SMILE

Answer Book 1601-1918

Front cover illustration:

Sonya Kovalevsky (1850 - 1891) was an outstanding mathematician. She was born in Moscow.

Her work in mathematics included a famous theorem on partial differential equations called the 'Cauchy- Kovalevsky Theorem', and also important work on the rotation of a rigid body about a fixed point. For this she won the Prix Bordin which was the highest award of the French Academy of Sciences.

Despite being a brilliant mathematician , it was not until quite late in her career that she was able to get a job as a Professor of Mathematics at a university - Stockholm University. This was because other universities would not allow women to become mathematics professors. Indeed they would not allow women to study maths at all.

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SMILE

ANSWER BOOK

1601-1918

SMILE

Answer Book 1601-1918

1601 Growing Squares

The process of growing squares is very similar to the natural growth of snowflakes. You will see the connection more readily if you try growing triangles.

You may like to make a wall-display or folder of your patterns.

1602 A Surveying Problem

No answers required.

1603 Man-Trap

No answers required.

1604 Nim

No answers required

1605 Guess

No answers required

1606 GuessD

No answers required

1607 Elephant

No answers required

1608 Reverse

No answers required

1609 Maze

No answers required

1610 Maggots

No answers required

1611 Cheap at Half the Price

A 1. Standard: 100gm costs 7p; Economy: 100gm costs 6p; Jumbo: 100gm costs 5p

2. 3 Standard boxes = 1 Jumbo Pack

3. 3 Standard boxes would cost £1.05
So buying 1 Jumbo pack saves 30p

B 1/2 (a) In the car accessories shop the cost is £48
(b) In the catalogue the cost is £54 (20 x £2.70)
(c) In the radio shop the cost is £55.90 (£43 + £12.90 interest)
Monthly repayments = £2.33 (24 months)

1612 Re-fit

All of the shapes can be made ...
... and many of them can be made in more than one way.

(This is possible because the two larger shapes can each be made from two small triangles.)

1613 Calculating Kitty

It may be best to choose £1000 now because even Mr Moneybanks will not be able to pay the reward in a month's time!

1614 Probability Kitty

It seems likely that Kitty is bound to win....eventually!
But supposing that tails comes up ten times in a row, how much would Kitty be owing?

1615 Logical Kitty

1st: Betty
2nd: Carol
3rd: Alan

1616 Numbers Banned

No answers required

1617 Square Versatiles

No answers required.
You may like to use the micro program "Tiles 1617A" to explore more patterns like those you have made.

1618 Number Names

A - 36	C - 45	E - 36
B - 25 (or 1)	D - 13	F - 8

1619 How Many Squares?

Looking at one rectangle you may have been able to find a systematic way to count the different size squares. For example, in a 6x4 rectangle, start with squares 1x1, then squares 2x2, and so on.....

1x1 squares	6x4	24
2x2 squares	5x3	15
3x3 squares	4x2	[redacted]
4x4 squares	[redacted]x[redacted]	[redacted]
	Total:	50

continued/

1619 How Many Squares? (cont)

A systematic approach like this will enable you to make a table of numbers of squares for different size rectangles:

Width of rectangle	Length of rectangle 1	2	3	4	5	6	7
1	1	2	3	4	5	6	7
2	2	5	8	11	14	17	20
3	3	8	14				
4	4	11					
5	5						

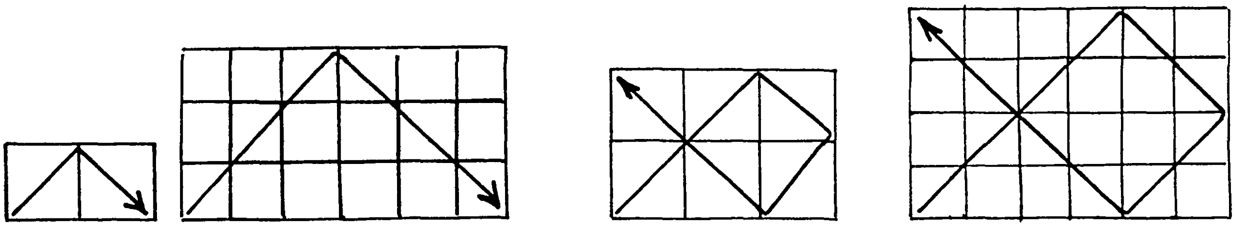
Can you predict how many squares are contained in a 5x8 rectangle?

Can you test your prediction?

In fact, there are several patterns within the table. With these you may be able to use algebra to predict how many squares would be contained in an m×n rectangle.

1620 Billiards

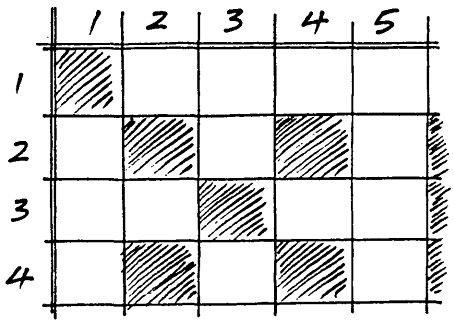
Doubling or trebling the sides of the billiard table does not change which pocket the ball goes in - why not?



A 4x6 table gives the same results as a 2x3 table.

If the sides of the table have a common factor, is there always a smaller table which will give the same result?

One way to continue the investigation is to study billiard tables whose sides have no common factor.



However there are many other avenues of research. For example
- what happens on square tables?
- what happens when sides have a common factor of 2? .... or 3? .... or 4?
- what happens on an m×n table?

1621 Rhino

No answers required

1622 Vectmeet

No answers required

1623 Reflect

No answers required

1624 Snooker

No answers required

1625 Boxes

No answers required

1626 Boat

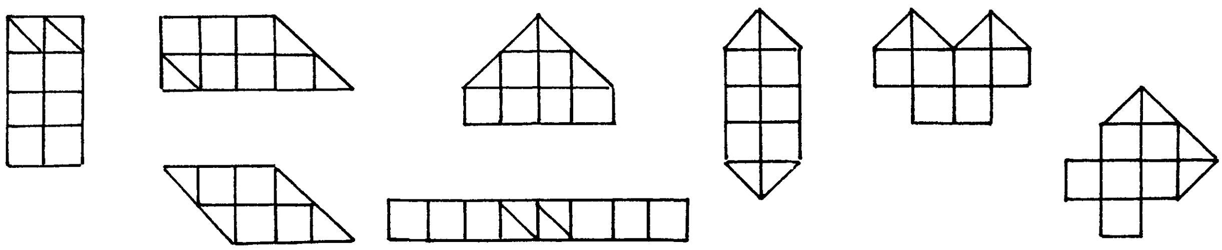
No answers required

1627 Self Portrait

DEC	NOV	JAN
MAR	AUG	JULY
JUNE	SEPT	FEB
APR	OCT	MAY

1628 Eight Squares

Many answers are possible.
Here are some:



1629 Pentagons

No answers required

1630 Along the Line

There are many possible ways to do all the numbers from one to twenty except 8, 12, 14, 16 and 19.
Here are some examples:

1 = 9 - 8	6 = 3 x 2	13 = 8 x 5
2 = 2 ÷ 1	7 = 5 + 2	15 = 9 + 6
3 = 9 - 6	9 = 6 + 3	17 = 8 + 9
4 = 4 x 1	10 = 5 x 2	18 = 6 x 3
5 = 2 + 3	11 = 7 + 4	20 = 5 x 4

Using diagonal neighbours as well 8(= 2 x 4), 12(= 2 x 6) and 14(= 8 x 6) can be done. It is not possible to do 16 or 19.

1631 Target 100

No answers required

1632 Marked Buttons

There are many sets of three numbers you could use but they all need $\boxed{9}$.

Here are examples using 9,2 and 3 (as well as 6,7 and 8).

15 = 7 + 6 + 2	18 = 9 + 7 + 2	21 = 7 + 8 + 6
16 = 8 + 6 + 2	19 = 9 + 8 + 2	22 = 7 + 9 + 6
17 = 8 + 7 + 2	20 = 9 + 8 + 3	23 = 8 + 9 + 6

How many other sets will work?
Which sets will not work?

1633 Up the Thames

No answers required

1634 Colouring the Dots

No answers required

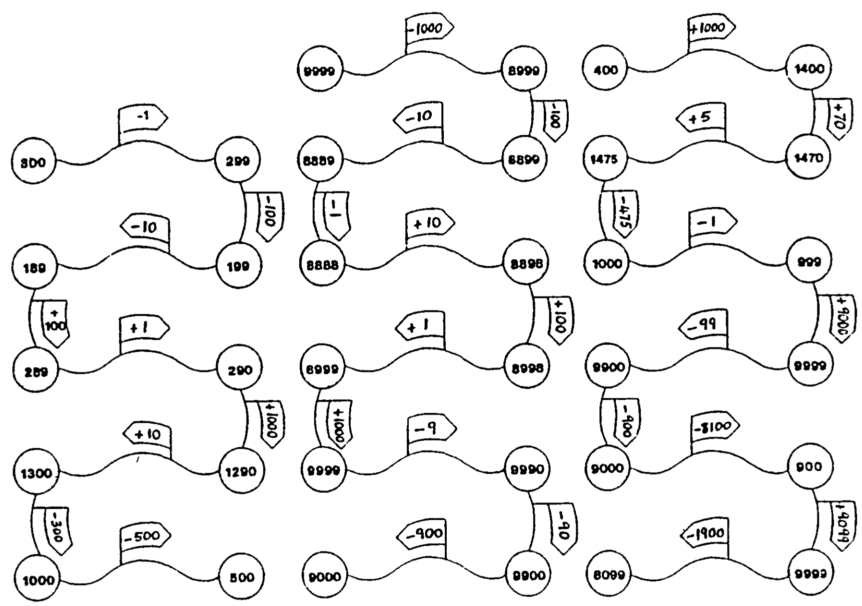
1635 The Key to Success

Several different answers are possible in some cases.
For example, 6 = 8 - 2
or 6 = 2 x 3

So if your answer is different - you may still be correct!

3 - 2 = 1	3 - 3 = 0	3 + 6 = 9
2 + 2 = 4	9 - 5 = 4	7 - 6 = 1
2 + 3 = 5	5 + 3 = 8	6 + 6 = 12
2 x 3 = 6	(9-5)x3=12	6 + 7 = 13
2 + 8 = 10	9 + 5 = 14	3 x 6 = 18
3 + 8 = 11	3 x 5 = 15	3 x 7 = 21
2 x 8 = 16	3 x 9 = 27	6 x 6 = 36
3 x 8 = 24	5 x 9 = 45	6 x 7 = 42

1636 Calculator Flags



1637 Squares and other Powers

$1^3 = 1^2 - 0^2$
$2^3 = 3^2 - 1^2$
$3^3 = 6^2 - 3^2$
$4^3 = 10^2 - 6^2$

There are some curious patterns in these columns.
You will probably have noticed that in the equations

[redacted]$^3 = a^2 - b^2$

the b^2 in one row is the same as the a^2 in the previous row... and both of these are triangle numbers. Are there similar patterns in the other columns?

If you choose to investigate the underlying feature of these powers, one particular identity is very useful. This is the identity $a^2 - b^2 = (a+b)(a-b)$.

In the equation $4^4 = 34^2 - 30^2$
the right hand side can be rewritten $(34+30)(34-30)$
which is 64×4 and you will notice that 64 is 4^3 and 4 is obviously 4^1.

Similarly in the equation $5^4 = a^2 - b^2$
with the right-hand side written as $(a+b)(a-b)$
it would be necessary for $(a+b)$ to be 5^3 and $(a-b)$ to be 5^1.

Can you choose $(a+b)$ to be 125 and $(a-b)$ to be 5?

This question may suggest one fruitful approach to this investigation.
Can you generalise it further?

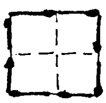
1638 Triumph

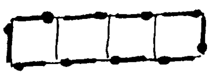
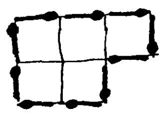
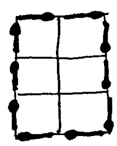
No answers required

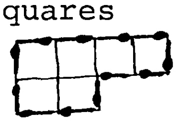
1639 Quarto

No answers required

1640 S & N

Not all size 4 diagrams need 10 sticks.
This one needs only 8 

10 sticks can enclose
4 squares...  or 5 squares...  or 6 squares 

There are several ways to enclose 6 squares with 12 sticks. Here is one example : 

If S = N you can make...a square when S = N = 16
...a rectangle or another shape when S = N = 18

1641 Lines

No answers required

1642 Triangles from Straws

These 6 triangles have longest side 4cm: (4,4,4) (4,3,3)
(4,4,3) (4,3,2)
(4,4,2)
(4,4,1)

There are 9 triangles with longest side 5cm, and so there is a pattern which begins

Longest side		No of different triangles
3	⟶	4
4	⟶	6
5	⟶	9

but this pattern is not quite so straightforward because there are less than 13 triangles with longest side 6cm...

1643 Lucky Dip

1. The 6th guess should always be correct because there is only one card left to turn over.
3. When there are less choices, the chance of being correct gets better.
5. In the 5th column, there are only 2 choices, so you would expect to be correct half the time: 50 out of 100.

1644 Initial Survey

S-Z	A-D	E-K	L-R

1645 Phone Books

S-Z	A-D	E-K	L-R

To decide how to regroup the directory into three volumes try measuring the four volumes together (or count the pages). Then divide by three and choose the letters which fit nearest to these divisions.

1646 Probability Kitty 2

No need to feel upset, Sarah. The reason that Kitty visits you only one Sunday out of every 10 is not personal, it is mathematical. You see, the way the timetable is arranged the train in your direction comes exactly 1 minute after the train in Irene's direction. If Kitty arrives in a 9 minute period between trains she will catch Irene's train and if she arrives in the one minute interval between trains, she will catch your train. This means that she is 9 times more likely to catch Irene's train than your train.

1647 Weaving

	A
B	C
D	E
F	G

C and G represent the same pattern from opposite sides.

1648 Number Clues

a = 8 b = 6
c = 5 d = 9
e = 4

1649 Walking to school

1. B lives near to school. (see diagram)
2. No, we do not have enough information yet.
3. Yes. If B lives 500m away from school, then A lives 1000m away.
4. Yes.
5. B lives nearer so will get to school before A.
6. A lives further away so will have to travel faster than B. (perhaps B walked and A took a bus!)
7. Steven is A and Lindsay is B because Steven lives twice as far as Lindsay.
8. Paula is C and Robyn is D because Paula lives four times farther than Robyn.

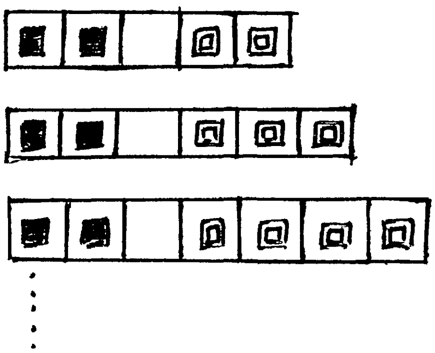
1650 Makehalf

No answers required

1651 Frog Investigation

If you can do the puzzles then the investigation will be worthwhile.

Make sure you work systematically - you might want to start with 2 counters on one side and change the number on the other side:



Or you might prefer to have the same number of counters on both sides...

1652 Jugs

No answers required

1653 Master

No answers required

1654 Racegame

No answers required

1655 The Factor Game

No answers required

1656 The Lost Divide

4 ÷ 17 = 0.2352941

1657 The Great Divide

56 ÷ 73 = 0.7671232

1658 The Smith Family

You will know when you have been successful because the final message spells "SOLVED".

1659 Mind Reversal

The clue to finding the connecting link is 9.
The multiples of 9 are 18,27,36,45,.... and the number to be added each time will be a repitition of these: 181818, 272727, 363636,....

The final question which needs answering is 'what is the connection between the multiple of 9 and the 2 original digits?'

1660 The Champion Flea

1. 1.5mm.
2. £60,000
3. a — 14.81 metres
   b — 20.25 acres
   c — 248.8 (249) people
4. 2.83 seconds
5. 5059 jumps

1661 Puzzlement

Ten Piles of Coins

If you take 1 coin from the first pile, 2 coins from the second pile, 3 coins from the third, and so on.....

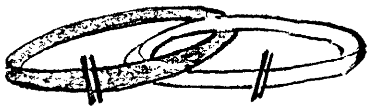
What would happen when you weighed the group of 55 coins?

Sand-timers

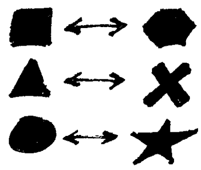
Time 7 minutes + 4 mins together. When the 4 mins runs out, there are 3 mins left on the 7 timer...

Chain links

If you cut both loops in this double link...



Three Cubes in One



40lbs

1lbs
3lbs
9lbs
27lbs

Jugs

You start with
9 + 7 + 5
and empty the 5 litre jug.
9 + 7 + 0
Fill the 5 litre jug from the 7 litres :
9 + 2 + 5
How can you get another 4 in the middle jug?

1662 Get to One

The example can be done in 3 operations: $28 \rightarrow \boxed{+3} \rightarrow \boxed{+3} \rightarrow \boxed{-33} \rightarrow 1$

Here are possible solutions for the three problems:

Problem 1 $\quad 55 \rightarrow \boxed{+6} \rightarrow \boxed{+6} \rightarrow \boxed{-66} \rightarrow 1$

Problem 2 $\quad 28 \rightarrow \boxed{+3} \rightarrow \boxed{+3} \rightarrow \boxed{-33} \rightarrow 1$

Problem 3 $\quad 24 \rightarrow \boxed{+9} \rightarrow \boxed{+9} \rightarrow \boxed{-99} \rightarrow \boxed{-99} \rightarrow \boxed{\div 99} \rightarrow 1$

1663 Largest and Smallest

1. There are 6 different numbers possible : 182 218 812
   128 281 821

   (a) The largest is 821
   (b) The smallest is 128

2/3. You will find that the largest number always has the largest digit first, then the next largest, etc.
The smallest number is always the reverse of the largest number.

4. The largest number is 9742
   The smallest number is 2479.

1664 Knights

5	4	5	4	5	4	5	6
4	3	4	3	4	5	4	5
3	4	3	4	3	4	5	4
2	3	2	3	4	3	4	5
3	2	3	2	3	4	3	4
2	1	4	3	2	3	4	5
3	4	1	2	3	4	3	4
0	3	2	3	2	3	4	5

1665 $(x+1)^2$

2. $3^2 + (2x3) + 1 = 4^2$
   $4^2 + (2x4) + 1 = 5^2$
   $5^2 + (2x5) + 1 = 6^2$
   $6^2 + (2x6) + 1 = 7^2$
   $7^2 + (2x7) + 1 = 8^2$
   $8^2 + (2x8) + 1 = 9^2$

3. $20^2 + (2x20) + 1 = 21^2$

4. $54^2 + (2x54) + 1 = 55^2$

5. $x^2 + 2x + 1 = (x+1)^2$

7. Yes, the identity is always true because
   $$\begin{aligned}(x+1)^2 &= (x+1) \text{ x } (x+1)\\ &= x(x+1) + 1(x+1)\\ &= x^2 + x + x + 1\\ &= x^2 + 2x + 1\end{aligned}$$

1666 Tower

No answers required

1667 Goldhunt

No answers required

1668 Mapping Puzzle

5	15	14	2
60	3	18	9
6	10	30	5

6	4	10
24	12	24
36	20	3
2	4	36

8	2	7	13
9	18	14	52
54	6	42	50

1669 Sim

No answers required

1670 Find the Fakes

Here is the answer which Anne Barnes and Yvonne Dove (from Lewisham School) wrote. *Do you agree with them?*

	groups of 1's	groups of 2's	groups of 3's	groups of 4's	groups of 5's	groups of 6's	groups of 7's
Artie	6	6	3	3	1	1	0
Joe	16	5	4	3	0	0	0
Maria	32	9	0	0	0	0	0
Simon	13	3	2	2	2	0	1
Debbie	8	11	2	1	2	0	0
Bob	26	12	0	0	0	0	0

from these results we can see that Maria and Bob have faked their results because their results only show groups of one or two heads or tails. They have no other types of groups. Everyone else has different types of groups.

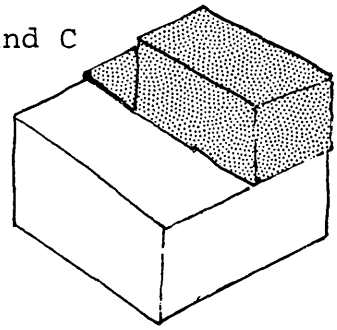
1671 20-piece jigsaw

You will know you are correct when you have completed the multiplication square!

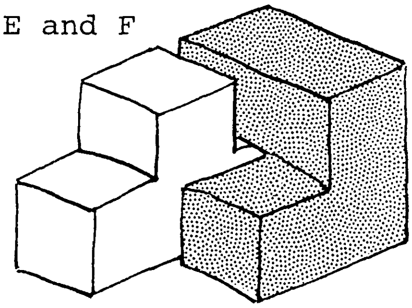
1672 Soma Solids

2.

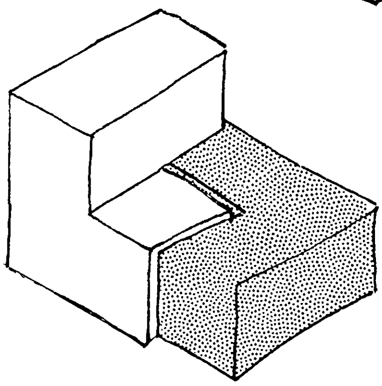
B and C



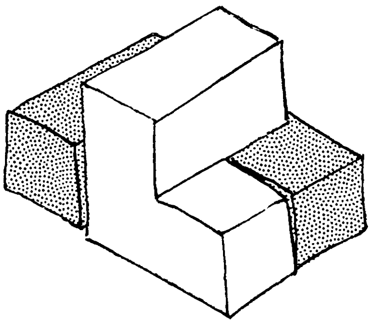
E and F



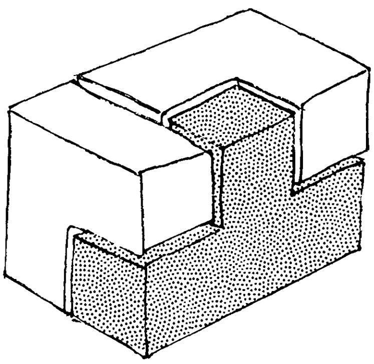
B and G



3. B and G. (You might have drawn them looking from the opposite direction.)



4.

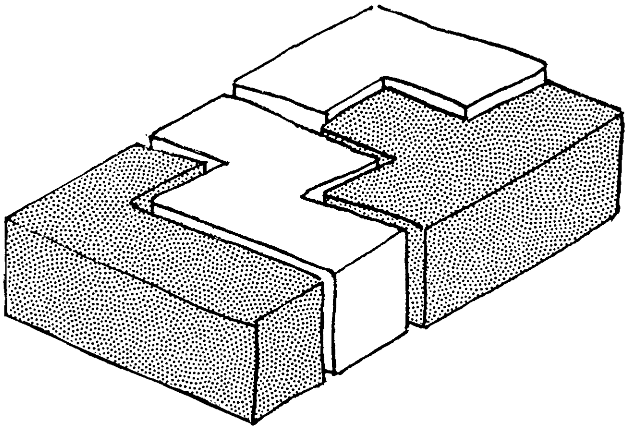


This is one arrangement of D,E and F which will make the cuboid.

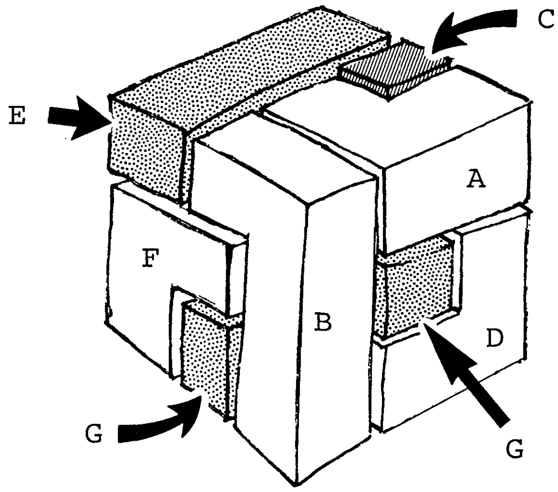
1672 Soma Solids (cont)

5. You might have already recognised that you cannot use D,F and G for this puzzle. In fact, you have to use all the others (A,B,C and E). *Why*?

   Here is one arrangement that is possible:



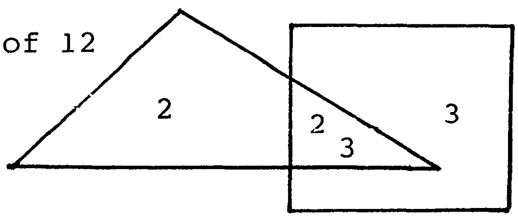
6. There are many solutions possible. Here is one:



1673 HCF and LCM

HCF	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1
2	1	2	1	2	1	2	1	2	1	2
3	1	1	3	1	1	3	1	1	3	1
4	1	2	1	4	1	2	1	4	1	2
5	1	1	1	1	5	1	1	1	1	5
6	1	2	3	2	1	6	1	2	3	2
7	1	1	1	1	1	1	7	1	1	1
8	1	2	1	4	1	2	1	8	1	2
9	1	1	3	1	1	3	1	1	9	1
10	1	2	1	2	5	2	1	2	1	10

Factors of 12

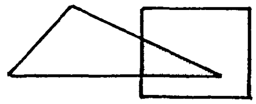


Factors of 18

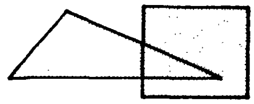
continued

1673 HCF and LCM (cont)

HCF is product of numbers in intersection



LCM is product of numbers in union

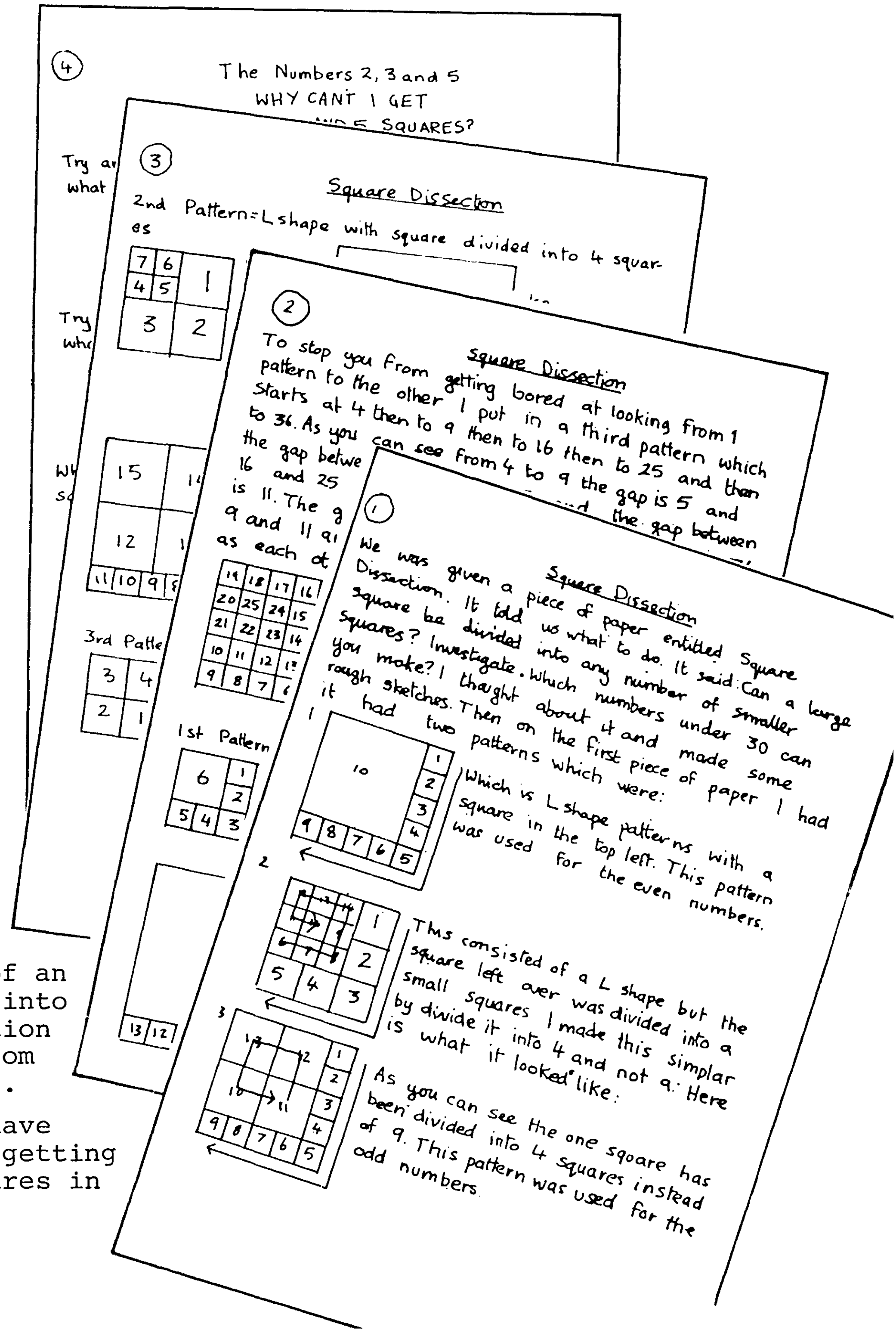


The product of the numbers 12 and 18 is product of their LCM and their HCF.

When you multiply LCM x HCF the numbers in intersection are multiplied twice.

1674 Square Dissection

Dissection into squares:



This account of an investigation into Square Dissection is by Gavin from Crofton School.

Did you also have difficulty in getting 2,3 and 5 squares in a square ?

1674 Square Dissection (cont)

Dissection of Triangles

Equilateral triangles can be dissected into equilateral triangles with the exception of sides 2,3 and 5.

Can isosceles triangles be dissected into isosceles triangles?

1675 Board Order

No answers required

1676 Pythagorean Triples

These are hints rather than complete explanations for each statement:

(A) All non-primitive Pythagorean triples are <u>multiples</u> of primitive Pythagorean triples.
(B) $(\text{odd})^2 + (\text{odd})^2 \neq (\text{odd})^2$
(C) $(\text{even})^2 + (\text{odd})^2 \neq (\text{even})^2, (\text{even})^2 + (\text{even})^2 \neq (\text{odd})^2$
(D) If x,y and z were all even they would have a common factor of 2, and then (x,y,z) would not be primitive.
(E) This is hard. If z were the even number, it would be equal to the sum of two odd squares, say $2k + 1$ and $2\ell + 1$

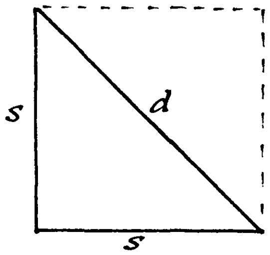
$$(2k + 1)^2 + (2\ell + 1)^2 = 4k^2 + 4k + 1 + 4\ell^2 + 4\ell + 1$$
$$= 2 \text{ in mod } 4$$

but z is the square of an even number and therefore $z = 0$ in mod 4, so z is not the even one.
(F) We haven't assumed anything special about x or y so it doesn't matter which "name" we give the even number.
(G) $x^2 + y^2 = z^2 \Rightarrow x^2 = z^2 - y^2$ (See SMILE 0818 if you're still not sure).
(H) (i) x is even by definition (See F)
(ii) $z + y = \text{odd} + \text{odd}$
$z - y = \text{odd} - \text{odd}$
(I) $x^2 = (z + y)(z - y)$ from statement G
$(2u)^2 = (2v)(2w)$
(J) z and y have no common factor
so $z + y$ and $z - y$ have no common factor apart from 2
so $\frac{z+y}{2}$ and $\frac{z-y}{2}$ have no common factor.
(K) This is hard. Imagine writing our v and w as products of their prime factors. Each prime must appear an even number of times (since vw is square) but if any prime appears in v's list it cannot appear in w's and vice-versa (since H.C.F. of v and w is 1).
(L) The prime factors of p are the same as the prime factors of v.
(M) $z + y = 2v$
$z - y = 2w$
$2z = 2v + 2w$ (by adding)
$z = v + w$
(N) $x^2 = z^2 - y^2 = (p^2 + q^2)^2 - (p^2 - q^2)^2$
$= p^4 + 2p^2q^2 + q^4 - q^4 + 2p^2q^2 - q^4$
(O) $p = \sqrt{v}$, $q = \sqrt{w}$, $v = \frac{z+y}{2}$, $w = \frac{z-y}{2}$

$\frac{z+y}{2} > \frac{z-y}{2}$ since y is positive.
(P) If p and q had the same parity then y would be either $(\text{odd})^2 - (\text{odd})^2$ or $(\text{even})^2 - (\text{even})^2$ and so would be even.

1677 Proof by Contradiction

These are hints rather than complete answers:

1. 

$$d^2 = s^2 + s^2$$
$$d^2 = 2s^2$$
$$\frac{d^2}{s^2} = 2$$
$$\frac{d}{s} = \sqrt{2}$$

2. $p^2 = 2q^2$ so p^2 is even.... an odd number squared is still odd so p must be even.

4. $$\left.\begin{matrix} p^2 = 4k^2 \\ \text{and} \\ p^2 = 2q^2 \end{matrix}\right\} \Longrightarrow 4k^2 = 2q^2$$

   By the same argument as in (2), q must be even.

5. By assuming that $\sqrt{2} = \frac{p}{q}$, we have shown that $\frac{p}{q}$ would have a common factor, so $\sqrt{2}$ can never be written as a "cancelled down" fraction.

1678 Build

No answers required

1679 Spheres

Straight Lines

page 5: The two parallel rails are not straight lines on the sphere. Straight lines on a sphere are defined as being "great circles" of the sphere — Otherwise they do not give the shortest distance between 2 points.

page 6: Flat surface

Lines	1	2	3	4	5	6	7	8	9
Regions	2	4	7	11	16	22	29	37	46

page 7: Sphere

Lines	1	2	3	4	5	6	7	8	9
Regions	2	4	8	14	22	32	44		

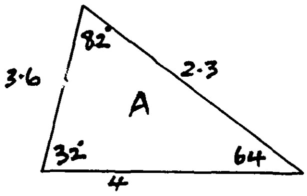
A new band (the nth band) cuts through 2(n-1) regions and misses $2(T_{n-3})$ regions (where T_n is the nth triangle number).
The total no. of regions for the nth band is therefore $4(n-1) + 2T_{n-3}$ which simplifies to $(n^2 - n+2)$.

page 8: The thinner the ruler, the more accurate it will be. To be absolutely accurate it would have to have zero thickness, so in practical terms it could never be completely accurate.

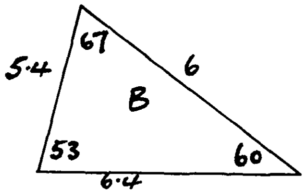
Triangles

page 2: The curved triangle has 3 right angles and so the angle sum of this triangle is 270^0.

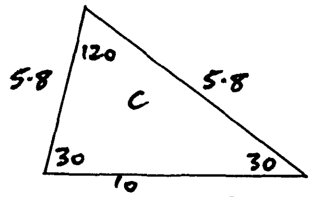
page 3:



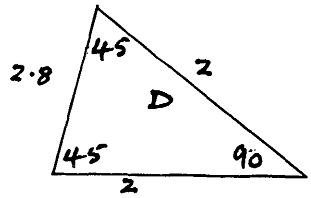
Area = 4 cm²



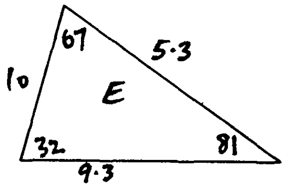
Area = 15 cm²



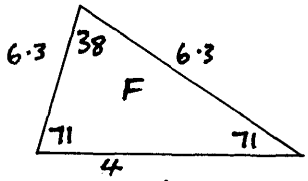
Area = 15 cm²



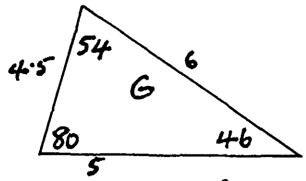
Area = 2 cm²



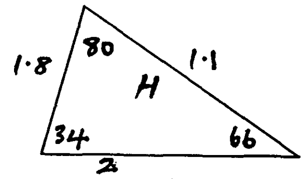
Area = 25 cm²



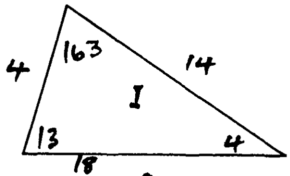
Area = 12 cm²



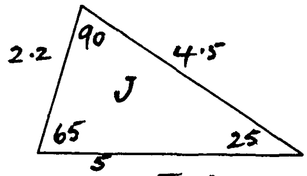
Area = 11 cm²



Area = 1 cm²



Area = 9 cm²



Area = 5 cm²

pages 6 and 7:

Statement	Triangles on Flat paper	Triangles on a sphere
1	True	False
2	False	False
3	True	True
4	True	True
5	True	True
6	False	False
7	True	True
8	False	True
9	True	False

page 8:

	A	B	C	D	E
Sum of angles	195	210	225	270	360
Difference from 180^0	15	30	45	90	180
No. of triangles to cover sphere	48	24	16	8	4

If the angle sum is 15^0 more than 180^0, the triangle is $\frac{1}{48}$ of the sphere's surface.

If the angle sum is 30^0 more than 180^0, the triangle is $\frac{1}{24}$ of the sphere.

This suggests that if the angle sum is 60^0 more than

continued

1679 Spheres (cont)

180^0, the triangle would be $\frac{1}{12}$ of the sphere's surface.

In algebraic language, if the triangle is 15n degrees more than 180^0 then its area is $\frac{n}{48}$ of the sphere's surface.

Maps

page 2: A cylinder can be made by bending.
A cone can be made by bending and cutting or by bending and stretching.
For a cube the paper would need cutting and folding.
A sphere would need a lot of stretching!

page 3: 1. (a) Saudi Arabia
(b) China
(c) USSR
(d) Tanzania
(e) Mauritania

2. No
3. Yes
4. No

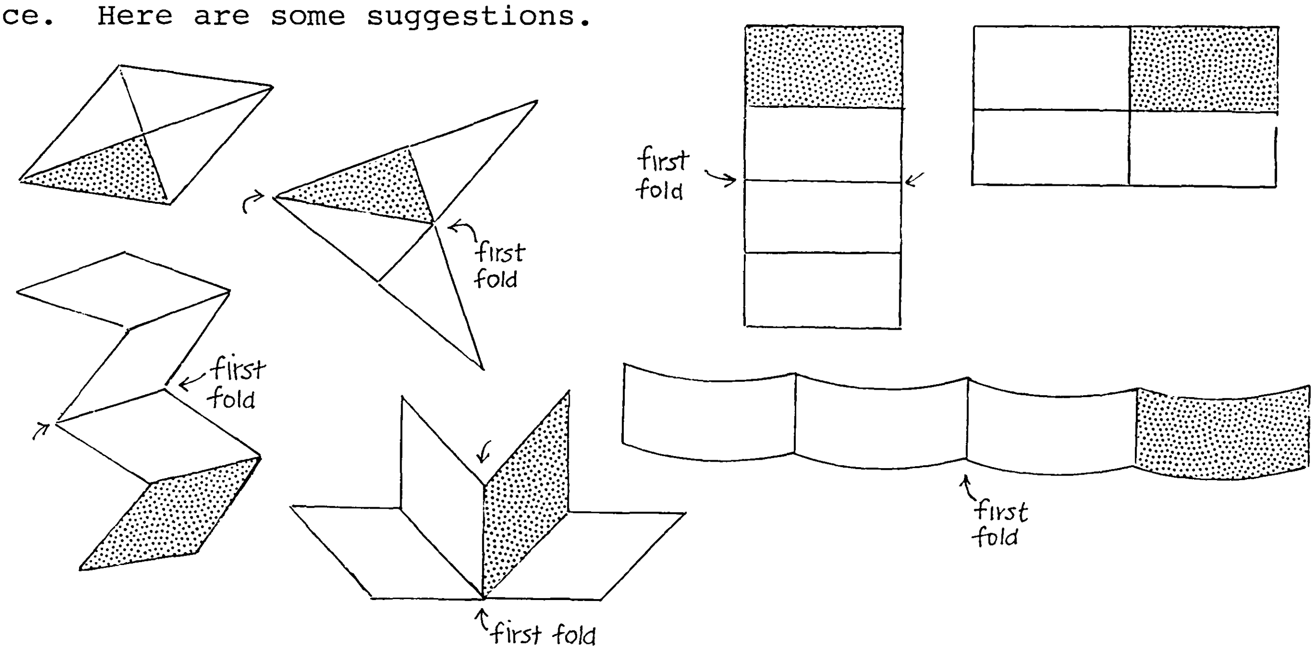
page 7: The distances are definetely not the same lengths on the maps and so none of them are reliable for measuring distances.
The polar map indicates the direct route from London to Tokyo but it would not be accurate for a route near the equator, like Georgetown to Mogadishu.
None of the maps are reliable for measuring all areas, angles or distances because, in each one, one part of the globe has been stretched more than the other.

1680 Reflect-a-bug

Pictures 1,3,5,6 and 10 can all be made by using the mirror.

1681 Folding

There are several ways of making the first 3 shapes by folding twice. Here are some suggestions.



The semi-circle cannot be made by folding a shape twice.

1682 Number Jumble

a = 8
b = 21
c = 5
d = 3
e = 11
f = 10
g = 16

1683 A Squares Puzzle

15	3
21	12

5	15
12	4

7	4
21	3

We'll leave you to work out which square is which !

1684 A Problem of Power

Powers

1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1
2	4	8	6	2	4	8	6	2
3	9	7	1	3	9	7	1	3
4	6	4	6	4	6	4	6	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	9	3	1	7	9	3	1	7
8	4	2	6	8	4	2	6	8
9	1	9	1	9	1	9	1	9

The 5th column is the same as the first and the columns repeat after that.

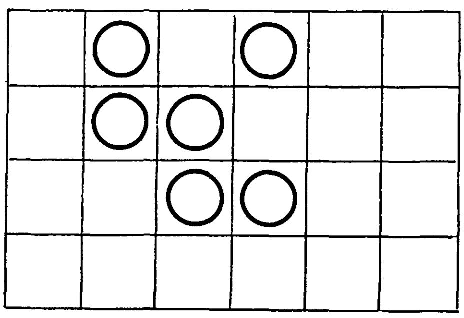
3111696 is a 4th power (42^4)

1601613 is a cube (117^3)

4464769 is a square (2113^2)

1685 Milk Crate

Here is one solution which shows the six empty places rather than the 18 bottles.



(Because there are an even number of spaces in each row of the crate, it is easier to move around 6 "empty" counters rather than 18 "bottle" counters.)

1686 Square

The designs on all four posters have all been developed from the Takehalf micro film (1650A) in which half the square is always black and half is white.

On posters 1,2 and 4 each individual square is divided in two equal areas by the two colours.

On poster 3 the areas have been divided into 4 colours which give a variety of "pyramid" effects. As far as their areas are concerned purple + red = orange + yellow.

1687 Change

Only 1p and 3p cannot be made using the coins.

1688 Square Jigsaw

If you have had any problems doing this difficult puzzle, take a look at SQUARE 4 in the poster pack 1686.

1689 Fraction Flags

1. Poland
   - $\frac{1}{2}$ red
   - $\frac{1}{2}$ white
2. Holland
   - $\frac{1}{3}$ red
   - $\frac{1}{3}$ white
   - $\frac{1}{3}$ blue
3. Mauritius
   - $\frac{1}{4}$ red
   - $\frac{1}{4}$ blue
   - $\frac{1}{4}$ yellow
   - $\frac{1}{4}$ green
4. Belgium
   - $\frac{1}{3}$ black
   - $\frac{1}{3}$ yellow
   - $\frac{1}{3}$ red
5. Dahomey
   - $\frac{1}{3}$ green
   - $\frac{1}{3}$ yellow
   - $\frac{1}{3}$ red
6. UAR
   - $\frac{1}{4}$ red
   - $\frac{1}{4}$ green
   - $\frac{1}{4}$ white
   - $\frac{1}{4}$ black
7. Nigeria
   - $\frac{2}{3}$ green
   - $\frac{1}{3}$ white
8. Uganda
   - $\frac{1}{3}$ black $(\frac{2}{6})$
   - $\frac{1}{3}$ yellow $(\frac{2}{6})$
   - $\frac{1}{3}$ red $(\frac{2}{6})$
9. Czechoslovakia
   - $\frac{1}{4}$ blue $(\frac{2}{8})$
   - $\frac{3}{8}$ white
   - $\frac{3}{8}$ red
10. Thailand
    - $\frac{1}{3}$ blue $(\frac{2}{6})$
    - $\frac{1}{3}$ white $(\frac{2}{6})$
    - $\frac{1}{3}$ red $(\frac{2}{6})$
11. Sharjah
    - $\frac{3}{10}$ red $(\frac{6}{20})$
    - $\frac{7}{10}$ white $(\frac{14}{20})$
12. Switzerland
    - $\frac{11}{16}$ red $\frac{44}{64}$
    - $\frac{5}{16}$ white $\frac{20}{64}$

1689 Fraction Flags (cont)

13. The flags spell GOOD TRY. It is not always necessary to spell words because special combinations of flags are used for common messages. The combination PYX means "GOOD VOYAGE".

With modern communications flags are unnecessary for relaying information but this extract from "The Observer Book of Flags" describes some signal flags still in use:

> *These flags enable an incredible variety of signals to to given, being displayed singly or in hoists of from two to five; the fewer their number the more urgent is the signal. The two swallow-tailed flags act as warnings, A that a vessel is undergoing a speed trial and B that it is taking in or discharging explosives. Flag D is also a warning that a ship is manoeuvring with difficulty. K warns another vessel that it should "stop instantly" and U that it is "standing into danger". O conveys the grim message "man overboard!" Other flags request help, F because a ship is disabled, V because it requires assistance, and W because it requires medical assistance.*
> *Two flags are frequently seen in harbours, Q which means that a vessel declares itself healthy and requests practique (permission to enter port), and H (or similar flag divided horizontally, white over red) that it carries a pilot. Probably the most widely known of all such flags is the famous "Blue Peter", the P which announces that a vessel is about to sail.*

14. F: $\frac{1}{2}$ is white

15. U: $\frac{1}{2}$ is red

16. N: $\frac{1}{2}$ is blue

17. A: $\frac{2}{5}$ $(\frac{8}{20})$ is blue

Kuwait: $\frac{1}{6}$ is black
$\frac{1}{4}$ is white
$\frac{7}{24}$ is red
$\frac{7}{24}$ is green

Guyana: $\frac{1}{4}$ is red
$\frac{1}{4}$ is yellow
$\frac{1}{2}$ is green

Brunei: $\frac{1}{4}$ is white
$\frac{1}{4}$ is black
$\frac{1}{2}$ is yellow

1690 Logical Kitty

Surprisingly enough only 4 socks are needed.
If the first three are all different then a fourth one will make a pair with one of them.

1691 Predict

No written answers required.

1694 Pools, Floods and Seeds

No answers required.
You will find it useful to make some notes about
(i) the idea of probability
(ii) relative frequencies

1696 Car Trial Results

		A	B	C	D
TRIAL 1	TIME	6	12	4	8
	SPEED	80	40	120	60
TRIAL 2	TIME	3	6	2	9
	SPEED	80	40	120	$26\frac{2}{3}$
TRIAL 3	TIME	9	18	12	15
	SPEED	80	40	60	48
TRIAL 4	TIME	7.2	14.4	10.8	9.6
	SPEED	96	48	64	72

1697 Motor Cycle Ratios

page 2: 6th gear
8000 rpm corresponds to 80 mph
3500 rpm corresponds to 35 mph
6500 rpm corresponds to 65 mph

1st gear
3000 rpm corresponds to 10 mph
1500 rpm corresponds to 5 mph
3750 rpm corresponds to $12\frac{1}{2}$ mph

In 6th gear 9500 rpm means a top speed of 95 mph.
In 1st gear 9500 rpm means a top speed of 31.6 mph.

page 3: In 2nd gear 9500 rpm means a top speed of 38 mph.

page 4: In 3rd gear the top speed is 47.5 mph.

page 5: In 4th gear the top speed is 59 mph.

1697 Motor Cycle Ratios (cont)

page 6:

Gear	Engine Speed at 30 mph
1	9000
2	7500
3	6000
4	4800
5	(3800 would be a good estimate)
6	3000

page 7:

Gear	Top Speed	Lowest Speed
1	31.6	6.6
2	38	8
3	47.5	10
4	59	12.5
5	75	15.8
6	95	20

Changing gears at the top speed for each gear would give the maximum acceleration.

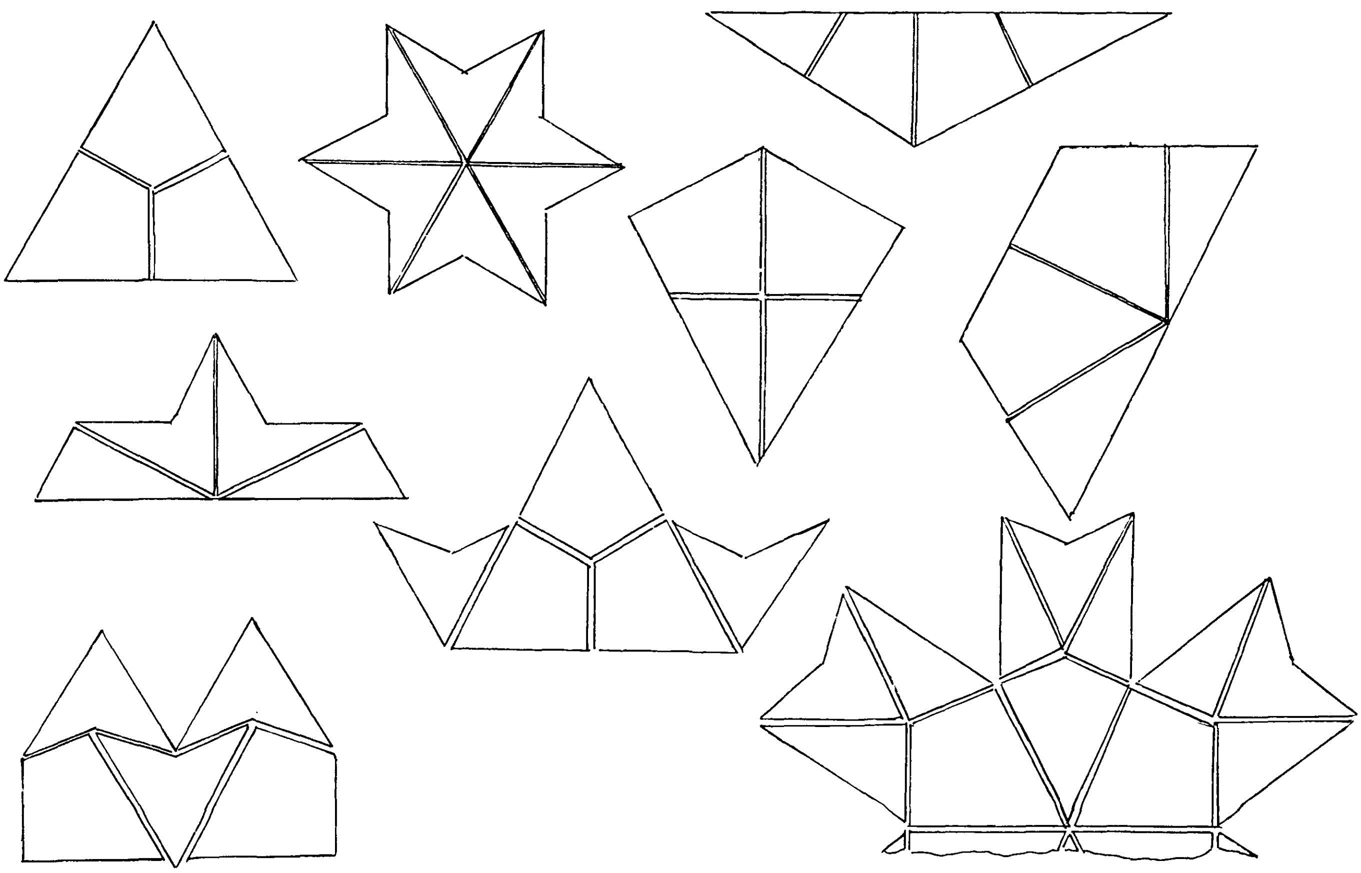
1698 Identikit

The test of your description is whether a stranger can identify what you have described!

1699 Fifteen Game

No answers required.

1700 Fitting



1701 Posthalf

No answers required

1702 Circle

No answers required

1703 Find the Murderer

The murderer is the man in the bottom left-hand corner.

1704 Combined Probability

1. Left fork at C : 200
   Through E : 1000
   Through D : $\frac{1400}{2400} = \frac{7}{12}$

2. Through L : $\frac{7}{24}$

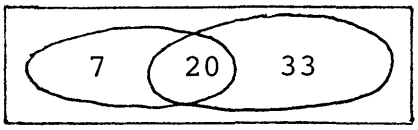
   Through P, $\frac{3}{16}$; through R $\frac{13}{16}$; through P or R is therefore 1.

3. If the leaf takes the left hand channel at A it has a probability of $(\frac{1}{5} \times \frac{1}{2})$ of being diverted to D at C. If the leaf takes the right hand channel at A it can only reach D. So the probability of the leaf reaching D is $\frac{9}{10}$ $\{= (\frac{1}{5} \times \frac{1}{2}) + (\frac{4}{5})\}$

   You could have worked this out by finding that the probability of the leaf reaching E is $\frac{1}{10}$ $(= \frac{1}{5} \times \frac{1}{2})$ and subtracting this from 1.

1706 Think!

1. Some people bought both fish and chips.



   As you can see from this Venn diagram 20 people bought both.

2. 5 of the triangular stamps were not European.
   9 of the European stamps were not triangular.

3. 7 people watched neither BBC 1 nor ITV.
   However some of these may have watched BBC 2 or Channel 4.

4. 8 patients might have complained of both sniffles and headaches - but they might have had different illnesses!

5. 102 passengers.

6. 32 chocs altogether.

7. 22%, unless any of them are dumb or too young to speak.

1707 Graph Matching

A - 4	E - 7
B - 3	F - 1
C - 8	H - 6

1708 Factor

No answers required.

1709 Ratio Problems

A Holiday Villa

3 weeks costs £165...
so 1 week costs £55...
so 4 weeks would cost £220.

B Paint

$24m^2$ is covered by 2.5 litres...
so $12m^2$ would be covered by 1.25 litres...
so $36m^2$ would be covered by 3.75 litres...
They would need to buy 4 litres (perhaps a 5 litre can would be cheaper).

cont/

1709 Ratio Problems (cont)

C Boat Trip

126 Francs was about £12
so 58 Francs would be roughly £6.

D Exchange Rate

£42 is just under three times £15...
so 3 x 2790 would be a good guide.
3 x 2790 is less than 9000
so 8000 pesetas would be a good approximation for £42.

E Phone Call

45 seconds costs 5p...
$1\frac{1}{2}$ minutes costs 10p...
so 15 minutes would cost £1.

F Petrol

135 miles used 18 litres...
so 1 mile uses $\frac{18}{135}$ litres... that's $\frac{2}{15}$
so 75 miles would need $\frac{2}{15} \times 75 = 10$.

Assuming that the petrol is used up at the same rate, they would need at least 10 litres.

1710 Pencils

1. Latifa's pencil is F.
2. Chris' pencil is D.
3. Kate's pencil is A.
4. Pencil E belongs to Sean.
5. Latifa's pencil is half the length of Chris'.

The shortest pencil belongs to Kim. So Kim's pencil must be B. It is half the length of Sean's pencil. Abdul's pencil must be C. This is the longest pencil. It is twice as long as Chris' pencil.

1711 Missing Digits

1. 93 x 86 = 7998
2. 837 x 49 = 41013
3. 8013 - 4138 = 3875
4. 1058 ÷ 23 = 46
5. 276 x 843 = 232668
6. 383 x 37 = 14171
7. 3984 ÷ 83 = 48
8. Impossible
9. 23 x 34 x 17 = 13294
10. 23 x 7 x 7 x 34 = 38318

1712 Four Signs

1. (37x21) + 223 = 1000
2. (756 ÷18) x 29 = 1218
3. 27 + (36x18) = 675
4. 31 ~~37~~ x (87-19) = 2108
5. 476 - (2040 ÷ 24) = 391
6. (3461+276) ÷ 101 = 37
7. (967+34)x(1023-654) = 369369
8. $(2^9+8^2) \div 9 = 64$
9. 619 - 316 + 425 + 196 = 924
10. 6975 ÷ (36 + 39) = 93

1713 Subzero

No answers required.

1714 Queens

These problems are quite difficult. You will find the micros program "Queens" very helpful. It enables you to place queens and to move them around different sized boards. As well as colouring in squares which are under the queen's control, the program also allows you to wipe out any queen which you want to change.

1715 Locate

No answers required.

1716 Unibond Mixtures

Problem A

1. Kristy only needs 2 pints (not 6 pints)
2. Yes. $1\frac{2}{3} : \frac{1}{3}$ is the same as $\frac{5}{3} : \frac{1}{3}$ or 5:1
3. 5 parts glue x 1 part water = 6 parts solution.
   So fraction of water = $\frac{1}{6}$.

Problem B

1. 5 pints water.
2. 1 pint glue.
3. glue is $\frac{1}{6}$ of solution.
4. for 3 pints solution
   $2\frac{1}{2}$ pints water
   $\frac{1}{2}$ pint glue

Problem C

1. 3 pints solution C
   $2\frac{1}{4}$ pints glue
   $\frac{3}{4}$ pint water

Problem D

1. 6 pints solution A
   5 pints water
   1 pint glue

   for stiffening
   20 pints water
   1 pint glue

   So $7\frac{1}{2}$ pints of water must be added.
2. 20 pints water
   1 pint solution
   21 pints solution

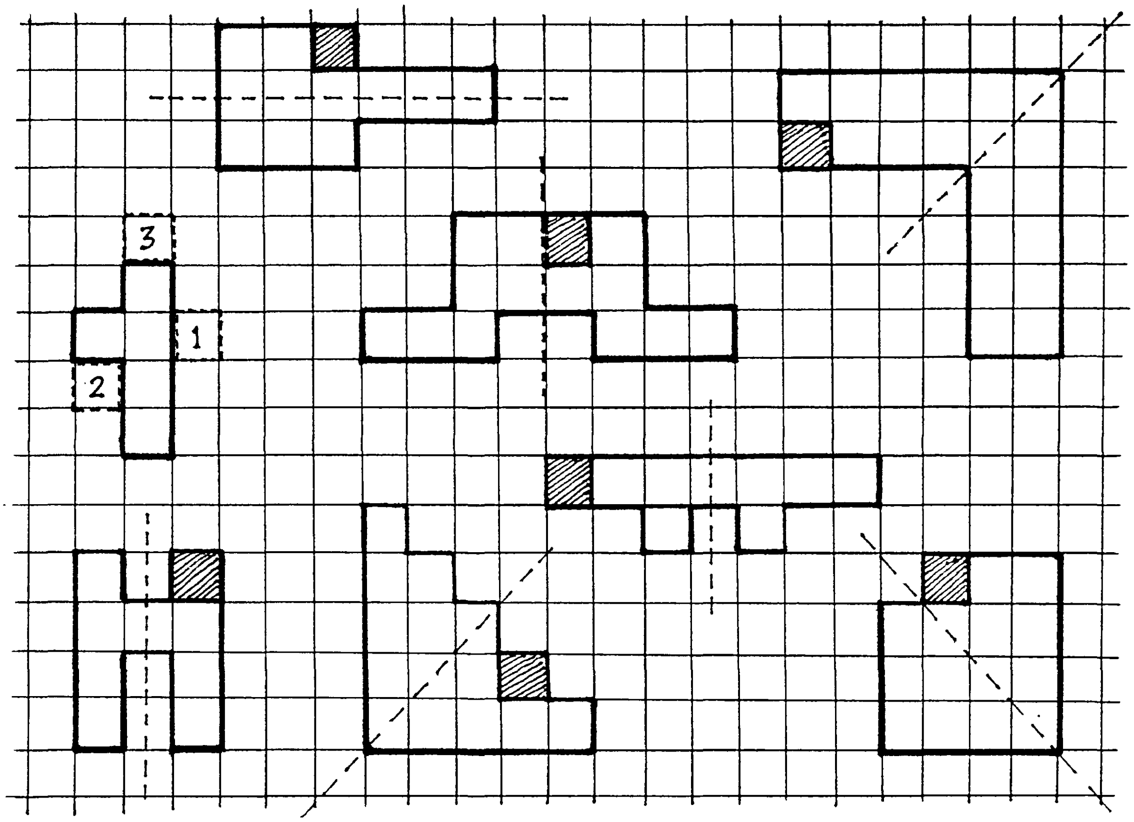
Problem E

1 litre weak solution (1:5)
$\frac{1}{6}$ L glue
$\frac{5}{6}$ L water

6 litres bonding solution
$4\frac{1}{2}$ L glue
$1\frac{1}{2}$ L water

1. glue used $(\frac{1}{6} + 4\frac{1}{2})$L = $4\frac{2}{3}$L
2. glue bought :5L would probably be the best **buy**.

1717 Add a Square



Take a square

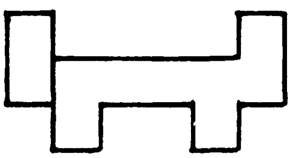
When you have found which square to remove to make each of the shapes symmetrical...

... is the line of symmetry different?

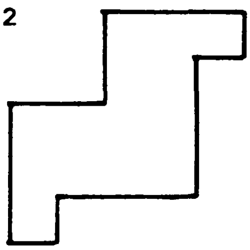
... what connection is there between the square added in part 1 and the square taken away in part 2?

1718 Symmetry Puzzles 1-4

1



2



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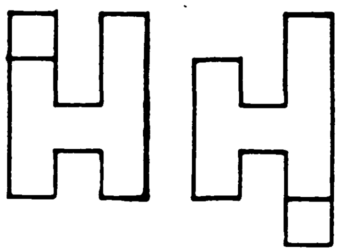

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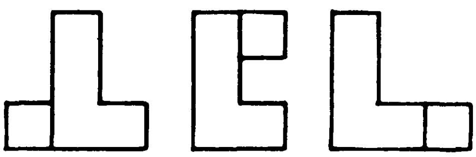
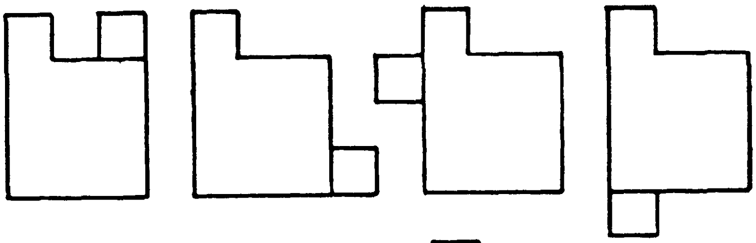
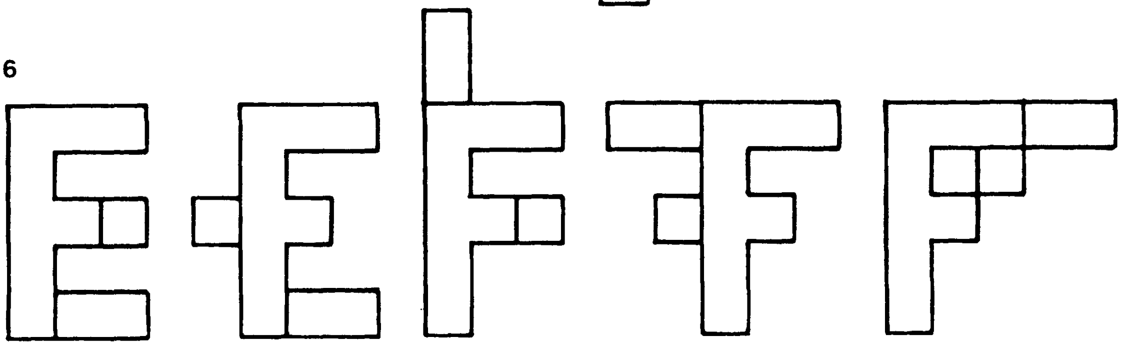


1719 Symmetry Puzzles 5-10

5



6



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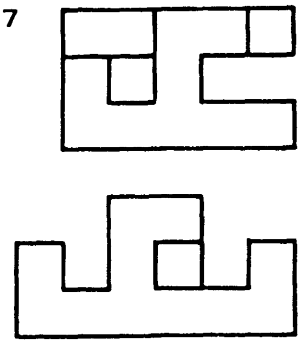

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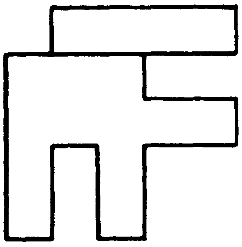

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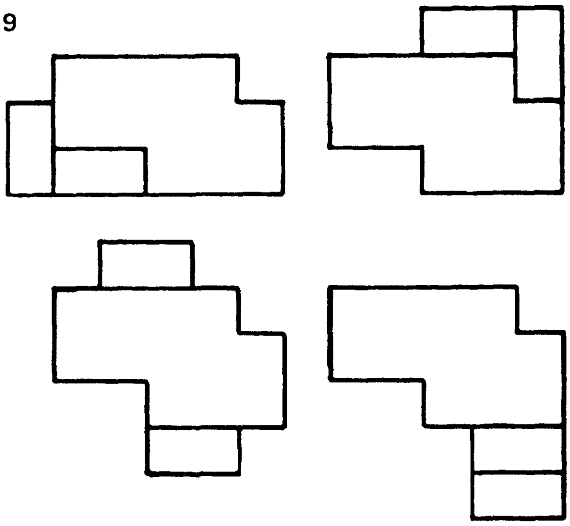

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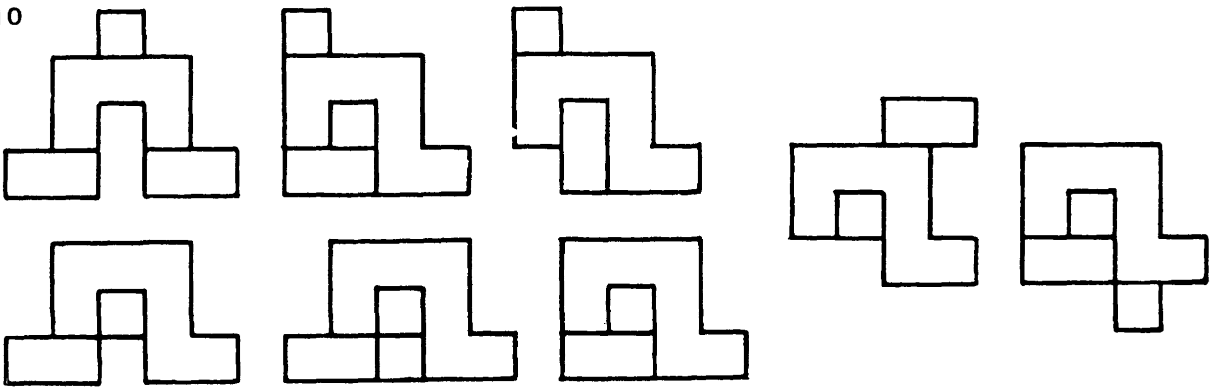


1720 Centi-cube Surprise

1. (a) 27
   (b) 27g
   (c) 40½p

2. (a) 1000
   (b) 1kg (=1000g)
   (c) £15

3. (a) If you placed 1 cube per second continuously for 1,000,000 seconds it would take 277.77 hours or 11.57 days to build it!
   (b) It would weigh 1 tonne so it would be impossible for a person to lift.
   (c) Probably not - it would cost £15000.

1721 Rhino

No answers required

1722 How Many Cubes?

1.	4	5.	4	9.	4	13.	5
2.	2	6.	6	10.	5	14.	8
3.	4	7.	6	11.	7	15.	4
4.	4	8.	6	12.	9	16.	4

1723 Getting Closer

No answers required.

1724 Digit Division

There are several answers for a 5 digit number even if you are limited to starting with 7...

72365 74125
72645 ...
72685 ...
72965

It gets progressively more difficult the longer the number which you try to make. Here is one nine-digit number - 381654729.

1725 Closest Product

No answers required.

1726 Dividing Pairs

No answers required.

1727 Point Circles

You will need to organise your investigation; a table of results should be useful...

12 point circles (Starting Number 1)			
	Jump size	Shape	Points Visited
	4	Triangle	3
	5	Star	12
	9	Square	4

cont/

1727 Point Circles (cont)

... in fact you may need several tables to help you.

8 point circles (Starting Number 1)			
	Jump size	Shape	Points Visited
	1	Octagon	8
	2	Square	4
	3	Star	8

There is a definite connection between the number of points on the circle and the jump size which produces regular geometric shapes. You may be able to see how this can be extended more abstractly to describe the star patterns too.

1728 Boxed

No answers required.

1729 Minimax

No answers required.

1730 Wall

No answers required.

1731 Rose

No answers required.

1732 3D Maze

No answers required.

1733 An Even Code

1. USE THE CODE
2. KEEP SMILING
3. TWO AND TWO MAKE FOUR
4. I CAN USE THE EVEN CODE
5. EVEN NUMBERS ARE ALL MULTIPLES OF TWO

1734 An Islamic Design

There are 3 squares for every 2 triangles.
The ration of hexagons : squares : triangles is 1:3:2

1735 Centimetres

(a) 3cm
(b) 6cm
(c) 5cm
(d) 6cm
(e) 7cm
(f) 8cm
(g) 8cm
(h) 8cm
(i) 8cm
(j) 12cm
(k) 12cm
(l) 40cm

1736 Algebra Pairs

A

1. There is no possible answer
2. $x = 1\frac{1}{3}$
3. $x = 3$
4. $y = 20$
5. No answer
6. Any number will work for p
7. $q = 0$
8. No answer
9.
10.
11.
12. many possible answers

B

1. Sometimes
2. Always
3. Sometimes
4. Always
5. Sometimes
6. Always
7. Sometimes
8. Sometimes
9. Sometimes

C

$$\left.\begin{aligned} \tfrac{1}{2}(a+b) &= \tfrac{1}{2}a + \tfrac{1}{2}b \\ \tfrac{1}{2}a + b &= \frac{a}{2} + b \\ a + \tfrac{1}{2}b &= \frac{b}{2} + a \end{aligned}\right\}$$ these are always equal

---

1737 Route Six

There are three routes which give a product of 6:

(a) 2, $\frac{1}{2}$, $\frac{1}{2}$, 4, 3

(b) 2, $\frac{1}{2}$, 2, 3

(c) $1\frac{1}{2}$, 1, 4

1738 Calcumaze

No answers required

1739 Again and Again

There is a limit which each process like $\boxed{\div 5}$ $\boxed{+1}$ approaches. You can more easily recognise a connection between the limit and the two numbers which govern the process if you write it as a fraction rather than a decimal.

PROCESS	Decimal Limit	Limit as fraction
÷5 + 1	1.25	$\frac{5}{4}$
÷5 + 2	2.5	$\frac{10}{4}$
÷5 + 3	3.75	$\frac{15}{4}$
÷11 + 3	3.3	$\frac{33}{10}$

1740 About How Much?

Milk bottle	1kg	Adult person	80kg
Hair	0.1g	Baby	3kg
Jacket	1kg	Cup of sugar	30g
Bus	8 tonnes	Ruler	20g
Paper clip	1g	Calculator	150g
4 10p coins	50g	Shoe	500g
Chair	2kg		

1741 Make-Half

No answers required.

1742 The Game of 20

No answers required.

1743 Decimal Products

The biggest product for 2 numbers which add up to 1 is 0.25
That's 0.5 x 0.5.

The smallest product is zero - that's 0 x 1.

If you have continued the investigation for 2 $\begin{pmatrix}\text{biggest } 1\times1\\ \text{smallest } 0\times2\end{pmatrix}$, for 3 $\begin{pmatrix}\text{biggest } 1.5\times1.5\\ \text{smallest } 0\times3\end{pmatrix}$ and for decimals 1.5 $\begin{pmatrix}\text{biggest } 0.75\times0.75\\ \text{smallest } 0\times1.5\end{pmatrix}$ you will not need much convincing that the smallest product is always zero. After a few more numbers, you will probably know how to find the biggest product for any number split into 2 decimal parts - what is the rule?

What is the rule for finding the biggest product if the number is split into 3 decimal parts? - or 4 decimal parts?

1744 Yes/No

You might have tried some of these questions:

Game 1
Is it a triangle?
Does it have four sides?
Does it have a right angle?
Does it have symmetry?

Game 2
Does it have two nodes (dots)?
Does it have five arcs?
Does it have any 5-nodes?

Game 3
This is difficult. Have you tried games 1 and 2?

1745 Identify

No answers required.

1746 Define

No answers required.

1747 Darts

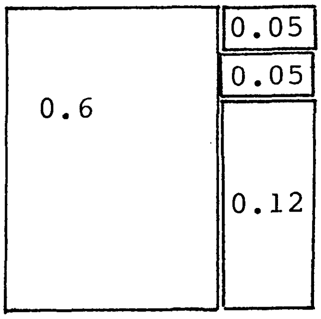
No answers required

1748 Taxi

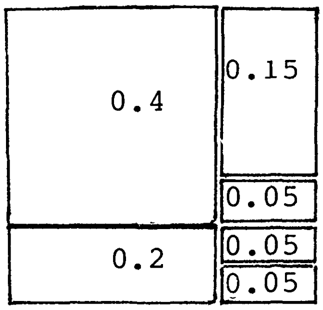
No answers required

1749 Decimal Jigsaw

1. Here are two ways of making a 0.9 square:

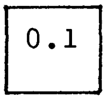


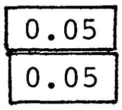
0.6
0.2
0.05
0.05
0.9



0.4
0.2
0.15
0.05
0.05
0.05
0.9

2. Here are two ways to make a 0.1 square:

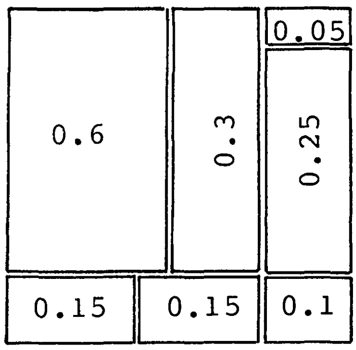




3. Here are all the ways to make a 0.4 square:

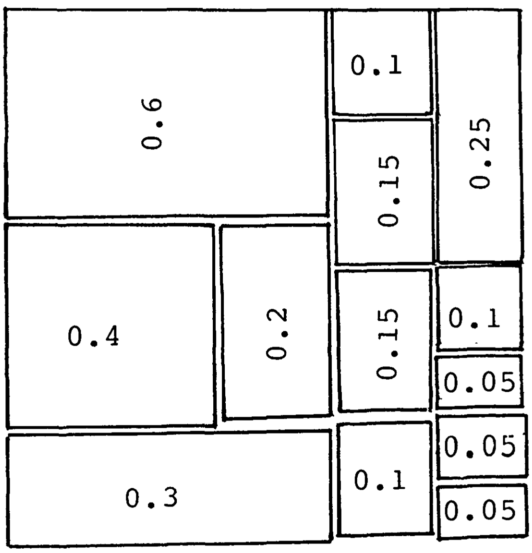
0.4	0.2	0.2	0.2	0.15	0.15	0.15	0.1
	0.15+	0.1	0.1	0.15	0.1	0.1	0.1
	0.05	0.1	0.05	0.05	0.1	0.05	0.1
	0.4	0.4	0.05	0.05	0.05	0.05	0.05
			0.4	0.4	0.4	0.05	0.05
						0.4	0.4

4. Here is one way to make a 1.6 square:



0.6
0.3
0.25
0.15
0.15
0.1
+ 0.05
1.6

5. A 2.5 square: this example uses all the pieces.



0.6
0.4
0.3
0.25
0.2
0.15
0.15
0.1
0.1
0.1
0.05
0.05
+ 0.05
2.5

1750 Layers

1.	6		8.	16	(*four times q.6*)
2.	12	(*double q.1*)	9.	28	(*seven times q.6*)
3.	18	(*three times q.1*)	10.	5	
4.	11		11.	30	(*six times q.10*)
5.	33	(*three times q.4*)	12.	10	
6.	4		13.	80	(*eight times q.12*)
7.	8	(*twice q.6*)	14.	200	

1751 Decimal Lists

1. 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2
2. 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6
3. 0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 2.8, 3.2, 3.6, 4.0, 4.4, 4.8, 5.2
4. 0.3, 0.6, 0.9, 1.2, 1.5, 1.8, 2.1, 2.4, 2.7, 3.0, 3.3, 3.6, 3.9
5. 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2
6. 1.5, 3, 4.5, 6, 7.5, 9, 10.5, 12, 13.5, 15

7.	8	13.	12
8.	10	14.	5
9.	12	15.	6
10.	10	16.	4
11.	13	17.	12
12.	12	18.	6

1752 Under a magnifying glass

beetle	1cm or 10mm.
caterpillar	1.2cm (12mm)
spider	0.6cm (6mm)
blowfly	0.8cm (8mm)
ant	0.3cm (3mm)
ladybird	0.5cm (5mm)
flea	0.2cm (2mm)
snail	1.1cm (11mm)

1753 Matching Pairs

A - 70	E - 73	J - 70
B - 72	F - 72.5	K - 73
C - 75	G - 75	
D - 72	H - 73.5	

So the four pairs are
A and J
B and D
C and G
E and K

1754 Chinese Number Puzzle

The puzzle contains the Chinese numbers 1 to 35 written in rows of 5.

三
十
五 is the number 35. 63 is written 六
十
三

1755 Hopslide

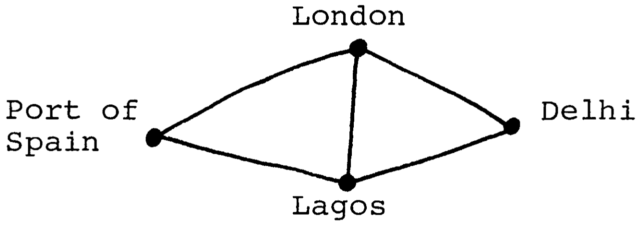
No answers required

1756 Tadpoles

No answers required

1757 Airline Networks

1.



2.

to / from	Delhi	Lagos	London	Port of Spain
Delhi	0	1	1	0
Lagos	1	0	1	1
London	1	1	0	1
Port of Spain	0	1	1	0

3.

to / from	Bombay	Calcutta	Delhi	Madras
Bombay	0	0	1	1
Calcutta	0	0	1	1
Delhi	1	1	0	0
Madras	1	1	0	0

to / from	Caracas	Georgetown	Miami	Pt of Spain
Caracas	0	1	0	1
Georgetown	1	0	0	1
Miami	0	0	0	1
Pt of Spain	1	1	1	0

to / from	Delhi	London	Moscow	Tehran
Delhi	0	1	1	1
London	1	0	1	2
Moscow	1	1	0	0
Tehran	1	2	0	0

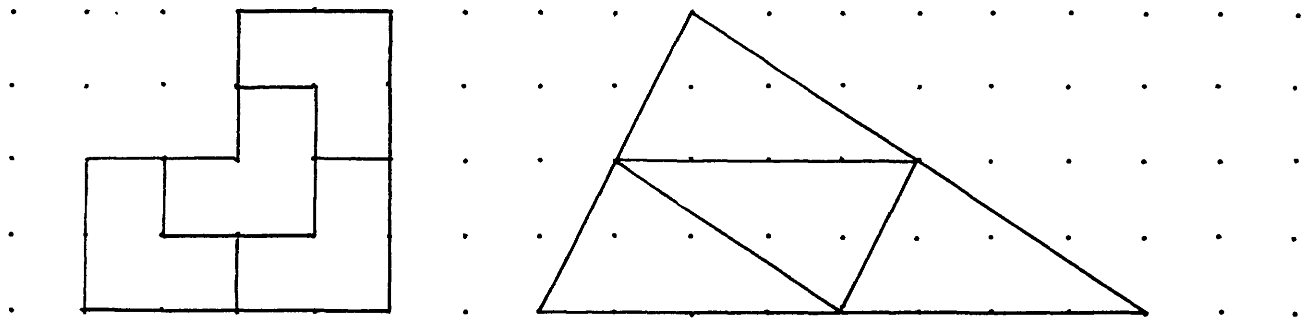
from	London	Miami	New York
London	0	1	3
Miami	1	0	2
New York	3	2	0

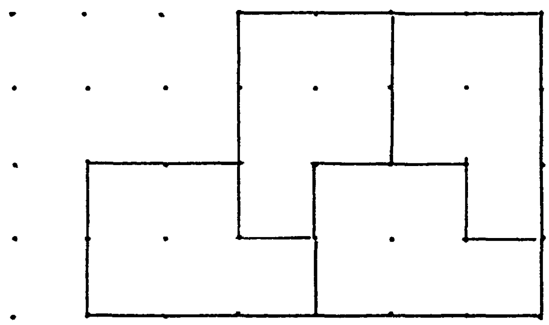
1758 Co-ordinate Messages

Did you understand the message?

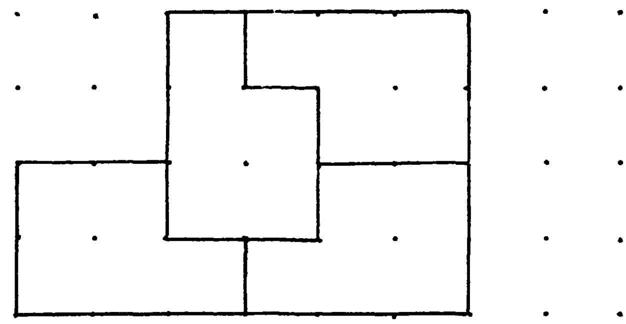
1759 Shapes that can grow

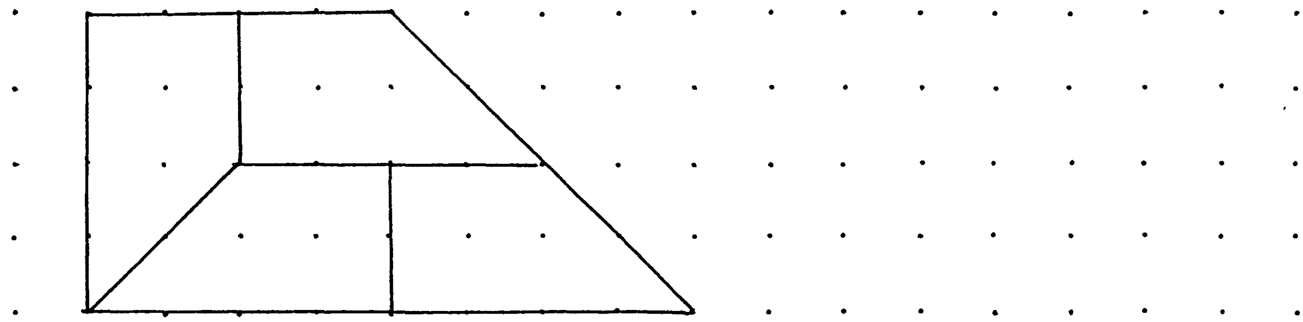
Side 1



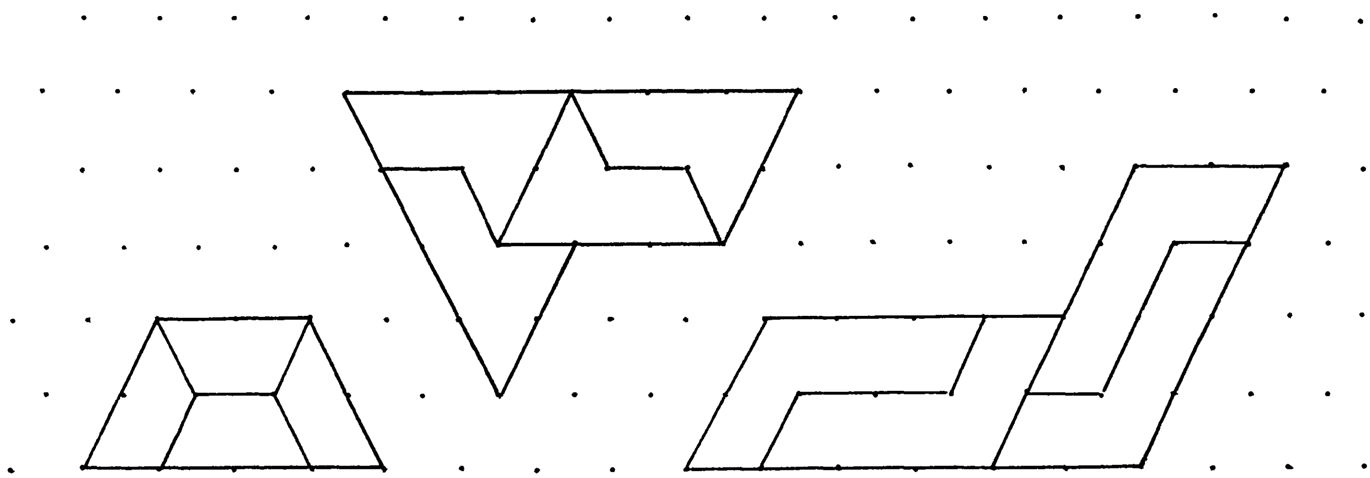


or

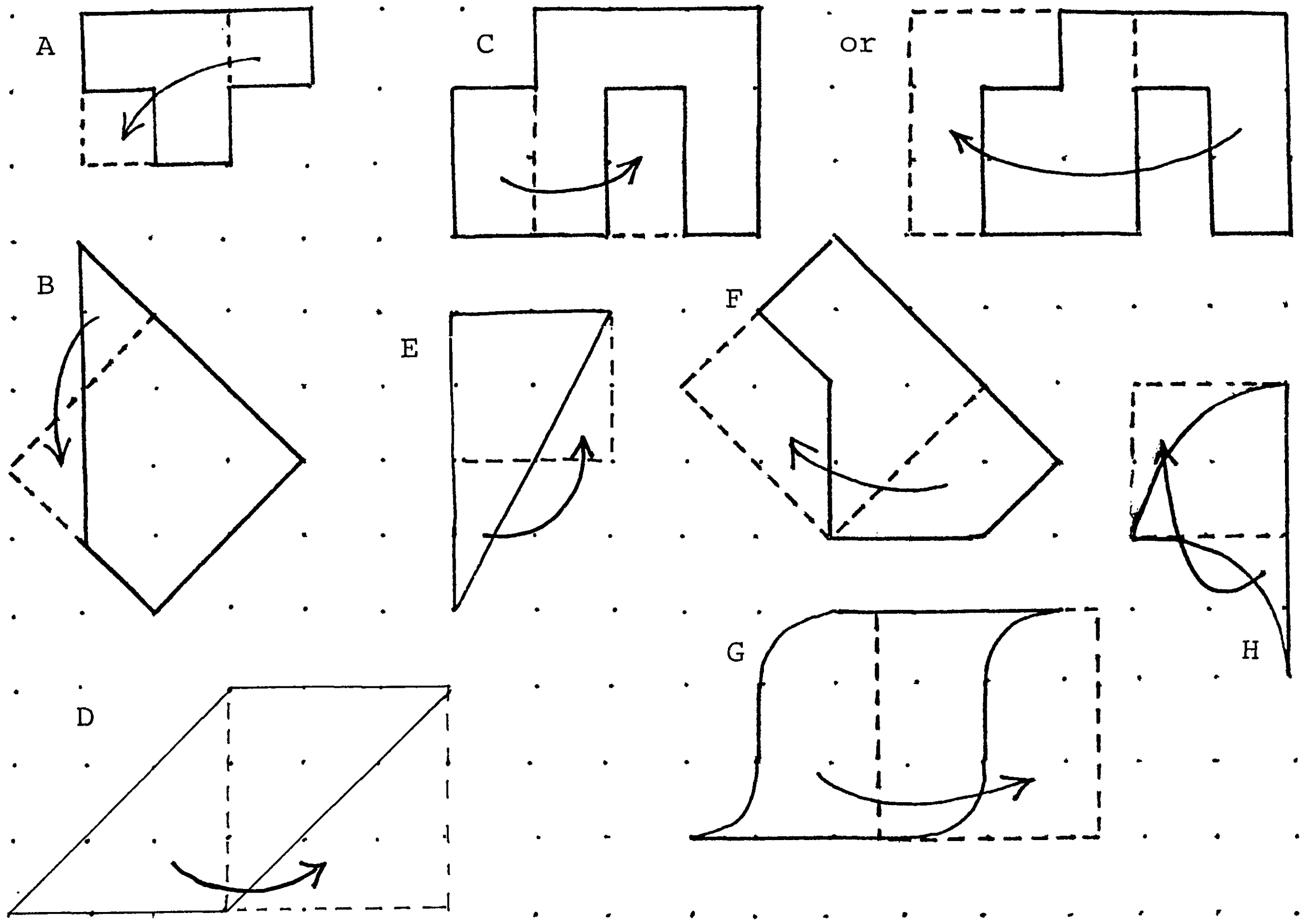




Side 2



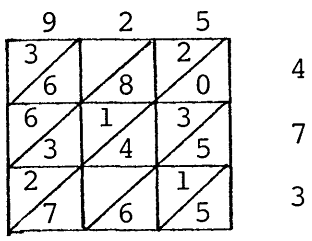
1760 One Straight Cut



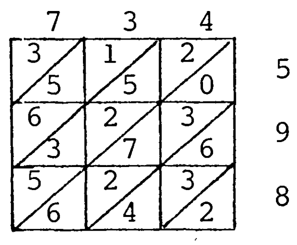
1761 Gelosia Problems

The multiplications are 925 x 473 and 734 x 598.

1.



2.



1762 From A to B

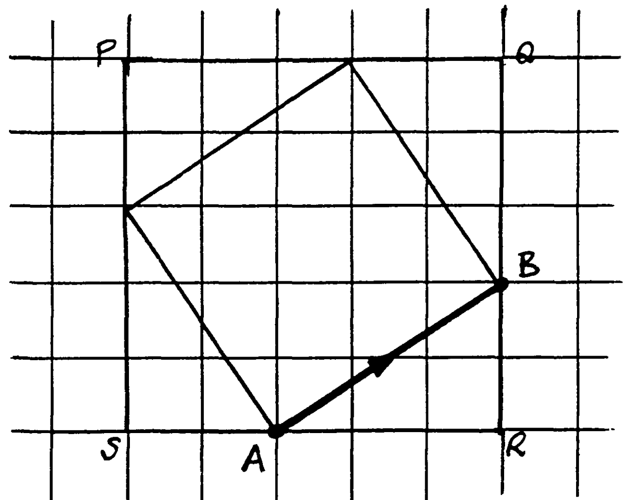
The area of the square is $13cm^2$.

One method of finding the area is illustrated on the right.

Area of PQRS = $25cm^2$

Area of each triangle = $3cm^2$

Area of original square = 25 -12
= $13cm^2$



continued

1762 From A to B (cont)

Try finding the areas of squares on vectors with one number fixed.

eg. $\begin{pmatrix}1\\1\end{pmatrix}$, $\begin{pmatrix}2\\1\end{pmatrix}$, $\begin{pmatrix}3\\1\end{pmatrix}$, $\begin{pmatrix}4\\1\end{pmatrix}$ ......

or $\begin{pmatrix}1\\2\end{pmatrix}$, $\begin{pmatrix}2\\2\end{pmatrix}$, $\begin{pmatrix}3\\2\end{pmatrix}$ ......

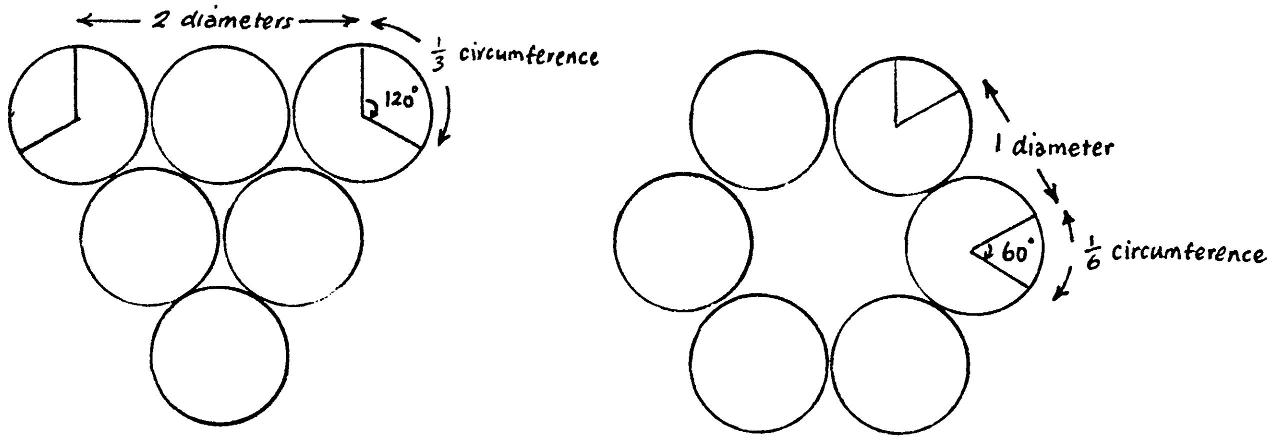
Try vectors with negative numbers like $\begin{pmatrix}-3\\1\end{pmatrix}$.

Try vectors with zero $\begin{pmatrix}4\\0\end{pmatrix}$.

The area of the square drawn on the vector AB is closely associated with the most famous of mathematical theorems. This theorem was known to many scholars long before the time of the famous Greek mathematician it is named after. Have you discovered what it is?

1763 Circles, Triangles and Hexagons

The elastic band would have the same perimeter in each case.



Perimeter = 6 diameters + 1 circumference for both arrangements.

1764 Tangled Quadrilaterals

1. Square HOSM — Trapezium ACJZ
   Rhombus QNUX — Rectangle FGWV
   Kite DEKR — Parallelogram BPYL

   The letter T is not used.

2. Square BKYQ — Trapezium ADTL
   Rhombus HEOR — Rectangle FGWV
   Kite CPXM — Parallelogram NUZS

   The letter J is not used.

3. Square FHSQ — Trapezium DKZG
   Rhombus BJUM — Rectangle MOTR
   Kite EXVL — Parallelogram ACYW

   The letter P is not used.

1764 Tangled Quadrilaterals (cont)

4. | | | | |
|---|---|---|---|
| Square | GOXQ | Trapezium | CEZV |
| Rhombus | KTWN | Rectangle | ABML |
| Kite | JUYS | Parallelogram | DPRF |

The letter H is not used.

5. Many possible answers.

1765 Two by Two

The pairs are

A and E   B and I
F and J   C and G

H and D are left over.

1766 Flying Engineers

These answers are based upon prices in 1985. You may have used more up-to-date prices.

The cheapest way of moving the engineers is to send 1 from Frankfurt to Athens (£80);
and then 2 from London - one to Athens (110) and one to Madrid (£50) ; a total cost of £240.

Later in the year

The costs for organising the conference in each city are

London		Madrid		Frankfurt		Athens	
	50		100		60		220
	90		180		60		100
	440		400		320		240
	£580		£680		£440		£560

so Frankfurt would be cheapest for air travel.

For accommodation the hotel charges would be

London: $10 \times £28 = £280$

Madrid: $\frac{10 \times 4080}{217} = £188.02$

Frankfurt: $\frac{10 \times 165}{3.88} = £425.06$

Athens: $\frac{10 \times 2130}{174} = £122.$

so Athens has the cheapest hotels.

Overall costs are:

London - £860 Madrid - £868.02 Frankfurt - £865.26 Athens - £682.41

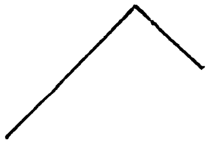
so the conference should be held in Athens.

1767 Adds up to

No answers required.

1768 Zig Zag

Each line is the same length!
You can check by counting the number of steps up and the number of steps down on each line.
For example,

Up 6 and down 3  is the same length as

up 2, down 1 repeated three times.

1769 Lines, Squares and Circles

The red circles occupy the same area in each of the yellow squares... the sets of lines on each poster are equal in length..

... and... what did you discover about the squares ?

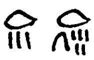
1770 The Lewis Family

Peter - 3 years	Mrs Lewis - 36 years
Laurie - 13 years	Mr Lewis - 41 years
Clara - 18 years	Grandma - 66 years

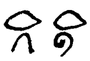
1771 Early Egyptian Fractions

1. (a) and (c) are unit fractions.
2. (a) $\frac{1}{20}$ (b) $\frac{1}{7}$ (c) $\frac{1}{100}$ (d) $\frac{1}{45}$ (e) $\frac{3}{4}$ (f) $\frac{1}{2}$ (g) $\frac{1}{42}$ (h) $\frac{2}{3}$
3. (a)  (b)  (c)  or  (d) 
4. (a)  is $\frac{1}{2} + \frac{1}{4}$

   (b)  is $\frac{1}{2} + \frac{1}{8}$
5. (a) $\frac{6}{15}$

    is $\frac{1}{3} + \frac{1}{15}$ which is the same as $\frac{5}{15}+\frac{1}{15}$ or $\frac{6}{15}$ (or $\frac{2}{5}$).

   (b) $\frac{11}{100}$

    is $\frac{1}{10}+\frac{1}{100}$ which is the same as $\frac{10}{100}+\frac{1}{100}$ or $\frac{11}{100}$

   (c) $\frac{7}{30}$

    is $\frac{1}{5} + \frac{1}{30}$ which is the same as $\frac{6}{30} + \frac{1}{30}$ or $\frac{7}{30}$

6. (a) $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

(b) $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$

(c) $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

(d) $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

7. 

$= \frac{1}{202} + \frac{1}{303} + \frac{1}{606}$

$= \frac{3}{606} + \frac{2}{606} + \frac{1}{606}$

$= \frac{6}{606}$

$= \frac{1}{101}$ = 

8. The three fractions drawn in hieroglyphics for $\frac{7}{24}$ represent $(\frac{1}{6} + \frac{1}{9} + \frac{1}{72})$, $(\frac{1}{24} + \frac{1}{4})$ and $(\frac{1}{6} + \frac{1}{8})$.

The Egyptians would have prefer $(\frac{1}{4} + \frac{1}{24})$ because $\frac{1}{4}$ is the unit fraction which is nearest in size to the required amount.

9. They are all equivalent to $\frac{2}{35}$.

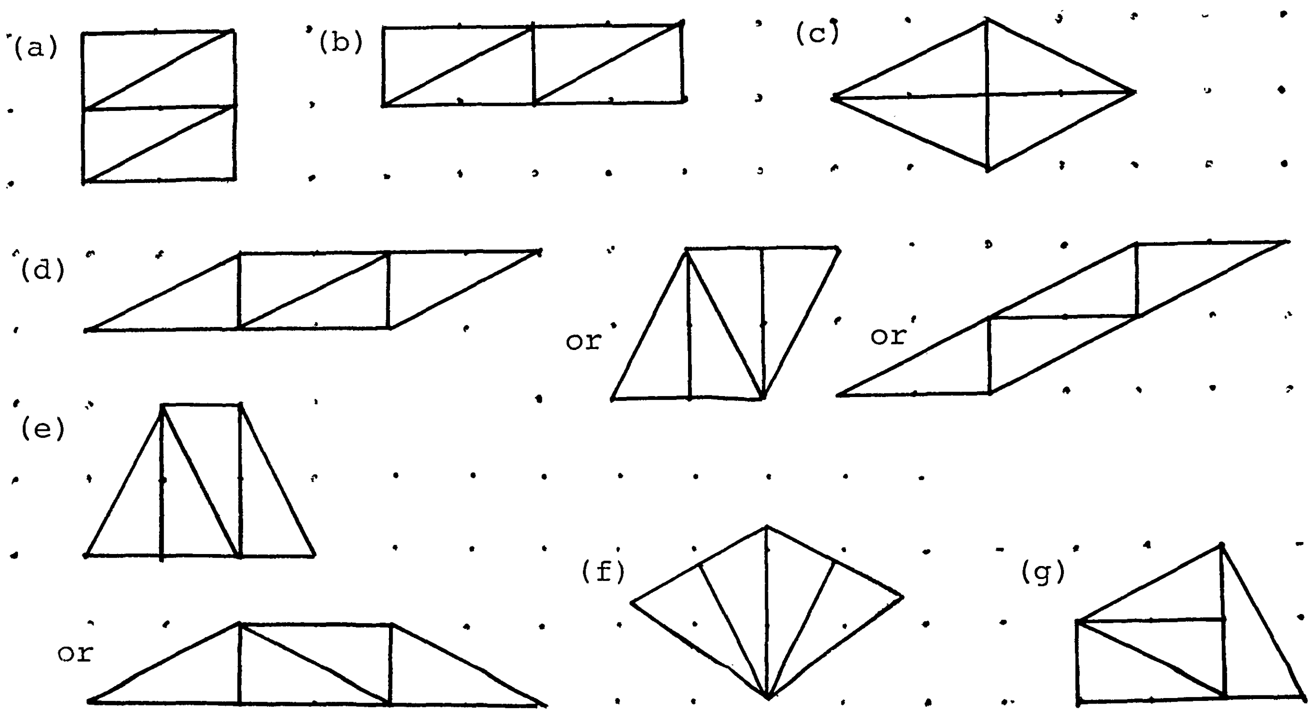
10. $\frac{29}{80}$ is the same as $\frac{20}{80} + \frac{8}{80} + \frac{1}{80}$

This is equilvalent to $\frac{1}{4} + \frac{1}{10} + \frac{1}{80}$.

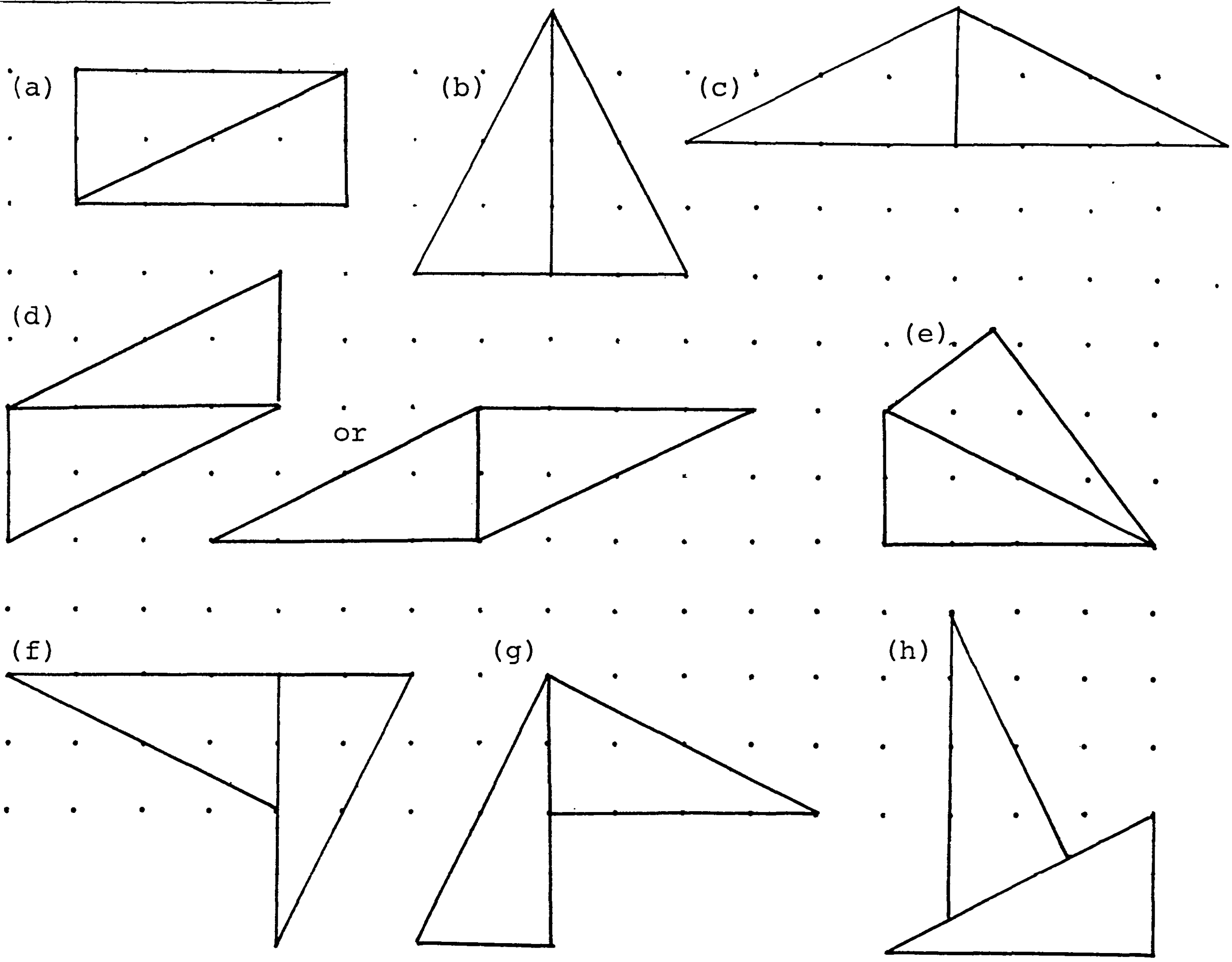
$\frac{1}{80}$ is so small in comparison to the other unit fractions that $\frac{1}{4} + \frac{1}{10}$ would be a good approximation.

11. $\frac{3}{4}$

1772 Four Triangles



1773 Two Triangles



1774 Modelling with Graphs

In each case you can only choose what seems to be the most appropriate graph. Here are some suggestions.

1774A Raising Bricks
D is most likely as the labourer pulls with one hand after the other. However, B is also possible if she is able to maintain a steady pull.

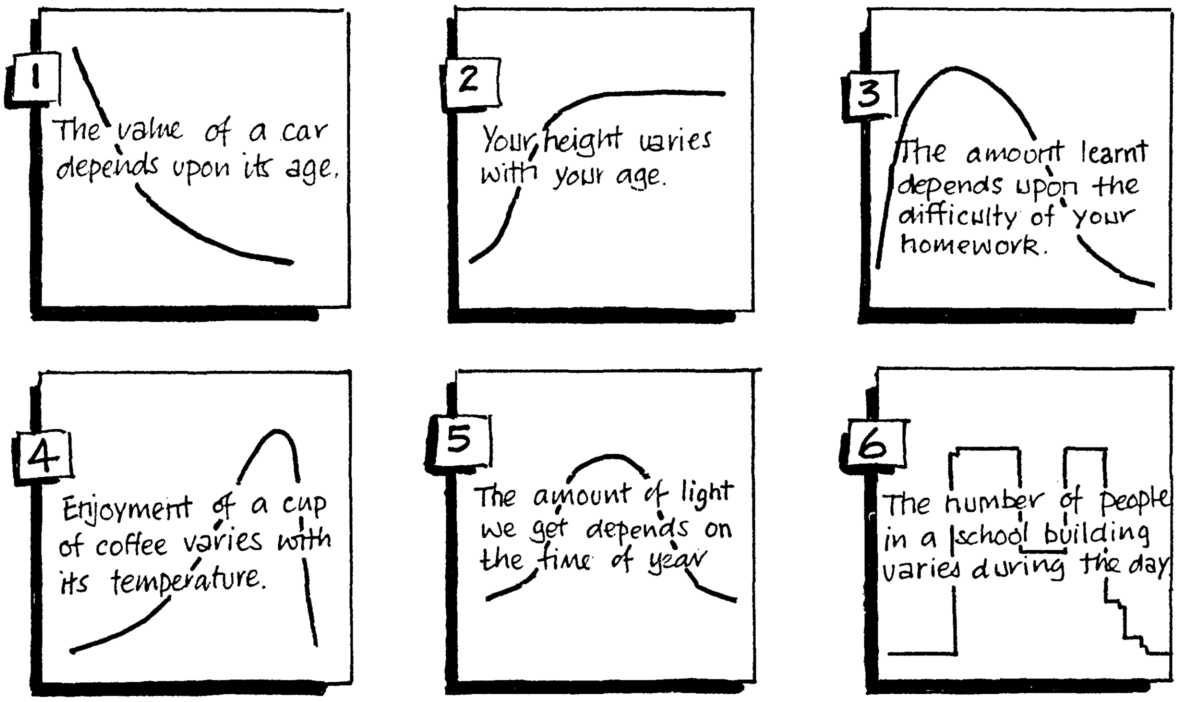
1774B Cinema admission prices
B is most likely because there will be no profit until the charge for admission is big enough to cover the cost of running the cinema.

1774C Digging up the road
The time taken will be shorter as more people start working. However, if there are too many people working, they may get in each others' way so, although C and D are possible, D is the most likely.

1774 Modelling with Graphs (cont)

1774D Suggestions

Here are some sketches for situations 1 to 6. Do you agree with them?



1775 Partners

Here are some more partners:

(2,2)

$(\frac{6}{5}, 6)$

$(\frac{7}{6}, 7)$

$(\frac{8}{7}, 8)$

$(\frac{9}{8}, 9)$ or you may write this as $(1\frac{1}{8}, 9)$

You may find it easier to find a partner for $1\frac{5}{6}$ if you think of it as $\frac{11}{6}$.

Its partner is $\frac{11}{5}$.

$\frac{11}{6} \times \frac{11}{5} = \frac{121}{30}$ and $\frac{11}{6} + \frac{11}{5} = \frac{55}{30} + \frac{66}{30}$

$$= \frac{121}{30}$$

For any number $\frac{a}{b}$, its partner is $\frac{a}{a-b}$

1776 Spirals

No answers required.

1777 Avoiding Each Other

Here is one possible answer:

						Q	
	Q						
					Q		
		Q					
Q							
			Q				
							Q
				Q			

1778 Jumping

No answers required.

1779 Lineover

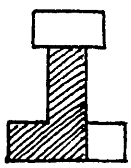
No answers required.

1780 Symmetry Investigation

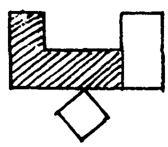
The number of shapes you can find depends on what you allow and what you don't allow .....

Did you always use all the shapes?

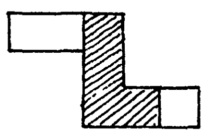
Did you allow shapes like this?



Did you allow shapes like this ?



Did you allow point (rotational) Symmetry?



What rules did you use?

1781 Three out of nine

By shading 3 of the squares in a 3 x 3 grid, it is possible to create 22 different patterns (*if rotations are not allowed*).

Shading 4 squares in a 3 x 3 grid it is possible to make 33 different patterns.

Shading 5 squares is the same as shading 4 squares. Why?

Shading 6 squares is the same as shading squares? ....

1782 To be continued

No answers required.

1783 Calculator Booklet

pages 2 & 3 Mystery Isle: After completing each calculation, turn your calculator upside-down to read where the next clue is.

page 4 Mappings:

A → b	E → g
B → d	F → e
C → c	G → h
D → a	H → f

page 5 Magic Squares: One of the figures 16 should be 14 — which one ?

100 Up: Try 37 + 73 - 7 - 3

page 7 A-maze-ing: Try 5 x 4 - 4 x 6 + 4

page 8 Missing Digit:

35 x 57 = 1995	48 x 47 = 2256
13 x 59 = 767	or 48 x 57 = 2736
131 x 23 = 3013	14 x 29 = 406
36 x 47 = 1692	can't be done
56 x 72 = 4032	73 x 57 = 4161
59 x 17 = 1003	37 x 43 = 1591

page 9 Operations: Several answers are possible.

page 11 Ticket Sales: 5 in the stalls and 3 in the balcony.

page 14 Productive: With the first five digits, the largest product is 431 x 52.
What is the largest product with 1,2,3,4,5 & 6 ?

page 15 Hot and Cold: California

page 16 Target 200: You could score 200 with 8 arrows (4x11,2x23, 1x37,1x73). Could you do it with less than 8 ?

page 17 Missing Signs:

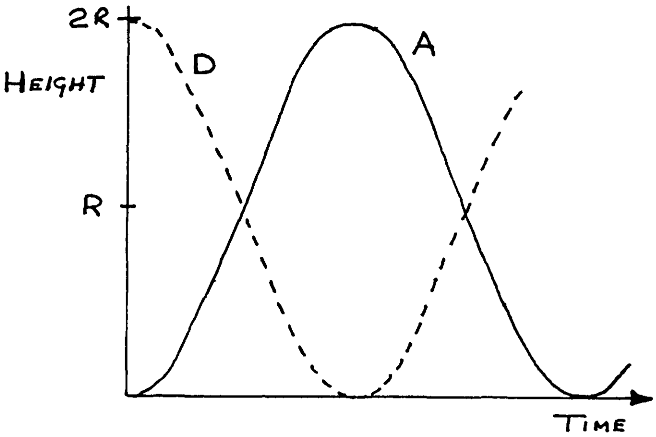
17 x 17 + 17 = 306	(437 x 2) + 126 = 1000
38 -47 + 58 = 49	(91 x 7) ÷ 13 = 49
(47 + 53) x 10 = 1000	47 + 23 + 27 + 15 = 112
27 x (5 x 5) = 675	768 ÷ (43 - 37) = 128
34 + (37 x 18) = 700	1116 - (23 x 47) = 35

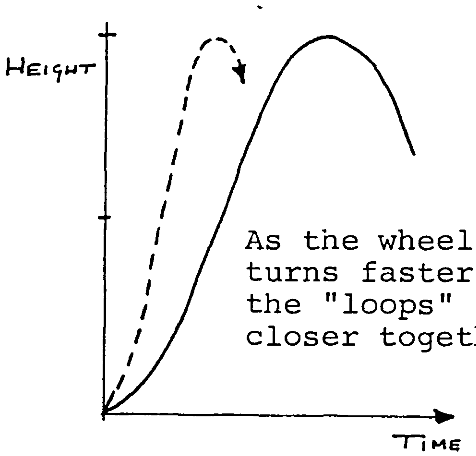
page 22 Problems:

1. About 100 years!
2. 100 kg - about the same weight as the largest man you have ever seen.
3. At least $1\frac{1}{4}$ hours.
4. Non-stop at about 4 mph — more than 7 years !
5. At least once around the world.
6. Nearly 16 000 million.
7. You would have a party every 3 years 2 months.

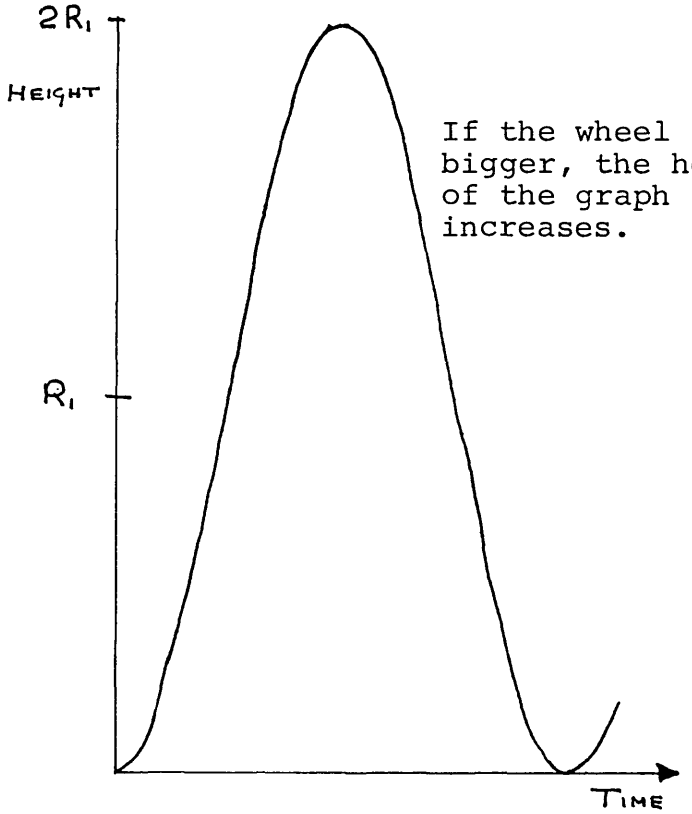
page 24 MiniMagic: Each distinct set of 3 x 3 squares is magic.

1784 Big Wheel

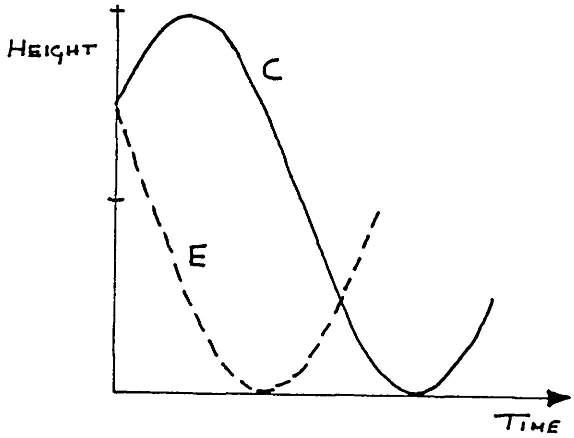


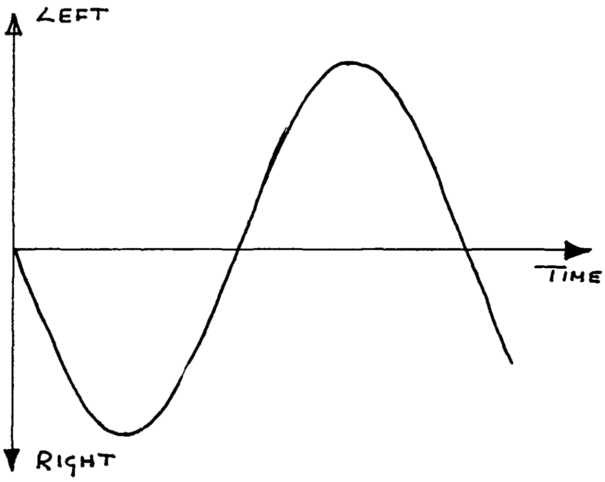


As the wheel turns faster, the "loops" get closer together.



If the wheel is bigger, the height of the graph increases.





1785 Investigating Queens

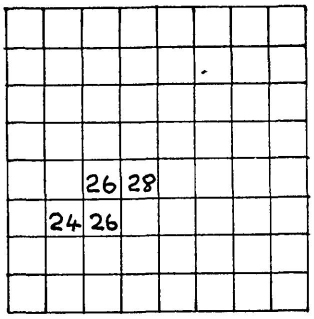
The largest number of squares which can be protected by a queen on an 8x8 chess board is 28.

There are many questions that can be asked -

Can a queen protect an odd number of squares?

What is the smallest number of squares a queen can protect?

1785 Investigating Queens (cont)



Drawing and numbering the board might help you to see patterns and to decide on the best position for the queen.

To find a rule for squares it might help to look at odd and even sized boards, but it may be easier to get a general rule for rectangles which are not squares.

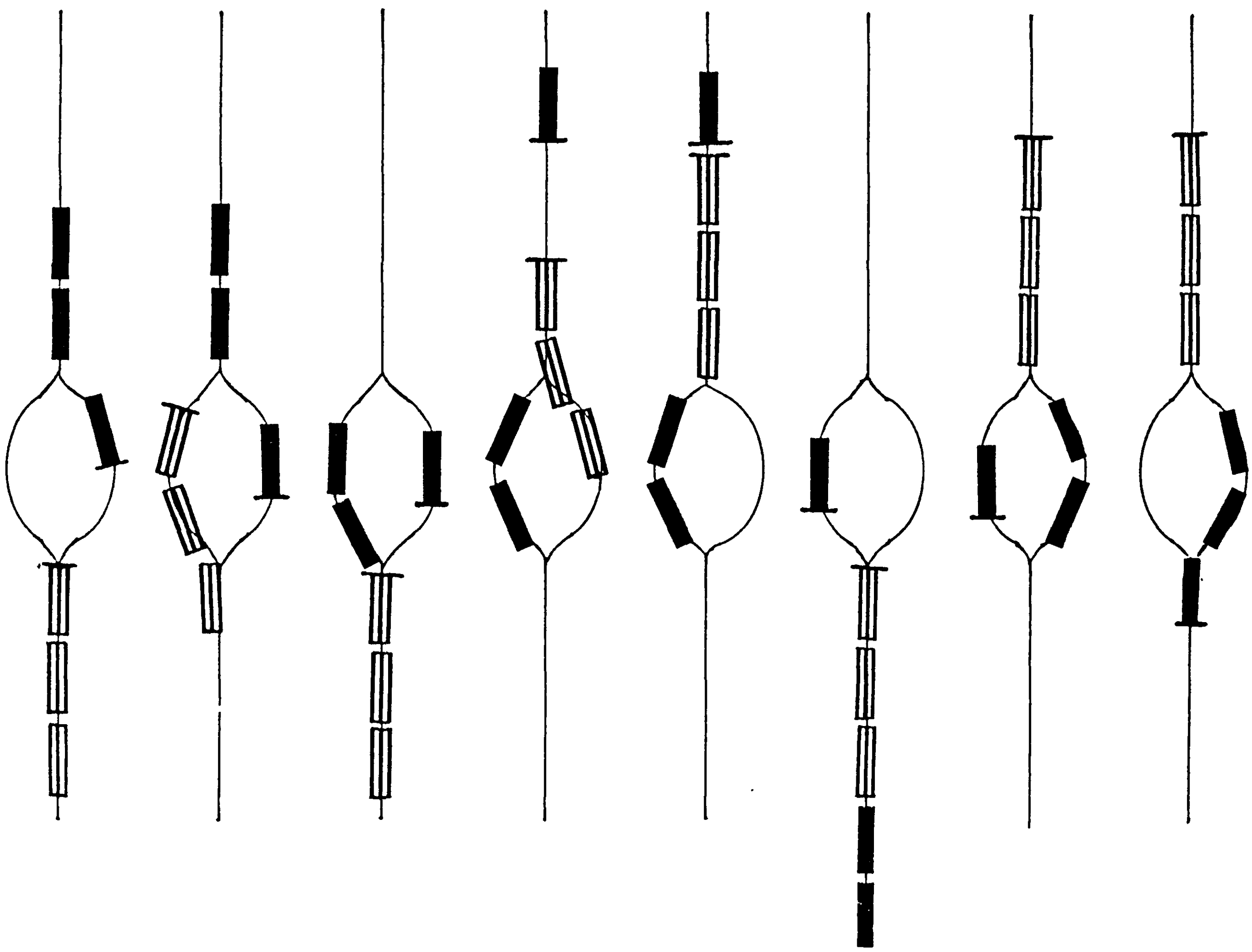
1786 Which number?

The number is 292. Explain your ways of writing 23 to a friend.

1787 Angle 360^0

No answers required.

1788 Blocked



1789 Fraction Loop

Get a friend to check your loop.

1790 The Chinese Triangle

No answers required.

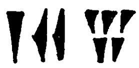
1791 Getting into Shape

There are two puzzles - one with shapes and one with words. Try to solve them both.

1792 Feeling Hungry?

Everybody's day is different so you will need to check your answers with a friend.

1793 Cuneiform Numbers

front : The tablets show the 9 times table. The Babylonians used a mixed base system with both base 10 and base 60. For example 85 would be 

back : A-ra means multiply. The example shows part of the 7 times table.

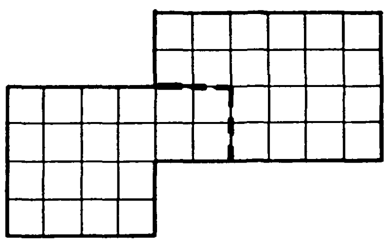
1794 Building Cubes

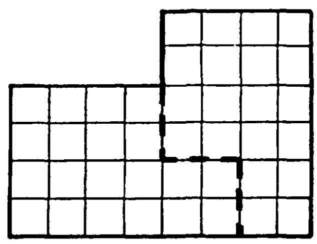
A - F
D - I
H - K
B - E

G and J will both fit with C or L.

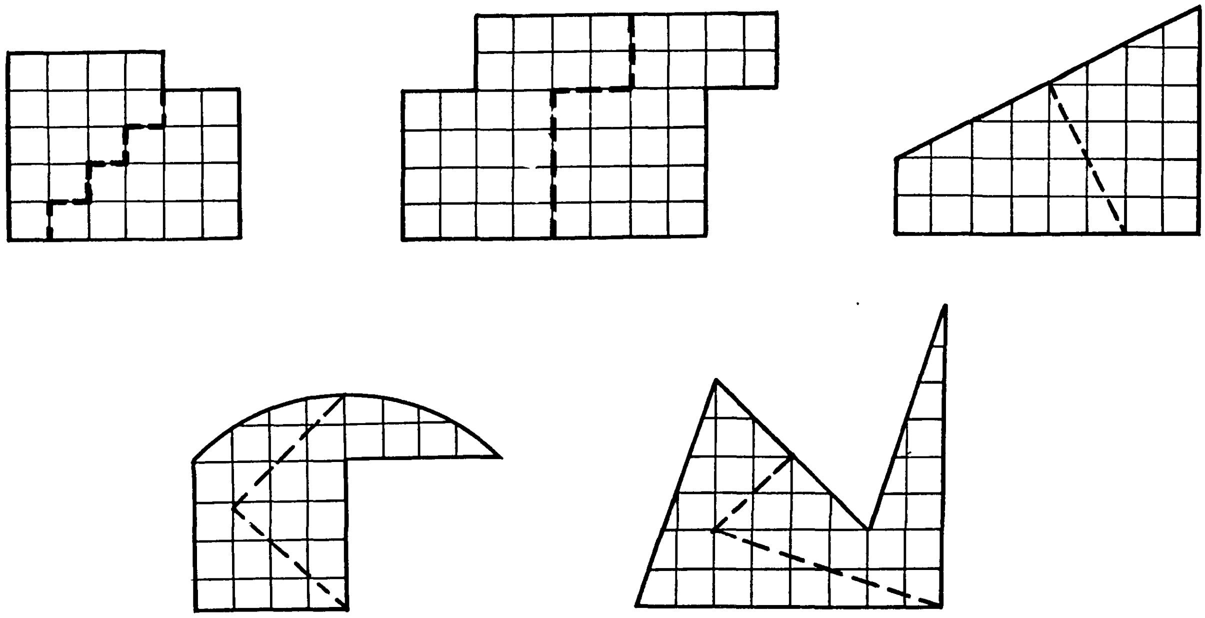
Challenge. Draw the 3D shape that will fit with M to make a cube.

1795 Identical Halves





1795 Identical Halves (cont)



Some of these pairs are mirror images, which ones?

1796 Plotter (MICRO)

This is a resource program so your answers will depend on how you have used it.

1797 Newtiles (MICRO)

Ask your teacher for a print-out of your floor patterns.

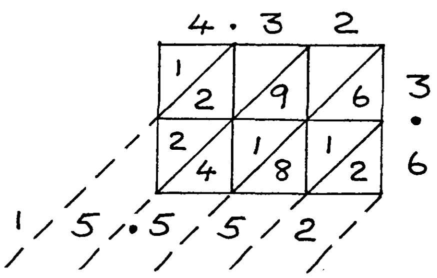
1798 Quilts (MICRO)

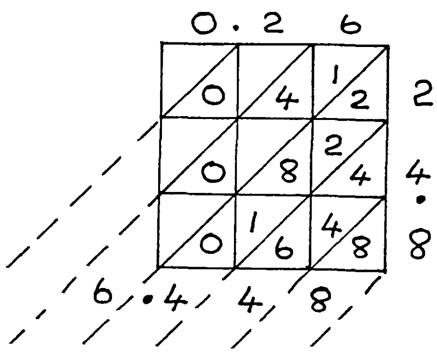
If you are trying to find out about the numbers of threads you need, try thinking hard about factors.

1799 Boxes

No answers required.

1800 Gelosia for decimals





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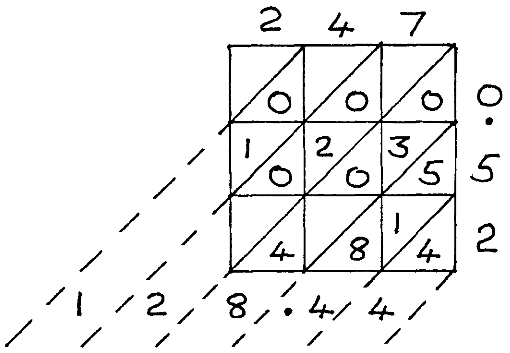
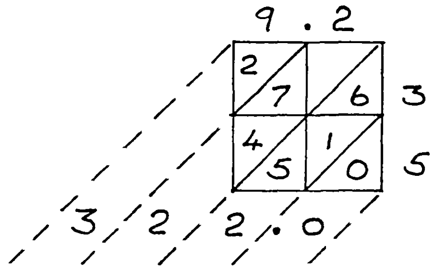
1800 Gelosia for Decimals (cont)





1801 The Limit of a Sequence (ATM)

(i) $a_1 = 3.1$, $a_2 = 3.01$, $a_3 = 3.001$.

(ii) $b_1 = 0$, $b_2 = 0.3465...$, $b_3 = 3662...$, $b_4 = 0.3465...$,
$b_5 = 0.3218...$, $b_{30} = 0.1183...$, $b_{1,000,000} = 0.00001381...$

Thinking of the function $\frac{1}{n}$, the function $\log_e n$ and the function b_n extended to cover other than integer values and then sketching their curves will help you understand what is happening, particularly near the beginning of the sequence.

(iii) $c_1 = 0.3333...$, $c_2 = 0.4285...$, $c_3 = 0.5384...$,
$c_{50} = 0.9607...$

(iv) This function is the reciprocal of b_n.
$d_2 = 2.8860...$

(v) $e_1 = -1$, $e_2 = 1$, $e_3 = -1$.

(vi) $f_1 = 0.8414...$, $f_2 = 1.8185...$, $f_3 = 0.4233...$,
$f_4 = -3.0272...$, $f_5 = -4.7946...$, $f_6 = -6.764...$,
$f_7 = 4.5989...$, $f_8 = 7.9148...$, $f_9 = 3.7090...$,
$f_{10} = -5.4402...$

(vii) $g_1 = 0.8414...$, $g_2 = 0.4546...$, $g_3 = 0.0470...$,
$g_4 = -0.1892...$, $g_5 = -0.1917...$, $g_6 = -0.0465...$,
$g_7 = 0.0938...$, $g_8 = 0.1236...$, $g_9 = 0.0457...$,
$g_{10} = -0.0544...$

(viii) $h_1 = 20$, $h_2 = 10$, $h_3 = 6$, $h_4 = 5$, $h_{20} = 1$, $h_{30} = 0$,
$h_{50} = 0$.
$j_1 = 1$, $j_2 = 0$, $j_3 = -0.33...$, $j_4 = 0$, $j_5 = 0.2$,
$j_6 = 0$, $j_7 = -0.142...$

Some classifications that occur to us are :

e_n, f_n, g_n, j_n oscillate.

(Thinking of the functions f_n, g_n, j_n extended to cover other than integer values then sketching their curves and the sine curve will help you understand what is happening.)

e_n, h_n, give only integer values.

g_n, j_n have a limit but f_n does not.

b_n, d_n change very slowly.

a_n, c_n tend quickly to an obvious limit. Can you explain why in each case?

There are many other classifications you may have found. Which seem the most useful?

1802 A Matrix Sequence (ATM)

It is possible to carry out this investigation mechanically at quite a low level but its interest lies in the use of an iterative process to generate an irrational number to any degreee of accuracy.

* What happens with a negative guess?
* Try to find $\sqrt{4}$ using this method.
* Does the process ever fail to converge?
* What happens with a matrix like $\begin{pmatrix} 1 & -3 \\ 1 & 1 \end{pmatrix}$?
* The result for $\sqrt{2}$ can be proved as follows.

Let $\begin{pmatrix} x_n \\ y_n \end{pmatrix}$ be the n^{th} vector produced

then $\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix}$

which gives $x_n + 2y_n = x_{n+1}$

and $x_n + y_n = y_{n+1}$

dividing gives $\dfrac{x_{n+1}}{y_{n+1}} = \dfrac{x_n + 2y_n}{x_n + y_n} = \dfrac{x_n/y_n + 2}{x_n/y_n + 1}$

This will converge to a limit ℓ when two successive terms are equal, that is $\dfrac{x_n}{y_n} = \dfrac{x_{n+1}}{y_{n+1}} = \ell$

so $\ell = \dfrac{x_{n+1}}{y_{n+1}} = \dfrac{x_n/y_n + 2}{x_n/y_n + 1} = \dfrac{\ell + 2}{\ell + 1}$

$\Rightarrow \ell(\ell + 1) = \ell + 2$

$\Rightarrow \ell^2 + \ell = \ell + 2$

$\Rightarrow \ell^2 = 2$

$\Rightarrow \ell = \sqrt{2}$

continued

1802 A Matrix Sequence (ATM) (cont)

Try to produce a similar proof using the matrix which will generate $\sqrt{a}$. where a is any positive number.

1803 The Fibonacci Sequence (ATM)

(i) Each term is found by adding the two previous terms.

(ii) $U_n = U_{n-1} + U_{n-2}$ and $U_1 = 1$, $U_2 = 1$

$$V_1 = \frac{U_2}{U_1} = \frac{1}{1} = 1$$

$$V_2 = \frac{U_3}{U_2} = \frac{2}{1} = 2$$

$$V_3 = \frac{U_4}{U_3} = \frac{3}{2} = 1.5$$

$$V_4 = \frac{U_5}{U_4} = \frac{5}{3} = 1.\dot{6}$$

.
.
.

$$V_{12} = \frac{U_{13}}{U_{12}} = \frac{233}{144} = 1.61805\ldots$$

As n gets larger, Vn tends towards a limit known as the Golden Ratio. The SMILE pack 0824 has more work on this and also contains a list of book references which you should find in your school library.

Is the limit the same if one of the starting values is negative?

Try forming sequences using building rules like

$$p_n = \frac{U_{n+2}}{U_n}, \quad q_n = \frac{U_{n+3}}{U_n} \quad \ldots\ldots$$

and describe their behaviour.

1804 Arithmetic Progressions (ATM)

It will be worthwhile spending some time finding a general rule and testing it for simple cases before trying the examples.

2 + 4 + ...+ 100 = 2550

2 + 4 + ...+1000 = 250500

3 + 6 + ... +99 = 1683

1 + 3 + 5 +..+ 999 = 250000

1 + 4 + 7 +..+ 100 = 1717

1 + 5 + 9 +... +97 = 1225
2 + 7 +12 +..+ 102 = 1092

100 +97 +94 +.... +7 = 1712

-2579 + -2569 + ...+ -19 = -333843

Any 'A' level textbook will give proofs, formulae and further examples of Arithmetic Progressions.

1805 Geometric Progressions (1) (ATM)

$S_1 = 1$ For large n, S_n tends to a limit of 2.

$S_2 = \frac{3}{2}$

$S_3 = \frac{7}{4}$ You should be able to use the patterns in

$S_4 = \frac{15}{8}$ these fractions to find a formula for

$S_5 = \frac{31}{16}$ S_n.

.
.
.

Listing S_n and $2S_n$ may help you towards a proof.

The formula for the second series should have a similiar form. This should help you to generalise.

1806 Geometric Progressions (2) (ATM)

Try to use number patterns to find general rules. You will find more formal methods explained in SMILE card 1439.

1807 Geometric Progressions (3) (ATM)

(a) $Sn = 2^n - 1$

(b) $3 + 6 + 12 + 24 + \ldots\ldots = 3\times(1 + 2 + 4 + 8 + \ldots\ldots$ so ....

(c) $Sn = 3^n - 1$

(d) $5 + 15 + 45 + \ldots\ldots = 5\times(1 + 3 + 9 + \ldots\ldots$ so ....

(e) The G.P. which starts at 3 with a common ratio 4 is $3 + 12 + 48 + \ldots\ldots$

Any 'A' level textbook will give formulae, proofs and further examples of Geometric Progressions.

1808 Sums of Squares (1)(ATM)

You might have realised that to find the number of squares on an n x n board involves the sum $1^2+2^2+\ldots\ \ldots+n^2$.
Finding a formula for this sum is a difficult task. You could either persevere,with the following hint, or do SMILE task 1809 which introduces a new method for tackling such problems.

Hint: If you write the number of squares as the product of its prime factors, some interesting patterns emerge : eg.

size of board	number of squares
11	2 x 11 x 23
23	2 x 2 x 23 x 47
29	5 x 29 x 59
41	7 x 41 x 83

size of board	number of squares
18	3 x 19 x 37
30	5 x 31 x 61
36	2 x 3 x 37 x 73
78	13 x 79 x 157

Does either of these columns suggest a possible formula? If so, will it generalise?

1809 Sums of Squares (2) (ATM)

A. note : $1^2 + 2^2 + 3^2 + \ldots + 10^2$ can be written in shorthand as $\sum_{1}^{10} i^2$ where $\sum$ stands for "sum" ($\sum$ is the greek letter S).

so $\sum_{1}^{10} i^2 = 385$.

$$
\begin{array}{l}
2^3 - 1^3 = 3 \times 1^2 + 3 \times 1 + 1 \\
3^3 - 2^3 = 3 \times 2^2 + 3 \times 2 + 1 \\
4^3 - 3^3 = 3 \times 3^2 + 3 \times 3 + 1 \\
\vdots \\
10^3 - 9^3 = 3 \times 9^2 + 3 \times 9 + 1 \\
11^3 - 10^3 = 3 \times 10^2 + 3 \times 10 + 1 \\
\hline
11^3 - 1^3 = 3 \times \sum_{1}^{10} i^2 + 3 \times \sum_{1}^{10} i + 10
\end{array}
$$

Adding the identities cancels out most terms

Just three times what we want

This is the 10th triangle number. You know a formula to evaluate this.

To generalise : $(n+1)^3 - 1^3 = 3 \times \sum_{1}^{n} i^2 + 3 \times \sum_{1}^{n} i + n$.

You might like to simplify this your own way eg. substitute a formula for the triangle number term.

This difference method is a very powerful one for deriving formulae. It is based on the identity

$$(n+1)^3 - n^3 \equiv 3n^2 + 3n + 1$$

Check this identity by expanding $(n+1)^3$

B. Use a similiar method for $(n+1)^4$ to see where the pattern comes from.

1810 Two series on the Computer (ATM)

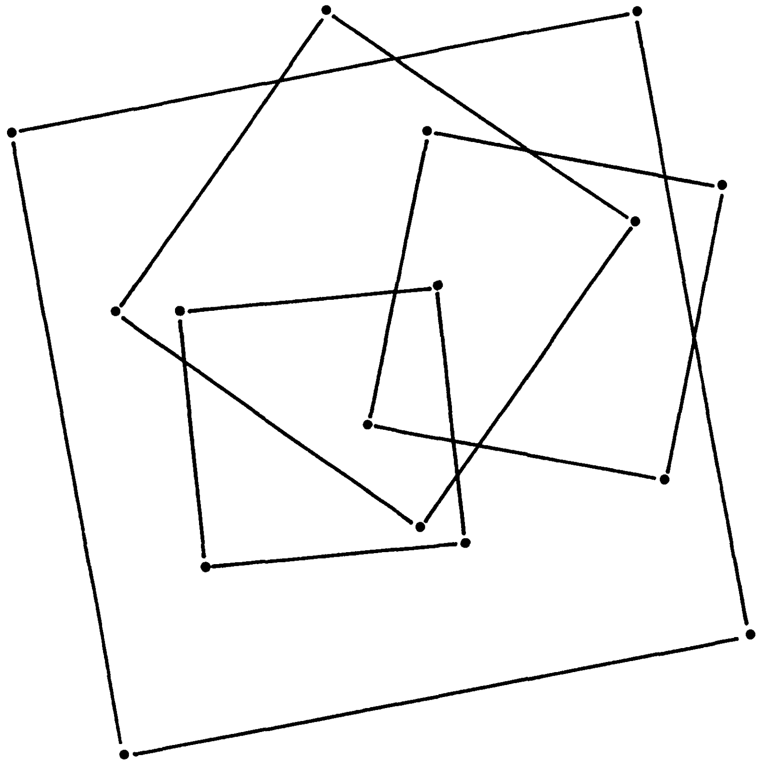
One of the series converges to a finite limit. What do you think happens to the other?

You should be able to adapt your programs to find the sum of similiar series. The following may give interesting results.

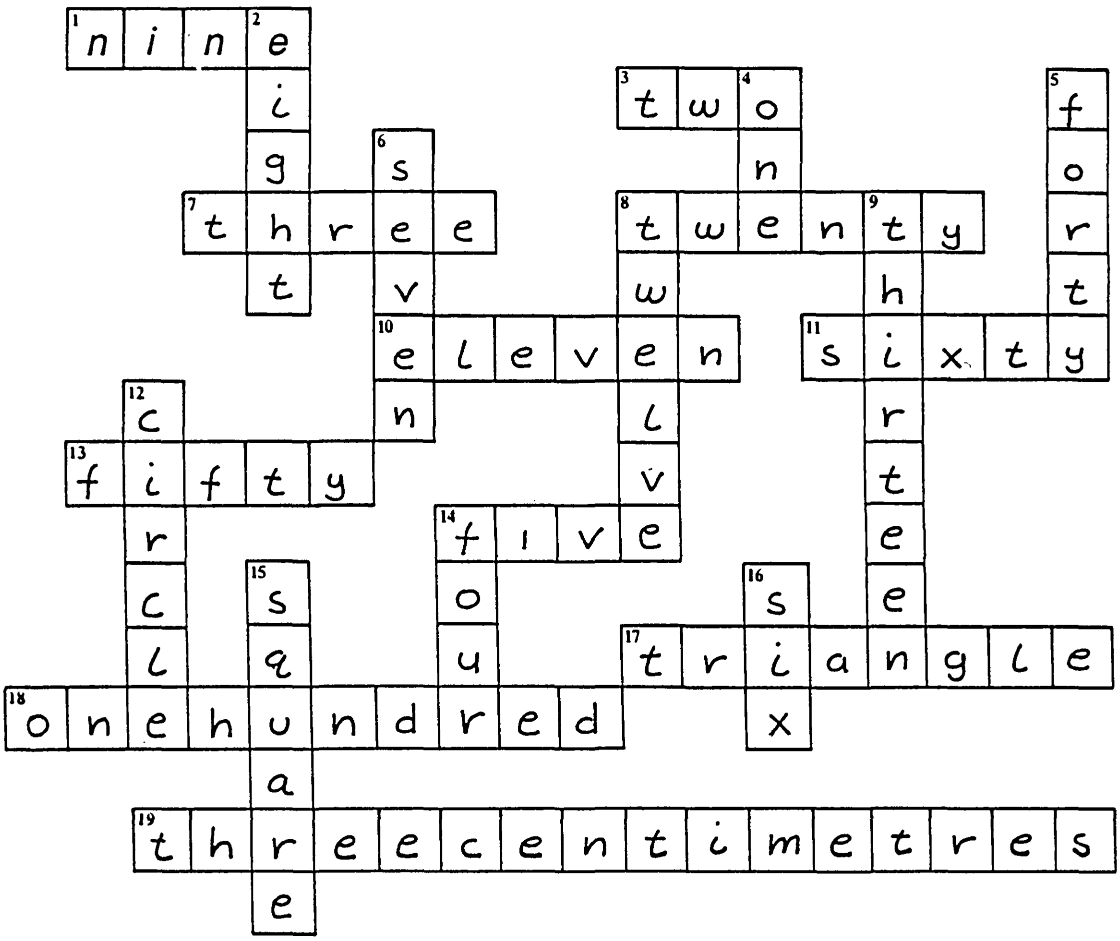
$$Sn = 1 + \frac{1}{8} + \frac{1}{27} + \ldots + \frac{1}{n^3}$$

$$Sn = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots \frac{1}{\sqrt{n}}$$

1812 Find four squares



1813 Crossword



1814 Proofs for Sequences (ATM)

The sequence given tends to a limit of 2.

(i) Changing box 1 does not affect the final result.

(ii) Changing box 3 while keeping box 2 fixed should allow you to find a simple general rule.

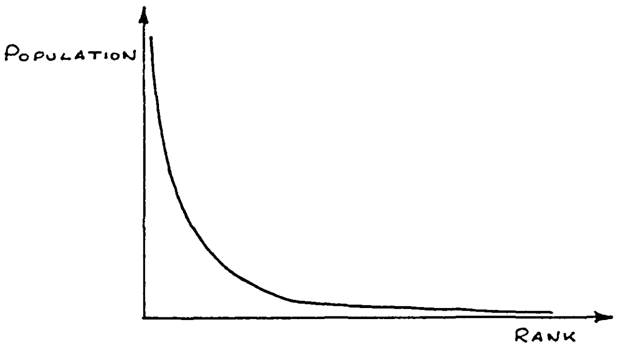
(iii) Changing box 2 alters the sequence in a more complicated way. Expressing your results as fractions may help you to see a pattern.

It is possible to produce fairly simple proofs for some of these results. It is important to be able to convince others that your proofs are valid.

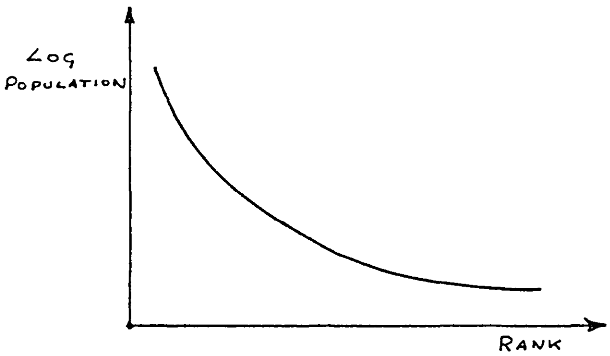
You may like to reconsider your work on this investigation after you have studied Geometric Progressions.

1815 Towns in Italy (ATM)

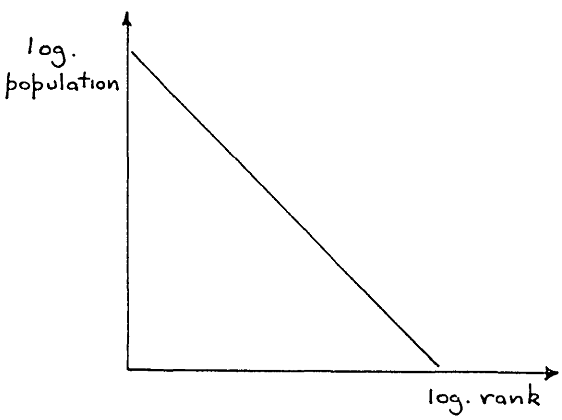
a)



b)



c)



1815 Towns in Italy (ATM) (cont)

Your third graph should have a gradient of around -1 and an intercept on the log. population axis of around 6.5 (log 2833000 = 6.45 and log 1 = 0)

These results give the approximate formula

$$\log p = -1 \times \log r + 6.5$$

$$\text{so} \quad p = \frac{1}{r} \times 10^{6.5}$$

$10^{6.5}$ is approximately the population of the largest city, Rome.

Geographers use the approximate formula

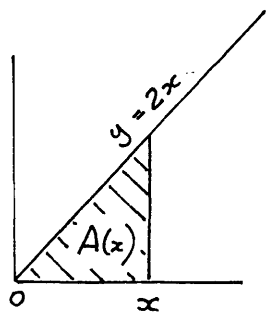
$p = \frac{P}{r}$ where P is the population of the largest city.

This is known as Zipf's Law.

1816 Areas under Graphs (ATM)

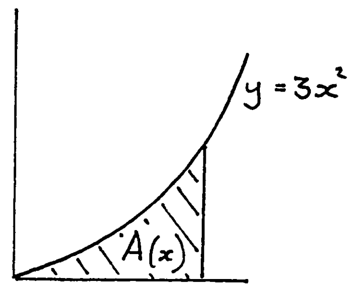
for $y = 2x$

$A(0) = 0$
$A(1) = 1$
$A(2) = 4$
$A(3) = 9$
.
.
.
so $A(x) = x^2$



for $y = 3x^2$

$A(0) = 0$
$A(1) = 1$
$A(2) = 8$
$A(3) = 27$
.
.
.
so $A(x) = x^3$



As you have counted squares under the curve do not expect your answers to be exact. If your answers are very different from these then check the scales you have used and make them as simple as possible.

continued

1816 Areas under Graphs (ATM) (cont)

for $y = 3x$ $\quad A(x) = \frac{3x^2}{2}$

for $y = 4x$ $\quad A(x) = \frac{4x^2}{2}$

which suggests $y = ax$ gives $\quad A(x) = \frac{ax^2}{2}$

for $y = x^2$ $\quad A(x) = \frac{x^3}{3}$

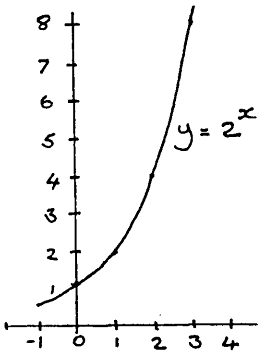
for $y = 2x^2$ $\quad A(x) = \frac{2x^3}{3}$

which suggests $y = ax^2$ gives $\quad A(x) = \frac{ax^3}{3}$

In general for $y = ax^n$ $\quad A(x) = \frac{ax^{n+1}}{n+1}$

1817 Some New Functions (ATM)

(a)



x	0	1	2	...
y	1	2	4	...

(b) The chord-slope function should be the same shape as $y = 2^x$ but below it. This is how you could calculate some points on the chord-slope function.

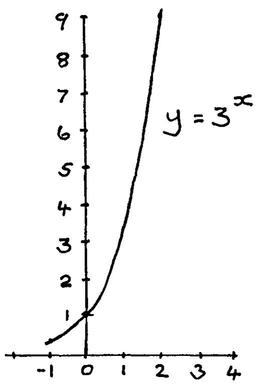
when $x = 1$, $f'(x) = \frac{2^{1.001} - 2^1}{.001}$

$= \frac{2.00013868 - 2}{.001}$

$= 1.3868$

x	1	2	...
y	1.39	2.77	...

when $x = 2$, $f'(x) = \frac{2^{2.001} - 2^2}{.001}$



The chord-slope function should be the same shape as $y = 3^x$ but above it.

when $x = 2$, $f'(x) = \dfrac{3^{2.001} - 3^2}{.001}$

Given that the chord-slope function of $y = 2^x$ is below $y = 2^x$ and the chord-slope function of $y = 3^x$ is above $y = 3^x$ you should find some function $y = a^x$ $(2 < a < 3)$ where the chord-slope function is roughly equal to a^x.

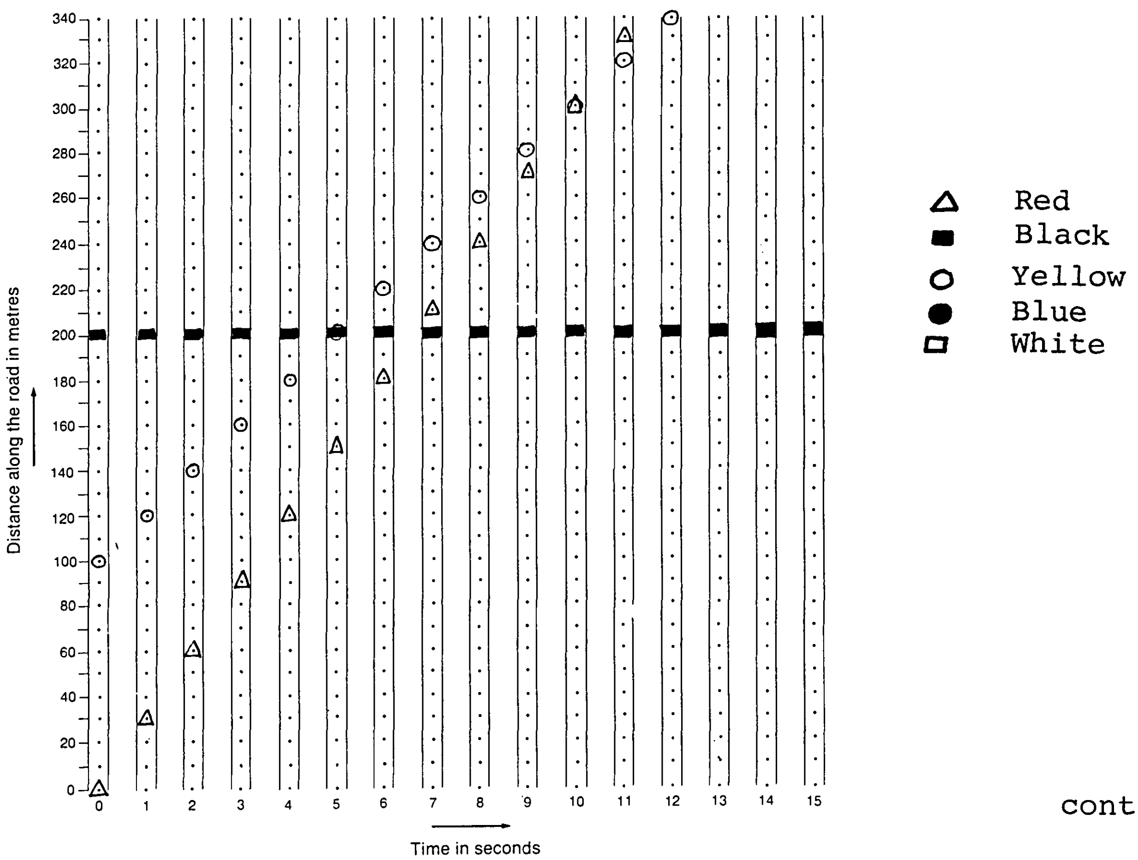
Hint: chord slope function for $y = 2^x \simeq 0.7 \times 2^x$
chord slope function for $y = 3^x \simeq 1.1 \times 3^x$

1818 Helicopter Photographs

Car driving is much more complicated than the card suggests. For example, cars usually slow down, then speed up when overtaking. Real life situations are often deliberately simplified so that we can use maths to help us understand them. This is known as mathematical modelling.

1. The red car will pass the black car then the yellow one will. After this, the yellow car will overtake the red one.

2.



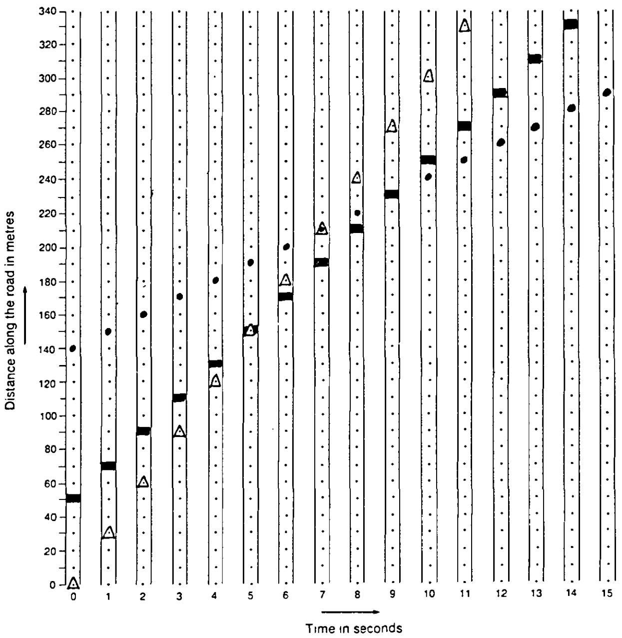
continued

1818 Helicopter Photographs (cont)

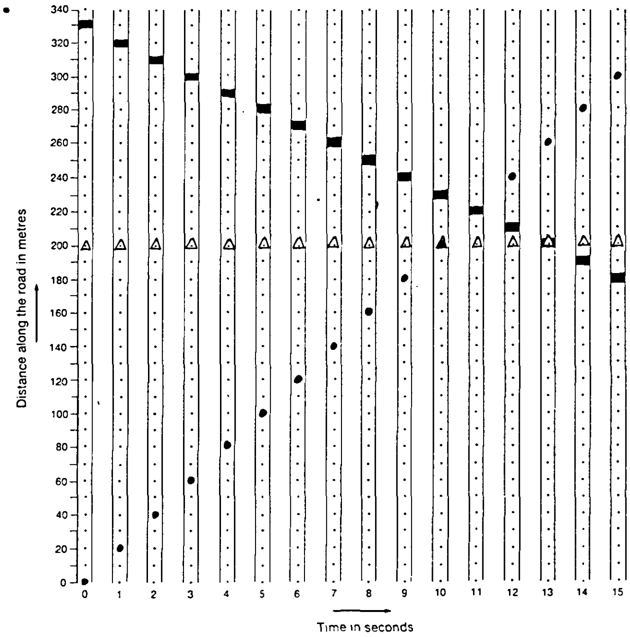
3. Your answer should be similiar to question 1, but you should now be able to give more details.

4. The cars would crash after 10 seconds because the lane is only wide enough for 2 cars.

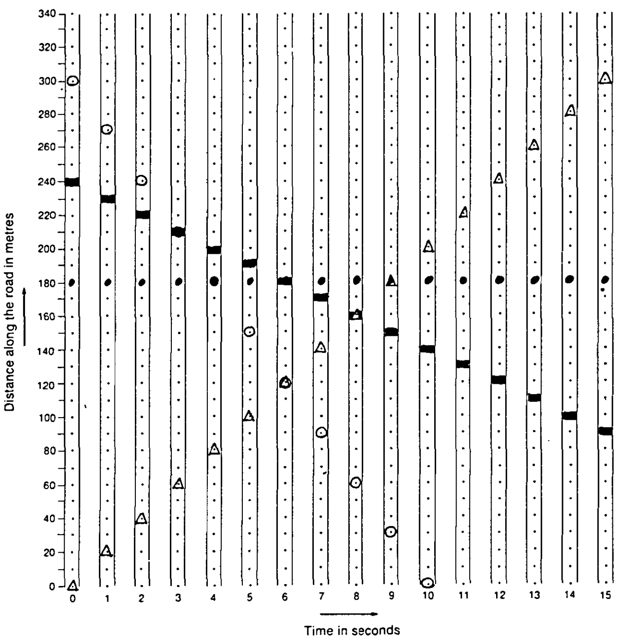
5.



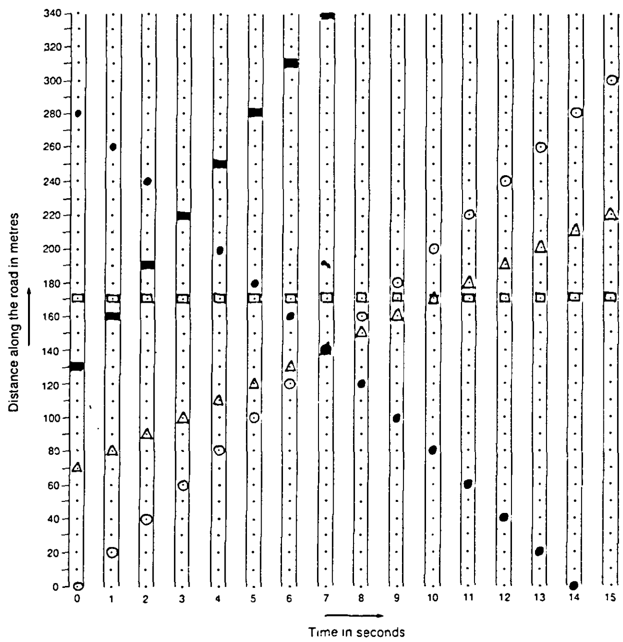
6.



7.



8.



5.-8. None of the cars crash but there are some near misses.

9. Fastest ○ △ ● ✕ ⊙ slowest

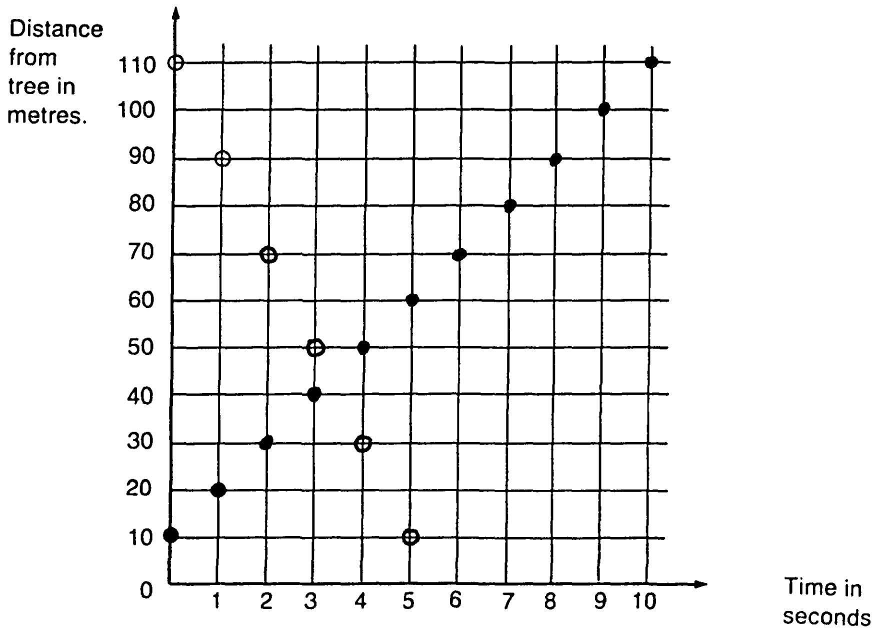
The fastest car is the one which gives the steepest line.

1820 Parallels

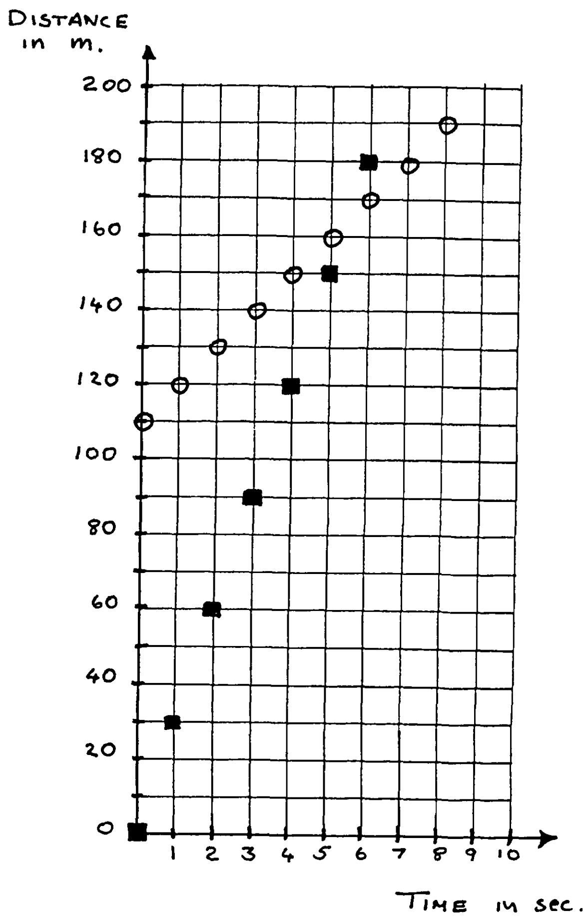
No answers required

1821 Overtaking

1. 10 seconds
2. 5 seconds
3. $3\frac{1}{3}$ seconds
4. $43\frac{1}{3}$ metres
5.



6. Your answers should be close to those for 3) and 4)



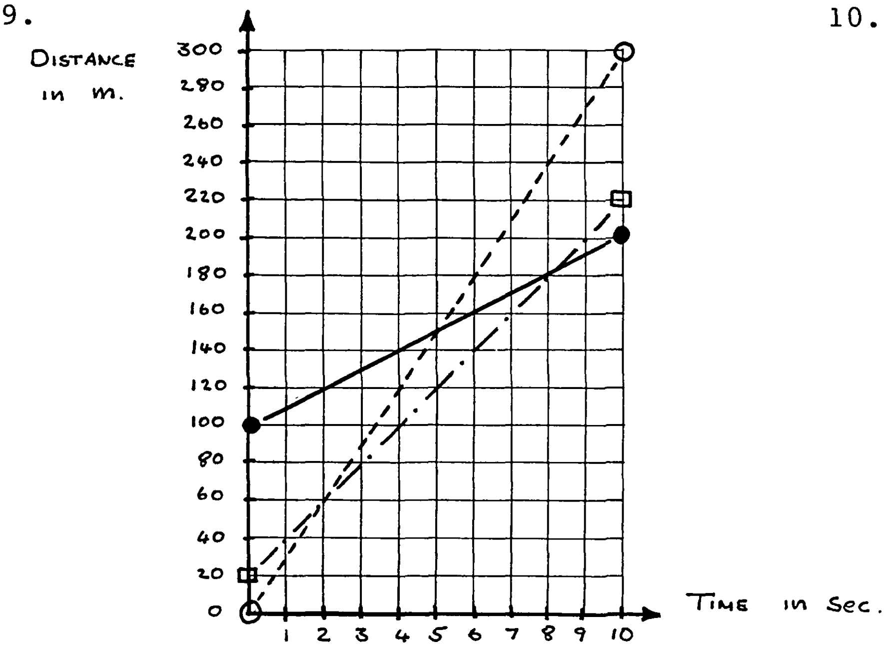
7. $5\frac{1}{2}$ seconds
8. 165m

   The answers to 7) and 8) can be read from the graph.

continued

1821 Overtaking (cont)

9.



10. The white car overtakes the grey car and then overtakes the black car. Later the grey car overtakes the black car.

1822 Product of Primes

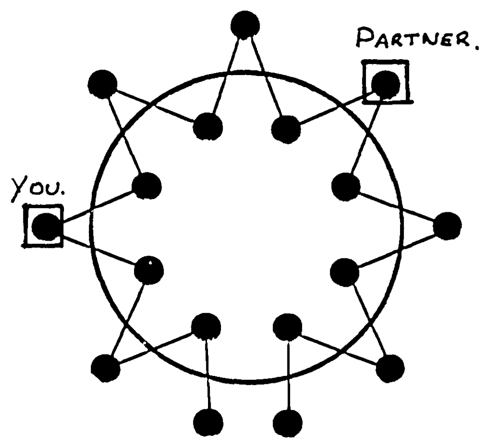
1. a) 3 and 13
   b) 19 and 5
   c) 17 and 11
   d) 13 and 19
   e) 131 and 211
2. Many possible answers

1823 In and Out

This is a very difficult puzzle to solve and the following strategy is probably not the only one. It is in several connected parts which get harder. You may like to read part one and then see if you can go back and use it to devise your own winning strategy.

1. Show that if it's your turn and you can close the gap between you and your partner (by landing on the blob next to them), then your partner must retreat and lose.

2. Suppose now that it's your turn but you cannot close the gap between you and your partner. Show that to prevent her from winning you must leave at least four blobs between you. What happens if you can leave exactly four blobs between you and your partner? Try several positions and see what happens.



You to move.

How can you win?

3. You may have found in part (2) that if you can put exactly four blobs between you and your partner then you will win. Why does putting exactly eight blobs between you also allow you to win?

4. Extend this agument to show that you can win if you put exactly twelve blobs between you and your partner. Does this suggest you should go first or second? Does this help with looking at different size boards.

Different Sized Moves

In this game you move up to three blobs at a time and the winning moves seemed to consist of looking at groups of four. If you are allowed to move up to n blobs at a time (where n=1,2,3,4 etc) then can you write down a rule which will help you to win.

Does your strategy for this game help with the SMILE program Jumping?

1824 Silver Earrings

1. £3	2. £8	3. £7	4. £2	5. £1
6. £4	7. £8	8. £4.50		

1825 Exactly Ten

No answers required.

1826 $y = mx$

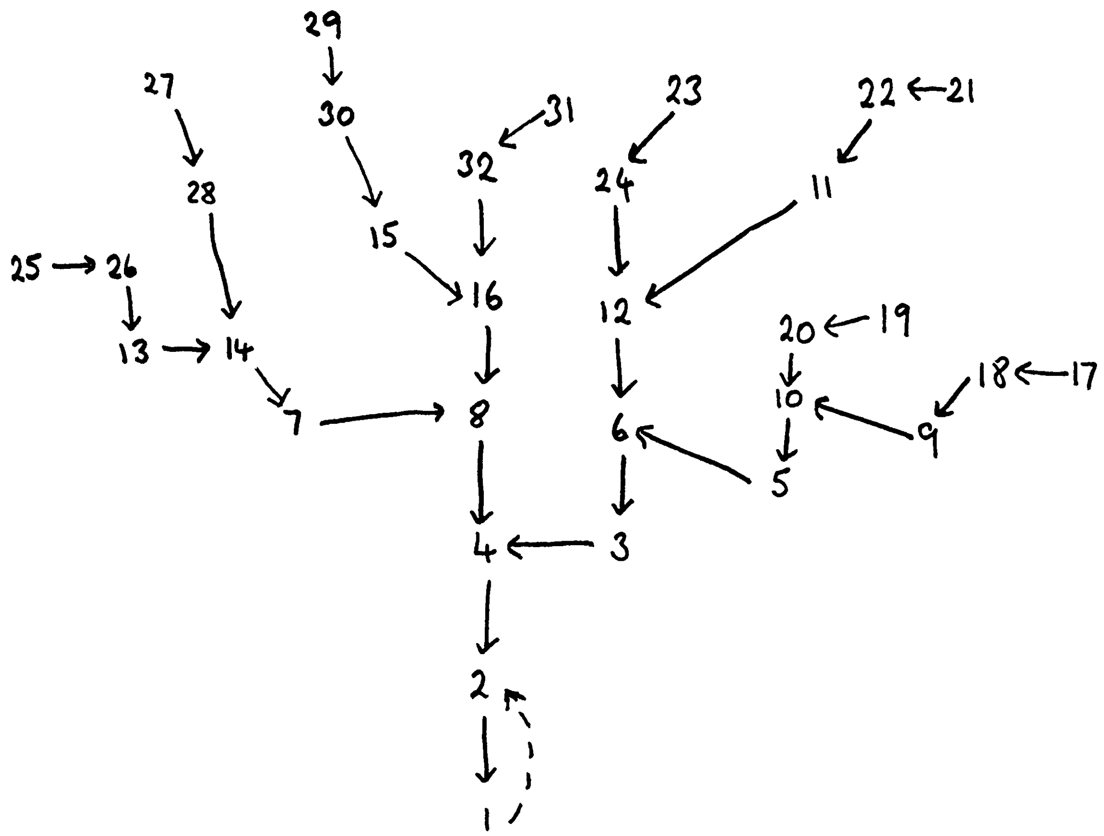
- As m gets larger, the line becomes steeper.
- The line is steepest when it coincides with the y axis.
- The line is least steep when $m = 0$ and the line coincides with the x-axis.
- The y axis is the same as the line $x = 0$ and the x-axis is the same as the line $y = 0$.
- $y = -mx$ is a mirror image of $y = mx$.
- A further question : What line is at right angles to $y = mx$?

1827 Odd and Even Chains

The chain for 9 continues like this:

9 → 10 → 5 → 6 → 3 → 4 → 2 → 1 → 2 → 1 and so on.

If you have looked at the numbers from 1 to 32, the chains can be put together into one diagram.



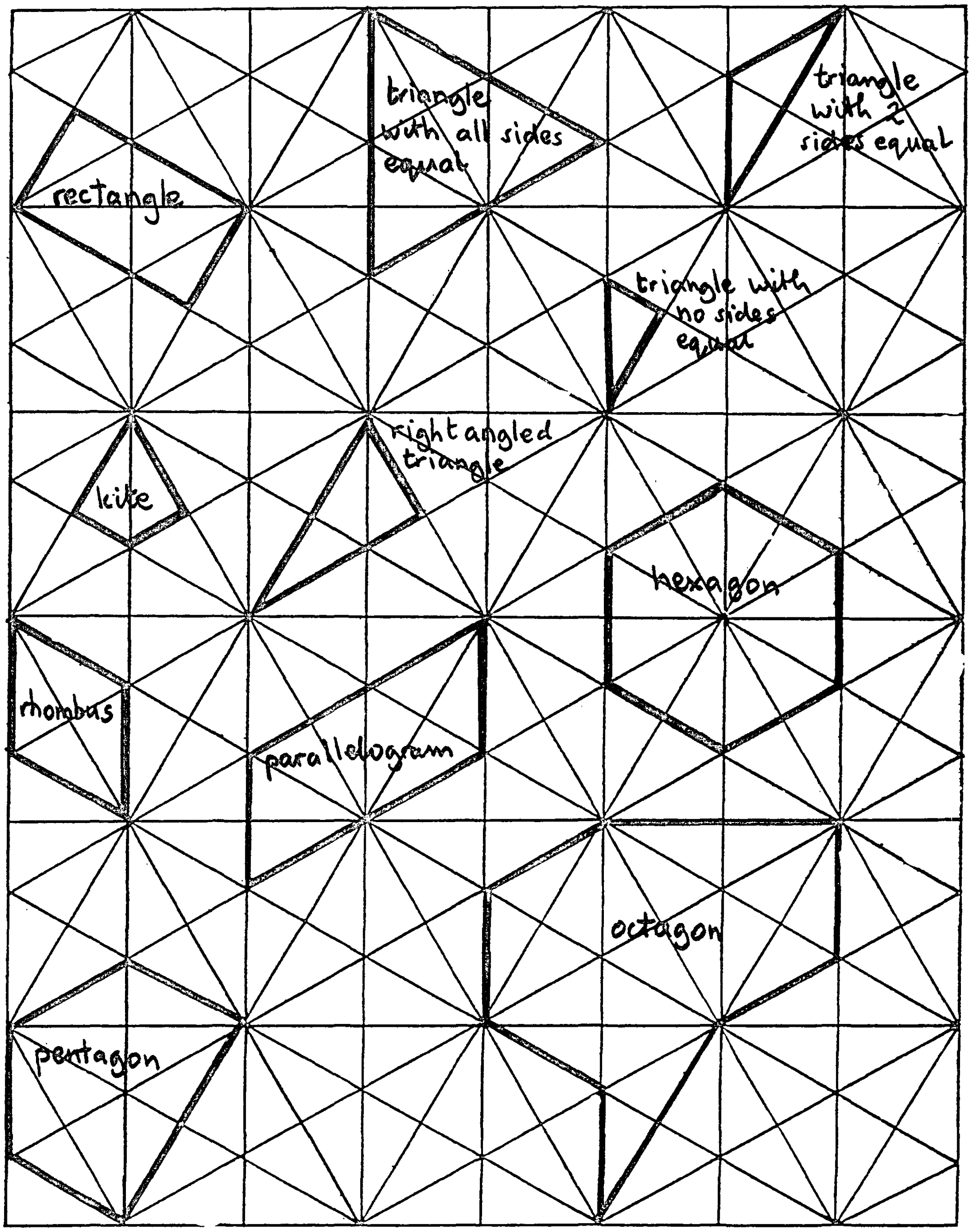
1,2,4,8,16,32 ... give the shortest chains. Why? What are these numbers?

If you work out the chain lengths for the numbers 1 to 32, you may find a pattern which you can use to predict which numbers give the longest chains and how long those chains are.
Does 128 give a long or short chain? What about 129?

You might like to investigate chains made with different rules.

1828 Find the Shape

There are many different answers for each shape.
Here is one possible answer for each shape.



You may like to see how many different answers you can find.

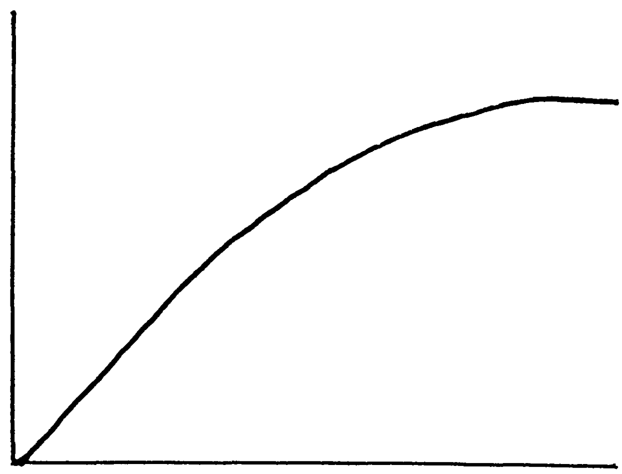
1829 Two Rectangles

If you have a problem with this puzzle, try working with someone else.
Did you find both rectangles?

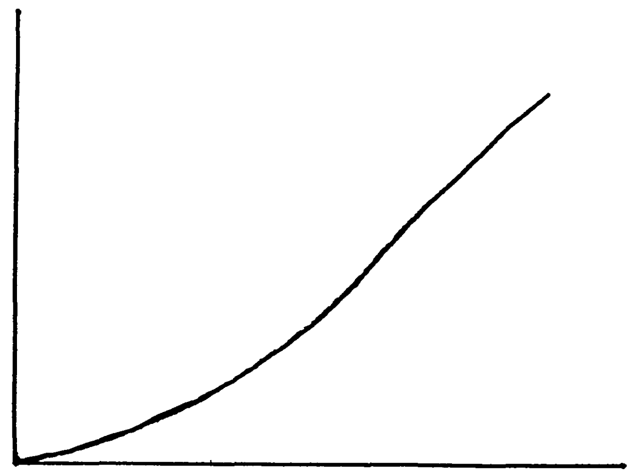
1830 The 'smoothing out' principle

1. a. after 3 seconds
   b. after 7.5 seconds
   c. 4.5 seconds

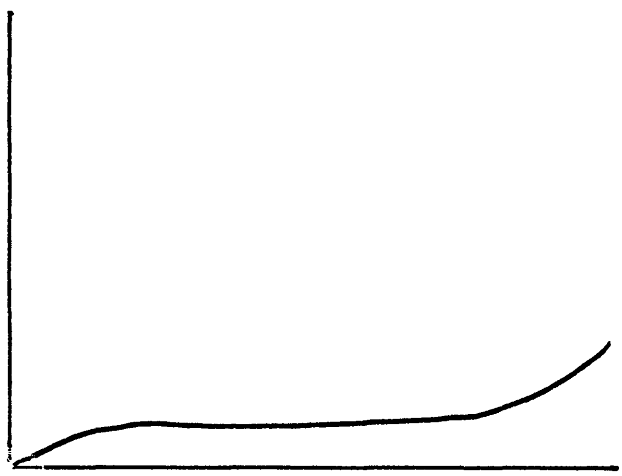
2.



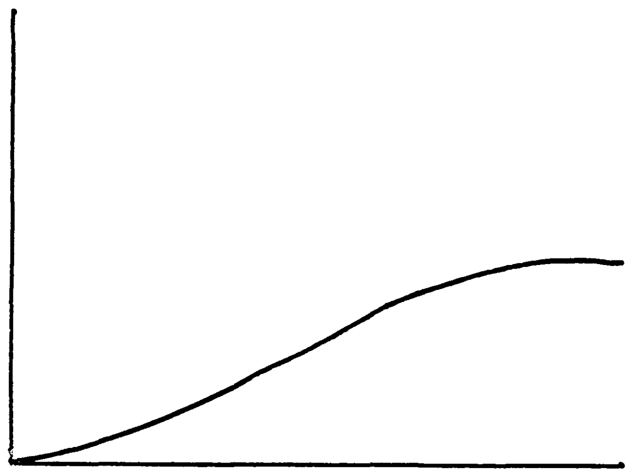
3.



4.

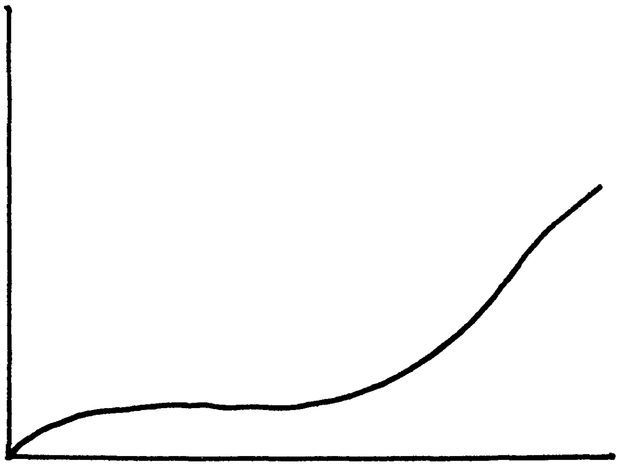


5.



1830 The 'smoothing out' principle (continued)

6.



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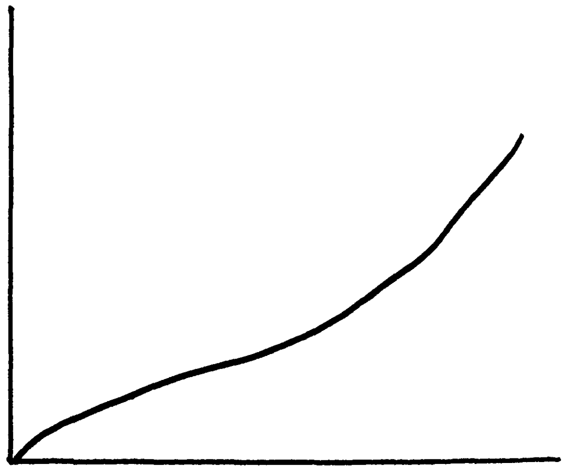
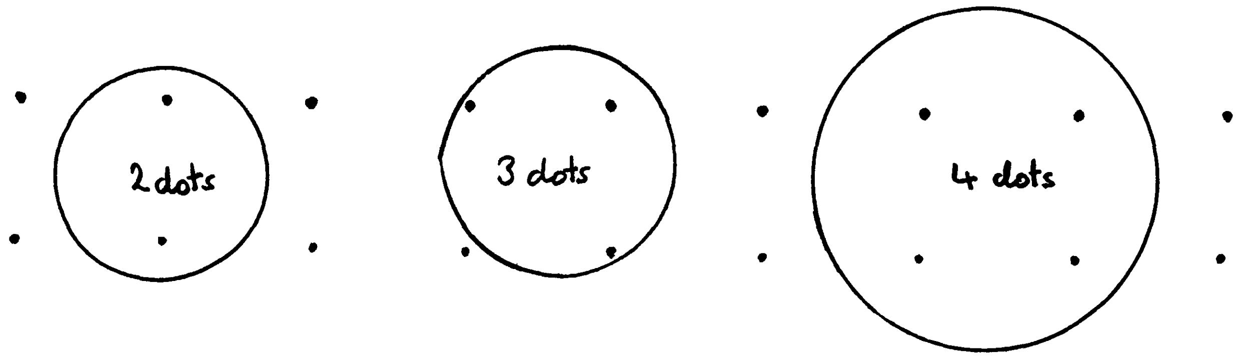


Your graphs will be different from these but should have the same general shape.

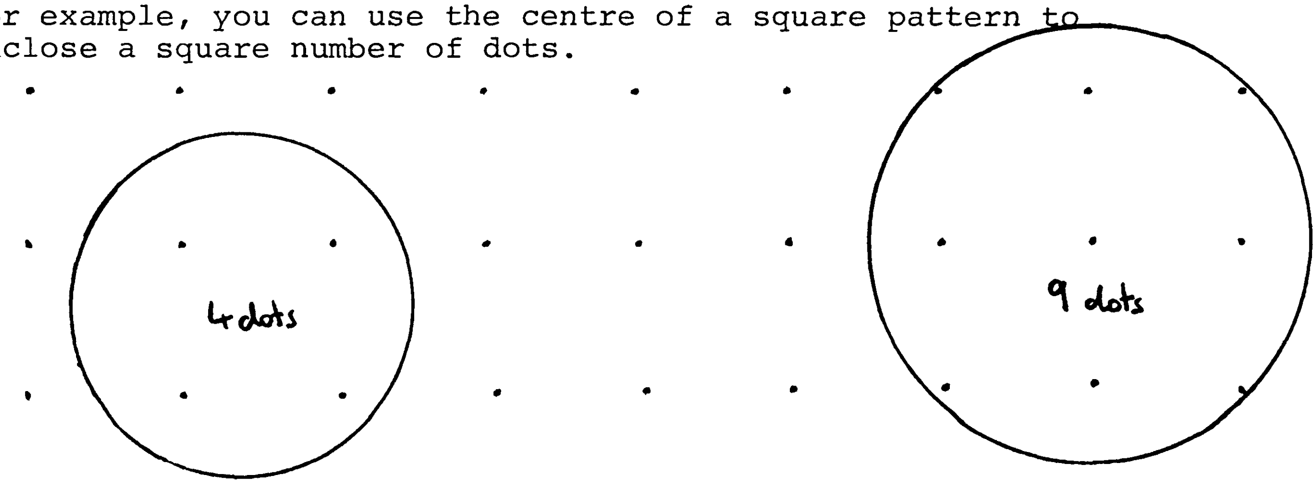
1831 Circles and Dots

Did you manage to enclose 2,3,4 dots?



Some numbers are easier to enclose than others. Can you explain why?

For example, you can use the centre of a square pattern to enclose a square number of dots.

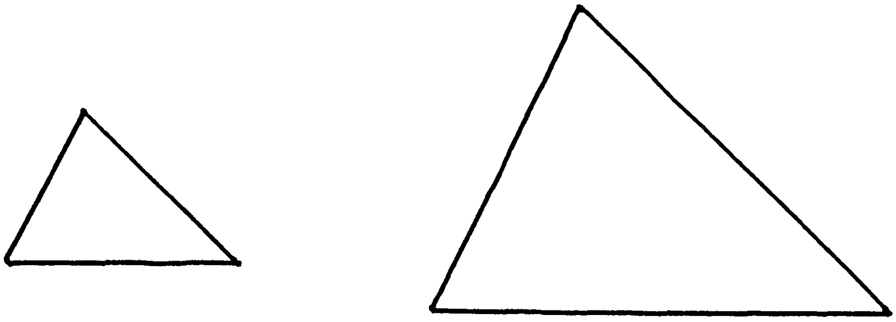


Which numbers are easy to enclose on isometric paper?

1832 Minimum Information

Sui does not have enough information. Triangles with the same angles will be the same shape but can be different sizes.

eg.



However it is possible for Sui to be certain by asking just three questions. She can ask for *two sides and the angle between them* or *two angles and a side* or *three sides*. Try out these possibilities to convince yourself.

1833 Magic (MICRO)

No answers required.

1834 Tenners (MICRO)

No answers required.

1835 Magnify (MICRO)

No answers required.

1836 Three-in-a-Line (MICRO)

No answers required.

1837 Mirror (MICRO)

No answers required.

1838 0^0

$0.1^{0.1} = ?$

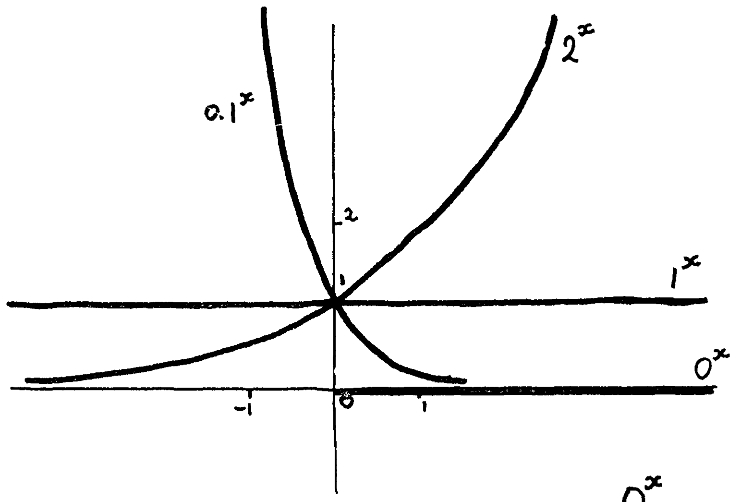
$0.01^{0.01} = ?$

$0.001^{0.001} = ?$

$0.0001^{0.0001} = ?$ so it looks as if $0^0 = 1$

Or try sketching graphs of $y=2^x$, 1.5^x, 1^x, 0.5^x, 0.1^x and deduce the graph of 0^x.

1838 0^0 (continued)



0^x appears to jump up at the origin to pass through (0,1)

1839 Which Card is Missing?

There are 4 suits. These are spades ♠, hearts ♥, diamonds ♦ and clubs ♣.
There are 13 cards in each suit. These are ace (one), 2,3,4,5,6,7,8,9, Jack, Queen, King.
This should help you to sort the cards.

How many cards in a complete pack?

How many smiling faces are there on the back of card 1839.

1840 A Point and Many Lines

Discuss your report with your teacher or with someone else.

1841 Interlocking Squares (DIME)

No answers required.

1842 Shapes Jigsaw (DIME)

No answers required.

1843 Polygons and Right Angles

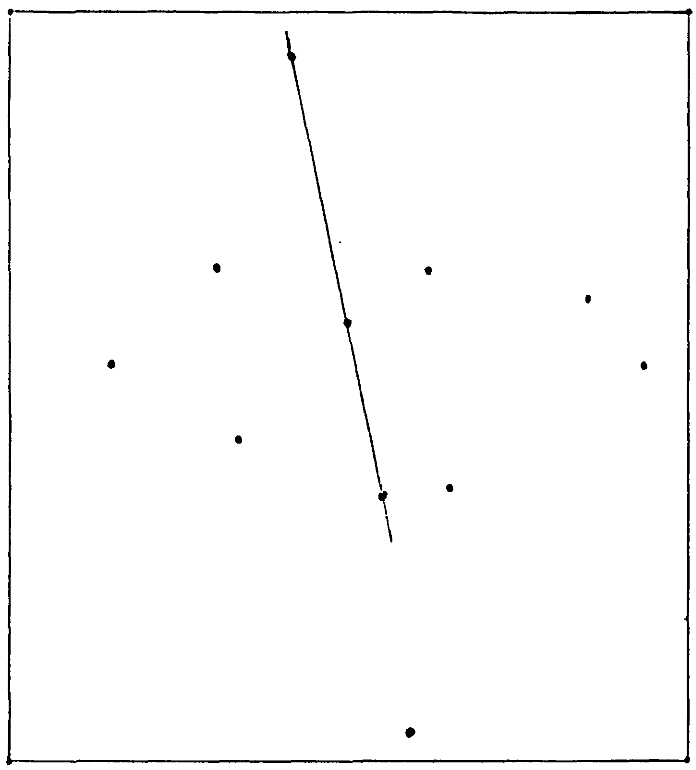
If all six angles of a hexagon were 90^0 then the angle sum would be 540^0. But the angle sum of a hexagon is 720^0 so a hexagon cannot have 6 right angles.

To justify the results of your investigation into other polygons with right angles you may like to consider other angle sums. You will need to be systematic. Can you find a generalisation for the maximum number of right angles possible for each polygon?

Challenge: draw a polygon with 13 right angles.

1844 Straight Lines

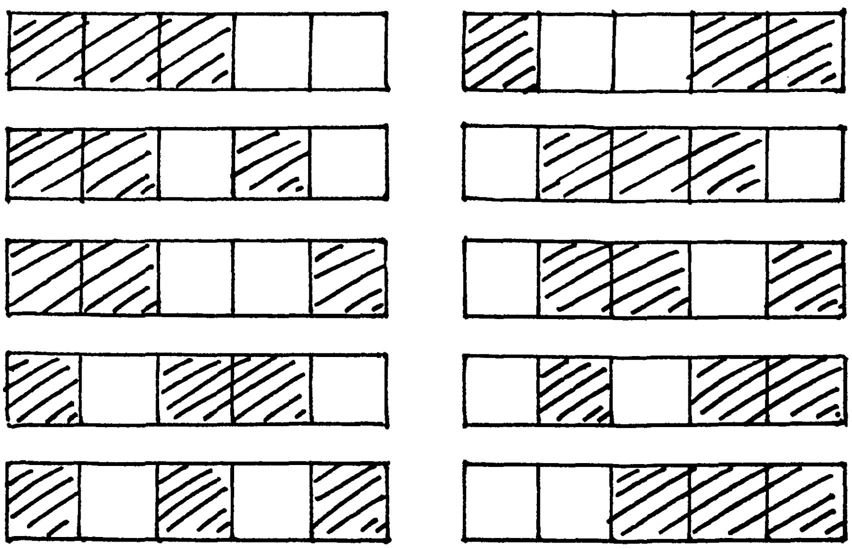
1.



2. It is possible to find 8 different lines.

1845 Shading Strips

There are 10 different ways of shading $\frac{3}{5}$ of the strip.



It helps to be systematic.

Shading $\frac{2}{5}$ will be easy now. Just change the white squares to shaded squares. You might also try $\frac{1}{5}$ and $\frac{4}{5}$.

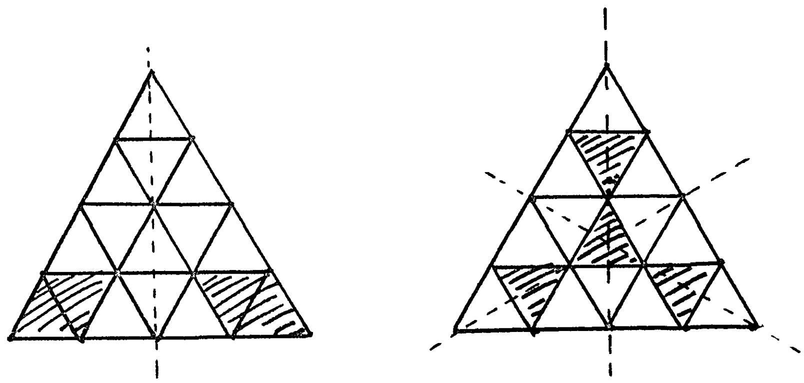
Try other sets of fractions e.g. $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$ or $\frac{1}{6}$, $\frac{2}{6}$ $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$ and see if you can spot any patterns.

1846 'L' an adventure game (MICRO ATM)

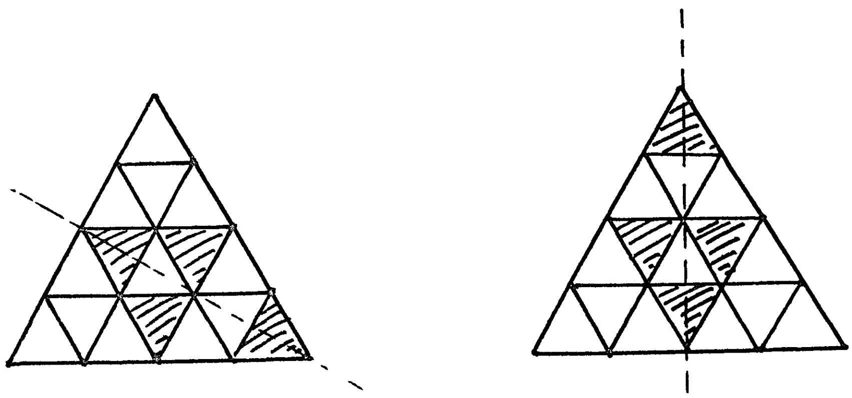
No answers required.

1847 Symmetrical Triangles

There are many possible answers.
Some have one line of symmetry and some have three lines of symmetry.



What rules did you make for shapes being different?
Did you count these as different?



Have you tried changing the number of triangles shaded?
Or have you tried changing the size of the large triangle?
If you want to investigate this you will need isometric paper.

1848 Three by three

No answers required.

1849 100 Search

10	25	5	50	5	5	10	10	3	5
25	20	2	3	10	20	5	10	85	10
35	25	30	5	15	5	35	10	45	5
65	20	2	40	25	10	60	10	2	35
2	2	5	2	5	5	45	10	50	90
65	2	3	15	85	10	3	10	10	5
15	35	2	15	10	5	5	10	3	3
15	3	90	3	15	10	5	10	75	2
75	3	2	5	2	5	5	10	70	3
2	3	55	30	20	55	12	20	80	97

1850 Alice (MICRO)

No answers required.

1851 Regions (MICRO)

This is a resource program so your answers will depend on what you used it to do.

1852 Foxes (MICRO)

No answers required.

1853 Pinball (MICRO)

This is a resource program so your answers will depend on what you used it to do.

1854 Matrices (MICRO)

This is a resource program so your answers will depend on what you used it to do.

1855 Quadratic Mappings (DIME)

There are many possible ways of explaining the mappings. The numerical and algebraic answers are given below.

1.
$1 \to 1$
$2 \to 4$
$3 \to 9$
$4 \to 16$
$5 \to 25$
$6 \to 36$
$10 \to 100$

$k \to k^2$

2.
$1 \to 0$
$2 \to 2$
$3 \to 6$
$4 \to 12$
$5 \to 20$
$6 \to 30$
$11 \to 110$

$x \to x(x-1)$
or
$x \to x^2-x$

3.
$1 \to 4$
$2 \to 9$
$3 \to 16$
$4 \to 25$
$5 \to 36$
$6 \to 49$
$11 \to 144$

$a \to (a+1)^2$
or
$a \to a^2+2a+1$

4.
$1 \to 3$
$2 \to 8$
$3 \to 15$
$4 \to 24$
$5 \to 35$
$6 \to 48$
$10 \to 120$

$n \to n(n+2)$
or
$n \to n^2+2n$

5.
$1 \to 5$
$2 \to 13$
$3 \to 25$
$4 \to 41$
$5 \to 61$
$6 \to 85$
$10 \to 221$

$\ell \to \ell^2+(\ell+1)^2$
or
$\ell \to 2\ell^2+2\ell+1$

6.
$1 \to 4$
$2 \to 12$
$3 \to 24$
$4 \to 40$
$5 \to 60$
$6 \to 84$
$10 \to 220$

$n \to 2n(n+1)$
or
$n \to 2n^2+2n$

7.
$1 \to 1$
$2 \to 6$
$3 \to 15$
$4 \to 28$
$5 \to 45$
$6 \to 66$
$10 \to 190$

$y \to y(2y-1)$
or
$y \to 2y^2-y$

8.
$1 \to 6$
$2 \to 15$
$3 \to 28$
$4 \to 45$
$5 \to 66$
$6 \to 91$
$10 \to 231$

$w \to (w+1)(2w+1)$
or
$w \to 2w^2+3w+1$

9.
$2 \to 5$
$3 \to 14$
$4 \to 27$
$5 \to 44$
$6 \to 65$
$7 \to 90$
$10 \to 189$

$x \to 2x^2-x-1$

continued

10. $1 \rightarrow 1$
$2 \rightarrow 2$
$3 \rightarrow 5$
$4 \rightarrow 10$
$5 \rightarrow 17$
$6 \rightarrow 26$
$10 \rightarrow 82$

$k \rightarrow k+(k-1)(k-2)$
or
$k \rightarrow k^2-2k+2$

Target test - Standard

Height	Small squares	Horizontal matches	Vertical matches
$1 \rightarrow$	2	4	3
$2 \rightarrow$	6	9	8
$3 \rightarrow$	12	16	15
$4 \rightarrow$	20	25	24
$5 \rightarrow$	30	36	35
$10 \rightarrow$	110	121	120
$k \rightarrow$	$k(k+1)$ or k^2+k	$(k+1)^2$ or k^2+2k+1	$k(k+2)$ or k^2+2k

4. $3k^2$

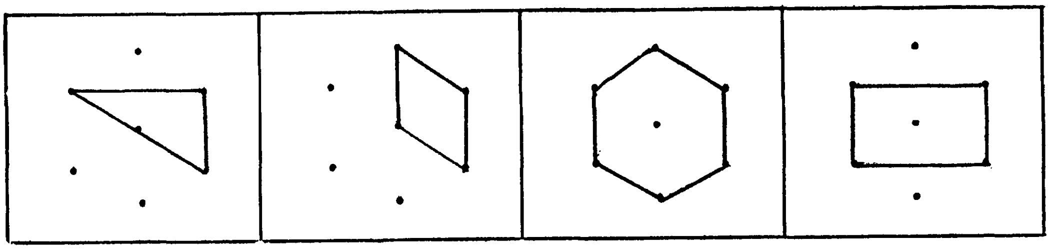
Target test - Advanced

Height	Small squares	Number of	Number of matches
$1 \rightarrow$	1	0	4
$2 \rightarrow$	6	2	17
$3 \rightarrow$	15	8	38
$4 \rightarrow$	28	18	67
$5 \rightarrow$	45	32	104
$10 \rightarrow$	190	162	409
$n \rightarrow$	$n(2n-1)$ or $2n^2-n$	$(n-1)(2n-2)$ or $2n^2-4n+2$	$2n^2+(n+1)(2n-1)$ or $4n^2+n-1$

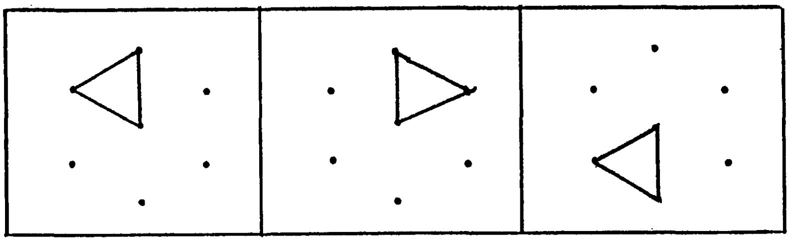
4. $n(21-n)$ or $21n-n^2$

1856 What Shapes?

Here are some of the shapes you can make.

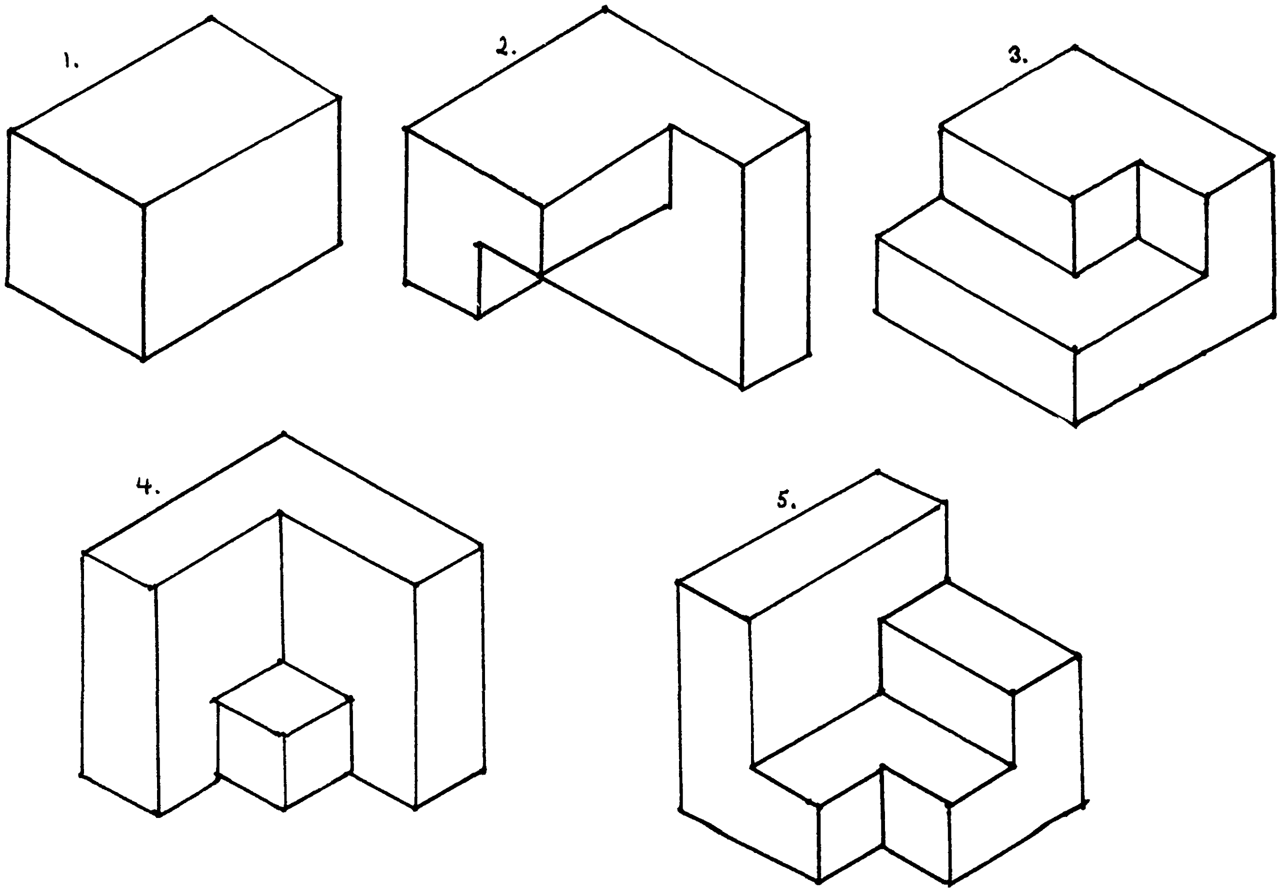


In your work, have you counted these as being the same or different?



1857 The Other Side

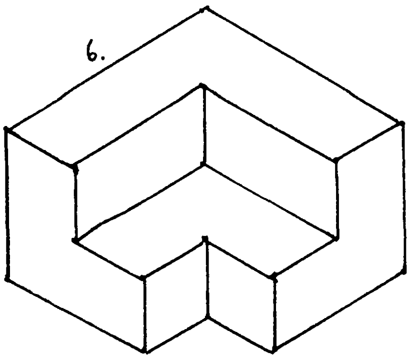
Your answers may look different if you have drawn them from a different view.



continued

1857 The Other Side (continued)

6.



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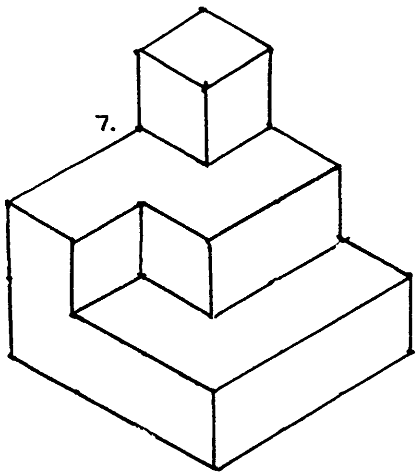

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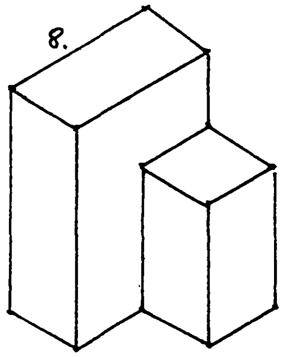


1858 Bengali ১৯ Piece Puzzle

No answers required.

1859 Co-ordinate Jigsaw (MICRO MWAS)

No answers required.

1860 Colouring Puzzle (MICRO MWAS)

No answers required.

1861 Dipsticks

1 → e
2 → c
3 → a
4 → f
5 → d
6 → b

1862 Even Animal

You should have drawn a bird.

1863 Magic Egg (BRISTOL)

No answers required.

1864 Block Buster (BRISTOL)

No answers required.

1866 Mirror Match (DIME)

1. Yes.

2

View	A	B	C	D
Diagram	2	3	1	4

3

View	A	B	C	D	E	F	G	H
Diagram	4	2	6	5	3	7	4	1

4

View	A	B	C	D	E	F	G	H
Diagram	6	1	5	4	3	2	7	8

5

View	A	B	C	D	E	F	G	H
Diagram	5	2	4	4	2	5	1	3

6

View	A	B	C	D	E	F
Diagram	4	3	5	6	1	2

7

View	A	B	C	D	E	F	G	H
Diagram	5	6	3	7	4	1	3	2

8

View	A	B	C	D	E	F	G	H
Diagram	4	8	5	2	7	1	3	6

9

View	A	B	C	D	E	F	G	H
Diagram	3	2	6	3	1	5	5	4

10

View	A	B	C	D	E	F	G	H
Diagram	1	7	6	4	8	2	5	3

11

View	A	B	C	D	E	F	G	H	I	J
Diagram	1	9	7	5	3	10	6	8	2	4

12

View	A	B	C	D	E	F	G	H
Diagram	5	6	2	1	4		3	3

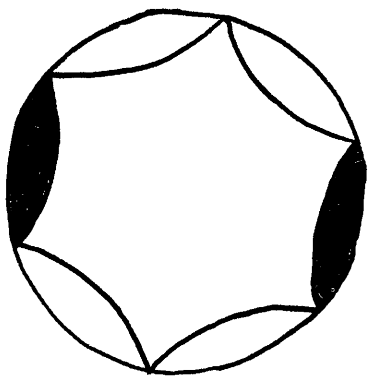
Target Test: Part 1

View	C	F	A	G
Diagram	1	2	3	4

Part 2

C	B	J	A
1	2	3	4

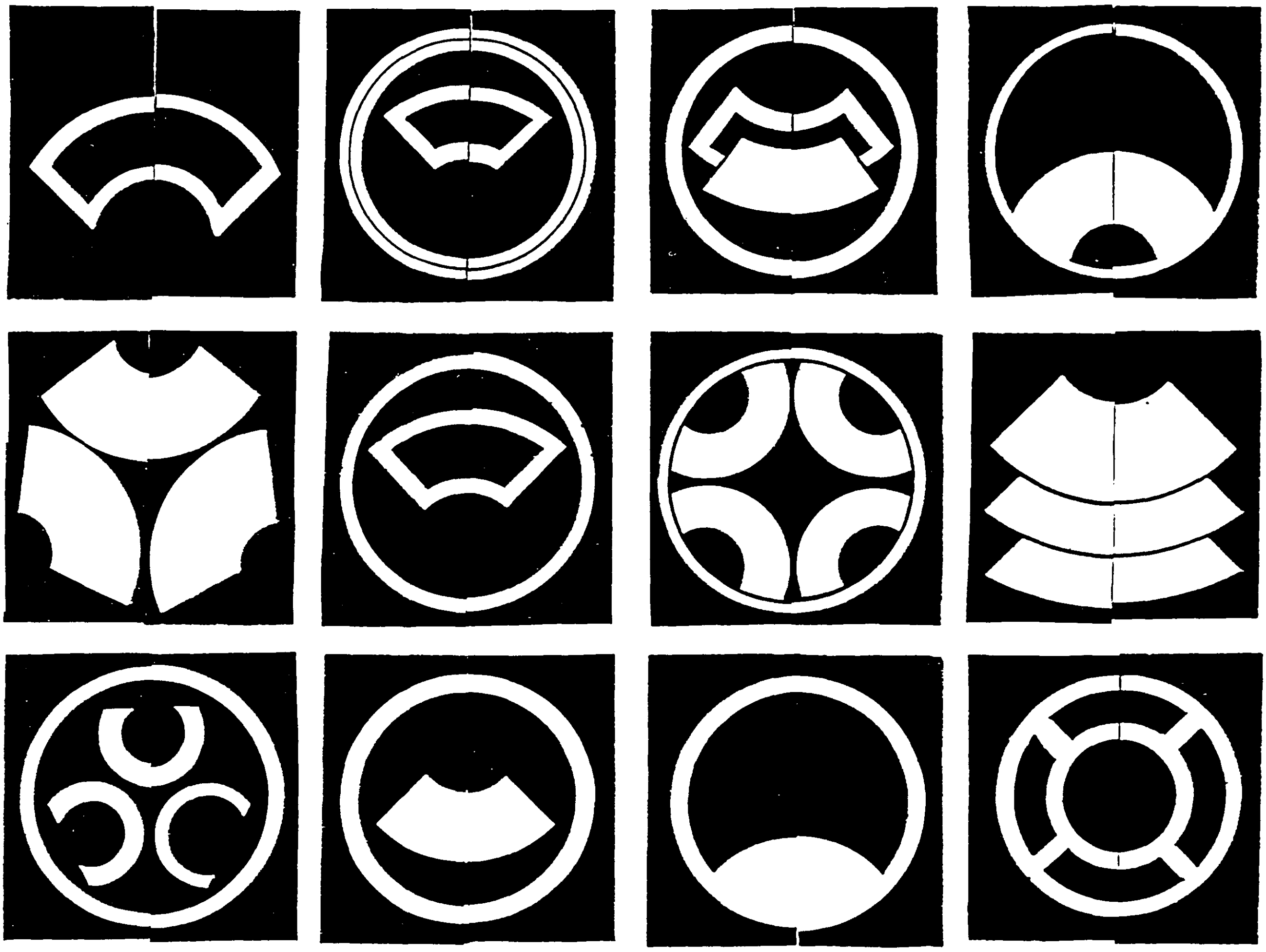
The missing diagram is



1867 Four Cubes

Each solid fits with the one that is the same to make a cube.

1868 Symmetry Match



1869 The Chinese Triangle

You will find more information about the Chinese Triangle in your school or local library although it may be listed under Pascal's Triangle.
Further work on the Triangle is contained in SMILE cards 0746 and 1438.
The patterns given on the poster are for modulo 2,5 and 6. The modulo 5 pattern was made by writing the numbers in modulo 5 (divide by 5 and write down the remainder) and then colour-coding them.

1870 Cheating

No answers required.

1871 Mayan Numbers

Having zero as a 'space-filler' means that we can easily tell the difference between 2 9 and 209. The Mayan symbol for zero was  .

An efficient number system must have a fixed base and a symbol for every value in the base, *including zero*. So in our base 10 system the Units, Tens, Hundreds .... go up in powers of 10 and we have 10 distinct symbols - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Although the Mayans had a symbol for zero they worked in a mixture of base 5 and base 20 and did not have separate symbols for each numeral.

You may like to investigate the features of other number systems eg. Hindu, Babylonian, Egyptain, Roman, Greek etc. Those cultures with an efficient number system tended to develop more mathematics and science.

The system first developed by the Hindus and extended by the Arabs (called the Hindu-Arabic decimal system) had all the features of a powerful number system. It replaced many other systems and is the one we use now.

1872 Back to back

No answers required.

1873 Polygon Symmetries

A quadrilateral can have 0, 1, 2 or 4 lines of symmetry but not 3.

What happens with triangles?

. . . pentagons?

. . . hexagons?

Can you find any general results?

Can you justify them for hexagons, octagons . . . ?

1874 Sevens Out

There are 81 numbers between 1 and 100 which do not have a '7' in them.

You may like to find how many numbers there are without a '4' between 1 and 100.

* How many without a '7' between 1 and 200?

  . . . between 1 and 500?

  . . . between 1 and 1000?

* Higher level students may like to consider how many numbers can be made between 1 and 10^n and perhaps attempt a proof. How many between say, 1 and 5336?

* It is known that the sum of the reciprocals of all positive numbers is infinite.

  $$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \quad . \; . \; .$$

  Is the same true with '7's out?

1875 Urdu Multiples.

The first is the multiple pattern of 3's.

The second is the multiple pattern of 7's.

* What is '0' in Urdu? You may find the answer by asking other people in your class, or by looking it up in the library.

1876 Fill the shape (DIME).

No answers required.

1877 Add a cube or two (DIME).

No answers required.

1878 Two blocks (DIME).

No answers required.

1879 Build and balance (DIME).

No answers required.

1880 More than two blocks (DIME).

No answers required.

1881 Hindi Addition.

Hint

$$\begin{array}{r} ८ \\ +\ ८ \\ \hline १६ \\ \hline \end{array}$$

* Get a friend to check that both additions are correct.

1882 Wedges 1 (DIME).

No answers required.

1883 Wedges 2 (DIME).

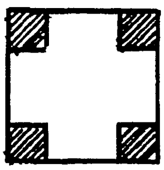
No answers required.

1884 4 Cube Shapes (BRISTOL).

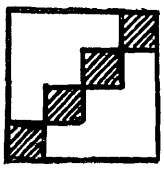
You should have 16 pieces. These can be divided into 2 identical sets. A 4 x 4 x 4 cube can be made in many ways using all the pieces.

Hints. Use the 4 long pieces

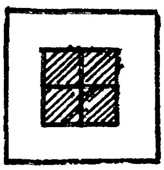
a) as corners



b) as a diagonal



c) as the centre core



1885 Optimising

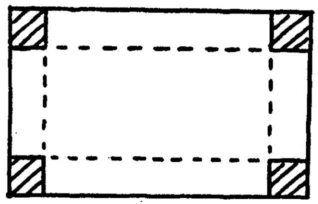
The problems in this pack may require a lot of calculations of a repetitive nature. Efficient use of your calculator can cut down the effort significantly. If you have access to a Micro and can write simple programs you will be able to generate examples very quickly. There are many different ways to tackle these problems, so only hints have been given.

Note.

When using your calculator, try to always work to full accuracy and only round off your answers at the end.

1885a Boxes

* The easiest way to make an open box is to remove squares from each corner and bend up the sides to make a box.



* By varying the size of square you can change the volume of the box.

* You could make models, work out the volumes, tabulate your results and then draw a graph of volume against square size.

* You could also write a simple program to avoid doing so many calculations.

* The largest volume found was just over $1128 cm^3$.

1885b Baked Beans.

* There are four quantities which you have to deal with in this problem - radius, height, volume and surface area. The only constant is the volume.

* You could try different values for the radius, calculate the heights and surface areas that this would produce and then use trial and improvement methods to refine your answers.

continued

<u>1885 Optimising (cont)</u>

* An algebraic method would be to combine the formula for the volume and for the surface area into one formula relating the surface area to one of the other variables.

* The minimum area found was $348.73cm^2$ (to 2 d.p.).

* There are three reasons why tins are not usually this shape.

  1) The model given is over simplified, as lids and sides are made of different metals and the length of the edges which have to be joined is a factor in the cost.

  2) The tin which has the largest volume does not often look the largest.

  3) The tin should also be easy to pick up with one hand.

1885c Books.

* You could find a first approximation for the maximum profit by working out the profit for selling the given forecast sales.

* If you draw a graph of the forecast sales against price you can use this to find a relationship between price and forecast sales which may help you to calculate the maximum profit.

* The optimum price will depend upon the assumptions and methods used but should be around £5.50.

* Seasonal variation and advertising play an important part in the selling of books plus the pricing of items as £4.95 instead of £5.00.

1885d Cones.

* This problem can be tackled by looking at various size angles of the sector being removed and finding the volume of the resultant cone.

* An algebraic solution might start with the relationship between the angle of the sector cut away and the radius of the cone. Tabulating results and using a graph may help to find an approximate maximum volume.

1885 Optimising (cont)

* It is also possible to find the radius of the cone which gives the largest volume and then to calculate the angle of the sector cut away.

* The largest volume found was 403.06cm^3(to 2 d.p.) when radius = 8.16cm (to 2 d.p.). Angle of the sector cut away = 66.24^{o}.

GENERAL COMMENTS

All the problems in this pack can be solved using calculus methods, which are covered in 'A' level Mathematics courses.

* You may like to return to these problems after you have studied calculus.

1886 World View

*** Which is bigger?**

China is bigger than the United States of America.

South Africa is bigger than Bolivia.

Cuba is bigger than Iceland.

Bangladesh is bigger than Portugal.

A more accurate guide to the area of these countries can be found in an Atlas. The following figures are from "The New State of the World Atlas" by Michael Kidron and Ronald Segal IBSN 0330284320.

China..............................9595000km^2
United States of America...........9363000km^2
South Africa.......................1221000km^2
Bolivia............................1099000km^2
Cuba...............................115000km^2
Iceland............................103000km^2
Bangladesh.........................144000km^2
Portugal...........................92000km^2

*** How many?**

Approximately 11 "Britains" will fit into Zaire.

Approximately 13 "Britains" will fit into India.

Algeria, Greenland and Saudi Arabia are approximately 5 times the size of Morocco.

Approximately 130 - 140 "Nicaraguas" in South America.

continued

1886 World View (cont)

*** Countries**

We found 48 countries on the mainland of Africa. There are also 11 offshore islands. These figures were checked by looking in Websters' New English Dictionary.

Sudan is tha largest with 2506000km^2
Algeria .... 2382000km^2
Zaire ....2345000km^2

We found 13 countries in South America.

French Guiana is the smallest with an area of 91000km^2.

*** Another Country**

Uganda, Upper Volta and Ghana are roughly the same size as Britain.

Ecuador and Guyana are roughly the same size as Britain.

Laos is roughly the same size as Britain.

West Germany, Romania and Yugoslavia are roughly the same size as Britain.

1887 Logo can calculate (LOGO)

RML 380Z, 480Z and Nimbus

The commands do this:-

```
PRINT add 2 2
4
PRINT subtract 3 2
1
PRINT multiply 2 add 2 3
10
PRINT add 2 3
5
PRINT multiply 2 3
6
```

Missing numbers less than 10 are 2,3,7,8,9,

Here are ways to make logo print these missing numbers.

```
PRINT add 2 subtract 3 3
2
PRINT add 2 subtract 2 2
3
PRINT add 2 add 2 3
7
PRINT add 2 multiply 2 3
8
PRINT multiply 3 3
9
```

Atari, BBC and Logotron

The commands do this:-

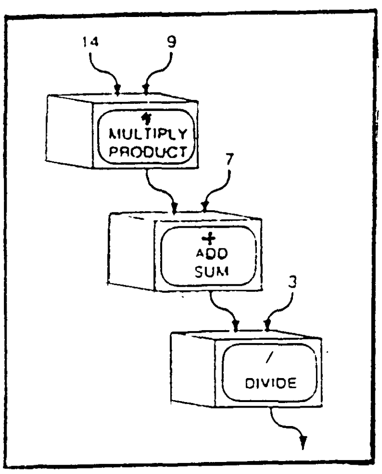
```
PRINT 2+2
4
PRINT 3-2
1
PRINT 2*(2+3)
10
PRINT 2+3
5
PRINT 2*3
6
```

Missing numbers less than 10 are 2,3,7,8,9.

Here is one way to make logo print these missing numbers.

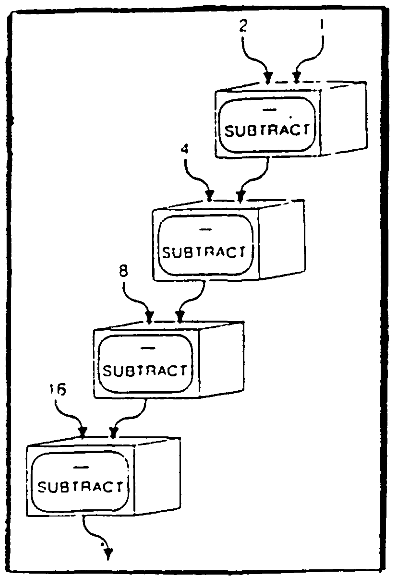
```
PRINT 2+(3-3)
2
PRINT 2+(3-2)
3
PRINT 2+(2+3)
7
PRINT 2+(2*3)
8
PRINT 3*3
9
```

1888 Operations (LOGO)



(14*9 + 7)/3

DIV ADD MUL 14 9 7 3

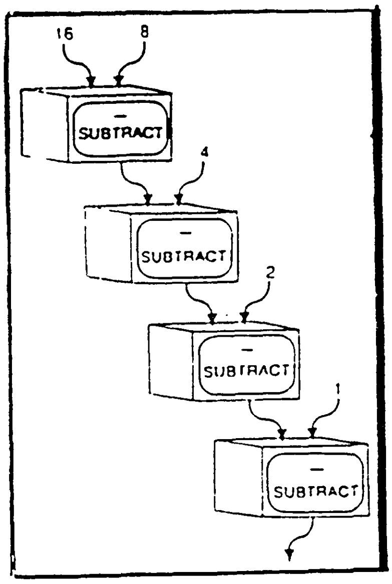


SUB 16 SUB 8 SUB 4 SUB 2 1

16 − (8 − (4 − (2 − 1)))

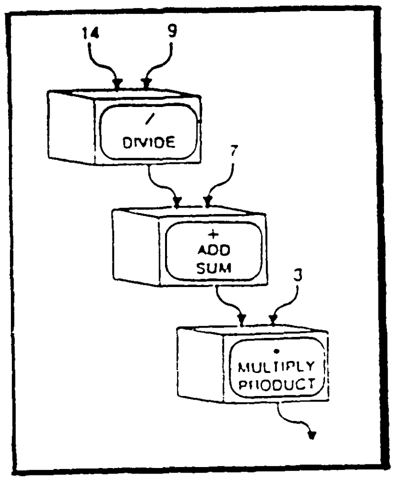
continued

1888 Operations (LOGO) (cont)



SUB SUB SUB SUB 16 8 4 2 1

16 – 8 – 4 – 2 – 1



MUL ADD DIV 14 9 7 3

(14/9 + 7)*3

1. Sub Sub Sub Sub 16 8 4 21 = 1
2. 16 - (8 - (4 -(2-1))) = 11
3. div add mul 14 9 7 3 = 44.3
4. 16 - 8 - 4 - 2 - 1 = 1
5. Sub 16 Sub 8 Sub 4 Sub 2 1 = 11
6. ($\frac{14}{9}$ + 7) * 3 = 25.6
7. (14 * 9 + 7) /3 = 44.3
8. Mul add div 14 9 7 3 = 25.6

1889 Regular Tilings 1 (DIME)

Many possible answers.

1890 Regular Tilings 2 (DIME)

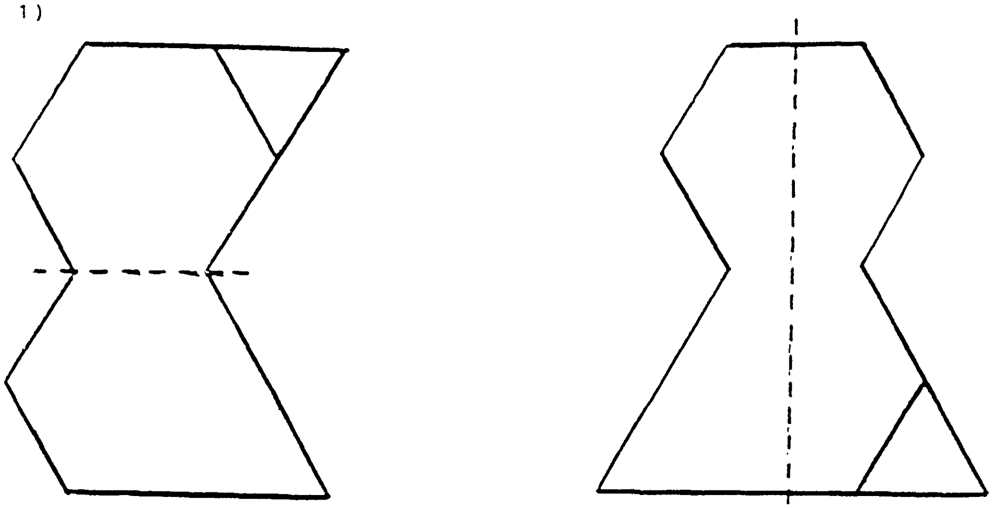
Many possible answers.

1891 Regular Tilings 3 (DIME)

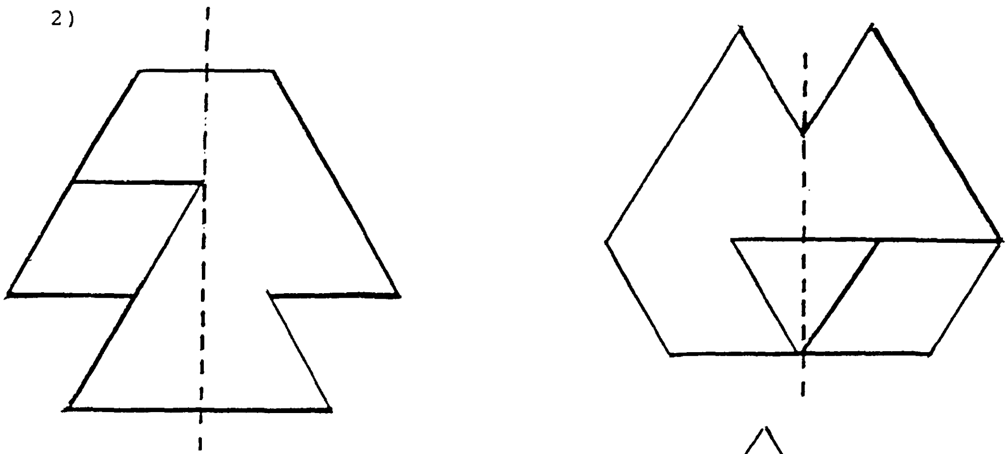
Many possible answers.

1892 Symmetry Puzzle B 1-3 (DIME)

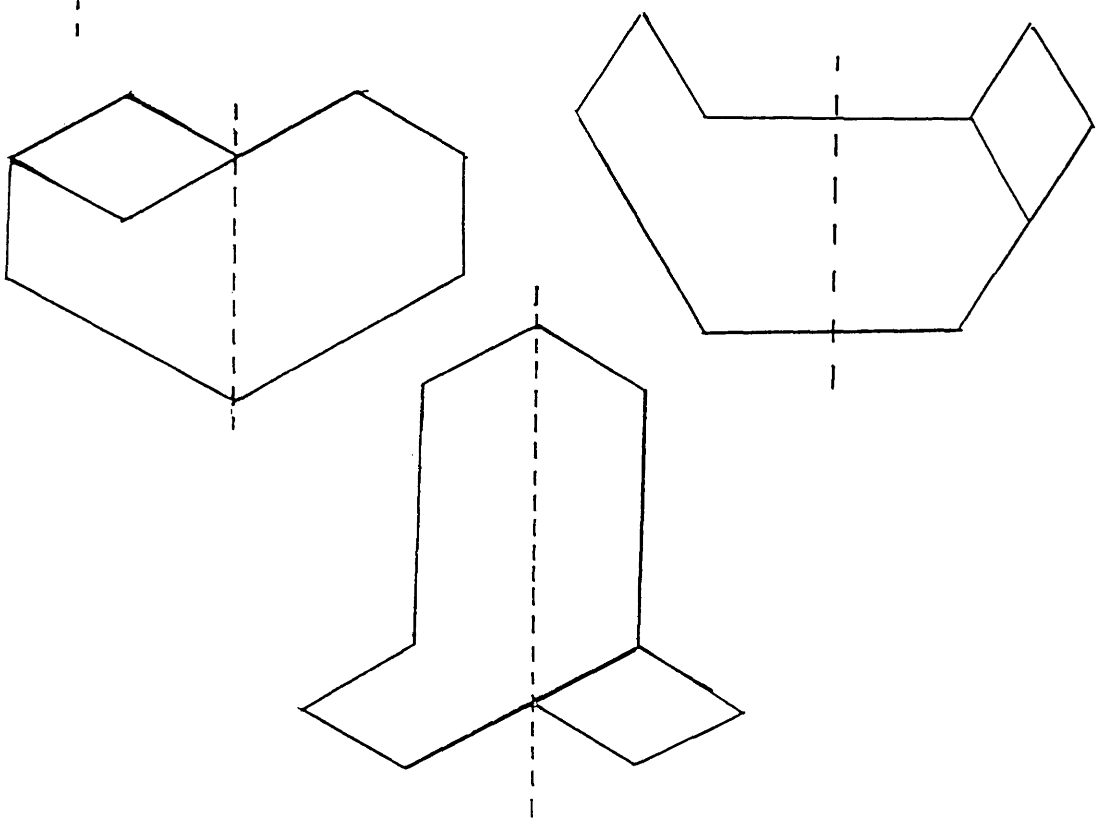
1)



2)

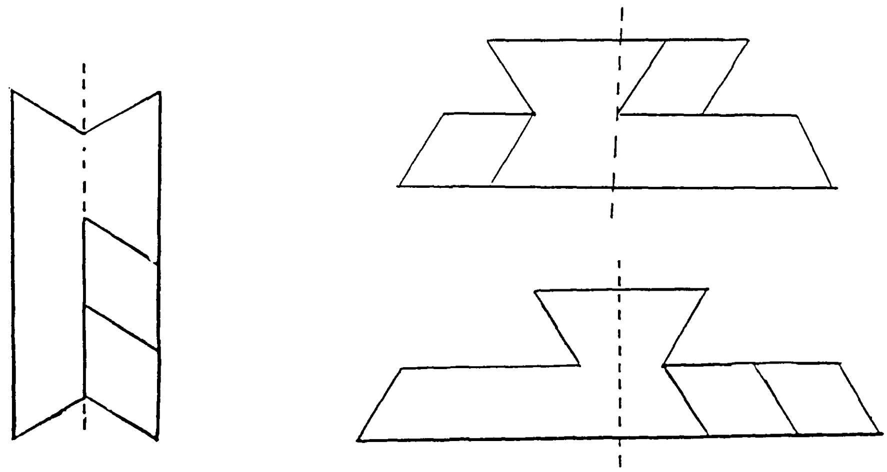


3)

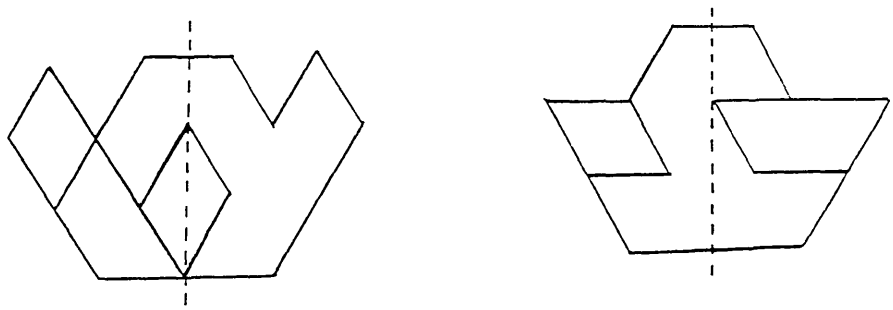


1893 Symmetry Puzzle B 4-6 (DIME)

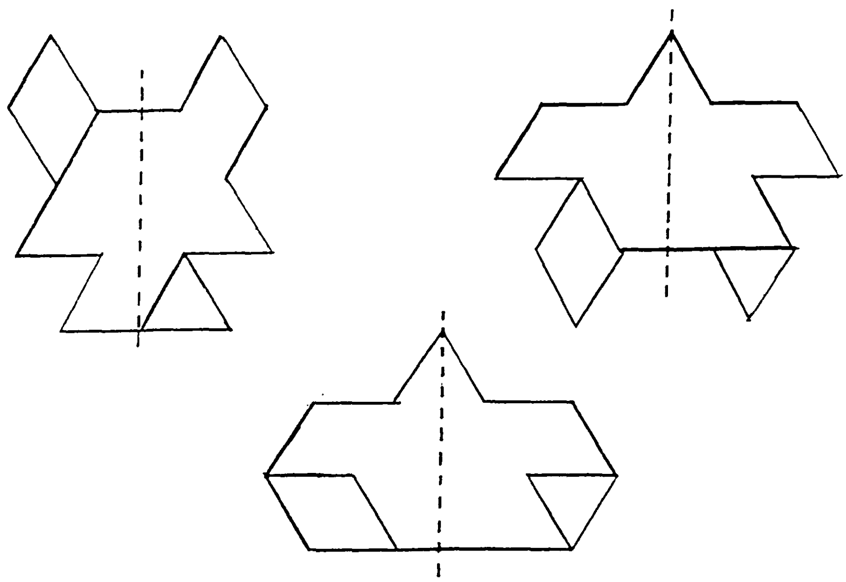
4)



5)

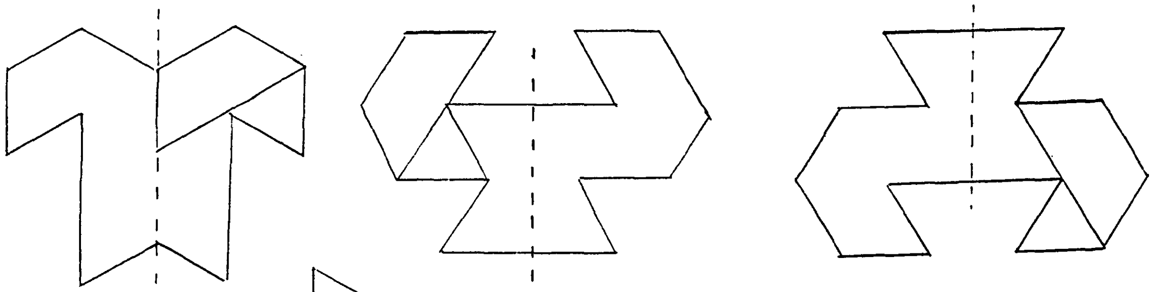


6)

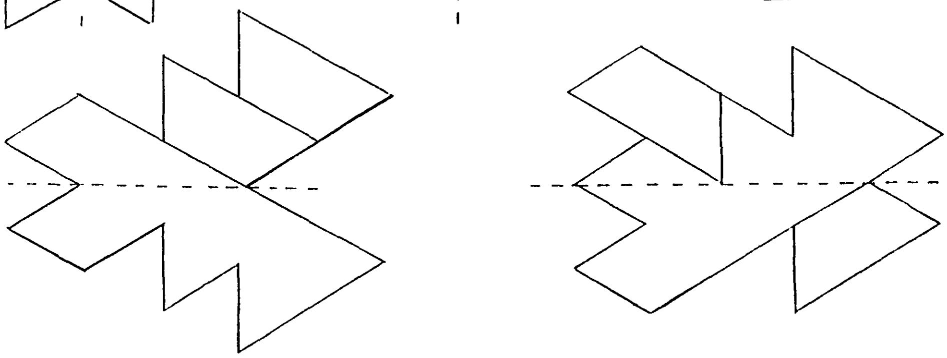


1894 Symmetry Puzzle B 7-10 (DIME)

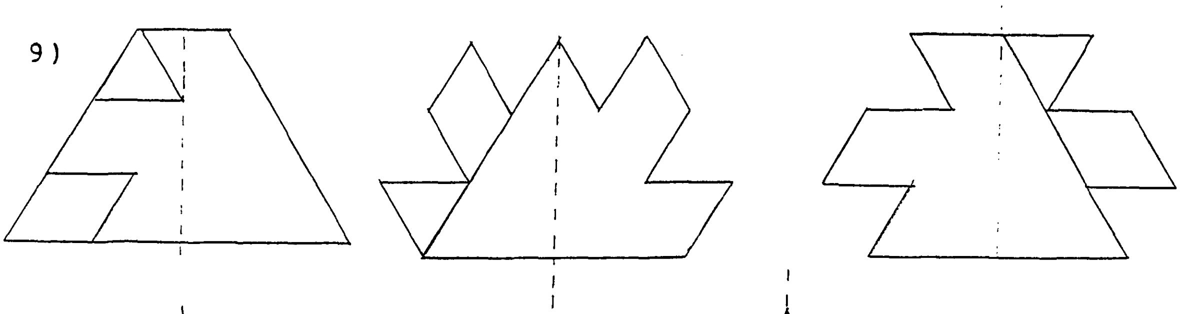
7)



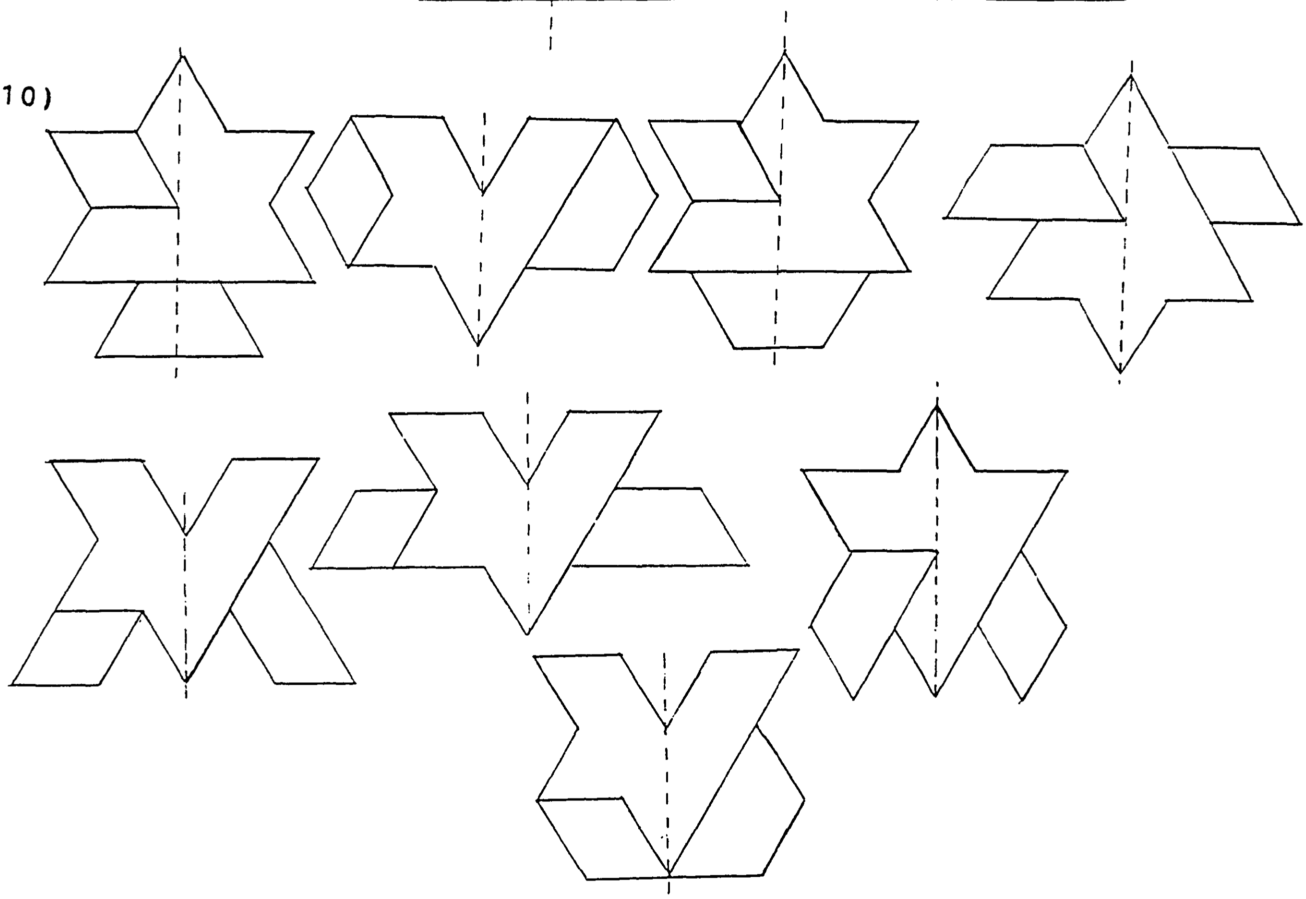
8)



9)



10)



1895 Flat Patterns

No answers required

1896 Spatial Reasoning (DIME)

No answers required.

1897 Who is the schoolkeeper?

Mee Fing is the schoolkeeper.

How did you get the answer?

* You might like to write an explanation to help other people solve puzzles like these.

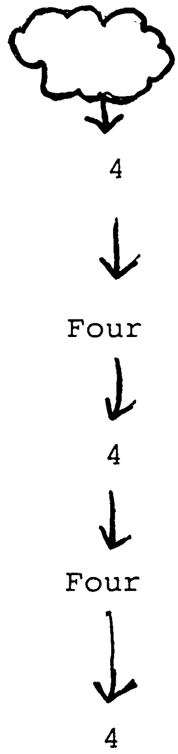
* You might like to make up a similar puzzle for a friend.

1898 Who has the micro-computer?

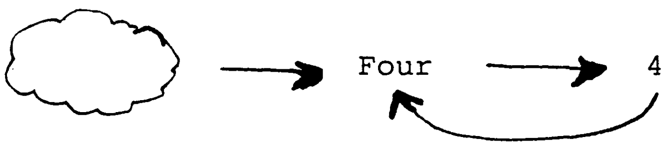
	Door	Girl	Equipment	Drinks	Subject
Top	white	Deepika	Tape-recorder	Shandy	Chemistry
3	green	**Esther**	**Micro-computer**	Tea	Engineering
2	red	Amandep	Stero	Coffee	Accountancy
1	blue	Bronwen	Remote/ control T.V.	Coke	Maths
Bottom	yellow	Carol	Video recorder	Lemonade	Physics

<u>1899 Number Words</u>

In English all the chains end



An easier way to write this is



<u>1900 Rotatel (MICRO)</u>

The shape has been repeatedly rotated anticlockwise by 30°.

$$\cos 30^\circ = 0.866$$
$$\sin 30^\circ = 0.5$$

This pattern could equally be obtained by the shape being rotated through 150°, 210° and 330°.

continued

1900 Rotatel (MICRO) (cont)

$$\begin{bmatrix} \cos 150^\circ & -\sin 150^\circ \\ \sin 150^\circ & \cos 150^\circ \end{bmatrix} = \begin{bmatrix} -0.866 & -0.5 \\ 0.5 & -0.866 \end{bmatrix}$$

$$\begin{bmatrix} \cos 210^\circ & -\sin 210^\circ \\ \sin 210^\circ & \cos 210^\circ \end{bmatrix} = \begin{bmatrix} -0.866 & 0.5 \\ -0.5 & -0.866 \end{bmatrix}$$

$$\begin{bmatrix} \cos 330^\circ & -\sin 330^\circ \\ \sin 330^\circ & \cos 330^\circ \end{bmatrix} = \begin{bmatrix} 0.866 & 0.5 \\ -0.5 & 0.866 \end{bmatrix}$$

1901 Flip and turn (MICRO+)

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ This matrix gives the reflection on the front page.

Rotations $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Reflections $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Neither $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

1902 Short Middle Long

Your answer may not be **exactly** the same as these, but they should be very nearly the same.

All measurements given in millimetres.

Short	Middle	Long
76	175	190
52	120	130
25	55	60

S ÷ M	S ÷ L	M ÷ L
0.43	0.4	0.92
0.43	0.4	0.92
0.45	0.42	0.92

1902 Short Middle Long (cont)

Short	Middle	Long
61	77	98
46	59	75
37	48	60

S ÷ M	S ÷ L	M ÷ L
0.79	0.62	0.79
0.78	0.61	0.79
0.77	0.62	0.8

Similar triangles are enlargements of each other. They have the same angles as each other.

Similarities are transformations in which angles are invariant.

1903 Numbers (MICRO)

"Numbers" is a resource program and your answers will depend upon how you used it.

1904 Find the Operation w/s.

1)

b

a

*	0	1	2	3	4	5
0	0	1	2	3	4	5
1	3	4	5	6	7	8
2	6	7	8	9	10	11
3	9	10	11	12	13	14
4	12	13	14	15	16	17
5	15	16	17	18	19	20

Numbers increase by one as you go right.

Numbers increase by three as you go down.

Are there any other patterns in the table?

2) $a * b = 2ab$ $\quad a * b = a^2 - b$ $\quad a * b = (a + b) \text{ Mod} 4$

Note. Mod 4 means modulo 4. This is a mathematical name for clock arithmetic with a 4 hour clock. You may have already seen this in 0461 Venus Clock.

1905 Sorting Triangles

You might like to find out what a Maths dictionary says about shapes that are similar.

Similar triangles are enlargements of each other. They have the same angles as each other.

1906 Enlarging Flags (Micro+)

1) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

2) To make the pattern you need to use

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

1907 About How Long? w/s

No answers required.

1908 Pattern Pack A(DIME)

No answers required.

1909 Pattern Pack B(DIME)

No answers required.

1910 Shape Discrimination (DIME)

No answers required.

1911 Dissection Pairs

Shape 2 can be cut to give Shape 6.
Shape 3 can be cut to give Shape 7.
Shape 4 can be cut to give Shape 9.
Shape 5 can be cut to give Shape 11.
Shape 8 can be cut to give Shape 12.

1912 Painted Tyres

This may look like a trivial problem but if you try to answer the questions given in the hints you will find that it is quite complex.

Try to convince another student that your explanations and scale drawings are correct.

1913 Bengali Numbers

The number in the bottom right-hand corner should give you a clue.
It must be 10, so ১ = 1 and ৫ = 5

There are no more '1's in the table, so the pairs that make 5 must be 2 and 3.

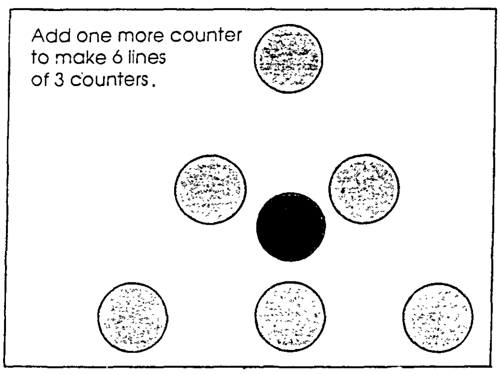
+	2	3	4	5
2	4	5	6	7
3	5	6	7	8
4	6	7	8	9
5	7	8	9	10

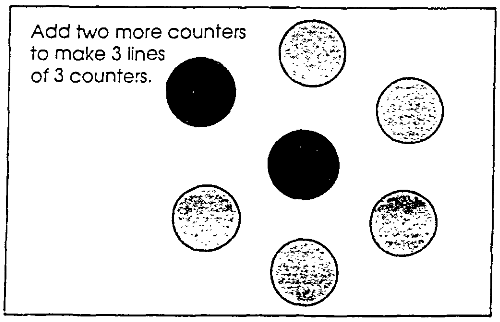
This gives ২ = 2
and ৩ = 3
৭ = 7

Using symmetry the rest of the table can be completed giving
৪ = 4
৬ = 6
৮ = 8
৯ = 9

You now have all the Bengali numbers from 0 to 9 to extend the table.

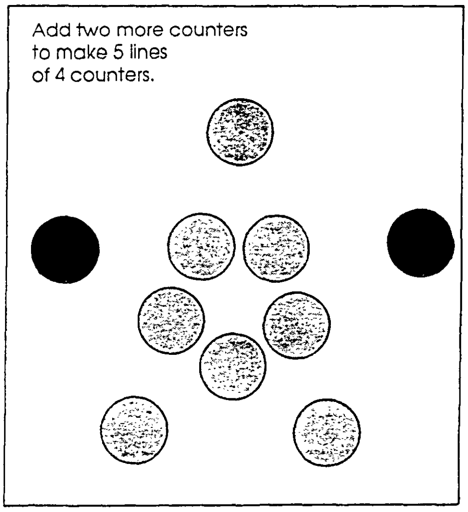
1914 Adding Counters

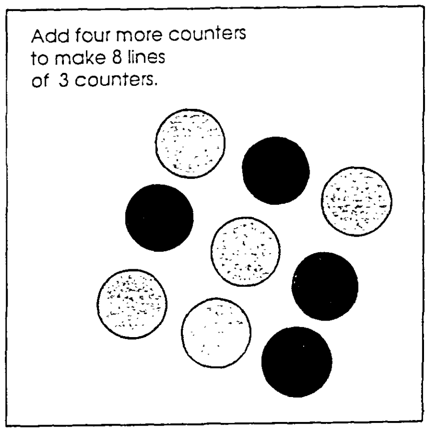




continued

1914 Adding Counters (cont)





1915 Drawing from Memory

No answers required.

1916 A Domino Trick

* Yes.
* Let the domino be $\boxed{n \mid m}$

Multiply one of the numbers by 5	$\rightarrow$	$5n$
Add 8	$\rightarrow$	$5n + 8$
Multiply by 2	$\rightarrow$	$2(5n + 8)$
	$=$	$10n + 16$
Add m	$\rightarrow$	$10n + 16 + m$
Subtract 16	$\rightarrow$	$10n + 16 + m - 16$
	$\rightarrow$	$10n + m$

1917 Rising Gradients

ANGLE	GRADIENT
0^0	0
10^0	0.18
20^0	0.36
30^0	0.58
40^o	0.84
50^o	1.20
60^o	1.73
70^o	2.75
80^o	5.67

2. An angle of 45^o.

   Note. The height and base of the triangle must be equal.

3. The gradient increases towards 90^o.

4. The gradient increases at a faster rate as the angle approaches 90^o.
   What happens at 90^o?
   What happens between 89^o and 91^o?
   Try to explain what happens to the gradient between 89^o and 91^o.

1918 The Coin Problem

Any number of coins can be changed from heads to tails by turning over an odd number of coins at each stage.

However, if you turn over an even number of coins at each stage, you can only change from heads to tails with an even number of coins.

* Here is one method of recording showing 4 coins starting at Heads and turning over 3 at a time.

	H	H	H	H
1st stage	1	1	1	0
2nd stage	0	1	1	1
3rd stage	1	1	0	1
4th stage	1	0	1	1
	T	T	T	T

'1' means "change the state"
'0' means remain the same.

Note.

Changing the state of the coin means changing from heads to tails, or the other way around.

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