
The gradient is -15 litres per minute. Why do this graph and the one in part (g) have negative gradients?

f) The graph shows a car gaining speed (accelerating).
The gradient shows that the speed is increasing by 20km/h in each minute.

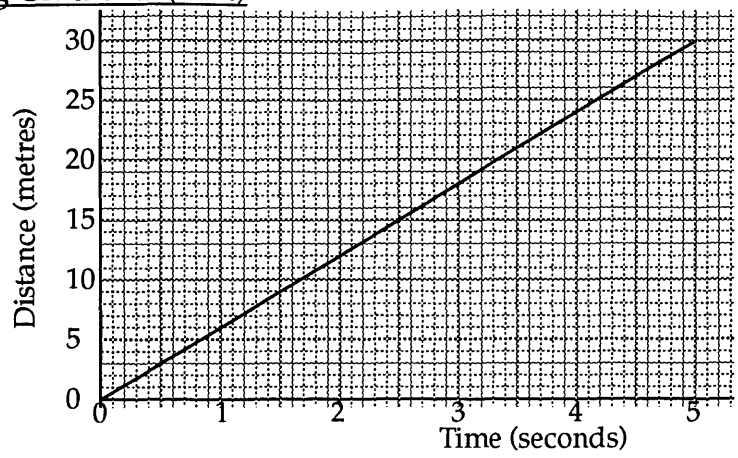
g) The graph shows a car slowing down (decelerating).
The gradient shows that it is losing speed at a rate of 3km/h in each second.
The gradient is -3km/h per second.

h) The graph shows a car travelling at a constant speed of 34km/h.
The gradient is 0, which shows that the speed is not changing.

continued/

1281 Using Gradients (cont)

2. a)



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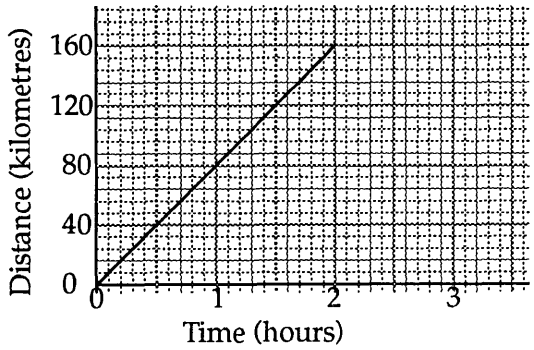

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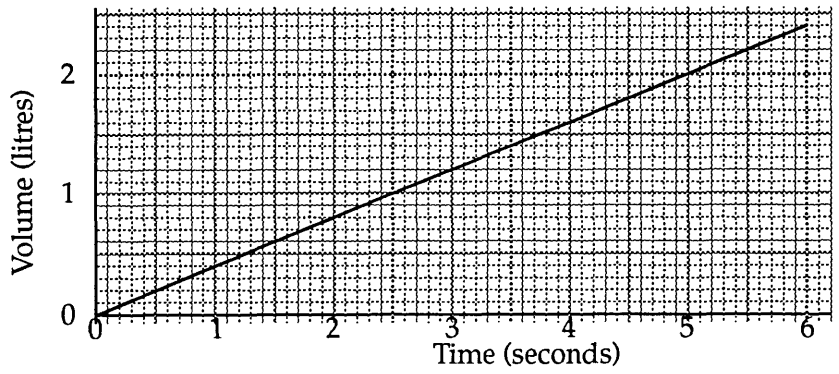
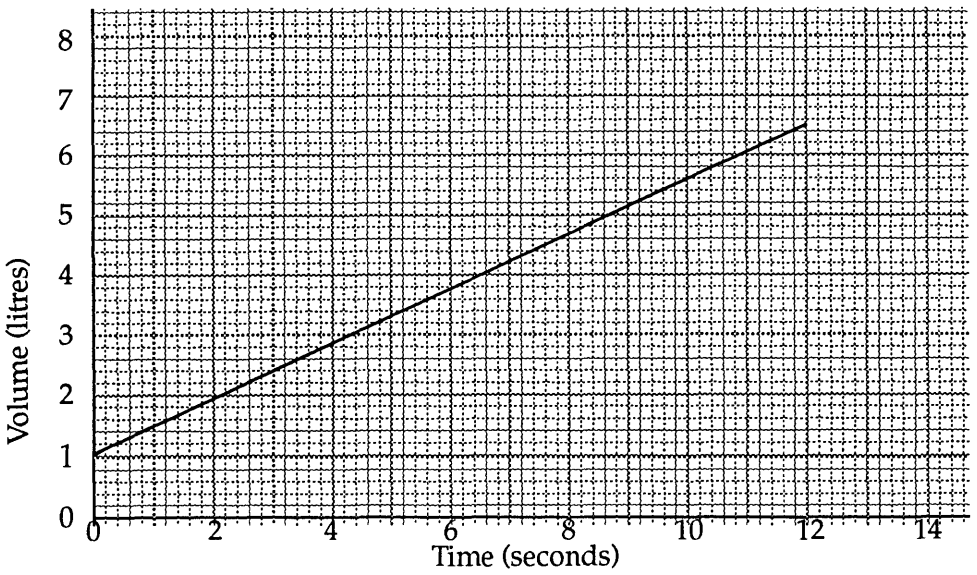


d)



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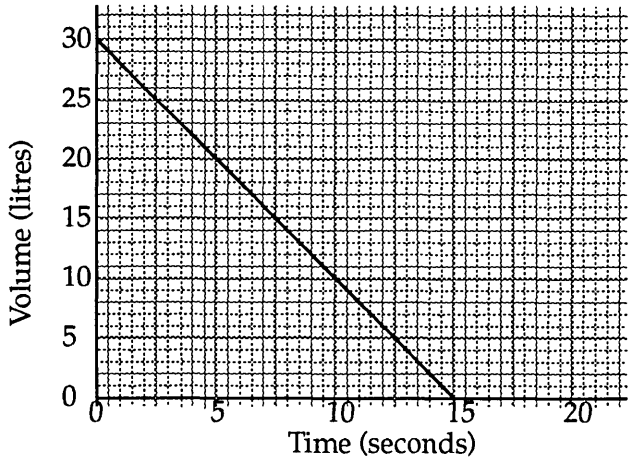
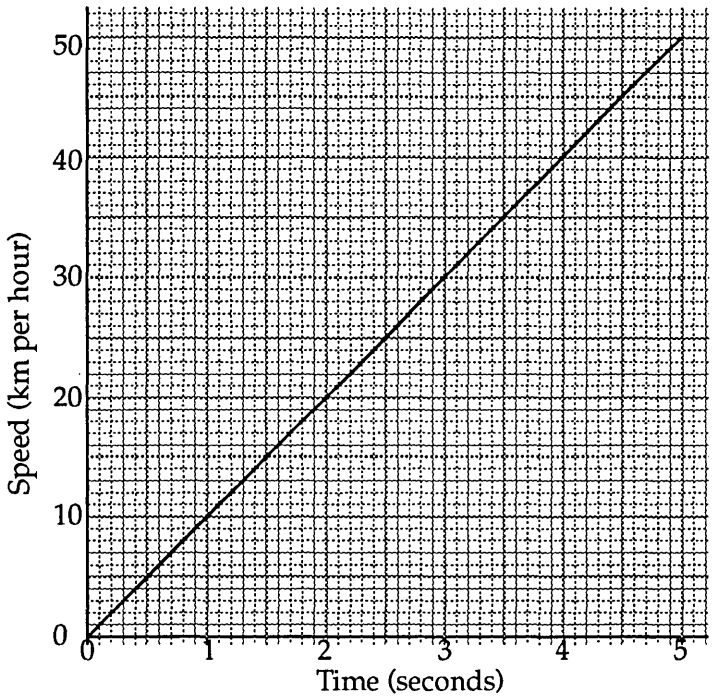
1281 Using Gradients (cont)

2. e)



f)



1287 Equilateral Construction

1. Measure the sides and angles of your equilateral triangle. The sides should each measure 4cm. The angles should each measure 60°.

2. Measure the sides and angles of your equilateral triangle. The sides should each measure 5.3cm. The angles should each measure 60°.

3. The triangles are equilateral because the point R is the same distance from P and from Q, and this distance is the same as PQ. PQ = PR = RQ
'equi'-'lateral' means equal sided (Latin)

4. If R is the same distance from P and from Q but this is not the same as PQ then the triangle will be isosceles.
'iso'-'sceles' means equal legs (Greek)
To draw the isosceles triangle accurately, first mark off 3cm on PQ, then use the compass to draw two arcs of length 6cm.

1289 Organising a Party

Answers to this will vary.

1290 Which Buy?

1. Mail order catalogue £27.99
   Comet £19.84
   Argos £19.95

2. Comet charges the least but the difference between Comet and Argos prices is very small so you would probably choose the nearest shop.

3. Maybe she lived nearest to an Argos shop.

4. 10 weeks

5. a) £28
   b) £28.12

6. Slightly cheaper and she could finish paying for the recorder more quickly.

7. You could save all the money in 7 weeks and buy it direct from Argos or Comet (if you didn't spend any money on anything else!)

1291 Classified Ad's

1.

HONDA	CJ	250T	5000	MLS
GOOD	COND.	MANY	EXTRAS	£400
ONO	004 £2.40	4622 £3.60	MR £3.60	DAVIE £3.60
AFT £3.60	7 £3.60	PM £3.60	£4.80	
			£4.80	

2. £3.60

3. Did you work out the cost of your bicycle advert? Show your advert and the cost to your teacher.

4. To have a box number for the advert in question 1 would make a total cost of £3.40 because you could leave off the last seven 'words' the advert itself would cost £2.40 plus the £1 for the box number. *Was your bicycle advert cheaper with a box number?*

continued/

1291 Classified Ad's (cont)

5. The Ascot advert is 36 words (counting the postal district 'SW2' as one word). It cost £3.24

6. 'Half as much again' or '$1\frac{1}{2}$ times'.

7. Two insertions cost £5.04 (36 x 14p).

8. The fishing advert has 14 words at 9p each and 2 words in capitals at 18p each. That's £1.62 altogether.

9. All in capitals would be £2.16 (16 x 9p x 150%)

10. Because they are most noticeable when placed among small letters.

11. 3cm

12. £4.50

13. £1.65 (15 words in capitals at 18p, 5 words at 9p).

14. £2.85

15. £10.80 (6 lines at £1.80 each, 1978 prices)

16. Show your advert to your teacher. You might decide to pay more because one paper has a bigger circulation and your advert would therefore be seen by more people.

17. Show your advert to your teacher.

1292 Sampling Shoes

Your results will depend upon which year you are in at school.

You should have collected the shoe sizes of a sample of about 100 people in your year (or all the year if it is less than 100). Make sure that your sample includes; people of a range of heights, a mixture of girls and boys (if your school is mixed).

A graph of the results of this survey, in addition to showing the most frequent size, will show the proportion of people in each size. You should choose the 50 pairs of shoes so that their sizes are in approximately the same proportions as your survey.

1294 Cooking Numbers

1. Ann's recipe for 2:

50g pastry
225g leeks
20g butter
salt and pepper
50g bacon
1 egg
75ml cream

3. Tom's recipe for 6:

150g pastry
675g leeks
60g butter
salt and pepper
150g bacon
3 eggs
225ml cream

3. *This table gives the quantities to the nearest 5g or 25ml. You may have given your answers to the nearest g or ml.*

No. of People	Pastry (g)	Leeks (g)	Butter (g)	Bacon (g)	Eggs	Cream (ml)
1	25	115	10	25	1	50
2	50	225	20	50	1	75
3	75	340	30	75	2	100
4	100	450	40	100	2	150
5	125	565	50	125	2	200
6	150	675	60	150	3	225
7	175	790	70	175	3	250

4. For cream, a little extra in the sauce would be fine, so measurements to the nearest 25ml would be okay. Similarly an extra half an egg wouldn't make very much difference, so the nearest whole egg would be okay.

5. a) No . . .
   b) . . . you might need a little longer because the quiche for 8 people would be much bigger, but $1\frac{1}{2}$ hours at most.
   c) The same
   d) An 11 or 12 inch tin would contain twice as much as an 8 inch tin (Why?) . . . . . about 28 or 30cm in metric sizes. (1 inch = 2.54cm)

1295 Second-Hand Cars

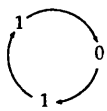
- £3995 seems to be a fair price. Fiestas in 1991 cost between £2750 and £5000.
- Jackie's scattergraph shows a negative correllation. The older the car, the cheaper it is. There are some exceptions to this, but it is generally true.

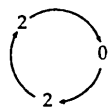
Show your scattergraph to your teacher and discuss your answers to the questions.

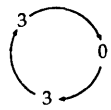
1297 Three Numbers Investigation

The differences usually end up with a repeating cycle of the form . . .

110
011
101

which can be summarised as 

You can also get 

or 

and so on.

Can you predict the figures in the repeating cycle from the first line of numbers?

The only 3-figure numbers which do not give a repeating cycle are those with 3 identical digits, for example 444 which gives 000 or 

straight away.

If you add the figures in each row of an investigation, for example

904	→	13
945	→	18
514	→	10
431	→	8
. . .		. . .

You will notice that the totals are always even (except in the first line). The explanation of this using algebra is not easy because, in order to show how the differences of the 3 digit number "abc" are calculated you will need to consider when a > b > c, when b > a > c and so on.

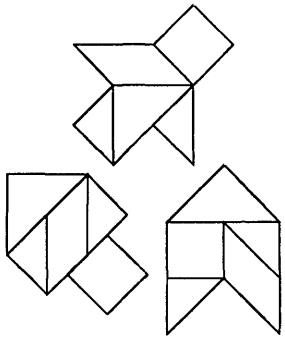
What happens if you start with a 4-figure number like 3714? Is the result different from that starting with 2468?

Five-figure numbers also produce characteristics of their own. Are they more like the 3-figure or the 4-figure starting numbers? What differences can you observe when starting with an odd or an even number of figures?

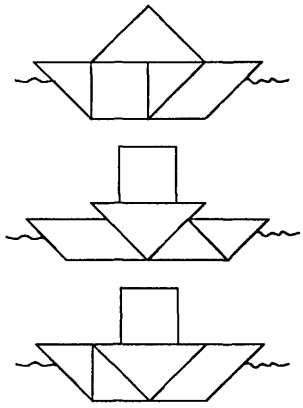
1299 Tangram Arrows

There are several ways to do each one.

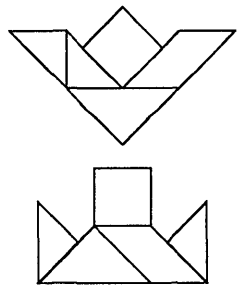
Page 1



Page 2



Page 3



Show your own tangram designs to your teacher.

1300 Measuring Windows

1. 2cm = 20mm
   5cm = 50mm
   20cm = 200mm

2. 2cm 6mm = 26mm
   5cm 4mm = 54mm
   20cm 7mm = 207mm

3. 70mm = 7cm
   40mm = 4cm
   210mm = 21cm

4. 25mm = 2cm 5mm = 2.5cm
   75mm = 7cm 5mm = 7.5cm

5. 3.7cm = 37mm
   1.2cm = 12mm
   12.8cm = 128mm
   0.6cm = 6mm

6. 41. 4cm

7. a) 41.4cm x 34cm b) 52.3cm x 35.7cm c) 20.9cm x 82.8cm

Answers • Answers • Answers

0501 to 1300

Answers