Answers • Answers • Answers

1301 to 1600

SMILE MATHEMATICS

SMILE

ANSWERS

1301 - 1600

Answers

1301 to 1600

MATHEMATICS

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This book contains answers to all the SMILE activities between 1301-1600, in numerical order.

As well as giving the answers there are also:

- explanations about how solutions have been arrived at,
- hints or prompts if you get stuck,
- ideas for extending some activities.

Use this book after you have completed each activity, so that you have immediate feedback on your work. You will remember the work more clearly and be able to identify any difficulties or misconceptions more easily. If you have made errors, look through your work again to see if you can spot where you have made an error. If you then do not understand why your answer is incorrect always seek help from your teacher so that she can help you to clarify any mis-understandings.

You can also use this book while you are working on an activity as it contains hints if you get stuck, or want to know how continue.

Remember, using the answer book to check your work or to help you if you are stuck is not cheating.

1301 Three in a Line

Show the board to your teacher when you have finished and explain who has won.

1302 Logipuzzle

In the puzzle on the card, 2 attributes change each time but this is not enough information to complete the pattern. You will need to have noticed that:

a) **Thick** is on top of **thin**.
Thin is on top of **thick**.

b) **Blue** is on top of **yellow**.
Yellow is on top of **red**.
Red is on top of **blue**.

c) **Rectangle** is on top of **circle**.
Triangle is on top of **rectangle**.
Circle is on top of **triangle**.

d) **Small** is on top of **large**.

These rules mean that:

Small thick blue circle should be on top of the **large thin yellow triangle**.

Small thick yellow rectangle should be on top of the **large thin yellow circle**.

Show one of your own puzzles that you made up to your teacher.

1303 Paraffins

1. Propane has 8 hydrogen atoms.

2. The formula for propane is C_3H_8.

3. Methane

```
     H
     |
H -  C  -  H
     |
     H
```

Butane

```
     H    H    H    H
     |    |    |    |
H-   C-   C -  C -  C -   H
     |    |    |    |
     H    H    H    H
```

Ethane

```
     H     H
     |     |
H -  C  -  C  -  H
     |     |
     H     H
```

Pentane

```
     H    H    H    H    H
     |    |    |    |    |
H -  C -  C -  C -  C -  C - H
     |    |    |    |    |
     H    H    H    H    H
```

continued/

1303 Paraffins (cont)

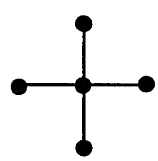
3. (cont)

Name	Carbon atoms	Hydrogen atoms	Formula
Methane	1	4	CH_4
Ethane	2	6	C_2H_6
Propane	3	8	C_3H_8
Butane	4	10	C_4H_{10}
Pentane	5	12	C_5H_{12}
Hexane	6	14	C_6H_{14}

4.

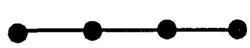
Name	Carbon atoms	Hydrogen atoms	Formula
	27	56	$C_{27}H_{56}$

5. To find the number of hydrogen atoms, double the number of carbon atoms and add 2.
The general formula is C_nH_{2n+2}

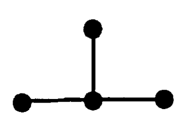
6. The third pentane isomer is 

7. There is only 1 form of methane, ethane and propane.

Butane has 2 isomers:



n-butane

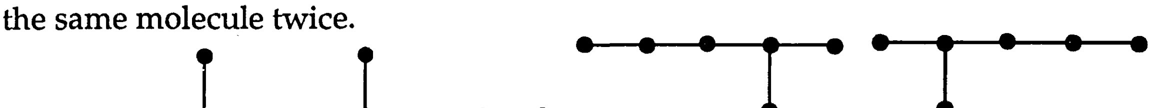


iso-butane

Pentane has 3 isomers (see question 6)
Hexane has 5 isomers.

After this the number of isomers increases rapidly.
Decane ($C_{10}H_{22}$) has 75 isomers.
Eicosane ($C_{20}H_{42}$) has 366319 isomers.

Only a few of these forms have been isolated but, theoretically, they could all exist. When you count paraffins with many carbon atoms there is a danger of counting the same molecule twice.

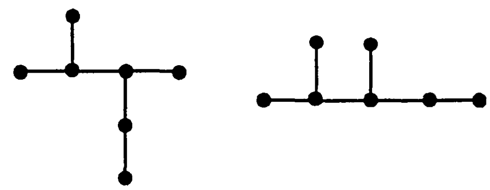


The 4 diagrams above all show **exactly the same** form of hexane.

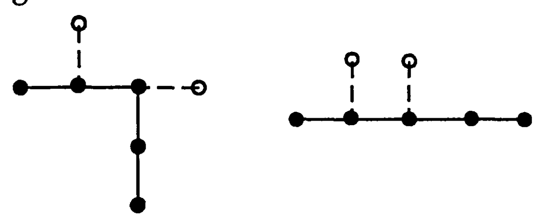
continued/

1303 Paraffins (cont)

There are less obvious repeats. Can you see why these show the same form of heptane?



Look at the longest chain (in this case 5 carbon atoms) and see why they are the same:



The details of the isomers of simple paraffins can be found in the Organic Chemistry section of most GCSE science books.

8. To find out about which isomers exist and what their different properties are, you should ask your science teacher to recommend a good chemistry book.

1304 An Honourable Problem

This is one solution.

A	K	Q	J
Q	J	A	K
J	Q	K	A
K	A	J	Q

Can you complete it so that each row column or diagonal has 4 different suits as well?

1305 Factorials!

1. 5! = 5 x 4 x 3 x 2 x 1 = 120

2. 6! = 6 x 5 x 4 x 3 x 2 x 1 = 720

3. a) 3! + 4! = 6 + 24 = 30

   b) 3! x 4! = 6 x 24 =144

   c) (3 + 4)! = 7! = 5040

   d) 3 x 4! = 3 x 24 = 72

   e) 4 x 3! = 4 x 6 = 24

continued/

4. a) $\frac{4!}{4} = \frac{4 \times 3 \times 2 \times 1}{4} = 6$

b) $\frac{4!}{3} = \frac{4 \times 3 \times 2 \times 1}{3} = 8$

c) $\frac{4!}{3!} = \frac{4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 4$

d) $\frac{4!}{4!} = 1$

5. (3!)! = 6!
= 720

6. The obvious factors of 6! are {1, 2, 3, 4, 5, 6}.
Any combination of these will also give factors of 6!
For example 20 (4 x 5), 24 (2 x 3 x 4) and so on.
There are some less obvious factors too. Can you find some of them?

7. 19! is even because 2 is a factor of 19!

8. 3 is a factor of 19! as it contains . . . x 3 . . .

9. 19! cannot be prime because it has more than two factors.

10. 19! is even so 19! + 2 must also be even, therefore it cannot be a prime number.

11. a) There are two zeros at the end of 10!
These are the result of '10' and 'x 5, x 2' appearing in the number.

b) There are 6 zeros at the end of 25!
These are the result of:
. . . 25 times a factor of 4 (giving two zeros)
. . . 20 times something
. . . 15 times an even number
. . . 10 times something
. . . 5 times an even number

Will there be enough even numbers?

c) In parts (a) and (b) you will have noticed that it is the multiples of 5 which produce zeros. You will need therefore to find all the numbers which contain a factor of 5.
For 100!
- Multiples of 5 will give a zero when multiplied by an even number.
- Multiples of 25 will give two zeros when multiplied by a multiple of 4.

For 1000! you will also need to consider multiples of 125.

1306 Decimal Estimation

1. How did you guess 24 ÷ 5. Did you work it out in your head?

2. 4.8

3. Your guesses to the sums in the table should be similar to the ones given. If you are unsure about your guesses, show them to your teacher.

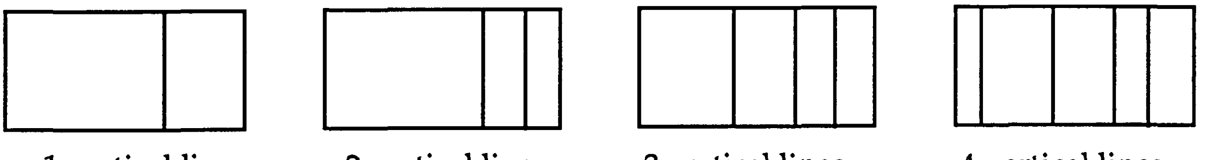
	GUESS	CALCULATOR
17 ÷ 4	4 and a bit	4.25
15 ÷ 4	nearly 4	3.75
17 ÷ 2	8 and a half	8.5
25 ÷ 4	6 and a bit	6.25
101 ÷ 10	10 and a bit	10.1
7 ÷ 2	3 and a half	3.5
16 ÷ 5	3 and a bit	3.2
19 ÷ 5	just less than 4	3.8
18 ÷ 8	2 and a bit	2.25
19 ÷ 8	2 and a bit more	2.375
23 ÷ 3	nearly 8	7.6666666
29 ÷ 7	4 and a bit	4.1428571

4. The answer should be 24 because multiplication is the inverse of division. The word inverse is explained on '0781 The Inverse'.

5. If you did not get the number you originally divided into by multiplying, check your method with your teacher.

1307 Sections

A sensible way to approach this investigation is to begin with a few simple examples. For instance you could start by looking at vertical lines only.



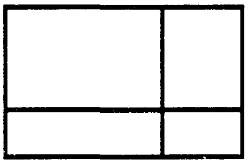
1 vertical line 2 sections | 2 vertical lines 3 sections | 3 vertical lines 4 sections | 4 vertical lines 5 sections

Can you describe the relationship between vertical lines and sections?

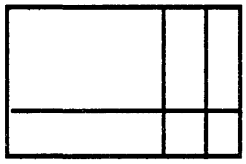
continued/

1307 Sections (cont)

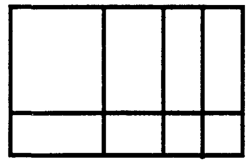
Then try 1 horizontal line:



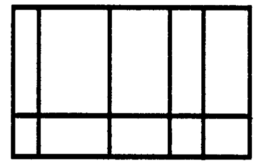
1 horizontal
1 vertical
4 sections



1 horizontal
2 verticals
6 sections



1 horizontal
3 verticals
8 sections



1 horizontal
4 verticals
10 sections

Can you describe the relationships this time?

Then try 2 horizontal lines, 3 horizontal lines, . . .

It is helpful to combine your results in a table.

Horizontal lines

Vertical lines

	0	1	2	3	4	5
0						
1	2	4	6			
2	3	6				
3	4	8	12			
4	5	10				
5						

If you cannot recognise any patterns in the table you will need to draw some more rectangles.

When you have enough numbers in the table you will recognise that it is symmetrical about the leading diagonal. e.g. 2 horizontals and 1 vertical give the same number of sections as 1 horizontal and 2 verticals. Why is this?

Predict how many sections are made by:

0 horizontals and 5 verticals
2 horizontals and 2 verticals?

Can you predict what numbers would be in the 'n horizontals' column?
Can you predict what numbers would be in the 'm verticals' row?

Try to generalise how many sections there will be in a rectangle with 'm' verticals and 'n' horizontals.

1308 Problems

A The fish is 72cm long.
You should have working out for the length of the body and tail.

B Farmer Brown has 5 cows, Farmer Giles has 7 cows.

C 1089

D Brown cows produce more milk.

1309 More Vector Messages

1. $\begin{pmatrix}+1\\+3\end{pmatrix}$ means 1 square right, 3 squares up from [M] to [I]

   $\begin{pmatrix}0\\-3\end{pmatrix}$ means 3 squares down from [I] to [L]

   $\begin{pmatrix}-3\\+2\end{pmatrix}$ means 3 squares left, means 2 squares up from [L] to [E]

2. VECTOR CODES ARE EASY

3. In VECTORS the top figure is for right (+) or left (-).

4.

$$\begin{pmatrix}-2\\+2\end{pmatrix} \begin{pmatrix}+3\\-1\end{pmatrix} \begin{pmatrix}-2\\-1\end{pmatrix} \begin{pmatrix}+4\\+3\end{pmatrix} \begin{pmatrix}-2\\-4\end{pmatrix} \begin{pmatrix}-2\\-1\end{pmatrix} \begin{pmatrix}-1\\+4\end{pmatrix} \begin{pmatrix}0\\0\end{pmatrix} \begin{pmatrix}+1\\-4\end{pmatrix} \begin{pmatrix}+2\\0\end{pmatrix} \begin{pmatrix}+2\\+5\end{pmatrix}$$

$$\begin{pmatrix}-4\\-2\end{pmatrix} \begin{pmatrix}+3\\0\end{pmatrix} \begin{pmatrix}-2\\0\end{pmatrix} \begin{pmatrix}-1\\+1\end{pmatrix} \begin{pmatrix}-1\\-2\end{pmatrix} \begin{pmatrix}+1\\0\end{pmatrix} \begin{pmatrix}+4\\+3\end{pmatrix} \begin{pmatrix}-2\\-5\end{pmatrix} \begin{pmatrix}-2\\+2\end{pmatrix} \begin{pmatrix}+2\\0\end{pmatrix} \begin{pmatrix}-1\\-2\end{pmatrix} \begin{pmatrix}-2\\+3\end{pmatrix} \begin{pmatrix}+5\\+2\end{pmatrix}$$

$$\begin{pmatrix}-4\\-1\end{pmatrix} \begin{pmatrix}-1\\-4\end{pmatrix} \begin{pmatrix}+5\\+5\end{pmatrix} \begin{pmatrix}-4\\-5\end{pmatrix} \begin{pmatrix}-1\\+2\end{pmatrix} \begin{pmatrix}+5\\+3\end{pmatrix} \begin{pmatrix}-4\\-4\end{pmatrix} \begin{pmatrix}0\\-1\end{pmatrix} \begin{pmatrix}+2\\+4\end{pmatrix} \begin{pmatrix}-1\\-4\end{pmatrix} \begin{pmatrix}-3\\+1\end{pmatrix}$$

1310 Planning a kitchen

Planning a kitchen

- How did you decide on whether there was enough space left for a person to work in the kitchen?
- Did you put the cooker near to a cupboard with a working surface?
- Which items did you leave out of your kitchen?

continued/

1310 Planning a kitchen (cont)

How much is your kitchen going to cost?

1. How much did you guess the price would be for a cooker?
   Do you think a gas cooker is cheaper than an electric cooker?

2. When you add up all your guesses you do not have to be very accurate, as this is just a rough estimate. A sensible answer would be £200 or £500 or £5000, rather than £203.45 or £498.60 or £5205.90.

3. Your answer will depend upon your choices.

4. Your answer to question 2 will probably be very different to your answer for question 3 because buying kitchen furniture and equipment is only done rarely.

5. Which items did you choose to buy second hand? Were they all electrical?

Your kitchen at home

The measurements will vary from kitchen to kitchen, so you will need to show your work to your teacher.

1311 Sorting Stamps

1. Norway (Norge)

2. 10 ore

3. 80 ore

4. 10 ore, 20 ore, 40 ore, 70 ore, 80 ore.

5. The $\frac{1}{2}$p stamp is the cheapest, but this was phased out in 1990.

6. 50p

7. To make sorting easier.

8. The charge for sending parcels and letters through the post depends upon the weight. The heavier the item, the more it costs.

The cheapest British stamp in 1995 is 1p, the most expensive stamp is £10. Your answers may be different.

1312 Matchstick Sequences

1.	4,	7,	10,	**13,**	**16,**	**19,**	**22,**	**25,**	...	The rule is add **3**
2.	3,	**5,**	**7,**	**9,**	**11,**	**13,**	**15,**	**17,**	...	The rule is **add 2**
3.	6,	11,	16,	21,	26,	31,	36,	41,	...	The rule is add 5.
4.	5,	9,	13,	17,	21,	25,	29,	33,	...	The rule is add 4.
5.	4,	7,	10,	13,	16,	19,	22,	25,	...	The rule is add 3.
6.	6,	11,	16,	21,	26,	31,	36,	41,	...	The rule is add 5.
7.	5,	9,	13,	17,	21,	25,	29,	33,	...	The rule is add 4.

1313 Match Patterns

1.	4,	12,	24,	40,	60,	84,	...
2.	3,	9,	18,	30,	45,	63,	...
3.	6,	16,	30,	48,	70,	96,	...
4.	6,	18,	36,	60,	90,	126,	...

1315 International Paper Sizes

1.

Paper Size	Width (mm)	Length (mm)	Area (mm^2)	Length ÷ width
A7	74	105	7770	1.42
A6	105	148	15540	1.41
A5	148	210	31080	1.42
A4	210	297	62370	1.41
A3	297	420	124740	1.41
A2	420	594	249480	1.41
A1	594	841	499554	1.42
A0	841	1189	999949	1.41

continued/

1315 International Paper Sizes (cont)

2. a) Each successive size doubles in area.
   b) Some of the successive areas are exactly double but not all.
      e.g. twice the area of A2 does not exactly equal the area of A1. This is because the length of A1 (to the nearest mm) is slightly more than double the width of A2 (to the nearest mm).
   c) The length and width are given to the nearest mm. They are not exact measurements. Therefore the area is not exactly $1m^2$ ($1000000mm^2$).

3. a) The ratio, length ÷ width, remains approximately the same.
   b) The front of the card will give you a hint on how to arrange the pieces.
   c) $\sqrt{2} = 1.41$ correct to 2 decimal places. Your results should be close to 1.41.

1316 Halving

1.

Original line (cm)	5
halved	2.5
halved again	1.25
halved again	0.625
halved again	0.3125
	0.15625
	0.078125
	0.0390625 . . .

0.0390625 may look bigger than 5 because it has more digits.

This 5 means $\frac{5}{10000000}$ whilst this 5 means 5 'whole ones'.

a) 5
b) 2.5
c) 0.3125
d) 0.625

2.

Original number	4
halved	2
halved again	1
and again	0.5
	0.25
	0.125
	0.0625
	0.03125 . . .

a) 0.5
b) 0.125
c) 1
d) 0.25

continued/

1316 Halving (cont)

3.		
Original line	20
10 times smaller	2
10 times smaller	0.2
10 times smaller	0.02
10 times smaller	0.002
10 times smaller	0.0002
10 times smaller	0.00002 . . .

If you did need to use a calculator can you now see a method for dividing by 10 in your head?

a) 2
b) 0.2
c) 0.002

4. a) 0.75
b) 0.1875
c) 1.5
d) 0.75

1317 Multiplying and Dividing by Ten

- Multiply by **10**

Th	H	T	U t	h	th	
	 7	7 **6**	6 · **0 ·**			
	 2	2 **5**	5 · 3 **3 ·**			
		 6	6 · 7 **7 · 2**	2 **3**	3	
		 5	5 · **0 ·**			
			0 · 0 **0 · 0**	0 **2**	2 **1**	1
 9	9 **7**	7 **0**	0 · **0 ·**			
	 8	8 **3**	3 · 2 **2 ·**			
	 1	1 **8**	8 · 4 **4 · 2**	2 **3**	3	
			0 · 2 **2 · 0**	0 **6**	6	
	 1	1 **2**	2 · **0 ·**			

You should notice that all the figures move one place to the left.
Get someone else to check that your own five numbers follow the rule:

Multiplying by 10 moves the figures **one** place to the **left**.

continued/

1317 Multiplying and Dividing by Ten (cont)

- Divide by **10**

Th	H	T	U t	h	th		
		7	6 · 7 · 6				
		2	5 · 3 2 · 5	 3			
			6 · 7 0 · 6	2 7	3 2	 3	
			5 · 0 · 5				
			0 · 0 0 · 0	0 0	2 0	1 2	 1
	9	7 9	0 · 7 ·				
		8	3 · 2 8 · 3	 2			
		1	8 · 4 1 · 8	2 4	3 2	 3	
			0 · 2 0 · 0	0 2	6 0	 6	
		1	2 · 1 · 2				

You should notice that all the figures move one place to the right.
Get someone else to check that your own five numbers follow the rule:

Dividing by ten moves the figures **one** place to the **right**.

- Multiply by **100**

Th	H	T	U t	h	th		
 7	 6	7 0	6 · 0 ·				
 2	 5	2 3	5 · 3 0 ·				
	 6	 7	6 · 7 2 · 3	2	3		
	 5	 0	5 · 0 ·				
		· · ·					

You should notice that all the figures move **two** places to the left.
Get someone else to check that your own five numbers follow the rule:

Multiplying by one hundred moves the figures **two** places to the **left**.

continued/

1317 Multiplying and Dividing by Ten (cont)

- Divide by **100**

Th	H	T	U t	h	th		
		7	6 · **0 · 7**	 **6**			
		2	5 · 3 **0 · 2**	 **5**	 **3**		
			6 · 7 **0 · 0**	2 **6**	3 **7**	 **2**	 **3**
		⋮					

You should notice that all the figures move **two** places to the right.
Get someone else to check that your own five numbers follow the rule:

Dividing by 100 moves the figures **two** places to the **right**.

- When multiplying by 1000 all the figures move **three** places to the **left**.
- When dividing by 1000 all the figures move **three** places to the **right**.

Copy this summary of your work.

- When multiplying by 10 all the figures move **one** place to the **left**.
- When multiplying by 100 all the figures move **two** places to the **left**.
- When multiplying by 1000 all the figures move **three** places to the **left**.
- When multiplying by 10000 all the figures move **four** places to the **left**.

⋮

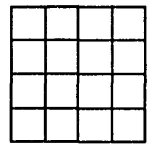
- When dividing by 10 all the figures move **one** place to the **right**.
- When dividing by 100 all the figures move **two** places to the **right**.
- When dividing by 1000 all the figures move **three** places to the **right**.
- When dividing by 10000 all the figures move **four** places to the **right**.

⋮

1318 Square Cover

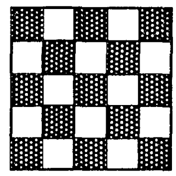
There are separate results for odd and even squares.

- **Even squares** e.g. 4 x 4 square.



You can start from any of the 16 small squares and cover the complete board.

- **Odd squares** e.g. 5 x 5 square.



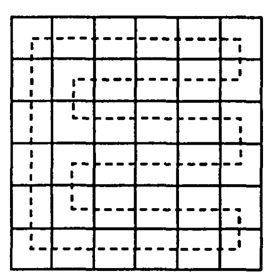
You can start from any of the 13 shaded squares but not from the 12 unshaded squares if you want to cover the complete board.
Make a table of your results and try to find a general rule.

When you have a general rule for squares you may like to move on to investigate rectangles. Try rectangles which are:

- even x even e.g. 6 x 4
- odd x odd e.g. 5 x 3
- even x odd e.g. 6 x 3

Can you find a general rule?
Can you justify your general rules?

- e.g. with the even squares you can show that there is always a continuous path through the square.
So it must be possible to start at any small square.



1319 Consecutives

- 6 x 7 x 8 is divisible by 24.
Which other sets of three consecutive numbers when multiplied together are divisible by 24? Can you explain why? Does your explanation cope with examples like 7 x 8 x 9?

You may like to use a spreadsheet. Here is the beginning of a spreadsheet to see the results of the product of three consecutive numbers which are divisible by 24.

	A	B	C	D	E
1	n	n + 1	n + 2	n(n+1)(n+2)	n(n+1)(n+2)/24
2	1	2	3	6	0.25
3	2	3	4	24	1
4	3	4	5	60	2.5
5	4	5	6	120	5

Why is one of the three consecutive numbers always a multiple of 3?

continued/

1319 Consecutives (cont)

Change the formula in the spreadsheet to see which products of consecutives are divisible by 20.

Try the product of **four** consecutive numbers.

- Which are divisible by 24?
- Which are divisible by 120?

Justify you findings.

- What can you say about the factors of the product of any set of four consecutive numbers?

Try **five** consecutive numbers.

1320 Rectangle Areas

1. $28cm^2$ 2. $65cm^2$ 3. $45m^2$ 4. $78m^2$

5. $22.5m^2$

6. 2km = 2000m. So the area is 160 000m^2.

7. Area of whole shape = Area A + Area B
$= (4m \times 3m) + (3m \times 2m)$
$= 12m^2 + 6m^2$
$= 18m^2$

8. Area of whole shape $= (6cm \times 7.5cm) + (3cm \times 2cm)$
$= 45cm^2 + 6cm^2$
$= 51cm^2$

9. Area of whole shape $= (2cm \times 10cm) + (4.2cm \times 2cm)$
$= 20cm^2 + 8.4cm^2$
$= 28.4cm^2$

10. Area of whole shape $= (9m \times 11m) + (7m \times 4m) + (6m \times 8m)$
$= 99m^2 + 28m^2 + 48m^2$
$= 175m^2$
You may have split the shape up into different rectangles, but your answer should be the same.

11. Area of whole shape $= (10m \times 5.2m) - (3m \times 2m)$
$= 52m^2 - 6m^2$
$= 46m^2$

12. Area of whole shape $= (7cm \times 11.3cm) - (3cm \times 3.5cm)$
$= 79.1cm^2 - 10.5cm^2$
$= 68.6cm^2$

1321 Prism or Pyramid?

Nets B and C make pyramids, and nets A and D make prisms.

1322 Solid Shapes

1. The cube has 6 faces.
2. The cube has 8 corners.
3. The cube has 12 edges.
5. Here are some of the solid shapes that you may have in your table.

SHAPE	FACES	CORNERS	EDGES
CUBE	6	8	12
TETRAHEDRON	4	4	6
CYLINDER	2	0	2
SQUARE PYRAMID	5	5	8
TRIANGULAR PRISM	5	6	9
CUBOID	6	8	12
SPHERE	1	0	0

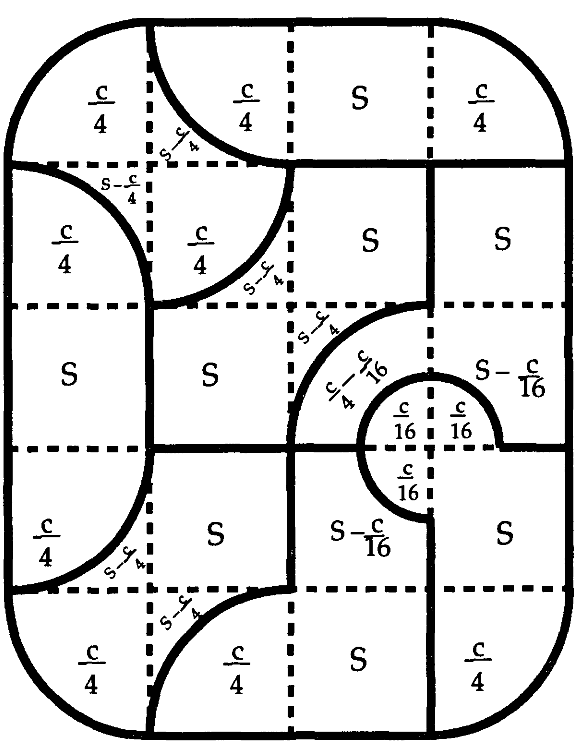
If your answers are different, check with your teacher.

6. The cylinder and the sphere have no corners.

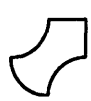
1323 Tak-Tile Areas

1. The area of the small circle is $\frac{c}{4}$.

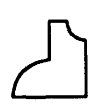
2.

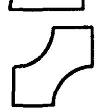



$\frac{c}{4} + \left(s - \frac{c}{4}\right) + \frac{c}{4} + \left(s - \frac{c}{4}\right) = 2s$


$\frac{c}{4} + \left(s - \frac{c}{4}\right) + s + \left(s - \frac{c}{4}\right) = 3s - \frac{c}{4}$


$\frac{c}{16} + \frac{c}{16} + \frac{c}{16} + s + \frac{c}{4} = s + \frac{7c}{16}$


$\frac{c}{4} + s + \left(s - \frac{c}{16}\right) = 2s + \frac{3c}{16}$


$s + s + \left(s - \frac{c}{4}\right) + \left(s - \frac{c}{4}\right) = 4s - \frac{c}{2}$


$\frac{c}{4} + s + \frac{c}{4} = s + \frac{c}{2}$


$s + \left(s - \frac{c}{16}\right) + \left(\frac{c}{4} - \frac{c}{16}\right) = 2s + \frac{c}{8}$


$\frac{c}{4} + s + \frac{c}{4} = s + \frac{c}{2}$ continued/

1323 Tak-Tile Areas (cont)

3 & 4. The total area is $16s + c$.

You can either:

- look at the whole shape which has area $16s + 4\frac{c}{4} = 16s + c$

or

- add all the tiles:

$$(2s) + (3s - \frac{c}{4}) + (s + \frac{7c}{16}) + (2s + \frac{3c}{16}) + (4s - \frac{c}{2}) + (s + \frac{c}{2}) + (2s + \frac{c}{8}) + (s + \frac{c}{2})$$

Total area = $16s + c$

5. Total area $= 16s + c$
$= 16r^2 + \pi r^2$
$= r^2(16 + \pi)$

6. $r^2 = s$
$r^2 = \frac{c}{\pi}$ (Rearranging $c = \pi r^2$)

Therefore $s = \frac{c}{\pi}$

1324 Pegboard Sums

4 + 3 = **7**
1 + **2** = **3**
3 + **3** = **6**

Get someone else to check your own sums.

1325 Sums on the Balance

5 + **3** = 8
5 + **1** = 6

Get someone else to check your own sums.

1326 Running Costs

- Most electricity bills consist of cost per unit, VAT and standing charges. Were there items on the bill that were different?
- The appliances which cost the most are washing machines, tumble driers and room heaters.
- There are many ways to save money in order to reduce the electricity bill. One way would be to use a cool wash. Discuss your answers with someone else, as they may be able to think of other ways.

Which room contained the most electrical appliances?

1327 Visiting the LEB

1. Your answers will vary from place to place.

2. Did you plan your journey from the school or from your home?

3. Make a display of the group's work.

5. Were you able to wire up a plug?

1328 Room to Move

- When you record the measurements of the greatest height that **you** can reach when sitting on the chair, remember that a disabled person may be unable to stretch so far.

  Which things were you able to reach?

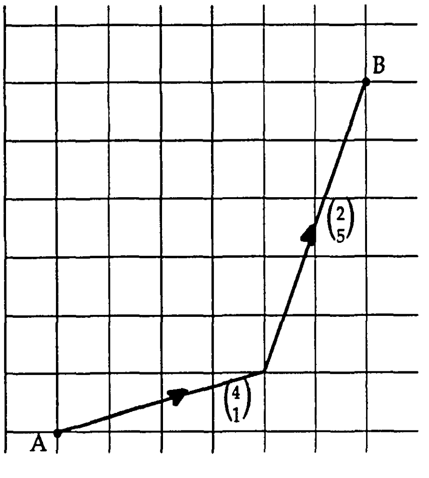
  Most light switches and door handles are at a suitable height for disabled people to reach.

- Is your school designed so that pupils confined to a wheelchair:
  a) have enough room to move around in a mathematics lesson?
  b) are able to get all their SMILE cards?
  c) are able to get to the equipment?

- Many public buildings now provide special facilities for disabled people.
  What facilities do they have?
  Can you give examples of shops and other public buildings which provide these facilities?

1329 Journeys

1.

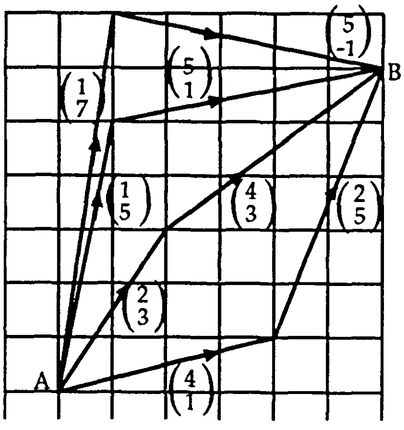


continued/

1329 Journeys (cont)

2. Here are 4 two-stage journeys which start at A and finish at B.

   Your answers may be different.



3. Here are some possible results.

Journey A to B		
Direct Vector	Two Stage Journey	
$\begin{pmatrix} 6 \\ 6 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 2 \\ 5 \end{pmatrix}$
$\begin{pmatrix} 6 \\ 6 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 3 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 3 \end{pmatrix}$
$\begin{pmatrix} 6 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 7 \end{pmatrix}$	$\begin{pmatrix} 7 \\ -1 \end{pmatrix}$
$\begin{pmatrix} 6 \\ 6 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -2 \end{pmatrix}$	$\begin{pmatrix} 6 \\ 8 \end{pmatrix}$
$\begin{pmatrix} 6 \\ 6 \end{pmatrix}$	$\begin{pmatrix} 6 \\ 5 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

   If you are uncertain about your results, show them to your teacher.

4. Each of the sets of two vectors add to give $\begin{pmatrix} 6 \\ 6 \end{pmatrix}$, the direct vector.

5. The vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ describes the journey 3 squares right and 2 squares down.

6. Many possible answers.

7. Each of the sets of three vectors add to give $\begin{pmatrix} 7 \\ 5 \end{pmatrix}$, the direct vector.

8. Many possible answers.

9. The set of vectors for each journey from E to F should add to $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$, the direct vector.

10. If you are uncertain about your results, show them to your teacher.

11. If you are uncertain about your results, show them to your teacher.

1330 Planning a Supermarket

1. 1 doz eggs ⟶ 1 large tin peaches ⟶ $\frac{1}{2}$ kg rice ⟶ 4oz coffee ⟶

   1 large white loaf ⟵ 1 fresh pineapple ⟵ $\frac{1}{2}$ lb butter ⟵ 1 tin dog food

2. Many possible answers. If each member of your group went to a different supermarket, were the plans very similar?

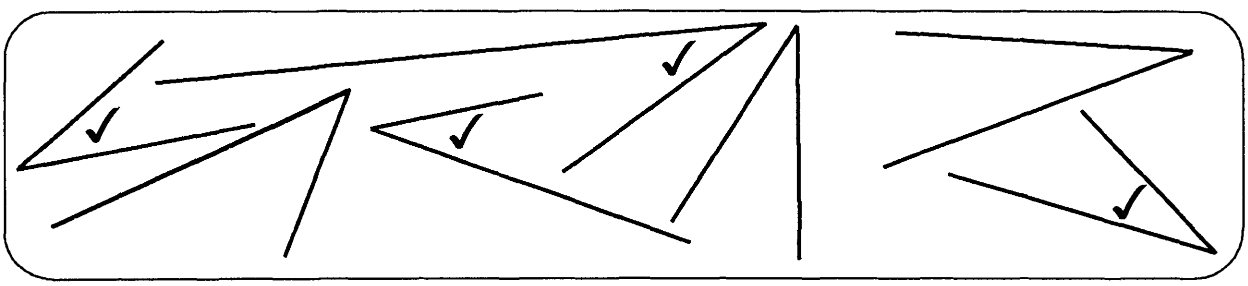
3. The order of the shopping list will depend on your local supermarket.

4. The way in which supermarkets display their goods is planned to encourage shoppers to buy more.

5. Make a display of your plan for a supermarket. What factors did you take into account when making your plan?

1331 Equal Angles

Page 1 - What are equal angles?



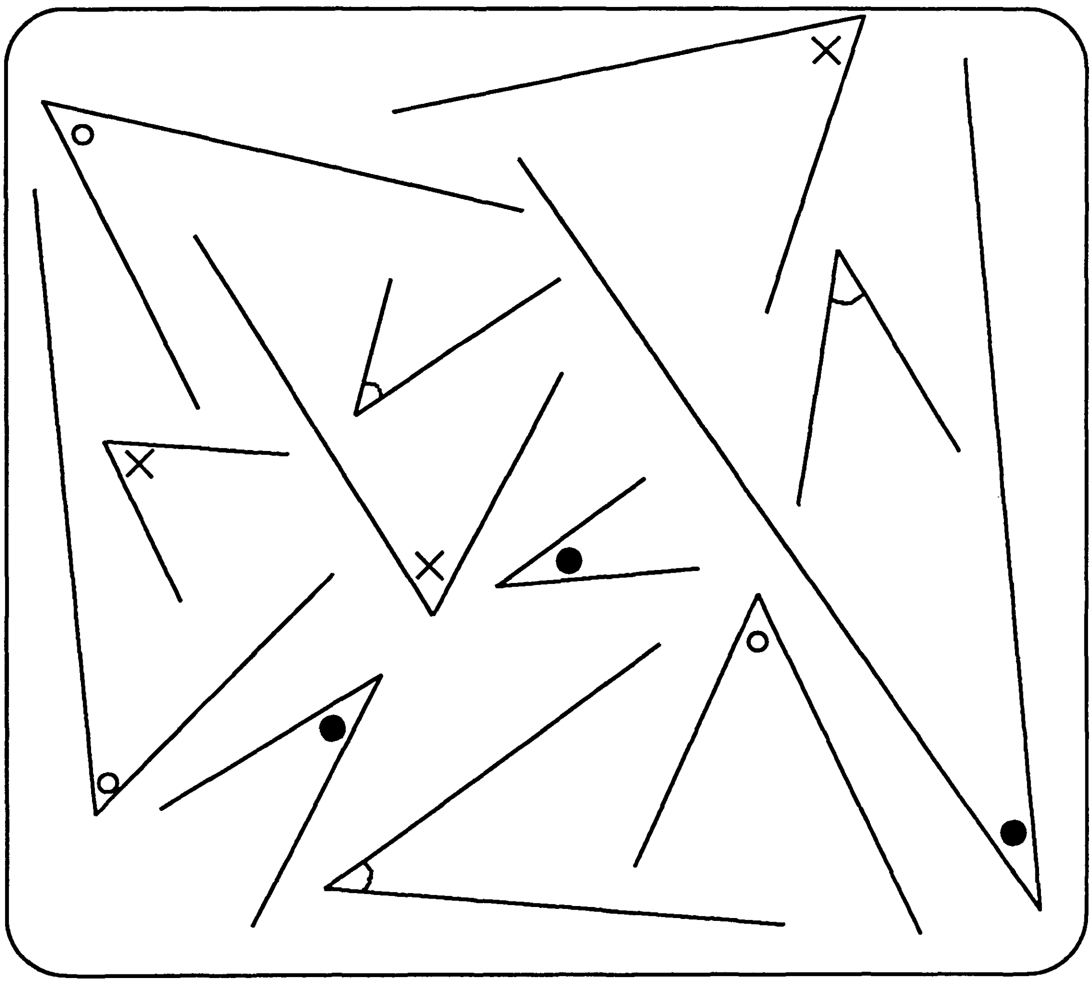
Page 2 - Pairing angles

$\hat{A} = \hat{G}$	This means → angle A equals angle G → or it can be written as ∠A = ∠G.
$\hat{B} = \hat{J}$	This means → angle B equals angle J → or it can be written as ∠B = ∠J.
$\hat{C} = \hat{M}$	This means → angle C equals angle M → or it can be written as ∠C = ∠M.
$\hat{D} = \hat{K}$	This means → angle D equals angle K → or it can be written as ∠D = ∠K.
$\hat{E} = \hat{F}$	This means → angle E equals angle F → or it can be written as ∠E = ∠F.
$\hat{F} = \hat{E}$	This means → angle F equals angle E → or it can be written as ∠F = ∠E.
$\hat{G} = \hat{A}$	This means → angle G equals angle A → or it can be written as ∠G = ∠A.
$\hat{H} = \hat{L}$	This means → angle H equals angle L → or it can be written as ∠H = ∠L.
$\hat{J} = \hat{B}$	This means → angle J equals angle B → or it can be written as ∠J = ∠B.
$\hat{K} = \hat{D}$	This means → angle K equals angle D → or it can be written as ∠K= ∠D.
$\hat{L} = \hat{H}$	This means → angle L equals angle H → or it can be written as ∠L = ∠H.
$\hat{M} = \hat{C}$	This means → angle M equals angle C → or it can be written as ∠M = ∠C.

continued/

1331 Equal Angles (cont)

Page 3 - Marking angles equal



Page 4 - Numbering angles

1 = 7 = 9

3 = 6 = 10

4 = 8 = 2

5 = 11 = 12

Page 5 - Zig-zags

$\hat{H} = \hat{L} = \hat{D} = \hat{P}$

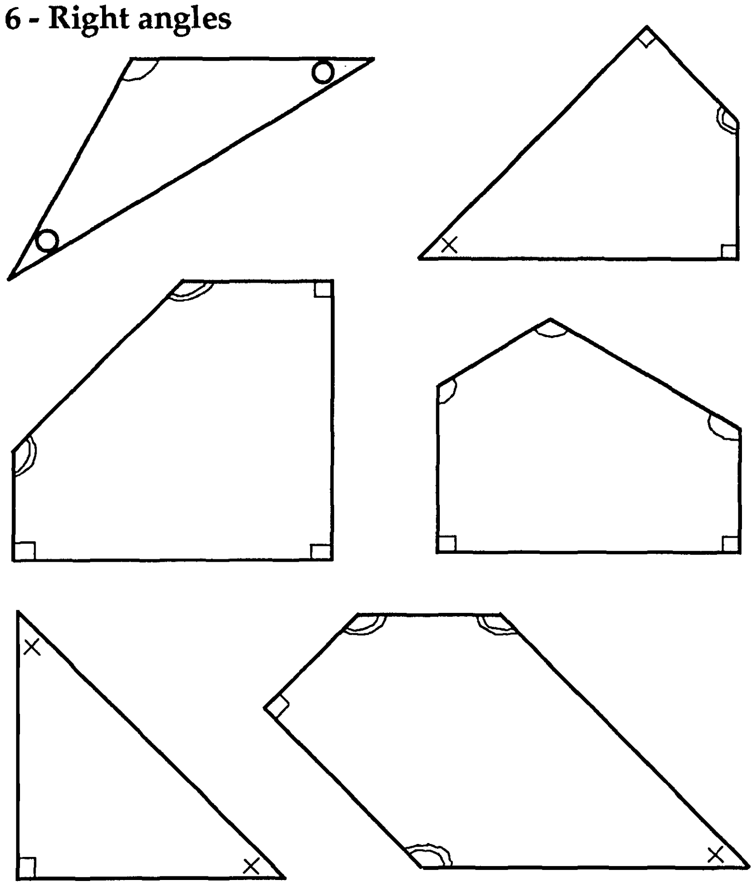
$\hat{A} = \hat{C} = \hat{G} = \hat{N}$

$\hat{Q} = \hat{B} = \hat{F} = \hat{J}$

$\hat{E} = \hat{K} = \hat{M} = \hat{R}$

continued/

Page 6 - Right angles



Page 7 - More difficult diagrams

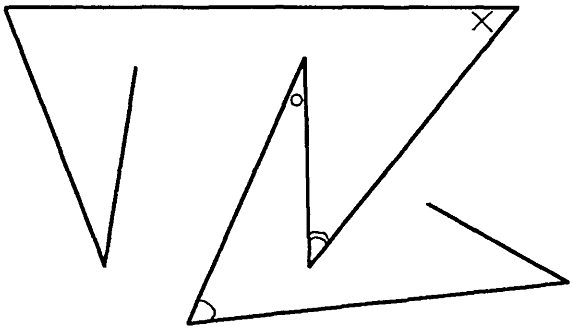
a) 3 = 4 = 8
2 = 6
1 = 7

b) 2 = 5 = 6 = 7
3 = 4
1 = 8

c) 3 = 2 = 10 = 11
5 = 8 = 1
7 = 6
9 = 4

continued/

Page 8 - Adjacent angles



Page 9 - Using numbers

a) (4) = (1) = (7)
(5) = (3) = (8)
(6) = (9)

b) (5) = (1) = (8)
(4) = (6) = (7)
(3) = (2)

c) (3) = (2) = (1)
(4) = (5)
(6) = (7)

Page 10 - Naming angles

Names of angle at H are: **$J\hat{H}L$** **$L\hat{H}J$** **$J\hat{H}K$** **$K\hat{H}J$**

List of equal angles are:
$H\hat{J}L$ = **$L\hat{K}J$**
$L\hat{H}J$ = **$L\hat{J}K$**
$H\hat{L}J$ = **$K\hat{L}J$** = **$H\hat{J}K$**

Target test - Standard

1. $S\hat{Q}R$ $R\hat{Q}S$ $P\hat{Q}R$ $R\hat{Q}P$
2. $P\hat{Q}R$ = $Q\hat{R}S$ = $S\hat{R}P$
   $Q\hat{P}R$ = $Q\hat{R}P$ = $P\hat{S}R$

continued/

1331 Equal Angles (cont)

3. 4 = 2 = 14 = 7 = 8 = 12
   5 = 1 = 10 = 11
   6 = 15 = 16 = 3
   13 = 9

Target test - Advanced

1. $P\hat{Q}U = P\hat{S}T = T\hat{Q}R$

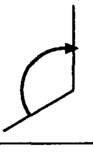
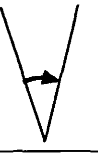
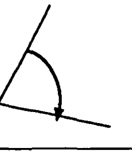
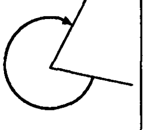
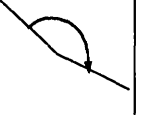
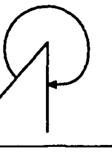
   $U\hat{Q}S = S\hat{R}U = T\hat{P}U$

   $P\hat{T}U = U\hat{S}Q = Q\hat{R}U$

2. 5 = 11 = 14
   1 = 9
   3 = 6
   4 = 15
   7 = 10

1332 Rotation

Page 1 - Turning a wheel

Amount of rotation						
Valve starts at F and rotates to:	X	G	H	D	A	E

Page 2 - Direction of rotation

The hands of a clock	C
Playing a record	C
Turning on the cold tap	A
Drilling a hole in wood	C
Traffic at a roundabout	C
Taking the cap off toothpaste	A
Steering a car to the left	A
Stirring porridge	A or C
Bath water down the plug hole	A or C

continued/

Page 3 - The Big Wheel

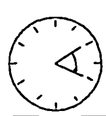
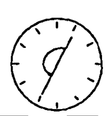
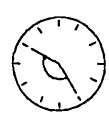
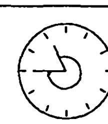
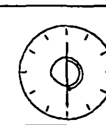
Which amount of rotation would take:	
Bob to the bottom?	3
Dick to the top?	5
Bob to the top?	7
Dick to the bottom?	1
Bob directly below Dick?	4
Dick directly below Bob?	8
Bob and Dick to the same level?	2
	6

Page 4 - A rotation code

Message	uncoded	WHAT	TIME	IS	THE	LANDING	TONIGHT
	coded	**FYLN**	**NVXH**	**VA**	**NYH**	**OLCTVCZ**	**NKCVZYN**
Answer	coded	YLOM	LC	YKRB	PHMKBH	YVZY	NVTY
	uncoded	**HALF**	**AN**	**HOUR**	**BEFORE**	**HIGH**	**TIDE**

Get someone else to uncode your message.

Page 5 - The hands of a clock

Amount of rotation:						
Time taken by:	hour hand	2 hours	**6 hours**	**5 hours**	**10 hours**	**18 hours**
	minute hand	10mins	**30mins**	**25mins**	**50mins**	**90mins**
	second hand	**10secs**	**30secs**	**25secs**	**50secs**	**90secs**

Page 6 - Big rotations

		Number of revolutions	
from	to	minute hand	second hand
2.00	4.00	2	120
6.30	9.30	**3**	**180**
8.20	**1.20**	5	300
3.00	5.30	$2\frac{1}{2}$	**150**
8.15	9.45	$1\frac{1}{2}$	**90**
10.10	10.25	$\frac{1}{4}$	**15**
1.00	**2.45**	$1\frac{3}{4}$	**105**

continued/

Page 7 - Small rotations

smallest first

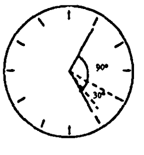
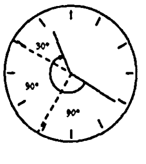
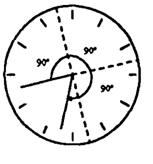
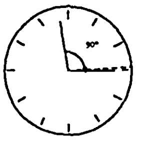
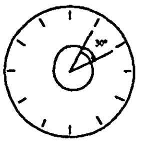
G	20°
B	30°
A	60°
D	80°
C	130°
H	170°
E	220°
F	280°

biggest last

Page 8 - Practical examples of rotation

Closing lid of box	105°
Closing pliers	15°
Movement of seesaw	40°
Closing door	110°
Folding stepladder	40°
Speedo needle from 0 to 70	105°
Turning switch to high	300°
Falling tree	90°

Page 9 - Clockface angles

Starting time	9.05	2.21	7.43	3.59	**6.05**
Final time	9.27	2.56	8.32	**4.15**	7.10
Rotation of minute hand					
Time taken	22mins	**35mins**	**49mins**	16mins	**65mins**
	= 15 + 5 + 2 **90°+ 30°+12°**	**= 15 + 15 + 15** **90°+ 90°+ 30°**	**= 15 + 15 + 15 + 4** **90° + 90°+ 90°+24°**	**= 15 + 1** **90°+ 6°**	**= 60 + 5** **360°+ 30°**
Angle of rotation	132°	**210°**	**294°**	**96°**	**390°**

continued/

Page 10 - Estimating rotations
Your answers may differ to these, but if you are unsure, check your own answers with your teacher.

Opening coffee jar	220°
Opening sardine tin	1000°
Switching on light	60°
Using bicycle brake	30°
Dialling 9	300°
Using corkscrew	1200°
Turning door handle	70°
Turning on tap	950°

Target test - Standard

1.
A 90° clockwise
B 120° anticlockwise
C 30° clockwise
D 0°
E 45° clockwise
F 45° anticlockwise
G 60° anticlockwise

2. D, A, F, E, G, C, B, H.

3. 156

Target test - Advanced

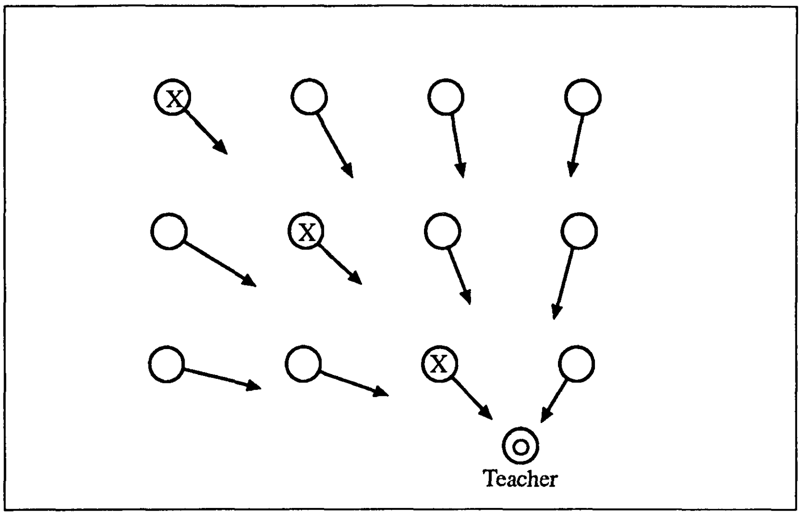
1. a) 120°
b) 150°
c) 45°

2. a) 3 hours
b) 2 minutes,
10 seconds.

3. A 180° approximately
B 90° approximately
C 150° approximately
D 250° approximately

1333 Directions

Page 1 - Directions



Page 2 - Compass directions

Inverness	is north of	Glasgow
Carlisle	is south of	**Dundee**
Oban	is west of	**Dundee**
Carlisle	is east of	**Stranraer**
Edinburgh	is NW of	**Newcastle**
Stranraer	is SW of	**Edinburgh**
Aberdeen	is NE of	**Glasgow**
Glasgow	is SE of	**Oban**

Page 3 - Name the girls

Who sits: north of J?	D
east of P?	Q & R
south of R?	**X**
west of T?	**S**
southwest of O?	T
northeast of V?	**Q & L**
northwest of Q?	**J & C**
north of M and west of J?	G
E of H and NE of U?	**K**
SE of C and SW of L?	**Q**

continued/

Page 4 - Directions from Bedford

From Bedford the bearing of:	is:
Cambridge	080°
London	**160°**
Peterborough	015°
Aylesbury	**210°**
Oxford	230°
Birmingham	**290°**
Southend	130°
Kings Lynn	**040°**
Grantham	350°

Page 5 - Finding your bearings

- The bearing of B from A is 070°
- The bearing of A from B is 250°

From:	the bearing of:	is:
Bedford	Cambridge	080°
Cambridge	Bedford	**260°**
Oxford	**Bristol**	250°
London	Cambridge	011°
Birmingham	Derby	**030°**
Kings Lynn	**Boston**	310°
Swindon	Kings Lynn	050°

Page 6 - A stretch of coast

letter	Name from map
A	**Radio Mast**
B	**Eagle Crag**
C	**Monument**
D	**Church**
E	**Yacht Club**
F	**Raven Tower**
G	**Costguard Lookout Post**
H	**Lighthouse**

continued/

Page 7 - Looking through a telescope

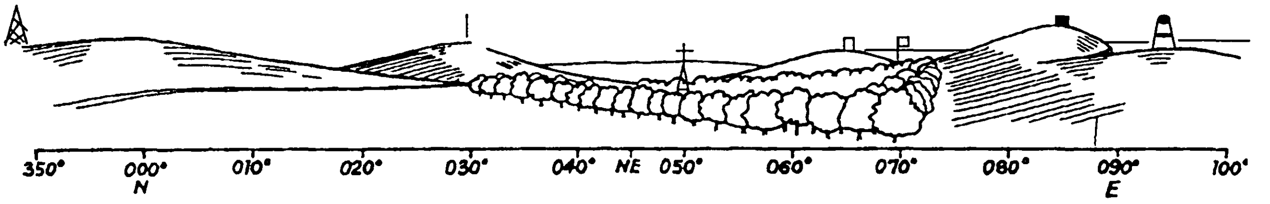
The bearing of the Coastguard from the radio mast is wrong. It should be 130°.

From Raven Tower the bearing of:	is:
Eagle Crag	030°
Church	050°
Monument	065°
Yacht Club Flagstaff	070°
Coastguard	085°
Lighthouse	095°
Radio Mast	355°

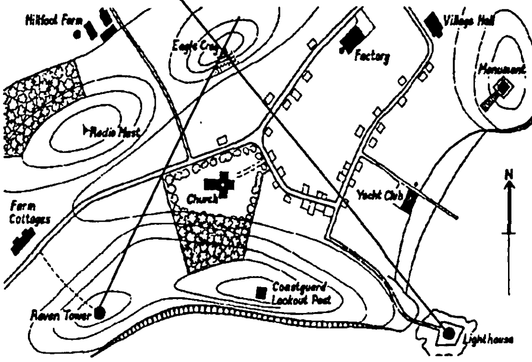
Page 8 - View from the radio mast

From the lighthouse the bearing of:	is
Raven Tower	270°
Coastguard	280°
Radio Mast	295°
Church	300°
Eagle Crag	315°
Factory	335°
Yacht Club Flagstaff	345°
Monument	015°

Page 9 - Drawing a panorama



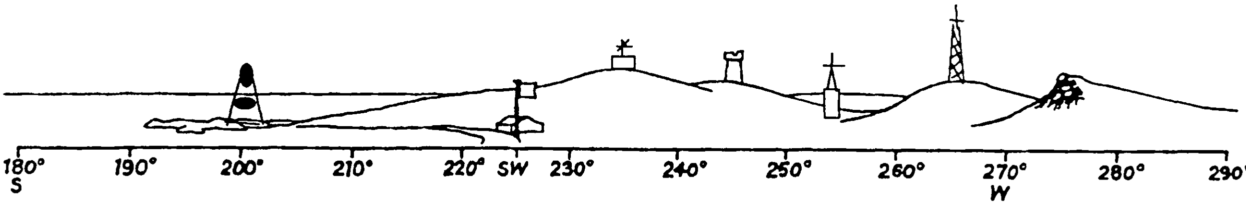
Page 10 - Puzzling panoramas



continued/

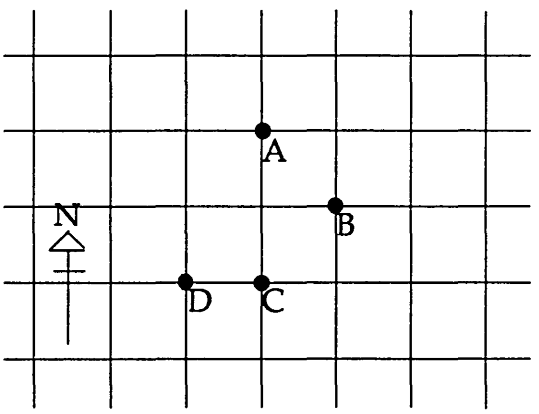
1333 Directions (cont)

This panorama was drawn from the monument.



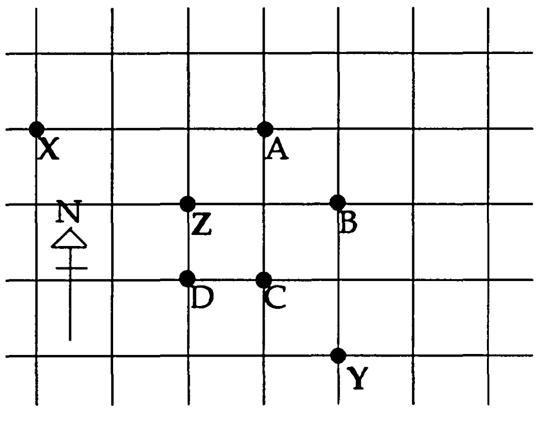
Target test - Standard

1.



From	To	Direction	Bearing
A	C	S	180°
C	A	N	000°
A	B	SE	135°
B	A	NW	315°
B	C	SW	225°
C	B	NW	045°
C	D	W	270°
D	C	E	090°

2.



From	To	Direction	Bearing
A	X	W	270°
D	X	NW	315°
Y	B	N	000°
Y	C	NW	315°
Z	C	SE	135°
Z	D	S	180°
Z	A	NE	045°
Z	B	E	090°

Target test - Advanced

C, A, D, B, E, H, G

D

G, F, C, D, A, B, E

<u>1334 Recognising Solids</u>

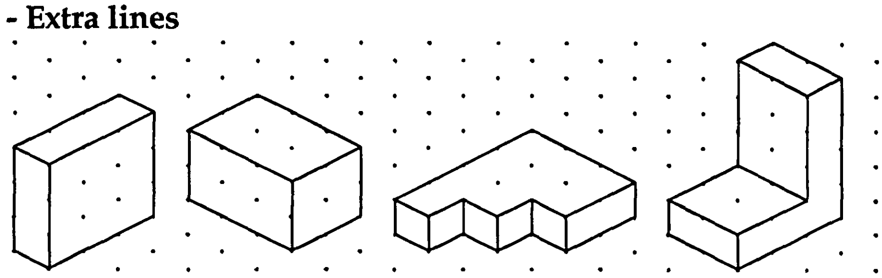
Page 1 - Join the dots and $\frac{1}{2}$cm isometric paper
Show your isometric drawings to your teacher.

Page 3 - Find the pairs

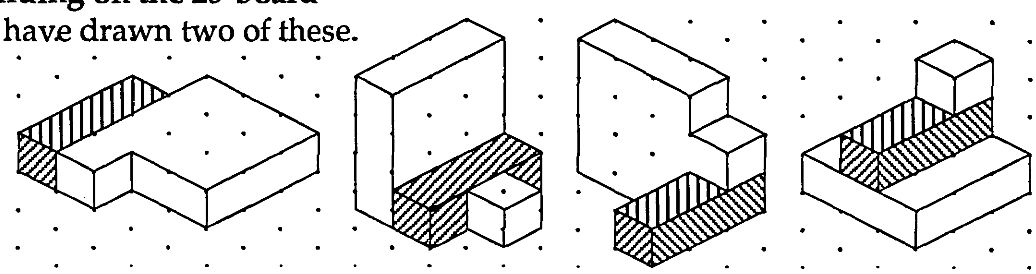
A	=	I
B	=	N
C	=	J
D	=	O
E	=	K
F	=	M
G	=	L
H	=	P
I	=	A
J	=	C
K	=	E
L	=	G
M	=	F
N	=	B
O	=	D
P	=	H

Page 4 - Using the 25-board
Show your isometric drawings to your teacher.

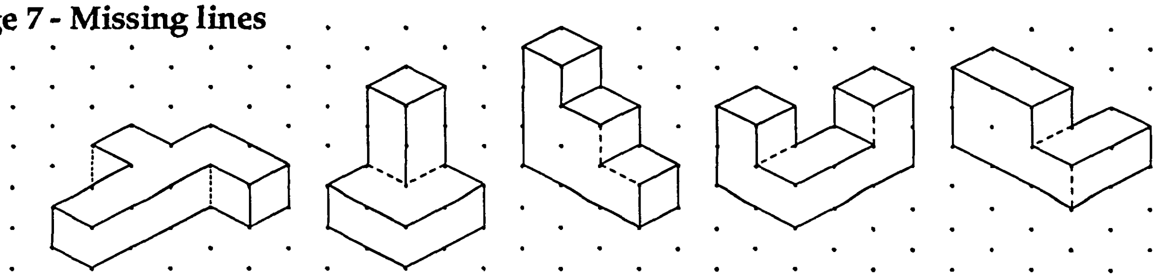
Page 5 - Extra lines



Page 6 - Building on the 25-board
You should have drawn two of these.

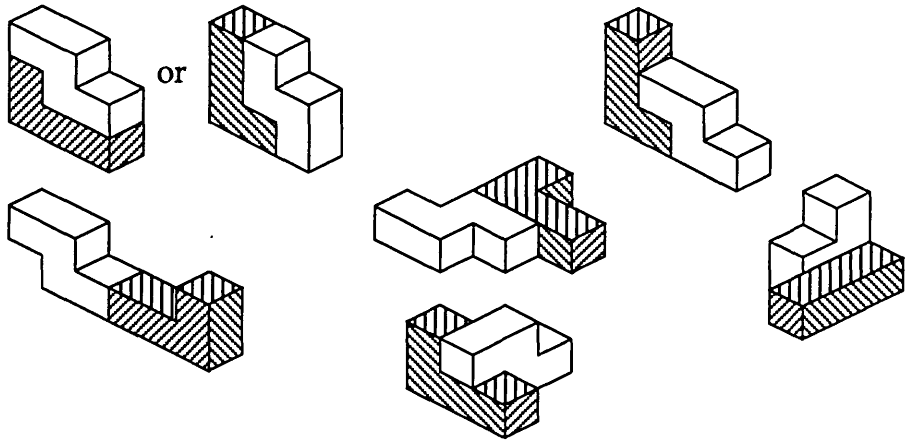


Page 7 - Missing lines

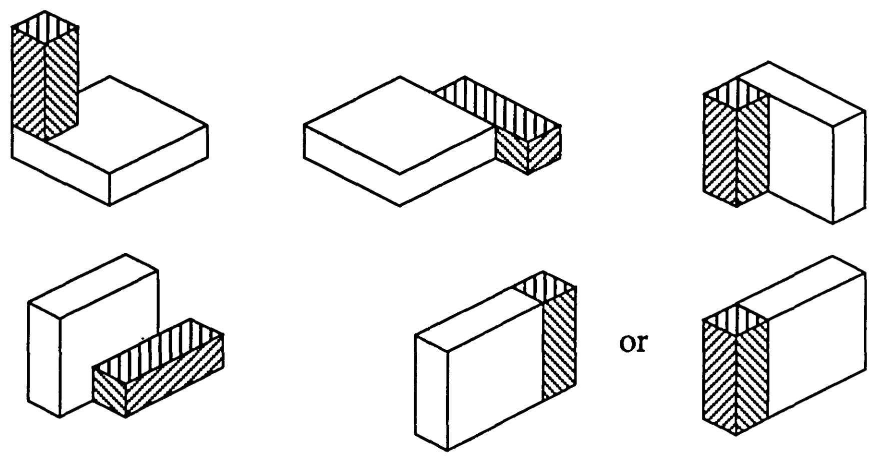


continued/

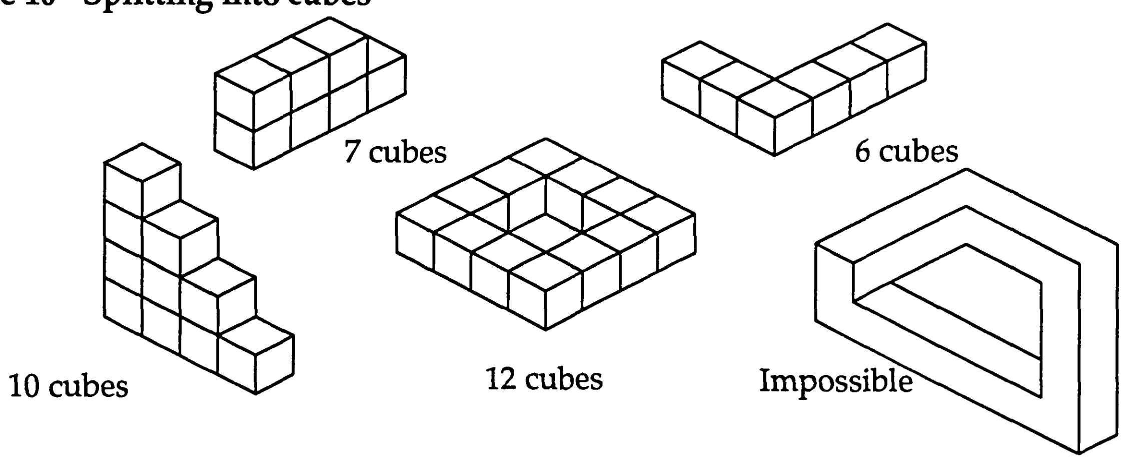
Page 8 - Shading the L-block



Page 9 - Shadows



Page 10 - Splitting into cubes



Target test - Standard

1.

2. A = I, B = D, C = G, D = B, E = H, F = J, G = C, H = E, I = A, J = F.

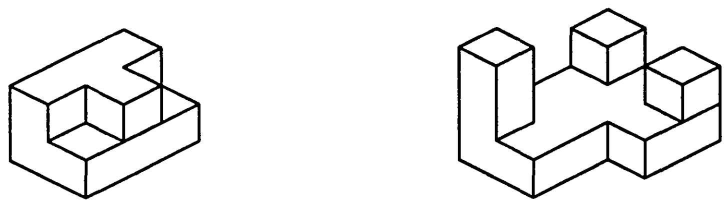
continued/

1334 Recognising Solids (cont)

3.

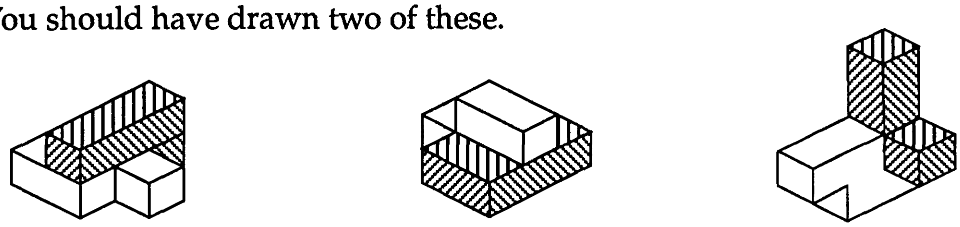
Target test - Advanced

1.



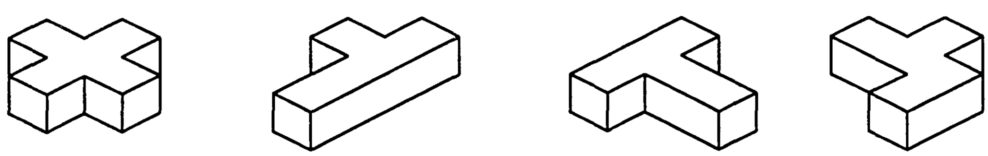
2. A = I, B = D, C = G, D = B, E = H, F = J, G = C, H = E, I = A, J = F.

3. You should have drawn two of these.

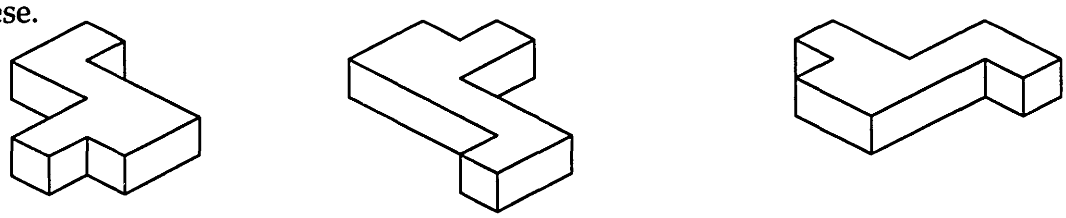


1335 Sketching Solids

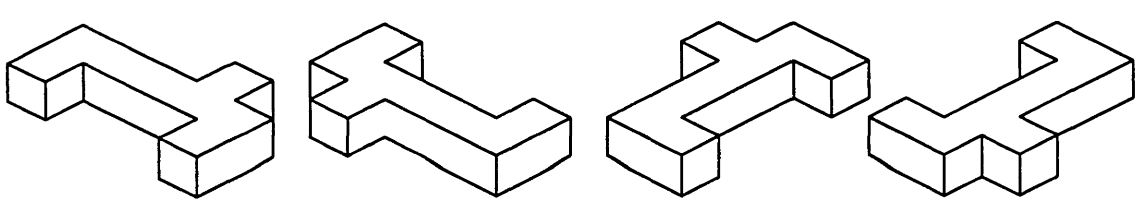
Page 1 - Five cubes
You should have drawn two of these.



Page 2 - The S and L on the 25-board
These show the same solid from the 3 different directions. You should have sketched one of these.

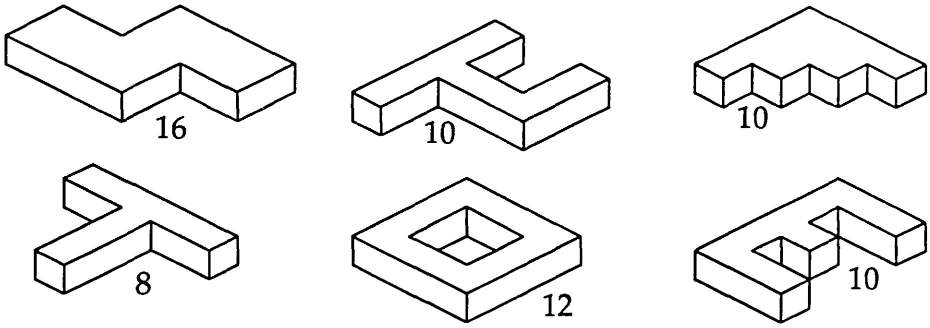


These show the same solid drawn from 4 different directions. You should have sketched one of these.

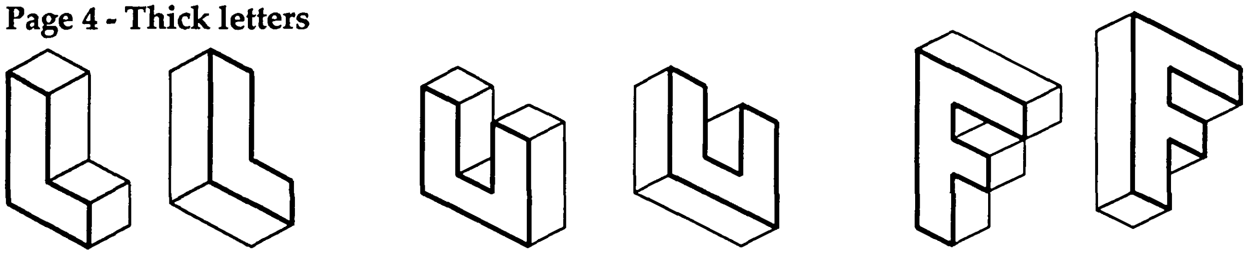


continued/

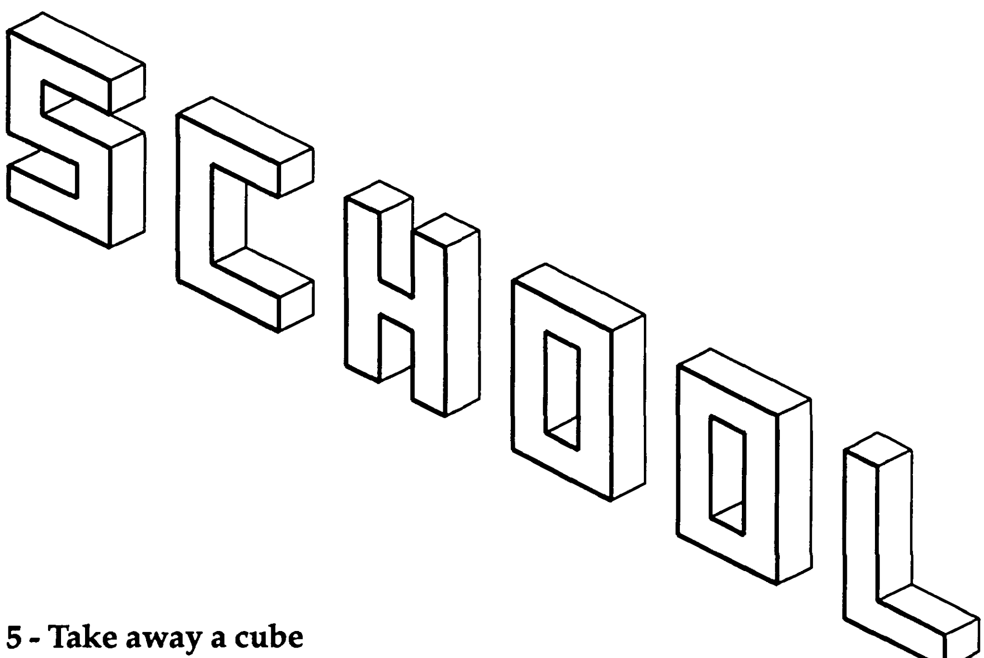
Page 3 - Single layer solids



Page 4 - Thick letters

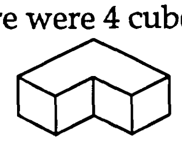


There are two ways to make the letters "thick".
Here is one of the answers.

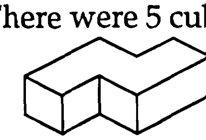


Page 5 - Take away a cube
You should have drawn three of these.

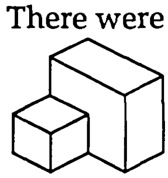
There were 4 cubes.



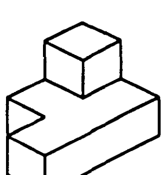
There were 5 cubes.



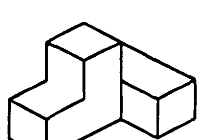
There were 6 cubes.



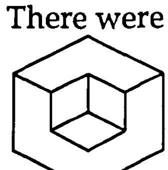
There were 7 cubes.



There were 6 or 7 cubes.



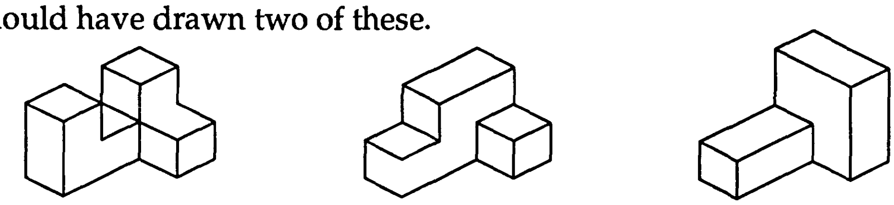
There were 8 cubes.



continued/

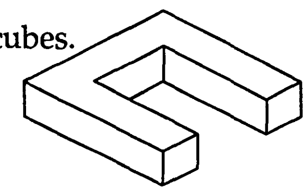
1335 Sketching Solids (cont)

Page 6 - Add a cube
You should have drawn two of these.

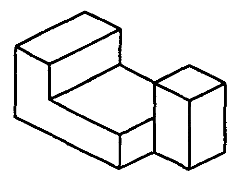


Page 7 - Two cubes less
You should have drawn three of these.

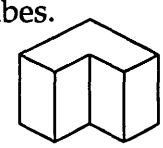
There were 12 cubes.



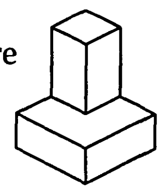
There were 12 cubes.



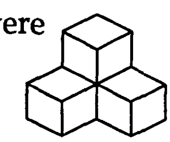
There were 8 cubes.



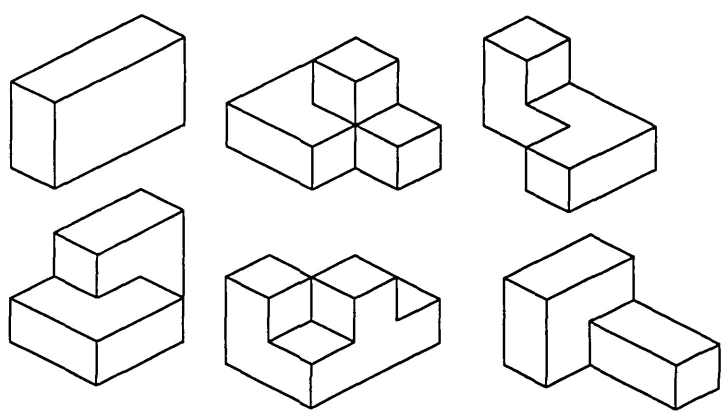
There were 8 cubes.



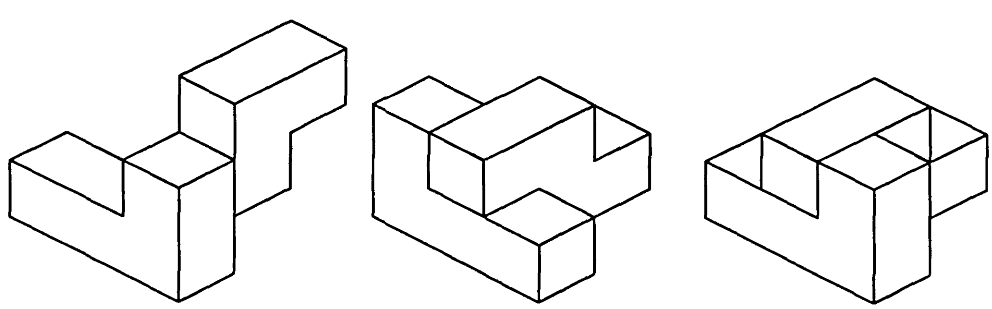
There were 6 cubes.



Page 8 - Two cubes more



Page 9 - A 25-board puzzle
You should have drawn two of these.



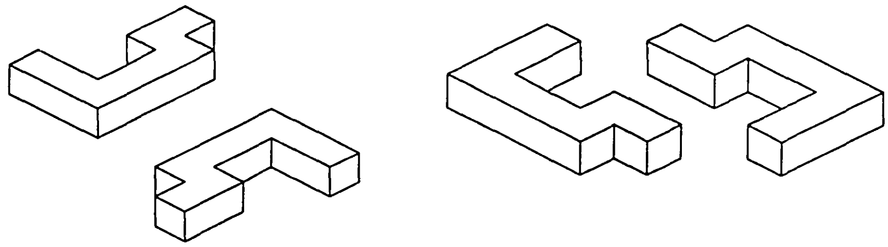
Page 10 - Making Solids
Many possible answers. Get someone else to check your drawings if you are not sure whether they are correct.

continued/

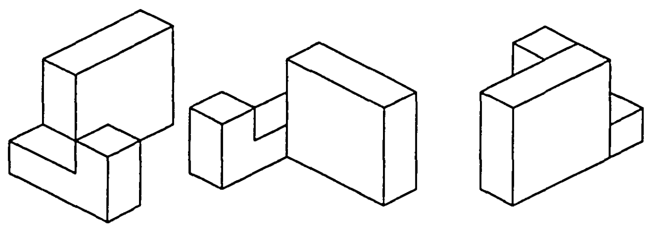
1335 Sketching Solids (cont)

Target test - Standard

1. You should have drawn two of these.

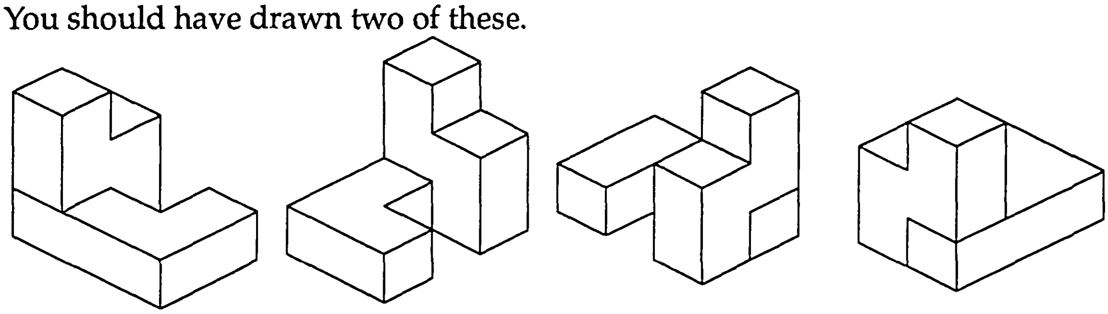


2. You should have drawn two of these.

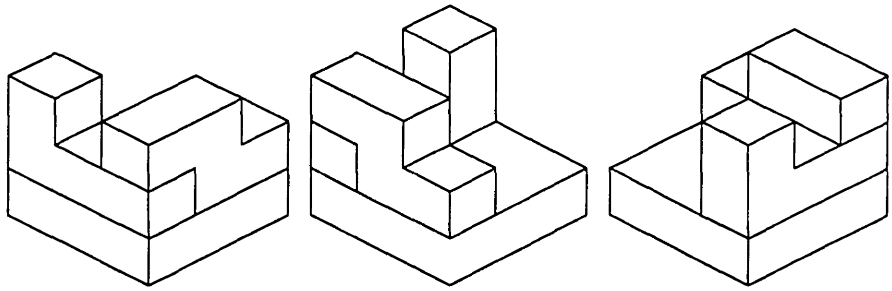


Target test - Advanced

1. You should have drawn two of these.



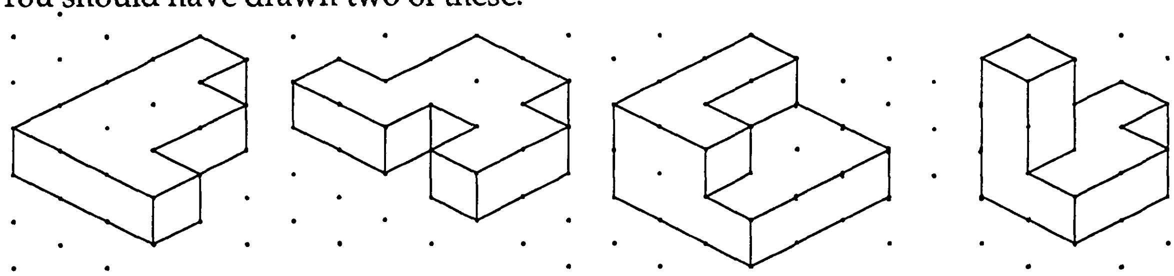
2. You should have drawn two of these.



1336 Turning and Toppling

Page 1 - Toppling

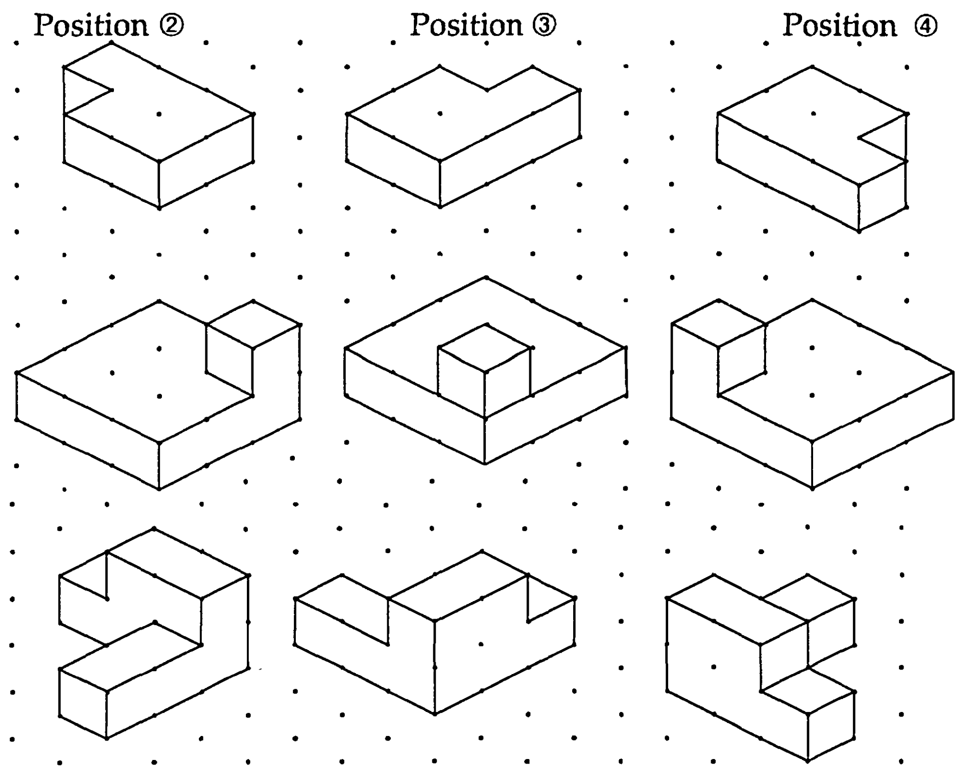
You should have drawn two of these.



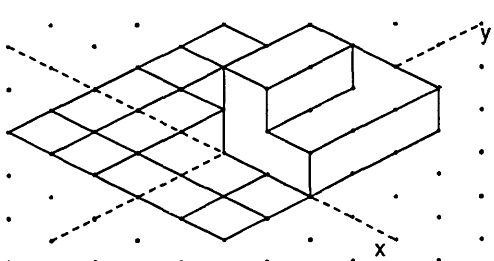
continued/

1336 Turning and Toppling (cont)

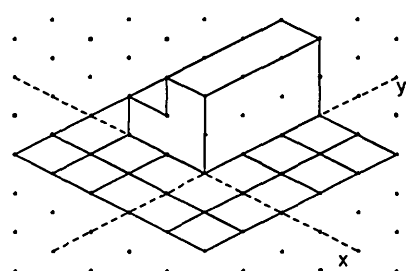
Page 2 - Turning solids round
You should have drawn one of these solids in the three positions.



Page 3 - Toppling on the 25 - board
Solid toppled about line x.

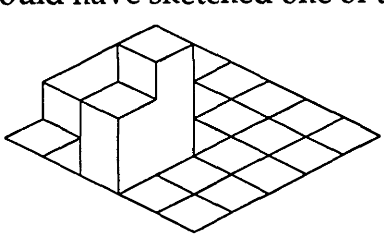


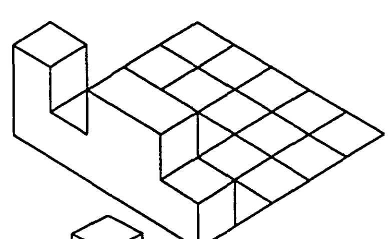
Solid toppled about line y.



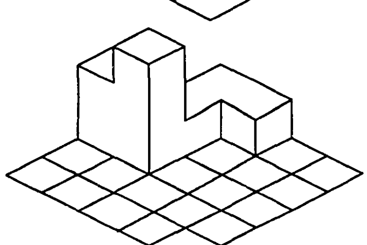
Page 4 - Turning the 25 - board
You should have sketched one of these.

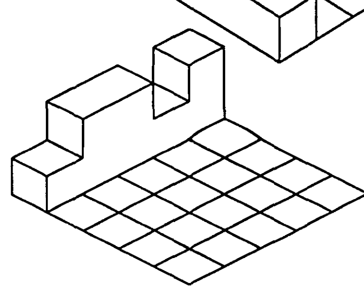
a)





b)



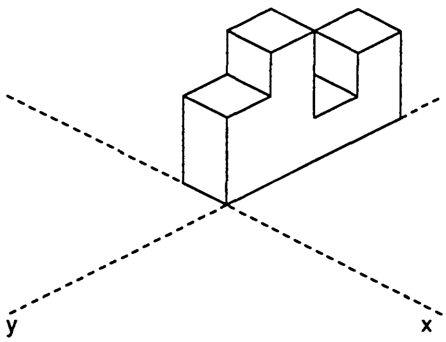


continued/

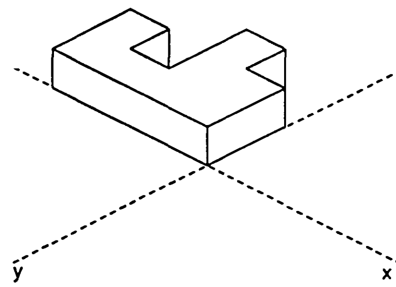
<u>1336 Turning and Toppling (cont)</u>

Page 5 - Double Topple

Toppled about the line x
and then about the line y.

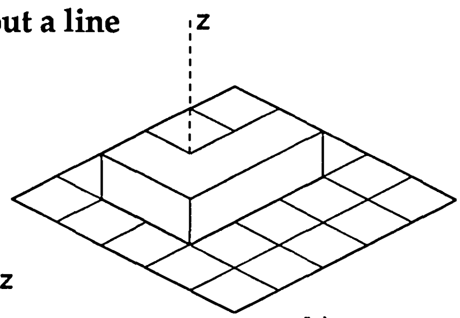


Toppled about the line y
and then about the line x.

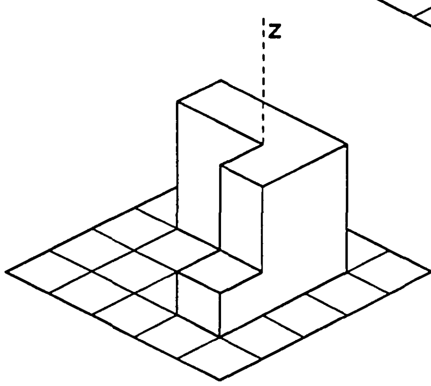


The results are different.

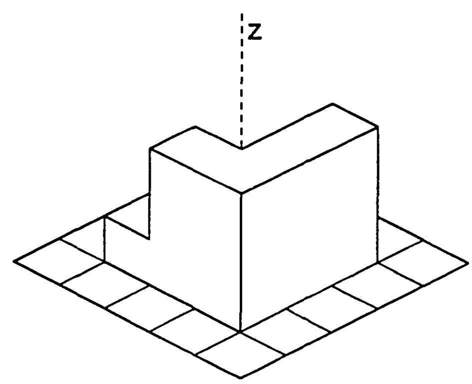
Page 6 - Turning about a line



a)



b)



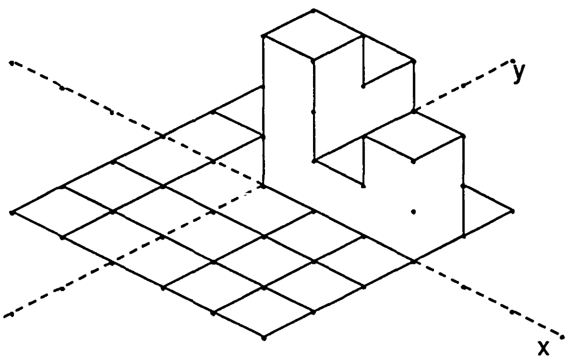
Page 7 - Spot the turns and topples

	Starting position	Final position	Name of Change
Example	⑥	④	quarter turn
1.	① or ❹	⑤ or ❸	topple to right ↗
2.	⑥ or ❷	① or ❹	topple to left ↖
3.	② ←→	⑤	half turn
4.	①	③	quarter turn
5.	① or ❹	⑤ or ❸	topple to right ↗
6.	⑥ or ❷	① or ❹	topple to left ↖

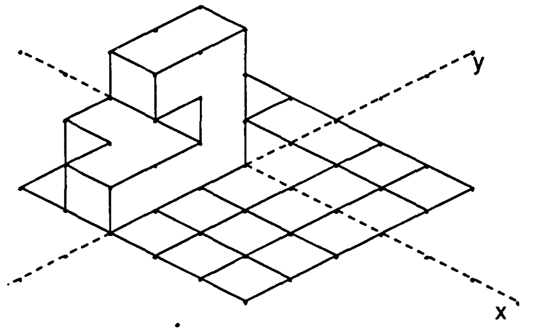
continued/

Page 8 - More toppling

a) Toppled about line x.

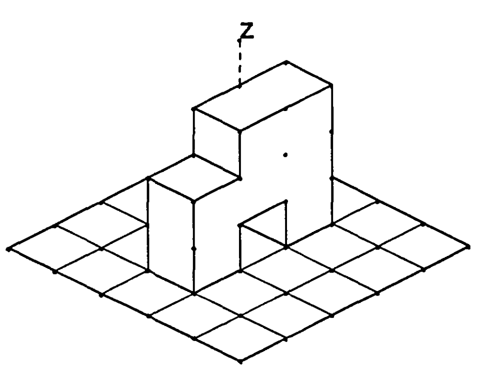


b) Toppled about line y.

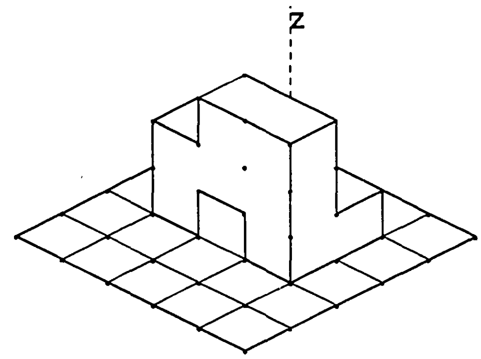


Page 9 - Turn again

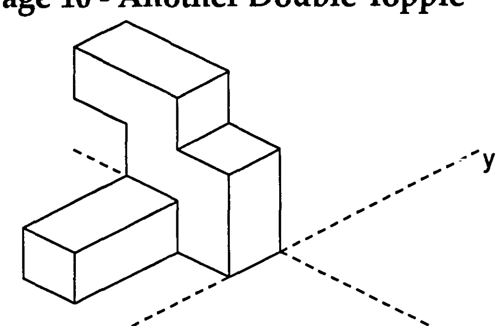
a) A quarter turn clockwise

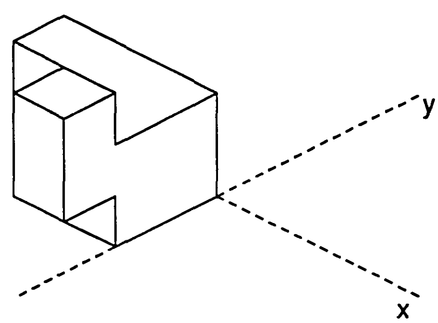


b) A half turn



Page 10 - Another Double Topple

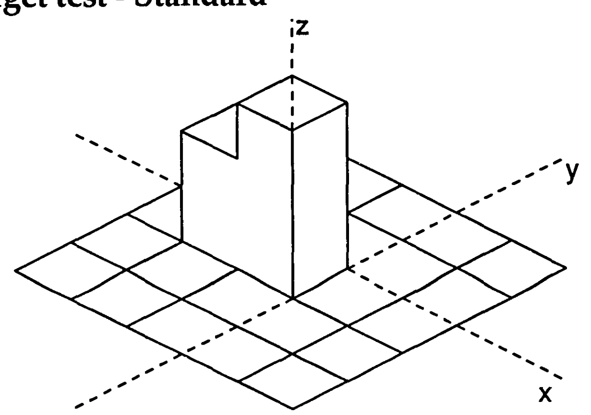




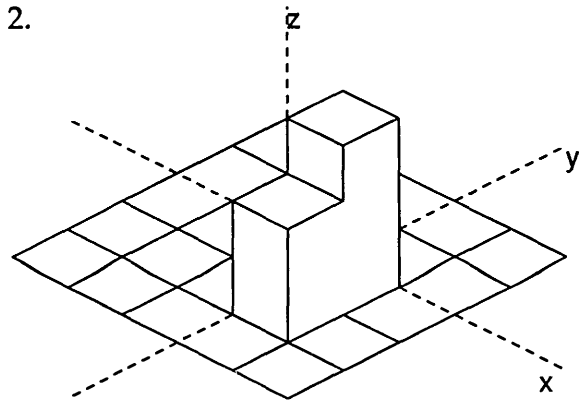
The final positions are not the same.

Target test - Standard

1.



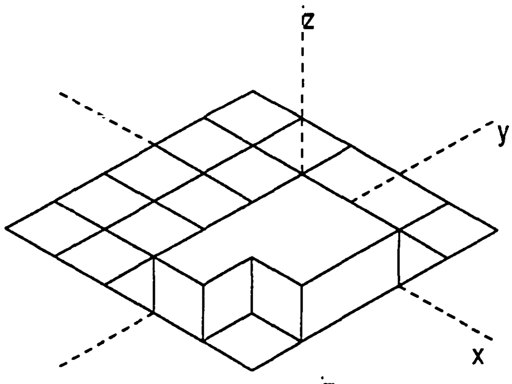
2.



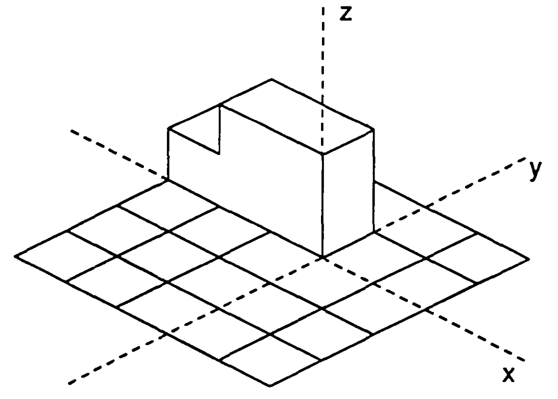
continued/

1336 Turning and Toppling (cont)

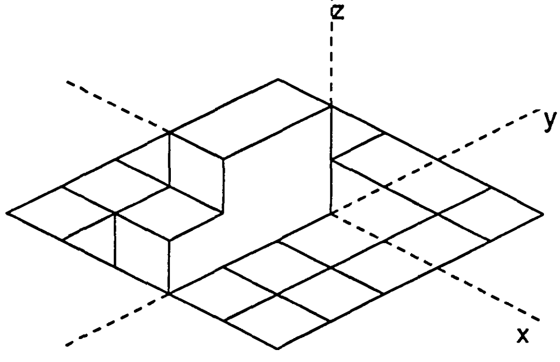
3.



4.

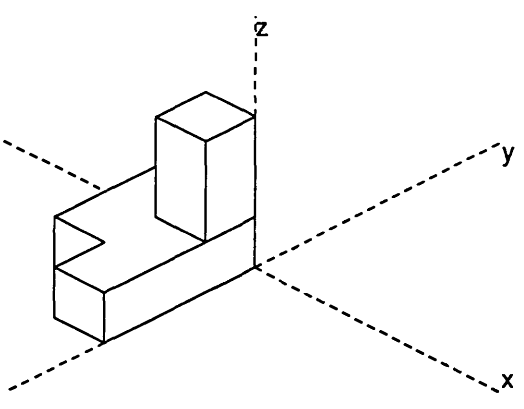


5.

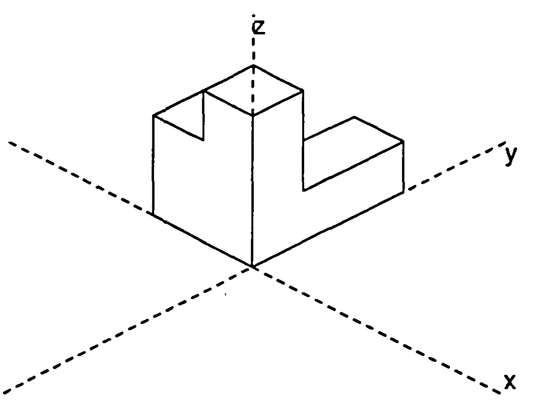


Target test - Advanced

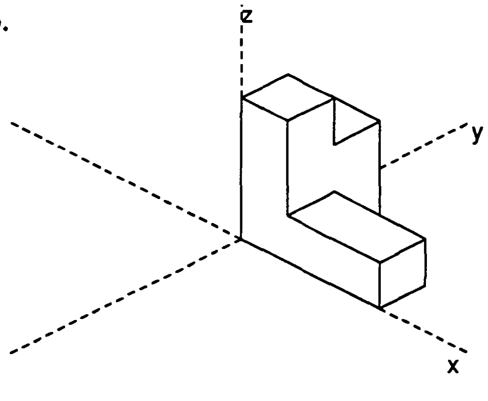
1.



2.

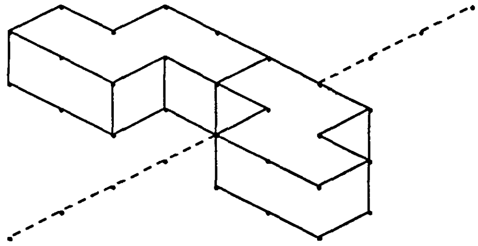


3.



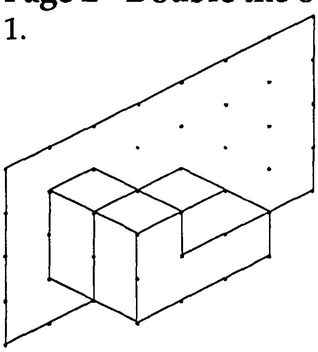
1337 Reflections

Page 1 - Reflections in a mirror

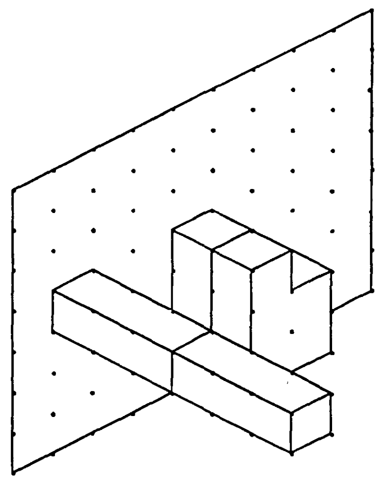


Page 2 - Double the object

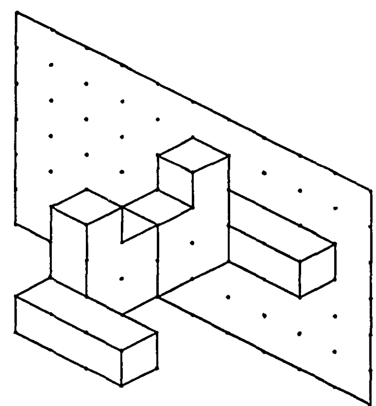
1.



2.

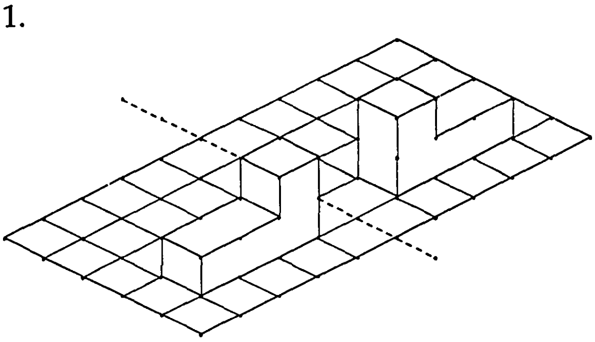


3.

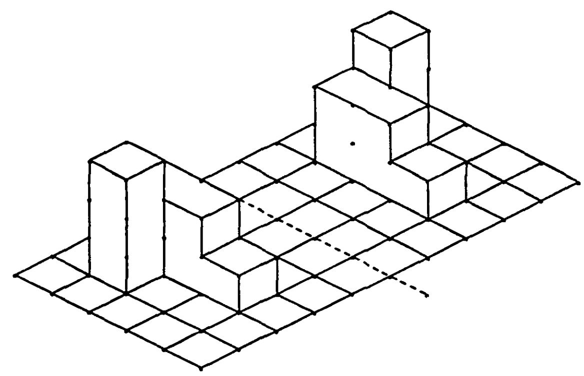


Page 3 - 25 - board and mirror

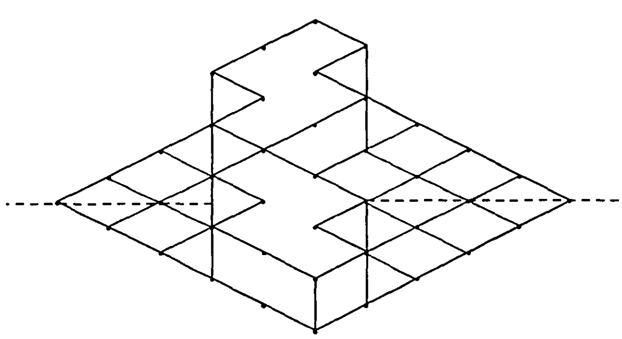
1.

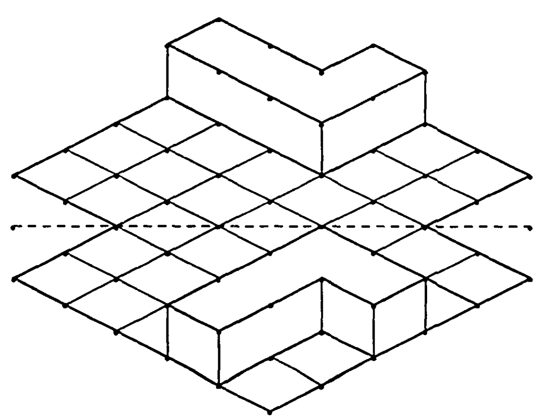


2.



Page 4 - Diagonal mirror

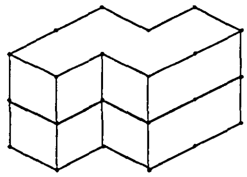


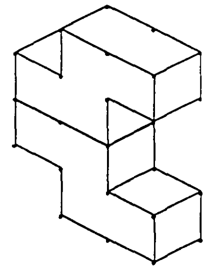


continued/

1337 Reflections (cont)

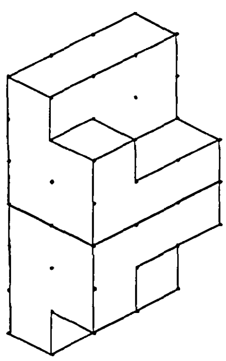
Page 5 - Sitting on a mirror



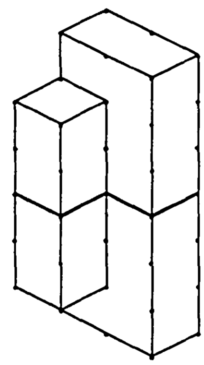


You should have drawn one of these.

1.

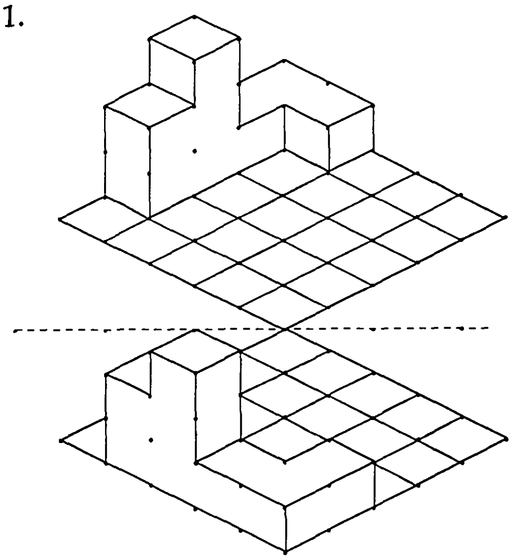


2.

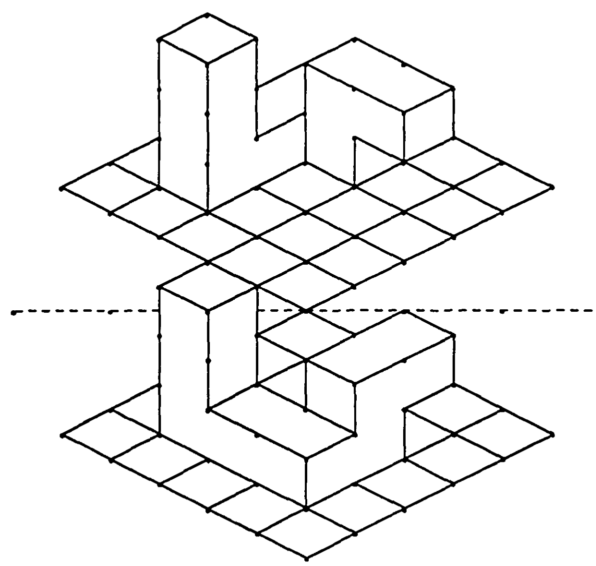


Page 6 - A mirror behind

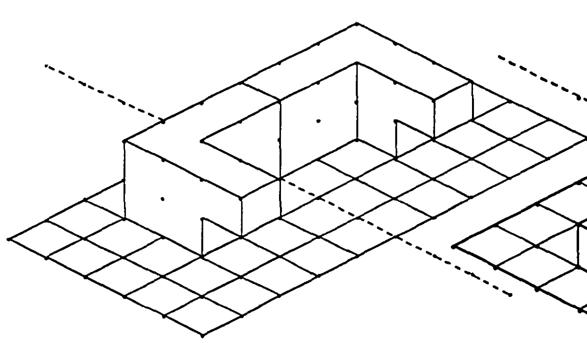
1.

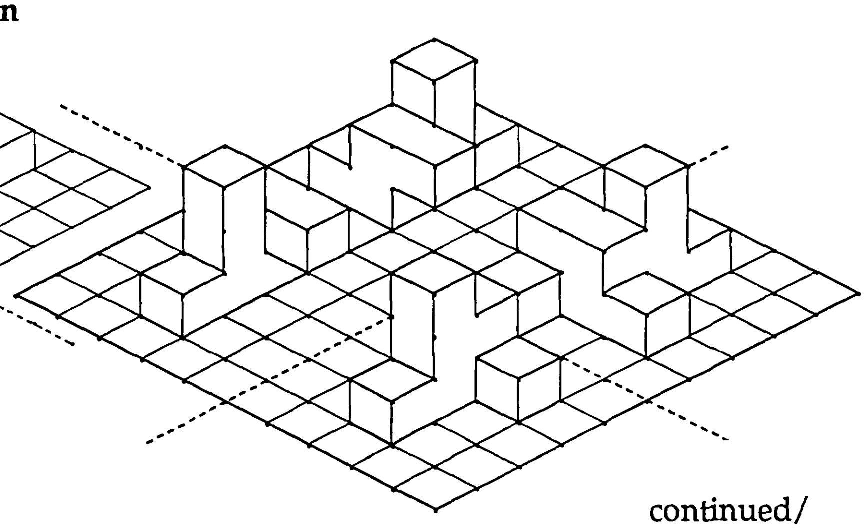


2.



Page 7 - Building a reflection



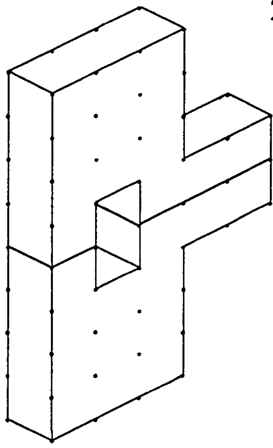


continued/

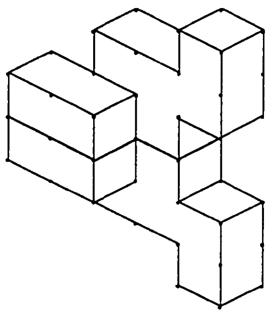
Page 8 - Solids on a mirror

You should have drawn two of these.

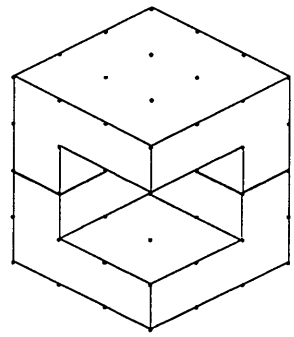
1.



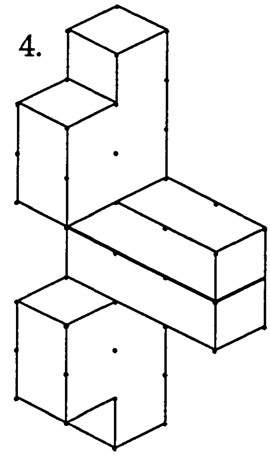
2.



3.



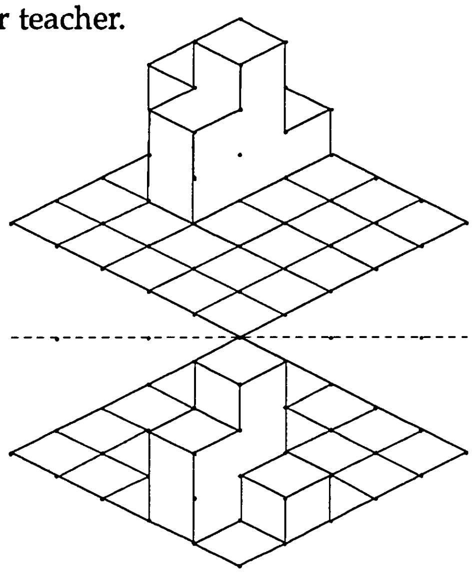
4.



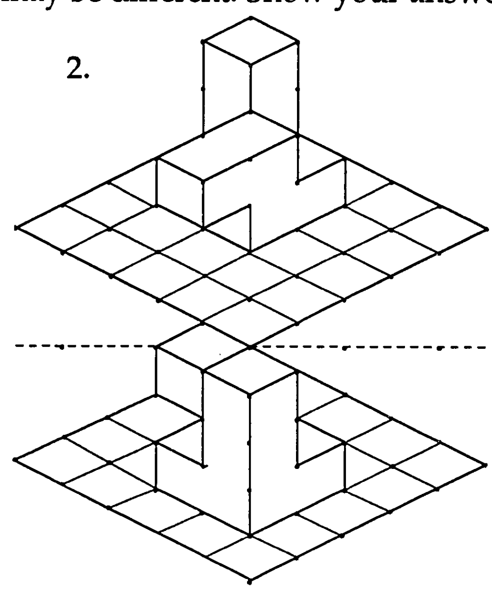
Page 9 - More difficult solids

You should have drawn two of these. In some cases it is possible to arrange the starting solids in more than one way. If so, your answers may be different. Show your answers to your teacher.

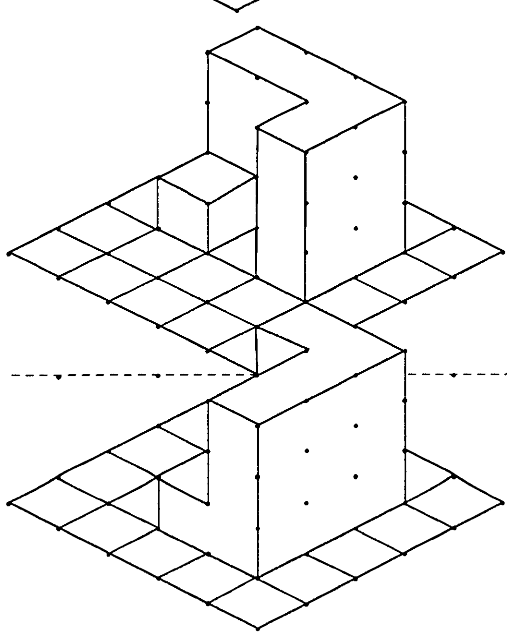
1.



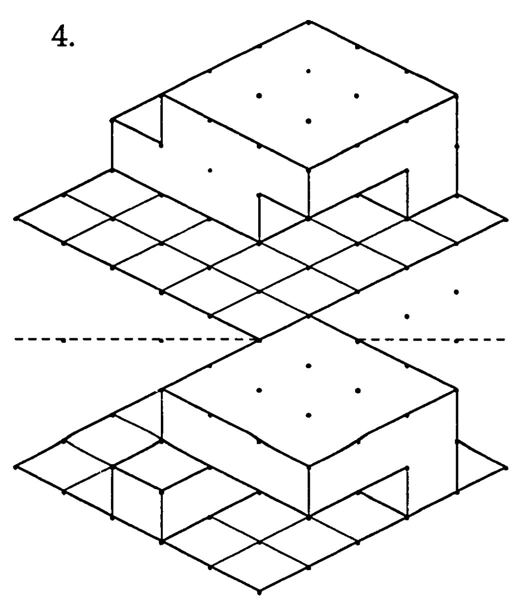
2.



3.

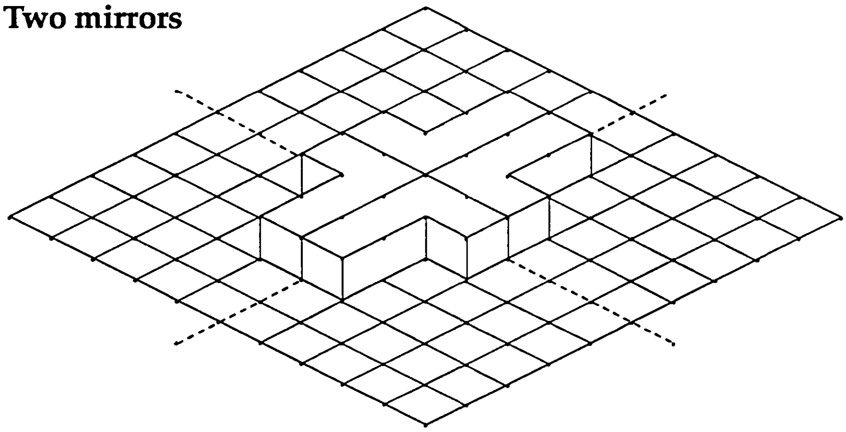


4.



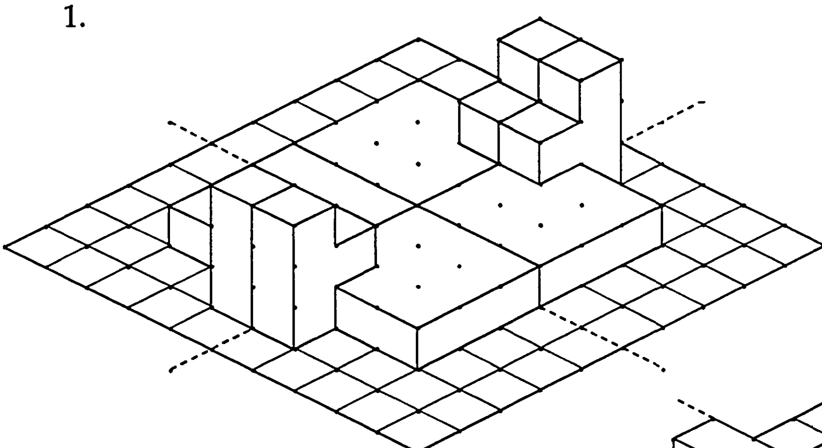
1337 Reflections (cont)

Page 10 - Two mirrors

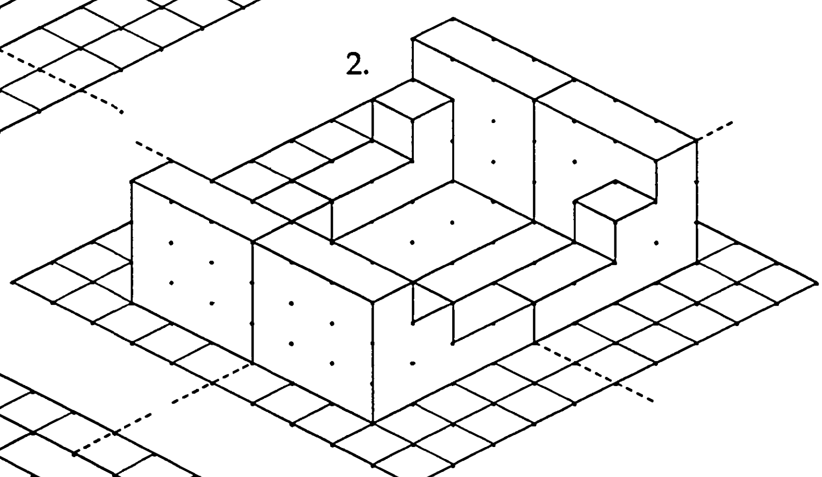


You should have drawn two of these.

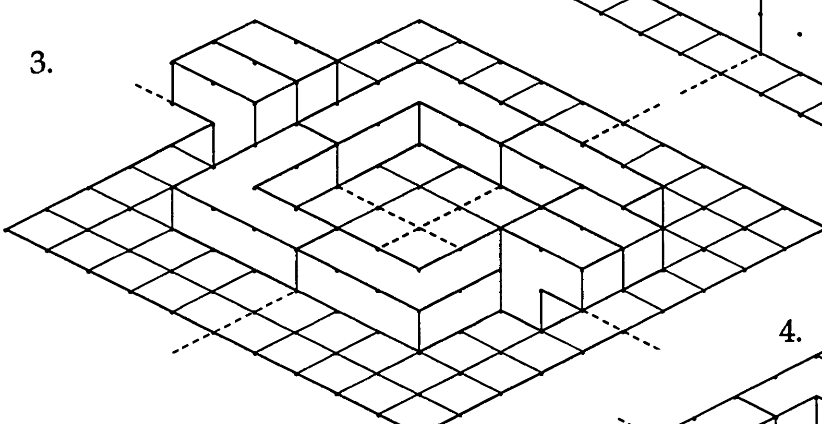
1.



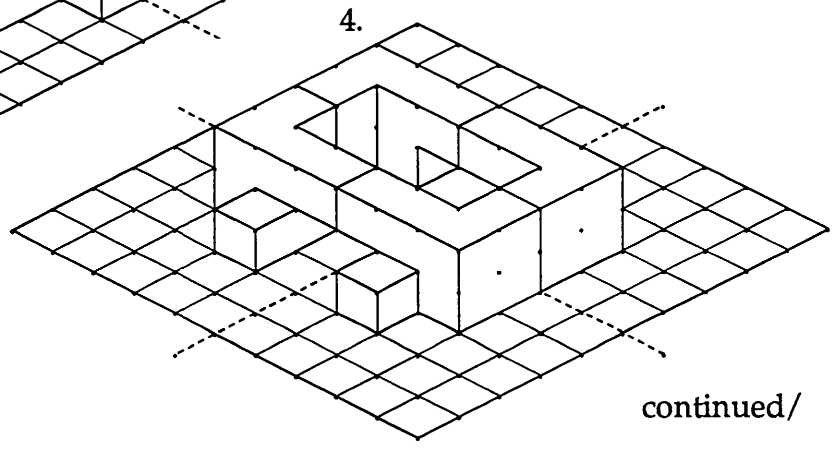
2.



3.



4.



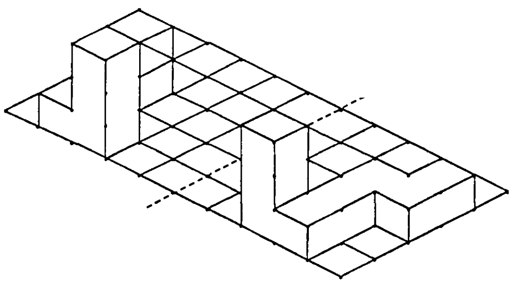
continued/

1337 Reflections (cont)

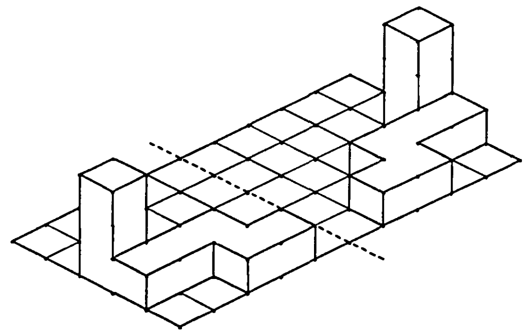
Target test - Standard

You should have drawn three of these.

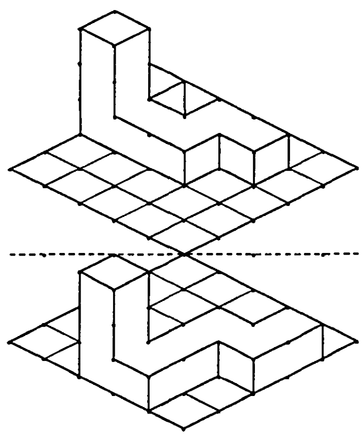
a)



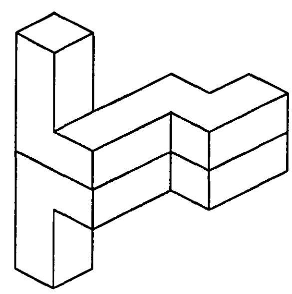
b)



c)

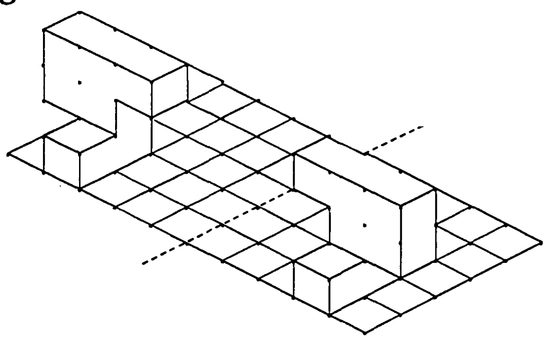


d)

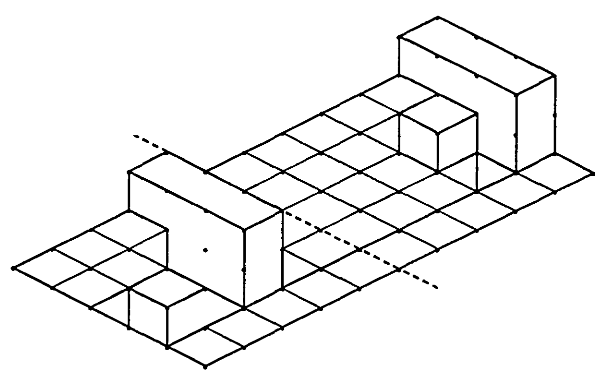


Target test - Advanced

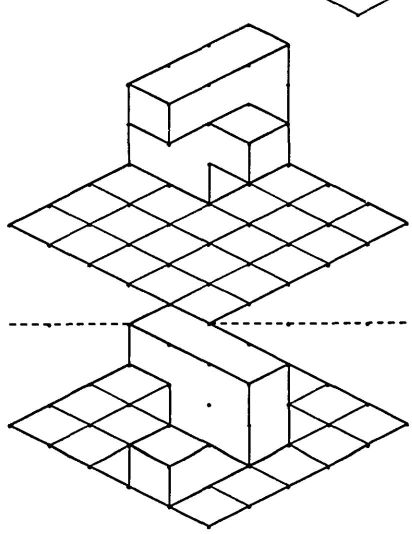
a)



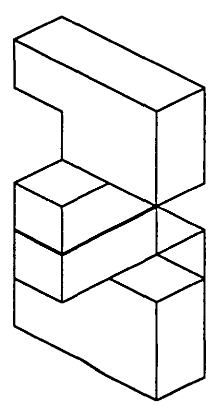
b)



c)



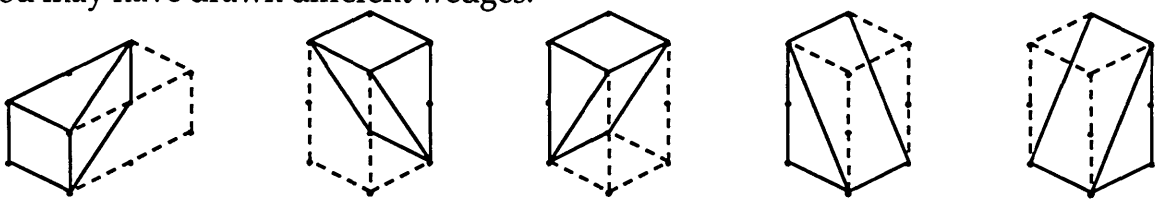
d)



1338 Wedges

Page 1 - How to draw a wedge

You may have drawn different wedges.



Page 2 - Making solids

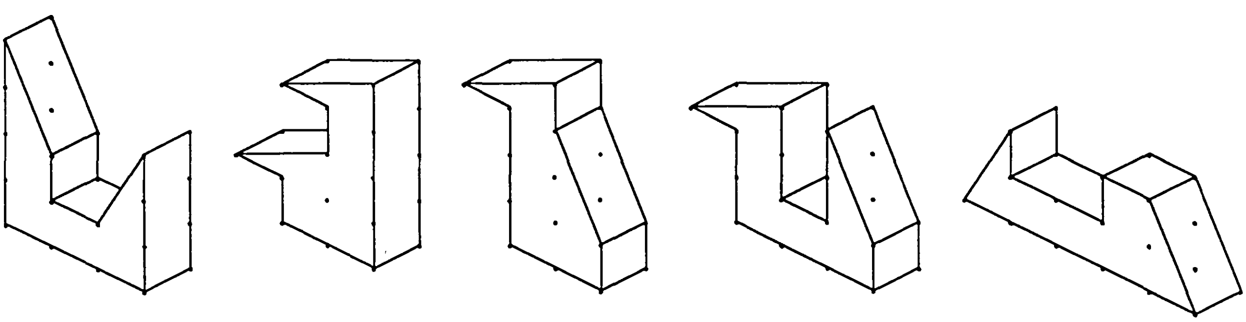
You should have drawn three of the solids.

Page 3 - Find the pairs

A	=	G
B	=	I
C	=	N
D	=	M
E	=	P
F	=	L
H	=	J
K	=	O

Page 4 - From plan to sketch

You should have sketched three of these.



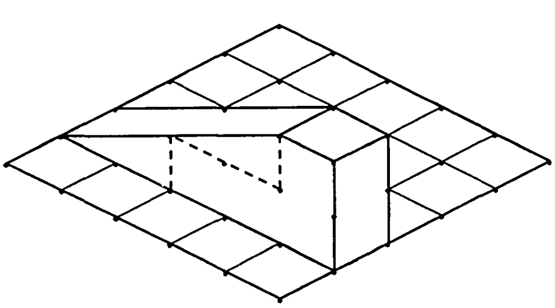
Page 5 - Make and draw

You should have sketched at least four different solids using a wedge and a long.

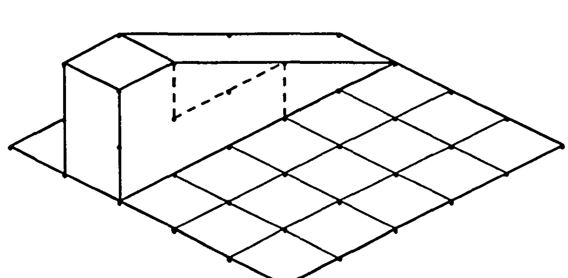
Page 6 -Turning the board

You should have drawn two of these.

a) a quarter turn



b) a half turn

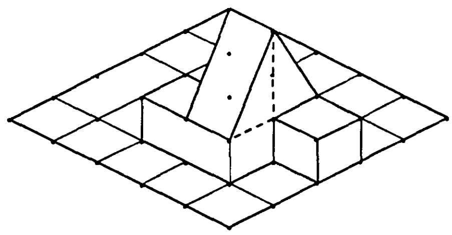


continued/

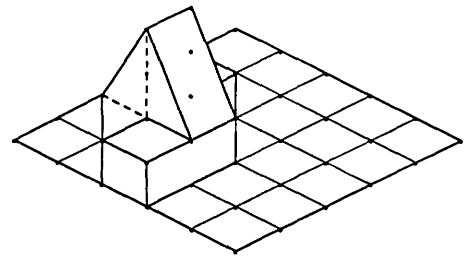
1338 Wedges (cont)

Page 6 - Turning the board (cont)

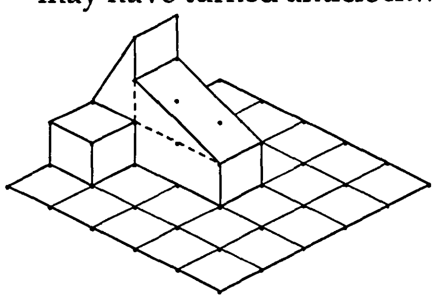
a) a quarter turn clockwise (you may have turned anticlockwise)



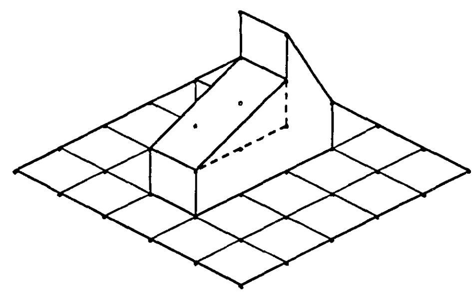
b) a half turn



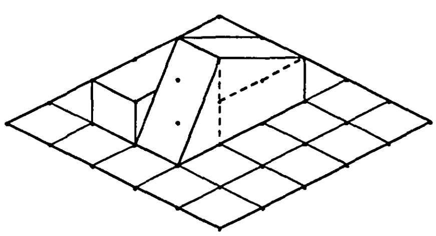
a) a quarter turn clockwise (you may have turned anticlockwise)



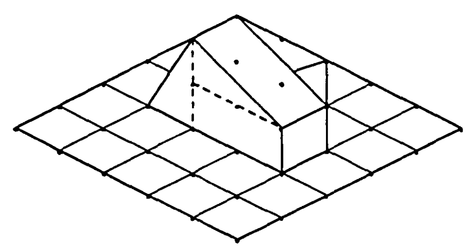
b) a half turn



a) a quarter turn clockwise (you may have turned anticlockwise)

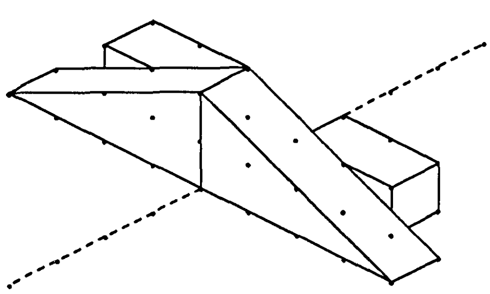


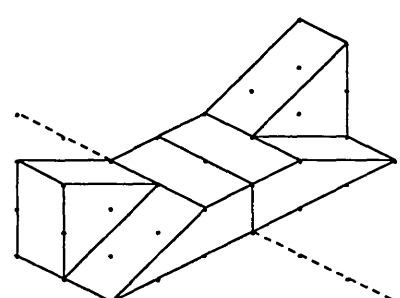
b) a half turn

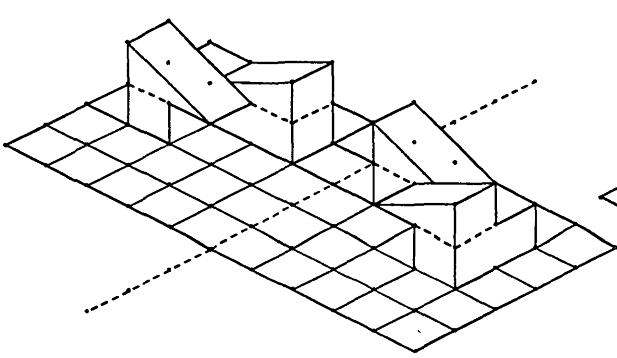


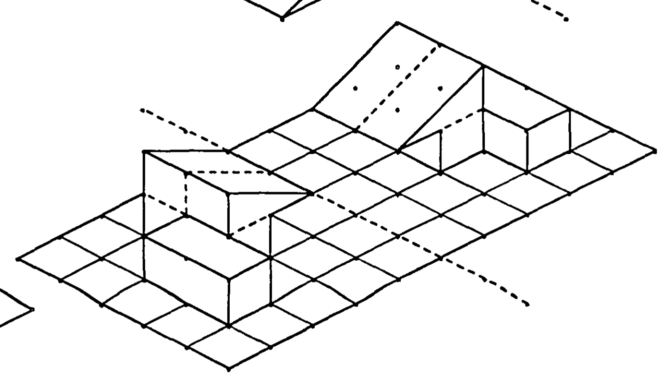
Page 7 - Reflections

You should have drawn two of these.





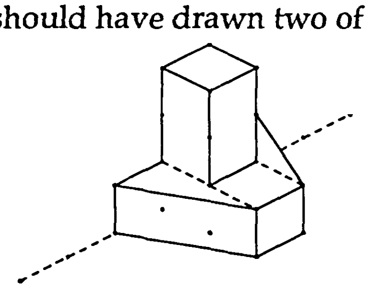


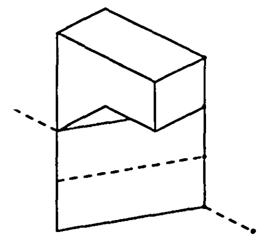


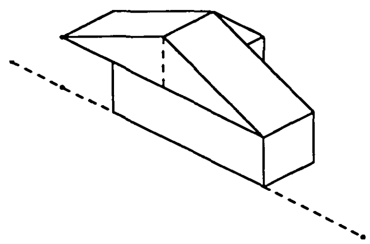
continued/

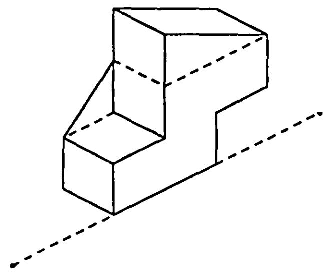
Page 8 - Topple

You should have drawn two of these.



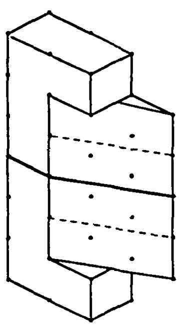


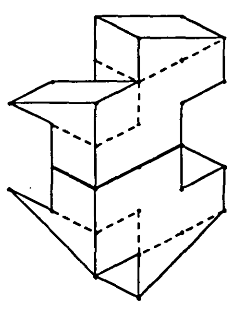


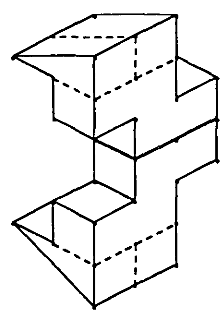


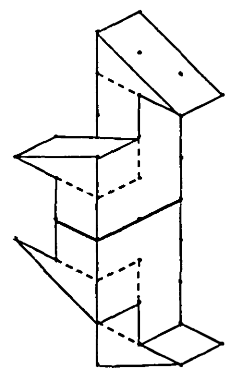
Page 9 - Horizontal mirror

You should have sketched two of these.



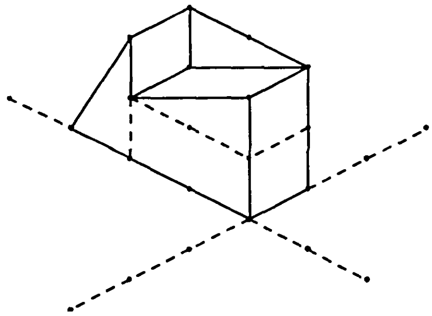




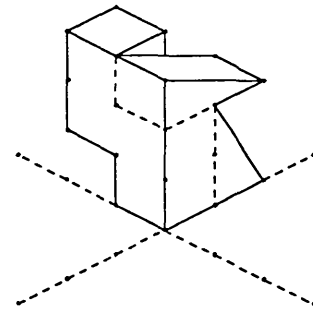


Page 10 - Double topple

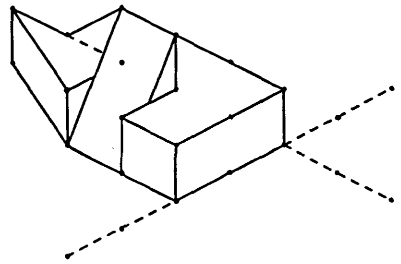
a) first about line x then about line y



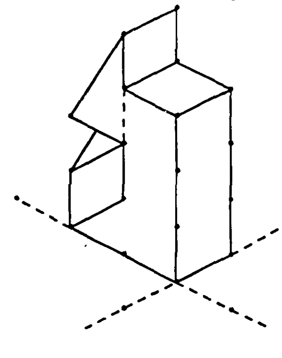
b) first about line y then about line x.



a) first about line x then about line y



b) first about line y then about line x.



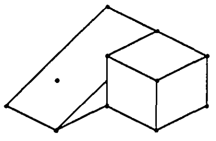
continued/

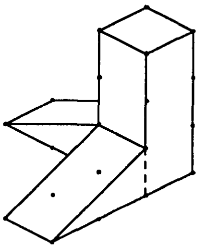
1338 Wedges (cont)

Target test - Standard

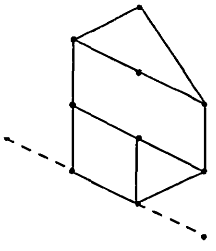
You should have drawn one set of these solids.

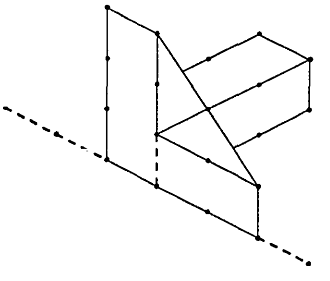
a) a quarter turn clockwise



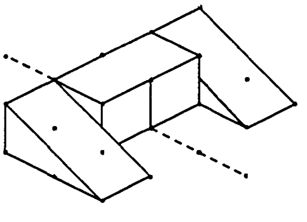


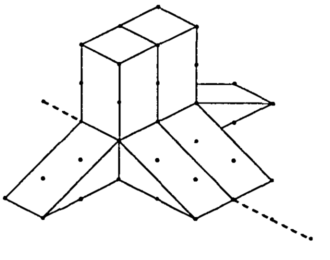
b) toppled about the line x



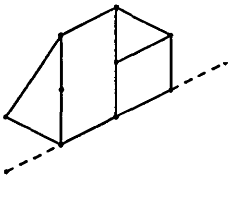


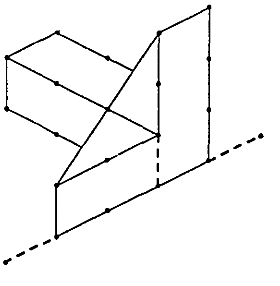
c) mirror put on line x



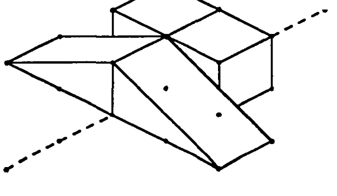


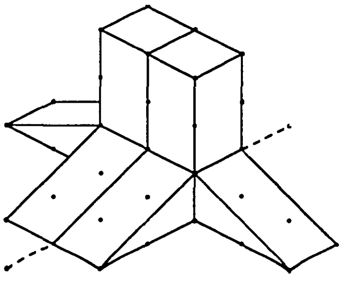
d) toppled about line y





e) mirror on line y



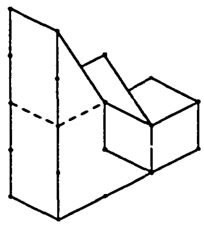


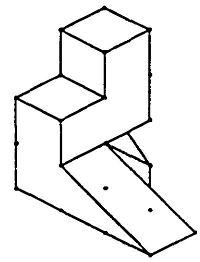
continued/

Target Test - Advanced

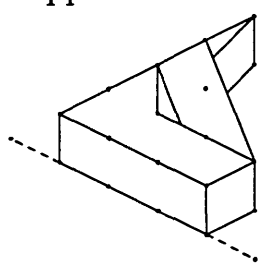
You should have drawn one set of these solids.

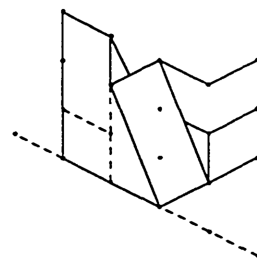
a) a quarter turn anticlockwise



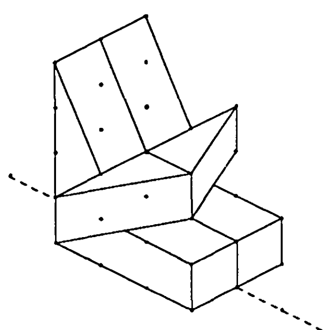


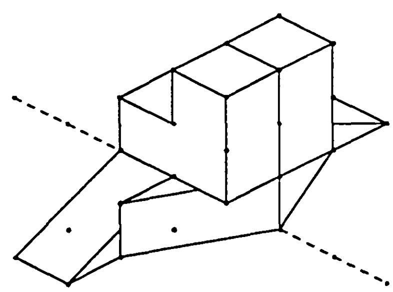
b) toppled about the line x



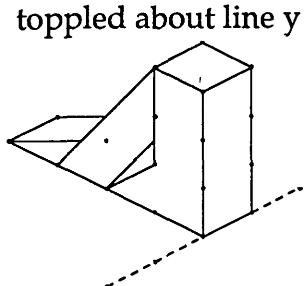


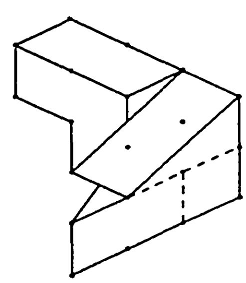
c) mirror put on line x



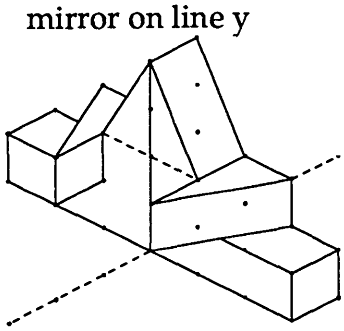


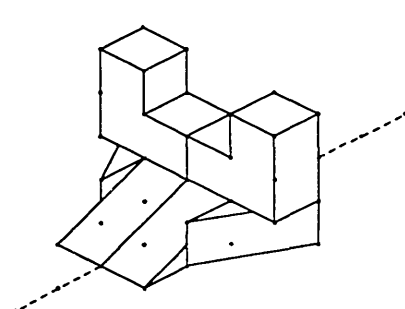
d) toppled about line y





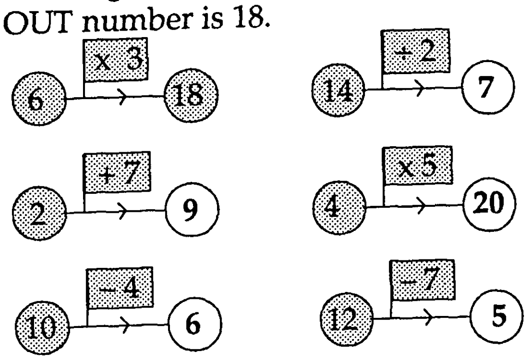
e) mirror on line y

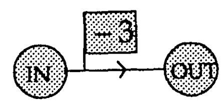




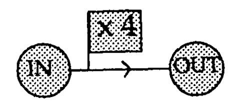
1339 Flags

Page 1 - Flags and number mappings
The OUT number is 18.



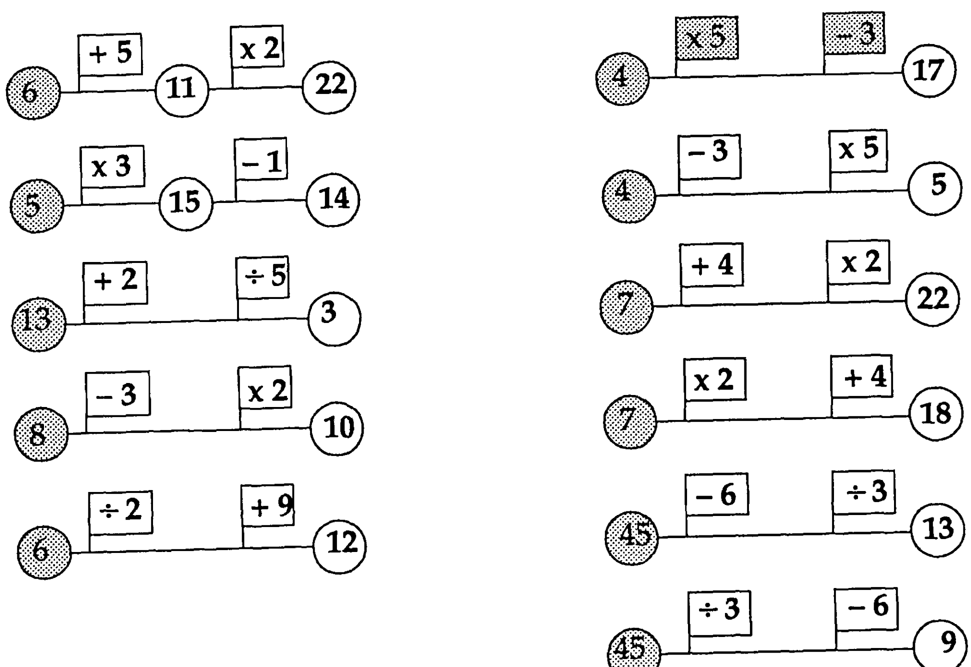


10	→	7
4	→	1
8	→	5
6	→	3

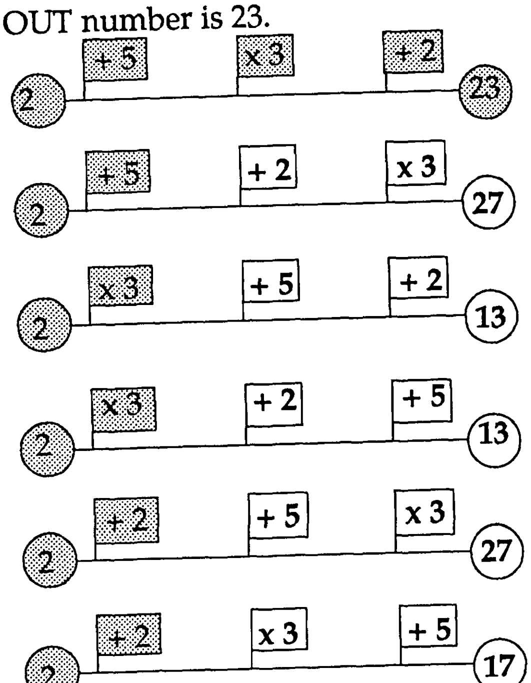


3	→	12
5	→	20
2	→	8
7	→	28

Page 2 - Double flags



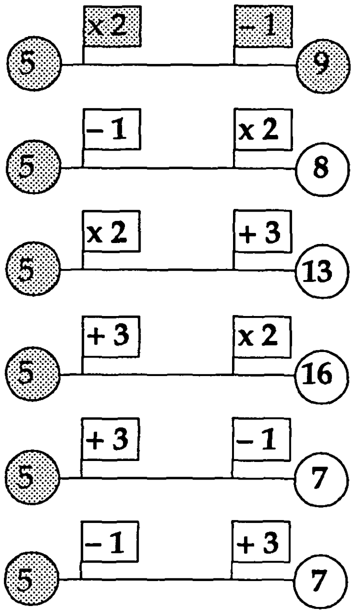
Page 3 - Three number machine
The OUT number is 23.



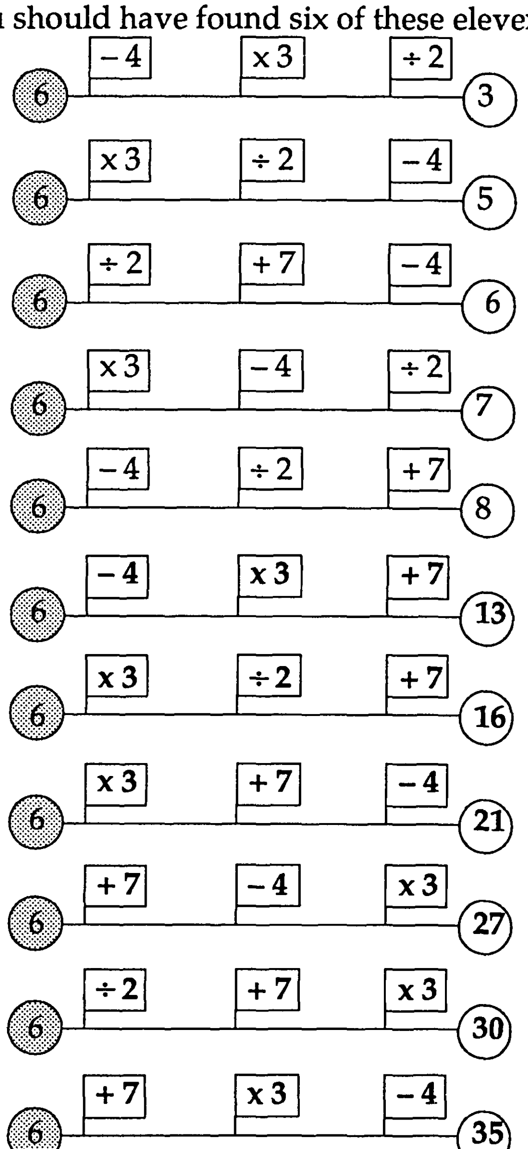
continued/

Page 4 - Puzzle page

You should have found five of these six possible ways.



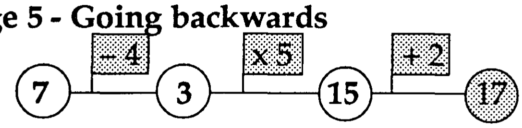
You should have found six of these eleven possible ways.



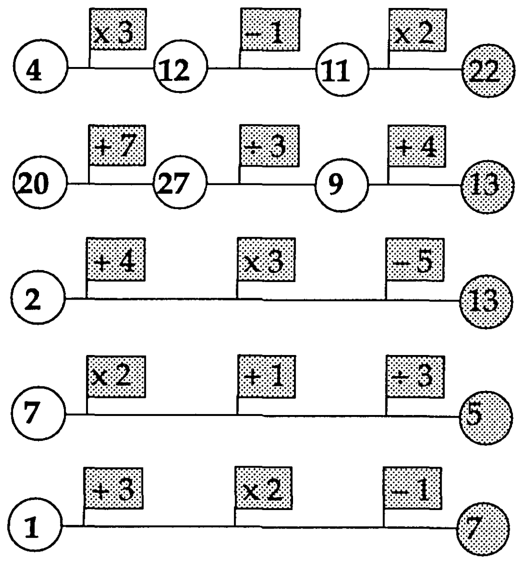
continued/

1339 Flags (cont)

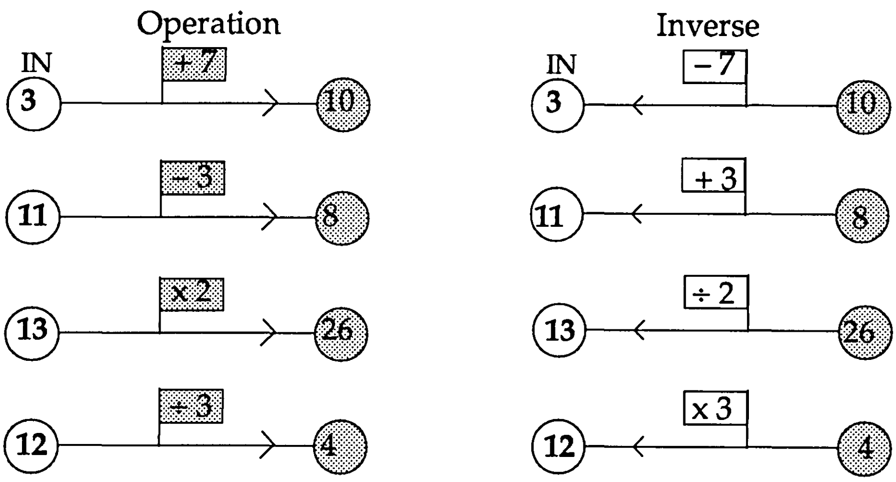
Page 5 - Going backwards



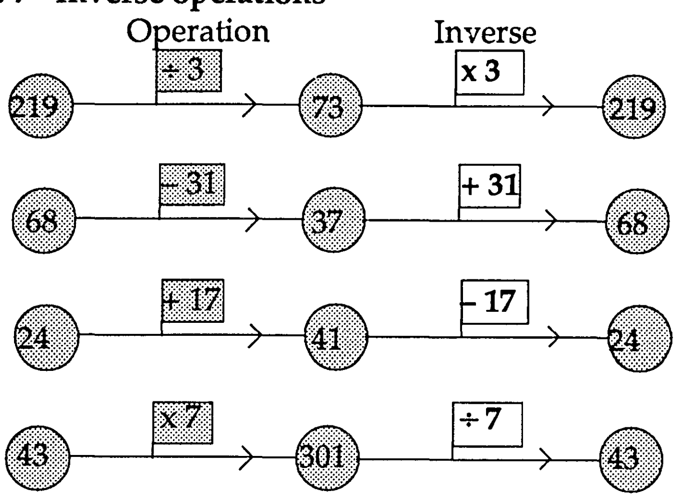
The IN number is 2.



Page 6 - Machines in reverse



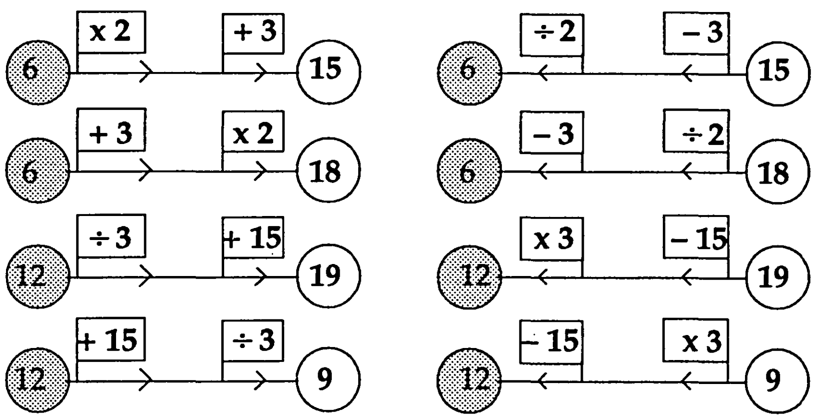
Page 7 - Inverse operations



continued/

Page 8 - Two-stage operations

Flag Diagram | Flags pointing left | Inverse Operation Program



Subtract 3, then divide by 2.

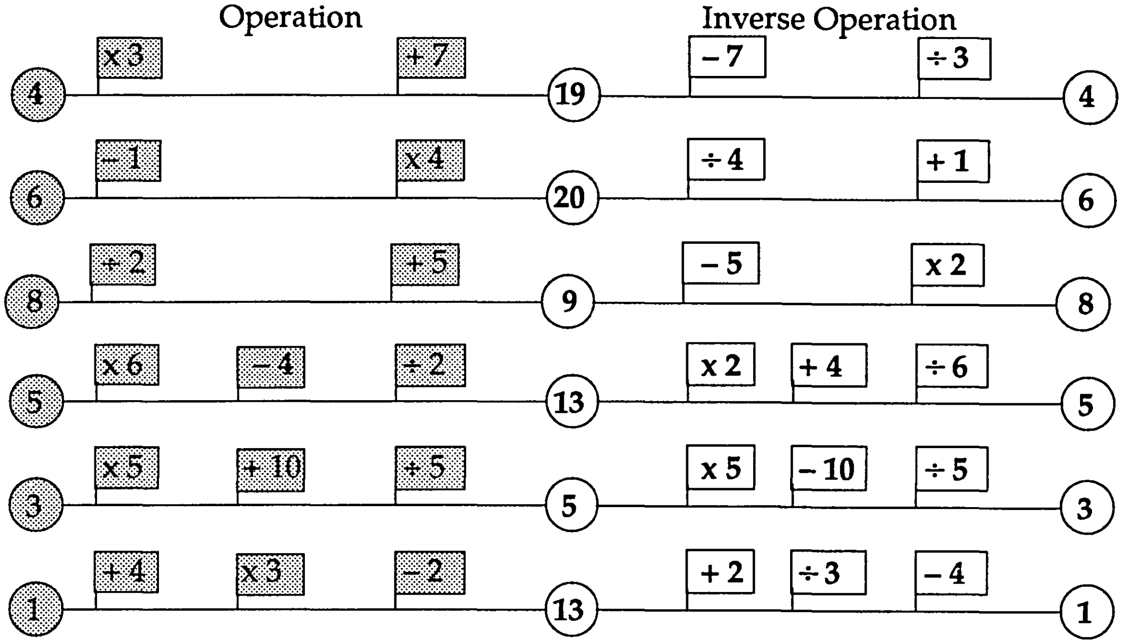
Divide by 2, then subtract 3.

Subtract 15, then multiply by 3.

Multiply by 3, then subtract 15.

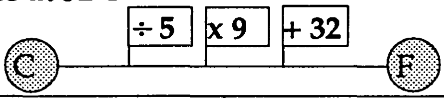
Page 9 - Understanding inverses

Operation | Inverse Operation

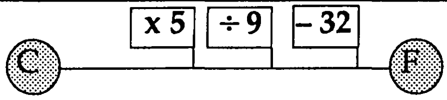


Page 10 - Celsius and Fahrenheit

Water freezes at 32°F

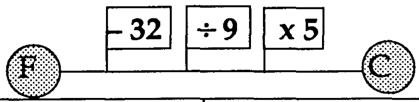


Celsius	Fahrenheit
100°C	212°F
75°C	167°F
50°C	122°F
25°C	77°F
0°C	32°F



The program for the machine is, 'subtract 32, divide by 9, then multiply by 5'.

continued/

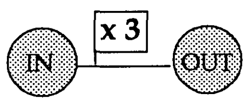


Fahrenheit	Celsius
50°F	10°C
68°F	20°C
95°F	35°C
140°F	60°C
158°F	70°C

Target test - Standard

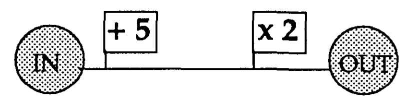
1. & 2.

A



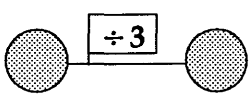
5	→	**15**
9	→	**27**
3	→	9
5	→	15

B

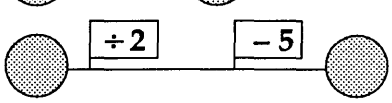


3	→	**16**
8	→	**26**
1	→	12
10	→	30

3. A



B

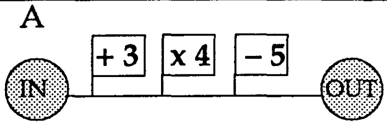


4. The inverse program for A is 'divide by 3'.
The inverse program for B is 'divide by 2, then subtract 5'.

Target test - Advanced

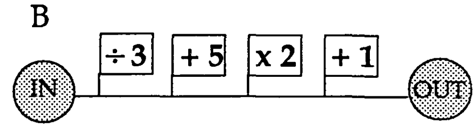
1. & 2.

A



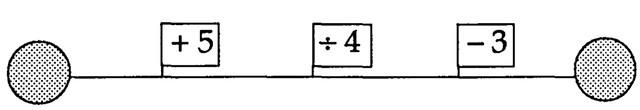
8	→	**39**
4	→	**23**
7	→	35
5	→	27
10	→	47

B

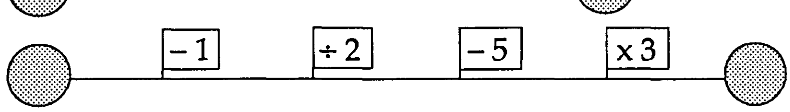


9	→	**17**
24	→	**27**
15	→	21
6	→	15
33	→	33

3. A



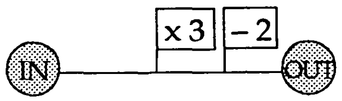
B



4. The inverse program for A is ' add 5, then divide by 4, then subtract 3'.
The inverse program for B is 'subtract 1, then divide by 2, then subtract 5, then multiply by 3'.

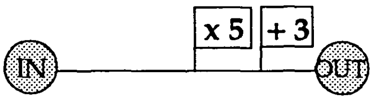
1340 Pattern and Notation

Page 1 - Showing the pattern



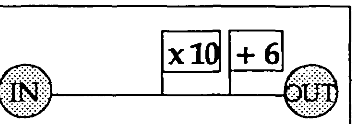
5	→	5 x 3 – 2 = 13
2	→	2 x 3 – 2 = 14
4	→	4 x 3 – 2 = 10
3	→	**3 x 3 – 2 = 7**
10	→	**10 x 3 – 2 = 28**

□ → □ **x 3 – 2**



4	→	4 x 5 + 3 = 23
10	→	10 x 5 + 3 = 53
3	→	3 x 5 + 3 = 18
8	→	**8 x 5 + 3 = 43**
5	→	**5 x 5 + 3 = 28**

□ → □ **x 5 + 3**



1	→	10 + 6 = 16
2	→	20 + 6 = 26
3	→	30 + 6 = 36
4	→	**40 + 6 = 46**
5	→	**50 + 6 = 56**

□ → □ **x 10 + 6**

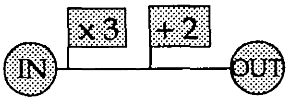
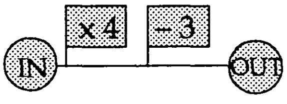
Page 2 - Describing patterns

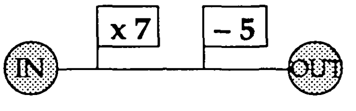
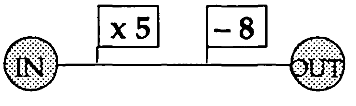
□ → □ **x 4 – 3**	
1 →	1 x 4 – 3 = 1
2 →	2 x 4 – 3 = 5
3 →	3 x 4 – 3 = 9

□ → □ x 6 + 2	
4 →	**4 x 6 + 2 = 26**
3 →	**3 x 6 + 2 = 20**
5 →	**5 x 6 + 2 = 32**

□ → □ x 10 – 6	
7	**→ 7 x 10 – 3 = 67**
2	**→ 2 x 10 – 3 = 17**
9	**→ 9 x 10 – 3 = 87**

Page 3 - Introducing *x*

Flag diagram	Pattern in algebra
	$x \to$ **3*x* + 2**
	$x \to$ **4*x* – 3**

Pattern in algebra	Flag diagram
$x \to 7x - 5$	
$x \to 5x - 8$	

Page 4 - Danger - ambiguity!

(7 + 4) x 2 = **22**
7 + (4 x 2) = **15**
(1 + 3) x 5 = **20**
1 + (3 x 5) = **16**
(5 x 4) – 3 = **17**
5 x (4 – 3) = **5**

2 x (3 – 1) = 4
(5 + 2) x 2 = 14
(2 + 7) ÷ 3 = 3
12 ÷ (4 – 2) = 6
(10 – 2) x 3 = 24
9 – (6 ÷ 3) = 7

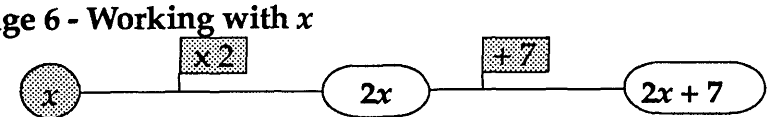
5 – (2 x 1) = 3 or
(5 – 2) x 1 = 3
5 x (2 – 1) = 5
(5 + 2) x 1 = 7
(5 x 2) – 1 = 9
(5 x 2) + 1 = 11
5 – 2 + 1 = 4 or
5 – (2 – 1) = 4

continued/

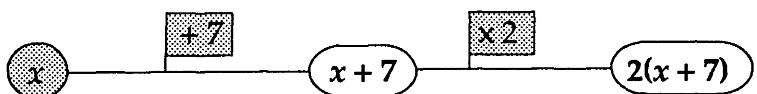
Page 5 - Getting things straight

Pattern in algebra	Example	Program in words
$x \rightarrow 2(x+3)$	$7 \rightarrow 20$	Add 3, then double
$x \rightarrow 2x+3$	$7 \rightarrow$ **17**	**Double, then add 3**
$x \rightarrow 7(x-1)$	$7 \rightarrow$ **42**	**Subtract 1, then multiply by 7**
$x \rightarrow x+3 \times 2$	$7 \rightarrow 13$	**Add 6** (the same as $x \rightarrow x+6$)
$x \rightarrow$ **$5x-4$**	$7 \rightarrow$ **31**	Multiply by 5, then subtract 4
$x \rightarrow$ **$6(x-2)$**	$7 \rightarrow$ **30**	Subtract 2, then multiply by 6

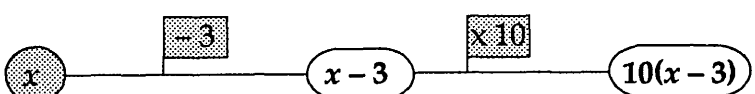
Page 6 - Working with x



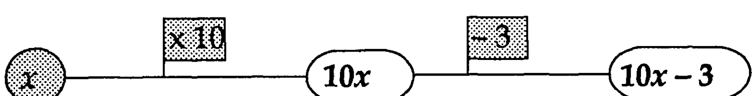
$x \rightarrow$ **$2x + 7$** $\qquad$ **$4 \rightarrow 15$**



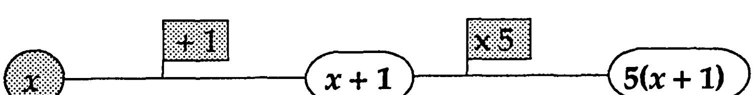
$x \rightarrow$ **$2(x + 7)$** $\qquad$ **$4 \rightarrow 22$**



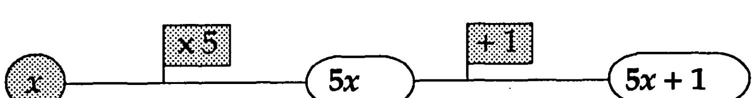
$x \rightarrow$ **$10(x - 3)$** $\qquad$ **$4 \rightarrow 10$**



$x \rightarrow$ **$10x - 3$** $\qquad$ **$4 \rightarrow 37$**

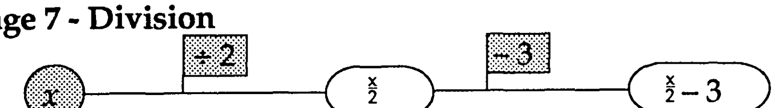


$x \rightarrow$ **$5(x + 1)$** $\qquad$ **$4 \rightarrow 25$**

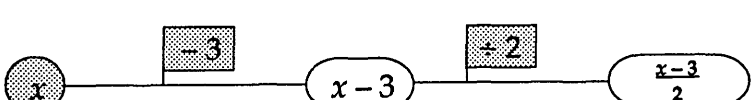


$x \rightarrow$ **$5x + 1$** $\qquad$ **$4 \rightarrow 21$**

Page 7 - Division



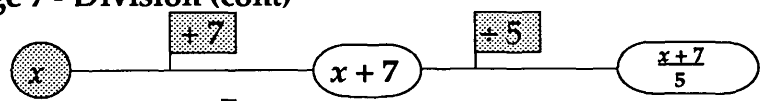
$x \rightarrow \frac{x}{2} - 3$



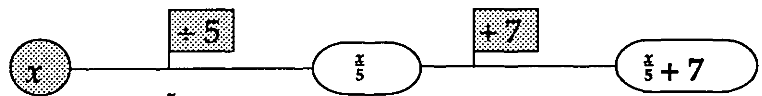
$x \rightarrow \frac{x-3}{2}$

continued/

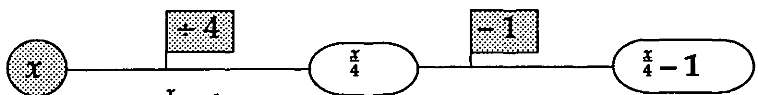
Page 7 - Division (cont)



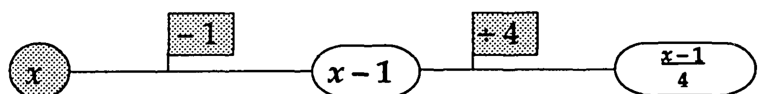
$x \to \frac{x+7}{5}$



$x \to \frac{x}{5} + 7$



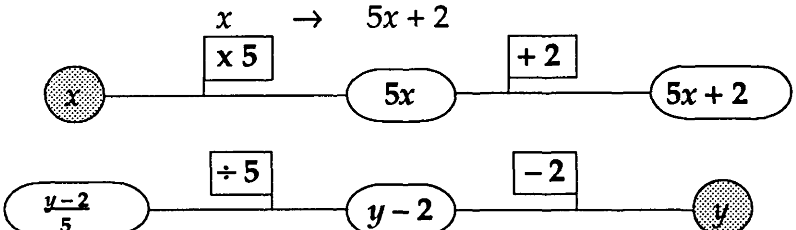
$x \to \frac{x}{4} - 1$



$x \to \frac{x-1}{4}$

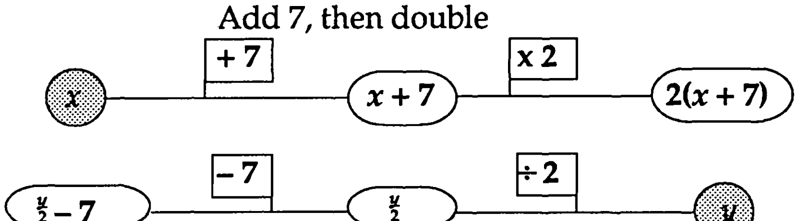
Page 8 - Working backwards

$x \to 5x + 2$



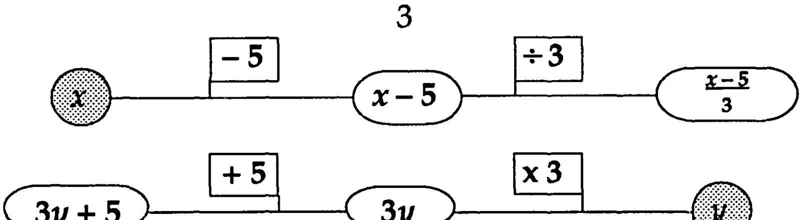
$y \to \frac{y-2}{5}$

Add 7, then double



$y \to \frac{y}{2} - 7$

$x \to \frac{x-5}{3}$



$y \to 3y + 5$

continued/

Page 9 - The equation of a machine

$y = \frac{x}{3}$, $x = 3y$

x	y
9	3
30	10
24	8
21	7
6	2
27	9

$y = 2(x - 1)$, $x = \frac{y}{2} + 1$

x	y
3	4
7	12
10	18
5	8
2	2
11	20

$y = \frac{x}{2} + 7$, $x = 2(y - 7)$

x	y
4	9
10	12
6	10
2	8
8	11
14	14

$y = 4x - 5$, $x = \frac{y+5}{4}$

x	y
2	3
7	23
4	11
5	15
3	7
10	35

$y = \frac{x+5}{2}$, $x = 2y - 5$

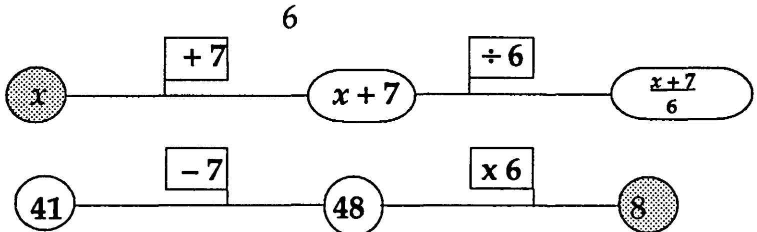
x	y
7	6
3	4
11	8
1	3
9	7
5	5

$y = 3x + 2$, $x = \frac{y-2}{3}$

x	y
1	5
6	20
10	32
2	8
5	17
3	11

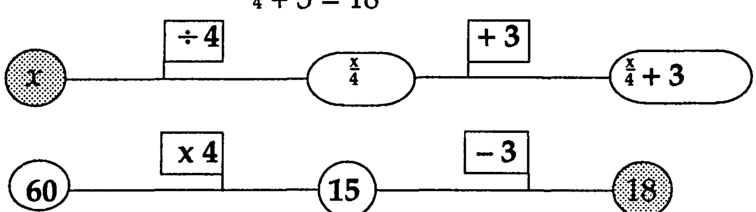
Page 10 - Solving equation in x

$$\frac{x+7}{6} = 8$$



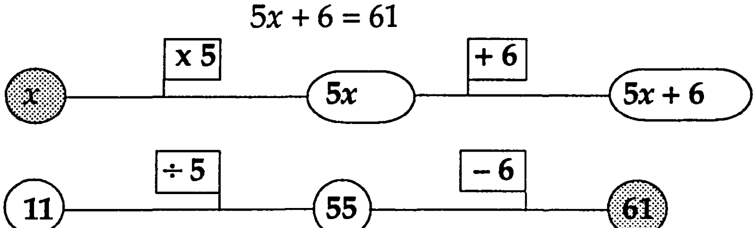
Solution: $x = 41$

$$\frac{x}{4} + 3 = 18$$



Solution: $x = 60$

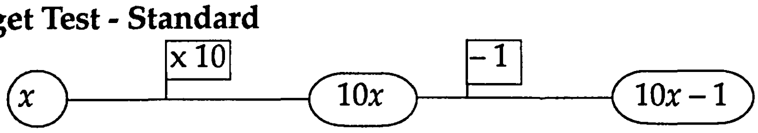
$$5x + 6 = 61$$



Solution: $x = 11$

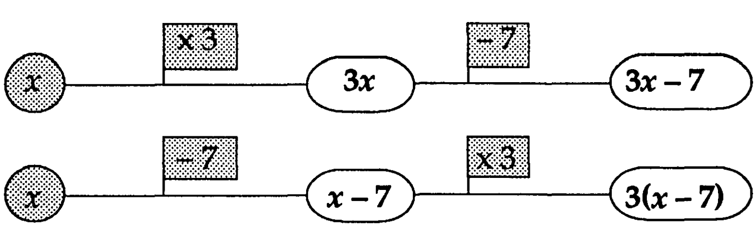
Target Test - Standard

1.



$x \rightarrow 10x - 1$

2.

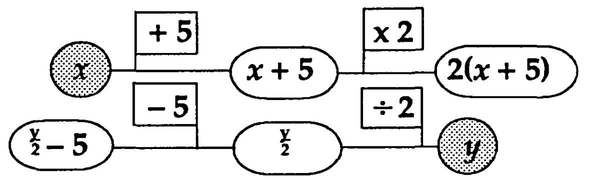


$x \rightarrow 3x - 7$

$x \rightarrow 3(x - 7)$

continued/

3.

Program		Equations
Add 5 then double		$y = 2(x + 5)$ $x = \frac{y}{2} - 5$

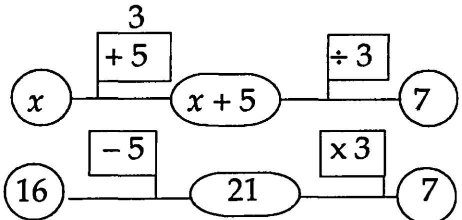
Target Test - Advanced

1. $x \rightarrow 3(x + 5)$

2. a) $y = 4(x + 6)$
   $x = \frac{y}{4} - 6$

   b) $y = \dfrac{x - 3}{7}$
   $x = 7y + 3$

3. $\dfrac{x + 5}{3} = 7$



Solution: $x = 16$

1341 Number Machines

Page 1 - Number mappings

Add 7	
$a \rightarrow a + 7$	
IN	OUT
2 →	**9**
10 →	**17**
31 →	**38**
5 →	**12**
8 →	**15**

Subtract 4	
$b \rightarrow b - 4$	
IN	OUT
7 →	**3**
29 →	**25**
85 →	**81**
12 →	**8**
23 →	**19**

Multiply by 5	
$c \rightarrow 5c$	
IN	OUT
8 →	**40**
20 →	**100**
3 →	**15**
100 →	**500**
12 →	**60**

Divide by 2	
$d \rightarrow \frac{d}{2}$	
IN	OUT
12 →	**6**
40 →	**20**
8 →	**4**
14 →	**7**
32 →	**16**

Page 2 - Spot the pattern

A Multiply by 4   B Subtract 7   C Divide by 3

Page 3 - Complicated programs

Multiply by 3 then subtract 2	
$e \rightarrow 3e - 2$	
IN	OUT
5 →	**13**
11 →	**31**
7 →	**19**
13 →	**37**
3 →	**7**

Take away from 20	
$f \rightarrow 20 - f$	
IN	OUT
5 →	**15**
11 →	**9**
7 →	**13**
13 →	**7**
3 →	**17**

Add 7 then divide by 2	
$g \rightarrow \dfrac{g + 7}{2}$	
IN	OUT
5 →	**6**
11 →	**9**
7 →	**7**
13 →	**10**
3 →	**5**

Add 1, then double, then subtract 2	
$h \rightarrow 2(h + 1) - 2$	
IN	OUT
5 →	**10**
11 →	**22**
7 →	**14**
13 →	**26**
3 →	**6**

continued/

Page 4 - Finding programs

A Multiply by 3 and then **subtract 4**
B Add 5, and then **multiply by 2**
C Take away from **15**

Page 5 - 5 → 15

a) 10 → **20** b) 10 → **30** c) 10 → **10** d) 10 → **25**

1. Add 12 — 4 → **16**
2. Take away from 30 — 4 → **26**
3. Double then add 3 — 4 → **11**
4. Multiply by 3 then subtract 6 — 4 → **6**

Add 12		
$i \to i + 12$		
IN		OUT
9	→	21
4	→	16
12	→	24
7	→	19
10	→	22

Take away from 30		
$j \to 30 - j$		
IN		OUT
9	→	21
4	→	26
12	→	18
7	→	23
10	→	20

Multiply by 3 then subtract 6		
$k \to 3k - 6$		
IN		OUT
9	→	21
4	→	6
12	→	30
7	→	15
10	→	24

Double, then add 3		
$l \to 2l + 3$		
IN		OUT
9	→	21
4	→	11
12	→	27
7	→	17
10	→	23

Page 6 - A new notation

Part 1

Multiply by 3 then add 5						
□	→	3 x □ + 5				
IN		OUT		IN		OUT
7	→	3 x 7 + 5	so	7	→	26
10	→	3 x 10 + 5	so	10	→	35
3	→	3 x 3 + 5	so	3	→	14
13	→	3 x 13 + 5	so	13	→	44
6	→	3 x 6 + 5	so	6	→	23

Part 2

Take away from 30		
□ → 30 − □		
IN		OUT
7	→	23
10	→	20
3	→	27
13	→	17
6	→	24

Multiply by 2 then add 3		
□ → 2 x □ + 3		
IN		OUT
7	→	17
10	→	23
3	→	9
13	→	29
6	→	15

continued/

Page 7 - Using letters

Add 5		
x	→	$x + 5$
4	→	**9**
13	→	**18**
2	→	**7**
6	→	**11**
9	→	**14**

Take away from 17		
x	→	$17 - x$
4	→	**13**
13	→	**4**
2	→	**15**
6	→	**11**
9	→	**8**

Divide by 2 then add 1		
x	→	$x \div 2 + 1$
4	→	**3**
14	→	**8**
2	→	**2**
6	→	**4**
10	→	**6**

Multiply by 3 then subtract 1		
x	→	$3 \times x - 1$
4	→	**11**
13	→	**38**
2	→	**5**
6	→	**17**
9	→	**26**

Page 8 - Introducing brackets

6 x (4 + 6) = **60**	2 + 3 x 4 + 5 = **19**	5 + 4 x 3 = 17	1 + 3 x (4 – 2) = 17
6 x 4 + 6 = **30**	(2 + 3) x 4 + 5 = **25**	(5 + 4) x 3 = 27	(1 + 3) x 4 – 2 = 14
9 – 4 x 2 = **1**	2 + 3 x (4 + 5) = **29**	(7 – 3) x 2 = 8	(1 + 3) x (4 – 2) = 8
(9 – 4) x 2 = **10**	(2 + 3) x (4 + 5) = **45**	7 – 3 x 2 = 1	1 + 3 x 4 – 2 = 11

Page 9 - An algebraic convention

Words	Boxes	Algebra
Add 3, then divide by 2	❑ → (❑ + 3) ÷ 2	$x \to (x + 3) \div 2$
Take away from 20	❑ → **20 – ❑**	$y \to$ **$20 - y$**
Subtract 7, then multiply by 5	❑ → **(❑ – 7) x 5**	$z \to 5(z - 7)$
Divide by 2, then add 1	❑ → **❑ ÷ 2 + 1**	$t \to$ **$t \div 2 + 1$**
Take away from 15, then multiply by 2	❑ →(15 – ❑) x 2	$g \to$ **$2(15 - g)$**
Double, then add 3, then double	❑ → **(2 x ❑ + 3) x 2**	$w \to 2(2w + 3)$
Double, then take away from 30	❑ → **30 – 2 x ❑**	$s \to$ **$30 - 2s$**

Target test - Standard

1.

Multiply by 3		
x	→	**$3x$**
7	→	21
2	→	6
5	→	15
10	→	**30**
8	→	**24**
15	→	**45**

2.

Subtract 2		
x	→	**$x - 2$**
4	→	2
10	→	8
7	→	5
12	→	**10**
5	→	**3**
16	→	**14**

3.

Double then add 1		
x	→	**$2x + 1$**
10	→	21
20	→	41
8	→	17
3	→	**7**
11	→	**23**
15	→	**31**

4.

Subtract from 23		
x	→	**$23 - x$**
12	→	11
9	→	14
14	→	9
3	→	**20**
20	→	**3**
7	→	**16**

continued/

1341 Number Machines (cont)

Target test - Advanced

1

Double then take away from 37

$x \to 37 - 2x$

IN		OUT
9	$\to$	**19**
6	$\to$	**25**
14	$\to$	**9**
5	$\to$	**27**
2	$\to$	**33**
16	$\to$	**5**

2.

Double then add 6, then halve

$x \to \dfrac{2x+6}{2}$

IN		OUT
9	$\to$	**12**
6	$\to$	**9**
14	$\to$	**17**
5	$\to$	**8**
2	$\to$	**5**
16	$\to$	**19**

3

Subtract from 18

$x \to 18 - x$

IN		OUT
9	$\to$	9
6	$\to$	12
14	$\to$	4
5	$\to$	**13**
2	$\to$	**16**
16	$\to$	**2**

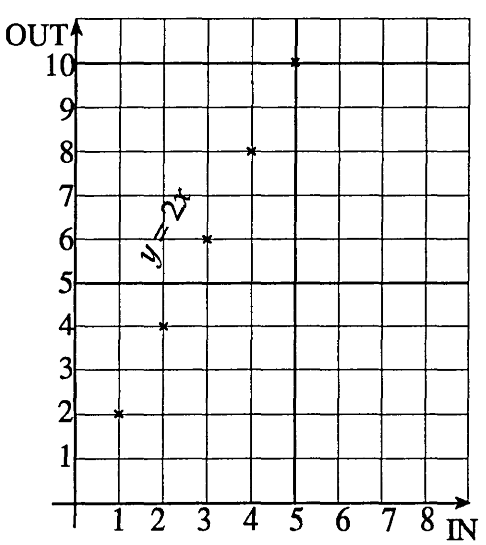
4. You should have four of these.

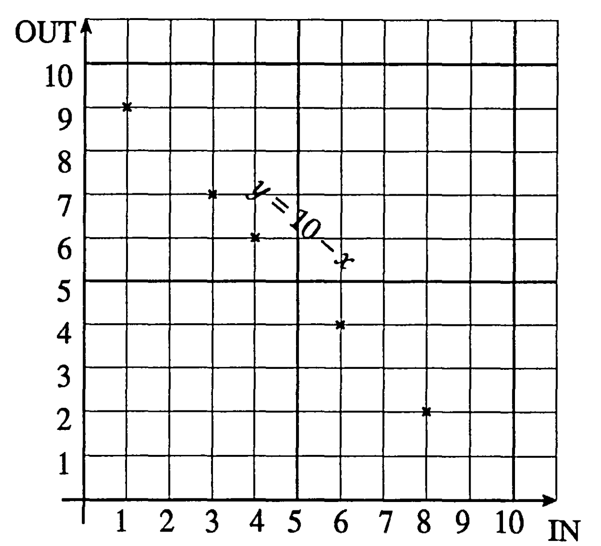
$x \to x + 10$
$x \to 24 - x$
$x \to 2x + 3$
$x \to 3x - 4$
$x \to 2(x + 1) + 1$
$x \to (5x - 1) \div 2$

If you have a different program from these, check it with your teacher.

1342 Mappings and Graphs

Page 1 - From mapping to graph





continued/

Page 2 - From graph to mapping

	IN		OUT
A	**7**	→	**6**
B	**3**	→	**4**
C	**9**	→	**7**
D	**1**	→	**3**
E	**5**	→	**5**

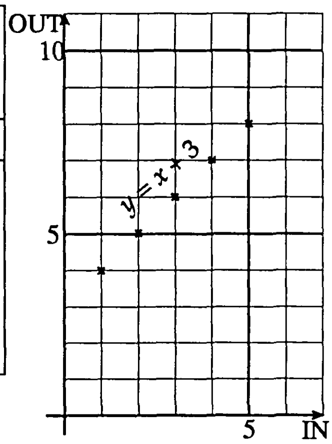
	IN		OUT
P	**5**	→	**6**
Q	**7**	→	**2**
R	**4**	→	**8**
S	**3**	→	**10**
T	**6**	→	**4**

Page 3 - From program to graph

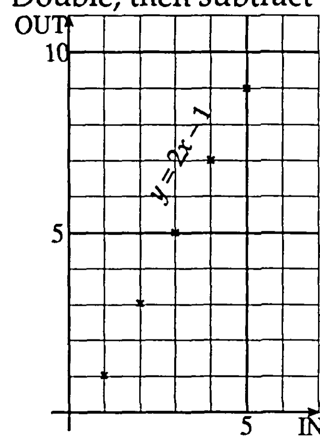
Add 3

$x \to x + 3$

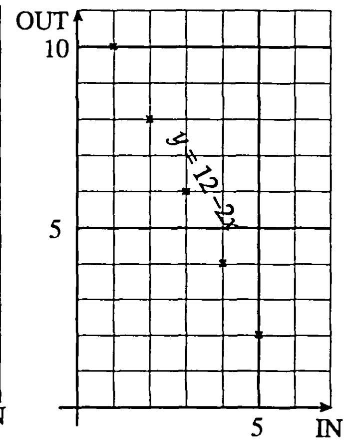
IN		OUT
1	→	4
2	→	5
3	→	6
4	→	7
5	→	8



Double, then subtract 1



$x \to 12 - 2x$

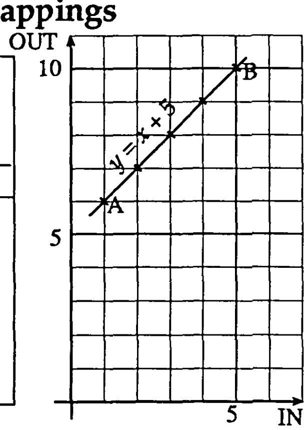


Page 4 - Linear mappings

Add 5

$x \to x + 5$

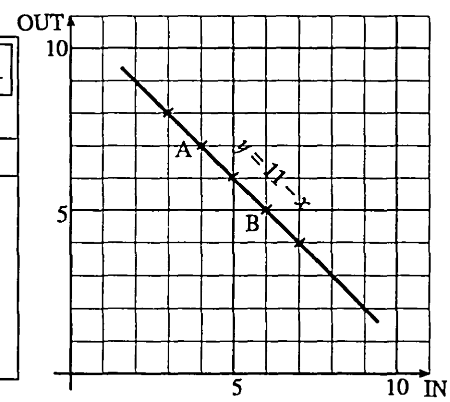
IN		OUT
1	→	6
2	→	**7**
3	→	**8**
4	→	**9**
5	→	10



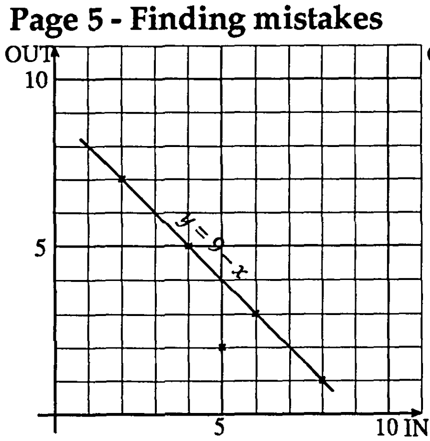
Subtract from 11

$x \to 11 - x$

IN		OUT
3	→	**8**
4	→	7
5	→	**6**
6	→	5
7	→	**4**

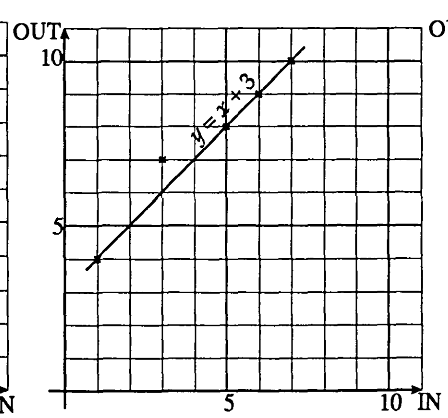


Page 5 - Finding mistakes



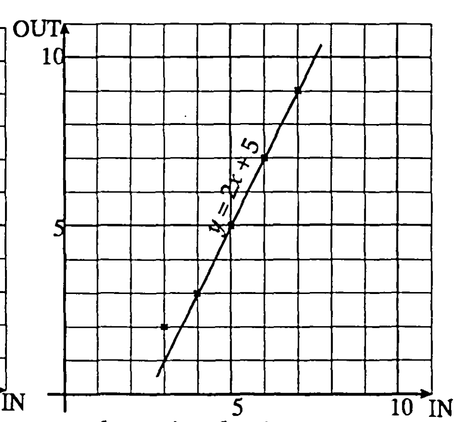
The mistake is $5 \to 2$

$5 \to 4$ $\quad x \to 9 - x$



The mistake is $3 \to 7$

$3 \to 6$ $\quad x \to x + 3$



The mistake is $3 \to 2$

$3 \to 1$ $\quad x \to 2x - 5$

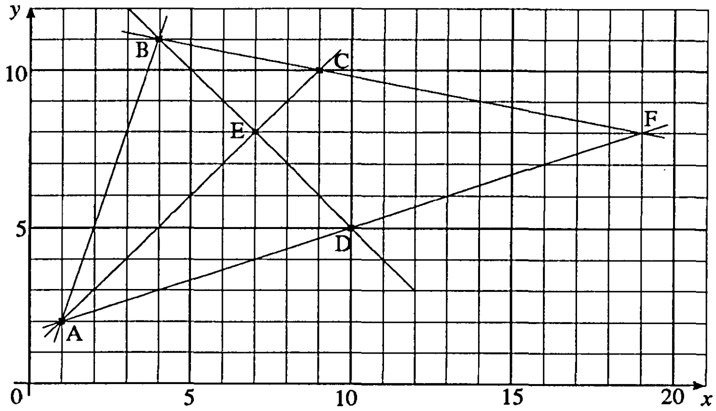
continued/

Page 6 - Co-ordinates

a)

A (2, 3)	P (7, 3)
B (7, 8)	Q (5, 4)
C (4, 5)	R (1, 6)
	S (9, 2)

b)

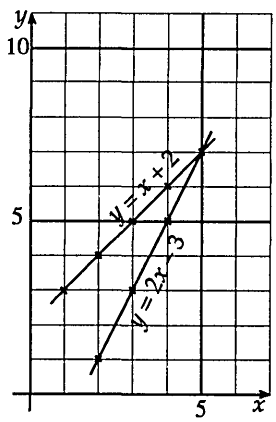


E (7, 8)
F (19, 8)

Page 7 - The equation of a line

$y = x + 2$	
x	y
1	3
2	4
3	5
4	6

$y = 2x - 3$	
x	y
2	1
3	3
4	5
5	7



Page 8 - Finding an equation

AC

$y = x + 1$	
x	y
1	2
4	5
7	8
9	10

BD

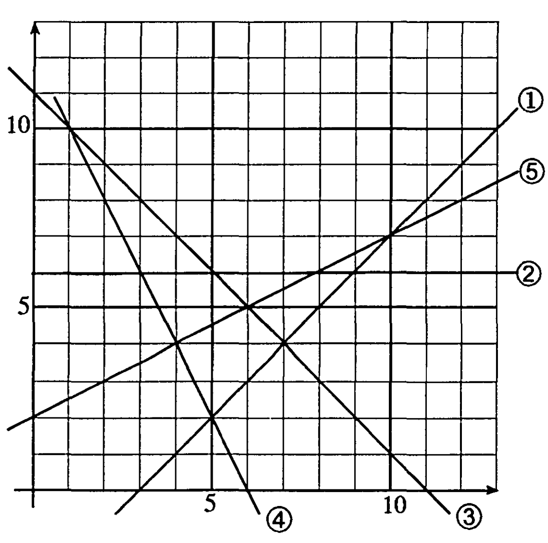
$y = 15 - x$	
x	y
4	11
5	10
7	8
10	5

AB

$y = 3x - 1$	
x	y
1	2
2	5
3	8
4	11

EF's equation is $y = 8$ because the y co-ordinate each time equals 8.

Page 10 - Intersecting lines

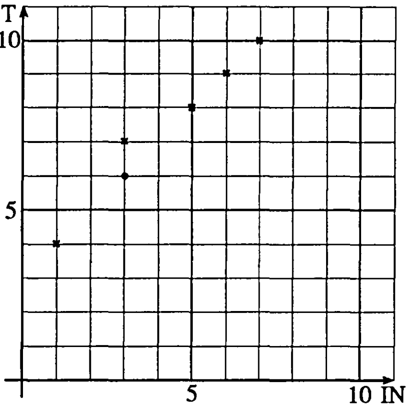


Number Machine	both give	Number Machine	both give
① and ②	9 → 6	③ and ④	1 → 10
① and ③	7 → 4	① and ⑤	10 → 7
② and ③	5 → 6	② and ⑤	8 → 6
① and ④	5 → 2	③ and ⑤	6 → 5
② and ④	3 → 6	④ and ⑤	4 → 4

continued/

Target test - Standard

1. a)



b) The mistake is 3 $\to$ 7, it should be 3 $\to$ 6.

c) add 3

d) $x \to x + 3$

e) $y = x + 3$

2. a)

$x \to y$
$1 \to 2$
$2 \to 2\frac{1}{2}$
$3 \to 3$
$5 \to 4$
$7 \to 5$
$9 \to 6$
$11 \to 7$

b) $y = 2x$

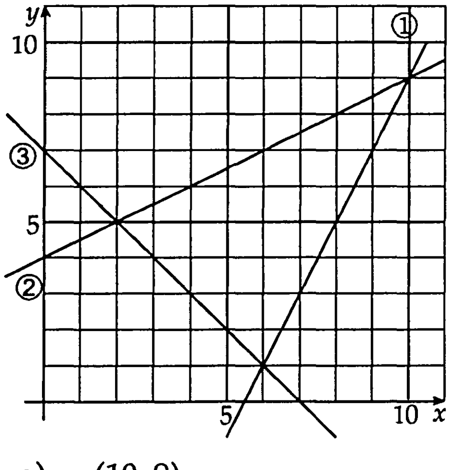
c) double or multiply by 2

d) $x \to 9 - x$

e) A (1, 2) B (3, 6) C (5, 4)

Target test - Advanced

1.



a) (10, 9)

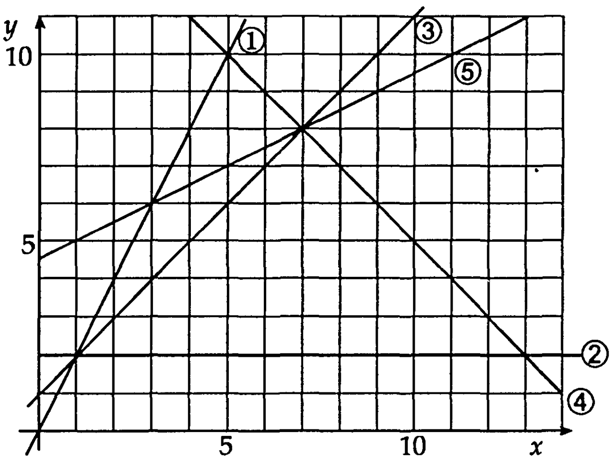
b) (2, 5)

c) (6, 1)

continued/

1342 Mappings and Graphs (cont)

2.



① $x \rightarrow 2x$
② $x \rightarrow 2$
③ $x \rightarrow x + 1$
④ $x \rightarrow 15 - x$

1343 Simple Mappings

Page 1 - Row of bricks

1. Bricks		Joins	
1	→	0	2. In words
2	→	1	**subtract 1**
3	→	2	
4	→	3	3. In algebra
5	→	4	
12	→	**11**	$\mathbf{x \rightarrow x - 1}$
100	→	**99**	

Page 2 - Equilateral triangles

1. Side		Matches	
1	→	3	2. In words
2	→	6	**multiply by 3**
3	→	**9**	
4	→	**12**	3. In algebra
5	→	**15**	
10	→	**30**	$\mathbf{n \rightarrow 3n}$
50	→	**150**	

Page 3 - Building a fence

1. Post		Rails	
1	→	0	2. In words **subtract 1 then multipy by 3**
2	→	3	**or**
3	→	6	**multiply by 3, then subtract 3**
4	→	**9**	
5	→	**12**	3. In algebra
12	→	**33**	
101	→	**300**	$\mathbf{p \rightarrow (3p - 1)}$ **or** $\mathbf{p \rightarrow 3p - 3}$

continued/

Page 4 - Row of squares

1. Squares		Matches	
1	→	4	2. In words
2	→	7	**multiply by 3 then add 1**
3	→	**10**	
4	→	**13**	3. In algebra
5	→	**16**	
11	→	**34**	**$m \rightarrow 3m + 1$**
60	→	**181**	

Page 5 - Rectangles

1. Length		Perimeter	
1	→	4	2. In words
2	→	6	**multiply by 2 then add 2**
3	→	**8**	or
4	→	**10**	**add 1 then multiply by 2**
5	→	**12**	3. In algebra
10	→	**22**	
50	→	**102**	**$y \rightarrow 2y + 2$ or $y \rightarrow 2(y + 1)$**

Page 6 - Diamond dots

1. Diamonds		Dots	
1	→	4	2. In words
2	→	7	**multiply by 3 then add 1**
3	→	**10**	
4	→	**13**	3. In algebra
5	→	**16**	
12	→	**37**	**$t \rightarrow 3t + 1$**
40	→	**121**	

Page 7 - Intersecting circles

1. Circles		Dots	
1	→	0	2. In words
2	→	2	**subtract 1 then multiply by 2**
3	→	**4**	or
4	→	**6**	**multiply by 2 then subtract 2**
5	→	**8**	3. In algebra
11	→	**20**	
52	→	**102**	**$k \rightarrow 2(k - 1)$ or $k \rightarrow 2k - 2$**

Page 8 - Dot pattern

1. Length		Dots	
1	→	5	2. In words
2	→	8	**multiply by 3 then add 2**
3	→	**11**	
4	→	**14**	3. In algebra
5	→	**17**	
10	→	**32**	**$d \rightarrow 3d + 2$**
100	→	**302**	

continued/

Page 9 - Row of hexagons

1. Hexagon		Matches	2. In words / 3. In algebra
1	→	6	2. In words
2	→	11	**multiply by 5 then add 1**
3	→	**16**	
4	→	**21**	3. In algebra
5	→	**26**	
12	→	**61**	$h \rightarrow 5h + 1$
100	→	**501**	

Page 10 - Overlapping triangles

1. Triangles		Matches	2. In words / 3. In algebra
1	→	6	2. In words
2	→	10	**multiply by 4 then add 2**
3	→	**14**	
4	→	**18**	3. In algebra
5	→	**22**	
11	→	**46**	$z \rightarrow 4z + 2$
110	→	**442**	

Target test - Standard

Number of triangles	Number of dots	Number of matches
1 →	3	3
2 →	4	5
3 →	**5**	**7**
4 →	**6**	**9**
5 →	**7**	**11**
10 →	**12**	**21**
100 →	**102**	**201**

a) add 2
b) multiply by 2 then add 1
a) $x \rightarrow x + 2$
b) $x \rightarrow 2x + 1$

Target test - Advanced

Number of squares long	Number of dots	Perimeter	Number of matches
1 →	6	6	7
2 →	9	8	12
3 →	**12**	**10**	**17**
4 →	**15**	**12**	**22**
5 →	**18**	**14**	**27**
10 →	**33**	**24**	**52**
100 →	**303**	**204**	**502**

continued/

1343 Simple Mappings

a) add 1 then multiply by 3 or multiply by 3 then add 3
b) double then add 4 or add 2 and then double
c) multiply by 5 then add 2

a) $y \to 3(y+1)$ or $y \to 3y+3$
b) $y \to 2y+4$ or $y \to 2(y+2)$
c) $y \to 5y+2$

1344 Further Mappings

Page 1 - Pattern with squares

1. Squares	Dots	Matches
1 →	4	4
2 →	7	8
3 →	**10**	**12**
4 →	**13**	**16**
5 →	**16**	**20**
10 →	**31**	**40**
100 →	**301**	**400**

2. Dots: **multiply by 3 then add 1**
   Matches: **multiply by 4**

3. Dots: $x \to 3x+1$
   Matches: $x \to 4x$

Page 2 - Pattern with triangles

1. Triangles	Dots	Matches
1 →	3	3
2 →	5	6
3 →	**7**	**9**
4 →	**9**	**12**
5 →	**11**	**15**
12 →	**25**	**36**
100 →	**221**	**330**

2. Dots: **multiply by 2 then add 1**
   Matches: **multiply by 3**

3. Dots: $y \to 2y+1$
   Matches: $y \to 3y$

Page 3 - Another triangle pattern

1. Triangles	Dots	Matches
1 →	3	3
2 →	6	7
3 →	**9**	**11**
4 →	**12**	**15**
5 →	**15**	**19**
11 →	**33**	**43**
80 →	**240**	**319**

2. Dots: **multiply by 3**
   Matches: **multiply by 4 then subtract 1**

3. Dots: $n \to 3n$
   Matches: $n \to 4n-1$

continued/

Page 4 - Hydrocarbons

1. C atoms	H atoms	Bonds	
1 →	4	4	2. H atoms: **multiply by 2 then add 2**
2 →	6	7	Bonds: **multiply by 3 then add 1**
3 →	**8**	**10**	
4 →	**10**	**13**	3. H atoms: $k \to 2k + 2$
5 →	**12**	**16**	
10 →	**22**	**31**	Bonds: $k \to 3k + 1$
100 →	**202**	**301**	

Page 5 - Row of houses

1. 'Houses'	Dots	Matches	
1 →	5	6	2. Dots: **multiply by 3 then add 2**
2 →	8	11	Matches: **multiply by 5 then add 1**
3 →	**11**	**16**	
4 →	**14**	**21**	3. Dots: $h \to 3h + 2$
5 →	**17**	**26**	
10 →	**38**	**61**	Matches: $h \to 5h + 1$
100 →	**302**	**501**	

Page 6 - Double row of squares

1. Crosses	Squares	Matches	
1 →	4	12	2. Squares: **multiply by 2 then add 2**
2 →	6	17	Matches: **multiply by 5 the add 7**
3 →	**8**	**22**	
4 →	**10**	**27**	3. Squares: $d \to 2d + 2$
5 →	**12**	**32**	
10 →	**22**	**57**	Matches: $d \to 5d + 7$
80 →	**162**	**407**	

Page 7 - Joined hexagons

1. Hexagons	Dots	Matches	
1 →	6	6	2. Dots: **multiply by 5 then add 1**
2 →	11	14	Matches: **multiply by 8 then subtract 2**
3 →	**16**	**22**	
4 →	**21**	**30**	3. Dots: $x \to 5x + 1$
5 →	**26**	**38**	
10 →	**51**	**78**	Matches: $x \to 8x - 2$
50 →	**251**	**398**	

continued/

Page 8 - Wall with spikes

1.

Spikes	Length	Matches
1 →	1	5
2 →	3	12
3 →	**5**	**19**
4 →	**7**	**26**
5 →	**9**	**33**
12 →	**23**	**82**
100 →	**199**	**698**

2. Length: **multiply by 2 then subtract 1**

Matches: **multiply by 7 then subtract 2**

3. Length: $s \rightarrow 2s - 1$

Matches: $s \rightarrow 7s - 2$

Page 9 - Square of squares

1.

Side	Edge Sq	Matches
1 →	4	12
2 →	8	24
3 →	**12**	**36**
4 →	**16**	**48**
5 →	**20**	**60**
11 →	**40**	**120**
51 →	**200**	**600**

2. Edge Square: **subtract 1 then multiply by 4**

Matches: **subtract 1 then multiply by 12**

3. Edge Square: $l \rightarrow 4(l - 1)$

Matches: $l \rightarrow 12(l - 1)$

Page 10 - Row of cubes

1.

Cubes	Dots	Matches
1 →	7	9
2 →	10	14
3 →	**13**	**19**
4 →	**16**	**24**
5 →	**19**	**29**
20 →	**64**	**104**
1000 →	**3004**	**5004**

2. Dots: **multiply by 3 then add 4**

Matches: **multiply by 5 then add 4**

3. Dots: $n \rightarrow 3n + 4$

Matches: $n \rightarrow 5n + 4$

Target test - Standard

Side of large square	Number of small squares	Perimeter of L-shape	Number of matches
2 →	3	8	10
3 →	5	12	16
4 →	7	16	22
5 →	9	20	28
6 →	11	24	34
10 →	19	40	58
100 →	199	400	598
x →	$2x - 1$	$4x$	$6x - 2$

continued/

1344 Further Mappings (cont)

Target test - Advanced

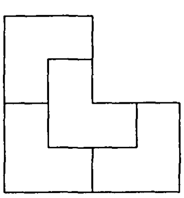
Number of dots	Perimeter	Number of small triangles	Number of matches
1 →	**6**	**6**	**12**
2 →	8	**10**	**19**
3 →	10	14	26
4 →	12	18	33
5 →	14	22	40
10 →	24	42	75
100 →	204	402	705
x →	$\mathbf{2x + 4}$	$\mathbf{4x + 2}$	$\mathbf{7x + 5}$

1345 Mastermind

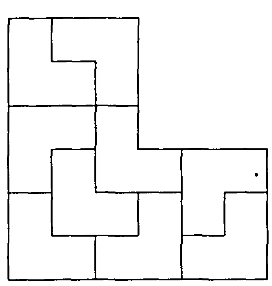
The second digit is either 8 or 6. Can you see why?

1347 Tromino

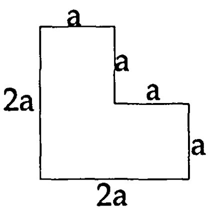
1. b)



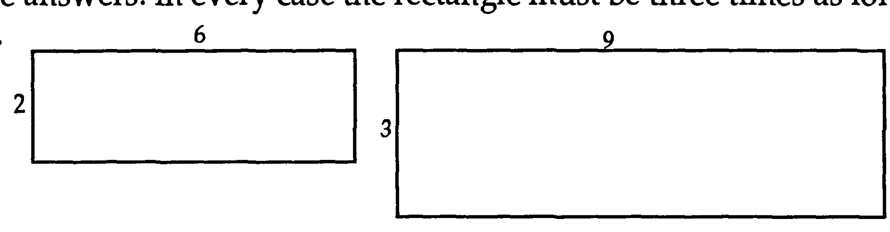
2. a) $27cm^2$
   b) $3cm^2$
   c) 9



3. Many possible answers.
   In every case the lengths
   must all be in the ratio shown.



4. Many possible answers. In every case the rectangle must be three times as long as it is wide. e.g.



1348 Look and Guess

1. D

2. B

3. D, J, H, A, E, C, G, F, B

4. F

5. E

6. F, C, H, A, G, B, D, E

7. J

8. E

9. J, C, G, D, A and B, F and H, E

10. a) Buckingham Palace

11. b) Houses of Parliament

12. b) Oxford Circus

13. b) Oxford Circus

14. a) via Oxford Circus

15. Either
Bow Street → Piccadilly Circus → Trafalgar Square → Houses of Parliament
or
Houses of Parliament → Trafalgar Square → Piccadilly Circus → Bow Street.

16. Victoria Station

1349 Time Line

1.-5. Get someone else to check your answers

6.-8. Show your answers to your teacher.

1350 Bases

1. a) 13 (base five)
   b) 23 (base five)
   c) 40 (base five)
   d) 103 (base five)

2. a) 32 (base four)
   b) 3 (base four)
   c) 113 (base four)
   d) 302 (base four)
   e) 1012 (base four)

3. 33 (base seven)

4. 111 (base eight)

5. 45 (base ten)

6. 1013 (base five)

1351 Base Three

2.

		threes	ones
5	=	1	2
4	=	1	1
8	=	2	2
3	=	1	0
2	=		2

4.

		nines	threes	ones
11	=	1	0	2
19	=	2	0	1
15	=	1	2	0
18	=	2	0	0
26	=	2	2	2

5.

		twenty-sevens	nines	threes	ones
30	=	1	0	1	0
42	=	1	1	2	0
51	=	1	2	2	0
54	=	2	0	0	0

		eighty-ones	twenty-sevens	nines	threes	ones
100	=	1	0	2	0	1

6. Three twenty-sevens make eighty-one, a new column.

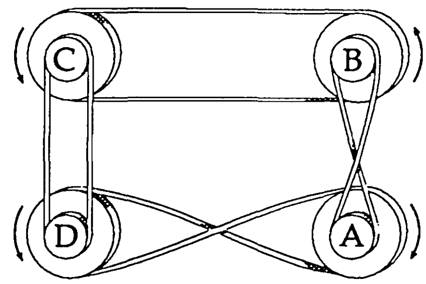
7. Eighty.

1352 Wheels

- In the first arrangement if wheel A turns clockwise, B turns anticlockwise, C turns clockwise and D turns clockwise.

- In the second arrangement A and C turn the same way and B and D turn the other way.

- Here is an arrangement so that wheel A turns clockwise and wheels B, C and D all turn anticlockwise.



- What other arrangements of belts did you find?

1353 A Number of Things

1. 24 portions of cheese. (6 x 4)

2. 32 legs. (4 x 8)

3. 22 legs. (11 x 2)

4. 5 felt tips in each pack. (15 ÷ 3)

5. 48 cans of coke. (12 x 4)

6. 3 apples each. (9 ÷ 3)

7. 21 darts. (7 x 3)

8. 50 squares of chocolate. (5 x 10)

9. 30 toes. (3 x 10 or 6 x 5)

10. 54 eggs. (9 x 6)

11. 5 players. (10 ÷ 2)

12. 21 buttons. (3 x 7)

13. 32 wheels. (4 x 8) or 40 wheels (5 x 8) if you include the spare tyre.

14. 5 boxes of pencils. (50 ÷ 10)

15. 64 sausages. (8 x 8)

1354 Euler Solids

- Were you able to make all of the five Platonic Solids?
  Did you check to see that Euler's Rule worked for these five solids?

- The small stellated dodecahedron, the great dodecahedron, the great icosahedron and the great stellated dodecahedron are now defined as regular because each side is equal.

- Were you able to make the great dodecahedron?

1355 Halves and Quarters

1. 4
2. 3
3. 5p
4. Many possible answers.
5. 30
6. 2p
7. 2
8. 3p
9. 15
10. 4

1356 How Much?

1. 21p
2. 40p
3. 21p
4. Yes, because 30p + 30p + 30p = 90p
5. 54p
6. 39p

continued/

1356 How Much? (cont)

7. 4p because the apples cost 36p altogether.

8. No because 7p + 7p + 7p = 21p

9. 42p

10. 63p

11. None. 2 pints = 80p
$\frac{1}{2}$ pints = 20p
80p + 20p = 100p = £1.00

12. No. 57p + 57p = 114p = £1.14

1357 Missing Signs

1.	60	÷	15	= 4	11.	12	x	13	= 156	
2.	60	+	15	= 75	12.	455	x	5	= 2275	
3.	60	x	15	= 900	13.	1246	+	39	= 1285	
4.	60	–	15	= 45	14.	1246	–	39	= 1207	
5.	456	+	3	= 459	15.	12	+	13	= 25	
6.	456	x	3	= 1368	16.	455	÷	5	= 91	
7.	456	–	3	= 453	17.	313	–	156	= 157	
8.	456	÷	3	= 152	18.	333	x	3	= 999	
9.	35	x	5	= 175	19.	924	–	154	= 6	
10.	260	x	10	= 2600	20.	924	÷	6	= 154	or
						924	–	770	= 154	

1358 Joining Multiples

- When you join up the multiples of 2 in order, you should draw the number 2.
- When you join up the multiples of 3 in order, you should draw the number 3.
- When you join up the multiples of 7 in order you should draw the number 7.

1359 Joining Odds and Evens

- When you join up all the odd numbers in order you should draw a flamingo.
- When you join up all the even numbers you should draw an eagle.

1360 Pictures from Multiples

- When you join up the multiples of 3 in order you should draw a hurdler.
- When you join up the multiples of 4 in order you should draw a footballer.
- When you join up the multiples of 5 in order you should draw a fencer.

1361 Three in Line

3 in line		back to front	Result
159	+	951	1110
789	+	987	1776
456	+	654	1110
123	+	321	444
753	+	357	1110
741	+	147	888
852	+	258	1110
963	+	369	1332

You may have seen these patterns.

- 1110 turns up four times. It comes from the lines with the 5 in the middle.
- 444 which is 444 x 1
  888 which is 444 x 2
  1332 which is 444 x 3
  1766 which is 444 x 4

1362 Visiting British Gas

1. The local gas showrooms have now changed their names to **Energy Centres**. Your answers will vary from place to place.

2. Did you plan your route from school or from your home?

3&4 Make a display of the group's work.

1363 Hexagon Grids

You should find that you never need more than 4 colours, and usually less.

1365 Number Snap

Copy the pairs you won in your book and show them to your teacher.

1366 Pairs

Copy the pairs you won in your book and show them to your teacher.

1367 Lines

Write down the numbers you were able to cover.
Which numbers were they multiples of?

1368 The Mobius Band

In this investigation you can vary:

- the number of cuts
- the placing of the cut and
- the number of twists.

With a systematic approach it should be possible to find patterns.

1369 Infinity

- **Between Fractions**

1.-3. There are many different answers for questions 1, 2 and 3. Make sure you checked your answers with a calculator.

4. There is always a fraction between two other fractions. However close the two fractions are, you can always squeeze another one between them. This means the number of numbers between 0 and 1 is infinite.

   There is a simple way to demonstrate this:
   Consider $\frac{a}{b}$ and $\frac{c}{d}$

   $$\text{Mean average} = \frac{1}{2}\left(\frac{a}{b} + \frac{c}{d}\right) = \frac{ad + bc}{2bd}$$

   The mean average is a fraction which must be half way between the other two fractions.

continued/

- **A Frog in a Pond**

1. After 1 jump the frog is 10 metres from the edge.
   After 2 jumps the frog is 5 metres from the edge.
   After 3 jumps the frog is 2.5 metres from the edge.
   After 6 jumps the frog is 0.3125 metres from the edge.

2. It takes 5 jumps to be within 1 metre of the edge.
   It takes 8 jumps to be within 10cm of the edge.

3. In theory, the frog **never** reaches the edge of the pond. It is always jumping half way and so there will always be some distance left to jump. This distance gets smaller and smaller. A spreadsheet can be used to show this.

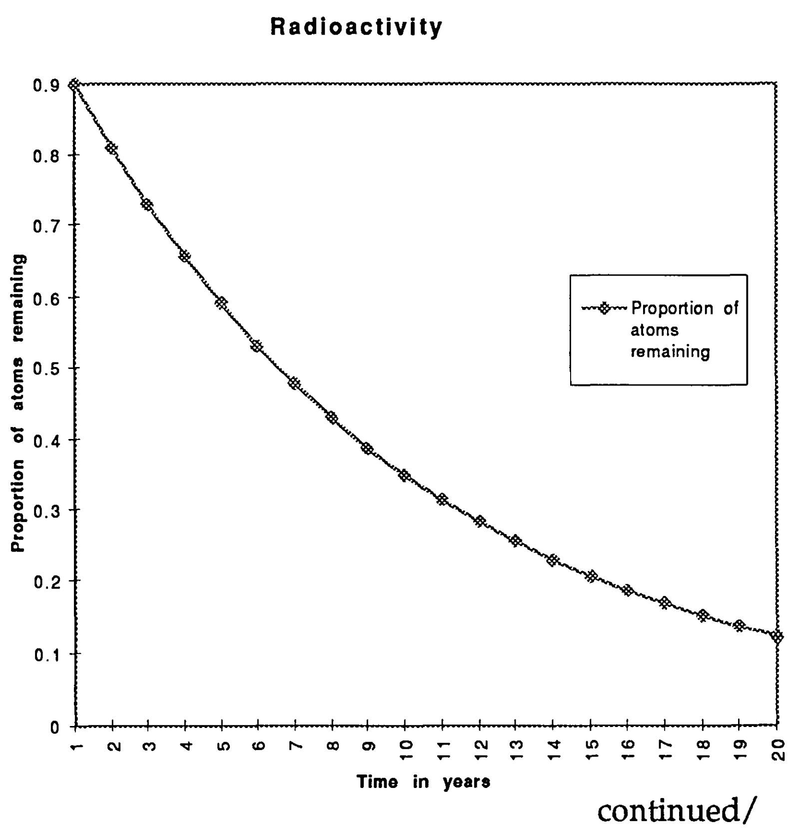
	A	B
1	Jumps	Distance to edge
2	0	20.0000000000
3	1	10.0000000000
4	2	5.0000000000
5	3	2.5000000000
6	4	1.2500000000
7	5	0.6250000000
8	6	0.3125000000
9	7	0.1562500000
10	8	0.0781250000
11	9	0.0390625000
12	10	0.0195312500

In practice, of course, the distance remaining gets so very small that the frog would be at the edge.

- **Radioactivity**

1.&2. A spreadsheet can be used to generate the information very quickly and can also create a graph to show the information.

Time in years	Proportion of atoms remaining
1	0.9
2	0.81
3	0.729
4	0.6561
5	0.59049
6	0.531441
7	0.4782969
8	0.43046721
9	0.38742049
10	0.34867844
11	0.3138106
12	0.28242954
13	0.25418658
14	0.22876792
15	0.20589113
16	0.18530202
17	0.16677182
18	0.15009464
19	0.13508517
20	0.12157665



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3. The half-life of smilephorus is approximately 6.7 years.

4. It will take approximately 13 years for 3/4 of the atoms to disintegrate.

5. By extending the spreadsheet you can quickly see:
   a) After 25 years approximately 0.07 of the atoms will remain.
   b) After 30 years approximatley 0.04 of the atoms will remain.
   c) After 40 years approximatley 0.01 of the atoms will remain.

6. The smilephorus will never all disintegrate (or at least not until there is only one atom left). If a certain proportion disintegrates each year, there must always be something remaining. (But the amount remaining after 40 years is very, very small).

   Half-life is an exact measure which physicists use. There must be a definite time when exactly half of the atoms have disintegrated and half still remain.
   If the half-life of smilephorus is 6.7 years, whereas the half-life of frownphorus is only 2 years then frownphorus must be much more radio-active than smilephorus. It disintegrates more quickly.

- **Infinite Series**

1. a) 1 + 1/4 + 1/16 + 1/64 + 1/256 + 1/1024 + . . .
   b) The series is infinite. It continues for ever.
   c) 1 + 1/4 = 1.25
      1 + 1/4 + 1/16 = 1.3125
      1 + 1/4 + 1/16 + 1/64 = 1.3281
      1 + 1/4 + 1/16 + 1/64 + 1/256 = 1.3320
      1 + 1/4 + 1/16 + 1/64 + 1/256 + 1/1024 = 1.3330
      (These answers are correct to 4 decimal places)
   d) $4/3 = 1.\dot{3}$ (= 1.3333 . . .)
   e) The answers in c) get closer and closer to the answer in d).

2. a) 1 + 1/5 + 1/25 + 1/125 + 1/625 + . . .
   b) The series is infinite.
   c) If you sum the first 2 terms, then the first 3 terms, then first 4 terms and so on, the answers get closer and closer to 1.25.
   d) The sum of the series is equal to 5/4.

3. 8/7 = 1 + 1/8 + 1/64 + 1/512 + . . .
   Try to explain why the denominators are powers of 8 this time.
   a) 1 + 1/8 = 1.125
      1 + 1/8 + 1/64 = 1.1406
      1 + 1/8 + 1/64 + 1/512 = 1.1426
   b) 8/7 = 1.1429
      (These answers are correct to 4 decimal places)

continued/

- **To Think About**
A circle has an infinite number of lines of symmetry: however many you think there might be it is always possible to imagine some more even if it is not always possible to draw any more.

The number of integers (whole numbers) is infinite: you can always make a larger integer by adding one, so there cannot be a largest one.

Similarly, the number of even numbers is infinite and so is the number of multiples of 5. It is interesting to see that there are as many even numbers as there are integers:

$$\begin{array}{ccc} 1 & \longleftrightarrow & 2 \\ 2 & \longleftrightarrow & 4 \\ 3 & \longleftrightarrow & 6 \\ 4 & \longleftrightarrow & 8 \\ 5 & \longleftrightarrow & 10 \\ 6 & \longleftrightarrow & 12 \\ \vdots & & \vdots \end{array}$$

Using this mapping, for every integer there is exactly one even number.

Can you see why this is surprising?
The same is true for multiples of five.

There is no biggest number which is smaller than 2. Whichever number you try, you can always find a bigger one:
e.g. if you thought 1.999999 was the largest you would be wrong because 1.9999991 is larger.

- **Achilles and The Tortoise**

1. $\frac{1000 + x}{50} = \frac{x}{10}$ Multiply both sides of the equation by 50.

$1000 + x = \frac{50x}{10}$

$1000 + x = 5x$ Subtract x from both sides.

$1000 = 4x$ Divide both sides by 4.

$250 = x$

2.
$200 = 200$
$200 + 40 = 240$
$200 + 40 + 8 = 248$
$200 + 40 + 8 + 1.6 = 249.6$
$200 + 40 + 8 + 1.6 + 0.32 = 249.92$
$200 + 40 + 8 + 1.6 + 0.32 + 0.064 = 249.984$

continued/

3. 

Achilles travels 2000 + x metres. Achilles' speed is 4.04m/sec
So Achilles' journey time is $\frac{2000 + x}{4.04}$ secs.
The tortoise travels x metres. The speed of the tortoise is 0.04m/sec.
So the tortoise's journey time is $\frac{x}{0.04}$
When Achilles catches the tortoise, the journey times are equal.

So $\frac{2000 + x}{4.04} = \frac{x}{0.04}$ Multiply both sides by 100.

$\frac{2000 + x}{404} = \frac{x}{4}$ Multiply both sides by 404.

$2000 + x = 101x$ Subtract x from both sides.

$2000 = 100x$ Divide both sides by 100.

$20 = x$

So Achilles runs 2020 metres and catches the tortoise after it has run 20 metres.

4. This spreadsheet allows each calculation to be shown to 17 decimal places.

	A
1	2000.00000000000000000
2	19.80198019801980000
3	0.19605920988138400
4	0.00194118029585529
5	0.00001921960688966
6	0.00000019029313752
7	0.00000000188409047
8	0.00000000001865436
9	0.00000000000018470
10	0.00000000000000183
11	0.00000000000000002
12	0.00000000000000000

a) 19.80198 + 0.19606 + 0.00194 + 0.000019 + 0.00000019 + 0.0000000019

b) 19.80198
19.99804
19.99998
19.99999
19.99999
19.99999

5. The answers to the partial sums in 4 get closer and closer to the answer in 3.

1370 Stepping Stones

1. 16
2. 28
3. 16
4. 27
5. 14
6. 48

1374 Nine Links

1. a)

31	81	63	72	54
−13	−18	−36	−27	−45
18	63	27	45	9

The chain is $31 \rightarrow 18 \rightarrow 63 \rightarrow 27 \rightarrow 45 \rightarrow 9$

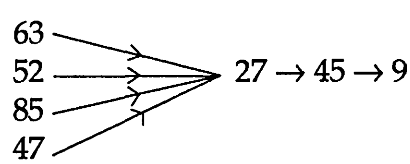
b) $67 \rightarrow 9$

c) $25 \rightarrow 27 \rightarrow 45 \rightarrow 9$

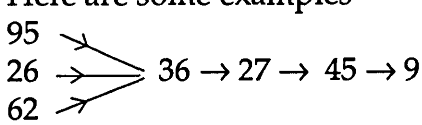
d) $39 \rightarrow 54 \rightarrow 9$

2. Except for the starting number, each number in the chain is a multiple of 9.

3. $63 \rightarrow 27 \rightarrow 45 \rightarrow 9$
$52 \rightarrow 27 \rightarrow 45 \rightarrow 9$
$85 \rightarrow 27 \rightarrow 45 \rightarrow 9$
$47 \rightarrow 27 \rightarrow 45 \rightarrow 9$



4. Here are some examples



5.

Digit difference	Example	Chain
1	23	$\rightarrow 9$
2	24	$\rightarrow 18 \rightarrow 63 \rightarrow 27 \rightarrow 45 \rightarrow 9$
3	25	$\rightarrow 27 \rightarrow 45 \rightarrow 9$
.	.	.
.	.	.
.	.	.

To explain why they work, start by thinking of a number like 34 with digit difference of 1.

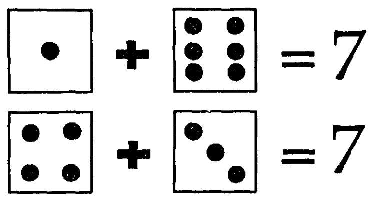
- By changing 3 into 4, you add 10.
- By changing 4 into 3, you subtract 1 . . .

1376 Jobs in Order

Each person has their own way of doing these jobs. Here is one set of sensible answers.

Washing hands	Making a phone call	Making a cup of coffee	Filling a car with petrol
Roll up sleeves	Pick up receiver	Put water in kettle	Switch off engine
Put plug in	Listen for dialling tone	Heat water	Remove petrol cap
Turn taps on	Dial number	Put instant coffee in cup	Choose grade of petrol
Turn taps off	Listen for pips	Add sugar	Pick up pump nozzle
Wash with soap	Put money in coin box	Turn off heat	Put nozzle into filler hole
Rinse off soap	Speak into phone	Add hot water	Fill with petrol
Pull out plug	Replace receiver	Add milk	Replace petrol cap
Dry hands		Stir	Pay cashier

1377 Dice

1. 

3. Show the net of your dice, with the dots marked on it, to your teacher.

5. Check that the dots always add up to 7.

1378 Mappings

1.

Cars		Tyres
4	→	**20**
5	→	**25**
12	→	**60**
100	→	**500**
n	→	**5n**

2.

Insects		Legs
4	→	**24**
5	→	**30**
12	→	**72**
100	→	**600**
n	→	**6n**

3.

Triangles		Matches
4	→	**9**
5	→	**11**
12	→	**25**
100	→	**201**
n	→	**2n + 1**

4.

Posts		Rail
4	→	**9**
5	→	**12**
12	→	**33**
100	→	**297**
n	→	**3(n – 1) or 3n – 3**

5. 50 → **200**
n → **4n**

6. 50 → **201**
n → **4n + 1**

7. 50 → **25**
n → **$\frac{1}{2}$n**

8. (a), (b) and (d).

9. There are many possible answers. Some possible answers are:
n → 3n   n → n + 8   n → $n^2 - 4$ n → 4(n – 1)
Check your answers with your teacher if they are different.

1379 Fishing

2. (0, 0) → (0, 6) → (3, 6) → (3, 12) → (7, 12) →
(7, 10) → (11, 10) → (11, 12) → (15, 12) → (15, 4) →
(11, 4) → (11, 6) → (7, 6) → (7, 2) → (10, 2) →
(10, 0) → (15, 0)

1381 Money

1. $4 \times 9p = 36p$ — Four times 9p is 36p.

2. $21p \div 3 = 7p$ — 21p divided among 3 people, is 7p each.

3. $50p - 28p = 22p$ — 22p more is needed.

4. $5 \times 7p = 35p$ — Five times 7p is 35p.

5. $26p \div 2 = 13p$ — 26p divided among 2 people, is 13p each.

6. a) $4 \times 13p = 52p$ — 4 bars at 13p cost 52p.
   b) £1.00 – 52p = 48p — 48p change.

7. $52p - 33p = 19p$ — The can costs 19p more than the bottle.

8. $7p + 19p + 13p = 39p$ — 39p altogether.

9. $73p - 57p = 16p$ — 16p more.

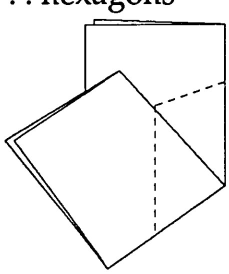
10. $6 \times 17p = 102p$ = £1.02 — Six bags at 17p each cost £1.02, so £1 is not enough.

1382 Paper Folding

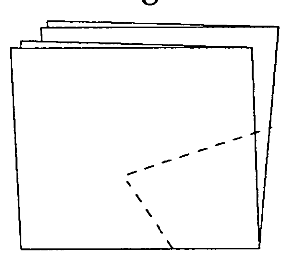
1382A What shape do you get?

- You may have found lots of possibilities with two folds and one or two cuts including

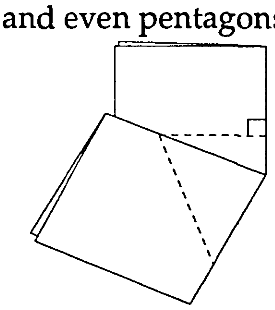
. . . hexagons



. . . and octagons

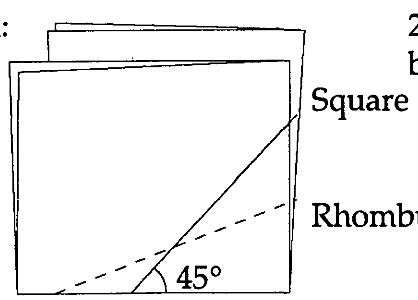


. . . and even pentagons.

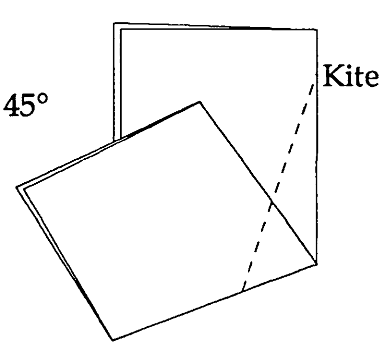


- All the shapes, except the parallelogram can be made by folding and cutting.
  Some of the shapes can be made in several ways.
  Here are some suggestions, all with one cut.

2nd fold:
90°



2nd fold:
between 90° and 45°



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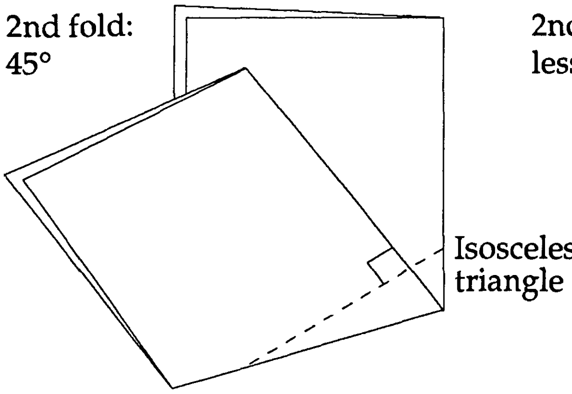
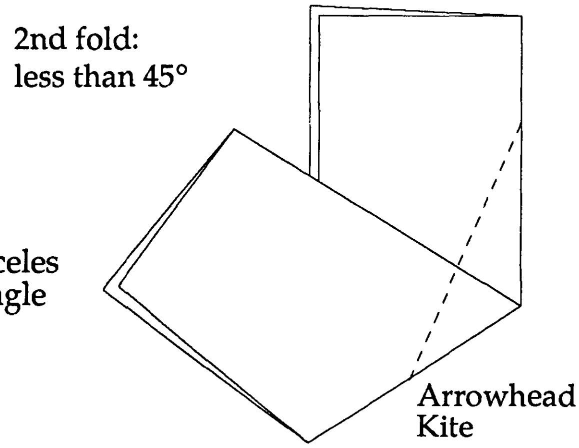
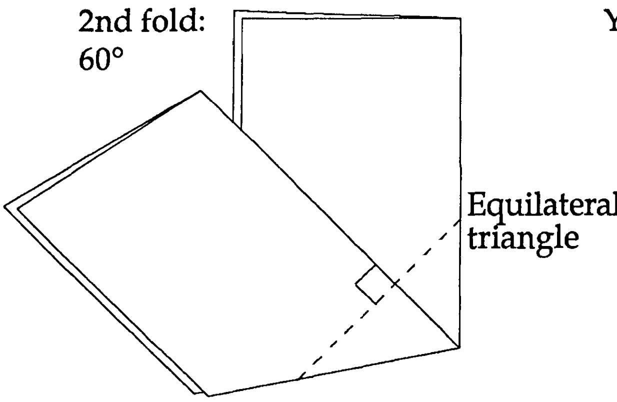
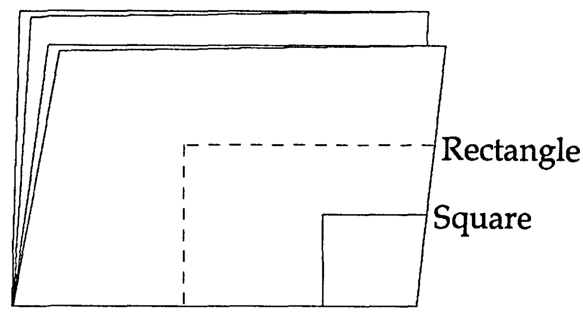




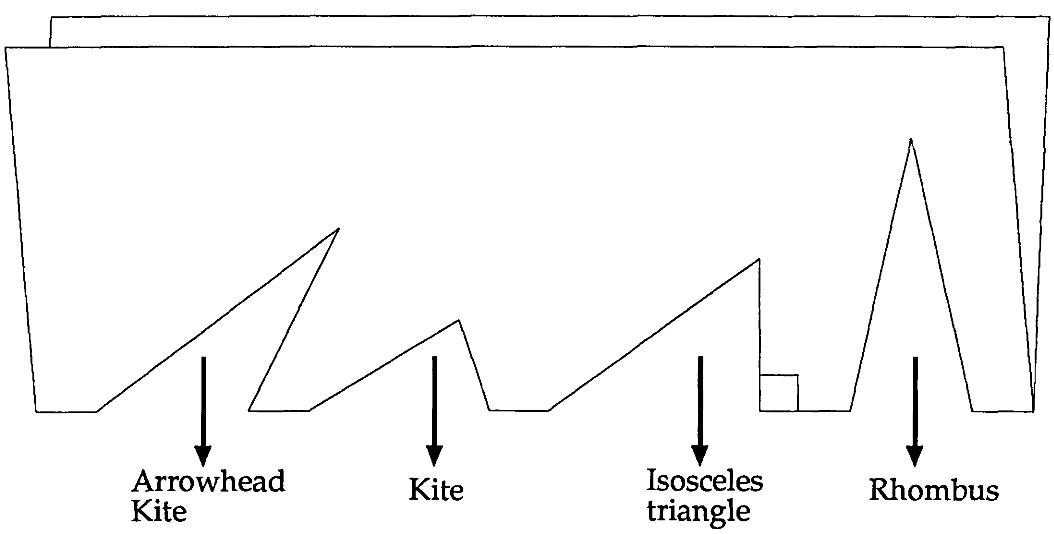


You will need two cuts to make a rectangle:

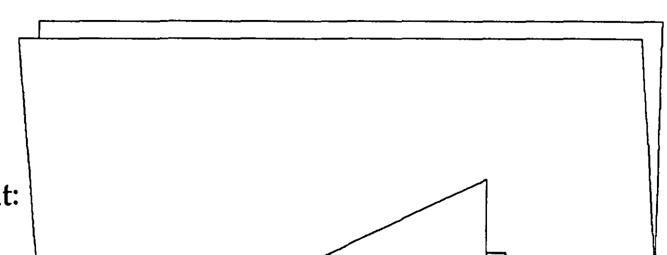


1382B One Fold

Cutting a triangle from one fold gives a variety of shapes with one line of symmetry.



You may also be able to make the isosceles triangle into an equilateral triangle by adjusting the angle of the cut:



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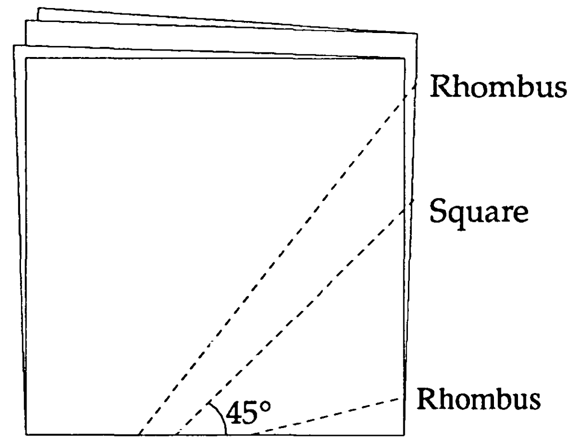
1382 Paper Folding (cont)

1382C Right-angle fold
One cut across 2 folds gives 4 equal sides, so you will be able to make a rhombus.



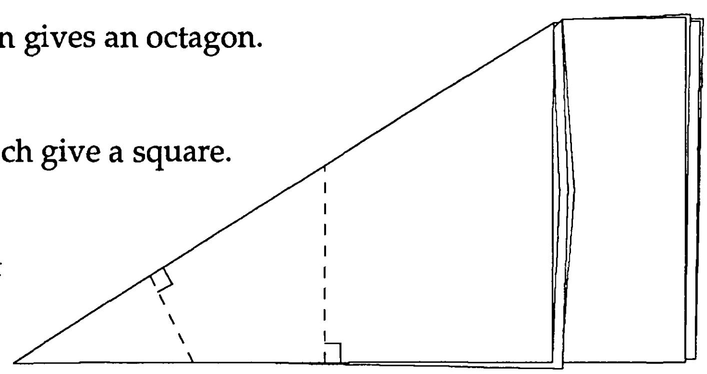
You will need to make the corners of the rhombus 90° for it to be a square.

1382D Two Folds
Most of the possibilities with one cut across two folds are described in the answers to 1382A.

1382E Straight Cut
One cut across 3 folds most often gives an octagon.
Can you see why?

There are two different cuts which give a square.
Did you find them both?

It is between these two cuts that you will get a convex hexagon.



1382F Three Folds
One cut across 3 folds most often gives an octagon. Even if the 3 folds do not all meet at one point your cut will still give an octagon. Can you see why?

What variations did you find by making your cut at right angles to one of the folds?

1383 Good Guesswork

1. Jim estimates that the table is about half his height.
   Half of 172cm is between 80cm and 90cm.

2. Jim estimates that the bar is about double his height.
   2 x 172cm is about 350cm.

3. 5 spans would measure about 1 metre because 5 x 20cm = 100cm.
   $2\frac{1}{2}$ spans would measure about 50cm because it is half of 5 spans.

continued/

1383 Good Guesswork (cont)

4. a) Your answer will depend upon the size of your door, but the average height of a door is 2m.
   b) Your answer will depend upon the size of your room, but the average height of a room is 2.5m.
   c) Your answer will depend upon the width of your filing cabinet, but the average width of a filing cabinet is about 50cm.

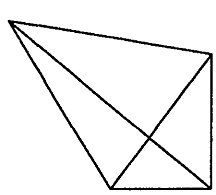
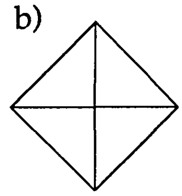
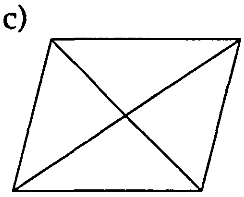
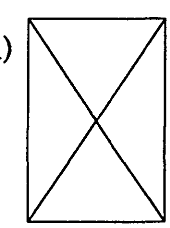
5. a) The tablecloth comes up to Jim's shoulders. A good estimate would be 150cm by 150cm or $1\frac{1}{2}$m by $1\frac{1}{2}$m. Jim's table is 1m x $\frac{1}{2}$m (see question 3).
   So the tablecloth is much too big.
   b) $4\frac{1}{2}$ spans would be about 90cm because
   4×20cm = 80cm and $\frac{1}{2} \times 20$cm = 10cm
   The picture is too big for the alcove.

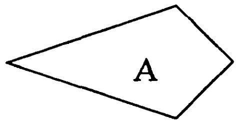
6. If you are unsure about your answers, get someone else to check them.

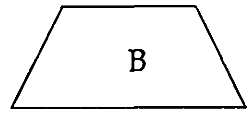
7. Approximate measurements are:

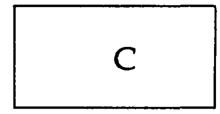
Length of a Mini Car	3 metres
Perimeter of the card	80cm folded (102cm opened out)
Size of a LP cover	120cm
Height of 12 storey building	40 metres
Height of double decker bus	4 metres

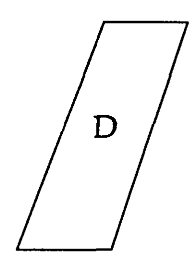
1384 Diagonals

1. a) 
   b) 
   c) 
   d) 

2. 

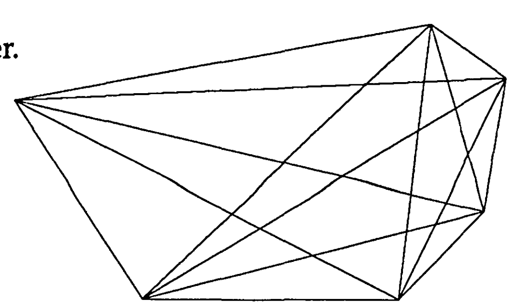






3. Five

4. Nine diagonals altogether.



1385 Times Square

- Which scores came up most often?
- Could you always use the scores?
- Were any squares impossible to cover?

1387 3-D Noughts and Crosses

- Did you develop a strategy? Is it best to go first?

1388 Double Up

1. $4cm^2$ → Double the sides → $16cm^2$
2. $8cm^2$ → Double the sides → $32cm^2$
3. $4cm^2$ → Double the sides → $16cm^2$
4. $8cm^2$ → Double the sides → $32cm^2$
5. $10cm^2$ → Double the sides → $40cm^2$
6. No. Doubling the length of the sides does not double the area.
7. Four.
8. Four.
9. Show your shapes to your teacher.
10. When I double the sides of a shape, the area becames **4** times as big.

- If you continue the investigation to trebling the sides of shapes you will find the results even more surprising!
  If you make the sides three times as long, the area becomes 9 times as big.
- What do you think would happen if you made the sides four times as long?

1389 Converging Sequences

1. Each sequence in A, B and C is obtained from the first two numbers. For an explanation of this look at the back of the card. Once you understand how the sequences are generated you could use a spreadsheet to continue the sequences.

A

p	q
1	1
2	3
5	7
12	17
29	41
70	99
169	239
408	577
985	1393
2378	3363

B

p	q
4	7
11	15
26	37
63	89
152	215
367	519
886	1253
2139	3025
5164	7303
12467	17631

C

p	q
7	10
17	24
41	58
99	140
239	338
577	816
1393	1970
3363	4756
8119	11482
19601	27720

2. The ratios of q/p in sequences A, B and C are:

A

p	q	q/p
1	1	1
2	3	1.5
5	7	1.4
12	17	1.41666667
29	41	1.4137931
70	99	1.41428571
169	239	1.41420118
408	577	1.41421569
985	1393	1.4142132
2378	3363	1.41421362

B

q/p
1.75
1.36363636
1.42307692
1.41269841
1.41447368
1.41416894
1.41422122
1.41421225
1.41421379
1.41421352

C

q/p
1.42857143
1.41176471
1.41463415
1.41414141
1.41422594
1.41421144
1.41421393
1.4142135
1.41421357
1.41421356

The sequence of the ratios q/p converges to $\sqrt{2}$.

3. Using the same rule to generate your own sequences you should find that the ratio of the q/p converges to $\sqrt{2}$.

4. For sequence D the rule for q is 'add the new p and twice the old p'.

Sequence D

p	q
1	1
2	4
⋮	⋮
x	y
$x+y$	$3x+y$
⋮	⋮

Successive ratios q/p will be of the form $\frac{y}{x}$, $\frac{3x+y}{x+y}$, ...

The sequence q/p again converges this time to 1.7320508.

continued/

Using the same rule to generate your own sequences you should find that the ratio of the q/p converges to √3.
The justification of this is:

$$\frac{y}{x} = \frac{3x+y}{x+y}$$

then $y(x+y) = x(3x+y)$

$xy + y^2 = 3x^2 + xy$

therefore $y^2 = 3x^2$

therefore $\frac{y^2}{x^2} = 3$

therefore $\frac{y}{x} = \sqrt{3}$

5. You may have used rules for q such as:
   - 'add the new p and three times the old p' when the ratio q/p tends to the limit √4, or
   - 'add the new p and four times the old p' when the ratio q/p tends to the limit √5 . . .

   You should be able to understand what is happening to the ratio q/p using the back of the card.

6. The sequence $\frac{y}{x}$, $\frac{2x+y}{x+y}$ . . . tends to the limit √2.

   and $\frac{y}{x}$, $\frac{3x+y}{x+y}$ . . . tends to the limit √3.

   and $\frac{y}{x}$, $\frac{4x+y}{x+y}$ . . . tends to the limit √4.

   This suggests that a sequence of the form

   $\frac{y}{x}$, $\frac{nx+y}{x+y}$ . . . will tend to the limit √n.

   You can generate the square roots of any number in this way (to whatever accuracy you require).

   e.g. to find an approximation for √7, construct the sequence,

p	q
.	.
.	.
.	.
x	y
$x+y$	$7x+y$
.	.
.	.
.	.

p	q	q/p
1	4	4
5	11	2.2
16	46	2.875
62	158	2.5483871
220	592	2.69090909
812	2132	2.62561576
2944	7816	2.6548913
10760	28424	2.64163569
39184	103744	2.64761127
142928	378032	2.64491212
520960	1378528	2.64613022
1899488	5025248	2.64558028
6924736	18321664	2.64582852
25246400	66794816	2.64571646

The ratio q/p tends to a limit of 2.64571646 ∝ √7

1390 Table Facts

Here is a completed table. You should learn the table facts which you do not know already.

1x1	2x1	3x1	4x1	5x1	6x1	7x1	8x1	9x1	10x1
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1x2	2x2	3x2	4x2	5x2	6x2	7x2	8x2	9x2	10x2
2	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**	**20**
1x3	2x3	3x3	4x3	5x3	6x3	7x3	8x3	9x3	10x3
3	**6**	**9**	**12**	**15**	**18**	**21**	**24**	**27**	**30**
1x4	2x4	3x4	4x4	5x4	6x4	7x4	8x4	9x4	10x4
4	**8**	**12**	**16**	**20**	**24**	**28**	**32**	**36**	**40**
1x5	2x5	3x5	4x5	5x5	6x5	7x5	8x5	9x5	10x5
5	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**
1x6	2x6	3x6	4x6	5x6	6x6	7x6	8x6	9x6	10x6
6	**12**	**18**	**24**	**30**	**36**	**42**	**48**	**54**	**60**
1x7	2x7	3x7	4x7	5x7	6x7	7x7	8x7	9x7	10x7
7	**14**	**21**	**28**	**35**	**42**	**49**	**56**	**63**	**70**
1x8	2x8	3x8	4x8	5x8	6x8	7x8	8x8	9x8	10x8
8	**16**	**24**	**32**	**40**	**48**	**56**	**64**	**72**	**80**
1x9	2x9	3x9	4x9	5x9	6x9	7x9	8x9	9x9	10x9
9	**18**	**27**	**36**	**45**	**54**	**63**	**72**	**81**	**90**
1x10	2x10	3x10	4x10	5x10	6x10	7x10	8x10	9x10	10x10
10	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**

1394 Turn the Tables

1. Here are some of the multiplication facts for numbers which appear several times each.

4	**6**	**8**	**10**	**12**	**16**
1 x 4	2 x 3	2 x 4	2 x 5	1 x 12	2 x 8
4 x 1	3 x 2	4 x 2	5 x 2	12 x 1	8 x 2
2 x 2	1 x 6	1 x 8	1 x 10	2 x 6	4 x 4
	6 x 1	8 x 1	10 x 1	6 x 2	
				3 x 4	
				4 x 3	

18	**20**	**24**	**30**	**36**	**40**
2 x 9	2 x 10	2 x 12	3 x 10	3 x 12	4 x 10
9 x 2	10 x 2	12 x 2	10 x 3	12 x 3	10 x 4
3 x 6	4 x 5	4 x 6	5 x 6	4 x 9	5 x 8
6 x 3	5 x 4	6 x 4	6 x 5	9 x 4	8 x 5
		3 x 8		6 x 6	
		8 x 3			

2. The numbers 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121 and 144 appear an odd number of times. These are the **square** numbers.
Where do they occur?
Why do they appear an odd number of times?

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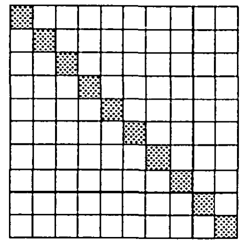
3. The line of symmetry is the leading diagonal and goes through the square numbers.



The table is symmetrical about this line because the multiplication fact for two numbers is the same, no matter which way round you write them.
e.g. 3 x 7 gives the same result as 7 x 3.

This is called **commutativity**. The result is not changed by altering the order of the numbers.
Multiplication of numbers is commutative.

4. The numbers which do not appear in the table are 13, 17, 23, 31, 37, 41, 43, 47. These are **prime** numbers. They do not appear in the table because a prime number only has two factors, itself and 1. The only prime numbers which appear in the table are those less than twelve. Why is this?

1395 Multiplication Table Patterns

1.

4
6
8
10

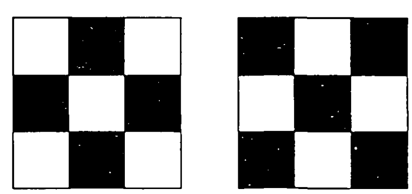
16	18
24	27
32	36

12	18	24	30	36
14	21	28	35	42
16	24	32	40	48
18	27	36	45	54

4	5	6
8	10	12

32	36
40	45

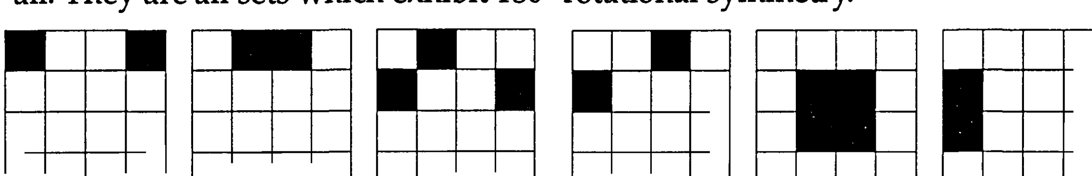
2. a) 9 + 16 + 20 + 27 = 72
8 + 10 + 24 + 30 = 72



18 x 4 = 72

The sum of the four 'corner' numbers and the sum of the four 'middle' numbers are both the same as 4 x the 'corner' numbers.
This is true for all 3 x 3 squares taken from this table.

b) There are five other sets of four numbers which add up to 75, making six in all. They are all sets which exhibit 180° rotational symmetry.



continued/

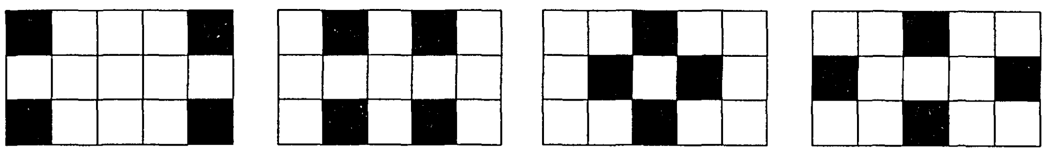
1395 Multiplication Table Patterns (cont)

2. c) Sets of four numbers with the same sum can be found in all sizes of rectangles. The sets always exhibit 180° rotational symmetry.

e.g. For this rectangle

1	2	3	4	5
2	4	6	8	10
3	6	9	12	15

the following sets of four numbers sum to 24.



In the case of rectangles with an odd number of squares, the sum is always 4 times the 'centre' number, e.g. all the sets of four add up to 4 x 6.

3. 15 x 42 = 630
30 x 21 = 630
Opposite corners always give the same product, no matter what the size of the rectangle is. This can be explained by remembering that any number in the table is the multiple of two numbers.

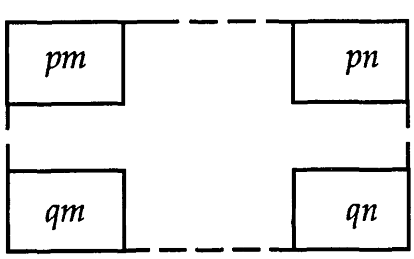
e.g.

15	20	25	30
18	24	30	36
21	28	35	42

so if you multiply 15 x 42 you are, in fact multiplying (5 x 3) x (7 x 6).

If you multiply 21 x 30 you are multiplying (7 x 3) x (5 x 6).

- **Multiplication of numbers is commutative,** so these products are the same. In general, any rectangle from the table is of the form



and so product of the opposite corners will be *pqmn*.

1396 Two Digit Sums

3 digits

The ratio $\frac{x}{y}$ is always 22.

Here is the proof using a, b and c to stand for the 3 digits.

There are 6 different possible numbers:

$10a + b$	$10b + a$	$10c + a$
$10a + c$	$10b + c$	$10c + b$

so $x = 10a + b + 10a + c + 10b + a + 10b + c + 10c + a + 10c + b$

$= 22a + 22b + 22c$

$= 22(a + b + c)$

$y = a + b + c$

So $\frac{x}{y} = \frac{22(a + b + c)}{a + b + c}$

$= 22$

4 digits or more

With 4 digits there are 12 different numbers possible two digit numbers and $\frac{x}{y} = 33$.

With 5 digits there are 20 different numbers possible two digit numbers and $\frac{x}{y} = 44$.

This table gives fuller details.

Number of digits	Possible 2-digit numbers	x	y	$\frac{x}{y}$
2	$10a + b$ $10b + a$	$11(a + b)$	$a + b$	11
3	$10a + b$ $10b + a$ $10b + c$ $10c + b$ $10a + c$ $10c + a$	$22(a + b + c)$	$a + b + c$	22
4	$10a + b$ $10b + a$ $10b + c$ $10c + b$ $10a + c$ $10c + a$ $10a + d$ $10d + a$ $10b + d$ $10d + b$ $10d + c$ $10c + d$	$33(a + b + c + d)$	$a + b + c + d$	33
5 . . . n	$10a + b$ $10b + a$ $10b + c$ $10c + b$ $10a + c$ $10c + a$ . . . . . . $n(n - 1)$	$44(a + b + c + d + e)$ . . . $11(n - 1)(a + b + \ldots)$	$a + b + c + d + e$ . . . $(a + b + \ldots)$	44 . . . $11(n - 1)$

- If you repeat digits it could confuse the results. For instance, if you start with the digits 3, 3 and 9 the only different two digit numbers are 33, 39 and 93. If you treat each digit quite separately (imagine three pieces of paper), then you will get all the possible combinations, 33, 33, 39, 39, 93 and 93.

1398 Trigg

- Did it matter who went first?
- Can you describe the strategy you used to win?

1399 Babylonian Method

Original Problem

L x W = 192
L + W = 28

Nowadays you are likely to solve problems like this using a spreadsheet. Here are the formulas used to solve the original problem.

	A	B	C	D
1	length (L)	width (W)	L + W	L x W
2	1	=28-A2	=A2+B2	=A2*B2
3	=A2+1	=28-A3	=A3+B3	=A3*B3

⇓ Fill down ⇓ Fill down ⇓ Fill down ⇓ Fill down

Here is part of the spreadsheet.

	A	B	C	D
1	length (L)	width (W)	L + W	L x W
2	1	27	28	27
3	2	26	28	52
4	3	25	28	75
5	4	24	28	96
6	5	23	28	115
7	6	22	28	132
8	7	21	28	147
9	8	20	28	160
10	9	19	28	171
11	10	18	28	180
12	11	17	28	187
13	12	16	28	192 ⇐
14	13	15	28	195
15	14	14	28	196
16	15	13	28	195
17	16	12	28	192
18	17	11	28	187
19	18	10	28	180

Looking at the spreadsheet you can see the unique solution where L x W = 192 is L = 12 and W = 16.

1. You can use your original spreadsheet to find that the unique solution where L x W = 180 is L = 18cm and W = 10cm. Remember to check your solution by putting the values you have found back into the original statement.

   Alternatively, you can use the Babylonian Method
   i) Half of 28 = 14
   ii) 14 x 14 = 196
   iii) 196 – 180 = 16
   iv) Square root of 16 = 4
   v) 14 + 4 = 18; Length = 18cm
   vi) 14 – 4 = 10; Width = 10cm Check: 18 x 10 = 180

continued /

1399 Babylonian Method (cont)

2. By changing the formula for *width* you can adapt your spreadsheet.

   Using the Babylonian Method
   i) Half of 37 is 18.5
   ii) 18.5 x 18.5 = 342.25
   iii) 342.25 – 336 = 6.25
   iv) Square root of 6.25 = 2.5
   v) 18.5 + 2.5 = 21 Length = 21cm
   vi) 18.5 – 2.5 = 16 Width = 16cm Check: 21 x 16 = 336

3. The missing instruction is 'square' or 'multiply by itself'.

4.
   59
   29.5
   870.25
   20.25
   4.5
   29.5 + 4.5 → Length = 34cm
   29.5 – 4.5 → Width = 25cm
   850

5.
   s
   $\frac{s}{2}$
   $\frac{s^2}{4}$
   $\frac{s^2}{4} - p$
   $\sqrt{\frac{s^2}{4} - p}$
   $\frac{s}{2} + \sqrt{\frac{s^2}{4} - p}$ → Length
   $\frac{s}{2} - \sqrt{\frac{s^2}{4} - p}$ → Width

continued/

1399 Babylonian Method (cont)

6. $W = \frac{s}{2} - \sqrt{\frac{s^2}{4} - p}$

7. $L \times W = \left(\frac{s}{2} + \sqrt{\frac{s^2}{4} - p}\right)\left(\frac{s}{2} - \sqrt{\frac{s^2}{4} - p}\right)$

This line is equivalent to $(x + y)(x - y)$ which equals $x^2 - y^2$ so you might have omitted line 2.

$$= \frac{s^2}{4} + \frac{s}{2}\sqrt{\frac{s^2}{4} - p} - \frac{s}{2}\sqrt{\frac{s}{4} - p} - \left(\frac{s^2}{4} - p\right)$$

$$= \frac{s^2}{4} - \left(\frac{s^2}{4} - p\right)$$

$$= p$$

8. a)

$s = 27\text{cm}$

$p = 180\text{cm}$

$L = \frac{27}{2} + \sqrt{\frac{729}{4} - 180}$

$= 15\text{cm}$

$W = \frac{27}{2} - \sqrt{\frac{729}{4} - 180}$

$= 12\text{cm}$

b)

$s = 6.3\text{cm}$

$p = 8.82\text{cm}^2$

$L = \frac{6.3}{2} + \sqrt{\frac{39.69}{4} - 8.82}$

$= 4.2\text{cm}$

$W = \frac{6.3}{2} - \sqrt{\frac{39.69}{4} - 8.82}$

$= 2.1\text{cm}$

c)

$s = 64.5\text{cm}$

$p = 845.46\text{cm}^2$

$L = \frac{64.5}{2} + \sqrt{\frac{4160.25}{4} - 845.46}$

$= 46.2\text{cm}$

$W = \frac{64.5}{2} - \sqrt{\frac{4160.25}{4} - 845.46}$

$= 18.3\text{cm}$

continued/

1399 Babylonian Method (cont)

9. Equation 1 $L + W = s$
   Equation 2 $LW = p$

From equation 1, $W = s - L$

Substitute this for W in equation 2.

$$L(s - L) = p$$
$$Ls - L^2 = p$$
$$L^2 - Ls + p = 0$$

Using the quadratic formula gives two solutions.

$$L = \frac{s \pm \sqrt{s^2 - 4p}}{2}$$

Similarly for W there are two solutions.

$$W = \frac{s \pm \sqrt{s^2 - 4p}}{2}$$

Our modern algebraic equations give both answers as the root of one equation. The length and width are the two solutions of

$$L = \frac{s \pm \sqrt{s^2 - 4p}}{2}$$

Dividing the numerator by 2 gives the combined solution for the Babylonian method.

$$L = \frac{s}{2} \pm \sqrt{\frac{s^2}{4} - p}$$

- The Babylonians did not recognise that a square root could be positive or negative. They had no concept of negative numbers. Negative numbers were not accepted until after 1500AD.

1400 A Transformation Technique

1. a) You should have found that pre-multiplication by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ has the effect of changing the signs of both the *x* and the *y* co-ordinate.

   b) This is the general case, describing the operation on any pair.

2. a) $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $\quad \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

   b) So the transformation matrix is $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

   c) This should confirm that $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ describes the transformation.

continued/

1400 A Transformation Technique (cont)

3. Some examples are:

$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ The identity matrix causing no change

$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ Reflects in the line $y = x$.

$\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$ Rotation of 180° about the origin.

$\begin{pmatrix}a & 0\\ 0 & a\end{pmatrix}$ Enlargement with scale factor a and centre (0, 0).

There are many other matrices so ask your teacher to check any others.

4. $\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \begin{pmatrix}\mathbf{a}\\ \mathbf{c}\end{pmatrix}$

$\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} = \begin{pmatrix}\mathbf{b}\\ \mathbf{d}\end{pmatrix}$

1404 Action Equations

A	1.	$n = 3$	6.	$n = 7$
	2.	$n = 5$	7.	$n = 6$
	3.	$n = 11$	8.	$n = 7$
	4.	$n = 8$	9.	$n = 10$
	5	$n = 8$	10.	$n = 6$
B	1.	$n = 8$	6.	$n = 22$
	2.	$n = 6$	7.	$n = 20$
	3.	$n = 5$	8.	$n = 19$
	4.	$n = 8$	9.	$n = 18$
	5.	$n = 6$	10.	$n = 19$

1405 Jump Equations

A	1.	$n = 5$	6.	$n = 4$
	2.	$n = 4$	7.	$n = 7$
	3.	$n = 7$	8.	$n = 6$
	4.	$n = 3$	9.	$n = 9$
	5.	$n = 3$	10.	$n = 5$
B	1.	$n = 9$	6.	$n = 24$
	2.	$n = 13$	7.	$n = 17$
	3.	$n = 14$	8.	$n = 14$
	4.	$n = 18$	9.	$n = 15$
	5.	$n = 12$	10.	$n = 17$

1406 Equality and Inequality

A	1.	$8 + 3 = 3 + 8$	6.	$3 \times 6 = 6 \times 3$
	2.	$4 + 9 = 9 + 4$	7.	$10 \div 2 \neq 2 \div 10$
	3.	$7 - 4 \neq 4 - 7$	8.	$18 - 10 \neq 10 - 18$
	4.	$6 \times 7 = 7 \times 6$	9.	$21 \div 3 \neq 3 \div 21$
	5.	$4 + 0 = 0 + 4$	10.	$14 - 6 \neq 6 - 14$
B	1.	$27 + 14 = 14 + 27$	6.	$\frac{1}{2} + \frac{1}{4} = \frac{1}{4} + \frac{1}{2}$
	2.	$36 - 49 \neq 49 - 36$	7.	$\frac{1}{2} - \frac{1}{4} \neq \frac{1}{4} - \frac{1}{2}$
	3.	$15 \div 5 \neq 5 \div 15$	8.	$0.6 + 0.3 = 0.3 + 0.6$
	4.	$15 \times 5 = 5 \times 15$	9.	$0.5 - 0.2 \neq 0.2 - 0.5$
	5.	$100 \div 10 \neq 10 \div 100$	10.	$1 \div 2 \neq 2 \div 1$

- The operation of **subtraction** is **not commutative.**
- The operation of **multiplication** is **commutative.**
- The operation of **division is not commutative.**

1408 Thermometer Readings

A = 7°C	B = 14°C	C = 21°C	D = 29°C
E = 38°C	F = 48°C	G = 56°C	H = 63°C
J = 82°C	K = 94°C		
L = 44°F	M = 58°F	N = 86°F	P = 92°F
Q = 108°F	R = 124°F	S = 134°F	T = 154°F
U = 172°F	V = 198°F		

The Fahrenheit scale goes up in 2's and the Celsius scale goes up in 1's, so you need to be very careful when reading off the scales.

		a)		b)			a)		b)
A	=	18°F	=	-8°C	B	=	32°F	=	0°C
C	=	46°F	=	8°C	D	=	64°F	=	18°C
E	=	76°F	=	24°C	F	=	90°F	=	32°C
G	=	102°F	=	39°C	H	=	126°F	=	52°C

1409 The Mean

1. 31
2. 32
3. 86.333333 or 86.3 which to the nearest car is 86.
4. 25p

1411 Roman Numerals

1. 2	3. 7	5. 35	7. 8
2. 12	4. 20	6. 26	8. 38

9. The most likely explanation seems to be that V is half of the symbol X. Perhaps the symbol X was used first.

10. 200	11. 3000
12. 150	13. 155

continued/

1411 Roman Numerals (cont)

14. 1251   15. 1361

16. 1666   17. 2008

18. CM means *100 less than 1000.*

19. 90 because it means *10 less than 100.*

20. MCM means *1000 and 100 less than 1000.*

21. 19   23. 2900   25. 79

22. 190   24. 2923   26. 1559

27 Normally the symbols are written from left to right in decreasing order with the higher symbols first. With 9, 90 or 900 it seems as if one of the smaller symbols is out of order.

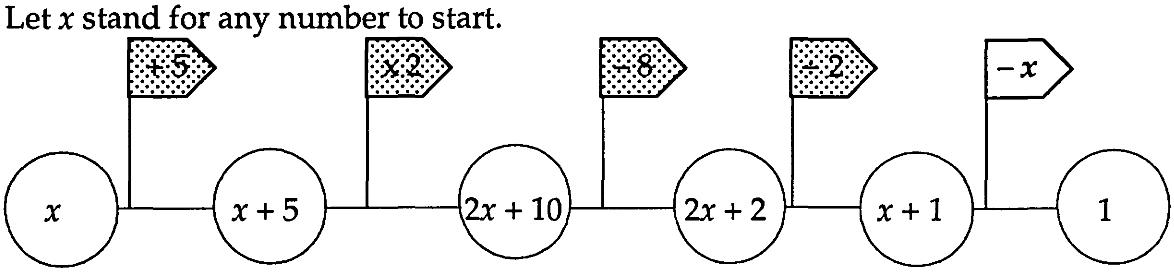
28. IV means *1 less than 5.*

29. 94   30. 1984

1412 Algebra Puzzle

1. Whatever number you start with, the answer is always 1.
Questions 2 and 3 will help explain why.

3. Let x stand for any number to start.



Add 5 $\rightarrow$ $x + 5$

Multiply by 2 $\rightarrow$ $2(x + 5) = 2x + 10$

Subtract 8 $\rightarrow$ $2x + 2$

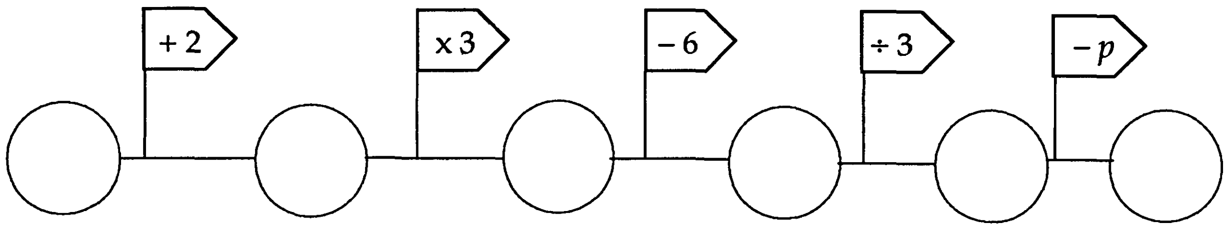
Divide by 2 $\rightarrow$ $\frac{2x + 2}{2} = x + 1$

Subtract x $\rightarrow$ 1

continued/

1412 Algebra Puzzle (cont)

4. Your flag chart should look like this. Let p stand for any number.



Whatever value you start with the answer is always 0.

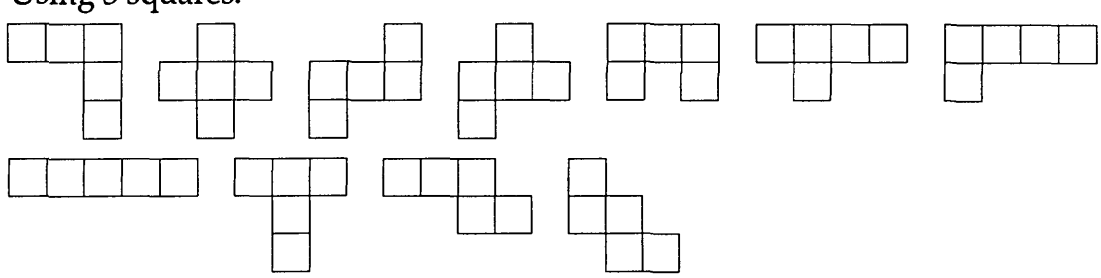
Add 2	→	$p + 2$		
Multiply by 3	→	$3(p + 2)$	=	$3p + 6$
Subtract 6	→	$3p$		
Divide by 3	→	$\frac{3p}{3}$	=	p
Subtract p	→	0		

So, whatever number you start with, the answer will always be zero.

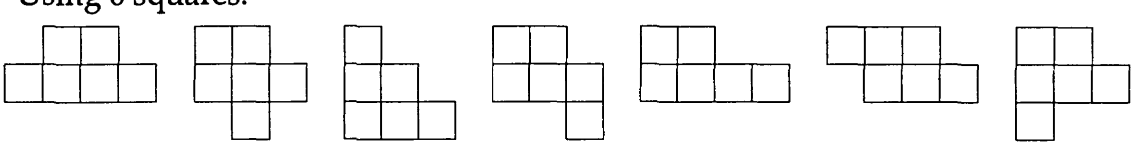
5. There are many possible answers.
Check your game by testing it with an integer, a fraction or decimal, a negative number and a letter.

1413 Twelve Inch Perimeter

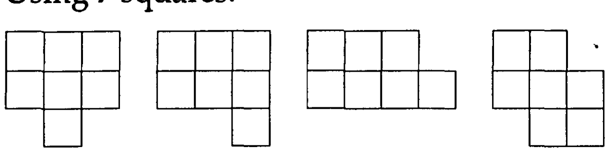
1. There are 25 different shapes, all with a perimeter of 12 inches.
Using 5 squares:



Using 6 squares:



Using 7 squares:



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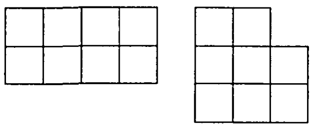
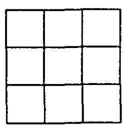
1413 Twelve Inch Perimeter (cont)

Using 8 squares:



Using 9 squares:



2. There are two shapes using 8 squares which have a perimeter of 12 inches.
There are seven shapes using 6 squares which have a perimeter of 12 inches.

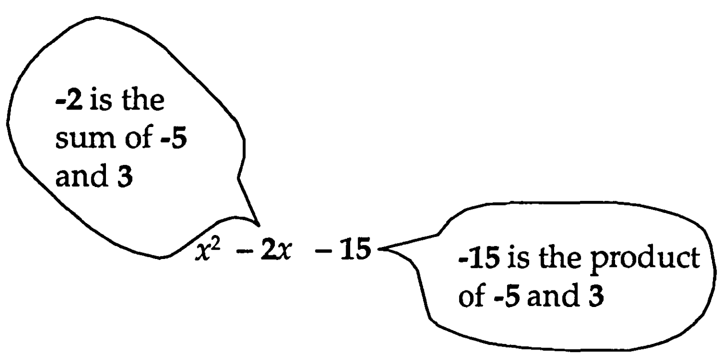
3. The biggest shape has 9 squares.
The smallest shapes have 5 squares.

1415 Simple Quadratics

1&2. $(x + k)(x + m) = x^2 + (k + m)x + km$ and so the multiplication gives a quadratic expression of the form $x^2 + bx + c$.
Occasionally, the result is of the form $x^2 + c$.

- What values of k and m cause the term 'bx' to disappear?

3.



4.

$x^2 + 29x + 100$	=	$(x + 25)(x + 4)$
$x^2 - 29x + 100$	=	$(x - 25)(x - 4)$
$x^2 - 52x + 100$	=	$(x - 50)(x - 2)$
$x^2 - 48x - 100$	=	$(x - 50)(x + 2)$
$x^2 + 15x - 100$	=	$(x + 20)(x - 5)$
$x^2 - 20x + 100$	=	$(x - 10)(x - 10)$ or $(x - 10)^2$
$x^2 - 100$	=	$(x - 10)(x + 10)$

5.

For $x^2 + 5x + 6$	$k = 3$ and $m = 2$ or vice versa
For $x^2 + 8x + 16$	$k = 4$ and $m = 4$
For $x^2 - 8x + 16$	$k = -4$ and $m = -4$
For $x^2 - 16$	$k = -4$ and $m = 4$ or vice versa
For $x^2 - 25$	$k = -5$ and $m = 5$ or vice versa

continued/

6. The values of b and c depend upon k and m.
The coefficient of x, b is the sum of k and m $(k + m)$.
The constant term, c is the product of k and m $(k \times m)$.

7. If k and m are equal but opposite in sign, b will be zero.
e.g. if $k = 7$ and $m = -7$ $\quad (x + 7)(x - 7) \quad = \quad x^2 - 49$

8. If either k or m are zero, then c will be zero.
e.g. if $k = 3$ and $m = 0$ $\quad (x + 3)(x + 0) \quad = \quad (x + 3)x \quad = \quad x^2 + 3x$

9. Both k and m have to be zero to make b and c both zero.

10. $x = 0$ or $x = -11$

11. $(x + 7)(x + 4) = 40 \quad \rightarrow \quad x$ could be 1 (or -12)
$(x + 7)(x + 4) = 70 \quad \rightarrow \quad x$ could be 3 (or -14)
$(x + 7)(x + 4) = 18 \quad \rightarrow \quad x$ could be -1 (or -10)
$(x + 7)(x + 4) = 4 \quad \rightarrow \quad x$ could be -3 (or -8)

12. $(x + 7)(x + 4) = 70 \quad \rightarrow \quad x$ could also be -14 (or 3)

13. $(x + 7)(x + 4) = 0 \quad \rightarrow \quad x$ could be -7 or x could be -4

14. Any 6 pairs of numbers in the form
number x zero = zero e.g. $5 \times 0 = 0$
or zero x zero = zero

15. $(x + 7)$ is 3 more than $(x + 4)$ so they both can't be zero.
a) $x = -7$ would make the left-hand bracket zero.
b) $x = -4$ would make the right-hand bracket zero.

16. a) $x = -3$
b) $x = -5$

17. a) $x = -3$ or $x = -5$
b) $x = 3$ or $x = 5$
c) $x = 3$ or $x = -5$

18. $(x + k)(x + m) = 0$ if $x = -k$ or $x = -m$

19. $(x + 3)(x + 12) = 0 \quad x = -3$ or $x = -12$
$(x - 3)(x - 12) = 0 \quad x = 3$ or $x = 12$
$(x - 5)(x + 7) = 0 \quad x = 5$ or $x = -7$
$x^2 + 5x + 6 = 0 \rightarrow (x + 3)(x + 2) = 0 \quad x = -3$ or $x = -2$
$x^2 + 15x - 100 = 0 \rightarrow (x + 20)(x - 5) = 0 \quad x = -20$ or $x = 5$
$x^2 + 8x + 12 = 0 \rightarrow (x + 6)(x + 2) = 0 \quad x = -6$ or $x = -2$

continued/

1415 Simple Quadratics (cont)

20. $x^2 + 8x + 14 = 2$
    Subtract 2 from each side of the equation.
    $x^2 + 8x + 12 = 0$ $\quad x = -6$ or $x = -2$

21. $x^2 + 15x = 100$
    Subtract 100 from each side of the equation.
    $x^2 + 15x - 100 = 0$ $\quad x = -20$ or $x = 5$

22. a) $x^2 - 48x = 100$
    Subtract 100 from each side.
    $x^2 - 48x - 100 = 0$ $\quad x = 50$ or $x = -2$

    b) $x^2 + 100 = 29x$
    Subtract $29x$ from each side.
    $x^2 - 29x + 100 = 0 \rightarrow (x - 25)(x - 4) = 0$ $\quad x = 25$ or $x = 4$

    c) $x^2 + 5x - 84 = 0 \rightarrow (x - 7)(x + 12) = 0$ $\quad x = 7$ or $x = -12$

    d) $x^2 + 5x = 50$
    $x^2 + 5x - 50 = 0 \rightarrow (x + 10)(x - 5) = 0$ $\quad x = -10$ or $x = 5$

    e) $x^2 = 11x - 10$
    $x^2 - 11x + 10 = 0 \rightarrow (x - 10)(x - 1) = 0$ $\quad x = 10$ or $x = 1$

    f) $x^2 - 8x + 16 = 0 \rightarrow (x - 4)^2 = 0$ $\quad x = 4$

    g) $2x^2 - 14x + 24 = 0$
    Divide each side by 2
    $x^2 - 7x + 12 = 0 \rightarrow (x - 3)(x - 4) = 0$ $\quad x = 3$ or $x = 4$

- Read the Next Step carefully and check you understand it. You may find the Summary on the last page useful in helping you to produce revision notes.

1417 Tens

Were you able to make two lines of 10 by placing one counter?

❸		
❺		
❷	❷	❻

e.g. by placing a ❷ here → (bottom left square)
you would get
two lines of ten.

1418 Series Geometrically

Page 1

Area B = $\frac{1}{4}$ Area C = $\frac{1}{8}$

$$\text{Area A} + \text{Area B} + \text{Area C} + \text{Area D} + \ldots = 1$$
$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots = 1$$

- The *n*th term of this series is $(\frac{1}{2})^n$.
- The series never stops. It is infinite.

Page 2

B is $\frac{3}{4}$ of the area that is not A.

So B is $\frac{3}{4}$ of $\frac{1}{4}$, which is $\frac{3}{16}$.

C is $\frac{3}{4}$ of the area that is not A and not B.

So C is $\frac{3}{4}$ of $\frac{1}{16}$, which is $\frac{3}{64}$.

	Area A	:	Area B
=	4	:	1
∴	lengths A	:	lengths B
=	$\sqrt{4}$	:	$\sqrt{1}$
=	2	:	1

The scale factor of enlargement of B to A is **x 2**

A scale factor of **x 4** would enlarge C to A.

$$\text{Area A} + \text{Area B} + \text{Area C} + \text{Area D} + \ldots = 1$$
$$\frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \frac{3}{256} + \ldots = 1$$

The general form of this series is $\frac{3}{4^n}$

Page 3

A scale factor of x $\frac{1}{2}$ would reduce A to B.
A scale factor of x $\frac{1}{4}$ would reduce A to C.
A scale factor of x $\frac{1}{2}$ would reduce B to C.

continued/

Page 3 (cont)

Area B = $\frac{1}{2}\left(\frac{1}{4} \times \frac{1}{4}\right) = \frac{1}{32}$ B is $\frac{1}{4}$ of A.

Area C = $\frac{1}{128}$ C is $\frac{1}{16}$ of A.

Area D = $\frac{1}{512}$

The whole triangle has area $\frac{1}{2}$.

$$3(\text{Area A}) + 3(\text{Area B}) + 3(\text{Area C}) + 3(\text{Area D}) + \ldots = \frac{1}{2}$$

$$\frac{3}{8} + \frac{3}{32} + \frac{3}{512} + \frac{3}{2048} + \ldots = \frac{1}{2}$$

Page 4

You may find it quicker to use a spreadsheet to check that the series on pages 1, 2, and 3 never exceed the limit one.

e.g.

	A	B	C	D
1	Numerator	Denominator	Decimal	Cumulative Total
2	1	2	0.5	0.5
3	1	4	0.25	0.75
4	1	8	0.125	0.875
5	1	16	0.0625	0.9375
6	1	32	0.03125	0.96875
7	1	64	0.015625	0.984375
8	1	128	0.0078125	0.9921875
9	1	256	0.00390625	0.99609375
10	1	512	0.001953125	0.998046875
11	1	1024	0.000976563	0.999023438
12	1	2048	0.000488281	0.999511719
13	1	4096	0.000244141	0.999755859
14	1	8192	0.00012207	0.99987793
15	1	16384	6.10352E-05	0.999938965
16	1	32768	3.05176E-05	0.999969482
17	1	65536	1.52588E-05	0.999984741
18	1	131072	7.62939E-06	0.999992371
19	1	262144	3.8147E-06	0.999996185
20	1	524288	1.90735E-06	0.999998093

⇓

The last six cells of the spreadsheet in column C display the number in Standard Form.

e.g. 6.10352E-05 = 6.10352×10^{-5} = 0.0000610352

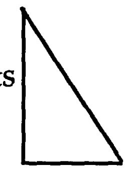
continued/

1418 Series Geometrically (cont)

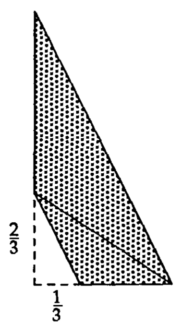
Page 5

Several answers are possible.

- If you assumed the dimensions of the triangle to have the height twice the base, the total area would be 1 square unit.



- One series can be made by considering lines parallel to the hypotenuse:



The first pair of triangles (shaded) together have area $\frac{8}{9}$ and the remaining triangle (unshaded) is $\frac{1}{9}$ of the original triangle.

The scale factor is therefore x $\frac{1}{3}$ ($\sqrt{\frac{1}{9}}$) and this gives the series:

$$\frac{8}{9} + \frac{8}{81} + \frac{8}{729} + \frac{8}{6561} = \ldots = 1$$

- Another series might start with the large shaded triangle of area $\frac{2}{3}$ . . .

1419 Versa-tiles

The angle combinations which are possible with the pentagon tile makes it very versatile indeed.

There are 11 different combinations which total to 360°.

These are:

(6 x 60°)	(3 x 100°) + 60°	(3 x 60°) + 100° + 80°
(2 x 100°) + 160°	(2 x 60°) + 100° + 140°	(2 x 100°) + (2 x 80°)
(2 x 60°) + 160° + 80°	(2 x 140°) + 80°	(2 x 60°) + (3 x 80°)
(2 x 80°) + 60° + 140°	60° + 160° + 140°	

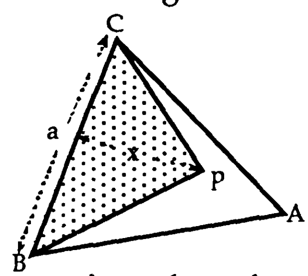
Make a wall display using your tiling patterns.

1420 Perpendicular Proof

You may want to use a geometry drawing computer package for your initial exploration.

For any point P inside the equilateral triangle ABC:

- Area of $\Delta BPC = \frac{1}{2}ax$
- Area of $\Delta APB = \frac{1}{2}ay$
- Area of $\Delta APC = \frac{1}{2}az$

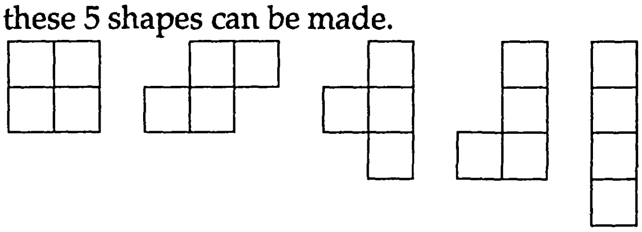


Therefore $\Delta APC + \Delta APB + \Delta BPC = \frac{1}{2}ax + \frac{1}{2}ay + \frac{1}{2}az = \frac{1}{2}a(x+y+z)$
Therefore area of $\Delta ABC = \frac{1}{2}a(x+y+z)$.

We know that the area of a given triangle does not change.
Therefore $\frac{1}{2}a(x+y+z)$ is constant value for a given equilateral triangle.
We also know that $\frac{1}{2}a$ is a constant value for this triangle.
Therefore $(x+y+z)$ must also be constant.
This is sufficient proof for any equilateral triangle.

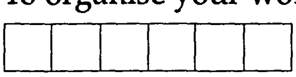
1421 Shapes from Squares

With 4 squares these 5 shapes can be made.

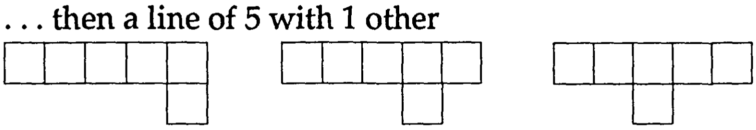


With 5 squares, 12 shapes can be made.
You will need to organise your work carefully to find them all.

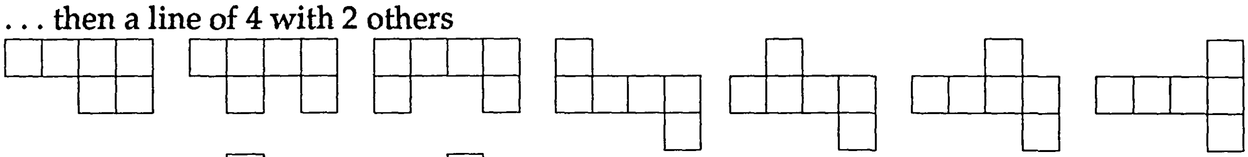
There are many more shapes which can be made with 6 squares.
To organise your work, one way is to start with a line of 6 . . .

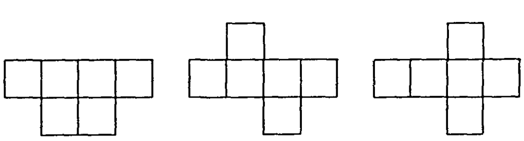


. . . then a line of 5 with 1 other



. . . then a line of 4 with 2 others





. . . and so on.

continued/

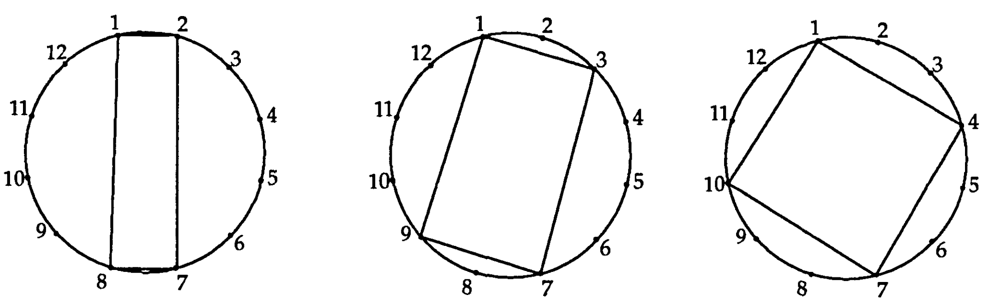
1421 Shapes from Squares (cont)

This mapping shows the results collected.

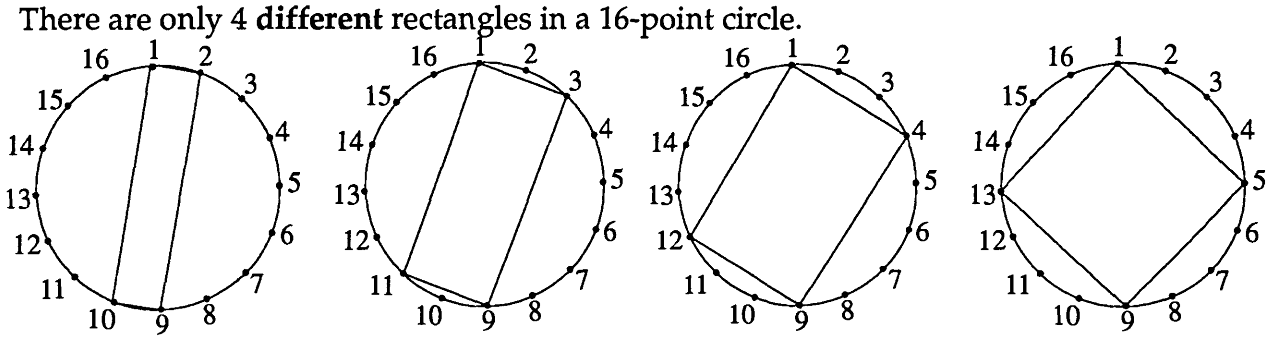
No. of squares used		No. of different shapes made
1	→	1
2	→	1
3	→	2
4	→	5
5	→	12
6	→	?

1422 Rectangle in Circles

It is possible to draw many rectangles in a 12-point circle, but there are only 3 **different ones.**



There are only 4 **different** rectangles in a 16-point circle.



1423 Calculator Guesses

1. 137 x **7** = 685
2. **7** x 21 = 147
3. 19 x **13** = 247
4. **23** x 23 = 529
5. 24 x **16** = 384
6. **21** x 46 = 966
7. 4956 = 354 x **14**
8. **12** x 214 = 2568
9. 25 x **25** = 625
10. 25 x **250** = 6250

1424 Dividing by Guessing

1. 64 ÷ **16** = 4
2. 104 ÷ **8** = 13
3. 84 ÷ **7** = 12
4. 54 ÷ **9** = 6
5. 105 ÷ **15** = 7
6. 52 ÷ **4** = 13
7. 81 ÷ **9** = 9
8. 75 ÷ **5** = 15
9. 90 ÷ **6** = 15
10. 56 ÷ **7** = 8
11. 168 ÷ **3** = 56
12. 144 ÷ **24** = 6
13. 520 ÷ **10** = 52
14. 136 ÷ **17** = 8
15. 136 ÷ **8** = 17

1425 A Rich Aunt

A table is a good way to compare the amount of money you would get from each scheme in each year. A spreadsheet can be used to create a table and then to graph the results.

This spreadsheet shows the amount of money Scheme (a) will generate up to the time when Aunt Lucy reaches 80 years of age.

	A	B	C
1		Scheme (a)	
2	Aunt's age	Amount each year	Cumulative Total
3	70	£ 100.00	£ 100.00
4	71	£ 90.00	£ 190.00
5	72	£ 80.00	£ 270.00
6	73	£ 70.00	£ 340.00
7	74	£ 60.00	£ 400.00
8	75	£ 50.00	£ 450.00
9	76	£ 40.00	£ 490.00
10	77	£ 30.00	£ 520.00
11	78	£ 20.00	£ 540.00
12	79	£ 10.00	£ 550.00
13	80	£ 0.00	£ 550.00

⇓

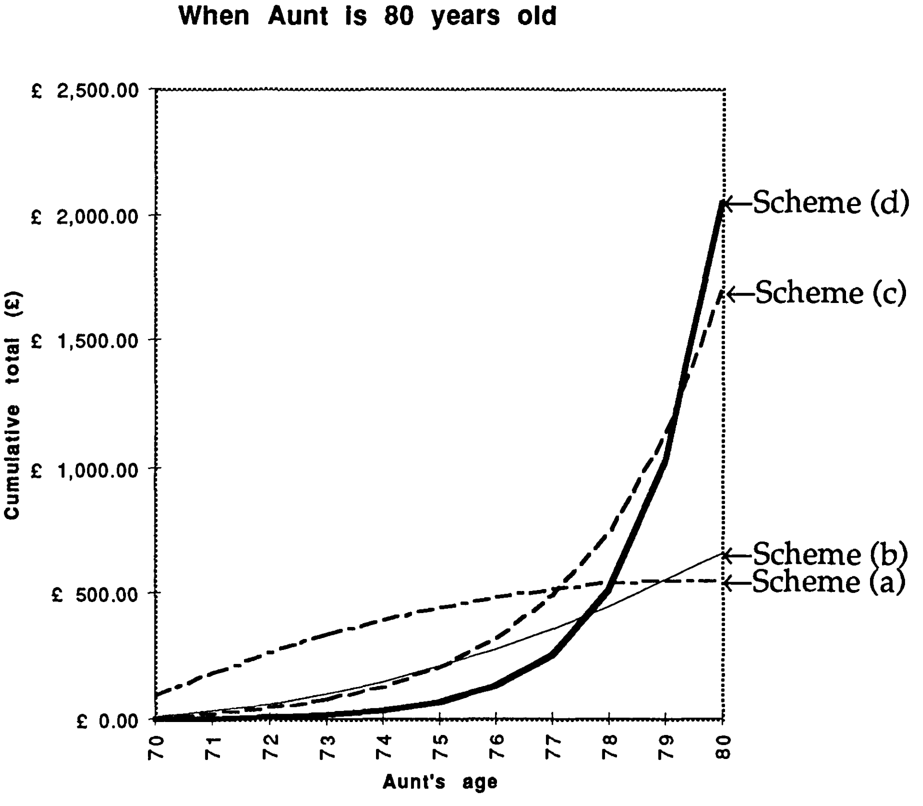
What happens if Aunt Lucy lives beyond 80?

When Aunt Lucy reaches 81 will you continue to receive £0.00, or will you have to give Aunt Lucy £10.00?

continued/

1425 A Rich Aunt (cont)

This graph shows all the cumulative totals that each scheme will generate up to the time when Aunt Lucy reaches 80 years of age.



The scheme you choose will depend on how long you think Aunt Lucy is likely to live. If you compare the cumulative totals each year, you will see that, although scheme (c) and scheme (d) start off slowly, they accumulate rapidly in years to come. Scheme (c) overtakes (a) and (b) after about 7 years. Scheme (d) overtakes them all after 10 years.

It might therefore be wise to choose scheme (d) and to wish Aunt Lucy a long and healthy retirement!

1426 Decimal Lines

1. 1.6
2. 3.8
3. 0.6
4. 6.3
5. 0.2
6. 3.1
7. 1.6 + 1 = 2.6
8. 1 + 1.3 = 2.3
9. 1.7 + 1.2 = 2.9
10. 2.2 + 0.4 = 2.6
11. 0.9 + 1.6 = 2.5
12. 2.5 + 0.5 = 3
13. 0.9 + 0.4 + 0.8 = 2.1

continued/

1426 Decimal Lines (cont)

14. 1.6 + 0.3 = 1.9

15. 0.7 + 2.1 = 2.8

16. 1.4 + 0.6 = 2

17. 1.3 + 1.7 = 3

18. 2.4 + 1.7 = 4.1

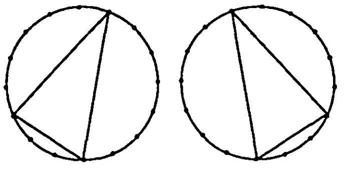
19. 1.5 + 2.8 = 4.3

20. 3.3. + 1.8 = 5.1

21. 0.6 + 0.8 + 0.7 = 2.1

1427 Triangles in Circles

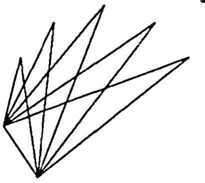
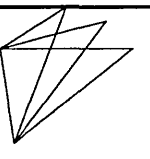
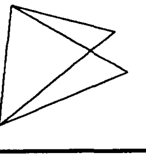
Because triangles like these are congruent (identical in shape and size) there are surprisingly few different triangles which can be drawn.



This mapping shows the results for the investigation up to 7-point circles:

No. of points on circle		No. of different triangles
3	→	1
4	→	1
5	→	2
6	→	3
7	→	4

However, the sequence is not so simple as it seems.
For example, in an 11-point circle there are 10 different triangles possible:

		Total triangles
Different triangles with shortest side of length 1.		5
Different triangles with shortest side of length 2.		3
Different triangles with shortest side of length 3.		2
Total number of different triangles	5 + 3 + 2 =	10

continued/

1427 Triangles in Circles (cont)

In a 16-point circle there are 7 + 6 + 4 + 3 + 1 = 19 different triangles

This suggests that for a 12-point circle the number of different triangles could be
5 + 4 + 3 + 1 = 13 *or* 5 + 4 + 2 + 1 = 12 *or* 5 + 4 + 2 = 11

- Can you decide which it will be?
- Is your prediction correct?
- Does this lead to any generalisations?

A more fruitful way to show the relationship is:

No. of points on circle		No. of triangles
9	→	4 + 2 + 1
10	→	4 + 3 + 1
11	→	5 + 3 + 2
12	→	5 + 4 + 2 + 1

You will more readily understand this relationship if you can extend this mapping in both directions, and if you can answer these questions for an *n*-point circle:

- what difference does it make if *n* is even?
- what difference does it make if *n* is odd?
- what difference does it make if *n* is a multiple of 3?

1428 Sum and Product

These are the pairs of numbers up to 30 for which their sum is a factor of their product.

2, 2	6, 6	9, 18	14, 14	20, 30	28, 28
3, 6	6, 12	10,10	15, 30	21, 28	30, 30
4, 4	6, 30	10, 15	16, 16	22, 22	
4, 12	8, 8	12, 12	18, 18	24, 24	
5, 20	8, 24	12, 24	20, 20	26, 26	

In most cases one of the numbers is a multiple of the other. You could investigate which number pairs work when one of the numbers is double the other:

1, 2	Sum = 3;	Product = 2	3 is not a factor of 2
2, 4	Sum = 6;	Product = 8	6 is not a factor of 8
3, 6	Sum = 9;	Product = 18	9 is a factor of 18
4, 8	Sum = 12;	Product = 32	12 is not a factor of 32
5, 10	Sum = 15;	Product =	

Which number pairs work when one of the numbers is:

- treble the other?
- 4 times the other?
- equal to the other?
- . . . ?

continued/

1428 Sum and Product (cont)

If you need to generate higher number pairs, the following computer program will help:

```
10 FOR N = 1 TO 100
20 FOR M = N TO 100
30 S = N+M
40 P = N*M
50 IF P/S = INT(P/S) THEN PRINT N;M
60 NEXT M
70 NEXT N
```

If you have developed a successful, systematic approach for 2 numbers, you might be able to adapt the same approach for 3 numbers. You might also be able to adapt the computer program.

1429 Multiples of 3 and 9

1.

3				
6				
9				
12	→	1 + 2 = 3		
15	→	1 + 5 = 6		
18	→	1 + 8 = 9		
21	→	2 + 1 = 3		
24	→	2 + 4 = 6		
27	→	2 + 7 = 9		
30	→	3 + 0 = 3		
33	→	3 + 3 = 6		
36	→	3 + 6 = 9		
39	→	3 + 9 = 12	→	1 + 2 = 3
42	→	4 + 2 = 6		
45	→	4 + 5 = 9		
48	→	4 + 8 = 12	→	1 + 2 = 3
51	→	5 + 1 = 6		
54	→	5 + 4 = 9		
57	→	5 + 7 = 12	→	1 + 2 = 3
60	→	6 + 0 = 6		
63	→	6 + 3 = 9		
66	→	6 + 6 = 12	→	1 + 2 = 3
69	→	6 + 9 = 15	→	1 + 5 = 6
72	→	7 + 2 = 9		
.		.		
.		.		
.		.		
99	→	9 + 9 = 18	→	1 + 8 = 9

Adding up the digits of a number and if necessary, repeating the process until a single digit is formed is called finding the **digital root**.

By adding the digits to give the digital root you get a pattern which goes 3, 6, 9, 3, 6, 9, . . . The pattern works for multiples of 3 up to 100.

continued/

2.

102	→	$1 + 0 + 2 = 3$		
105	→	$1 + 0 + 5 = 6$		
.		.		
.		.		
.		.		
222	→	$2 + 2 + 2 = 6$		
225	→	$2 + 2 + 5 = 9$		
228	→	$2 + 2 + 8 = 12$	→	$1 + 2 = 3$
231	→	$2 + 3 + 1 = 6$		

- Yes, the pattern still works.

3.

223	→	$2 + 2 + 3 = 7$
224	→	$2 + 2 + 4 = 8$

- The pattern does not work unless the number is a multiple of 3.

4.

9				
18	→	$1 + 8 = 9$		
27	→	$2 + 7 = 9$		
36	→	$3 + 6 = 9$		
.				
.				
.				
99	→	$9 + 9 = 18$	→	$1 + 8 = 9$
108	→	$1 + 0 + 8 = 9$		
117	→	$1 + 1 + 7 = 9$		
126	→	$1 + 2 + 6 = 9$		
.				
.				
.				
747	→	$7 + 4 + 7 = 18$	→	$1 + 8 = 9$
756	→	$7 + 5 + 6 = 18$	→	$1 + 8 = 9$
765	→	$7 + 6 + 5 = 18$	→	$1 + 8 = 9$

The digital root for the multiples of 9 gives the pattern 9, 9, 9, . . .

5. $2 + 9 + 7 + 1 + 1 + 4 + 2 + 3 + 6 = 35 \rightarrow \quad 3 + 5 = 8$
297 114 236 is **not** a multiple of 3 because its digital root is not a multiple of 3.

6. $6 + 7 + 4 + 2 + 1 + 5 + 0 + 2 = 27 \rightarrow \quad 2 + 7 = 9$
67 421 502 is a multiple of 9 because its digital root is a multiple of 9.

7. If the digital root of your three numbers did not give you a multiple of 3, then show your work to your teacher.

8. If the digital root of your numbers did not give you a multiple of 9, then show your work to your teacher.

1430 Bounce

This is a game of luck, rather than skill.

- How many times did you bounce on 10?
- How many times did you bounce back?

1432 Triangle Patterns

1. a)

1 x 1	=	1
11 x 11	=	121
111 x 111	=	12321
1111 x 1111	=	1234321

b) There are several patterns which will help you continue the pattern, without using a calculator.

- The number of digits is always odd.
- The number of digits increases by two each time.
- The centre digit increases by 1 each time.
- The centre digit is the same as the number of ones in the first number.
- Each row is a palindromic number.
- Each number starts and ends with a 1.
- The digits increase by one until the centre digit is reached.
- The sum of the digits is a square number.

c) The next number will have a 5 in the middle.

1 2 3 4 5 4 3 2 1
1 2 3 4 5 6 5 4 3 2 1
1 2 3 4 5 6 7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 9 8 7 6 5 4 3 2 1

d) Most calculators can only display numbers with 8 digits or less, so in order to display the answer to 11111 x 11111, a calculator will show the answer in standard form.
123454321 is displayed as [1.2345 08] so it is hard to check your answers accurately.
A spreadsheet allows for more numbers to be displayed.

e) The first number has 10 digits, but you cannot have 10 in the middle as 10 has two digits.
The answer is 1 2 3 4 5 6 7 0 0 9 8 7 6 5 4 3 2 1 and is no longer a palindrome.
Can you explain this answer?

continued/

1432 Triangle Patterns (cont)

2. a) This pattern gives

9
1089
110889
11108889

b) This pattern gives

10
1100
111000
11110000

c)

9
108
1107
11106
111105

d)

99
1188
12177
122166
1222155

e)

8
96
984
9872
98760
987648
9876536

f)

42
4422
444222
44442222
4444422222

3. Show your own patterns to your teacher.
How many ways could you describe them?

1433 Base $^{-}2$

A good start to this investigation is to build up a list of numbers in Base $^{-}2$

$(^-2)^5$	$(^-2)^4$	$(^-2)^3$	$(^-2)^2$	$(-2)^1$	$(-2)^0$	Base Ten
-32	16	-8	4	-2	1	Number
					1	1
			1	1	0	2
			1	1	1	3
			1	0	0	4
			1	0	1	5
	1	1	0	1	0	6
	1	1	0	1	1	7
	1	1	0	0	0	8
	1	1	0	0	1	9
	1	1	1	1	0	10
	1	1	1	1	1	11
	1	1	1	0	0	12
	1	1	1	0	1	13
	1	0	0	1	0	14
	1	0	0	1	1	15
	1	0	0	0	0	16
	1	0	0	0	1	17

This is sufficient to see patterns of 0's and 1's in the columns.
Significant numbers are 1, 5, 21, 85 . . .Why?

continued/

1433 Base $^{-}2$ (cont)

- Changing from a base $^{-}2$ representation to a base 10 number is not too difficult if you remember the column headings.
  e.g. 1101101_{-two} is equivalent to $64 - 32 - 8 + 4 + 1$ and so it is 29_{ten}

- Changing from a base 10 number to a base $^{-}2$ representation is more difficult.
  e.g. to translate 27_{ten} which is in the range 22 to 85, it is necessary to choose the power of $^{-}2$ which corresponds to that range . . . $(^{-}2)^6$ or 64.
  Choosing smaller powers of $^{-}2$ will allow you to obtain the correct combination for 27_{ten} . . . $64 - 32 - 8 + 4 - 2 + 1$
  Hence, 27_{ten} is equivalent to 1101111_{-two}

Addition

In any addition in base $^{-}2$, you will need to recognise that the sum of 1 and 11 is zero.

So in the sum of

	1	1	0
+	1	1	1

this part is zero

	1	1	0
+	1	1	1

and the sum is

	1	1	0
+	1	1	1
	1	0	1

In

	1	1	0	1	1
+	1	1	1	0	1
					0
				11	

there is a second group of zero which the first column addition produces.
As in ordinary addition of base 10, there are several techniques which you can use.

The latter example could also be 'cancelled' in this way:

	1	1	0	1	1
+	1	1	1	0	1
	1	0	1	0	0

Subtraction

Since 11 is equivalent to -1, any subtraction of the form 0 – 1 will be equivalent to adding 11.

e.g.

	1	1	0	0	0
–					1
	1	1	0	1	1
				1	

and

	1	1	0	0	0
–			1	0	1
			1	1	1
		1		1	

Multiplication

Multiplications are reasonably straightforward because using the long multiplication method, the problem is reduced to an addition.

e.g.

		1	1	0
		× 1	1	0
1	1	0	0	0
	1	1	0	0
		1	0	0

so $110^{110} = 100$

continued/

1433 Base $^{-}2$ (cont)

Division

This follows traditional base 10 long-division

e.g.

$$\begin{array}{r|rrrrr} & & & 1 & 0 & 1 \\ \hline 111 & 1 & 0 & 0 & 1 & 1 \\ & -1 & 1 & 1 & & \\ \hline & & & 1 & 1 & 1 \\ & & & -1 & 1 & 1 \\ \hline & & & & & 0 \end{array}$$

except that divisions like 11010 ÷ 110 present a problem. *Why*?

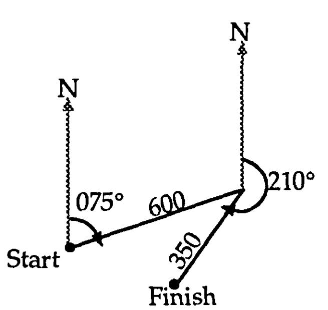
You might like to think about these.

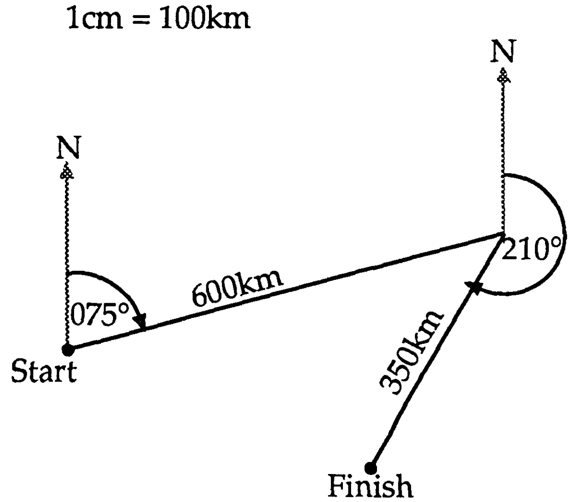
- What is the difference between base $^{-}2$ numbers which have an even number of digits, and those which have an odd number of digits?
- At what stages do the number of digits increase when counting in base $^{-}2$?

1434 Bearings and Scale Drawing

1. Your answers may vary slightly. If they are very different check with your teacher.
   a) 270km on a bearing of 140°.
   b) 440km on a bearing of 320° followed by 390km on a bearing of 252°.
   c) 60km on a bearing of 229° followed by 505km on a bearing of 087°.
   d) 350km on a bearing of 343° followed by 465km on a bearing of 122°.
   e) 400km on a bearing of 037° followed by 420km on a bearing of 207°.
   f) 270km on a bearing of 090° followed by 530km on a bearing of 270°.
   g) 270km on a bearing of 043° followed by 390km on a bearing of 084° followed by 410km on a bearing of 150°.
   h) 250km on a bearing of 360° followed by 250km on a bearing of 090° followed by 250km on a bearing of 180°.

2. Ask someone else to check your scale drawings. They should look similar to these.
   a) Rough sketch — Scale drawing 1cm = 100km





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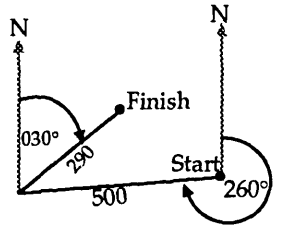
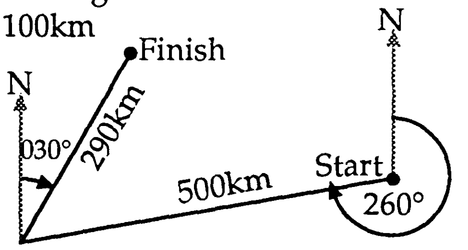
1434 Bearings and Scale Drawing (cont)

2. b) Rough sketch



Scale drawing
1cm = 100km



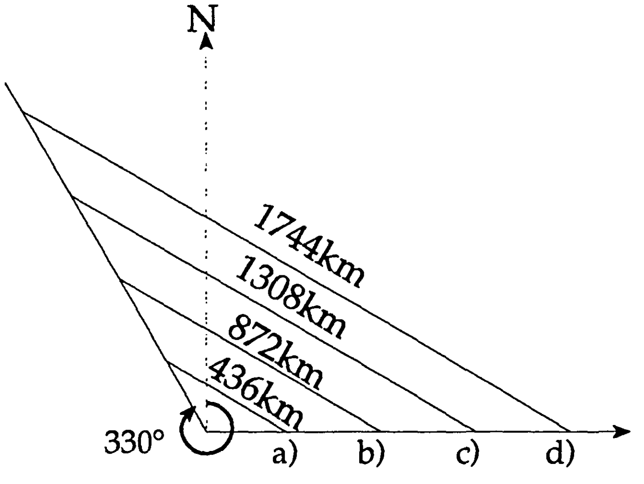
1435 Back Bearings

1. 280°

2. 300°

3. 82km on a bearing of 088°.

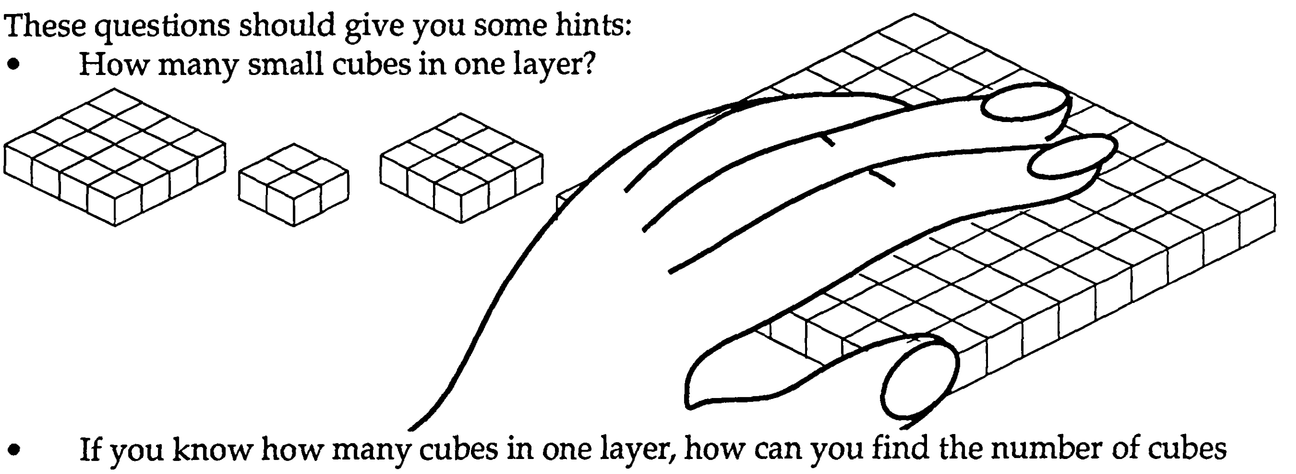
4.



1436 Block Problems

These questions should give you some hints:

- How many small cubes in one layer?



- If you know how many cubes in one layer, how can you find the number of cubes in each block?

1437 Four Consecutive Numbers

$(1 \times 2 \times 3 \times 4) + 1 = 25 \quad \rightarrow \quad 5^2$
$(2 \times 3 \times 4 \times 5) + 1 = 121 \quad \rightarrow \quad 11^2$
$(3 \times 4 \times 5 \times 6) + 1 = 361 \quad \rightarrow \quad 19^2$
$(4 \times 5 \times 6 \times 7) + 1 = 841 \quad \rightarrow \quad 29^2$
. . . .
. . . .
. . . .

- The process always gives a square number.

- The square root of the square number is always one more than the product of the first and last numbers (and one less than the product of the middle pair).
  e.g. $(9 \times 10 \times 11 \times 12) + 1 =$
  $[(9 \times 12) + 1]^2 = 109^2$ or
  $[(10 \times 11) - 1]^2 = 109^2$
- This suggests the generalisation, "if you multiply any 4 consecutive numbers $n(n + 1)(n + 2)(n + 3)$ and add one, the result will always be the square of $n(n + 3) + 1$ or $(n+1)(n+2) - 1$".

To prove the generalisation $n(n + 1)(n + 2)(n + 3) + 1 = [n(n + 3) + 1]^2$ or $[(n + 1)(n + 2) - 1]^2$
First look at the left hand side. $n(n + 1)(n + 2)(n + 3) + 1$
Multiplying out $n(n + 1)(n^2 + 5n + 6) + 1$
$n(n^3 + 6n^2 + 11n + 6) + 1$
$n^4 + 6n^3 + 11n^2 + 6n + 1$

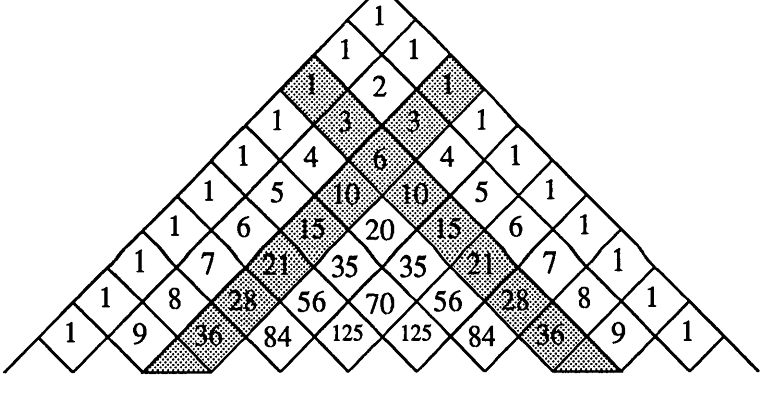
Then look at the right hand side. $[n(n + 3) + 1]^2$ or $[(n + 1)(n + 2) - 1]^2$
Multiplying out $[n^2 + 3n + 1]^2$ $[n^2 + 3n + 6 - 1]^2$
$n^4 + 6n^3 + 11n^2 + 6n + 1$ $n^4 + 6n^3 + 11n^2 + 6n + 1$

The three expressions are equal therefore we have proved that the left-hand side and the right hand side are equal. Our theory was correct. It will be true for any value of n.

1438 Patterns in Pascal's Triangle

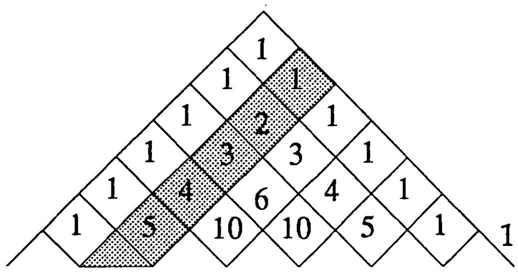
1. ... 91, 364, 1001, ... 1001, 364, 91 ...

2. The triangle numbers appear in a sequence along two lines.



continued/

3. The totals of each row give the sequence 1, 2, 4, 8, 16, 32, 64, ... which are the powers of 2. The power of 2 is the same as the second number in each row.



	1	=	2^0
$1 + 1 =$	2	=	2^1
$1 + 2 + 1 =$	4	=	2^2
$1 + 3 + 3 + 1 =$	8	=	2^3
$1 + 4 + 6 + 4 + 1 =$	16	=	2^4
$1 + 5 + 10 + 10 + 5 + 1 =$	32	=	2^5

4. In the row beginning 1, 7, 21, 35 . . . the numbers (except 1) are all multiples of 7. In the row beginning 1, 5, 10 . . . the numbers (except 1) are all multiples of 5. This property occurs in rows 3, 5, 7, 9, 11 . . . the odd numbers.

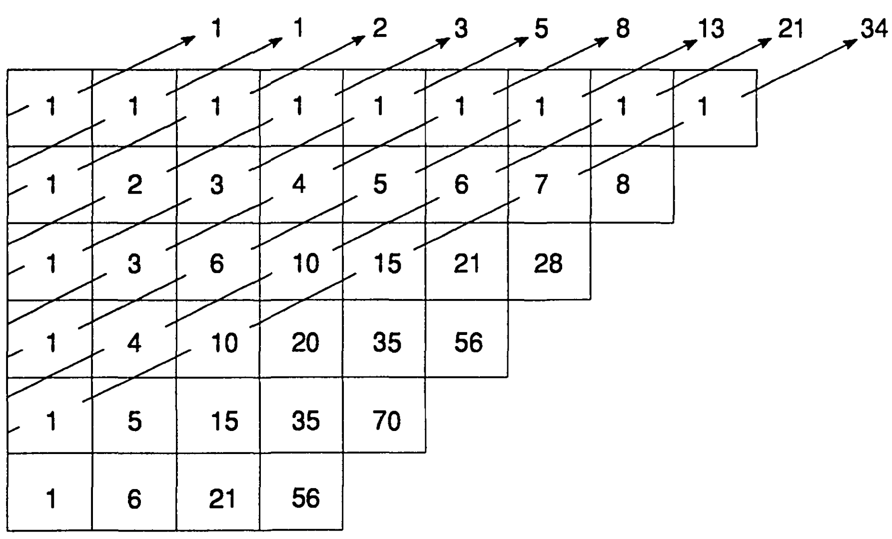
5. $11^2 = 121$
$11^3 = 1331$
$11^4 = 14641$
The power of 11 is the same as the second number in each row. You would need to carry the tens digit over into the preceding space
i.e. The 6th row is 1, 6, 15, 20, 15, 6, 1 and $11^6 = 1771561$
1 7 7 1 5 6 1

6. The previous diagram gives a useful ways of generating rows of larger numbers which would otherwise be very laborious to reach.
So the row beginning 1, 100 . . . will be:

$$1 \quad \frac{100}{1} \quad \frac{100 \times 99}{1 \times 2} \quad \frac{100 \times 99 \times 98}{1 \times 2 \times 3} \quad \ldots$$

$$1, \quad 100, \quad 4950, \quad 161700 \quad \ldots$$

7. The totals of the lines which are picked out in the last diagram give the Fibonacci sequence. A ruler will help you to pick out the appropriate numbers:



1439 Geometric Progressions

1. a) $S = 2^0 + 2^1 + 2^2 + 2^3 + \ldots 2^{14}$

Multiply both sides by 2.

$2S = 2^1 + 2^2 + 2^3 + 2^4 + \ldots 2^{15}$

Subtract the first equation from the second.

$2S - S = (2^1 + 2^2 + 2^3 + 2^4 + \ldots 2^{15}) - (2^0 + 2^1 + 2^2 + 2^3 + \ldots 2^{14})$

$S = 2^{15} - 2^0$

$S = 2^{15} - 1$

b) $S = 3^0 + 3^1 + 3^2 + 3^3 + \ldots 3^{15}$

$3S = 3^1 + 3^2 + 3^3 + 3^4 + \ldots 3^{16}$

$3S - S = (3^1 + 3^2 + 3^3 + 3^4 + \ldots 3^{16}) - (3^0 + 3^1 + 3^2 + 3^3 + \ldots 3^{15})$

$2S = 3^{16} - 3^0$

$= \dfrac{3^{16} - 1}{2}$

c) $S = 4^0 + 4^1 + 4^2 + \ldots 4^{15}$

$4S = 4^1 + 4^2 + 4^3 + \ldots 4^{16}$

$4S - S = 4^{16} - 4^0$

$3S = 4^{16} - 1$

$S = \dfrac{4^{16} - 1}{3}$

d) $S = \dfrac{5^{17} - 1}{4}$

2. a) The series in 1(d) was $5^0 + 5^1 + 5^2 + 5^3 + \ldots$

The series $2 + 10 + 50 + 250 + \ldots$ is twice as large because it can be written as:

$2 + 2(5^1) + 2(5^2) + 2(5^3) + \ldots$

The sum of the series for 17 terms will be double the sum of the series in 1(d).

The sum is $\dfrac{2(5^{17} - 1)}{4}$ which is $\dfrac{5^{17} - 1}{2}$

b) The series is 3 times the sequence in 1(a) because $3 + 6 + 12 + 24 + 48 \ldots$ can be written as $3 + 3(2^1) + 3(2^2) + 3(2^3) + 3(2^4) + \ldots$

The sum of the series for 16 terms will be 3 times that of 1(a)

The sum is $3(2^{16} - 1)$.

c) $2 + 6 + 18 + 54 + 162 + \ldots$ can be written as $2 + 2(3^1) + 2(3^2) + 2(3^3) + 2(3^4) \ldots$

The sum of the series for 20 terms is $\dfrac{2(3^{20} - 1)}{2}$ which is $3^{20} - 1$

3. In question 2(a) the series was written as $2 + 2(5^1) + 2(5^2) + 2(5^3) + \ldots$

Comparing this with $a + ar^1 + ar^2 + ar^3 + \ldots$ you can see that $a = 2$ and $r = 5$.

In question 2(b) $a = 3$ and $r = 2$.

In question 2(c) $a = 2$ and $r = 3$.

4. a) The sixth term is ar^5

b) The nth term is ar^{n-1}

continued/

1439 Geometric Progressions (cont)

5. a) $2(4^{14})$
   b) The sum of the series is $\frac{2(4^{15} - 1)}{3}$

6. a) $5 + 10 + 20 + 40 + \ldots = 5 + 5(2^1) + 5(2^2) + 5(2^3) + \ldots + 5(2^{19})$
   $= 5(2^{20} - 1)$

   b) $1 + r^1 + r^2 + r^3 + \ldots + r^9 = \frac{r^{10} - 1}{(r - 1)}$

   c) $1 + r^1 + r^2 + r^3 + \ldots + r^{n-1} = \frac{r^n - 1}{(r - 1)}$

   d) $a + ar^1 + ar^2 + ar^3 + \ldots + ar^{n-1} = \frac{a(r^n - 1)}{(r - 1)}$

1440 Locating the depot

The total distance travelled each day if the depot is built on the corner of Third Street and Third Avenue is 42km.

Distance from Shop A	→ 4km	→	Total distance	→	8km
Distance from Shop B	→ 5km	→	Total distance	→	10km
Distance from Shop C	→ 3km	→	Total distance	→	6km
Distance from Shop D	→ 3km	→	Total distance	→	6km
Distance from Shop E	→ 6km	→	Total distance	→	12km

The shortest total distance travelled each day is 30km. There are three possible sites:

- on the corner of Fifth Street and Third Avenue,

Distance from Shop A	→ 6km	→	Total distance	→	12km
Distance from Shop B	→ 3km	→	Total distance	→	6km
Distance from Shop C	→ 1km	→	Total distance	→	2km
Distance from Shop D	→ 1km	→	Total distance	→	2km
Distance from Shop E	→ 4km	→	Total distance	→	8km, or

- on the corner of Highway One and Fourth Avenue, or
- on the corner of Highway One and Second Avenue.

Shops on the same street

For this investigation it helps to use co-ordinates to define the positions of a single point.

- Distances W↔E are the x co-ordinate.
- Distances N↔S are the y co-ordinate.

The co-ordinates of Shop A could be (1, 2) and Shop B could be (5, 1), so the best place to build the depot would be (3, 2).

Work systematically, next with 3 shops, then 4 shops . . .

Shops on different streets

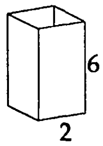
Once again you need to work systematically so that you can see a pattern in order to answer the questions on page 3.

1441 Max Box

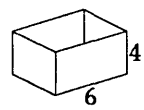
Where do you start?

A quick check with a sheet of card 14cm x 14cm will give a good estimate of desirable sizes:

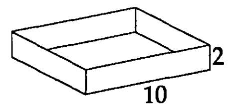
- Cut out a 6 x 6 . . . . . . 4 x 4 . . . . . . 2 x 2 . . . . . . 1 x 1 . . .
  square at each corner.



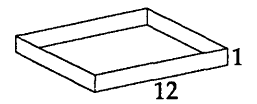
volume = 24cm^3



volume = 144cm^3



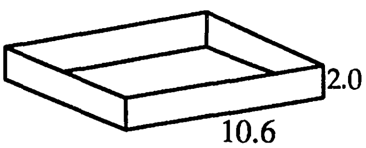
volume = 200cm^3



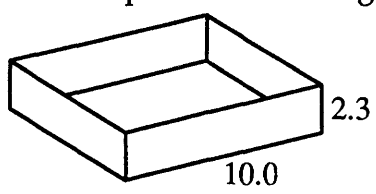
volume = 144cm^3

Narrowing down the possibilities

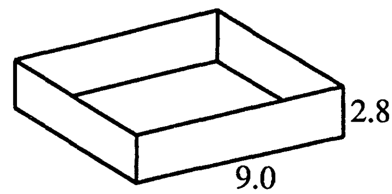
The largest volume so far, starting with a 14cm square is when 2cm x 2cm squares are cut out.
It is now sensible to use the 14.6cm square and investigate cut-outs close to 2cm x 2cm.



Volume = 224.72cm^3



Volume = 230cm^3



Volume = 226cm^3

It seems that the maximum volume will be when the sides are between 2cm and 2.8cm.

Using a spreadsheet

A spreadsheet can be used to generate a closer approximation to the maximum volume.
Here is part of a spreadsheet.

	A	B	C	D	E
1	Size of cut-out square	Length of box (cm)	Width of box (cm)	Height of box (cm)	Volume of box (cm^3)
2	2	10.6	10.6	2	224.72
3	2.1	10.4	10.4	2.1	227.136
4	2.2	10.2	10.2	2.2	228.888
5	2.3	10	10	2.3	230
6	2.4	9.8	9.8	2.4	230.496
7	2.5	9.6	9.6	2.5	230.4
8	2.6	9.4	9.4	2.6	229.736
9	2.7	9.2	9.2	2.7	228.528
10	2.8	9	9	2.8	226.8

The maximum volume will be when the sides are between 2.4cm and 2.5cm.

This is the next part of the spreadsheet.

	A	B	C	D	E
1	Size of cut-out square	Length of box (cm)	Width of box (cm)	Height of box (cm)	Volume of box (cm^3)
2	2.4	9.8	9.8	2.4	230.496
3	2.41	9.78	9.78	2.41	230.5126
4	2.42	9.76	9.76	2.42	230.5234
5	2.43	9.74	9.74	2.43	230.5283
6	2.44	9.72	9.72	2.44	230.5273
7	2.45	9.7	9.7	2.45	230.5205
8	2.46	9.68	9.68	2.46	230.5079
9	2.47	9.66	9.66	2.47	230.4895
10	2.48	9.64	9.64	2.48	230.4654

Continuing with this process will allow you to find that a maximum volume of 230.52859cm^3 when squares of 2.4334cm are removed from the corners of the 14.6cm square card.

1442 Nearly but not quite

The more terms in the series $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots$ that you take, the closer the sum gets to $\frac{1}{2}$.

$\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} \ldots$ gets closer and closer to $\frac{1}{3}$.

$\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} \ldots$ gets closer and closer to $\frac{1}{4}$.

The more terms you take in the series $\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} \ldots$ the closer the sum gets to $\frac{1}{x-1}$.

1443 π

- The Chinese approximation gave the lowest value for π in this list.
- The Greek approximation $\sqrt{10}$, gave the largest value of π in this list. The Greeks were unable to calculate $\sqrt{10}$ accurately two thousand years ago. They used Archimedes' calculation of 'between $3\frac{1}{7}$ and $3\frac{10}{71}$'.
- The value which is closest to that calculated by a computer is the first approximation, which was made by the Chinese mathematican Tsu Ch'ung-chih. He lived 430 - 501AD. The fraction $\frac{355}{113}$ shows him to have been an expert mathematician because this value is accurate to six decimal places - an accuracy of 1 in a million.

1444 Stars

- Make a display of your stars and write about any discoveries you have made.

1445 Flexagons

Further investigations with flexing shapes are given in
- SMILE 0145 Tetraflexagons
- Mathematical Curiosities 1 by Jenkins and Wild ISBN 0906212 138
- Mathematical Curiosities 2 by Jenkins and Wild ISBN 0906212 146
- Mathematical Curiosities 3 by Jenkins and Wild ISBN 0906212 251

1446 Knots

You need to be systematic with this investigation as the order in which the folds are completed gives different results.

e.g. VMVM gives a loop which has 2 complete twists.
VVMM gives a loop with no twists.

VVV	→	$\frac{1}{2}$ twist	VVVV	→	0 twists	VVVVV	→	$\frac{1}{2}$ twist
VVM	→	$\frac{1}{2}$ twist	VVVM	→	1 twist			
			VVMM	→	0 twists			
			VMVM	→	2 twists			

Some useful questions to ask at this stage are:

- What is the difference between VVM and VVV?
- Is there any difference between VVVM and MVMM?
- Would VVVVVM give 1 twist or $\frac{1}{2}$ twist?

1447 Deltahedra

If you enjoyed making solids, you will be interested in the 5 perfect solids, sometimes called the Five Platonic Solids, which are shown on SMILE 1354 Euler Solids.

You may also be interested to use

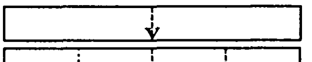
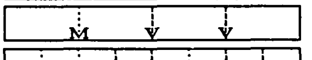
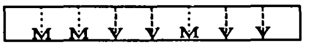
- Make Shapes Book 1 ISBN 0906212 006
- Make Shapes Book 2 ISBN 0906212 014
- Make Shapes Book 3 ISBN 0906212 022

which contain the nets for both simple and complicated solids.

Another further source is

- Mathematical Models by Cundy and Rollett ISBN 0906212 200.

1448 Folding a Strip

Folding	once	→		→	1 crease
	twice	→		→	3 creases
	three times	→		→	7 creases
	n times	→		→	$2^n - 1$ creases.

After n folds, the right half of the strip has the same creases as when the strip was folded $(n - 1)$ times.

e.g. With four folds, the right hand side will be identical to the strip with 3 folds.

The centre crease is always a valley fold.

The left half is the opposite of the right half.

So the strip will look like this →

→ 15 creases

continued/

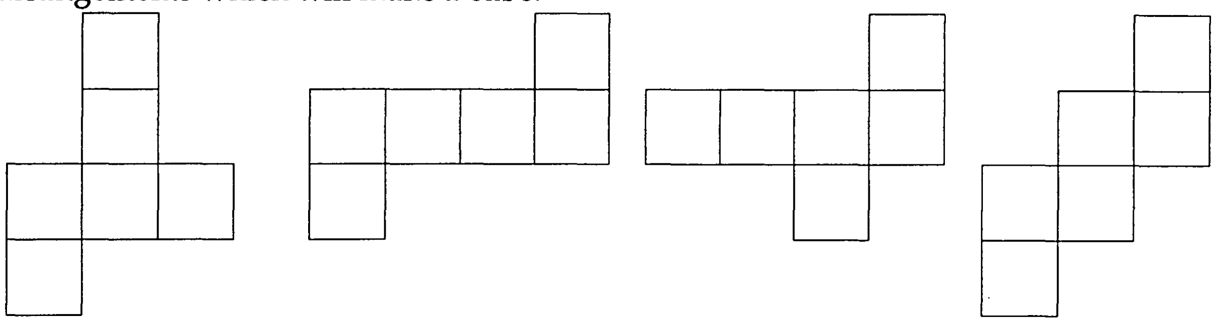
1448 Folding a Strip (cont)

Drawing

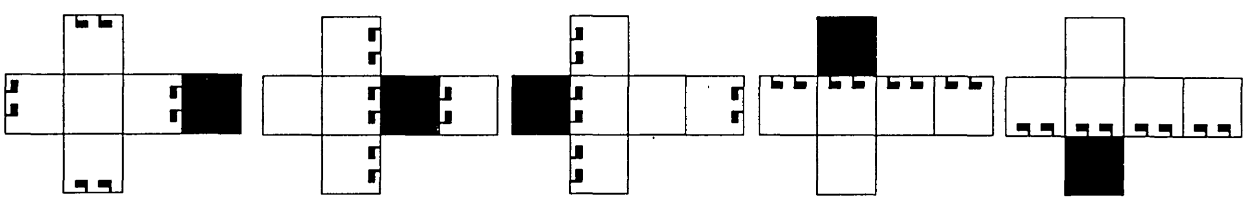
In transferring your folding results to paper, remember that the right hand side of the strip is the same as the whole of the previous strip.
By using scissors and overlapping drawings, you might save yourself some work and make some discoveries.

1449 Nets

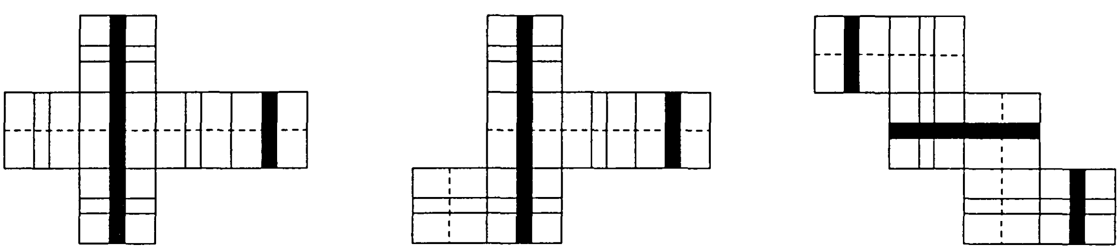
There are many possible answers, because each of these nets are just some of the arrangements which will make a cube.



For the cruciform shape the black face and flags could be arranged as follows.



Here are three possible arrangements of lines which will give the same picture as the cube on page 5.

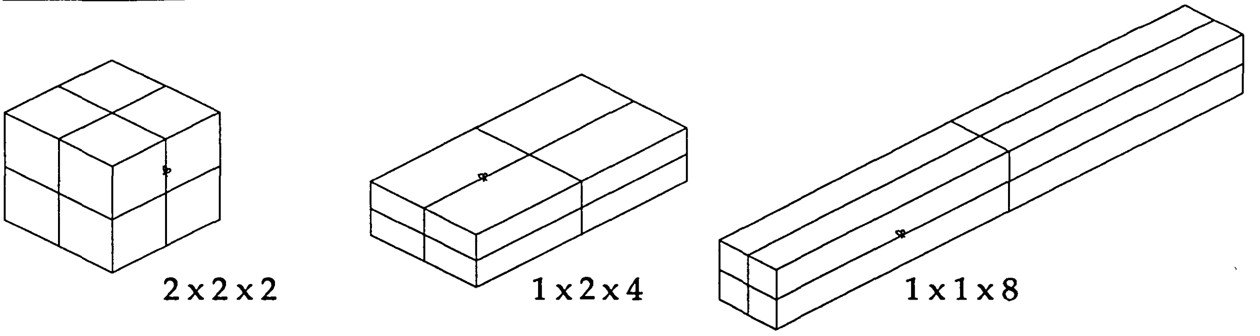


1450 Cuboids

You should be able to find a connection between the set of 3 numbers describing the box shape and the total.

e.g. (2, 3, 4) $\rightarrow$ 24
(2, 4, 5) $\rightarrow$ 40
(1, 1, 5) $\rightarrow$ 5

1451 Parcels



2 x 2 x 2 1 x 2 x 4 1 x 1 x 8

Wrapping

1. 24 squares 2. 28 squares 3. 34 squares

String

1. 24 edges 2. 28 edges 3. 40 edges

Larger parcels with 36 cubes	Wrapping	String
1 x 1 x 36	146	152
1 x 2 x 18	112	84
1 x 3 x 12	102	64
1 x 4 x 9	98	56
1 x 6 x 6	96	52
2 x 2 x 9	80	52
2 x 3 x 6	72	44
3 x 3 x 4	66	40

1452 Arrangements

Several arrangements are possible, but they are difficult.

1454 ISBN's and Errors

1. 0140057144 and 0298705576 are wrong.

2. a) 9
   b) 6
   c) 0

3. a) 10
   b) X is the Roman numeral for 10 and it can be used as a single digit.

4. a) Transposition error
   b) Random error
   c) Transcription error
   d) Double Transposition error

continued/

1454 ISBN's and Errors (cont)

5.

		Remainder	Will weighted modulo 11 test detect error?
Correct number	0 85985 051X	0	–
Transcription error	0 85985 057X	1	Yes
Transposition error			Yes
Double trans.error			Yes
Random error			Not necessarily

Many possible answers

6.

		Remainder	Will weighted modulo 11 test detect error?
Correct number	0453192132	0	–
Transcription error			Yes
Transposition error			Yes
Double transp. error			Yes
Random error			Not necessarily

Many possible answers

7. a) Transcription, Transposition and Double Transposition
   b) Random
   c) Many answers e.g. two transpositions: 085985051X correct number 089585015X

1455 Pinball

1.-4. Many possible answers.

5. 65p

6. Your answers will depend upon your results from question 1.

7. 65p

8. Most people perhaps thought that the amount you could win (10p and 15p) was not worth 10p a go.

9. About twice as much.

   Discuss how you would set up the prizes with your teacher.

1456 Matrices for Rotations

1. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

2. If the matrix did not give you the same co-ordinates as your drawing, check your results with your teacher.

3. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix}$

4. a) $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \end{pmatrix}$

   b) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}$

5. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

   a) A rotation of 180°
   b) Because a rotation of 90° followed by a rotation of 90° is equal to a rotation of 180°.

6. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

7. You would get the identity matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

8. Your results of multiplying other pairs of matrices together should demonstrate the combined effect of rotations.

1457 Combining Rotations

$$\begin{pmatrix} 0.94 & -0.34 \\ 0.34 & 0.94 \end{pmatrix}\begin{pmatrix} 0.34 & -0.94 \\ 0.94 & 0.34 \end{pmatrix} = \begin{pmatrix} 0 & -0.9992 \\ 0.9992 & 0 \end{pmatrix} \approx \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

The answer is always approximately $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

With pairs of angles which add up to 180° the answer = $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

With pairs of angles which add up to 270° the answer = $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

With pairs of angles which add up to 360° the answer = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

continued/

1457 Combining Rotations (cont)

$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ is the matrix which rotates 90°.

$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ is the matrix which rotates 180°.

$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is the matrix which rotates 270°.

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the matrix which rotates 360°.

1458 Reflection Matrices Investigation

The fact that $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ → reflected in the line $y = 2x$ → $\begin{pmatrix} -\frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$

and

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ → reflected in the line $y = 2x$ → $\begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix}$

suggests that this investigation has something to do with a right-angled triangle with sides 3, 4 and 5.

Similarly $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ → reflected in the line $y = \frac{1}{4}x$ → $\begin{pmatrix} \frac{15}{17} \\ \frac{8}{17} \end{pmatrix}$

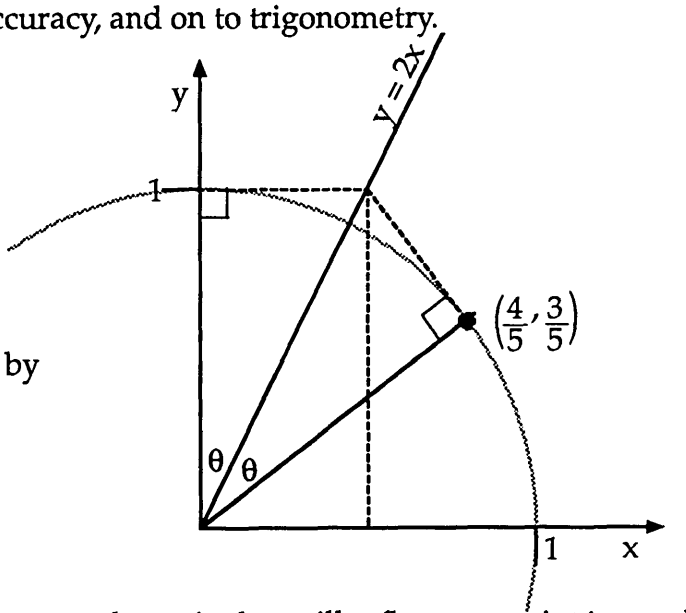
and

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ → reflected in the line $y = \frac{1}{4}x$ → $\begin{pmatrix} \frac{8}{17} \\ -\frac{15}{17} \end{pmatrix}$

suggests something to do with a right-angled triangle with sides 8, 15 and 17.

To continue this investigation, it is necessary to move away from scale drawings, because of the lack of accuracy, and on to trigonometry.

Look at the angle made by the line $y = mx$ and the unit vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$



It will help you to find a general matrix that will reflect any point in any line of the form $y = mx$.

1459 Matrices for Shears Investigation

Any matrix of the form $\begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix}$

produces shears where the points of the shape are shifted parallel to the x axis by m times the y co-ordinate.

$$\begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + my \\ y \end{pmatrix}$$

Any matrix of the form $\begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$

produces shears where the points of the shape are shifted parallel to the y axis by n times the x co-ordinate.

$$\begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ nx + y \end{pmatrix}$$

For investigating shears of other invariant lines it is best to look at y = x first, before moving on to lines of the form y = mx.

1460 Diophantine Problems

A. Original price is 20p. The man sold 14 pens.

B. a) 38
   b) 42
   c) 40

C. 6km per hour

D. They are either all good hens or all bad hen, but they lay 9 eggs each in either case.

1461 Figures for Words

2. 562

3. 0, because you end with 0.

4. a) 261
   b) 999
   c) 803
   d) 4056
   e) 7001
   f) 6090
   g) 5707
   h) 10010

1462 Missing Keys

There are many possible answers. Here are some examples:

1. [7] [−] [3] [−] [3] [=]
2. [3] [×] [3] [−] [7] [=]
3. [3] [×] [3] [−] [3] [−] [3] [=]
4. [7] [−] [3] [=]
5. [7] [−] [3] [×] [3] [−] [7] [=] *If you have a scientific calculator this will not give you 5.*
6. [3] [×] [3] [−] [3] [=]
7. [7] [=]
8. [3] [×] [7] [−] [3] [−] [7] [−] [3] [=]
9. [3] [×] [3] [=]

1463 Use Brackets

If you are unsure whether your questions are correct, show them to your teacher.

1464 Zero's the Limit

Pressing the number [5] allows the next player to use *all* the numbers.

Pressing the number [9] allows the next player to use large numbers [8] [5] and [6].

1465 Smallest on the Left

Smallest on the left, the fractions are:

$\frac{19}{58}$, C, $\frac{7243}{21586}$, A, $\frac{12}{35}$, D, $\frac{8}{23}$, B, $\frac{214}{607}$

where A, B, C and D are the missing ones that you chose.

1466 Patterns of Nines

This table should be enough to indicate the patterns:

1 x 9 = 09	1 x 99 = 099	1 x 999 = 0999	1 x 9999 = 09999	1 x 99999 = 099999
2 x 9 = 18	2 x 99 = 198	2 x 999 = 1998	2 x 9999 = 19998	2 x 99999 = 199998
3 x 9 = 27	3 x 99 = 297	3 x 999 = 2997	3 x 9999 = 29997	. 299997
4 x 9 = 36	. 396	. 3996	. 39996	.
5 x 9 = 45	. 495	. 4995	. 49995	
6 x 9 = 54	. 594	. 5994		
7 x 9 = 63				
8 x 9 = 72				
9 x 9 = 81	9 x 99 = 891	9 x 999 = 8991	9 x 9999 = 89991	9 x 99999 = 899991

1467 Patterns of Numbers

(1 x 8) + 1 = 9
(12 x 8) + 2 = 98
(123 x 8) + 3 = 987
. .
. .
. .

The explanation comes from the multiples of 8: 16, 24, 32, . . . where the 'tens' digit increases by 1 each time, and the 'units' digit decreases by 2 each time.

$(9 \times 1^2) + 1^2 = 10$
$(9 \times 2^2) + 2^2 = 40$
$(9 \times 3^2) + 3^2 = 90$
. .
. .
. .

This is equivalent to $10 \times x^2$ in each row.

(0 x 9) + 1 = 1
(1 x 9) + 2 = 11
(12 x 9) + 3 = 111
. .
. .
. .

The explanation comes from the multiples of 9: 18, 27, 36, . . . where the sum of the 'units' digits and the following 'tens' digit is always 10 . . .

(0 x 9) + 8 = 8
(9 x 9) + 7 = 88
(98 x 9) + 6 = 888
. .
. .
. .

. . . and, this time where the multiples of 9 are in descending order, the same sum is always 8.

1468 Remainders

$$17 \div 7 = 2.4285714$$

There are two different methods you could use.

Method 1

Multiply the whole number part of the answer (2) by the number you divided by (7). — $2 \times 7 = 14$

Subtract the answer (14) from the number you divided into (17). — $17 - 14 = 3$

The remainder is 3.

Remember that the inverse of division is multiplication.

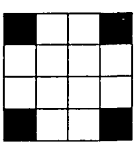
Method 2

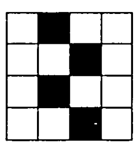
Take the decimal part (0.4285714) and multiply by the number you divided by (7). — $0.4285714 \times 7 = 3$

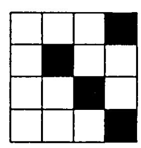
The remainder is 3.

On some calculators the answer is displayed as 2.9999998.
Why is this?

1469 Make a Thousand

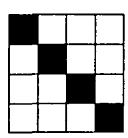
 → 694

 → 1116

 → 1056

The third pattern is nearest to 1000. It is too large by 56.

174 is fifty-six less than 230 so will give 1000 exactly.



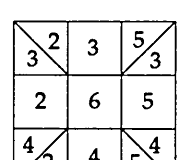
There are several other patterns which give 1000 exactly. How many did you find? Show your 3 x 3 square to your teacher.

1470 Make One

Did you play the game using numbers less than 1, negative numbers, . . . ?

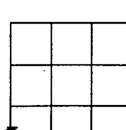
1471 Sixes

- This shows the numbers for all the squares by rolling the dice, starting from the middle square. It is possible to get two numbers in each of the corner squares. Can you see why?



- It is possible to get a 6 in every square:

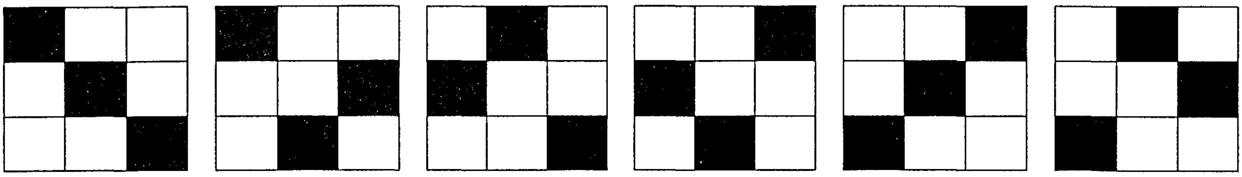
6 moves to get 6 in a corner.



3 moves to get 6 in here

1472 Patterns in Order

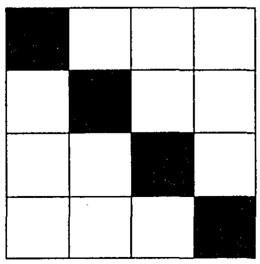
There are 6 different arrangements following the rules on page 1.



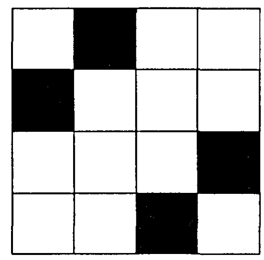
(1, 2, 3) (1, 3, 2) (2, 1, 3) (2, 3, 1) (3, 2, 1) (3, 1, 2)

By using the same recording system for a 4 by 4 square tile.

This is tile (1, 2, 3, 4).



This is tile (2, 1, 4, 3).



1473 Slides to Order

It is always possible to end up with the original order (1, 2, 3, 4, 5, 6, 7, 8).

Show your own puzzle to your teacher.

1474 Different Orders

With three things, it is possible to make 6 different orders.

GUARANTEED USED CARS	1	2	3	A	B	C
GUARANTEED CARS USED	1	3	2	A	C	B
USED CARS GUARANTEED	2	1	3	B	A	C
USED GUARANTEED CARS	2	3	1	B	C	A
CARS GUARANTEED USED	3	1	2	C	A	B
CARS USED GUARANTEED	3	2	1	C	B	A

1. Show your slogan to your teacher. It is possible to make 24 different orders. Did all your 24 different orders make sense?

number of things	1	2	3	4	5	6	. . .
number of orders	1	2	6	24	120	720	. . .

- You may like to make a folder or wall-display to show your results for some of the suggestions on pages 4 -7.

1475 Permutations

Here is one way of describing the permutations of four things. It is important to work methodically

- There are six permutations, with the first digit '1' kept stationary.

1	2	3	4
1	2	4	3
1	3	2	4
1	3	4	2
1	4	2	3
1	4	3	2

- There will be a further six different permutations with '2' kept stationary.

2	1	3	4
2	1	4	3
2	3	1	4...

- There will be a further six different permutations with '3' kept stationary.

3	1	2	4
3	1	4	2...

- There will be a further six different permutations with '4' kept stationary.

4	1	2	3
4	1	3	2...

There is a total of 24 permutations of four things.

1476 Doodles

Make a display of your doodle. Were you able to shade it with just one colour?

1477 Sprouts

What strategy did you use to win. Did it matter who went first?

1478 Zig-Zag

What strategy did you use to win. Did it matter who went first?

1479 Aggression

What strategy did you use to win. Did it matter who went first?

1481 String Knots

There have been many interesting investigations into String Knots. You can read about some of them in "Knots representing numbers", pages 42 - 45 of the Open University booklet Decimal Number Words; Tallies and Knots (ISBN 0 335 05017 4)

1482 Tricky Sum

How did Gauss solve the problem so quickly?

The method he used can be explained by considering

1 + 2 + 3 + 4 + 5 + . . . + 99 + 100.

- The first and last terms add up to 101.
- The second and last-but-one also add up to 101.
- So do the third and last-but-two . . .

1 + 100
2 + 99
3 + 98
4 + 97
. .
. .
. .

- There are 50 pairs which add up to 101, the last pair being 50 + 51.
  The sum of all the numbers from 1 to 100 is 50 times 101; in other words 5050.

For the sum 1 + 2 + 3 + 4 + . . . + 999 + 1000

- How many pairs are there which add up to 1001?

Gauss' method can be adapted for any regular series of numbers like

3 + 5 + 7 + 9 + . . . + 21 + 23 + 25

- By pairing numbers which add to 28 it is possible to find the sum very quickly:

3 + 25
5 + 23
7 + 21 There are 6 pairs, so the sum is 6 x 28; in other words 168.
. .
. .
. .
13 + 15

The method is a powerful tool, even for an odd number of terms.
What would you do in this case?

1483 Largest Product

Many products are possible

e.g. 1 x 2 x 3 x 4 = 24   21 x 34 = 714
2 x 134 = 268   1 x 234 = 234
24 x 13 = 312   21 x 43 = 903 . . .

The largest product using 1, 2, 3, and 4 is 41 x 32 = 1312

- It is difficult to find the largest product using 1, 2, 3, . . . 9 because the answer will not fit on to most calculators.

  You could try finding the largest product using: 1, 2, . . . 5
  1, 2, . . . 6
  1, 2, . . . 7

  until you can see a pattern.

1484 Decimal Patterns

1. 0.2, 0.4, 0.6, 0.8, 1.0, 1.2 . . . — add 0.2 each time
2. $0.1\dot{1}$, $0.2\dot{2}$, $0.3\dot{3}$, $0.4\dot{4}$, $0.5\dot{5}$, $0.6\dot{6}$ . . . — add $0.1\dot{1}$ each time
3. 0.5, 0.5, 0.5 . . . — all equal $\frac{1}{2}$
4. $0.3\dot{3}$, $0.6\dot{6}$, 1.0, $1.3\dot{3}$, $1.6\dot{6}$, 2.0 . . . — increase by $0.3\dot{3}$ *or* add on $\frac{1}{3}$.
5. 0.1, 0.01, 0.001, 0.0001 . . . — divide by 10 each time *or* the 1 moves to the right each time
6. 0.1, 0.2, 0.3, 0.4, 0.5 . . . — add 0.1 each time
7. 0.1, 0.1, 0.1. . . — all equal $\frac{1}{10}$
8. 0, $\dot{0}\dot{9}$, $0.\dot{1}\dot{8}$, $0.\dot{2}\dot{7}$, $0.\dot{3}\dot{6}$, $0.\dot{4}\dot{5}$, $0.\dot{5}\dot{4}$ . . . — each pair of repeated numbers adds to 9 *or* add on $0.\dot{0}\dot{9}$.
9. $0.33\dot{3}$, $0.33\dot{3}$, $0.33\dot{3}$ . . . — all equal $\frac{1}{3}$
10. 0.5, 0.05, 0.005, 0.0005 . . . — 5 moves to the right each time *or* divide by 10 each time
11. 0.2, 0.2, 0.2, . . . — all equal $\frac{1}{5}$
12. Many possible answers.

1485 Limits

1. a) $U_1 = \frac{1}{2}$

   b) $U_4 = \frac{1}{16}$

   c) $\frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \ldots \frac{1}{2^n},$

   d) Each term is half the previous one.

2. a) A spreadsheet will generate the sequence using this formula,

	A
1	1
2	=SQRT(8/A1)

⇓

and filling the formula down the column.

	A
1	1
2	2.82842712
3	1.68179283
4	2.18101547
5	1.91520656
6	2.0437943
7	1.97845603
8	2.0108598
9	1.99459211
10	2.00270944
11	1.99864666
12	2.00067702
13	1.99966158
14	2.00016923
15	1.99991539
16	2.00004231
17	1.99997885
18	2.00001058
19	1.99999471
20	2.00000264

$U_{19} = 1.99999471$
$U_{20} = 2.00000264$
The limit is 2.

   b) By changing the formula in the spreadsheet to;

	A
1	1
2	=SQRT(27/A1)

The limit is 3.

   c) By changing the formula in the spreadsheet to;

	A
1	1
2	=SQRT(125/A1)

The limit is 5.

   d) The limit is the cube root of the number above U_n.

   e) 2.1544345

continued /

1485 Limits (cont)

3. a) By changing the formula in the spreadsheet to;

	A
1	1
2	=SQRT(A1+2)

U13 = →

U14 = →

The limit is 2.

13	1.99999993
14	1.99999998
15	2

b) The limit is the number added to U_n.

4. √5 = 2.2360679.

1486 Threes and Sevens

In any investigation it is most important that you work in a systematic way so that you can see all your results clearly.
Can you see that all possible lengths that can be made from 3-rods and 7-rods would come somewhere in this table?

0	3	6	9	12	15	18	21	24 . . .
7	10	13	16	19	22	25	28	31 . . .
14	17	20	23	36	28	32	35	38 . . .
21	24	27	30	33	36	39	42	45 . . .

The largest impossible length is 11 because all numbers above 11 appear in the table. (In fact all the numbers above 11 appear in the top 3 rows and so the rest of the table is not needed).

Using 3-rods and 7-rods, the impossible lengths are 1, 2, 4, 5, 8 and 11.

You may have collected results for several pairs of rods and so a table will be useful:

	Number of Impossible Lengths					
	1	2	3	4	5	6 . . .
1						
2						
3						
4		∞	3			
5			4			
6						
7			6			

The table shows the results for 3-rods and 7-rods. It also shows that with 4-rods and 3-rods there are 3 impossible lengths and with 5-rods and 3-rods there are 4 impossible lengths. When more of the table is filled in you might see some patterns of number.

- ∞ is a symbol for infinity. Can you see why it has been used for 4-rods and 2-rods?

1487 Thinking in Three Dimensions

- $P \rightarrow Q \rightarrow U \rightarrow V = 13cm$

- $PR^2 = RQ^2 + PQ^2$
  $PR = \sqrt{(9 + 36)} = 6.708cm$ (3dp)

- $P \rightarrow R \rightarrow V = 10.708cm$

- $PV^2 = VR^2 + PR^2$
  $PV = \sqrt{(16 + 6.45)} = 7.810cm$ (3dp)

1. a) (i) $AC = \sqrt{(12^2 + 8^2)} = 14.422cm$ (3dp)
   (ii) $BG = \sqrt{(12^2 + 5^2)} = 13cm$
   (iii) $BE = \sqrt{(8^2 + 5^2)} = 9.434cm$ (3dp)
   b) $BH = \sqrt{(12^2 + 8^2 + 5^2)} = 15.264cm$ (3dp)

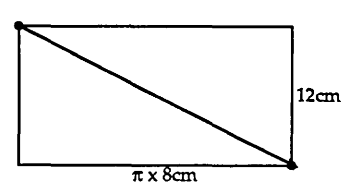
2. The longest distance that could be fitted into the garage is $\sqrt{(5^2 + 3^2 + 3^2)} = 6.557m$.
   A 7m pole will not fit in.

3. a)

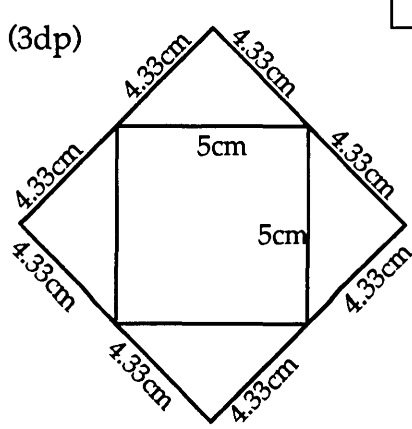
Route	Distance
$S \rightarrow E \rightarrow F$	
$S \rightarrow R \rightarrow F$	20.649cm
$S \rightarrow P \rightarrow Q \rightarrow F$	
$S \rightarrow R \rightarrow Q \rightarrow F$	24cm
$S \rightarrow P \rightarrow E \rightarrow F$	
$S \rightarrow E \rightarrow P \rightarrow Q \rightarrow F$	28.649cm
$S \rightarrow P \rightarrow Q \rightarrow R \rightarrow F$	44.649cm
$S \rightarrow R \rightarrow Q \rightarrow P \rightarrow E \rightarrow F$	40 cm
$S \rightarrow E \rightarrow P \rightarrow Q \rightarrow R \rightarrow F$	49.298cm

   b) $SF^2 = SE^2 + EF^2$
   $SF = \sqrt{(12^2 + 4^2 + 8^2)} = 14.967cm$ (3dp)

4. The length of the label is $\pi \times 8cm = 25.133cm$ (3dp)
   The length of the line is $25.132^2 + 12^2 = 27.851cm$ (3dp)



5. a) $\sqrt{(5^2 + 5^2 + 5^2)} = 8.660cm$ (3dp)
   b) $\frac{8.660}{2} = 4.330cm$ (3dp)



1488 Angles between planes

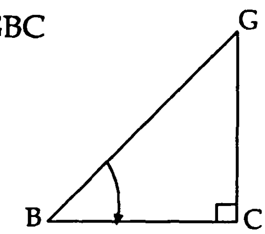
1. a) BC b) BC
   c) BD d) EH

2. a) 90° b) 45°
   c) 90° d) 90°

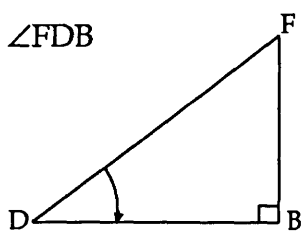
3. a) True b) False c) True d) False

4. a) ∠GBC or ∠HAD b) ∠CGB or ∠DHA
   c) ∠BDA or ∠FHE d) ∠AFE or ∠DGH
   e) ∠AFB or ∠DGC

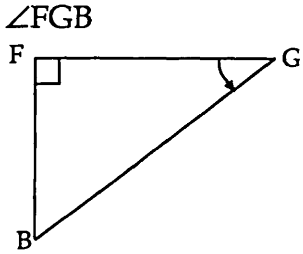
5. a) ∠GBC



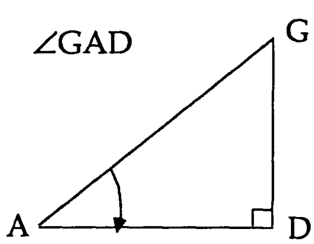
b) ∠FDB



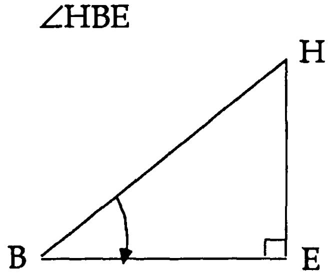
c) ∠FGB



d) ∠GAD



e) ∠HBE



continued/

1488 Angles between planes (cont)

6. a) 7.07cm
   b) 8.66cm
   c) 35.3°
   d) 3.54cm
   e) 45°

7. a) ON
   b) 90°
   c) ∠OMN

8. a) 7.07cm
   b) 3.54cm
   c) 7.18cm
   d) 63.8°
   e) 7.59cm
   f) 70.9°

1500 Subject of a Formula

- When a = 4, b = 6 and h = 7, $A = \frac{1}{2}(a + b)h$
  $A = \frac{1}{2}(4 + 6)7 = 35$

- Substituting a = 4, h = 7 and A = 35 into the formula where b is the subject should give b = 6. Did you find that a = 4 when you used the rearrangement with a as the subject and that h = 7 when you used the rearrangement with h as the subject?

- For $T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$, whatever values you substituted, the rearrangements should agree.

1. $b = \frac{A}{3}$
2. $l = \frac{v}{k}$
3. $t = \frac{v - u}{a}$
4. $x = 2m - y$
5. $v = \sqrt{\frac{Fr}{m}}$
6. $r = \frac{mv^2}{F}$
7. $h = \frac{d^2}{11.5}$
8. $p = \frac{A}{3} - 5$ *or* $p = \frac{A - 15}{3}$
9. $s = \frac{v^2 - u^2}{2a}$
10. $u = \sqrt{(v^2 - 2as)}$
11. $r = \frac{12}{6 - a}$
12. $v = \frac{2s}{t} - u$ *or* $v = \frac{2s - ut}{t}$
13. $c = \frac{144}{T^2}$ when T = 3, c = 16
    when T = 2.4, c = 25
14. $L = \frac{P}{2} - W$ length is 10.7cm
15. $h = \frac{S - 2\pi r^2}{2\pi r}$ height = 4.68cm (to 3 sig. fig.)

1501 Changing the Subject

1. $p = \frac{Z}{3 + q}$
2. $a = \frac{s}{2(c + 2b)}$
3. $h = \frac{d^2}{1 + 3R}$
4. $x = \frac{5}{y - 1}$
5. $m = \frac{2p}{v^2u^2}$
6. $p = \frac{100A}{100 + rt}$
7. $x = \frac{2Z}{1 - Z}$
8. $s = \frac{2R - 5}{R - 1}$
9. $f = \frac{uv}{u + v}$
10. $t = \frac{2W - 3}{3W - 2}$

1504 Areas under Graphs

1. a) 15 litres
   b) 90 dozen eggs
   c) $(10 \times 10) + \frac{1}{2}(5 \times 10) = 125$m
   d) 40cm^3 of gas
   e) 48 passengers
   f) A force of 56N
   g) 11p
   h) Base of cross-section is 350cm

2. a) Shaded square represents 20 000 litres.
   Area under graph is approximately 15 squares.
   Volume of water used is 300 000 litres approx.
   b) The area under the graph between 3pm and 6pm is larger than the area under the graph between 6am and 9am, so more water is used between 3pm and 6pm.
   c) Water entering the reservoir in the period under consideration is 336 000 litres. This exceeds the volume used. Therefore the water level will be higher at 6pm.

3. a) 10 pulses
   b) Area under the graph between 0 and 3.5 minutes approximates to a rectangle and a triangle.
   Area = $(3\frac{1}{2} \times 70) + \frac{1}{2}(3\frac{1}{2} \times 20) = 245 + 35 = 280$ pulses in 3.5 minutes.
   Average pulse rate for first 3.5 minutes = $\frac{280}{3.5} = 80$ pulses per minute.
   c) The normal resting pulse rate was 70.
   Area under the graph between 0 and 10 minutes approximates to a rectangle and two triangles.
   Area $\approx (10 \times 70) + \frac{1}{2}(3\frac{1}{2} \times 20) + \frac{1}{2}(3\frac{1}{2} \times 25) = 700 + 35 + 43.75 = 778.75$
   Average pulse rate for 10 minutes = $\frac{778.75}{10} = 78$ pulses per minute.

1511 Defining Regions

1.	i)	$x > 2$	matches graph d)
	ii)	$y \leq 6$	matches graph c)
	iii)	$x + y \geq 3$	matches graph g)
	iv)	$2x + 3y \leq 12$	matches graph b)
	v)	$y + 2x \leq 50$	matches graph e)
	vi)	$xy \leq 144$	matches graph a)
	vii)	$y \leq 2x$	matches graph h)
	viii)	$x \leq 2y$	matches graph f)

2. None of the graphs show all the inequalities.

Graph a) is defined by the inequalities	$y \geq 0$	but not by $x \geq 0$
	$2x + y > 4$	
	$x + 3y > 9$	
	$x + y < 6$	
Graph b) is defined by the inequalities	$x \geq 0$	
	$y \geq 0$	
	$2x + y < 4$	**not** $2x + y > 4$
	$x + 3y < 9$	**not** $x + 3y > 9$
		but not $x + y < 6$
Graph c) is defined by the inequalities	$x \geq 0$	but not by $y \geq 0$
	$x + 3y < 9$	**not** $x + 3y > 9$
	$x + y > 6$	**not** $x + y < 6$
		nor $2x + y > 4$
Graph d) is defined by the inequalities	$x \geq 0$	but not by $y \geq 0$
	$2x + y > 4$	
	$x + 3y < 9$	**not** $x + 3y > 9$
	$x + y < 6$	
Graph e) is defined by the inequalities	$x \geq 0$	
	$y \geq 0$	
	$x + 3y > 9$	
	$x + y > 6$	**not** $x + y < 6$
		but not $2x + y > 4$

1517 Trig Problems

1. Cosine rule $b^2 = 75^2 + 161^2 - 2(75)(161)\cos 100$
   b = 189m to the nearest metre.

2. Sine rule. $\dfrac{\sin 70}{285} = \dfrac{\sin 80}{x}$
   x = 299m to the nearest metre.

continued/

1517 Trig Problems (cont)

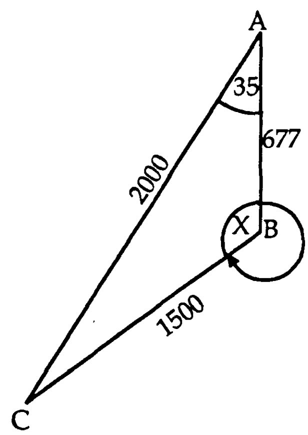
3. Sine rule $\frac{\sin 125}{250} = \frac{\sin 32.5}{b}$

b = 164m to the nearest metre.

4. Cosine rule $x^2 = 70^2 + 83.4^2 - 2(70)(83.4)\cos 42$

$x = 56.4$m to 3 sig. figs.

5. Sine rule



$\frac{\sin 35}{1500} = \frac{\sin X}{2000}$

$\sin^{-1}(0.765) = 50°$ to the nearest degree,
but X is obtuse so X = 180 – 50 = 130°.

The bearing of C from B is 360 – 130 = 230°

6. $\tan 16 = \frac{h}{AC + 30}$

h = 17m and AC = 30m

1520 Difference Game

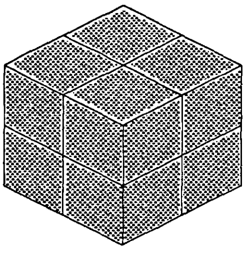
Who won?
Did you get better at working out the biggest differences you could make with your cards? What was the biggest difference you could make?

1521 Five Card Ent

Who won?
How many times did you have to deal the cards? Did you get better the more you played?

1522 Eight Cubes

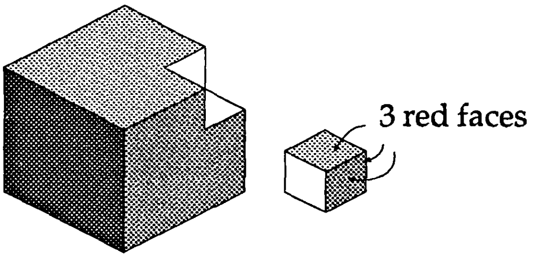
To make a yellow cube you must have no blue faces showing, even underneath!



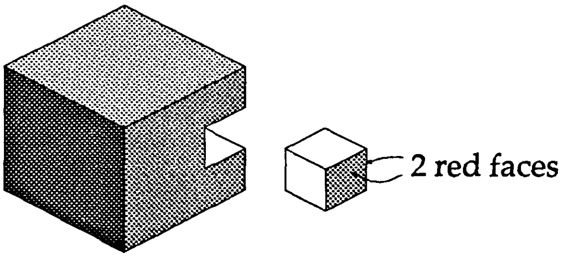
1523 A Red Cube

You will need to know that

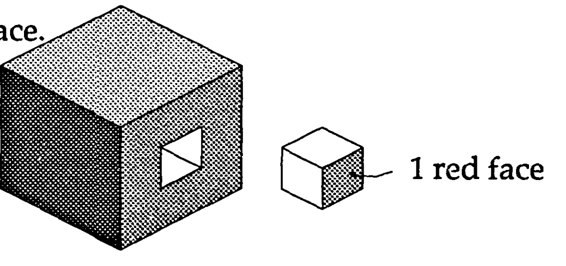
. . . a corner has 3 red faces. . .



. . . a side piece has 2 red faces. . .



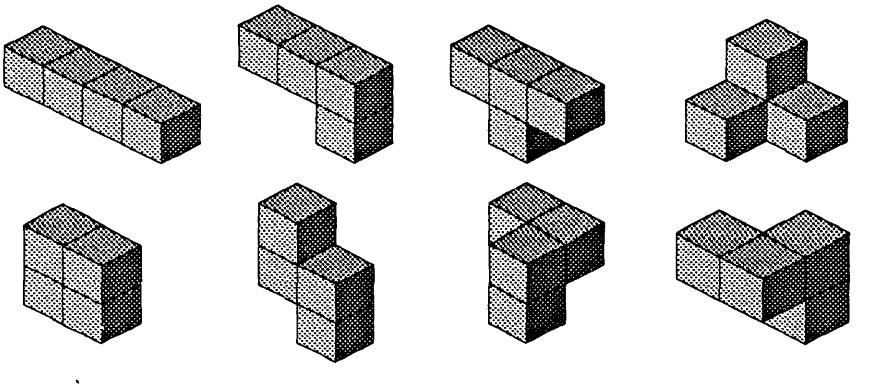
. . . and a middle piece has 1 red face.



Which piece has no red faces?

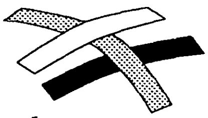
1524 4 Cube Solids

There are 8 different solids which cannot be turned around to look like one another.



1525 Economical Weaving

To develop the most economical colouring, you will need to imagine the same colours continuing under a cross-over.
For example:



You will also notice that the pattern repeats, like wallpaper. So make sure you use the same colours in corresponding positions.

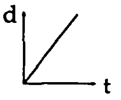
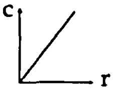
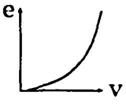
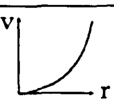
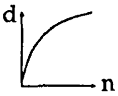
Some people have managed to colour the pattern using only four colours.

1528 Fraction Wall 2

1. $\frac{4}{8}$
2. $\frac{2}{8}$
3. $\frac{6}{8}$
4. $\frac{5}{8}$
5. $\frac{3}{8}$
6. $\frac{5}{8}$
7. $\frac{7}{8}$
8. $\frac{7}{8}$
9. $\frac{7}{8}$
10. $\frac{7}{8}$
11. $\frac{7}{8}$
12. $\frac{5}{8}$
13. $\frac{5}{8}$
14. $\frac{4}{8} = \frac{1}{2}$
15. $\frac{2}{8} = \frac{1}{4}$
16. $\frac{3}{8}$

1533 Proportion

1.

	(i) notation	(ii) formula	(iii) graph
a)	$d \propto t$	$d = kt$	
b)	$c \propto r$	$c = kr$	
c)	$e \propto v^2$	$e = kv^2$	
d)	$v \propto r^3$	$v = kr^3$	
e)	$d \propto \sqrt{n}$	$d = k\sqrt{n}$	

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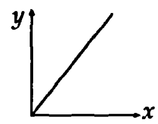
2. $y \propto x \qquad y = kx$

$12 = 2k \Rightarrow k = 6, \; y = 6x$

when $x = 5, y = 30$



3. $y \propto \frac{1}{x^2} \qquad y = \frac{k}{x^2}$

$3 = \frac{k}{16} \Rightarrow k = 48, y = \frac{48}{x^2}$

when $x = 8, y = {}^{48}/_{64} = {}^{3}/_{4}$

4. $0.1 = 8a \Rightarrow a = {}^{1}/_{80}$

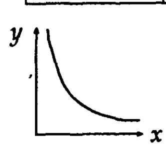
x	2	6	8	10
y	0.1	**2.7**	**6.4**	12.5

5. a) and c)

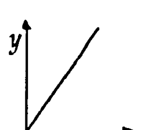
6. b) and d)

7.

x	20	17	14	10	8
y	**173.4**	240	**353.9**	693.6	**1084**



8. $y = 1^{3}/_{4}x$



1537 Simultaneous equations and inequalities

A solid line indicates that the boundary is included in the required region and a dotted line that it is excluded.

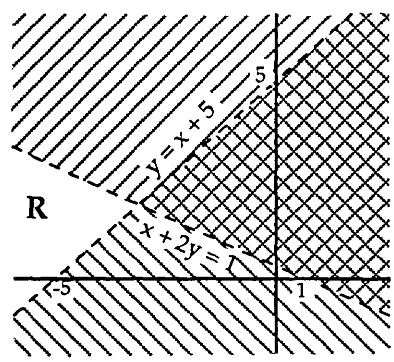
One point in the unshaded region is (3, -5).

Substituting into $2x + y < 12$
$6 + -5 < 12$
$1 < 12$
The coordinates satisfy the inequality.

Substituting into $x - y > 3$
$3 - -5 > 3$
$8 > 3$
The coordinates satisfy the inequality.

1. a) $x = -3, y = 2$

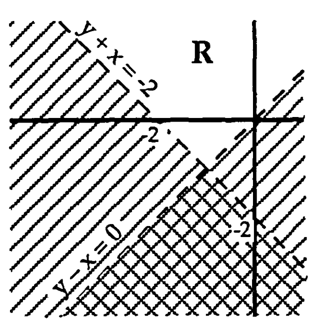
b)



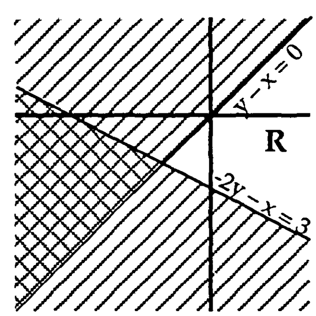
2. a) $x = {}^{1}/_{2}, y = 3$

b) $x = 1, y = -2$

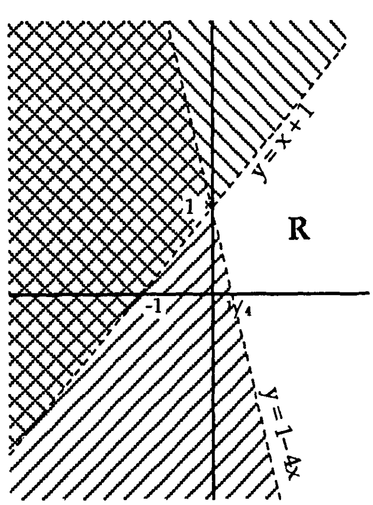
3 a)



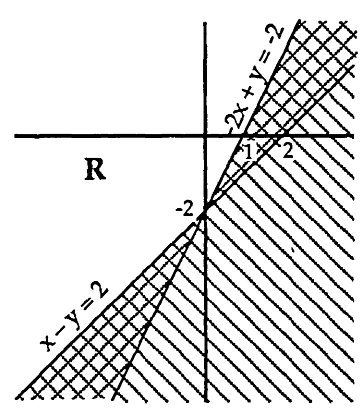
b)



c)



d)



1538 Solving Simultaneous Equations

Whichever method you used to solve the simultaneous equations you should have found these unique solutions. Which of the methods did you find the most useful?

1.	$x = 6$ $y = 2$	2.	$x = 4$ $y = 2$	3.	$x = -1$ $y = 3$
4.	$x = -^1/_2$ $y = 2$	5.	$x = 2^1/_2$ $y = 7^1/_2$	6.	$x = ^4/_{11}$ $y = ^1/_{11}$

1540 Is There a Solution?

a) There is an infinite number of solutions, the equations are represented by the same line.

b) There is no solution, the equations are represented by parallel lines which do not intersect.

c) There is a unique solution, the equations are represented by lines which intersect at the point $(^1/_2, ^1/_2)$.

d) There is an infinite number of solutions, the equations are represented by the same line

---

1541 Cones

All final answers are given correct to 3 significant figures although calculator accuracy has been used throughout the calculation using the π button. If you have used an approximation for π such as 3.14, your answers will vary slightly.

1. $(\pi \times 6 \times 11)\text{cm}^2 = 207.3451151\text{cm}^2 = 207\text{cm}^2$

2. Volume $V = 36 = {}^1/_3\pi(2.5)^2h$

   Height $h = \dfrac{3 \times 36}{2.5^2 \times \pi} = 5.500394833 = 5.50\text{cm}$

   Slant height, by Pythagoras, $l = \sqrt{(5.5^2 + 2.5^2)}\text{cm} = 6.041522987 = 6.04\text{cm}$
   Curved surface area $= (\pi \times 2.5 \times 6.04)\text{cm}^2 = 47.45001058 = 47.5\text{cm}^2$

3. Total surface area $= (\pi \times 4 \times 7.5) + (\pi \times 4^2) = 144.5132621 = 145\text{cm}^2$

4. a) ${}^1/_3\pi r^2 \times 12 = 400$
   $\pi r^2 = 100$ Base area $= 100\text{cm}^2$

continued/

1541 Cones (cont)

4. b) $r^2 = \frac{100}{\pi}$

$r = 5.641895835$ Base radius = 5.64cm

c) Slant height $= \sqrt{(5.64^2 + 144)}\text{cm} = 13.26012778 = 13.3\text{cm}$

Total surface area $= (\pi \times 5.64 \times 13.3) + (\pi \times 5.64^2)$

$= 335.0296454 = 335\text{cm}^2$

5. $A = \pi r^2 + \pi r l$

$A - \pi r^2 = \pi r l$

$\frac{A - \pi r^2}{\pi r} = l$

6. Think of this as 'the curved surface area of a cone of height $(16 + x)$cm' subtract 'the curved surface area of a cone of height $x$cm'.

The cones are similar so

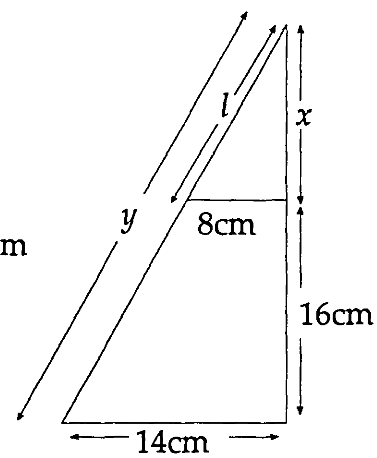
$\frac{x}{8} = \frac{16 + x}{14}$

$x = 21.\dot{3}\text{cm}$

$l = \sqrt{(21.3^2 + 8^2)} = 22.78400999 = 22.8\text{cm}$

$\frac{y}{14} = \frac{22.8}{8}$

$y = 39.87201748 = 39.9\text{cm}$



Surface area of lampshade $= (\pi \times 14 \times 39.9) - (\pi \times 8 \times 22.8)$

$= 1181.038293 = 1180\text{cm}^2$

1543 Composite Functions

1. (i) $x \rightarrow 3x + 2$

(ii) $x \rightarrow {}^{x}/_{3} + 2$

(iii) $x \rightarrow 3(x + 2)$

(iv) $x \rightarrow \frac{x + 2}{3}$

(v) $x \rightarrow x^2 - 7$

(vi) $x \rightarrow (x - 7)^2$

2. No.

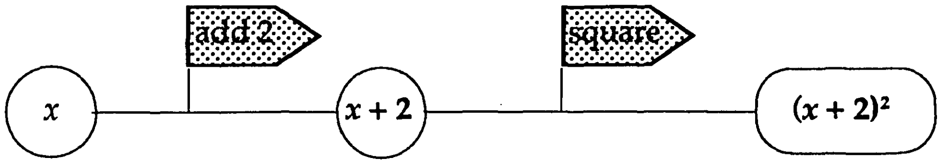
$x \rightarrow 3x^2$ is 'square and multiply by 3' function.

$x \rightarrow (3x)^2$ is 'multiply by three and square' function.

continued/

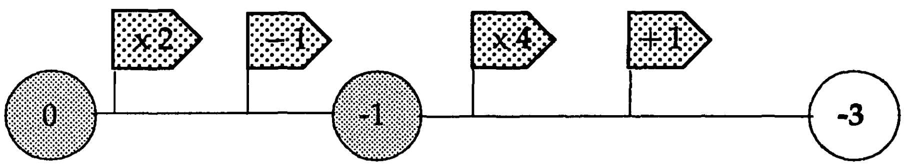
1543 Composite Functions (cont)

3.



$sd(x) = (x+2)^2$

4. (i)



(ii) -19 (iii) 1 (iv) -15

$fg(x) = 8x - 3$
$gf(x) = 8x + 1$

5. (i) 7 (ii) 7
(iii) -5 (iv) -5
(v) -2 (vi) -2

$fg(x) = 6x - 5$
$gf(x) = 6x - 5$
so $fg(x) = gf(x)$ for all x.

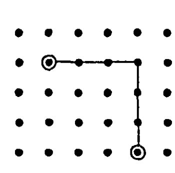
6. $fg(x) = \dfrac{1}{x+2}$

$gf(x) = \dfrac{1}{x} + 2$

7. $g(x) = x - 1$

8. $g(x) = \dfrac{x-1}{2}$

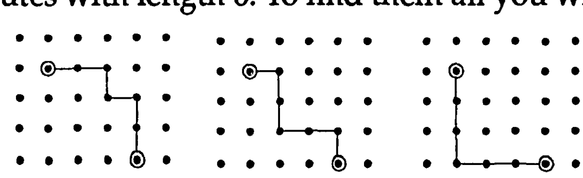
1544 Joins



The shortest line to join the two grey dots is 6 units.

This diagram will help you to explain why.

There are more than twelve routes with length 6. To find them all you will need to be systemmatic. For example,



. . . and you will have to decide whether you will count this last example as different from the first example above.

continued/

1544 Joins (cont)

You may like to investigate how many lines can be drawn with a different length.

Can you find a route between the two grey dots whose length is an odd number of units?

1545 Completing the Square

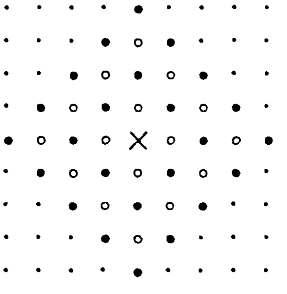
Because the points are arranged in a square lattice you should always be able to 'complete the square' if your first 2 points join to make one side.

If your first 2 points are to form opposite corners of a square there needs to be an even number of "hops" between them (see p4).

1546 Hops

The dots which are 3 hops from the cross are indicated with o

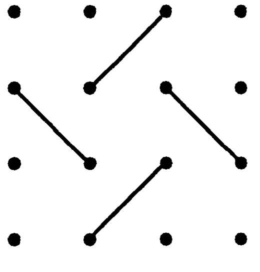
The dots which are 4 hops from the cross are indicated with •



When you investigate hopping distances, your dots will always lie on a line between the two crosses. This line is the perpendicular bisector of the line joining the two points.

1547 Link Patterns

This 16-dot square has only 4 links.



It is possible to make link patterns with 7, 8, 9 and 10 links on a 5 by 5 grid.

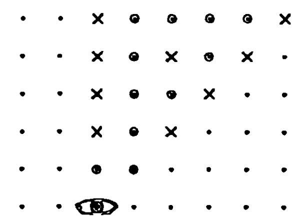
Does the pattern continue for 6 by 6?

1548 Link Pattern Tiles

You will be able to make an attractive poster with your results for this investigation. The SMILE pack 1617 will give you some further ideas or you may like to use MicroSMILE program "TILES".

1549 Lines of Sight

This shows $^1/_8$ of the 11 by 11 square array.



Why do you need only examine the posts in $^1/_8$ of an array?

You may find that labelling the posts by their co-ordinates (with reference to the eye) will help you decide which posts are always hidden and which are in sight.

1550 Turning and Shifting

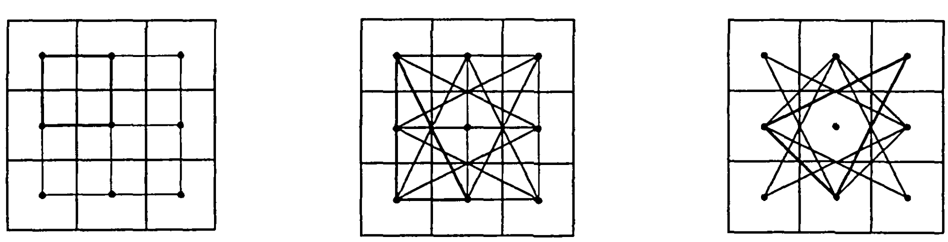
There are many patterns possible on the different size pin-boards. Did you find different starting figures which produced similar patterns? . . . the same patterns?

You may like to make a poster to display you results. Write about any observations that you have made.

1551 Changing Shapes

It would be a good idea for you to write your observations about the changing shapes. Perhaps you could make a poster of your shapes and observations.

1552 How many Tiles?



4 times | 8 times | 4 times

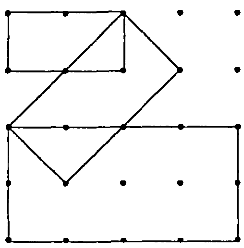
What difference does it make if the triangle is turned over as well as rotated?

1553 Halves and Quarters

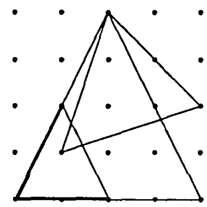
Make a display of your designs.

1554 Same Shape

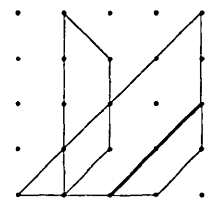
Three rectangles will fit so that the ratio of the sides is 1 : 2.



Three similar isosceles triangles with their base and height equal can be found.



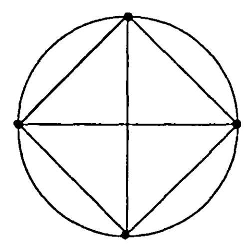
Three similar trapezia with ratio of a : b : h = 2 : 4 : 1 can be found.



1555 Mystic Rose

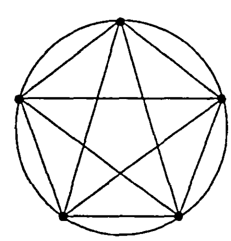
To find the number of lines in the pattern it is best to start with a simple pattern.

- A 4-point circle
  Each point has 3 lines coming to it, but there are not 12 lines. There are only 6 lines because each line goes to 2 points.

  Another way of explaining this is to draw the lines from one point first. You would draw 3. Then from the second point you would draw 2 more. From the third point you would only draw another 1. The fourth point would then be already drawn. So the number of lines is 3 + 2 + 1.

- A 5-point circle
  Each point has 4 lines coming to it. 5 x 4 = 20 ends.
  Each line has 2 ends so there are 10 lines.

  By the other method you would draw 4 from the first point, 3 from the second point, and so on.

- So, for the 16-point circle on the card, you can reach the total in two different ways:
  Each point has 15 lines coming from it. 16 x 15 = 240 ends. Each line has . . . etc.

  By the other method there are 15 lines from the first point, 14 more lines from the second, and so on. 15 + 14 + 13 + . . . + 2 + 1
  A total of . . . etc.

The circles with an odd number of points have a hole at the centre.
It is those with an even number of points which do not have a hole. Why?

1556 19 Piece Jigsaw

The 19 pieces should make a 100 squrare.

1557 Spirals

You might like to make a small poster to display the patterns which you have drawn. You could make other spiral patterns from your own starting shapes.

1559 Areas of Similar Shapes

a) Scale factor	b) original • corresponding length • new length	c) original area (cm^2)	d) new area (cm^2)	e) original • new area • area
$^1/_2$	$1 : {}^1/_2 = 2 : 1$	4	1	4 : 1
$1^1/_2$	$1 : 1^1/_2 = 2 : 3$	4	9	4 : 9
2	1 : 2	4	16	4 : 16 = 1 : 4
$2^1/_2$	$1 : 2^1/_2 = 2 : 5$	4	**25**	**4 : 25**
3	1 : 3	4	**36**	**4 : 36 = 1 : 9**
$3^1/_2$	$1 : 3^1/_2 =$ **2 : 7**	4	**49**	**4 : 49**

- The ratios in column e) are the squares of the corresponding ratios in column b). When the triangle is enlarged by, for example, scale factor 3, the base becomes 3 times larger and the height becomes 3 times larger, so the area becomes 9 times larger.

a) Scale factor	b) original • corresponding length • new length	c) original area (cm^2)	d) new area (cm^2)	e) original • new area • area
$^1/_2$	4 : 2 = 2 : 1	8	2	8 : 2 = 4 : 1
$1^1/_2$	4 : 6 = 2 : 3	8	18	8 : 18 = 4 : 9
2	4 : 8 = 1 : 2	8	32	8 : 32 = 1 : 4
$2^1/_2$	4 : 10 = 2 : 5	8	50	8 : 50 = 4 : 25
3	4 : 12 = 1: 3	8	72	8 : 72 = 1 : 9
$3^1/_2$	4 : 14 = 2 : 7	8	144	8 : 144 = 4 : 49

- The ratios in column e) are the squares of the corresponding ratios in column b).

continued/

1559 Areas of Similar Shapes (cont)

The hexagon
a) $16cm^2$
b) $1cm^2$
c) $49cm^2$

The pentagon
a) $24cm^2$
b) $1.5cm^2$
c) $73.5cm^2$

Summary

When a shape is enlarged by scale factor n
- the corresponding angles are **equal**
- the ratio of the sides is 1 : **n**
- the ratio of the areas is 1 : $\mathbf{n^2}$

1560 Similarity Problems

All answers are rounded correct to 2 decimal places.

1.

Radius (cm)	Diameter (cm)	Circumference (cm)	Area (cm^2)
2	4	12.57	12.57
4	8	25.13	50.27

With enlargement scale factor 2.

Ratio of diameter 4 : 8 = 1 : 2

Ratio of circumference 12.57 : 25.13 = 1 : 2

Ratio of area 12.57 : 50.27 = 1 : 4 = $1 : 2^2$

The results do agree with the summary.

2. $15cm^2$

3. a) 3.75cm
   b) $18.75cm^2$

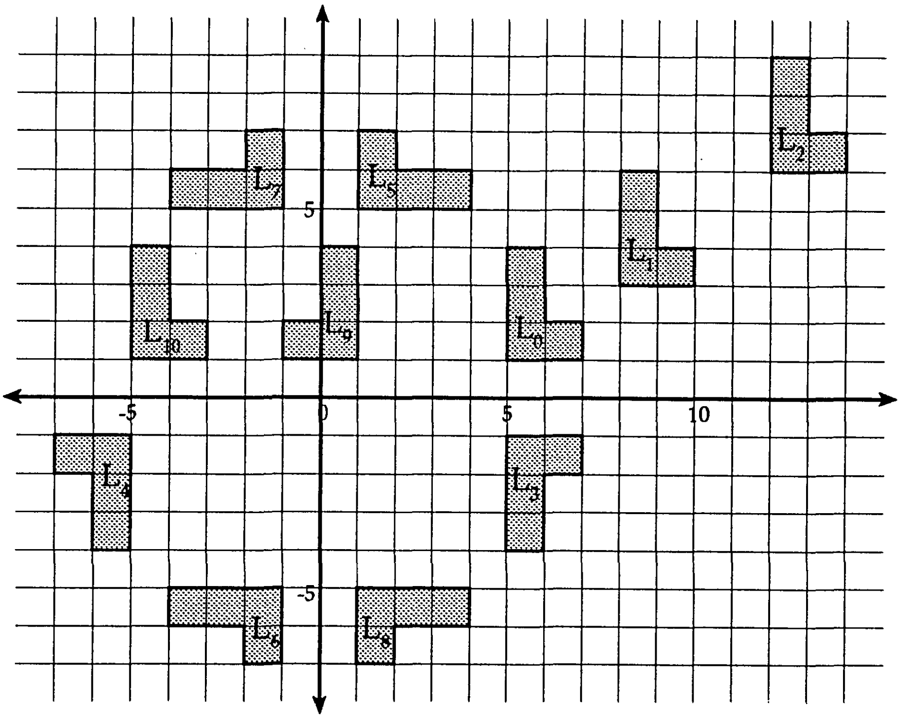
4. $10^2 \times 90 = 9000g = 9kg$

5. $4^2 \times 18 = 288$

6. Area on map is approximately $16cm^2$.

   Area of forest is $16 \times (50\,000)^2\ cm^2 = 4km^2$.

1561 Combining Transformations



a) Translation $\begin{pmatrix} -7 \\ -5 \end{pmatrix}$

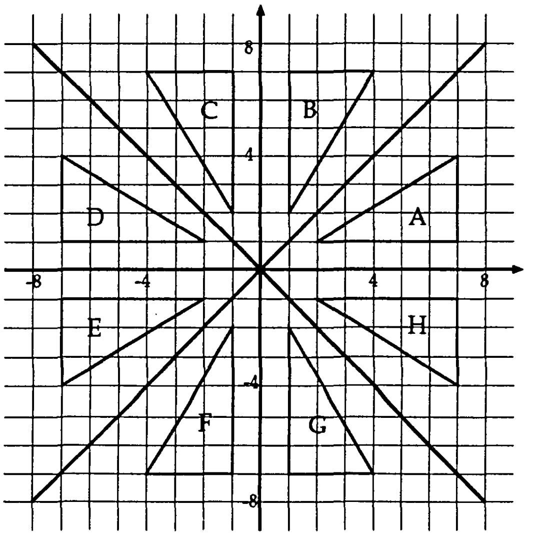
b) Rotation through 180° about (0, 0)

c) Reflection in $y = -x$

d) Rotation through 90° anticlockwise about (0, 0)

e) Translation $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$

1562 Combined Reflections



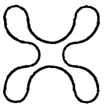
a) Rotation through 180° about (0, 0)

b) Reflection in $y = 0$, (x axis)

c) Rotation through 180° about (0, 0)

d) Reflection in $y = -x$

1564 Curvitiles

1. The other closed curve is bottom left. 

2. There are many possible answers which have no closed curves.

3. There are many possible answers which have more than 5 closed curves.

4. It is possible to make 11 closed curves.

5. Yes. Again there are many possible answers.

6. You may like to make a folder of your own designs.

1565 Symmetry

Use a mirror to check that your drawings are complete and correct.

Did you remember to answer the sums?

1566 Finding Square Roots

This method for finding square roots is called 'trial and improvement'.

You will know when your answer is correct because the check is to multiply the square root by itself:

square root x square root = number

This statement will remind you that the square of "the square root of n" is n. This can be written $(\sqrt{n})^2 = n$

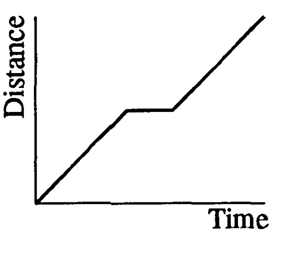
You should continue to make guesses until you get the target number correct to 3 decimal places.

e.g. $\sqrt{12}$ 3.464 x 3.464 = 11.999296

The guess 3.464 is good enough because the answer is equivalent to 12 (to 3 d.p.)

1568 Velocity from Distance-Time Graphs

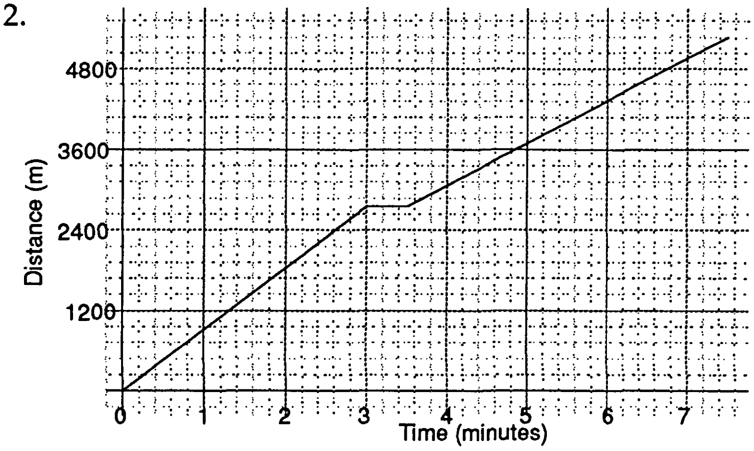
1. This is a possible example:



The gradient of the tangent to the curve when the car stops is 0.

continued/

2.

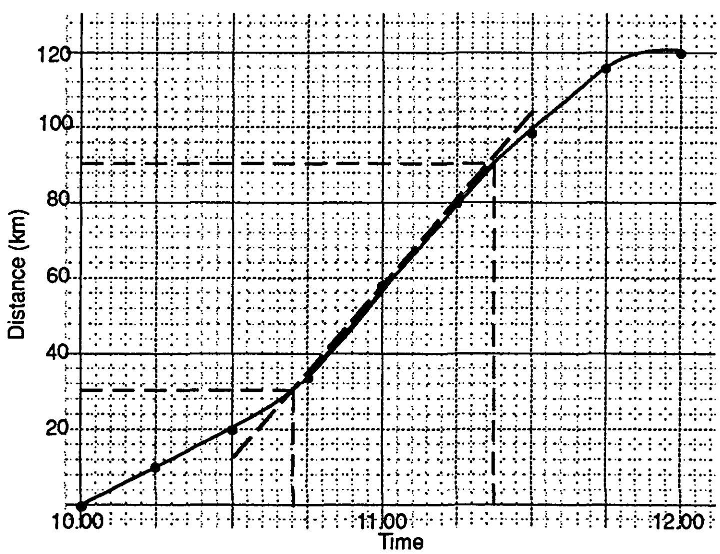


Average velocity for the journey

$$= \frac{\text{total distance (metres)}}{\text{total time (seconds)}}$$

$$= \frac{5100}{7.5 \times 60} = 11.3\text{m/s}$$

3.



Your answers may vary slightly from these.

a) 60km/h

b) 78km/h

c) From chord, time between distance 30km and 90km
$= 11.22 - 10.42$ h $=$ 40 minutes

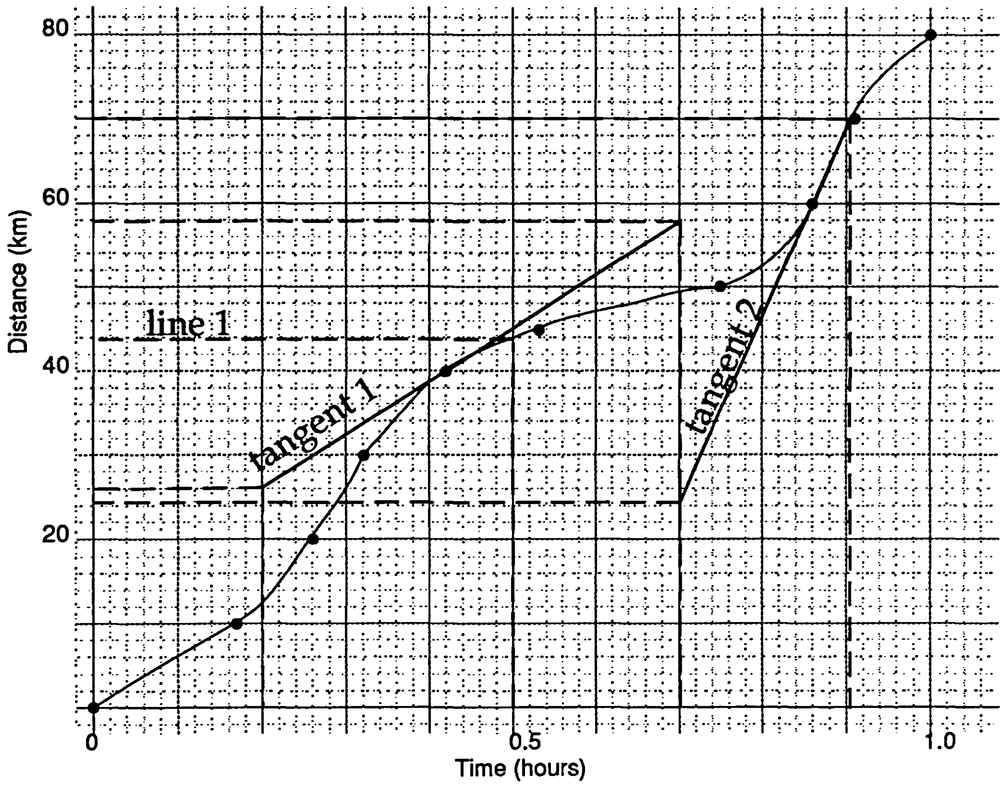
Gradient of chord joining y = 30 to y = 90
$\approx \frac{60}{40/60} = 90$km/h

d) maximum velocity from tangent at 11.00
$\approx \frac{104 - 12}{1} = 92$km/h

continued/

1568 Velocity from Distance-Time Graphs (cont)

4.



Your answers may vary slightly from these.

a) from line 1, velocity = $\frac{44-0}{0.5}$ = 88 km/h

b) from tangent 1, velocity ≈ $\frac{58-26}{0.5}$ = 64 km/h

c) from tangent 2, velocity ≈ $\frac{70-24}{0.2}$ = 230 km/h

5. *Your answers may vary slightly from these.*

a) 13m/s

b) 2.5s

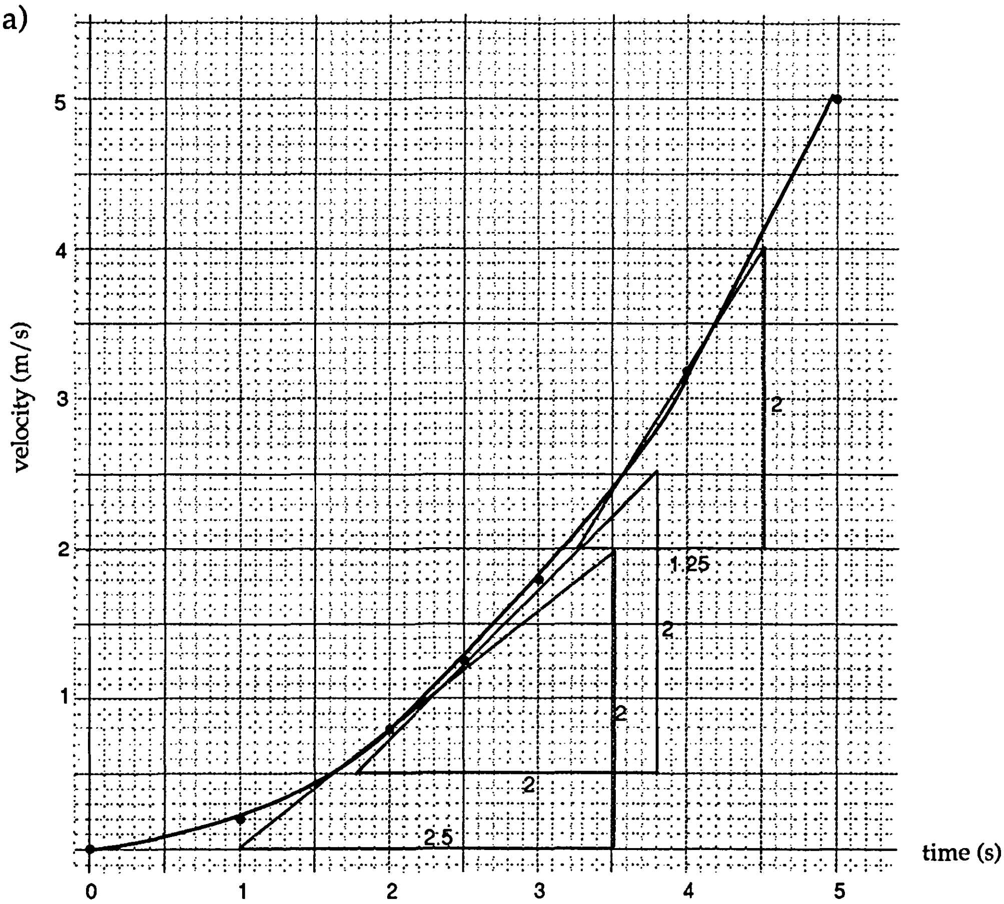
1569 Distance, Velocity and Acceleration

Section A

1. Graph ii) is the only one which could correspond to the acceleration-time graph. The acceleration-time graph shows constant acceleration which implies constantly increasing velocity.

continued/

2. a)



b) i) at t = 2 acceleration = $\frac{2}{2.5}$ = 0.8 m/s^2

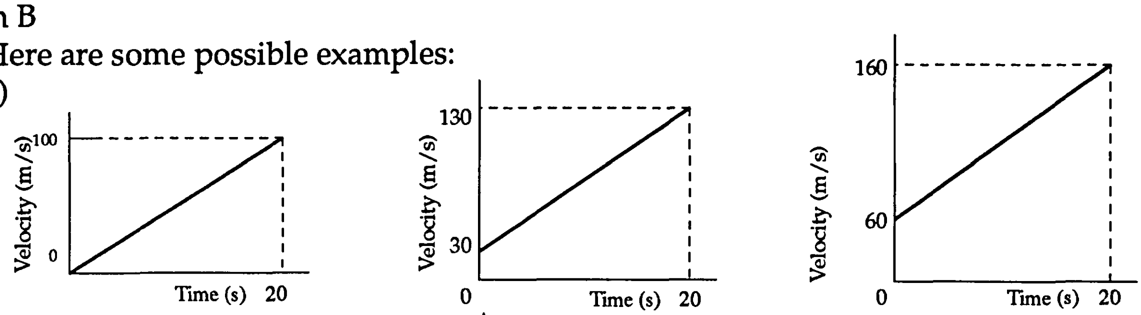
ii) at t = 4 acceleration = $\frac{2}{1.25}$ = 1.6 m/s^2

c) a = 1m/s^2 when t = 2.5s

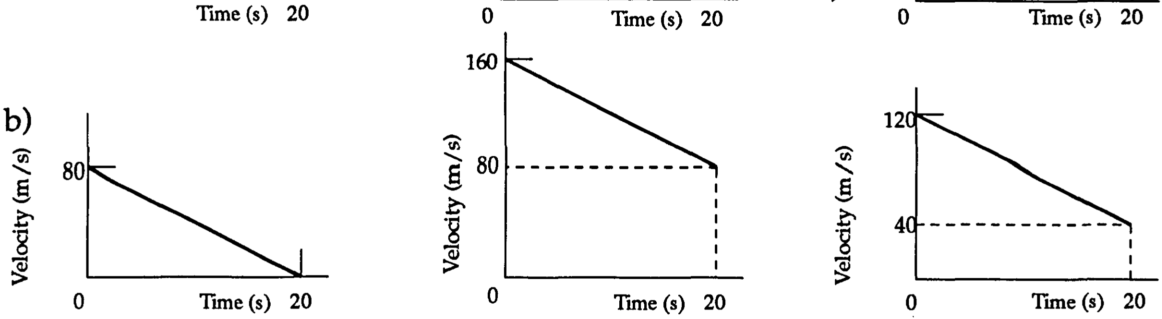
Section B

1. Here are some possible examples:

a)



b)

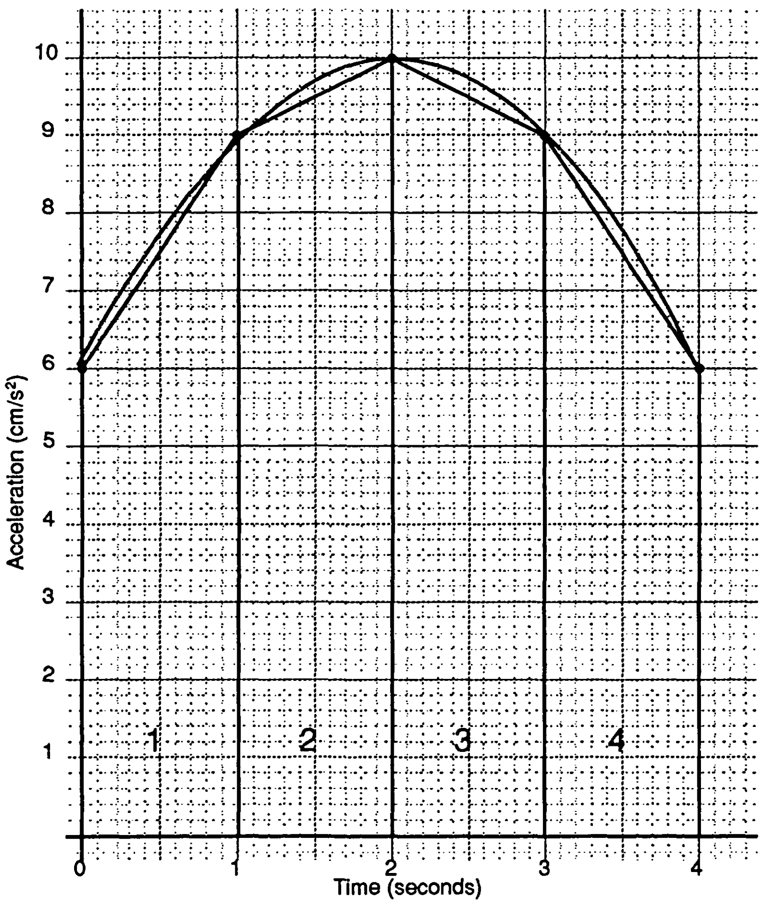


continued/

2. a) true
   b) false The correct answer is 275cm
   c) true
   d) false The correct answer is $-2cm/s^2$

3. a) The acceleration changes instantaneously from $10cm/s^2$ to $5cm/s^2$ when the object has a velocity of 15cm/s.
   b) The object is moving with a constant velocity of 20cm/s.
   c) The object is decelerating at $10m/s^2$ and has a velocity of 10cm/s.
   d) 20cm

4. a)



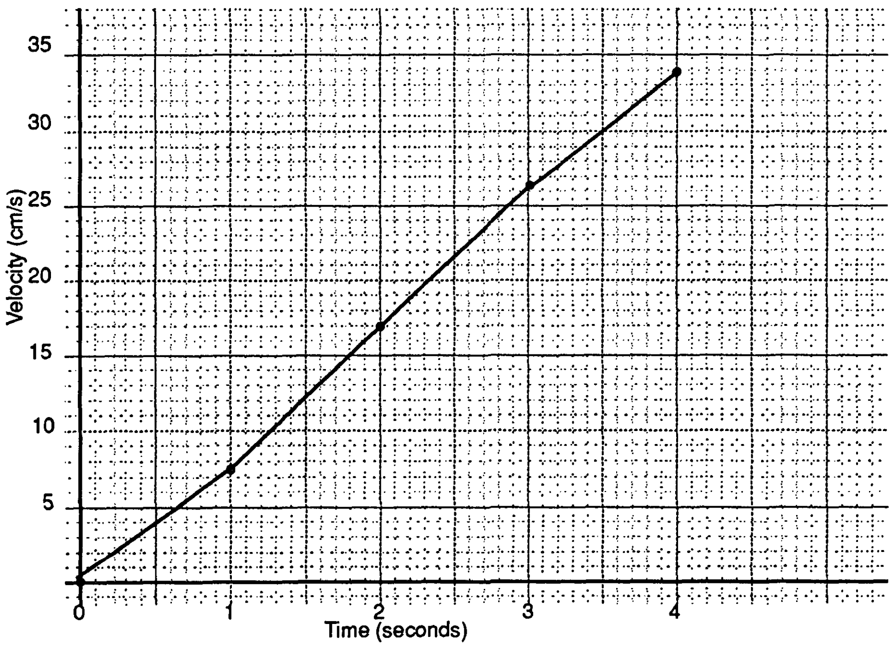
   b) Area of trapezium 1 = $^1/_2$ (6 + 9) = 7.5
      Area of trapezium 2 = $^1/_2$ (9 + 10) = 9.5
      Area of trapezium 3 = $^1/_2$ (9 + 10) = 9.5
      Area of trapezium 4 = $^1/_2$ (6 + 9) = 7.5

      At t = 1, velocity = 7.5cm/s
      At t = 2, velocity = 7.5 + 9.5 = 17cm/s
      At t = 3, velocity = 17 + 9.5 = 26.5cm/s
      At t = 4, velocity = 26.5 + 7.5 = 34cm/s

continued/

1569 Distance, Velocity and Acceleration (cont)

4. c)

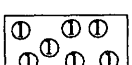
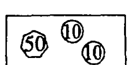
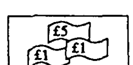
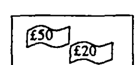


d) Total distance travelled by the object is represented by the area under the velocity-time graph, this is approximately a triangle.

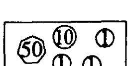
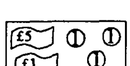
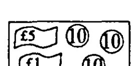
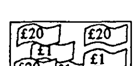
Distance travelled $\approx \frac{1}{2}(4 \times 34) = 68$cm.

1570 Pounds and Pence

1

			
7p	70p	700p	7000p
£0.07	£0.70	£7.00	£70
0.07	0.7	7.	70.

2.

			
63p	603p	630p	6300p
£0.63	£6.03	£6.30	£63
0.63	6.03	6.3	63

1571 Keyboard Patterns

1. This calculation always gives 27 for this keyboard.
2. This calculation always gives a multiple of 11.
3. Write about *your* patterns.

1572 "50% is Half Marks"

50% of £20 = £10
of £16 = £8
of £10 = £5
of £4 = £2

100% of £15 = £15
of £6 = £6
of £4 = £4

10% of £6 = 60p
of £4 = 40p
of £2 = 20p
of £1 = 10p

25% of £100 = £25
of £4 = £1
of £2 = 50p
of £1 = 25p

75% of £4 = £3
of £2 = £1.50
of £1 = 75p

1% of £7 = 7p
of £3 = 3p
of £1 = 1p

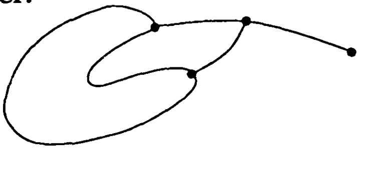
1578 Slicing a Triangle

Make a poster of your sliced triangle designs.

1579 Points and Buffers

1 point and 2 buffers cannot make a connected layout.

Here is one connected layout with 3 points and 1 buffer:



3 lines meet at a point, and 1 line at a buffer. You may be able to notice what connections are possible if you look at

. . . an even number of points with an even number of buffers?
. . . an odd number of points with an odd number of buffers?
. . . an odd and an even number of each?

1580 Rep-tiles Investigation

There are many possible answers to these questions, except Rep-3 and Rep-2.

It is not possible to make the T shape which is Rep-4. However, there are many examples of shapes that are Rep 4.

An example of a Rep-2 rectangle is A4 paper, but this cannot be drawn on the grid.

1581 Patterns and Shapes

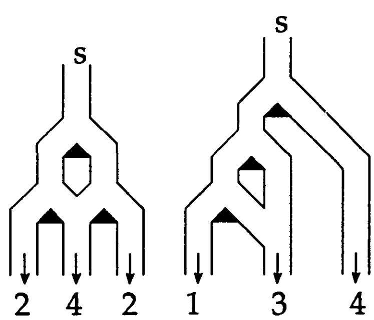
There are many shapes possible. Make a poster with your designs.

1582 Deal a Card Experiment

This is one way you could lay out your results:

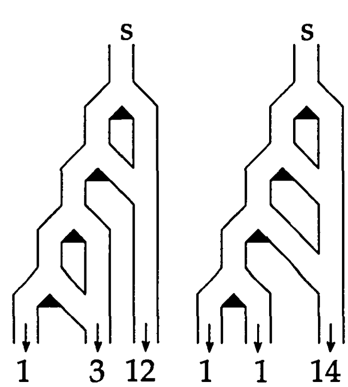
	One ace	6 or 7 Red	2, 3 or 4 Court cards	a 10 and a 6	5, 6 or 7 cards under 6
Hand 1	√	x	√	√	x
Hand 2	x	√	x	√	√

1583 Marbles



The second diagram, which gives 1, 3, 4 with 8 marbles would give 2, 6, 8 with 16 marbles.

These are possible networks,
you may have found different ones.



1589 Square Roots Investigation

For any number that you choose, the square root of the square root of the square root, etc . . . approaches 1.

This can be written $\ldots\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{x}}}}}} \to 1.$

To form the sequence where the number is doubled after you take the square root, $\ldots 2\sqrt{(2\sqrt{(2\sqrt{(2\sqrt{(6.8)})})})}\,..$ you may use a spreadsheet or a graphic calculator.

This spreadsheet shows the beginning of the sequence $\ldots 2\sqrt{(2\sqrt{(2\sqrt{(2\sqrt{(6.8)})})})}\,..$

	A
1	6.8
2	5.21536192
3	4.56743338
4	4.27431088
5	4.13488132
6	4.06688152
7	4.03330213
8	4.01661655
9	4.00829967
10	4.00414768

← Formula =2*SQRT(A1) (in cell A2)
Fill Down ↓

The same sequence may be formed on a Texas TI-81 graphic calculator.

Key press	Screen display
[6] [.] [8] [Enter]	6.8 6.8
[2] [√] [Ans] [Enter]	2√Ans 5.215361924
[Enter]	4.567433382
[Enter]	4.274310883
⋮	

What happens in the sequence $..2\sqrt{(2\sqrt{(2\sqrt{(2\sqrt{(x)})})})}$
. . . if x is more than 4?
. . . if x is less than 4?
. . . if x is equal to 4?

Answering these three questions will help you to investigate what happens when you multiply the square root by 3, 4, . . .k.

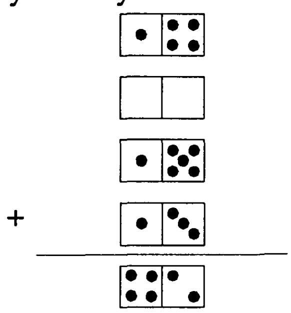
You may look at sequences formed from
cube roots $\sqrt[3]{}$
fourth roots $\sqrt[4]{}$
⋮
pth roots $\sqrt[p]{}$

1590 Squares

Describe the best strategy to win this game.

1591 Domino Sums

It is possible to make domino sums so that no dominoes are left over.
To do this, you may need to do some sums like



$$\begin{array}{r} 14 \\ 00 \\ 15 \\ + \underline{13} \\ 42 \end{array}$$

1592 Two Cuts Investigation

Use Smile 2163 Geometry Facts section on polygons to help you describe all the shapes you found in this investigation.

1594 Find the Objects

Objects you may have found are:

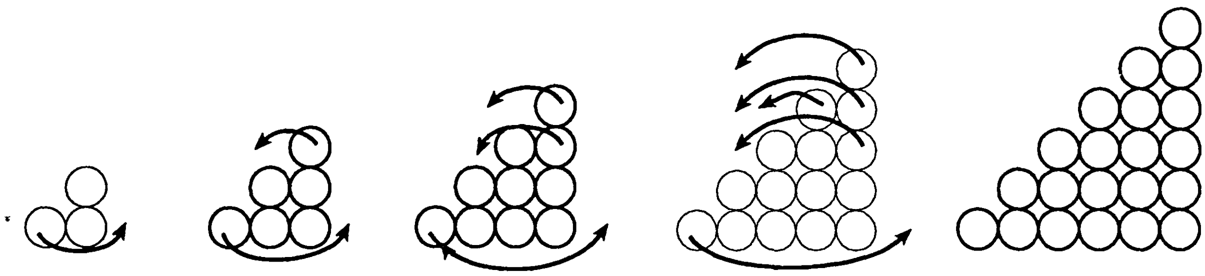
torch	telephone	pencil-sharpener	key
eraser	shuttlecock	camera	paint-brush
box of matches	lipstick	penknife	bottle of glue
spanner	kettle	table-tennis bat	mug
pair of scissors	paper-clip	whistle	

1595 Shunting

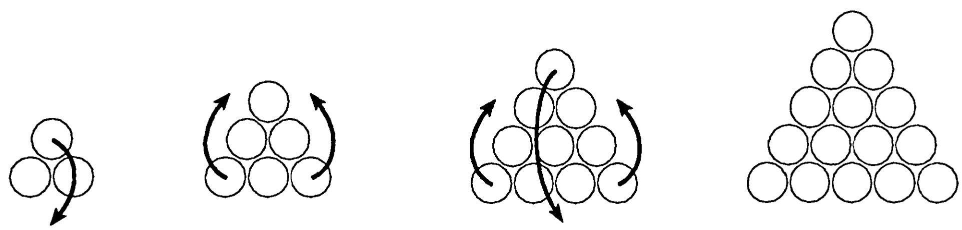
Make sure somebody else can understand how you have recorded the shunting steps.

1596 Count a Counter

There are several different ways of moving the counters, but these drawings may help you to see one way:



1 move   2 moves   3 moves   5 moves   . . . ? . . .



1 moves   2 moves   3 moves   . . . ? . . .

In what ways are these two series similar?

1597 Animals

There are several answers possible depending upon which shapes you make.

For example, with 3-counter-objects you could make  and  and  which are all animals.

This should enable you to decide which of the three statements on page 14 are true. Write a summary of your investigation.

1598 Animal Algebra

There are several different ways to reduce each combination but you should reach the same answer whichever you use.

Here is one example for each:

1. ACACACAC = CAC CAC AC
   = CACC AC AC
   = CA A CAC
   = C CAC
   = AC

continued/

1598 Animal Algebra (cont)

2. ACBCBC = AC CBC C
= ACC BCC
= AB

3. ACA BCB C = CAC CBC C
= CAC C BCC
= C A B

There are also many different ways to make longer routes. If you have difficulty with questions (4) and (5), the examples at the bottom of page 16 should help you.

1600 Slabs

Because there are so many systems to explore, your drawings will need some brief notes about the rules which you have used.

Answers • Answers • Answers

1301 to 1600

Answers