SMILE

Answer Book 1901-2178

SMILE

ANSWERS

1901 - 2178

SMILE

Answer Book 1901-2178

1901 Flip and Turn

The matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ gives the reflection on the front page.

Rotations $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Reflections $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Neither $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

1902 Short Middle Long

Your answers may not be exactly the same as these, but they should be very nearly the same.

All measurements given in millimetres.

Short	Middle	Long
76	175	190
52	120	130
25	55	60

S ÷ M	S ÷ L	M ÷ L
0.43	0.4	0.92
0.43	0.4	0.92
0.45	0.42	0.92

Short	Middle	Long
61	77	98
46	59	75
37	48	60

S ÷ M	S ÷ L	M ÷ L
0.79	0.62	0.79
0.78	0.61	0.79
0.77	0.62	0.8

Similar triangles are enlargements of each other. They have the same angles as each other.

Similarities are transformations in which angles are invariant.

1903 Numbers

"Numbers" is a resource program and your answers will depend upon how you used it.

1904 Find the Operation

1. b

	*	0	1	2	3	4	5
	0	0	1	2	3	4	5
	1	3	4	5	6	7	8
	2	6	7	8	9	10	11
a	3	9	10	11	12	13	14
	4	12	13	14	15	16	17
	5	15	16	17	18	19	20

Numbers increase by one as you go right.

Numbers increase by three as you go down.

Are there any other patterns in the table?

2. $a * b = 2ab$ $a * b = a^2 - b$ $a * b = (a + b) \text{ Mod } 4$

Note. *Mod 4 means modulo 4. This is a mathematical name for clock arithmetic with a 4 hour clock. You may have already seen this in SMILE 0461 Venus Clock.*

1905 Sorting Triangles

You might like to find out what a Mathematical Dictionary says about shapes that are similar.

Similar triangles are enlargements of each other. They have the same angles as each other.

1906 Enlarging Flags

1. $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

2. To make the pattern you need to use

$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ $\begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$ $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

1907 About How Long?

Check with some else that your answers are correct.

1908 Pattern Pack A

No answers required.

1909 Pattern Pack B

No answers required.

1910 Shape Discrimination

No answers required.

1911 Dissection Pairs

Shape 2 can be cut to give Shape 6.
Shape 4 can be cut to give Shape 9.
Shape 8 can be cut to give Shape 12.
Shape 3 can be cut to give Shape 7.
Shape 5 can be cut to give Shape 11.

1912 Painted Tyres

This may look like a trivial problem but if you try to answer the questions given in the hints you will find that it is quite complex.

Try to convince another student that your explanations and scale drawings are correct.

1913 Bengali Numbers

The number in the bottom right-hand corner should give you a clue.
It must be 10, so ১ = 1 and ৫ = 5.

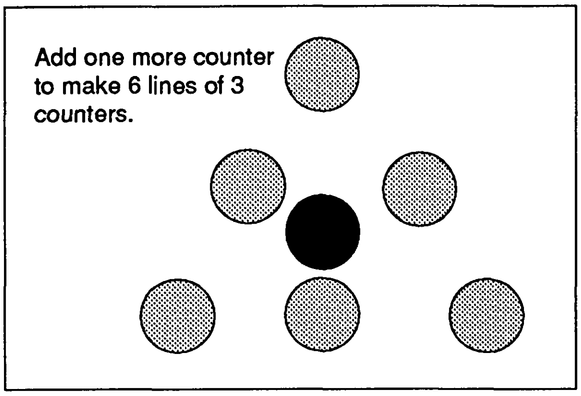
There are no more '1's in the table, so the pairs that make 5 must be 2 and 3.

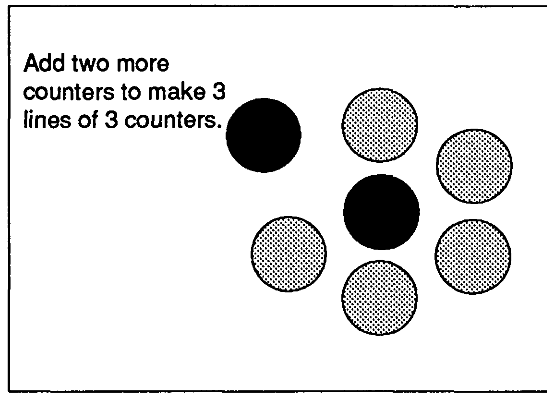
This gives ২ = 2 and ৩ = 3 ৭ = 7

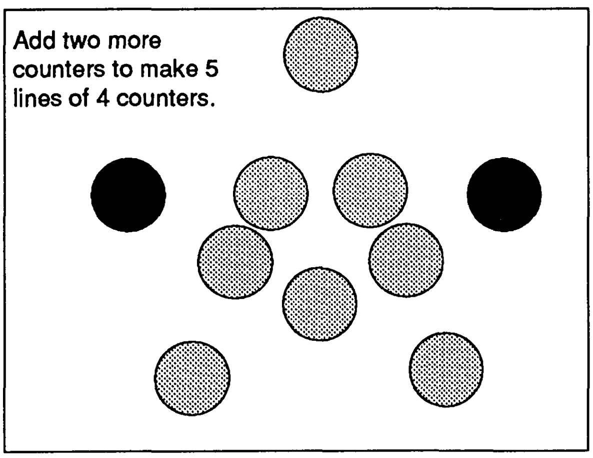
+	2	3	4	5
2	4	5	6	7
3	5	6	7	8
4	6	7	8	9
5	7	8	9	10

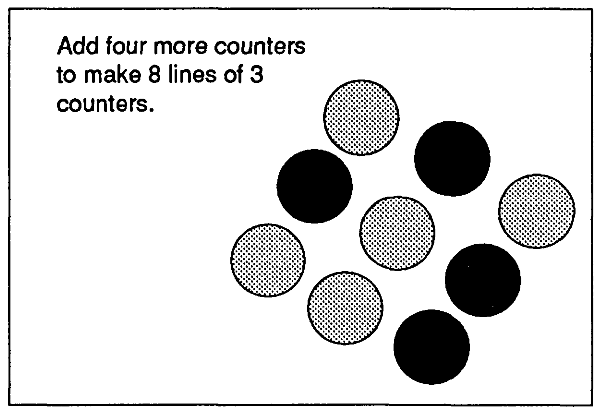
Using symmetry the rest of the table can be completed giving ৪ = 4, ৬ = 6, ৮ = 8 and ৯ = 9.
You now have all the Bengali numbers from 0 to 9 to extend the table.

1914 Adding Counters









1915 Drawing from Memory

No answers required.

1916 A Domino Trick

- Yes
- Let the domino be

n	m

Multiply one of the digits by 5	→	5n
Add 8	→	5n + 8
Multiply by 2	→	2(5n + 8) 10n + 16
Add m	→	10n + 16 + m
Subtract 16	→	10n + 16 + m – 16 10n + m

1917 Rising Gradients

1.

Angle	Gradient
0°	0
10°	0.18
20°	0.36
30°	0.58
40°	0.84
50°	1.20
60°	1.73
70°	2.75
80°	5.67

2. An angle of 45°.
   The height and base of the triangle must be equal.

3. The gradient increases towards 90°.

4. The gradient increases at a faster rate as the angle approaches 90°.
   What happens at 90°?
   What happens between 89° and 91°?
   Try to explain what happens to the gradient between 89° and 91°.

1918 The Coin Problem

Any number of coins can be changed from heads to tails by turning over an odd number of coins at each stage. However, if you turn over an even number of coins at each stage, you can only change from heads to tails with an even number of coins.

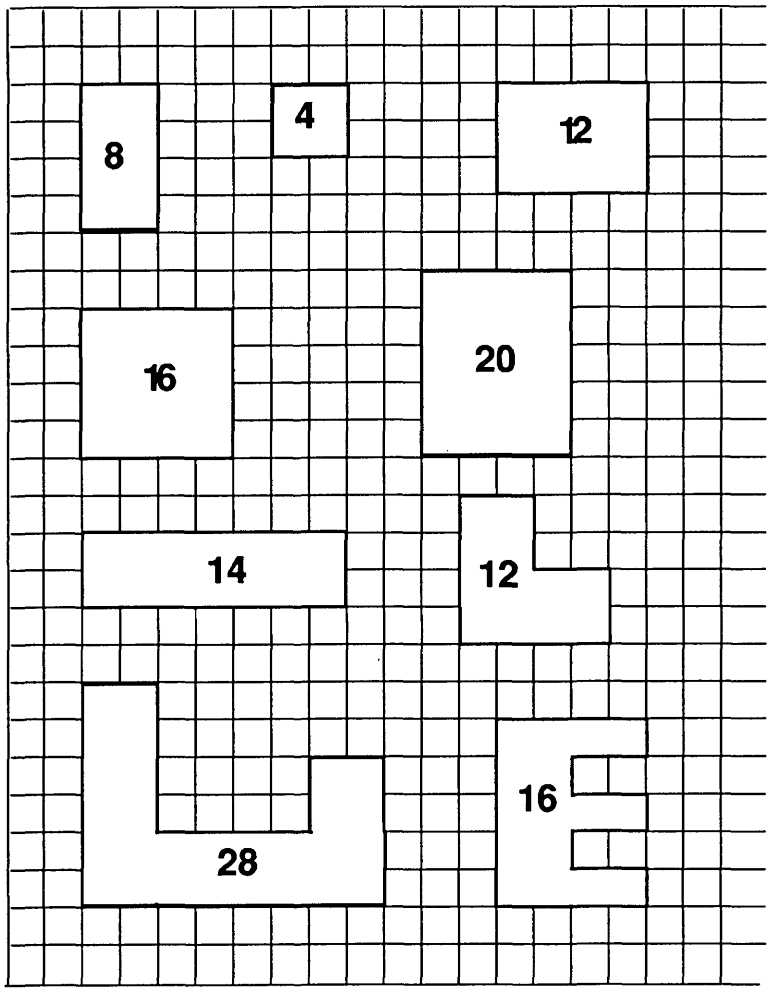
Here is one method of recording showing 4 coins starting at Heads and turning over 3 at a time.

	H	H	H	H
1st stage	1	1	1	0
2nd stage	0	1	1	1
3rd stage	1	1	0	1
4th stage	1	0	1	1
	T	T	T	T

'1' means "change the state"
'0' means "remain the same".

Changing the state of the coin means changing from Heads to Tails or the other way around.

1919 How Many Centimetre Squares?



1920 Pattern Spotting

If you start with this grid then

Pattern 1 is the multiples of 2.

Pattern 2 is the multiples of 3.

Pattern 3 is the multiples of 7.

Pattern 4 is the multiples of 11.

What happens if you change the starting number?

What happens if you change the size of the grid?

1921 Trig Lines

1. a) 0.82
   b) These are our answers. Your answers should be close to them.

Angle	Opposite Side	Adjacent Side
30	0.5	0.87
45	0.71	0.71
60	0.87	0.5
80	0.98	0.17

   c) Opposite side becomes larger.
   d) Adjacent side becomes smaller.

2. The answers are given to 2 decimal places.

	Angle	Opposite Side	Adjacent Side
a)	25	0.42	0.92
b)	83	0.99	0.12
c)	45	0.71	0.71
d)	40	0.64	0.77
e)	15	0.26	0.97

3. All the sides will be three times as large so the
   Opposite = 3 x 0.5 = 1.5 and the
   Adjacent = 2.60

4. These answers are also given to 2 decimal places.

	Angle	Opposite Side	Adjacent Side
a)	25	1.27	2.72
b)	63	5.35	2.72
c)	35	11.47	16.38

1922 Matrices & Area

$$\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 2 & 4 & 4 & 2 \\ 1 & 1 & 3 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 8 & 8 & 4 \\ 3 & 3 & 9 & 9 \end{pmatrix}$$

1. 4, 24, 6:1

2. $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 4 & 4 & 2 \\ 3 & 3 & 9 & 9 \end{pmatrix}$ $\begin{pmatrix} ½ & 0 \\ 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 2 & 1 \\ 1 & 1 & 3 & 3 \end{pmatrix}$

continued/

1922 Matrices & Area (cont)

$$\begin{pmatrix}3 & 0\\1 & 2\end{pmatrix} \rightarrow \begin{pmatrix}6 & 12 & 12 & 6\\4 & 6 & 10 & 8\end{pmatrix} \qquad \begin{pmatrix}1 & 3\\1 & 0\end{pmatrix} \rightarrow \begin{pmatrix}5 & 7 & 13 & 11\\2 & 4 & 4 & 2\end{pmatrix}$$

$$\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix} \rightarrow \begin{pmatrix}1 & 3 & 1 & -1\\3 & 5 & 7 & 5\end{pmatrix} \qquad \begin{pmatrix}3 & 1\\1 & 2\end{pmatrix} \rightarrow \begin{pmatrix}7 & 13 & 15 & 9\\4 & 6 & 10 & 8\end{pmatrix}$$

$$\begin{pmatrix}3 & 1\\2 & 2\end{pmatrix} \rightarrow \begin{pmatrix}7 & 13 & 15 & 9\\6 & 10 & 14 & 10\end{pmatrix} \qquad \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \rightarrow \begin{pmatrix}2 & 4 & 4 & 2\\1 & 1 & 3 & 3\end{pmatrix}$$

3.

Matrix	New Area	Ratio
$\begin{pmatrix}1 & 0\\0 & 3\end{pmatrix}$	12	3 : 1
$\begin{pmatrix}½ & 0\\0 & 1\end{pmatrix}$	2	½ : 1
$\begin{pmatrix}3 & 0\\1 & 2\end{pmatrix}$	24	6 : 1
$\begin{pmatrix}1 & 3\\1 & 0\end{pmatrix}$	12	3 : 1
$\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}$	8	2 : 1
$\begin{pmatrix}3 & 1\\1 & 2\end{pmatrix}$	20	5 : 1
$\begin{pmatrix}3 & 1\\2 & 2\end{pmatrix}$	16	4 : 1
$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$	4	1 : 1

The **determinant** of a matrix $\begin{pmatrix}a & b\\c & d\end{pmatrix}$ is defined as **ad – bc**. It is a useful measure of what effect the matrix will have. One of its uses is to determine how a matrix will alter the area of a shape.

1923 Days in a Month

There are two different solutions.

Either	Cube A	1,	2,	0,	3,	4,	5
	Cube B	1,	2,	0,	6 or 9,	7,	8

There must be 0, 1 and 2 on both dice.

continued/

1923 Days in a Month (cont)

Or, if it is acceptable to have one cube for a single digit date.

Cube A	1,	2,	3,	4,	5,	6
Cube B	1,	2,	7,	8,	9,	0

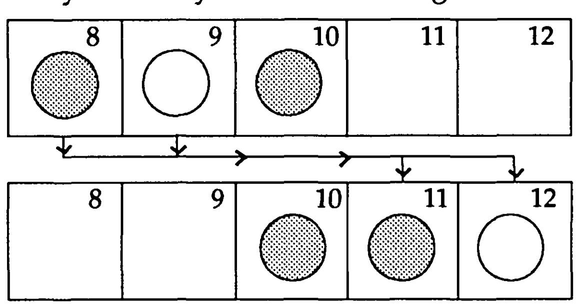
There must be a 1 and 2 on both dice. The digit 3 must be on a different dice to 0.

1924 Two Dice

Try different "top face" numbers and record your answers.
Try different arrangements with one as the top face. What did you notice?
What about if two is the top face?

1925 Hats

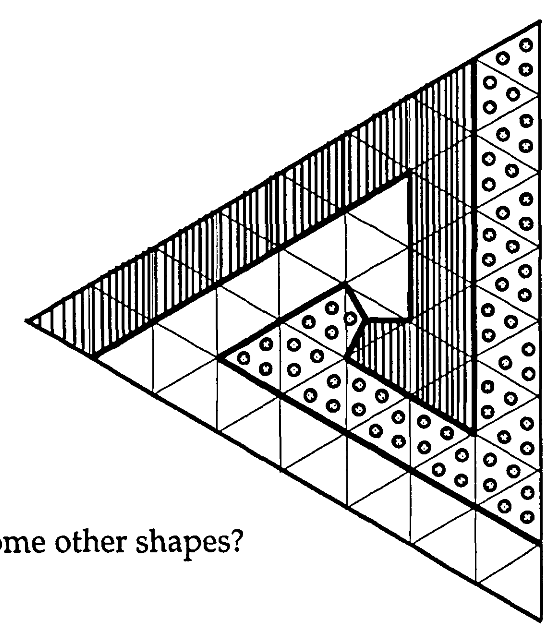
If you are really stuck try this as a starting move.



Try to record your answers.
What is the minimum number of moves? We managed to do it in six.
Can you do it with fewer moves?

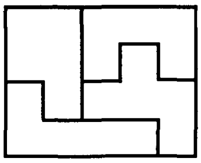
1926 Chess

Here is a clue.

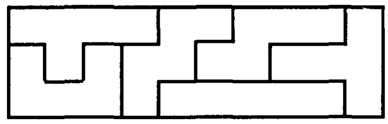


Can you extend this pattern for some other shapes?

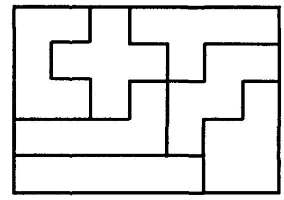
1927 Pentomino Puzzles

a) 

4 x 5 rectangle

b) 

3 x 10 rectangle

c) 

5 x 7 rectangle

If you enjoyed doing these, then you may like to try making the following rectangles.

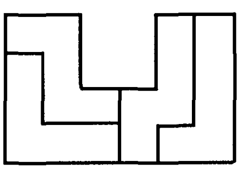
3 x 5 using 3 pieces
9 x 5 using 9 pieces
5 x 5 using 5 pieces
10 x 5 using 10 pieces
10 x 4 using 8 pieces
11 x 5 using 11pieces

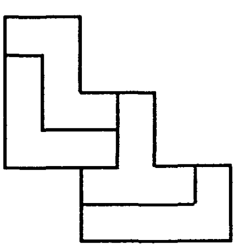
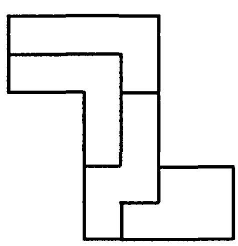
It is also possible to make rectangles using all of the pentominoes.

12 x 5 rectangle in 1010 ways
15 x 4 rectangle in 368 ways
20 x 3 rectangle in 2 ways

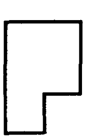
1928 Four Pentominoes

There are many possible answers. One is shown for each of the pentominoes.

1. 

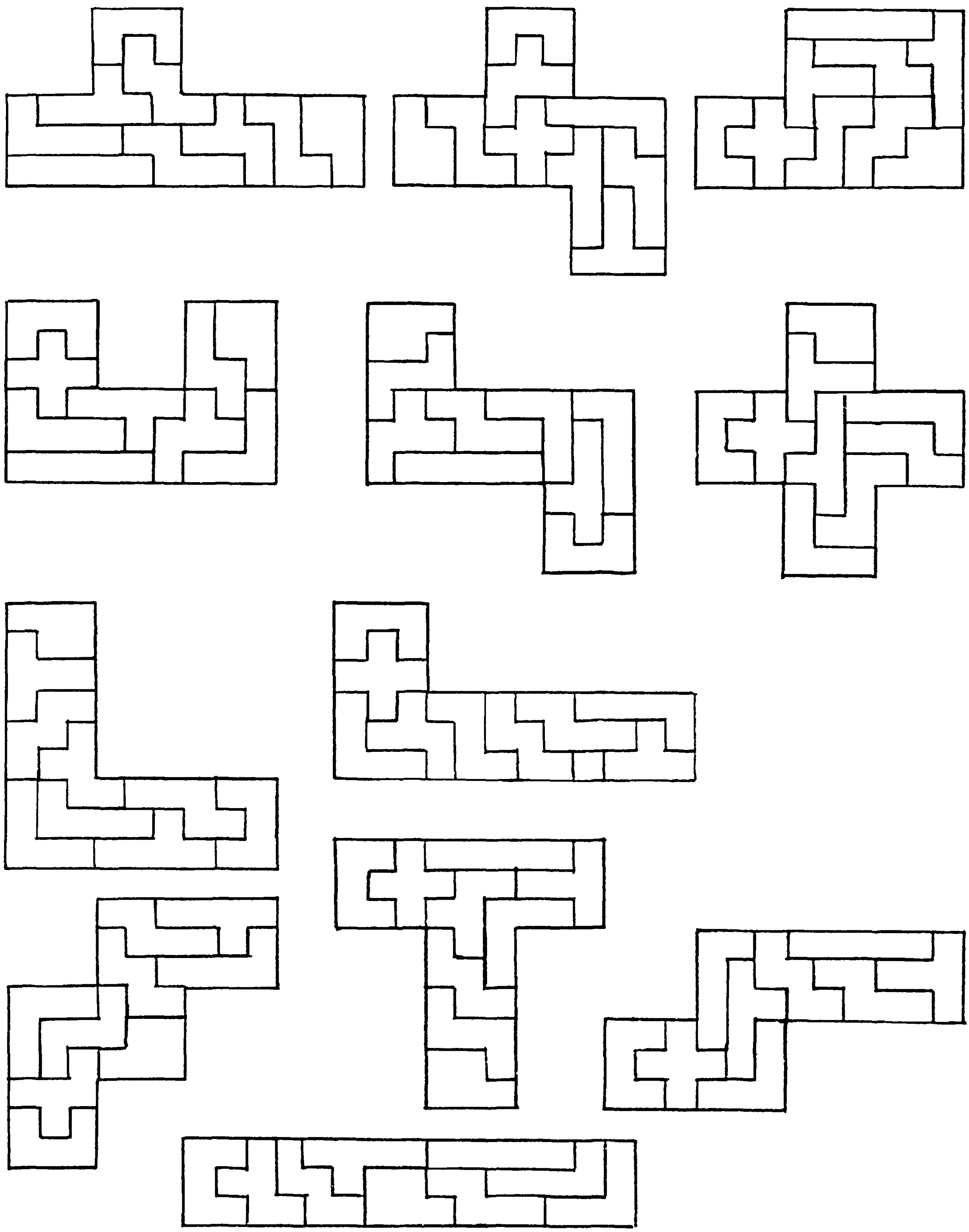
2.  

3. a) The sides are twice as long.
   b) The areas are four times as large.

4. 

<u>1929 Nine Pentominoes</u>

There are many different solutions. These are some of the possible ones.
It will help to calculate the scale factor and to draw an outline of the enlarged shape.



With 9 pentominoes the scale factor is 3.

1930 Opposite Corners

Here are the results for squares using the numbers arranged in 5 columns.

Number of Columns	5	5	5	5	.	.	.
Size of Square	2 x 2	3 x 3	4 x 4	5 x 5	.	.	.
Differences	5 (5 x 1)	20 (5 x 4)	45 (5 x 9)	80 (5 x 16)	.	.	.

Here are some results for squares with numbers arranged in 7 columns.

Number of Columns	7	7	7	.	.	.
Size of Square	2 x 2	3 x 3	4 x 4	.	.	.
Differences	7 (7 x 1)	28 (7 x 4)	63 (7 x 9)	.	.	.

Here are some results for squares with numbers arranged in 9 columns.

Number of Columns	9	9	9	.	.	.
Size of Square	2 x 2	3 x 3	.	.	.	
Differences	9 (9 x 1)	36 (9 x 4)	.	.	.	

By trying other squares using numbers arranged in different columns can you find a pattern?

From your results can you predict what the difference would be for numbers arranged in 11 columns with a square size of 6 x 6?

You may like to see if you can find a pattern using rectangles instead of squares.

1931 Which Script?

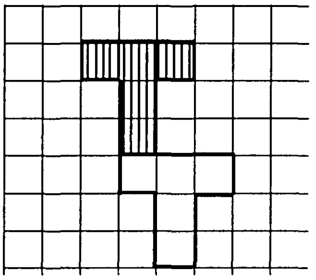
Chinese	Punjabi	Bengali	International	Urdu
二	੨	২	2	۲
十三	੧੩	১৩	13	۱۳
二十五	੨੫	২৫	25	۲۵
五十八	੫੮	৫৮	58	۵۸
八十三	੮੩	৮৩	83	۸۳
一百	੧੦੦	১০০	100	۱۰۰

五十一	੫੧	৫১	51	۵۱

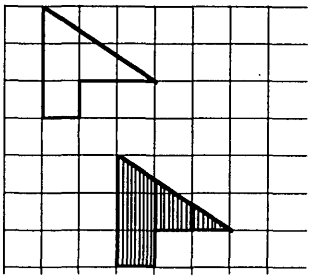
Can you write these numbers in any other scripts?

1934 Translations

1\.



2\.



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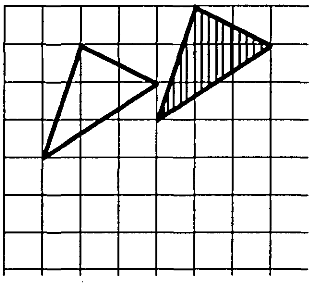
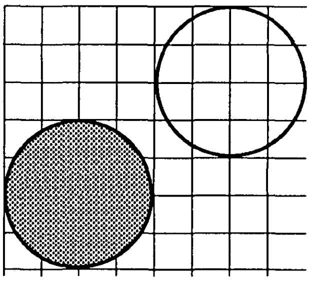
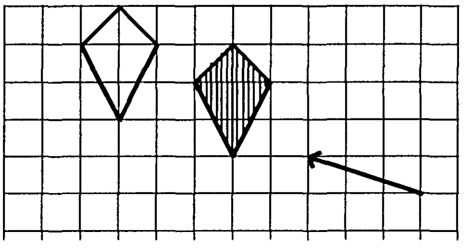


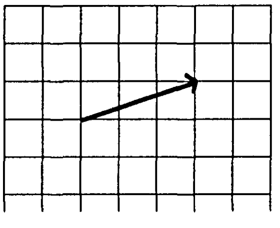
4\.



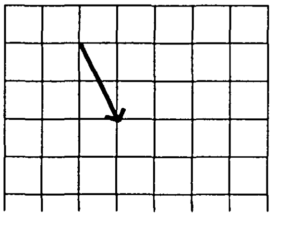
5\.



6\.



7\.



1935 Angles in Semi-Circles

You should have found that:

- If the 3rd point on your triangle (P) lies on the circle, the angle opposite the diameter is always a right-angle (90°).
  The size of the circle does not change this.

- When P is inside the circle the angle opposite the diameter is always greater than 90°.

- When P is outside the circle the angle opposite the diameter is always less than 90°.

1936 Many Grids

The different grids we found are:

3 x 12

8	9	10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39	40	41	42	43

4 x 9

8	9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24	25
26	27	28	29	30	31	32	33	34
35	36	37	38	39	40	41	42	43

6 x 6

8	9	10	11	12	13
14	15	16	17	18	19
20	21	22	23	24	25
26	27	28	29	30	31
32	33	34	35	36	37
38	39	40	41	42	43

2 x 18
1 x 36 } these grids are also possible but are too big for the screen.
All the grid sizes are factors of 36.

In order to find a grid size given the first and last number, work out
(last number – first number + 1)   e.g. (43 – 8 + 1) = 36
The grid sizes are all factors of the answer.
Why do you think you add on 1?
Does this work for other first and last numbers?

1937 Panjabi Numbers

੧ × ੯ + ੨ = ੧੧
੧੨ × ੯ + ੩ = ੧੧੧
੧੨੩ × ੯ + ੪ = ੧੧੧੧
੧੨੩੪ × ੯ + ੫ = ੧੧੧੧੧
੧੨੩੪੫ × ੯ + ੬ = ੧੧੧੧੧੧
੧੨੩੪੫੬ × ੯ + ੭ = ੧੧੧੧੧੧੧
੧੨੩੪੫੬੭ × ੯ + ੮ = ੧੧੧੧੧੧੧੧

1938 Olympic Medals

1. Great Britain and Italy.

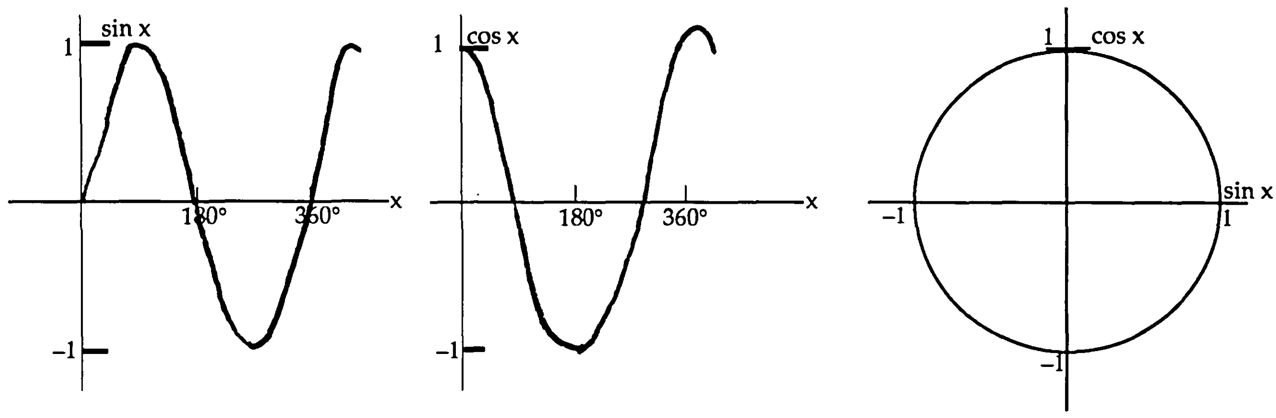
2. Many possible answers.

1939 Sin and Cos Graphs

Angle	Sin	Cos
0	0.00	1.00
10	0.17	0.98
20	0.34	0.94
30	0.50	0.87
40	0.64	0.77
50	0.77	0.64
60	0.87	0.50
70	0.94	0.34
80	0.98	0.17
90	1.00	0.00
100	0.98	–0.17
110	0.94	–0.34
120	0.87	–0.50
130	0.77	–0.64
140	0.64	–0.77
150	0.50	–0.87
160	0.34	–0.94
170	0.17	–0.98
180	0.00	–1.00
190	–0.17	–0.98
200	–0.34	–0.94
210	–0.50	–0.87
220	–0.64	–0.77
230	–0.77	–0.64
240	–0.87	–0.50
250	–0.94	–0.34
260	–0.98	–0.17
270	–1.00	0.00
280	–0.98	0.17
290	–0.94	0.34
300	–0.87	0.50
310	–0.77	0.64
320	–0.64	0.77
330	–0.50	0.87
340	–0.34	0.94
350	–0.17	0.98
360	0.00	1.00

continued/

1939 Sin and Cos Graphs (cont)



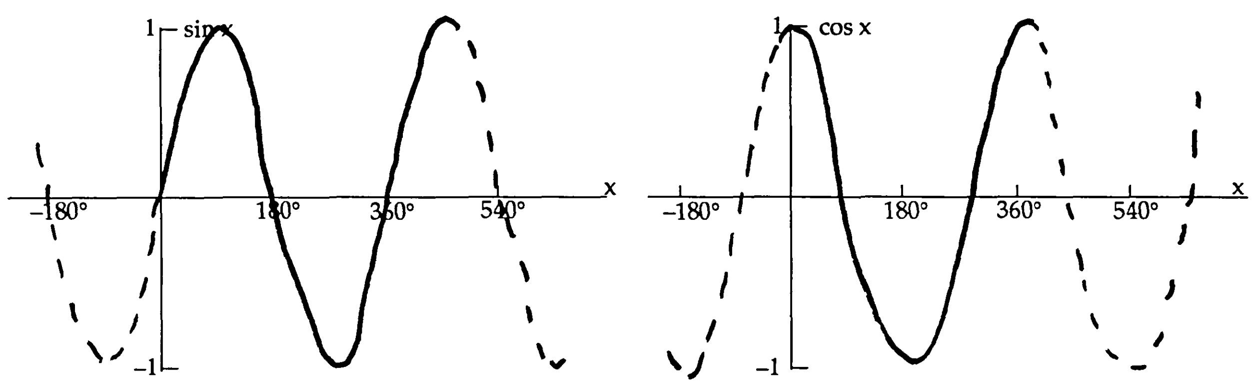
For angles greater than 360° both sin x and cos x are repeated as shown in the graph below.

e.g. sin 390 = sin(390 – 360) = sin 30 = 0.5
cos 510 = cos(510 – 360) = cos 150 = -0.87

For negative angles

sin(-x) becomes -sin x e.g. sin(-50) = -sin 50 = -0.77
cos(-x) stays the same, cos x e.g. cos(-20) = cos 20 = 0.94

For angles greater than 360° and negative angles, the graph of sin against cos will remain the same, i.e. always a circle.



1940 Dividing Investigation

When you divide by 4, the numbers that end in .25 are 1, 5, 9, 13 . . .

- Did you notice how these numbers continued?
- Could you predict some big numbers that, when divided by 4, end in .25?
- Did you find the other three endings when you divided by 4?
  What numbers gave each of these?

continued/

1940 Dividing Investigation (cont)

This spreadsheet shows the numbers from 1 - 16 when divided by 4.

The formula in this cell adds 1 to the number above.

=A2+1

The formula was copied down to A17 using the EDIT Menu to Fill Down.

	A	B
1	Number	Divided by 4
2	1	0.25
3	2	0.5
4	3	0.75
5	4	1
6	5	1.25
7	6	1.5
8	7	1.75
9	8	2
10	9	2.25
11	10	2.5
12	11	2.75
13	12	3
14	13	3.25
15	14	3.5
16	15	3.75
17	16	4

The formula in this cell divides the number in A2 by 4 .

=A2/4

The formula was copied down to A17 using the EDIT Menu to Fill Down..

This spreadsheet shows the numbers from 1 to 16 when divided by 4, 5, 8 and 3.

	A	B	C	D	E
1	Number	Divided by 4	Divided by 5	Divided by 8	Divided by 3
2	1	0.25	0.2	0.125	0.33333333
3	2	0.5	0.4	0.25	0.66666667
4	3	0.75	0.6	0.375	1
5	4	1	0.8	0.5	1.33333333
6	5	1.25	1	0.625	1.66666667
7	6	1.5	1.2	0.75	2
8	7	1.75	1.4	0.875	2.33333333
9	8	2	1.6	1	2.66666667
10	9	2.25	1.8	1.125	3
11	10	2.5	2	1.25	3.33333333
12	11	2.75	2.2	1.375	3.66666667
13	12	3	2.4	1.5	4
14	13	3.25	2.6	1.625	4.33333333
15	14	3.5	2.8	1.75	4.66666667
16	15	3.75	3	1.875	5
17	16	4	3.2	2	5.33333333

- How many different endings are there when you divide by 5?
  How many different endings are there when you divide by 8?
  How many different endings are there when you divide by 3?

- Can you predict large numbers which have each ending?

1941 Differences

There are many ways of investigating this. These are some of the questions you could have asked yourself.

- The number 3 in the mapping $n \rightarrow 3n^2 + 5n + 8$ is called the COEFFICIENT of n^2. What happens to the differences as the coefficient of n^2 changes?

continued/

1941 Differences (cont)

- The number 8 in the mapping $n \rightarrow 3n^3 + 5n + 8$ is called a CONSTANT. What happens to the differences as the constant changes?

- Can you predict the difference for the quadratic mapping $n \rightarrow 3n^2 + 5n + 8$?

- Can you generalise the difference for the quadratic mapping $n \rightarrow an^2 + bn + c$?

A spreadsheet is also an excellent way of generating results. This spreadsheet has been set up to show the differences of terms formed from the mapping $n \rightarrow 3n^2 + n$, and can be easily adapted to explore the sequences formed by other quadratic mappings such as: $n \rightarrow n^2 + 4$ $n \rightarrow 3n^2 + 5n + 8$ $n \rightarrow 10n^2 + 8$

The formula in this cell squares n, multiplies it by 3 and adds n.

=3*(A2^2)+A2

Edit to Fill Down

The formula in this cell calculates the differences.

=B3–B2

Edit to Fill Down

	A	B	C	D
1	n	Mapping		
2	1	4	Difference 1	
3	2	14	10	Difference 2
4	3	30	16	6
5	4	52	22	6
6	5	80	28	6
7	6	114	34	6
8	7	154	40	6
9	8	200	46	6
10	9	252	52	6

What is the formula to calculate the differences here?

You may like to explore mappings with higher powers of n such as cubics and quartics. If you are using a spreadsheet you will need to add extra columns until you reach a constant difference.

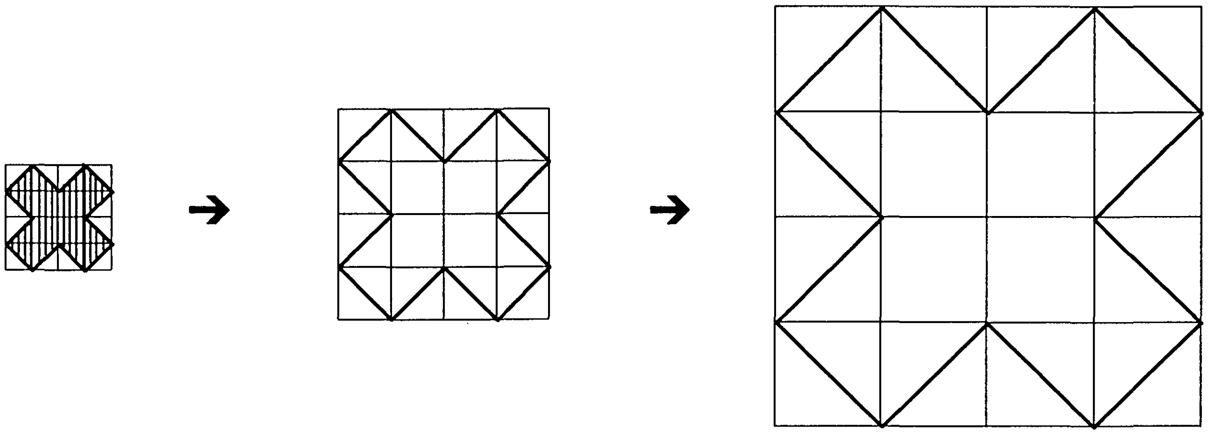
- What happens to the differences as the power of n increases?

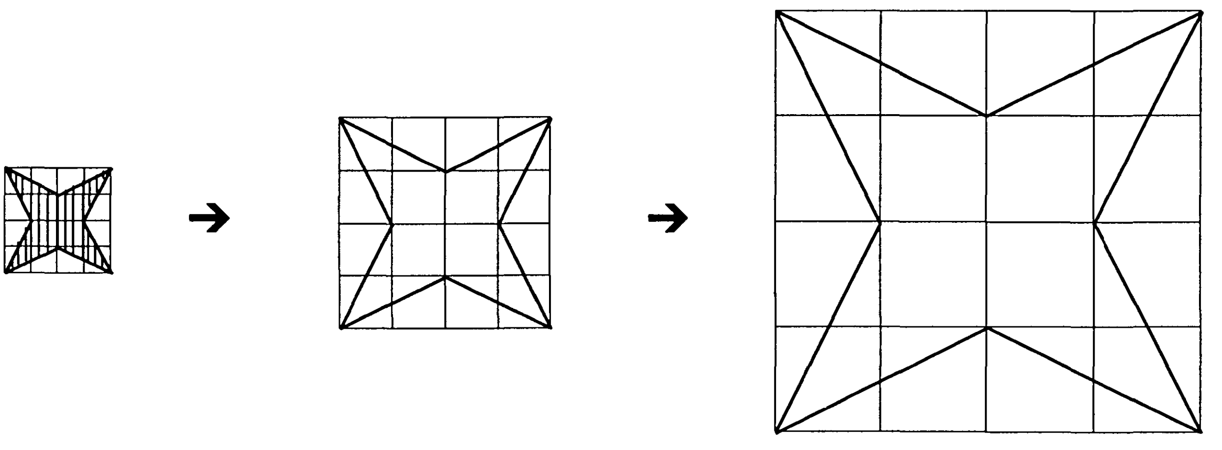
- How many rows does it take to reach a constant difference? Could you predict a mapping for which the constant difference is 24 after 4 rows?

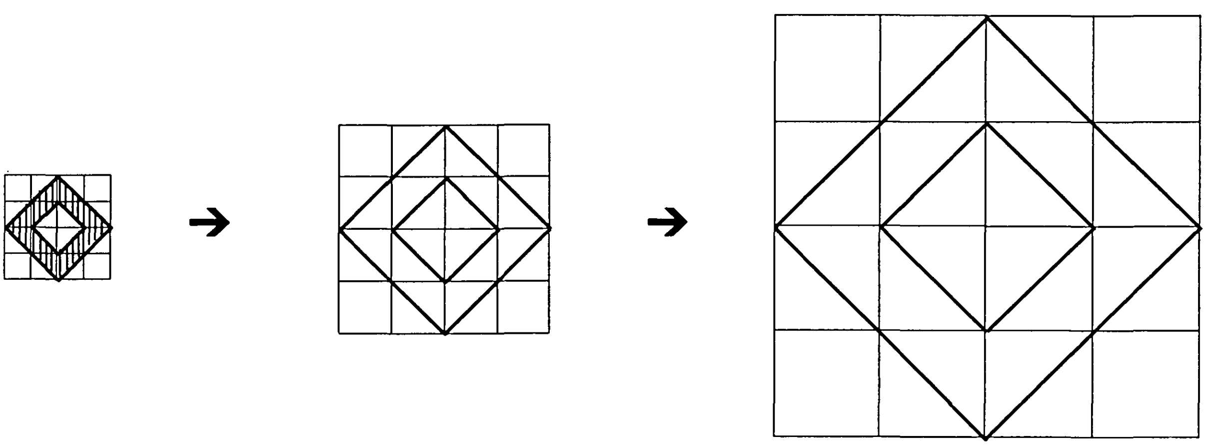
This spreadsheet shows if you start with a quartic mapping of $n \rightarrow 2n^4 + n^2$ a constant difference of 48 is reached in the 4th column of differences.

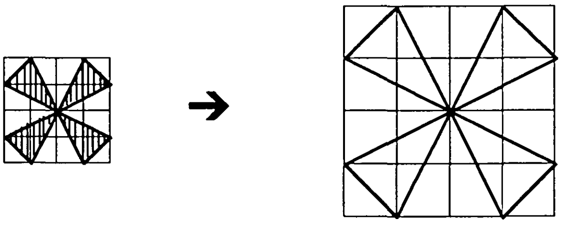
	A	B	C	D	E	F
1	n	Mapping				
2	1	3	Difference 1			
3	2	40	37	Difference 2		
4	3	189	149	112	Difference 3	
5	4	576	387	238	126	Difference 4
6	5	1375	799	412	174	48
7	6	2808	1433	634	222	48
8	7	5145	2337	904	270	48
9	8	8704	3559	1222	318	48
10	9	13851	5147	1588	366	48

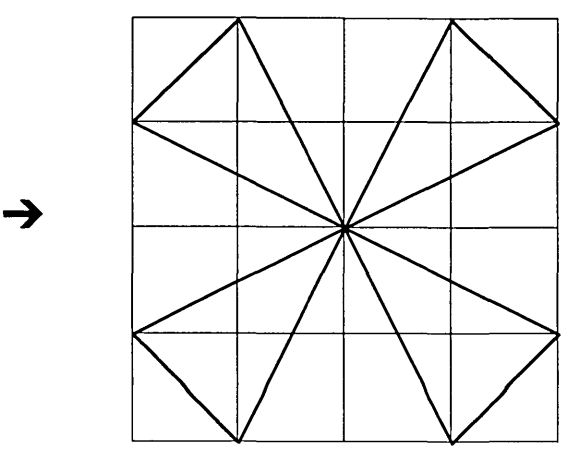
1942 Growing Patterns











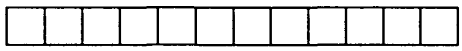
Check your own shapes with someone else.

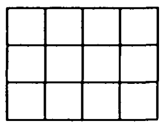
1943 Lines and Squares

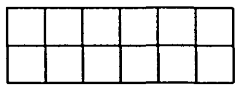
There seem to be 3 different approaches to this problem:

a) Choose the number of squares and look at the different rectangles you can make.

e.g. 12 squares

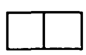
 12 x 1 — 15 lines

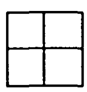
 4 x 3 — 9 lines

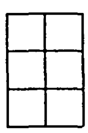
 6 x 2 — 10 lines

Which other rectangles can be made?
How many lines do they have?

b) Change the size of rectangle in an ordered sensible way and record the number of lines and squares.

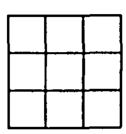
e.g. 2 x 1  5 lines — 2 squares

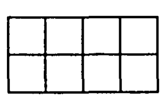
2 x 2  6 lines — 4 squares

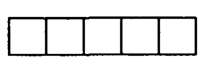
2 x 3  7 lines — 6 squares

c) Choose the number of lines and look at the different rectangles you can make.

e.g. 8 lines

 3 x 3 — 9 squares

 4 x 2 — 8 squares

 5 x 1 — 5 squares

For a given number of lines can you predict how many different rectangles can be made?

1944 Odd & Even Triangle Patterns

Here is the rule:

- If the two spots above are the same either ● ● or ○ ○ then the one below is ○.
- If the two above are different, either ● ○ or ○ ● then the one below is ●.

continued/

1944 Odd & Even Triangle Patterns (cont)

There are many ways of approaching this investigation.

- It may help to work out how many different first rows are possible.

e.g. ● ● ● ● ●
or ● ● ● ● ○
or ● ● ● ○ ●
or ● ● ○ ● ● . . . etc.

- One possible way is to work backwards . . .
  What could the 4th row be for a ● 5th row? etc.

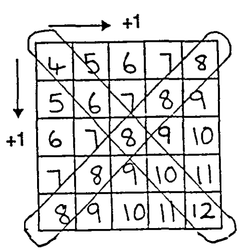
- Another way would be to try different size triangles.
  Start with 3 spots in the top row or even 2.

  ● ○ ○
  ● ○
  ● etc.

  This would be a possible direction to extend in ....

1945 Square Diagonals

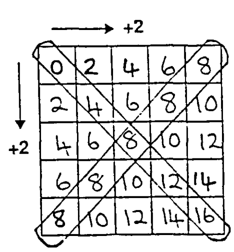
1.



The numbers on the diagonal 4, 6, 8, 10, 12 go up in twos. They are all even numbers.

The numbers on the other diagonal are all 8.

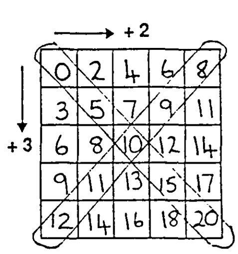
2.



The numbers on the diagonal 0, 4, 8, 12, 16 go up in fours.

The numbers on the other diagonal are all 8.

3.



The numbers on the diagonal 0, 5, 10, 15, 20 go up in fives.

The numbers on the other diagonal go up in ones.

4. What happens if you make the square bigger?
   What happens if you 'take away' instead of adding?
   Did you notice a connection between the rules and numbers on the diagonals?

1946 A Problem of Division

These are common methods:

Counting up in 13's

13	1
+ 13	
26	2
+ 13	
39	3
+ 13	
52	4
.	
.	
.	
221	17

Subtracting 13's

221	
− 13	
208	1
− 13	
195	2
− 13	
182	3
.	
.	
.	
0	17

Long Division

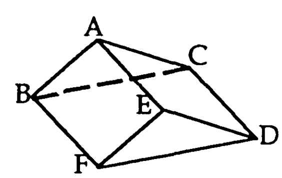
```
    17
13) 221
   - 13
     91
   - 91
     --
```

so $221 \div 13 = 17$
$266 \div 14 = 19$

1947 3-D Frameworks

Framework 1 → g   Framework 2 → a   Framework 3 → b
Framework 4 → c   Framework 5 → e   Framework 6 → d

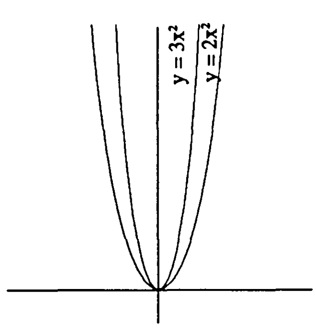
The matrix f describes this framework.

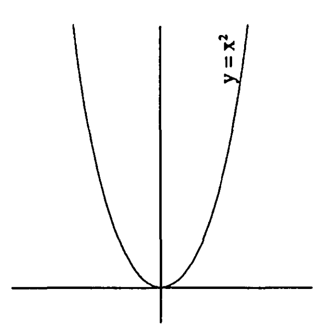


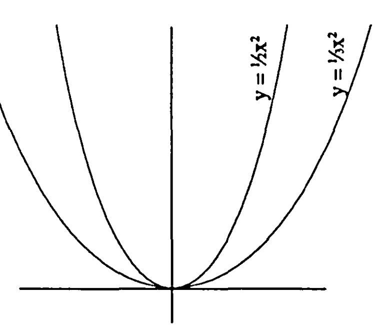
1948 $y = ax^2$

As 'a' increases the graph becomes narrower. e.g. $3x^2$ is narrower than $2x^2$.

As 'a' decreases the graph beomes wider. e.g. $\frac{1}{3}x^2$ is wider than $\frac{1}{2}x^2$.



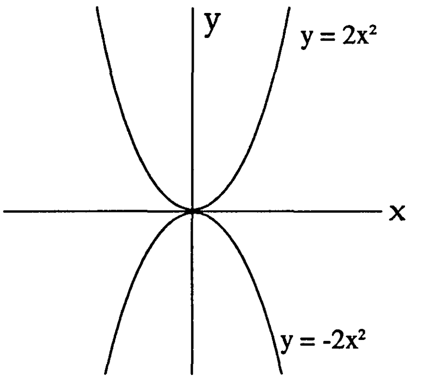




continued/

1948 $y = ax^2$ (cont)

When 'a' is negative the curve points downwards, e.g. $-2x^2$ is a reflection of $2x^2$ in the x axis.



1949 Compass Game

No answers required

1950 Diagonal Multiples

For a grid made of 7 columns, we found that multiples of 2, 3, 4, 6 and 8 gave diagonal patterns. We decided not to include 5. Did you?

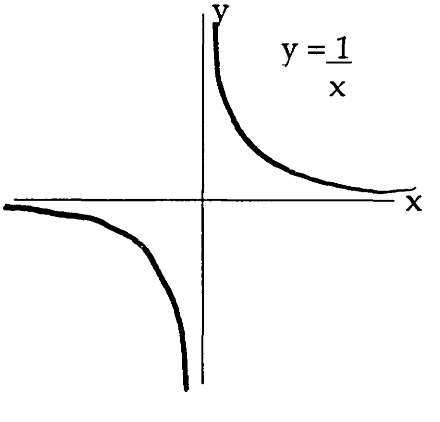
The diagonal pattern for multiples of three goes to the left. Which ways do the others go? Try using other numbers of columns.

The micro program Numbers will make grids of up to 10 columns. Multiple, another MicroSMILE program, allows you to explore with up to 12 columns.

1951 When X is ?

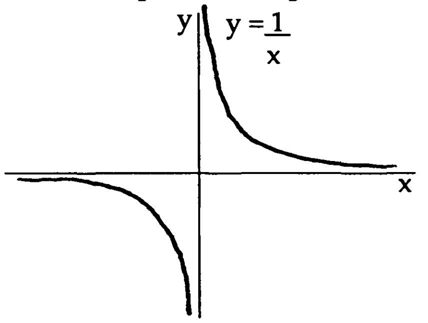
$y = \frac{1}{x}$

When x = 10	y = 0.1
x = 2	y = 0.5
x = 1	y = 1.0
x = 0.5	y = 2
x = 0.1	y = 10
x = 0.01	y = 100
.	.
.	.
.	.

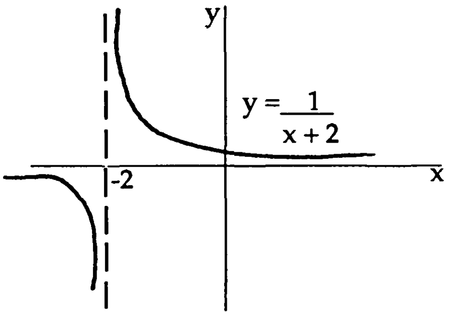


- When x is negative, the y values are the same as when x is positive but are negative as well.
- When x is very large, y gets closer to 0.
- When x is very close to zero, y becomes very large.
- When x is zero, y does not exist.

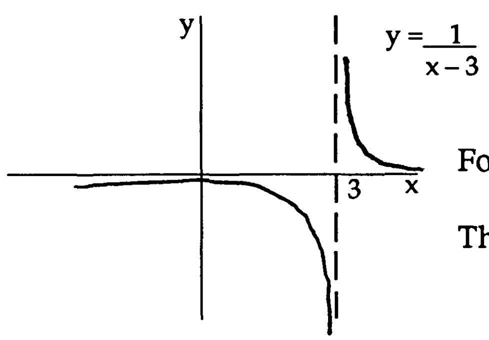
1952 Reciprocal Graphs



For $y = \frac{1}{x}$ you will notice that the graph is in two parts.
Each part gets closer to both the y and x axis but never meets them. (Here the x and y axes are called asymptotes). There is reflective symmetry in the line $y = -x$.



For $y = \frac{1}{x+2}$ the graph is the same shape as $y = \frac{1}{x}$ but is translated (moved) to the left. The asymptotes are now the x axis and the line $x = -2$.



For $y = \frac{1}{x-3}$ the graph is translated to the right.
The asymptotes are now the x axis and $x = 3$.

So graphs of the form $y = \frac{1}{x+c}$ translate the graph $y = \frac{1}{x}$ parallel to the x axis, -c spaces.
You should have similar types of description for other sets of reciprocal graphs.

1953 Sets of Signs

1. Many possible answers. Get someone in your class to check your answers.

2. A circle, red or blue, usually gives an order.
   A rectangle or a pentagon with:
   - a blue background is usually a motorway direction sign.
   - a green background is usually a primary route direction sign.
   - a white background and a black border is usually a non-primary route direction sign.
   - a white background and a blue border is usually a local direction sign.

1954 Line Symmetry

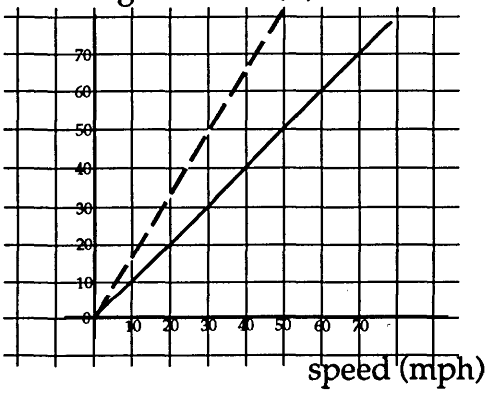
We found at least 15. How many did you find?
Get someone in your class to check your answers with a mirror.

1955 Rotational Symmetry

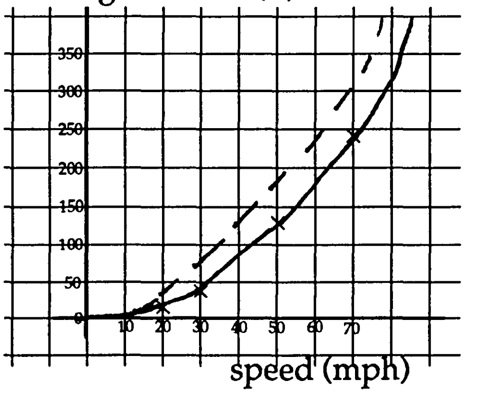
We found at least 5. How many did you find?
Get someone in your class to check your answers with tracing paper.

1956 Thinking & Braking

a) Thinking Distance (ft)



b) Braking Distance (ft)



Graph (a) would be different because a tired driver would take longer to react. The dotted line shows a possible answer.
If the car had worn brakes it would need a greater braking distance, the dotted line on graph (b) shows a possible answer.

1957 Shears

Using $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

How has the square changed? There are a variety of ways to describe the changes:

- The square has become a parallelogram.
- The height remains invariant (remains unchanged).
- Points on the x axis (the line y = 0) are invariant.
- The area remains invariant.
- The length of the slanting lines become $\sqrt{5}$.
- The angles are changed.

If the square starts at a different place, the shear produces the same parallelogram. The further away the square is from the x axis, the more the parallelogram is sheared. The amount of the shear is double the y co-ordinates.

Using $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

The above description is the same except the properties are now related to the y axis. So, for example, the further away the square is from the y axis, the more the parallelogram is sheared. The amount of the shear is double the x co-ordinate.

continued/

1957 Shears (cont)

The description on the previous page applies to any shape.

Matrices of the form $\begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix}$ produce shears where the points of the shape are shifted parallel to the x axis by m times the y co-ordinates.
This is because $\begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + my \\ y \end{pmatrix}$

Matrices of the form $\begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$ produce shears where the points of the shape are shifted parallel to the y axis by n times the x co-ordinates.

So in general terms, a shear is a translation parallel to one line where any point is moved parallel to the fixed line, proportional to its distance from that axis. You may have found an alternative definition.

1958 Ealing Broadway

Yes it is true. This assumes that the trains, when they reach West India Quay can go straight on to Island Gardens or to the right to Tower Gateway.

We found 4 other stations for which this is true. Can you find them?

1959 Making One

You should have found nine of these.

$\frac{1}{2} + \frac{1}{2}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{4}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8}$

$\frac{1}{2} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8}$

$\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

$\frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

1960 Make a Model

Have you made a plan?
What 'scale' have you used?

1961 One Million

A DOT IS A SECOND
1 000 000 seconds = 278 hours = 11.5 days.
If you count at the rate of one number per second, it would take 11.5 days counting day and night, to reach a million.

A DOT IS A PACE
Measure the length of a pace when a child is walking normally - say 60cm. 1 000 000 paces = 600 000m = 600km which is approximately the distance from London to Edinburgh.

A DOT IS A STAIR
A normal staircase rises 15 – 20cm per step. 1 000 000 steps is 150 000 – 200 000 metres, roughly twenty times the height of Mount Everest (8848m). It is best to measure an actual staircase in your school.

A DOT IS A PERSON
An adult needs about 0.16 square metres (m^2), a child about 0.1 square metres (m^2). A million people need between 10 – 16 hectares or 15 – 25 football pitches.

A DOT IS A TON OF SAND
A lorry might hold between 10 - 25 tons of sand, so between 40 000 and 100 000 lorries would be needed for a million tons.

A DOT IS A SUGAR LUMP
If a sugar lump was 1 cubic centimetre (cm^3), then a million would be a cubic metre. A million real sugar lumps would weigh between one and two tons and go easily on to a small lorry.

A DOT IS A COIN
A pound coin has a diameter of 24mm, so a million of them would stretch for about 25km. Larger coins would reach further.

A DOT IS "ONE MILLION"
It takes about a second to say "one million" and so it would take about 11.5 days to say it a million times.

1962 Spiralling Squares

One possible solution is

Spiral 6 0 3

Arms is a procedure which draws all the squares in each arm for you.

Arms procedure	**Explanations**
Arms 's	
If :s>60 [stop]	Stops the procedure when squares are too large.
Repeat 4[fd :s rt 90]	Draws square of size s.
rt 90	
fd :s	
lt 75	
arms :s*sqt(2)	Calls the procedure with s made larger.

Spiral is a procedure which places the arrow at the start of each arm.

Spiral Procedure	**Explanations**
Spiral 'a 'n 's	
if :n>11 [stop]	You only need 12 arms
Setx: a*sin(30 * :n)	Places the arrow at the starting point of each arm
Sety: a*cos(30 * :n)	
Seth -45 + 30 * :n	Points the arrow in the starting direction.
arms :s	Draws the nth arm.
Spiral :a :n+1 :s	Calls the procedure to draw the next arm.

The instructions above may differ when using different logo languages.

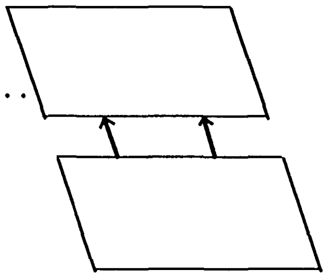
1963 Gift Box

No answers required.

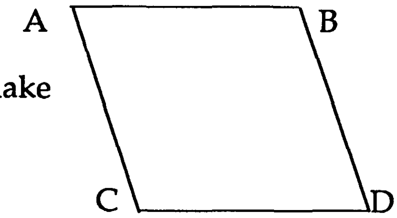
1964 Tube

This is one way of making a tube.

Join the two pieces like this . . .



. . . to make this.



Then roll so A meets B and C meets D.

Can you find another way?

1965 Fold and Hold

Compare your answers with a carrier bag from a shop.

1966 Curve Stitching

No answers required.

1967 One Dice

First Experiment
To win, the '6' must show on one of the two top faces out of the six positions possible. The likelihood of this happening is 2 out of 6 (or 1 out of 3). In 100 TRIALS you would expect to get about 33 WINS.

Alternatively, consider the diagram which shows ALL possibilities. Out of the 24 equally likely outcomes there are just eight WINS, giving a chance of a WIN of one in three.

Number on the top face

Number on other face	1	2	3	4	5	6
1	not possible					not possible
2		not possible			not possible	W
3			not possible	not possible		W
4			not possible	not possible		W
5		not possible			not possible	W
6	not possible	W	W	W	W	not possible

Probability of a win = $\frac{8}{24} = \frac{1}{3}$

continued/

Second Experiment

Various combinations of two adjacent numbers on a dice add up to 6, 7 or 8. If we list all of them in order starting with the '1', we get only four possibilities:

1 and 5,
2 and 4,
2 and 6,
3 and 5.

When we shake the tube, one of the twelve **edges** of the dice will end up on top. And each one comes between a different pair of adjacent numbers. So only four edges out of the possible twelve give the required total of 6, 7 or 8. And thus we can expect to get a WIN one third of the time.

Number on the top face

Number on other face	1	2	3	4	5	6
1	not possible				W	not possible
2		not possible		W	not possible	W
3			not possible	not possible	W	
4		W	not possible	not possible		
5	W	not possible	W		not possible	
6	not possible	W				not possible

Probability of a win = $\frac{8}{24} = \frac{1}{3}$

To see this in another way, we can turn again to the diagram which shows all the possible results. Once again we see that there are eight ways of finishing with a WIN out of the 24 possibilities, a likelihood of one in three again.

Third Experiment

As can be seen in the list above, adjacent numbers on dice never add up to seven. This is because OPPOSITE numbers on dice always total seven. It follows that you CANNOT FAIL to get seven as the sum of the side numbers, and thus you ALWAYS WIN!

In the diagram, ALL the squares except for the six shaded one which give a WIN turn out to be IMPOSSIBLE outcomes because of the way the dice are constructed. So YOU CANNOT LOSE!

Number on the top face

Number on other face	1	2	3	4	5	6
1	not possible	not possible	not possible	not possible	not possible	W
2	not possible	not possible	not possible	not possible	W	not possible
3	not possible	not possible	not possible	W	not possible	not possible
4	not possible	not possible	W	not possible	not possible	not possible
5	not possible	W	not possible	not possible	not possible	not possible
6	W	not possible	not possible	not possible	not possible	not possible

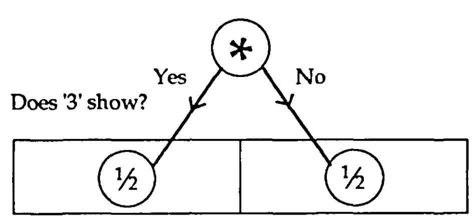
Probability of a win = $\frac{6}{6} = 1$

1968 Numbers Up

Three Numbered Counters

First Experiment

Here the '1' and '2' counters are simply distractors. All that matters is which way up the '3' counter lands. In other words it is just like tossing a coin, and the chance of success are one in two.

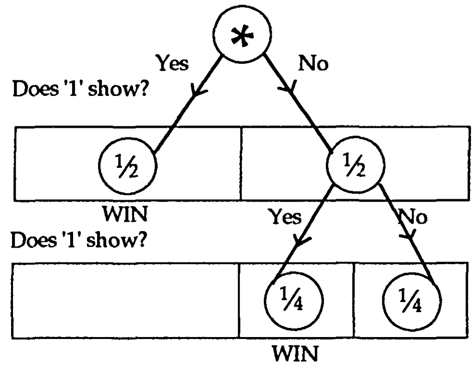


Probability of a WIN = ½

continued/

Second Experiment

Half the time you will win because the '1' is showing. But even if it is hidden, you can still win if the '2' shows up. So the likelihood of winning is 'one half plus (a half of a half)', in other words three quarters.



Probability of a WIN = ½ + ¼ = 3/4

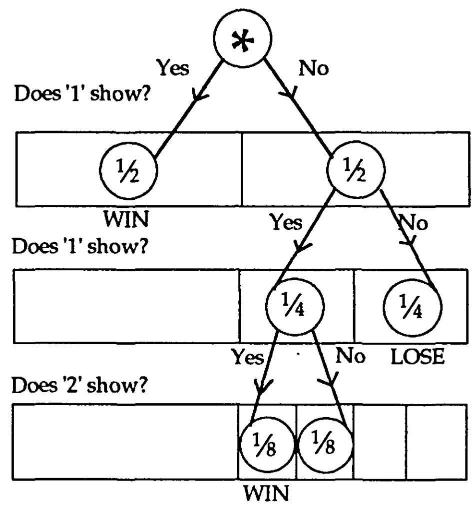
Alternatively, we can think of the likelihood of LOSING. This happens if we see NEITHER the '1' NOR the '2'. On half the TRIALS the '1' will be face down and on half these the '2' will be face down as well so we expect to LOSE a quarter of the time, which is the same as saying we expect to WIN three quarters of the time.

Third Experiment

This is a more difficult situation, but it can be broken down into easy steps. First we see that we definitely WIN if the '3' faces up. So half the time we will WIN in this way.

But even if we do not see the '3' we can still WIN provided BOTH the '1' AND '2' land face up. We can expect this to happen a quarter of the time, just as we can expect NEITHER of them to show a quarter of the time.

So we are likely to WIN a half of the time with the '3' showing and OF THE OTHER HALF we can still expect to win one out of four TRIALS. In short, in every 8 trials we can expect to win 4 times with the '3' face up, and once with both the '1' and the '2' showing.



Probability of a WIN = ½ + ⅛ = ⅝

First Experiment

Whatever way the pink dice falls, there are six ways that the white one can fall and they are equally likely. Only one of those ways will give a WIN, so the likelihood of a WIN is one in six whatever the pink number is.

Alternatively, the diagram shows all the combinations of two numbers you can obtain with two dice, and they are all equally likely. Six out of thirty-six give a WIN, so you can expect to win once in every six TRIALS.

Number on white dice (columns); Number on pink dice (rows)

	1	2	3	4	5	6
1	W					
2		W				
3			W			
4				W		
5					W	
6						W

Probability of a WIN = $6/36 = 1/6$

Second Experiment

Out of the 36 possible combinations of numbers, you will win if the numbers (pink dice first) are:

3 and 6
4 and 5, 4 and 6
5 and 4, 5 and 5, 5 and 6
6 and 3, 6 and 4, 6 and 5, 6 and 6

So on average you can expect to win 10 times in every 36.

Number on white dice (columns); Number on pink dice (rows)

	1	2	3	4	5	6
1						
2						
3						W
4					W	W
5				W	W	W
6			W	W	W	W

Probability of a WIN = $10/36 = 5/18$

Third Experiment

Once again we can list the possibilities:
1 and 2,
1 and 3,
1 and 4,
1 and 5,
1 and 6,
2 and 3,
2 and 4 etc but it is more convenient to put them in a diagram like this one.

Bearing in mind that all the combinations are equally likely, we can easily work out that we can expect to WIN 15 times in 36.

Number on white dice (columns); Number on pink dice (rows)

	1	2	3	4	5	6
1		W	W	W	W	W
2			W	W	W	W
3				W	W	W
4					W	W
5						W
6						

Probability of a WIN = $15/36 = 5/12$

1970 Five Beads

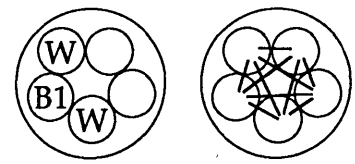
First Experiment

There are two blue balls and five equally likely positions for each of them to fall into, so the chance of a WIN is two in five.

Probability of a WIN = $\frac{2}{5}$

Second Experiment

Wherever the first blue ball, B1, falls there are four equally likely positions for B2 to take, of which two give a WIN. So the likelihood of a WIN is two in four, or 'evens'.

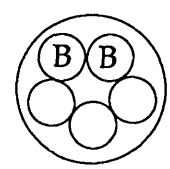


Probability of a WIN = $\frac{2}{4} = \frac{1}{2}$

Alternatively, there are ten ways of choosing a pair of two beads, and in five of these the two beads are touching.

Third Experiment

First, the likelihood that the two balls touch is, as we have seen, one in two. Now, for the two red balls to touch as well, the yellow ball MUST land next to a blue ball. And two positions of the yellow ball out of three give this.



Probability of a WIN = $\frac{2}{3}$

So, out of every six TRIALS we can expect the blue balls to touch in three, and the red ones to touch also in two of these. Thus the likelihood of a WIN is two out of six, or one in three

---

1971 Seven Beads

First Experiment

The red ball is equally likely to fall in any one of the seven positions in the base of the tube. So the likelihood of it falling in the centre is 1 in 7.

Second Experiment

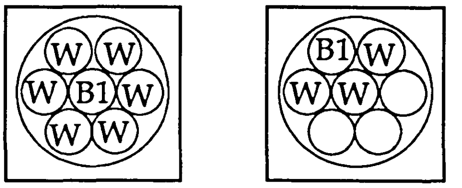
There are two blue balls, so there are two chances in seven that a trial will end up with a WIN.

continued/

1971 Seven Beads (cont)

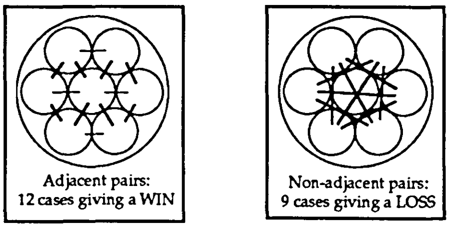
Third Experiment

Should the first ball, B1, land in the centre (likelihood 1 in 7), then you definitely WIN as B2 must touch it. But even if B1 does not land in the centre (likelihood 6 out of 7) you still WIN half the time as B2 will land next to it three times out of six. So in every 7 Trials, you can expect to WIN once with B1 in the centre, and another three times with B1 at the side.



Probability of a WIN = $\frac{1}{7} + (\frac{1}{2} \text{ of } \frac{6}{7}) = \frac{4}{7}$

Alternatively, count up all the possible 'PAIRS OF TWO BALLS'. In 12 of these the balls touch, while in the other 9 pairs the two balls are separate. As these cases are all equally likely, the chance of having the blue balls touching is 12 out of 21, or 4 out of 7.



Probability of a WIN = $\frac{12}{21} = \frac{4}{7}$

1972 Scenticube

No answers required.

1973 Drinka Pinta

No answers required.

1974 Turkish Delight

No answers required.

1975 Wrap a Ball

No answers required.

1976 Pillow Box

No answers required.

1977 Kit Kat

No answers required.

1983 T-Shirt

No answers required.

1984 Bags

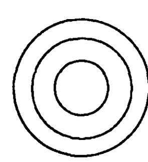
With a 3 x 3 square + a 4 x 4 square you can get a 5 x 5 square. There are lots of ways of doing this, one way is to unpick 6 squares from the 4 x 4 square.

1985 Style

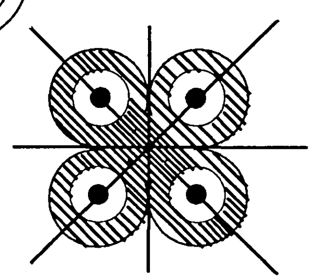
Approximately 125 - 135cm plain cloth.

1986 Adinkra

This design has many lines of symmetry.
How many?



This design has 4 lines of symmetry
and rotational symmetry order 4.



Ohene Anima	2 lines of symmetry
Foofoo	8 lines of symmetry and rotational symmetry order 8
Dwennimmen	2 lines of symmetry and rotational symmetry order 2
Aya	1 line of symmetry
Hwemudua	1 line of symmetry
Fi-Hankra	2 lines of symmetry and rotational symmetry order 2

1988 Seven & Ten

How many different ways did your group find to show 7 and 10?

- Did you write them in only one number script?
- Did you find some numbers easier to show?
- Did you think of your number having a special colour?

1989 Your Number Line

How many different number lines did your group find?
Were you able to fit all the numbers on your number line?

1990 Some Subtractions

No answers required.

1991 On the Bus

No answers required.

1992 Drinks Machine

The most money without being able to make exactly 20p is 23p.

10 + 5 + 2 + 2 + 2
10 + 5 + 2 + 2 + 2 + 2
5 + 5 + 5 + 2 + 2 + 2
5 + 5 + 5 + 2 + 2 + 2 + 2
2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 5
2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 5

1993 Calendar Sevens

Use this calendar sheet to check your answers.

Mon	Tues	Wed	Thur	Fri	Sat	Sun
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Did you find the 7 different patterns?

1994 Alphabet Numbers

No answers required.

1995 Pages in Books

16 pages
24 pages
199 pages
Many possible answers.

1996 Nine Times

Add the digits of any answers. You should always get NINE.

126 is in the nine times table because 1 + 2 + 6 = 9

In the 11 times table, add the digits of your answer. You should always get an even number.

1 x 11 = 11	1 + 1 = 2
2 x 11 = 22	2 + 2 = 4
3 x 11 = 33	3 + 3 = 6
.	.
.	.
.	.

What happens to this pattern after 9 x 11, 18 x 11, 27 x 11 . . . ?

1997 In the Playground

You may need to find out how many pupils are in your school.

- How many pupils use the playground at break time?
- Is the playground used at other times . . . fire drills, sporting activities?

1998 Archimedes Spirals

Cotton Reel Method

- With one cotton reel, there is only one possible spiral. The length of the strip of paper will affect the amount of the spiral that you can draw.

- The size of the cotton reel will affect the shape of the spiral. A smaller cotton reel will give a tigher spiral.

continued/

1998 Archimedes Spirals (cont)

Polar Graph Method

- The sequences increase by adding on a constant amount.

  1 in the first
  2 in the second
  ½ in the third.

Other Archimedes Spirals

- Boxes of the same size give the same spiral.
- Different sizes gives different spirals.

1999 Equiangular Spirals

No answers required.

2000 Fibonacci and Square Root Spirals

1, 1, 2, 3, 5, 8, 13, 21, ...
This sequence is made by adding the two previous numbers:

$1 + 1 = 2$
$1 + 2 = 3$
$2 + 3 = 5$
$3 + 5 = 8$
.
.
.

2001 Helix

No answers required.

2002 Real Spirals

No answers required.

2003 Birthday Dates

No answers required.

2004 54% is a little more than half marks

11% of £900	= £99
19% of £200	= £38
47% of £245	= £115.15
56% of £42	= £23.52
64% of £320	= £204.80
68% of £210	= £142.80
70% of £80	= £56
72% of £250	= £180
94% of £165	= £155.10
103% of £78	= £80.34

2005 Peerie

No answers required.

2006 A Mountain Walk

3 miles in 1 hour

Using 5 miles is about 8 km
1 mile is about 1.6 km
so 3 miles is about 3 x 1.6 = 4.8 km

You may have a slightly different result but it should still approximate to the same, easy to remember, rule of **5 km in 1 hour.**

2000 feet in 1 hour

Using 1 inch = 2.54 cm
12 inches = 1 foot = 12 x 2.54 = 30.48 cm
2000 feet = 60960 cm (609.6 m)
so approximately **600 m in 1 hour.**

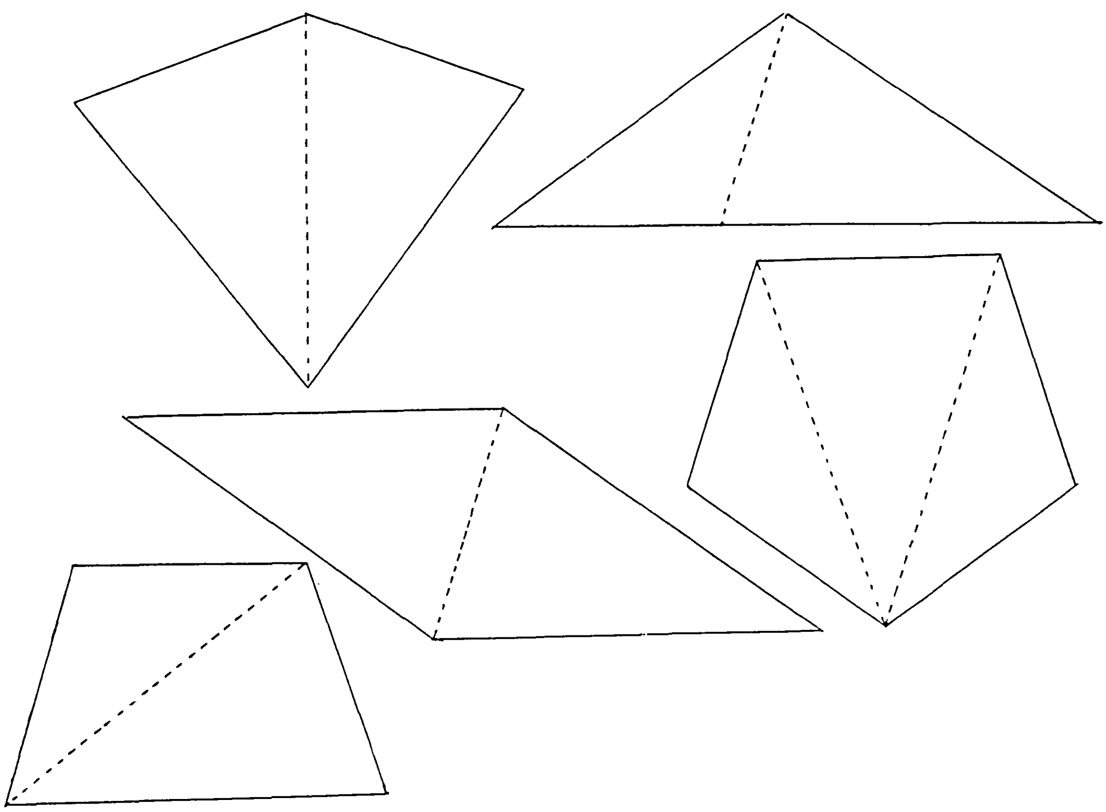
continued/

2006 A Mountain Walk (cont)

This is usually remembered as 300 m in half an hour because in Britain not many mountains involve a 600 m climb.

For information: Naismith's rule was devised by William Wilson Naismith (1856 - 1935) a pioneer Scottish mountaineer. In July 1916, aged 60, he walked from Glasgow to the mountain Ben Lomond and back - that's 62 miles in 20 hours! He was very religious and wouldn't climb mountains on Sundays.

2007 Triangle Shapes



Try to find out the names of your own shapes.

2008 Curves of Pursuit

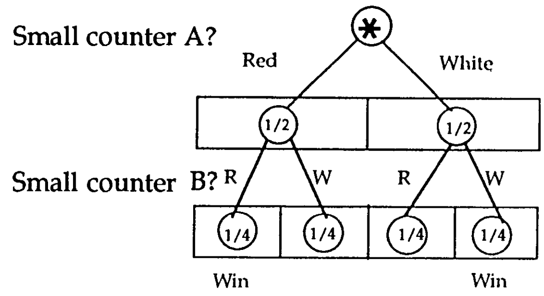
No answers required.

2009 Three Counters

These are answers for Dime Probability Pack A: Card 1

First Experiment

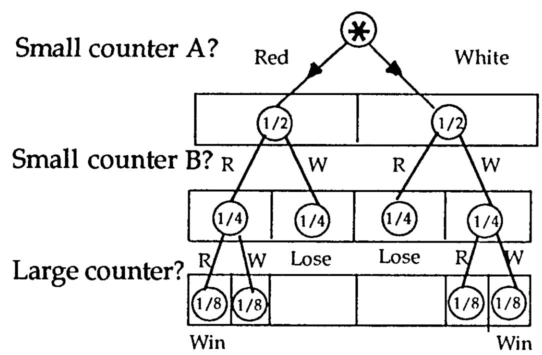
Call the two small counters A and B. Whichever colour A shows, half the time B will show the same colour. So in 100 trials we would expect to have about 50 cases of the two small counters showing the same colour. The large counter is nothing more than a distractor, because which way up it falls does not affect the outcome.



Probability of a win = 1/4 + 1/4 = 1/2

Second Experiment

Think of this as an extension of the first experiment. Half the time we can expect the two small counters to show the same colour, say red. On roughly **half** of these occasions the large counter will also show red. So we would expect all three counters to show the same colour, whether red or white, a quarter of the time.

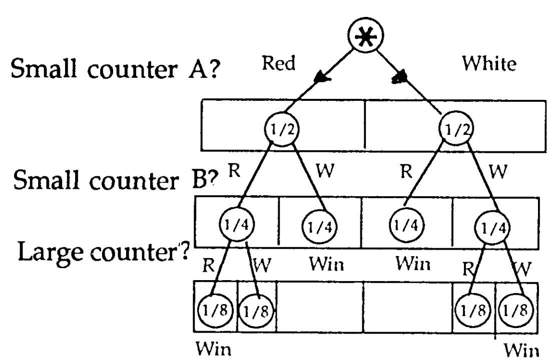


Probability of a win = 1/8 + 1/8 = 1/4

Third Experiment

We can deduce this from the first two experiments. Half the time the two small counters A and B will show the same colour, so the other half of the time A and B will be different. In these cases the large counter must be the same colour as one of them - giving a win.

But we can also win by having all three counters showing the same colour. This will happen, as we have seen, a quarter of the time. So altogether we can expect to win in this experiment three quarters of the time.



Probability of a win = 1/8 + 1/4 + 1/4 + 1/8 = 3/4

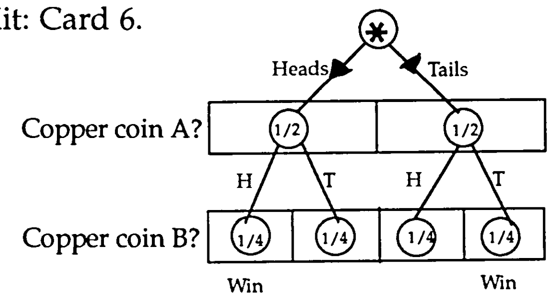
continued/

2009 Three Counters (cont)

These are answers for Dime Probability Kit: Card 6.

Experiment A

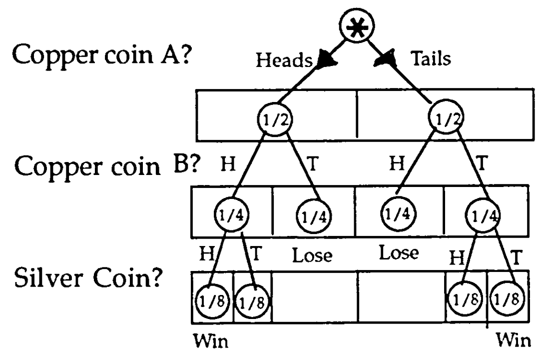
Call the two small copper coins A and B. If A shows a 'head' then half the time B will show a 'head'. So in 100 trials we would expect to have about 50 cases of the two small copper coins being the same. The silver coin is nothing more than a distractor, because which way up it falls does not affect the outcome.



Probability of a win = 1/4 + 1/4 = 1/2

Experiment B

Think of this as an extension of the first experiment. Half the time we can expect the two small copper coins to be the same. That is they are both 'heads' or both 'tails'. On roughly **half** of these occasions the silver coin will also be a 'head'. So we would expect all three coins to be the same, whether 'heads' or 'tails', a quarter of the time.

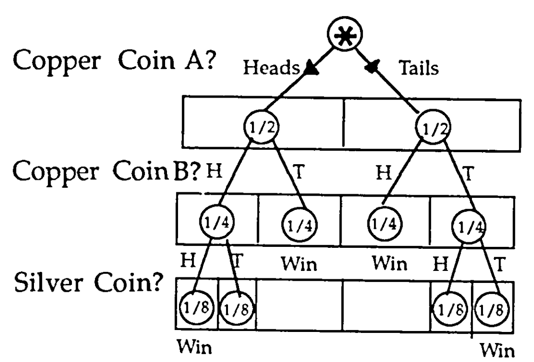


Probability of a win = 1/8 + 1/8 = 1/4

Experiment C

We can deduce this from the first two experiments. Half the time the two small copper coins A and B will be the same, so the other half of the time A and B will be different. In these cases the silver coin must be the same as one of the copper coins - giving a win.

But we can also win by having all three coins show the same. This will happen, as we have seen, a quarter of the time. So altogether we can expect to win in this experiment three quarters of the time.



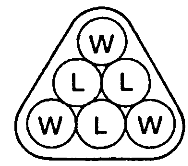
Probability of a win = 1/8 + 1/4 + 1/4 + 1/8 = 3/4

2010 Six Beads

These are answers for Probability Pack B: Card 2 **(the answers for Probability Kit: Card 3 are in brackets)**

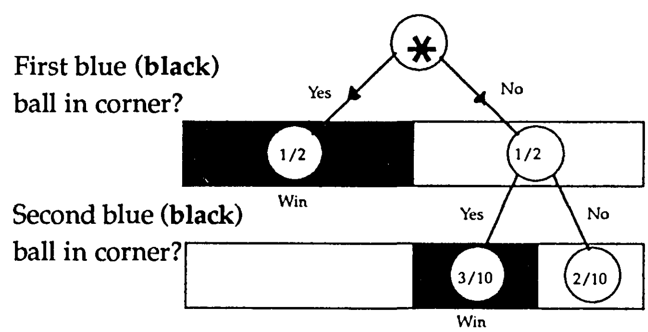
First Experiment

Of the six equally likely positions that the single yellow **(gold)** ball can take, three are in the corner and give a win. So the likelihood of a win is 3 out of 6, or 'evens'.



Second Experiment

It is understandable to think that with two blue **(black)** balls the likelihood is twice as great, which would make it 6 out of 6 or certain! As before, the likelihood of the first blue **(black)** ball, B1, landing in a corner is one in two. But if it takes an inner position then the likelihood that B2 will fall in a corner is now three out of five. Thus the chances of a win are a half plus 'three fifths of the other half', which makes eight out of ten, or four fifths altogether.

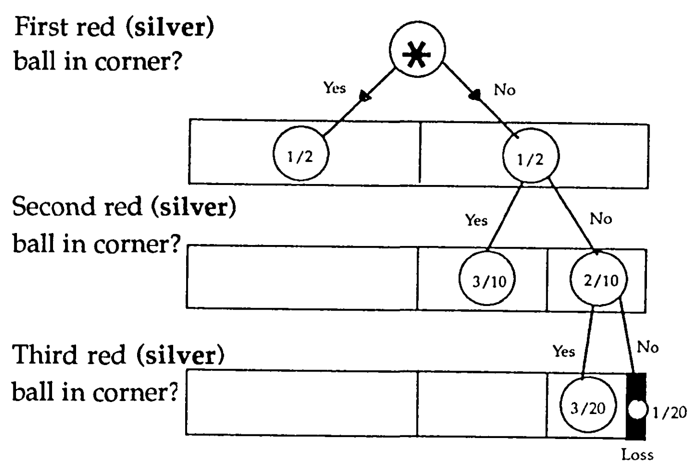


Probability of a win = 1/2 + 3/10 = 4/5

Third Experiment

The likelihood of a particular red **(silver)** ball landing in a corner is, like the yellow **(gold)** one, one in two or 'evens'. So thinking of all three of them, we might feel that it is more than 'certain' that one will do so. But in fact it is still possible that no red **(silver)** ball will land in a corner at all.

Instead of approaching this case with the previous type of argument, we will ask instead: 'What is the likelihood of losing?'



Probability of a loss = 1/2 x 2/5 x 1/4 = 1/20

Probability of a win = 19/20

continued/

2010 Six Beads (cont)

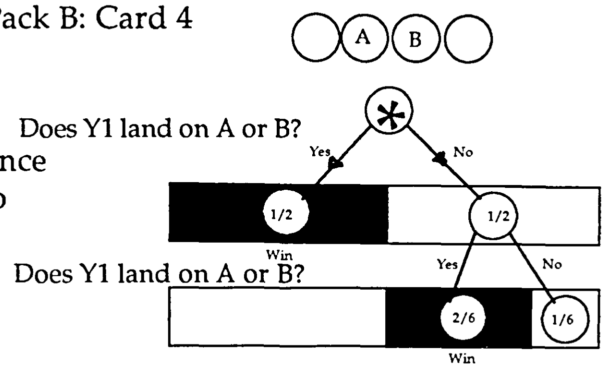
For this to happen, all three red **(silver)** balls must land in central positions. The first will do so half the time. The likelihood of the second doing so as well are two fifths of the half (or one fifth). With the first two red **(silver)** balls in central positions there are four equally likely places left for the third to go, only one of which is not in a corner. So the likelihood of losing with all three balls is one quarter of that fifth, or one twentieth. It follows that the chances of a win are nineteen twentieths.

2011 Four Beads

These answers are for the Dime Probability Pack B: Card 4

First Experiment

Call the two yellow balls Y1 and Y2. The chance that the first, Y1, will fall in either of the two centre spaces, A and B, is two out of four, as all positions are equally likely. If Y1 takes an end position, then the likelihood of Y2 making it a win is two out of three. So in each trial the likelihood of a win is 'a half' plus 'a half of two thirds', or five sixths.

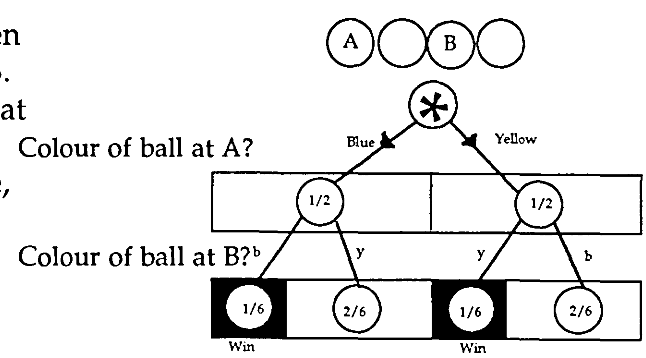


Probability of a win 1/2 + 1/3 = 5/6

Second Experiment

Suppose the left hand ball (at A) is blue. Then you win only if the other blue ball lands at B. Should the ball at A be yellow, then the ball at B must also be yellow for a win. You can expect to get the colours at A and B the same, and so win, about one time in three.

Alternatively list all the ways in which the balls could fall. There are only six of them, in how many do the colours alternate? Only two. Thus, once again, the likelihood of a win is one in three.

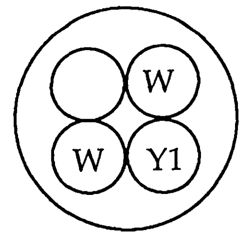


Probability of a win 1/6 + 1/6 = 2/6 = 1/3

Third Experiment

This is a more tricky example that people often get wrong. There are only the two possible ways in which the four balls can fall into the recess, and it is easy to assume that they are equally likely. But we can soon see that this is not so.

Suppose the first yellow ball, Y1, falls in the position shown in the diagram. Whether or not we get a win depends on the fall of Y2, and we can see that we get two wins to one loss. Thus the likelihood of getting a win is two out of three.

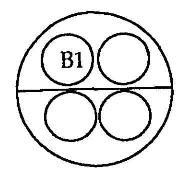


Probability of a win = 2/3

These answers are for the Dime Probability Pack: Card 1

Experiment A

Suppose a black ball, B1 lands on one side of the ridge. Then the likelihood of a silver ball landing next to it, making it a win, is two out of three. So in each trial the likelihood of a win is two thirds.

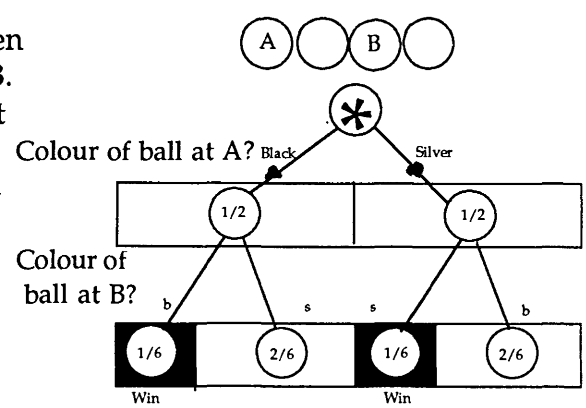


Probability of a win = 2/3

Experiment B

Suppose the left hand ball (at A) is black. Then you win only if the other black ball lands at B. Should the ball at A be silver, then the ball at B must also be silver for a win. You can expect to get the colours at A and B the same, and so win, about one time in three.

Alternatively list all the ways in which the balls could fall. There are only six of them, in how many do the colours alternate? Only two. Thus, once again, the likelihood of a win is one in three.



Probability of a win 1/6 + 1/6 = 2/6 = 1/3

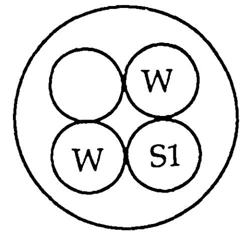
Experiment C

This is a more tricky example that people often get wrong. There are only the two possible ways in which the four balls can fall into the recess, and it is easy to assume that they are equally likely. But we can soon see that this is not so.

continued/

2011 Four Beads (cont)

Suppose the first silver ball, S1, falls in the position shown in the diagram. Whether or not we get a win depends on the fall of S2, and we can see that we get two wins to one loss. Thus the likelihood of getting a win is two out of three.



Probability of a win = 2/3

2012 Tessellating Patterns

No answers required.

2013 Round the Bend

There are two sets of answers, the first set of answers are obtained by using π **= 3.14**, the second set of answers are obtained **using the π button.**

Hint: **Diameter is twice the radius**

Answers using π = 3.14

1.	Diameter = 2 x 6400 km	= 12800 km	
	Distance around equator	= 3.14 x 12800 km	= 40192 km
2.	Outer diameter	= 2 x 100 m	= 200 m
	Outside edge	= 3.14 x 200 m	= 628 m
	Inner diameter	= (100 m – 8 m) x 2	= 184 m
	Inside edge	= 3.14 x 184 m	= 577.76 m
	How much further	= 628 m – 577.76 m	= 50.24 m
3.a)	**Hour hand** goes round twice in one day.		
	Once round	= 3.14 x 10 cm	= 31.4 cm
	So twice round	= 62.8 cm	= 62.8 cm

continued/

b) **Minute hand** goes round 24 times in one day.

Once round = 3.14 x 20 cm = 62.8 cm

24 times round = 24 x 62.8 cm = 1507.2 cm = 1.5 m (approx)

4. One lap = 2 straights + 2 semi-circles

400 m = 2 straights + 1 circle

400 m = 2 straights + (3.14 x 80 m)

400 m = 2 straights + 251.2 m

2 straights = 400 m – 251.2 m = 148.8 m

1 straight = 74.4 m

Answers using the π button.

1. Diameter = 2 x 6400 km = 12800 km

Distance around equator = π x 12800 km
= 40212.386 km
= 40212 km (to nearest km)

2. Outer diameter = 2 x 100 m = 200 m

Outside edge = π x 200 m = 628.318 m

Inner diameter = (100 m – 8 m) x 2 = 184 m

Inside edge = π x 184 = 578.053 m

How much further = 628 m – 578.053 m= 50.265 m

3.a) **Hour hand** goes round twice in one day.

Once round = π x 10 cm = 31.4159 cm

So twice round = 31.4159 cm x 2 = 62.83 cm

continued/

2013 Round the Bend (cont)

b) **Minute hand** goes round 24 times in one day.

Once round	$= \pi \times 20$ cm	$= 62.831853$ cm	
24 times round	$= 24 \times 62.83185$ cm $= 1507.96$ cm		$= 1.5$ m (approx)

4.

One lap	=	2 straights + 2 semi-circles	
400 m	=	2 straights + 1 circle	
400 m	=	2 straights + ($\pi \times 80$ m)	
400 m	=	2 straights + 251.132741 m	
2 straights	=	400 m – 251.132741 m	= 148.67259 m
1 straight	=	74.336294 m	= 74.336 m

2014 Probably Probable?

You should get an answer between these values

A. 0.94 - 0.87

B. 0.26 - 0.16

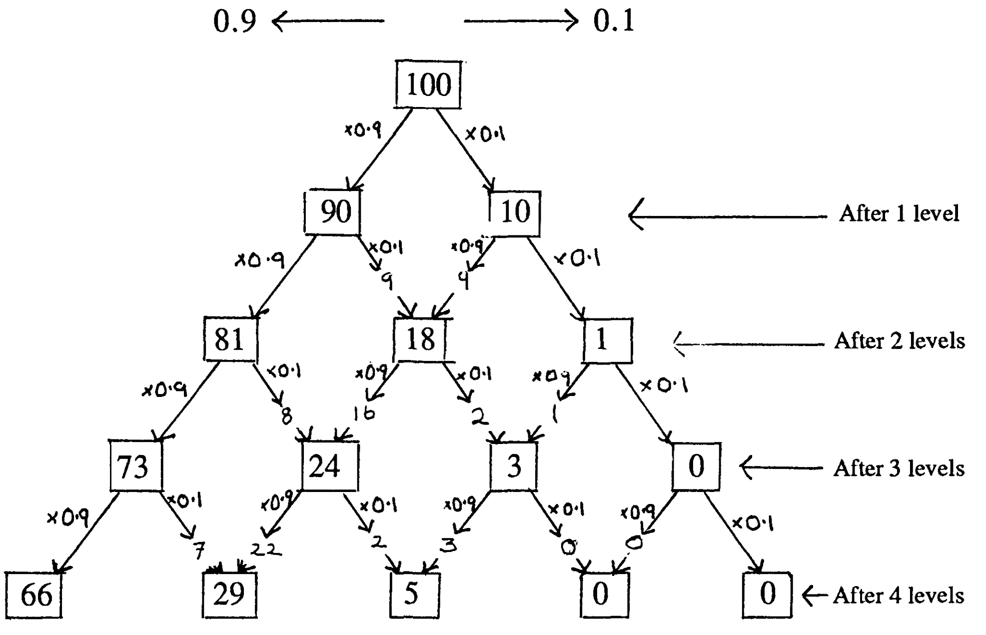
C. 0.37 - 0.21

You could have chosen a probability outside these ranges that is similar to the outcomes but you would have been lucky!

If you chose 0.9 as the probability in A you could predict the outcome without the computer.

continued/

2014 Probably Probable? (cont)

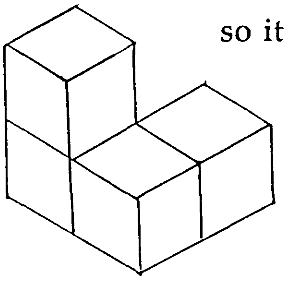
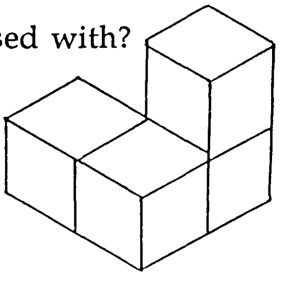


Find the expected answers for your probabilities for B and C. You may like to use a spreadsheet.

Do they compare with the outcomes? Discuss with your teacher.

2015 Making Cubes

What did you notice about the solids that fitted together?

How could you describe  so it isn't confused with? 

When you have designed your puzzle, give it to another group.

2016 Target 24 - a 3 Digit Problem

Some **possible** answers.

1. $[(1 + 1)^2]! \times 1 = 24$

2. $22 + 2 = 24$

3. $3^3 - 3 = 24$

4. $4! \times \frac{4}{4} = 24$

5. $5^2 - \frac{5}{5} = 24$

6. $6^2 - 6 - 6 = 24$

7. $\left[\frac{(7 + 7)^2}{7^2}\right]! = 24$

8. $8 + 8 + 8 = 24$

9. $\sqrt{9} \times 9 - \sqrt{9} = 24$

If your answers are different get somebody else to check them.

2017 Fair Play

The game is not fair. Evens usually win more games than odds.

Why? The highest possible even score is 6
The highest possible odd score is 5

The next highest possible even score is 4
The next highest possible odd score is 3

The lowest possible even score is 2
The lowest possible odd score is 1

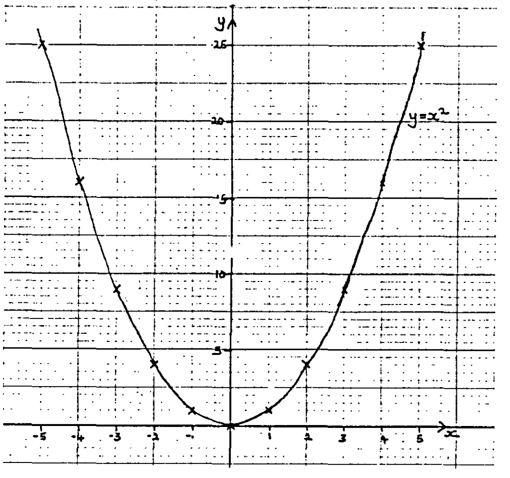
The chances of odd and even appearing are the same (there are three odd and three even numbers on a dice).

It is more likely that evens will take more counters because the even scores are higher than the odd.

2018 Drawing the Curve

1. $y = x^2$

x	y
+5	+25
+4	+16
+3	+9
+2	+4
+1	+1
0	0
−1	+1
−2	+4
−3	+9
−4	+16
−5	+25

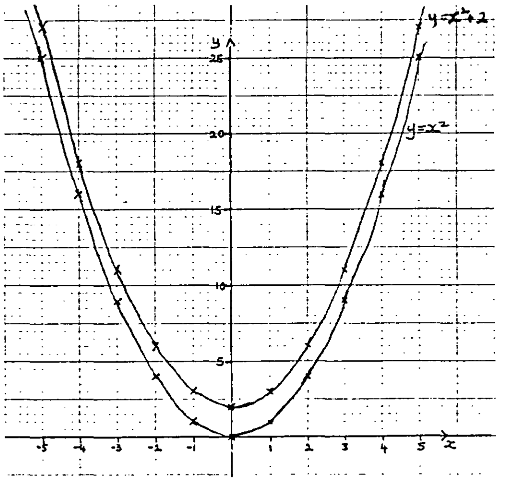


There are various observations you can make, such as

* the lowest point is (0,0)
* it is symmetrical about the y axis
* as x gets further away from 0 the curve gets steeper
* the curve is called a parabola.

2. $y = x^2 + 2$

x	y
+5	+27
+4	+18
+3	+11
+2	+ 6
+1	+ 3
0	+ 2
−1	+ 3
−2	+ 6
−3	+11
−4	+18
−5	+27



Your description of $y = x^2 + 2$ should be similar to that of $y = x^2$ except that the curve has shifted 2 units up, so that now the lowest point is (0, 2).

continued/

If you can't see that these two curves are the same shape **check with tracing paper.**

3. Curves of the form $y = x^2 + c$ are all the same shape. If c is positive they are all shifted upwards; if c is negative they are shifted downwards.
The lowest point is always (0, c).

2019 Power Match

The missing number is 2.

You should get the following pairs:

$4^2 = 16$

$32 = \mathbf{2}^5$

$\mathbf{2} \times 5 = 10$

$8 = \mathbf{2}^3$

$3^2 = 9$

$3 \times \mathbf{2} = 6$

2020 High Powered Matching

The missing number is 3.

You should get the following pairs:

$\mathbf{3} \times 10 = 3^3 + 3$

$24 \div \mathbf{3} = 2^3$

continued/

2020 High Powered Matching (cont)

$3^2 \times 3^2 \times 3 = 3^5$

$3 \times 10^3 = 3000$

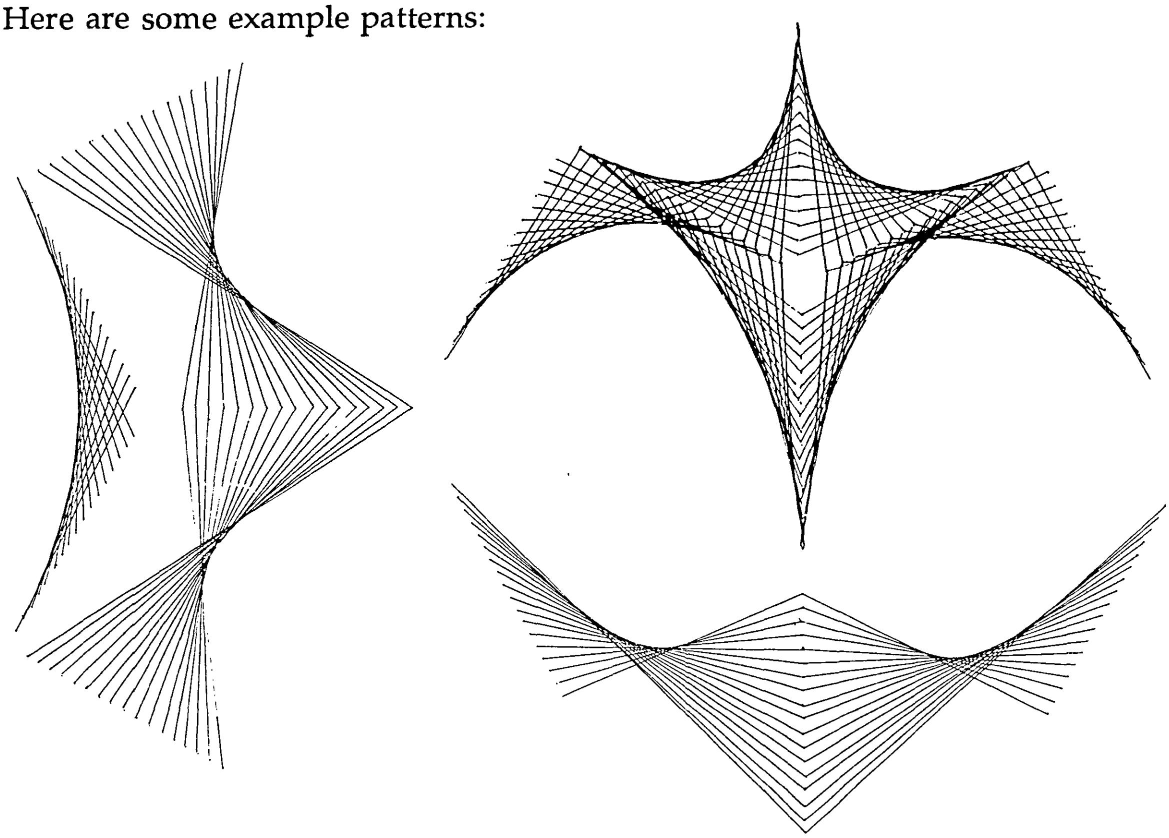
$3^2 = 9$

$9^3 + 271 = 997 + 3$

2021 Stitching Curves

Trace the dots on to card. It helps to make the holes first before you start stitching. Often the most effective patterns are created using thin cotton.

Here are some example patterns:



You can also get other ideas from the SMILE 1966 Curve Sketching poster (Tarquin). Your group might like to make a display of the different patterns they made.

2022 Fewest Keys

Your answers will depend upon the calculator you are using.

a) We managed to do this calculation using 14 key presses in 2 different ways.

5 . 2 + 3 . 6 = $\frac{1}{x}$ × 2 . 5 =

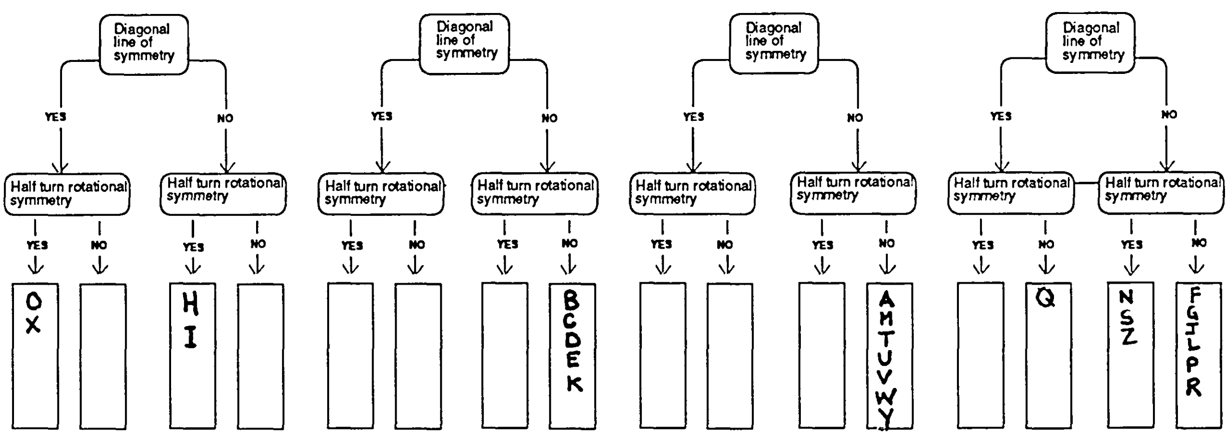
2 . 5 ÷ (5 . 2 + 3 . 6) =

You may have different answers.
You may have used fewer keys if you did some calculations in your head e.g. 5.2 + 3.6

These are our results. Did you use fewer keys?

b) 21 presses
c) 20 presses
d) 8 presses (we used the fraction key; without it we would have needed 9 presses)
e) Did you manage to do it in 10 presses? Get someone to check your solution. Using some mental arithmetic you could do this in 4 key presses. How?

2023 Alphabet Symmetry



* we got these answers by using the letters on the worksheet.
* you may have some different answers if you write your letters differently.

Most boxes are empty because there aren't many letters with more than two different types of symmetry.

2024 Excess Luggage

You may have given several different answers.

Example: planes would become too heavy to fly,
no room on the plane.
they want to make some more money . . .

Passenger	Destination	Class	Weight	Excess Luggage	Charge
Ngozi	New York	Economy	26 kg	6 kg	£ 94.26
Pritesh	Rome	First	26 kg	0 kg	Free *
Paul	Rio	Economy	27 kg	7 kg	£120.40
Yuen	Brussels	First	41 kg	11 kg	£ 10.45
Chris	Tokyo	Economy	28 kg	8 kg	£182.72
Maria	Addis Ababa	Economy	19 kg	0 kg	Free

* remember that first class passengers may take up to 30 kg free.

Luggage charges to Rome

Weight	First class passenger luggage charge	Economy class passenger luggage charge
15	Free	Free
17	Free	Free
19	Free	Free
21	Free	£4.06
23	Free	£12.18
25	Free	£20.30
27	Free	£28.42
29	Free	£36.54
31	£ 4.06	£44.66
33	£12.18	£52.78
35	£20.30	£60.90
37	£28.42	£69.02

20 kg is enough for most passengers - try collecting several objects which together weigh 20 kg.
How far could you carry them?
How many jumpers would you need to make 20 kg?

It depends; probably not for a short holiday but it may be worth going over the limit if you are going away for a long time.

continued/

2024 Excess Luggage (cont)

We think **no**. Why should first class passengers be allowed to take more luggage free of charge? What do you think?

Examples of fairer ways

* to increase the amount of excess luggage that can be carried by an economy class passenger.
* economy class passengers could pay excess luggage charges based on the economy fare **not** the first class fare.

Find a way to display the different amounts charged to economy and first class passengers for their excess luggage to be carried to Rome.

2025 Turning Into a Polygon

Shape	Number of sides	Turning angle
Triangle	3	120^o
Square	4	90^o
Pentagon	5	72^o
Hexagon	6	60^o
Septagon	7	51^{o*}
Octagon	8	45^o
.	.	.
.	.	.
.	.	.

Your turning angles should be close to these. Maybe you noticed that the number of sides x turning angle = 360 .

* This is an approximation to the nearest whole degree.

2026 Number Pyramids

You will find it easier to see patterns if you put your pyramids in order:

e.g.

```
      8          12           16
    3   5      5    7       7    9
  1   2   3   2   3   4   3   4   5
```

continued/

2026 Number Pyramids (cont)

Some suggestions for what you can investigate:

- * Predict the numbers at the peak from the bottom numbers.
- * Have 4 consecutive numbers in the base, 5 numbers, ...
- * Change the rule for the numbers in the base.

2027 Similar Triangles

Here are 3 possible ways of defining similar triangles:

- * they each have the same set of 3 angles.
- * they are enlargements of each other.
- * the ratios of corresponding sides are equal.

To find the groups of similar triangles you could either draw them accurately or use Pythagoras Theorem to find the missing side.

Group 1 A I E
Group 2 J D B
Group 3 F C H

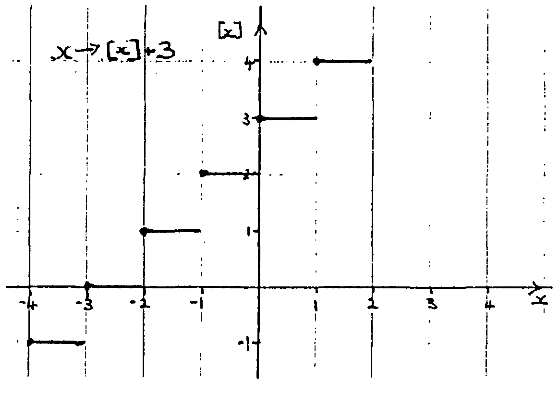
G is the odd triangle.

2028 Integer Graphs

1. The steps are not joined together because, for example, anything below 1 and above 0 will be 0 no matter how close it is to 1. When you reach 1 however, it is 1 and so there is a jump between the numbers before 1 and 1 itself. Therefore the steps are not joined.

   There are dots on the left hand end of each step because each line has a definite start but no definite end.

2.a)



The graph of $x \longrightarrow [x] + 3$ is the same as $x \longrightarrow [x]$ but it has shifted 3 spaces up.

Similarly, $x \longrightarrow [x] - 3$ is the same as $x \longrightarrow [x]$ but it has shifted 3 spaces down.

continued/

2028 Integer Graphs (cont)

b) $x \longrightarrow [x] + c$ is the same as $x \longrightarrow [x]$ with a vertical shift of c.
What can you say about $x \longrightarrow [x + c]$?

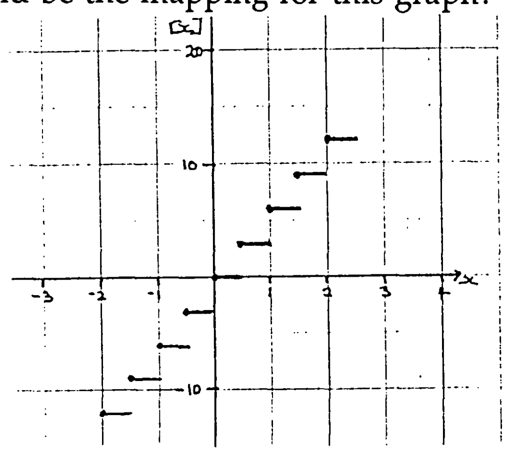
3.a) $x \longrightarrow 2\,[x]$ is the same as $x \longrightarrow [x]$ except the vertical gap between each step is 2 spaces.

$x \longrightarrow [2x]$ has vertical gaps of 1 space but this time the steps are $\frac{1}{2}$ the length.

b) $x \longrightarrow m[x]$ vertical gap m step length 1

$x \longrightarrow [mx]$ vertical gap 1 step length $\frac{1}{m}$

What would be the mapping for this graph?



4. $x \longrightarrow \left[\frac{x}{m}\right]$ vertical gap 1 step length m

$x \longrightarrow \frac{[x]}{m}$ vertical gap $\frac{1}{m}$ step length 1

2029 Strings

You can describe the $\left[\frac{1}{3}n\right]$ string 0, 0, 0, 1, 1, 1, 2, 2, 2, . . .

as a 3, 3, 3, . . . pattern.

Similarly the $\left[\frac{2}{3}n\right]$ string as a 2, 1, 2, . . . pattern.

How does this continue?

continued/

2029 Strings (cont)

What are the patterns for $\left[\frac{1}{4}n\right]$?

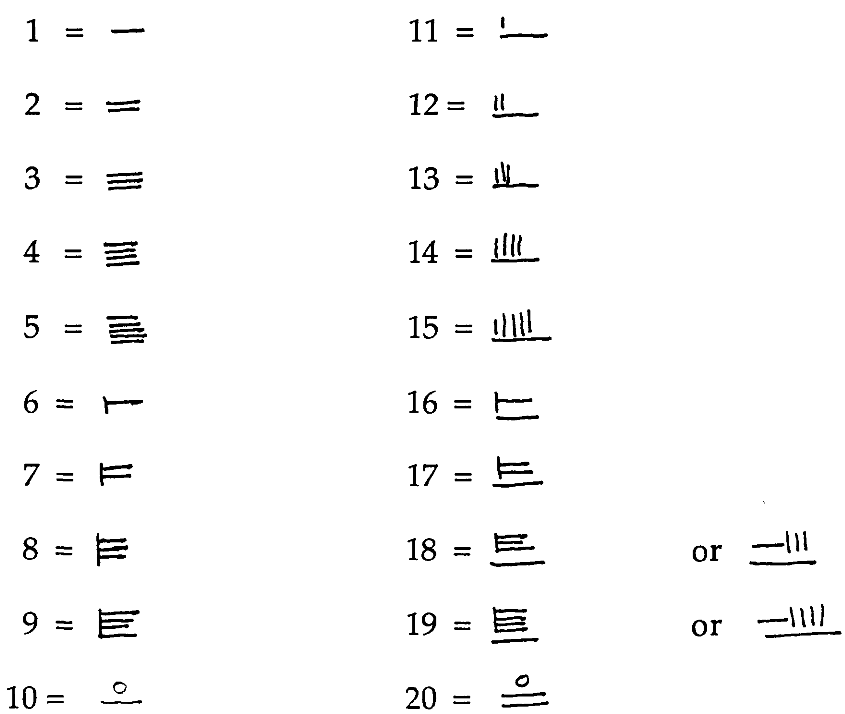
Can you describe these patterns in a way that allows you to predict them for any fraction?

What happens to the strings for fractions where the numerator is greater than the denominator e.g. $\left[\frac{3}{2}n\right]$?

Can you predict the number of zeros?

2030 Old Chinese Numbers

From the triangle:



continued/

2030 Old Chinese Numbers (cont)

This would then continue with

30 = 

35 = 

40 =  or 

41 =  or 

50 = 

If you disagree with any of these answers discuss them with your teacher.

2031 Spiralling Squares Patterns

Describe the way in which you started this activity. What mathematics did you need to use?

The next square would be 8 cm by 8 cm. Can you see a relationship between the size of the squares?

Some of the shapes we found were arrowheads, diamonds and 12-pointed stars. Can you find any more?

2032 D.I.Y. Earrings

Diameter (cm)	Radius (cm)	Area (cm^2)
1	0.5	0.7853982 *
1.5	0.75	1.7671459 *
2	1	3.1415927 *
4	2	12.566371 *
5	2.5	19.634954 *

* Using the π button on the calculator gives these answers.
If you used $\pi = 3.14$ they will vary slightly.

continued/

2032 D.I.Y. Earrings (cont)

Earring A is made up of
1 silver disc diameter 1 cm
+ 1 silver disc diameter 1.5 cm
+ 1 silver disc diameter 2 cm

Total area of 1 earring = 5.6941368 cm^2

Cost of one = 5.6941368 x 20p = 113.88274p

Cost of a pair = (113.88274p x 2) + 10p = £2.38 (to the nearest penny.)

Earring B is made up of
1 silver disc diameter 4 cm
– 1 silver disc diameter 1.5 cm

Cost of a pair = £4.42

Earring C is made up of
$\frac{1}{2}$ of a silver disc diameter 2 cm
+ $\frac{1}{2}$ of a silver disc diameter 4 cm

Cost of a pair = £3.24

Earring D is made up of
$\frac{7}{36}$ of a silver disc diameter 5 cm
+ $\frac{2}{36}$ of a copper disc diameter 5 cm

Cost of a pair = £1.65

Earring E is made up of
$\frac{1}{2}$ of a silver disc diameter 4 cm
+ $\frac{1}{2}$ of a copper disc diameter 4 cm

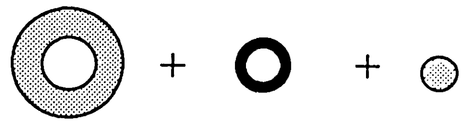
Cost of a pair = £2.74

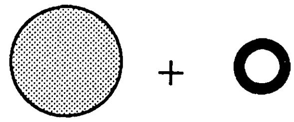
You may have thought of sticking some pieces of copper on top of a disc of silver, so your answers may be different.

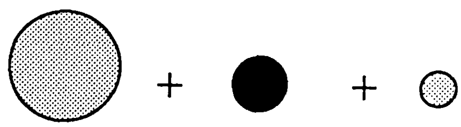
continued/

2032 D.I.Y. Earrings (cont)

Earring F can be made in various ways. According to which way you used, you will get one of these answers:

Cost of a pair = £4.60 

or £5.15 

or £5.90 

Cost of a pair of earrings C	=	£3.24
120% profit	=	£3.89
Price charged	=	£7.13

2033 Is it true?

Have you collected any information/data?

Have you used this information/data to draw a graph or chart (example: pie chart, frequency graph, pictogram, scatter diagram, bar chart,...)?

Try to convince someone in your class that your statement is true/false or neither.

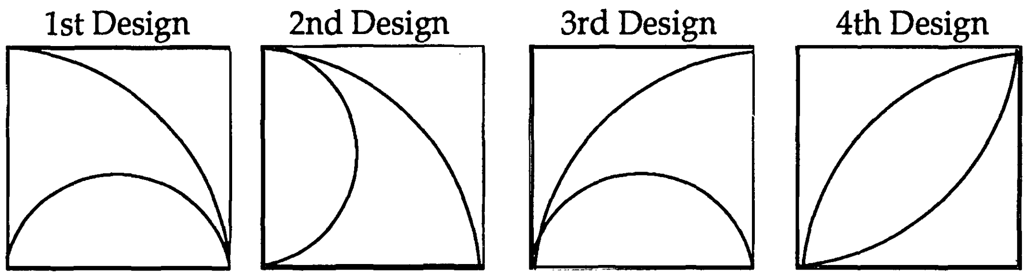
2034 Likely or Unlikely?

Many possible answers although it is **certain** that you will be older than you are today and it is **impossible** that you will grow a banana on an apple tree.

2035 Symmetry Codes

1. Code: 4 4
2. Code: 4 2
3. Code: 4 1
4. Code: 8 1
5. Code: 4 0
6. Code: 6 1
7. Ask someone to find the codes of your shapes. Do your answers agree?

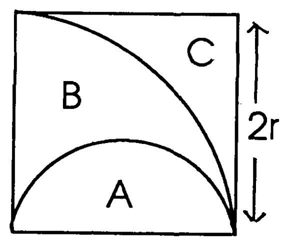
2036 Fabric Design



In order to compare whether more black or white dye is used, it is necessary to compare the area.

For each design there are many ways of isolating parts of the design. The working out and the answers given below are just one way of arriving at the solution. You may have used a different method, but still arrived at the same solution.

When dealing with complex area shapes, it is often useful to verbalise how you perceive the design has been created.



For the first design we have isolated this square.

This square is repeated throughout the fabric design.

The square is divided into 3 areas A, B and C.
For easy calculation we have made the length of the sides of the square equal to 2r.

$$\text{Area A} + \text{Area B} + \text{Area C} = 4r^2$$

Area A (black) is a semicircle. $\text{Area A} = \frac{\pi r^2}{2}$

Area C (black) is (area of square – quarter of a large circle)

$$\text{Area C} = 4r^2 - \frac{1}{4}(4\pi r^2)$$

$$= 4r^2 - \pi r^2$$

$$\text{Total area of black is A + C} = \frac{\pi r^2}{2} + 4r^2 - \pi r^2$$

$$= 4r^2 - \frac{\pi r^2}{2}$$

continued/

2036 Fabric Design (cont).

Area of white can be found by taking away the area of black from the total area of the square.

Area B $= 4r^2 - \left(4r^2 - \frac{\pi r^2}{2}\right)$

Area B $= 4r^2 - 4r^2 + \frac{\pi r^2}{2}$

Total area of white is the Area B = $\frac{\pi r^2}{2}$

By giving a value to **r** you can then get a value for the areas.

If r = 1 then

Area of white = 1.57 square units

Area of black = 2.43 square units

The ratio of dye required for the first design is:

Black dye	:	White dye
1	:	0.646

The design of the 2nd and 3rd are based on the same square as the first design.

In the 2nd design exactly $\frac{1}{2}$ black $\frac{1}{2}$ white.

Black dye	:	White dye
1	:	1

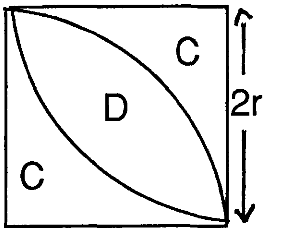
In the 3rd design the positions of black and white are reversed.

White dye	:	Black dye
1	:	0.646

continued/

2036 Fabric Design (cont).

In the 4th design we worked upon this square.



Length of side of the square equal to 2r.

Area C + Area D + Area C = $4r^2$

Area C (black) and Area C (black) are the same shape and area.
It has the same area as C in the first design.

2 x Area C	=	$2 \times (4r^2 - \pi r^2)$
	=	$8r^2 - 2\pi r^2$
Area D	=	Area of whole square – (2 x Area C)
Area D	=	$4r^2 - (8r^2 - 2\pi r^2)$
	=	$4r^2 - 8r^2 + 2\pi r^2$
	=	$-4r^2 + 2\pi r^2$

By giving a value of **r** you can then get a value for the areas.

If r = 1 then
Area of black = 1.72 square units

Area of white = 2.28 square units

The ratio of dye required for the 4th design is:

Black dye	:	White dye
1	:	1.33

2037 3 in 1 Maze

There are three ways to get out of the maze by following:-

a) all the shapes with a smiling face,

b) follow the shapes that are coloured blue, yellow, blue, yellow, blue, . . . (blue and yellow making green!),

c) the shapes which have sides of **3**, **3**, 4, **3**, 4, 5, **3**, 4, 5, 6, **3**.

continued/

2037 3 in 1 Maze (cont)

Can you find other ways?
Using the worksheet may help.

2038 Percentage Problems

1. Original price in November = £400.

Original price in November	=	£400.		
15% of £400	=	£400 x $\frac{15}{100}$	=	£60
New price in December	=	£400 + £60	=	£460
January Sale Price	=	£460 – (15% of £460)		
	=	£460 – £69	=	£391

The 15% reduction applies to the increase **and** the original amount.

2. 8.75% of the wood consumed each year is used by rich countries for paper.

3. Height of normal sized can is 110 mm.

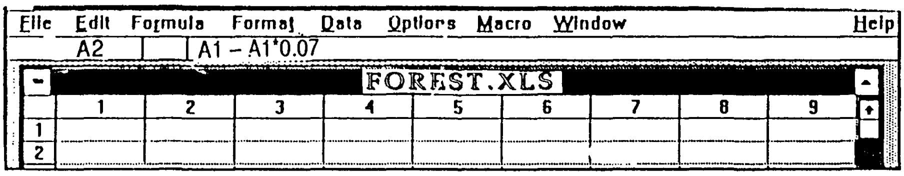
4. The following will show you how to use the spreadsheet Excel to solve the problem.

Excel Help

What to do	How to do it
1. Enter 100 into the first cell (A1)	Type [**100**] [↵]
2. Enter a formula to calculate (100 – 7% of 100)	Type [=] to enter a formula Click on A1 Type [–] Click on A1 Type [*** 0.07**]

continued/

2038 Percentage Problems (cont)

This will appear on the top part of the screen. 	
3. Copy this formula down the spread-sheet.	Click on A2 Edit menu to Copy Highlight 100 cells. Edit menu to Paste.

Here is part of the spreadsheet, showing:-

	A	B
1	100	Now
2	93	In 1 Year
3	86.49	
4	80.4357	
5	74.805201	
6	69.56883693	In 5 Years
7	64.69901834	
8	60.17008706	
9	55.95818097	
10	52.0411083	
11	48.398230	
12	45.	
13	4	
14		

in 5 years time 69.57% is left

.

.

.

in 10 years time 48.40% is left*

.

.

.

in 50 years time 2.66% is left

.

.

.

in 100 years time 0.07% is left.

* so how much has disappeared?

5. Maximum number of illegally parked cars is 500 000.

2039 Finding Equivalent Fractions

1. $\frac{4}{10}$ = 4 ÷ 10 = 0.4

$\frac{6}{15}$ = 6 ÷ 15 = 0.4

continued/

2039 Finding Equivalent Fractions (cont).

Some possibilities are $\frac{8}{20}$ $\frac{10}{25}$ $\frac{12}{30}$ $\frac{14}{35}$ etc

'Equivalent' means 'same as' or 'equal to'.

2. a) $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \ldots = 0.5$

b) $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \ldots = 0.75$

c) $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \ldots = 0.6666\ldots = 0.\dot{6}$

$0.\dot{6}$ is a shorthand notation for 0.6666...

3. Many possible answers. Get someone else to check yours.

2040 x^y Experiment

The calculator displays the number 8.
Many possible answers for other values of x and y including

[4] [x^y] [3]	=	4^3	=	4 x 4 x 4		= 64
[3] [x^y] [3]	=	3^3	=	3 x 3 x 3		= 27
[3] [x^y] [4]	=	3^4	=	3 x 3 x 3 x 3		= 81
[4] [x^y] [-2]	=	4^{-2}	=	$\frac{1}{4 \times 4}$		= 0.0625

The above examples show that:

1. $x^y = \underbrace{x \times x \times x \times \ldots \times x \times x}_{y \text{ times}}$

2. $y^x = \underbrace{y \times y \times y \times \ldots \times y \times y \times y}_{x \text{ times}}$

continued/

2040 x^y Experiment (cont)

3. You might like to try other values for x and y including negative numbers, decimals and zero.

2041 Going Scientific

1. At some stage the following should appear:

0.0002 x 0.0003	= 6 –08
0.002 x 0.003	= 0.000006
0.02 x 0.03	= 0.0006
0.2 x 0.3	= 0.06
2 x 3	= 6
20 x 30	= 600
200 x 300	= 60000
2000 x 3000	= 6000000
20000 x 30000	= 6 08

Your calculator might write 6 08 as 6^{08} or 6. 08 or 6.E08

Calculators record very large and very small numbers using standard form.

so 6 08 on your calculator means

$6 \times 10^8 = 6 \times 100\,000\,000$

$= 600\,000\,000$

and 6 –08 means

$6 \times 10^{-8} = 6 \times 0.00000001$

$= 0.00000006$

This is called STANDARD FORM because it uses a fixed standard for writing numbers:

$$a \times 10^n$$

where $1 \leq a < 10$ and n is an integer.

2. Check that you understand your calculator display for the three sequences.

3. $700000 \times 2900000 = 2.03 \times 10^{12}$

Writing this whole question in standard form:

$(7 \times 10^5) \times (2.9 \times 10^6) = 20.3 \times 10^{11}$

$= 2.03 \times 10^{12}$

2042 Ans and Exe

0, 1, 2, 3, 4, . . .
3, 7, 11, 15, . . .

The same number is added to the previous number.

0, –1, –2, –3, . . .
1, –1, –3, –5, . . .

The same number is subtracted from the previous number.

The sequences will be of the form:

a, a + b, a + b + b or a + 2b, a + 3b, . . . a + nb, where $0 \leq n$

and

a, a – b, a – 2b, a – 3b, . . . a – nb, where $0 \leq n$

1, 2, 4, 8, 16, . . . the powers of 2
1, 3, 9, 27, 81, . . . the powers of 3

Here is an example: if a = 3 and b = 2,

3, –1, 3, –1, 3, –1, . . .

1	EXE	ANS	+	2	EXE	EXE	EXE
4	EXE	ANS	–	2	EXE	EXE	EXE
1	EXE	11	ANS	EXE	EXE	EXE	
1	EXE	1	–	ANS	EXE	EXE	EXE

2043 Unit Fraction Patterns

You could check your answers by using equivalent fractions.

For example, to calculate $\frac{1}{7} + \frac{1}{2}$

continued/

2043 Unit Fraction Patterns (cont)

$$\frac{1}{7} = \boxed{\frac{2}{14}} = \frac{3}{21} = \frac{4}{28} = \ . \quad . \quad .$$

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \boxed{\frac{7}{14}} = \frac{8}{16} = \ . \ . \ .$$

so $\frac{1}{7} + \frac{1}{2} = \frac{2}{14} + \frac{7}{14} = \frac{9}{14}$

Similarly, if

$$\frac{1}{5} = \frac{2}{10} = \frac{3}{15} = \frac{4}{20} = \ldots$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \ldots$$

Can you see why $\frac{1}{5} + \frac{1}{3} = \frac{8}{15}$?

You may have spotted this rule.

$$\frac{1}{7} + \frac{1}{2} = \frac{9}{14} = \frac{7 + 2}{7 \times 2}$$

So, for $\frac{1}{5} + \frac{1}{6}$

$$\frac{1}{5} + \frac{1}{6} = \frac{5 + 6}{5 \times 6} = \frac{11}{30}$$

Non-unit Fractions

The rule no longer works.

To calculate $\frac{3}{7} + \frac{2}{5}$

$$\frac{3}{7} + \frac{2}{5} = \frac{(3 \times 5) + (7 \times 2)}{7 \times 5} = \frac{15 + 14}{35} = \frac{29}{35}$$

This method works for **any** pair of fractions.

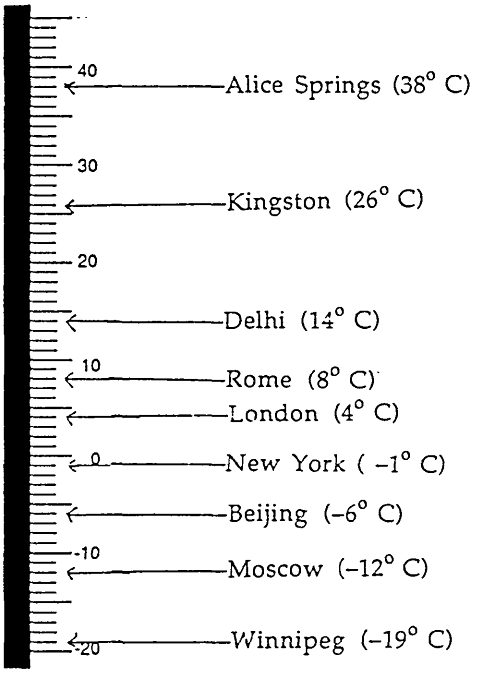
2044 Matching Graphs

1. $y = 5 - 2x$
2. $y = 2x^2 + 3x - 4$
3. $y = \frac{2 - x}{5}$
4. $y = x^3 - 3x^2 + 2$
5. $y = \frac{x}{3} + 5$
6. $y = -x^3$
7. $y = 4x - 2$
8. $y = -4x^2 + 6$
9. $y = 5x - x^2$
10. $y = x^3$
11. $y = x^3 - x$
12. $y = \frac{x^2}{2}$

2045 Hot and Cold

A = 42°, B = 34°, C = 28°, D = 14°, E = 9°, F = 3°, G = −2°, H = −6°, I = −12°, J = −18°.

1.



2. You could have given several different answers. It was probably a month between November and March.

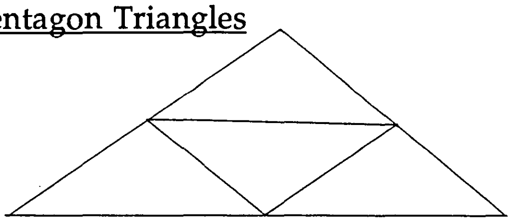
3. Moscow

4. 5° C

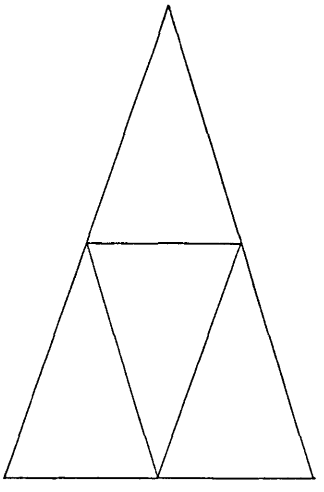
5. −10° C is colder.

2046 Enlarging Pentagon Triangles

4 triangles used.

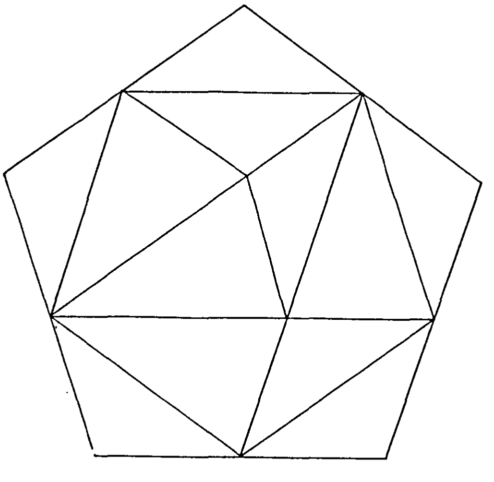


4 triangles used.



length of sides	number of triangles
2	4
3	9
4	16
.	.
.	.
.	.

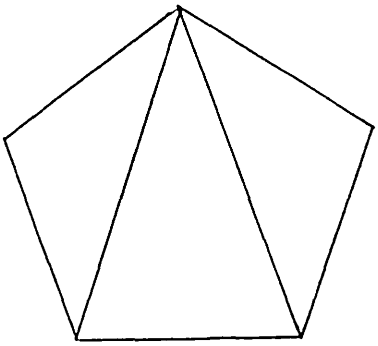
There is more than 1 way to make a pentagon which has sides twice as long.
This is one way.



continued/

2046 Enlarging Pentagon Triangles

Which ever way you found you would need 12 triangles.



x 4

2047 Pegs in Squares

1.

$$1 + 3 + 5 + 7 + 9 = 25 = 5^2$$
$$1 + 3 + 5 + 7 + 9 + 11 = 36 = 6^2$$
$$1 + 3 + 5 + 7 + 9 + 11 + 13 = 49 = 7^2 \ldots$$

2.

$$4 + 12 + 20 + 28 = 64 = 8^2$$
$$4 + 12 + 20 + 28 + 36 = 100 = 10^2$$
$$4 + 12 + 20 + 28 + 36 + 44 = 144 = 12^2 \ldots$$

3.

$$1 + 8 + 16 + 24 = 49 = 7^2$$
$$1 + 8 + 16 + 24 + 32 = 81 = 9^2$$
$$1 + 8 + 16 + 24 + 32 + 40 = 121 = 11^2 \ldots$$

4.

$$1 + 2 + 3 + 4 + 3 + 2 + 1 = 16 = 4^2$$
$$1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25 = 5^2$$
$$1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 = 36 = 6^2 \ldots$$

2048 A Growing Concern

No answers required.

2049 Unpredictable patterns?

By looking at the pattern it would be reasonable to expect a doubling pattern to continue. This works for 5 points, but the pattern breaks down for 6 points. It is impossible to find 32 regions with 6 points; 31 regions is the maximum.

Further terms can be predicted using differences

	1		2		4		8		16		31		57 ...
1st Difference		1		2		4		8		15		26	...
2nd Difference			1		2		4		7		11	...	
3rd Difference				1		2		3		4	...		
4th Difference					1		1		1	...			

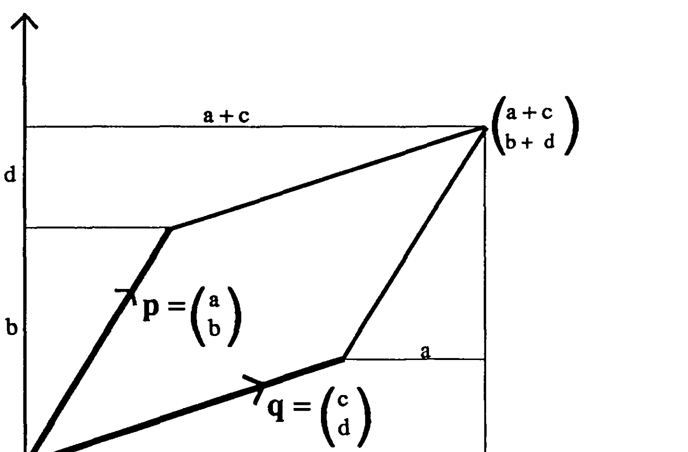
You may find a spreadsheet helpful.

2050 Vector Areas

1. a) Area of parallelogram = 12 square units.
   b) Area of parallelogram = 15 square units.

2. a) Area of parallelogram = 7 square units.
   b) Area of parallelogram = 11 square units.

3. The area of any parallelogram can be found by subtracting the areas of the two right-angled triangles and the two trapezia from the large rectangle.



The area of the parallelogram formed by vectors **p** and **q**, where $\mathbf{p} = \begin{pmatrix} a \\ b \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} c \\ d \end{pmatrix}$ is $bc - ad$.
(If you used the vectors $\begin{pmatrix} c \\ d \end{pmatrix}$ then $\begin{pmatrix} a \\ b \end{pmatrix}$ then the area = $ad - bc$.)

The area must always be positive, so the modulus of $bc - ad$ is taken.
This is written as $|bc - ad|$.

2051 The Log Button

1.

Number	Log
10	1
100	2
1000	3
10000	4
.	.
.	.
.	.

Number	Log
5	0.69897
50	1.69897
500	2.69897
.	.
.	.
.	.

(The results from your calculator may differ slightly.)

The log of a number is made up of two parts.
The part of the number to the left of the decimal point indicates the power of ten.
The part of the number to the right of the point is known as the mantissa.

2.

Number	Log
1	0
2	0.30103
3	0.4771213
4	0.60206
5	0.69897
6	0.7781513
7	0.845098
8	0.90309
9	0.9542425
10	1

a) 2.60206
b) 3.845098
c) 1.9542425
log 750 is somewhere between 2.845098 and 2.90309.

3. $\log x + \log y = \log (xy)$

$\log x - \log y = \log \frac{x}{y}$

$\log (x^n) = n \log x$

4. These show one way of arriving at each of the answers.

a) $\log 750 = \log (30 \times 25)$
$= \log 30 + \log 25$
$= \log 30 + \log 5^2$
$= \log 30 + 2 \log 5$
$= 2.8750613$

b) $\log 35 = \log \frac{70}{2}$
$= 1.544068$

continued/

2051 The Log Button (cont)

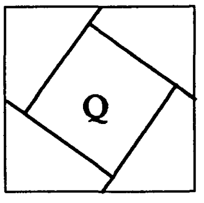
c) $\log 144 = \log (2 \times 6)^2$
$= 2.158362$

You may have shown alternative methods.

The log 375 =2.5740313 (You should have a method written down.)
Discuss your method with your teacher.

2052 Pythagoras Dissection

This diagram shows how square Q and the four pieces of square P will fit into square R.



The ratio of the dissection is important. Not all dissections work.
The ratio is dependent upon the average of the two smaller sides of the right-angled triangle.

2053 Odd Add

There are 6 ways of making 14 with 4 odd numbers.

1 + 1 + 1 + 11 = 14
1 + 1 + 3 + 9 = 14
1 + 1 + 5 + 7 = 14
1 + 3 + 3 + 7 = 14
1 + 3 + 5 + 5 = 14
3 + 3 + 3 + 5 = 14

There are 4 ways of making 14 with 2 odd numbers, did you find them all?

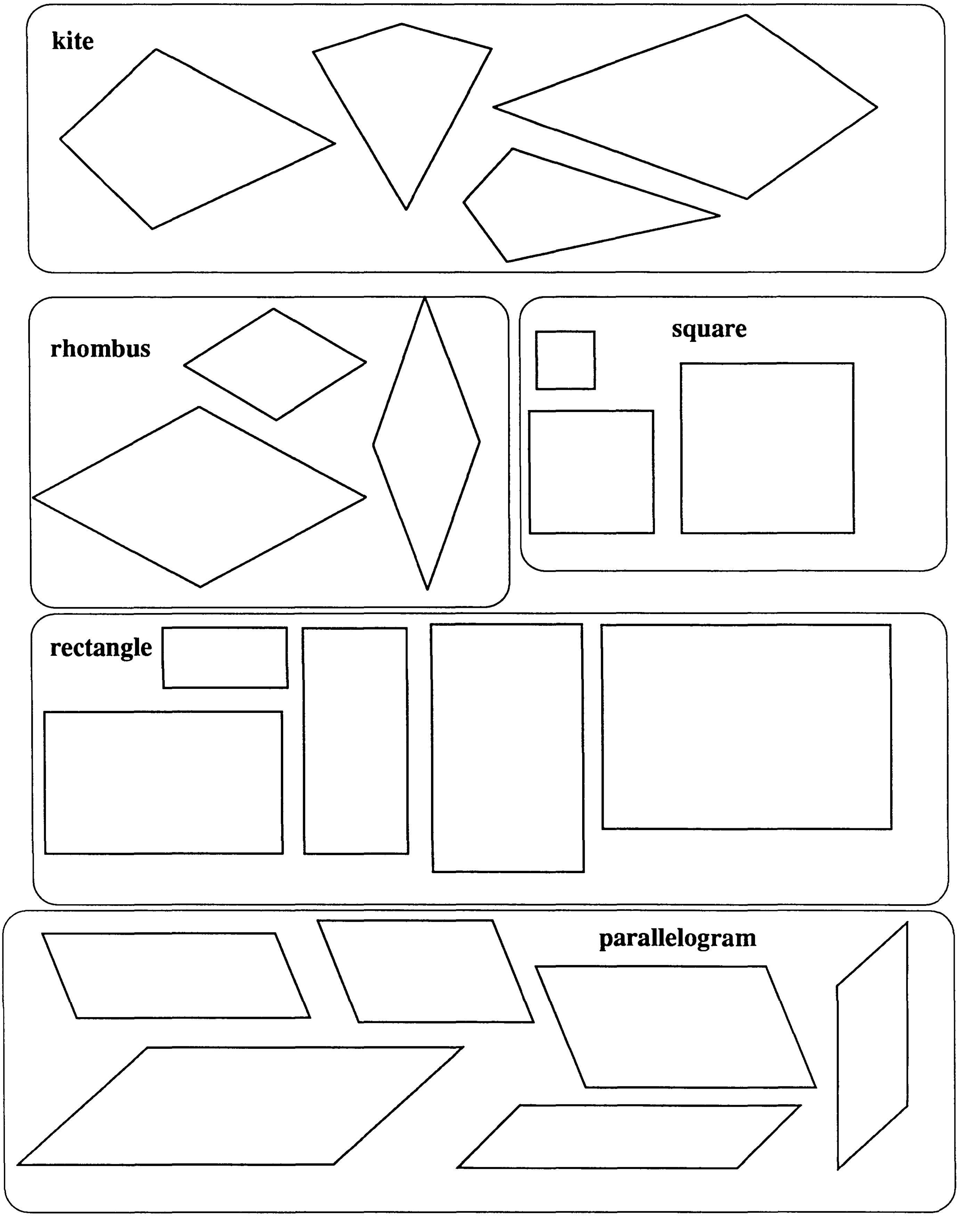
It is impossible to make 14 with 3 odd numbers because 3 odd numbers added together will always make an odd answer.

You might like to record your results using a mapping diagram.

Number of odd numbers		Number of ways of making 14
2	⟶	4
3	⟶	0
4	⟶	6
5		.
6		.
.		.
.		.
.		.
14		.

What did you find for other totals?

2054 4 sides



2055 Ellipses by folding

The nearer the dot is to the centre of the circle, the closer your ellipse will be to a circle.

The nearer the dot is to the circumference, the narrower your ellipse will be.

What do you notice about the longest line of symmetry?
What do you notice about the length of the widest part of the ellipse and your starting circle?

2056 Surrounding Right -Angled Triangles

Diagram	Area of P	Area of Q	Area of R
1.	9cm^2	4cm^2	13cm^2
2.	16cm^2	9cm^2	25cm^2
3.	8cm^2	8cm^2	16cm^2
4.	4cm^2	4cm^2	8cm^2
5.	2cm^2	8cm^2	10cm^2

The area of the square on the longest side (square R) is equal to the sum of the squares on the other two sides (squares P and Q).

This relationship is known as the Pythagoras' Theorem. Pythagoras was a Greek scholar born in 570BC who formed a secret society in southern Italy known as the Pythagoreans. They had what now might seem to be strange practices; they wouldn't wear wool, wouldn't touch a white cockerel or poke a fire with a poker.

Although this theorem is known as Pythagoras' Theorem it seems unlikely he discovered it himself, more likely one of the members of the society. Indeed the mathematics used was known in India and China at least 500 years before Pythagoras gave it his name.

You might like to find out more about Pythagoras.

2057 Fan

You may like to make a display of your designs.

2058 Tie

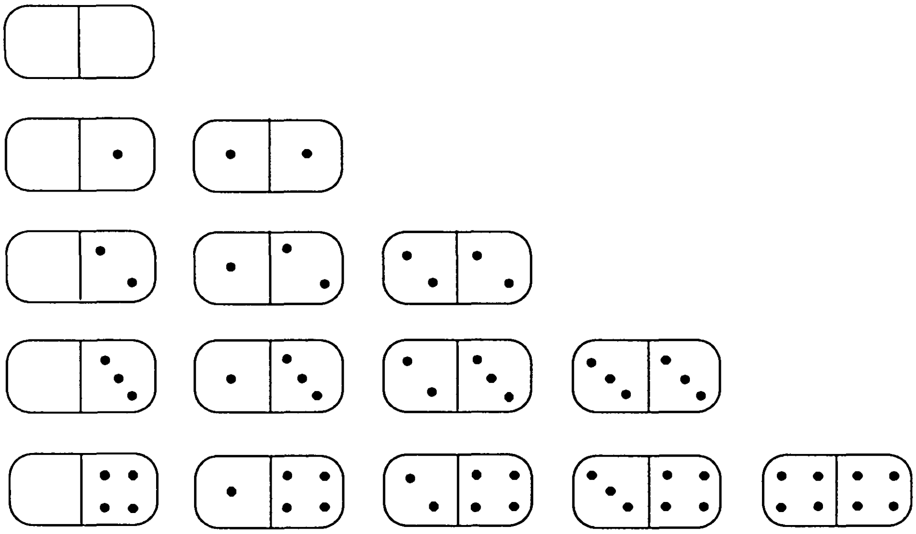
It is a good idea to cut four sets of templates and do the activity by trial and improvement as a group. Remember to keep the lines on the template pieces parallel when placing them on the material.

For a plain tie you would need about 120cm of 1m cloth to make 4 ties.

If you were using patterned material you would need more cloth. How much more material would you need for different types of patterned materials?

2059 Domino Patterns

1. This is a 4 set.



It has 15 dominoes.

2.

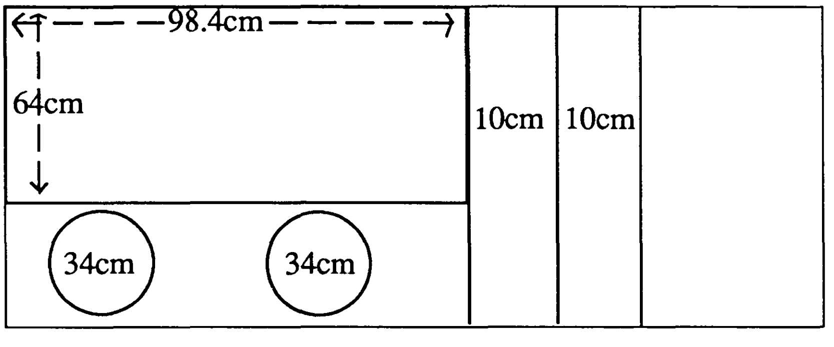
Domino Set		Number of Dominoes
0	→	1
1	→	3
2	→	6
3	→	10
4	→	15
5	→	21
6	→	28
7	→	36
8	→	45

3. There are many ways to describe how to get the number of dominoes in a set. Ask someone to check that your description works for your answers to the number of dominoes in the 10 and 15 sets.

Can you generalise for the number of dominoes in any size set?

2060 Kitbag

Here is one possible solution if you choose to use the same materials for the straps.

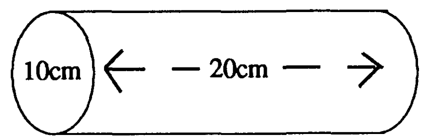


This solution requires 98cm allowing for 3cm wide straps. You may decide to choose an alternative material for the straps or to make them wider.

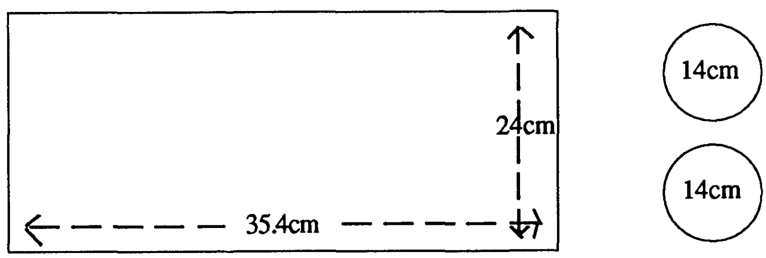
continued/

2060 Kitbag(cont)

Here are the dimensions of the pencil case without straps.



Here is the pattern, without straps.



2061 Convince Yourself

You do not **always** get a larger number when you multiply two numbers together.
You do not **always** get a smaller number when you divide one number by another.

By trying various numbers you should have found out you always get a larger number when you multipy . . .

. . . two positive numbers larger than one $12 \times 5 = 60$
. . . two negative numbers larger than one $-12 \times -5 = 60$
. . . two vulgar fractions (greater than 1) $\frac{3}{2} \times \frac{7}{5} = \frac{21}{10}$

You always get a smaller number when you multiply . . .

. . . two fractions together $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
. . . two decimals together $0.5 \times 0.5 = 0.25$
. . . a negative and a positive number $-2 \times 3 = -6$
. . . a negative and a positive fraction $\frac{-1}{2} \times \frac{1}{5} = \frac{-1}{10}$
. . . a negative and a positive decimal $-0.5 \times 0.2 = -0.01$

What did you decide about . . .

. . . a negative number and a fraction $-12 \times \frac{1}{2} = -6$

. . . a negative number and a decimal $-12 \times 0.5 = -6$

. . . a positive number and a fraction $12 \times \frac{1}{2} = 6$?

What happens if one of the numbers is 0 or 1?

You may decide to put all this information into a table or on a number line.

2062 Angles in Circles

1. Angle B must be 90° (angle at the circumference of a semi-circle).
   $x + 65° + 90° = 180°$ (angles of a triangle add up to 180°)
   $x = 25°$

2. $x = 16°$ This can be argued similarly to no. 1.

3. Angle A must be 90° (angle at the circumference of a semi-circle)
   $x = y$ (triangle is isosceles).
   $x + y + 90° = 180°$
   $x + y = 90°$
   $x = y = 45°$

4. In triangle DEG
   $x + 70° + 40° = 180°$ (angles of a triangle add up to 180°)
   $x = 70°$
   $x = y$ (angles on the same arc)
   $y = 70°$

5. $y = 90°$ (angle at the circumference of a semi-circle).
   $x = 38°$ (angles on the same arc QR)
   $x + y + z = 180°$ (angles of a triangle add up to 180°).
   $38° + 90° + z = 180°$
   $z = 52°$

Make sure you explain your reasoning for the following answers:

6. $x = 52°$ $y = 64°$ $z = 64°$

7. $x = 44°$ $y = 44°$ $z = 46°$

8. $x = 55°$

9. $x = 84°$ $y = 48°$ $z = 42°$

10. $x = 120°$ $y = 30°$ $z = 60°$

11. $x = 110°$ $y = 110°$ $z = 70°$ $w = 110°$

12. $x = 40°$ $y = 40°$ $z = 50°$

2063 Islamic Designs

You may like to make a display of your designs.

2064 Russian Multiplication

The Russian Multiplication method uses multiplication and division by 2 and addition of the un-crossed numbers at the end.

Here are some hints to help you understand the method.

odd number	49	x	423	
(49 ÷ 2 = 24 r 1)	~~24~~	~~x~~	~~846~~	(423 x 2)
(24 ÷ 2)	~~12~~	~~x~~	~~1692~~	(846 x 2)
(12 ÷ 2)	~~6~~	~~x~~	~~3384~~	(1692 x 2)
odd number	3	x	6768	
odd number	1	x	13536	
			20727	Total of all the numbers which have not been crossed out.

32 x any number gives you a Russian Multiplication where all but one pair of numbers are crossed out. Can you find some other numbers?
It is not possible to find a Russian Multiplication where all numbers are crossed out. Can you say why?

2065 Shrinking Earth

You will need to know the radius of the earth, the height of Mount Everest and the depth of the Marianas trench.

Radius of earth	≈	6350km = 6350000m
Height of Mt. Everest	≈	8848m
Depth of Marianas trench	≈	10843m
Radius of Golf Ball	≈	22mm = 0.022m

There are several ways of doing this, we chose the following way.

The ratio of the height of Mt. Everest to the radius of the earth will be the height of the shrunken Mt. Everest to the radius of the golf ball.

$$\frac{\text{Height Mt. Everest}}{\text{Radius of Earth}} = \frac{\text{Height of shrunken Mt. Everest}}{\text{Radius of Golf Ball}}$$

so
$$\frac{8848}{6350000} = \frac{\text{Height of shrunken Mt. Everest}}{0.022}$$

Therefore, height of shrunken Mt. Everest is approximately 0.03mm (if you used miles and feet you should get the same ratios).

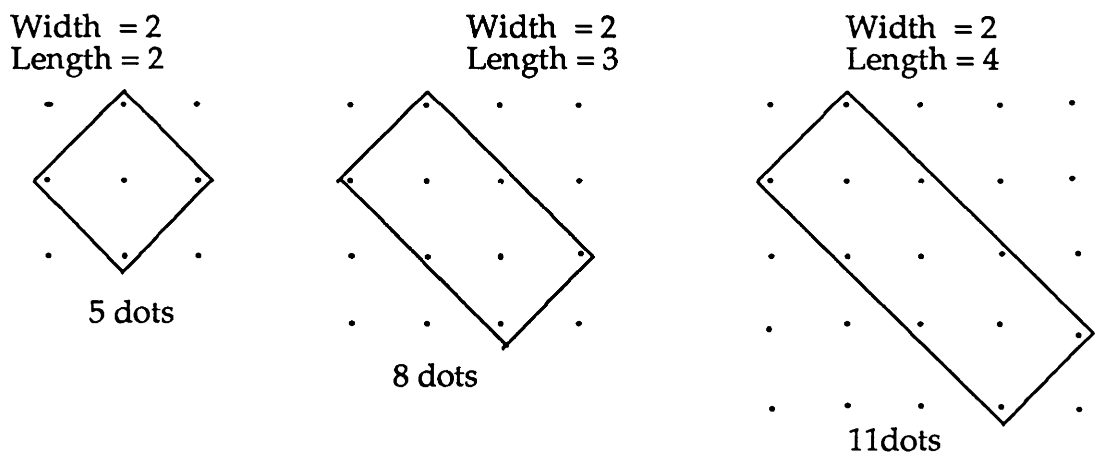
So would you be able to feel it?

Using the same method we worked out that the Marianas trench would be indented on the golf ball by 0.04mm.

So would you be able to feel it?

2066 Dots

Always start with a few simple examples when you investigate.



It is helpful to put your results in a table :

With width = 2 dots

Length	2	3	4	. . . .	n
Dots	5	8	11	. . . .	?

Can you see a pattern?

Now try with width 3 dots, 4 dots, . . .

Can you find a general pattern?

2067 Jeans

1. This spreadsheet was created to find (a) the correct quantity of raw materials needed and (b) to calculate the cost of the raw materials for one pair of jeans.

	A	B	C	D	E (a)	F	G	H (b)
1	**Materials**	**Materials for**	**Materials for**	**Raw mats**	**What to**	**Unit cost of raw**	**Total cost of5000**	**Cost for one**
2		**one pair**	**5000 doz**	**are sold**	**order**	**materials**	**dozen pairs**	**pair (£)**
3	Denim	1.6	96000	100	960	£ 250.00	£ 240,000	£ 4.00
4	Lining	0.2	12000	100	120	£ 106.00	£ 12,720	£ 0.21
5	Thread	230	13800000	5000	2760	£ 5.00	£ 13,800	£ 0.23
6	Labels	2	120000	1000	120	£ 15.00	£ 1,800	£ 0.03
7	Studs	5	300000	1000	300	£ 20.00	£ 6,000	£ 0.10
8	Zips	1	60000	100	600	£ 12.50	£ 7,500	£ 0.13
9	Buttons	1	60000	100	600	£ 3.00	£ 1,800	£ 0.03

2. a) By using the spreadsheet, an increase of 5% in the cost of denim would increase the cost of a pair of jeans by 20p.

   b) An increase in the cost of denim would increase the cost of a pair of jeans most.

2068 Quad

No answers required.

2069 Turn it Over!

The card with the E does not need to be turned over. (You are told the E's are found on the back of numbers bigger than 5. This does not mean you **cannot** find an 'E' where the front of the card is 5 or less!)

The card that is blank needs to be turned over. (If the number on the other side is more than 5, the statement is false.)

The card with the seven needs to be turned over. (If there is no 'E', the statement is false.) Do you need to turn over the card with the 4? Could there be a number more than 5 on the other side?

So the minimum number of cards you must turn over is 2.

2070 Card Towers

You would need 5612 cards for 61 levels. The height would be about 5 metres.
You might have decided to put your results in a table like this for your investigation.

Levels	1	2	3	4	5	...
Cards used	2	7	15	26	40	...

2071 Half a cuboid

Shapes **a, b, e, h, g,** and **j** can make cuboids.

2072 Nepali Numbers

The numbers used in Nepal are written in Hindi number script.

1. This Nepali mathematics lesson is about subtracting 4 over and over again.

   The picture of the children walking in 4's might have given you a clue.

   40 – 4 = 36
   36 – 4 = 32
   32 – 4 = 28
   28 – 4 = 24
   .
   .
   .
   4 – 4 = 0

2. The table for the number 2 will look like this.

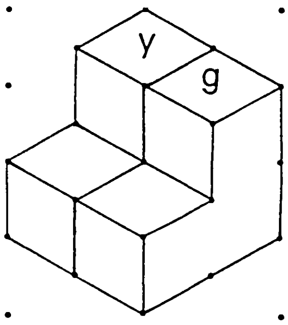
   २० – २ = १८
   १८ – २ = १६
   १६ – २ = १४
   १४ – २ = १२
   १२ – २ = १०
   १० – २ = ८
   ८ – २ = ६
   ६ – २ = ४
   ४ – २ = २
   २ – २ = ०

2073 Tricubes

r = red   y = yellow   b = blue   g = green

Sheets A2

B



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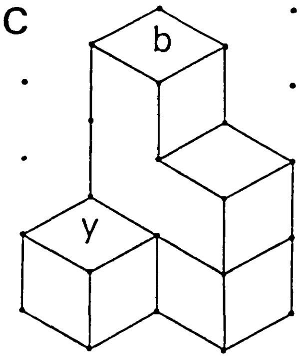

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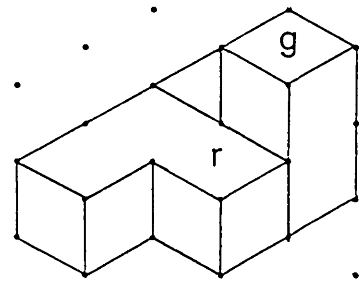
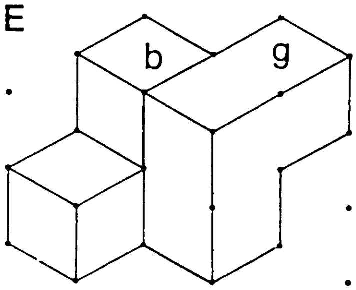


E



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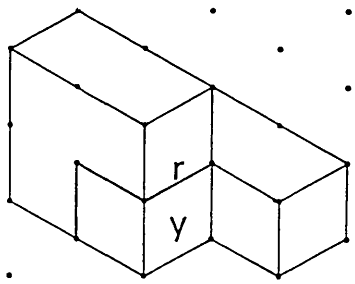

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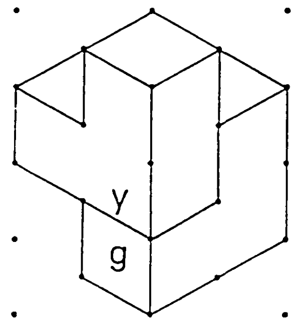

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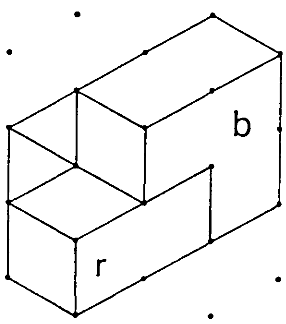
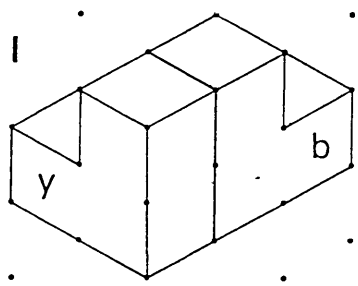
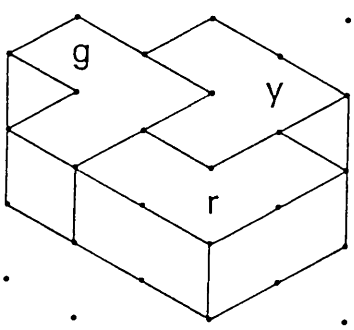


I



Sheet A3

B



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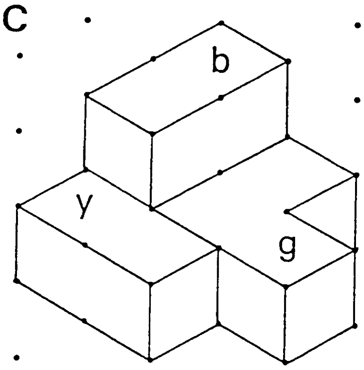

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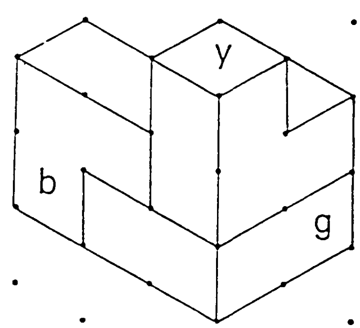

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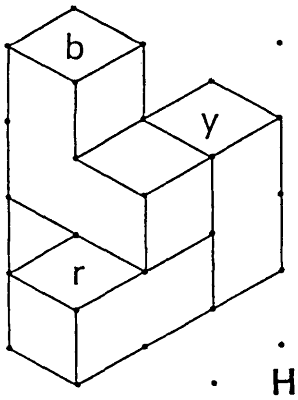
2073 Tricubes (cont)

E



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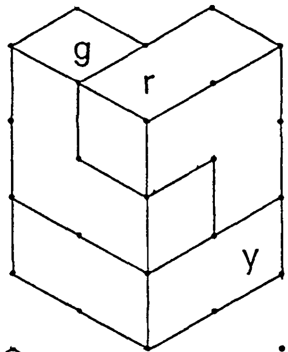

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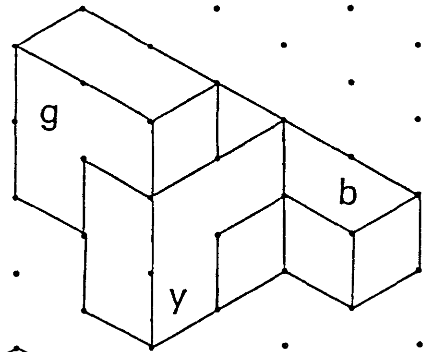

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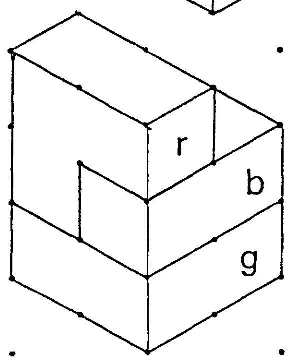

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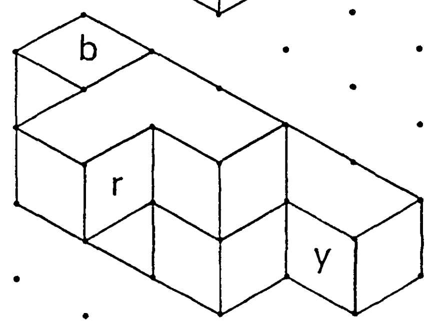
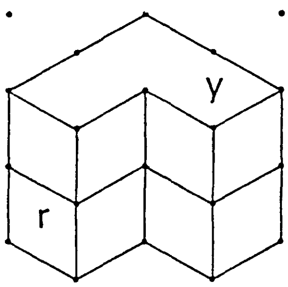


Sheet A4

A



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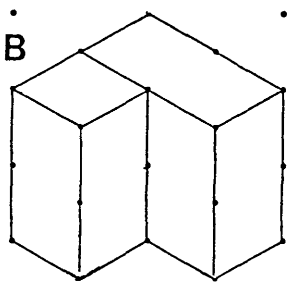

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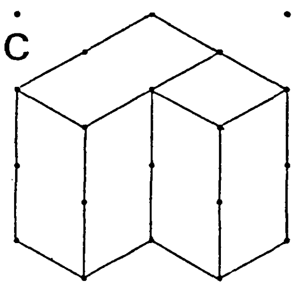
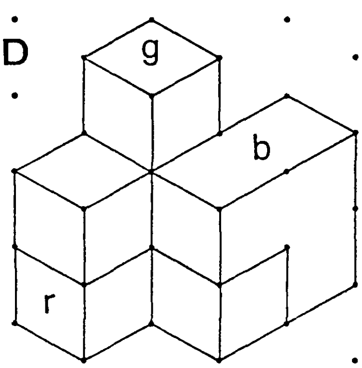
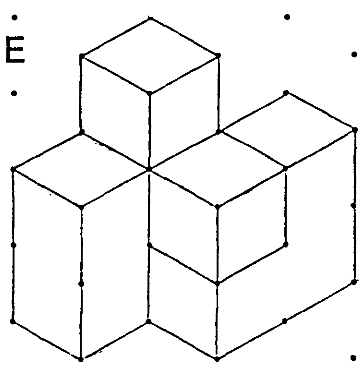


D



E



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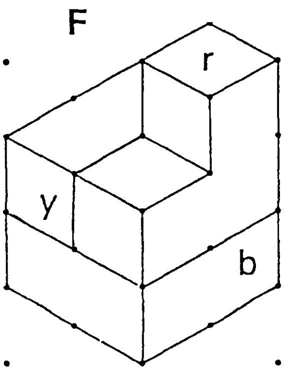

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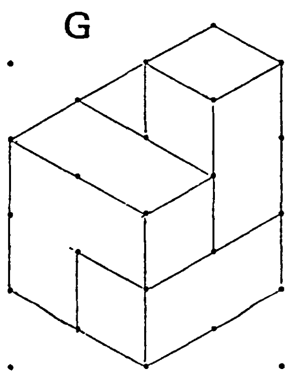

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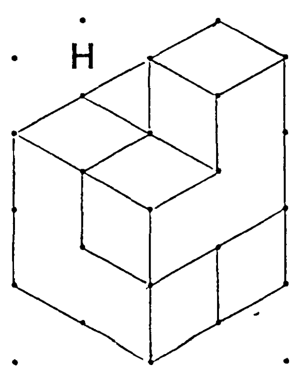

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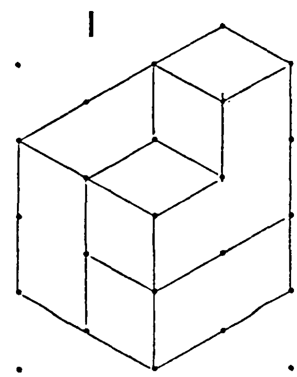
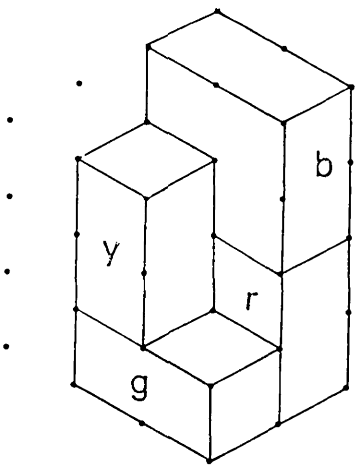


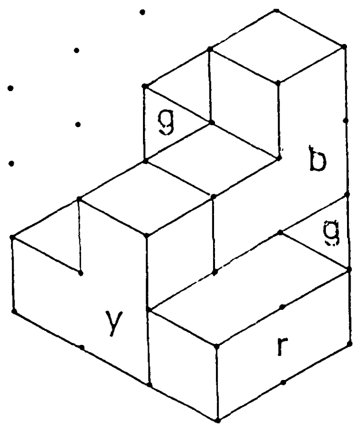
2074 Building Tricubes

r = red y = yellow b = blue g = green

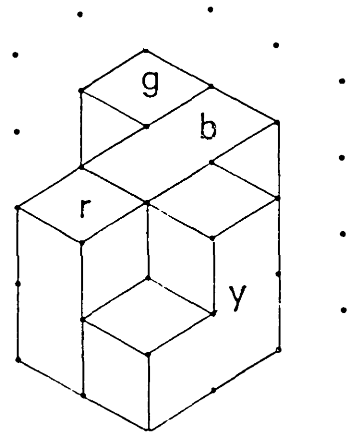
Sheet B2



B6



B10

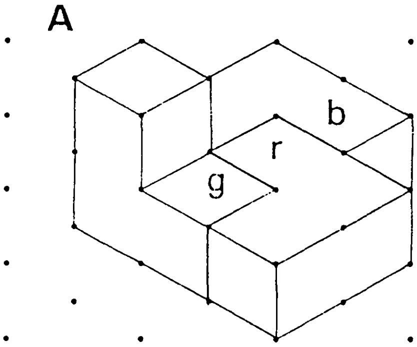


2075 Tricube Plans

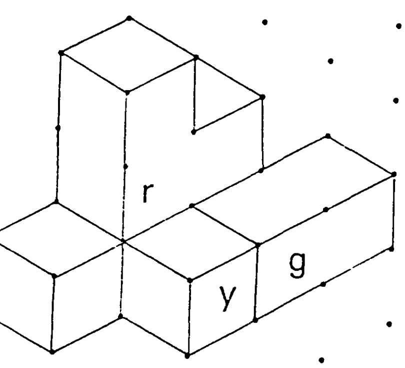
r = red y = yellow b = blue g = green

Sheet C1

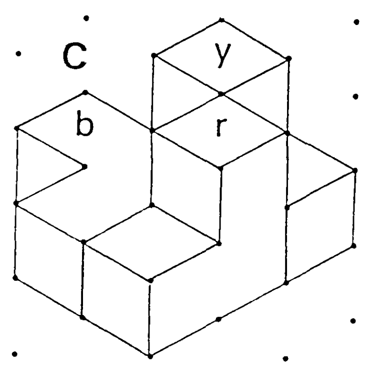
A



B

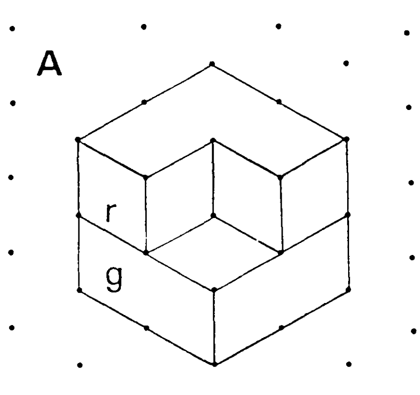


C

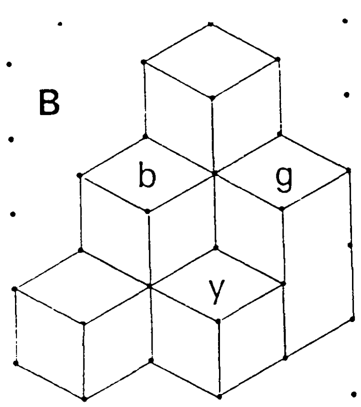


Sheet C5

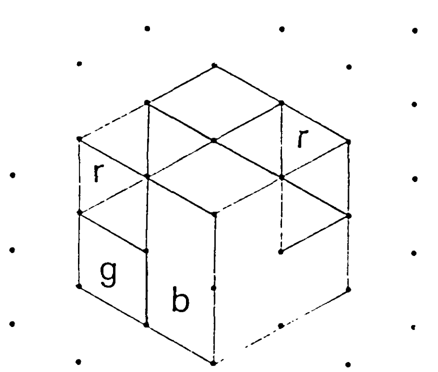
A



B



C



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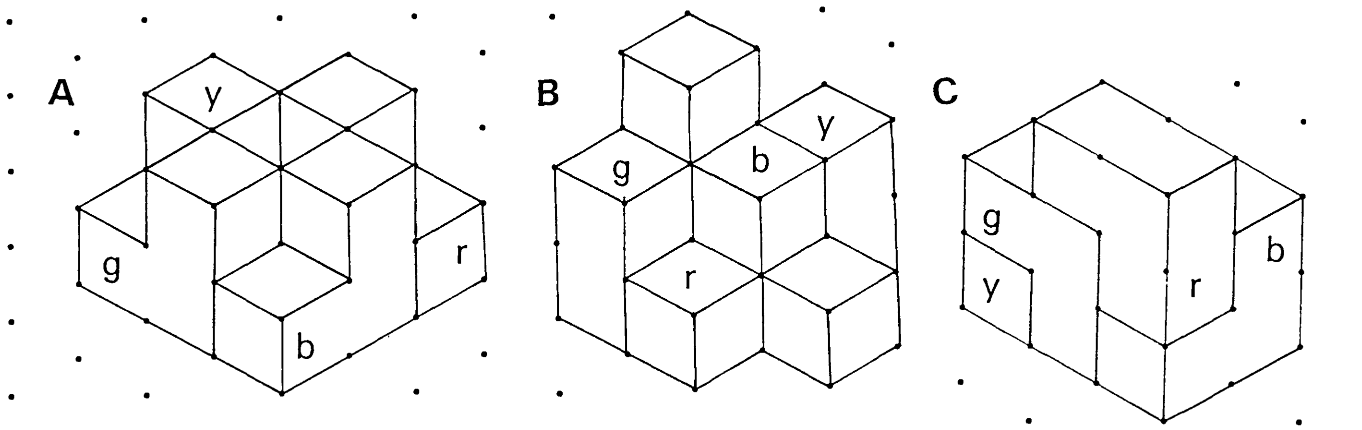
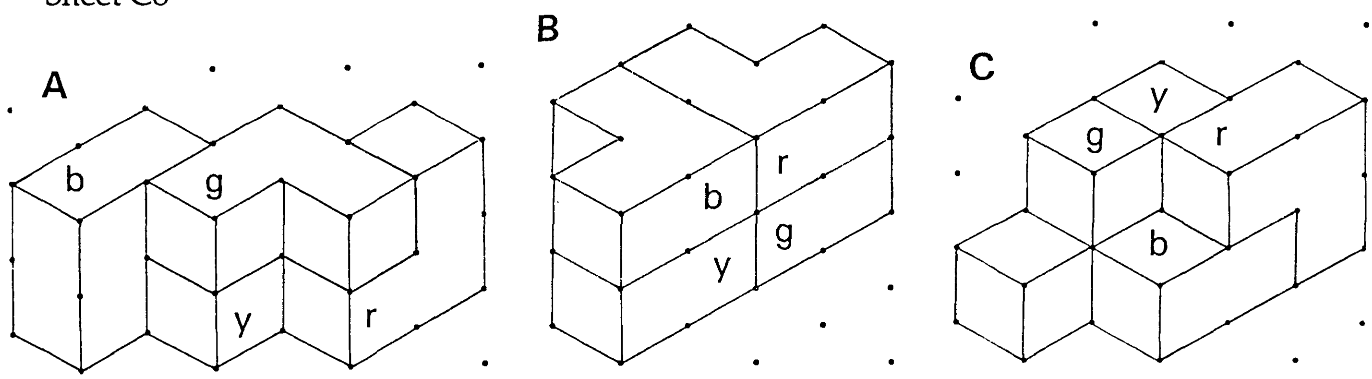
2075 Tricube Plans (cont)

Sheet C6



Sheet C8

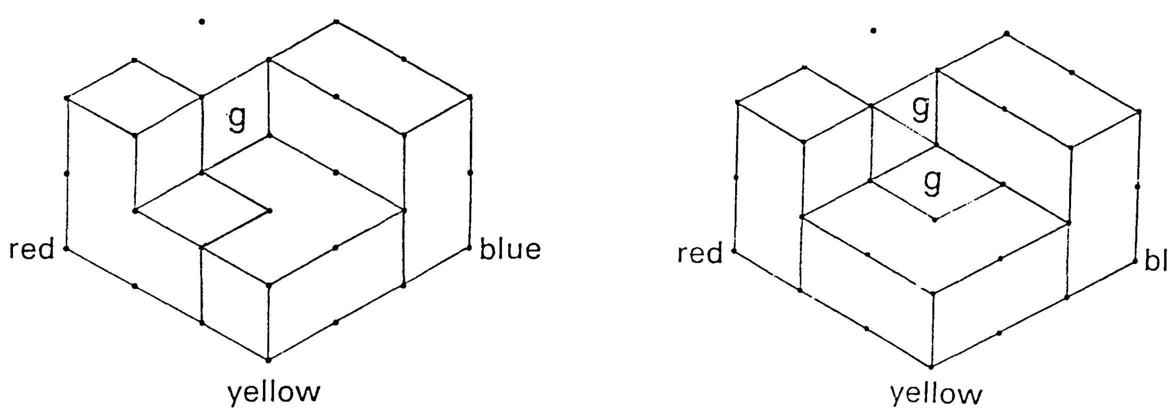


These show one way of fitting the four tricubes to match the coded plan.

2076 Building on a Square

r = red y = yellow b = blue g = green

Sheet D1



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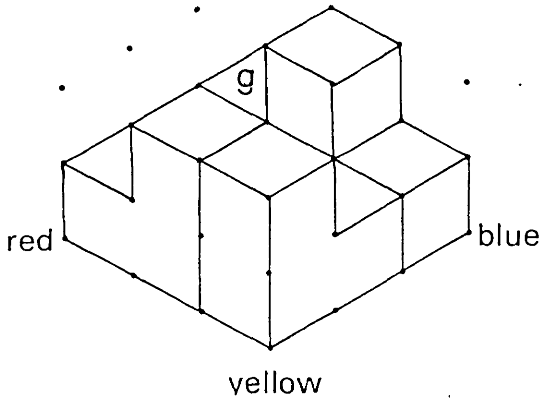
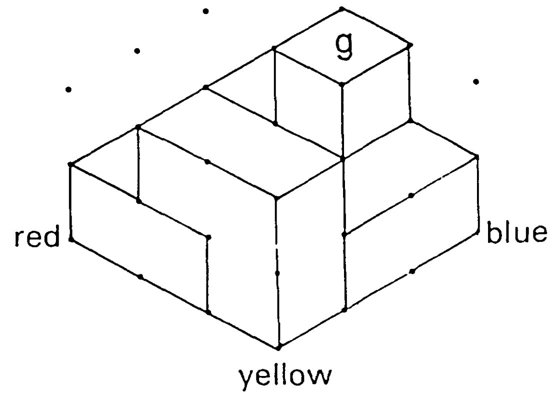
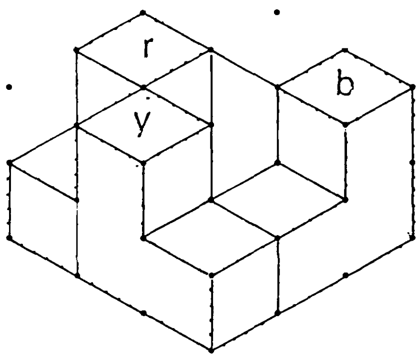
2076 Building on a Square (cont)

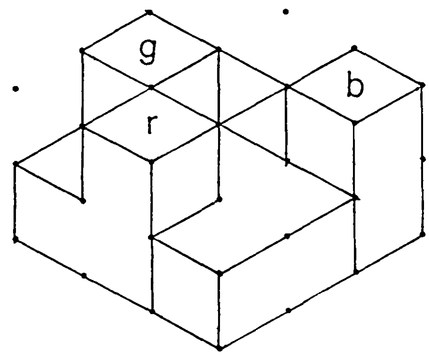
Sheet D5



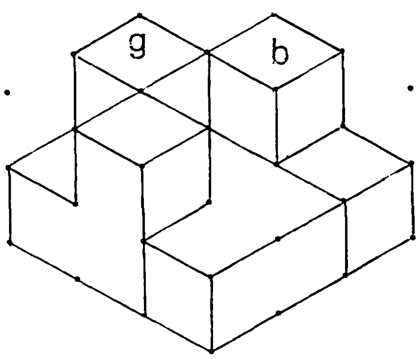


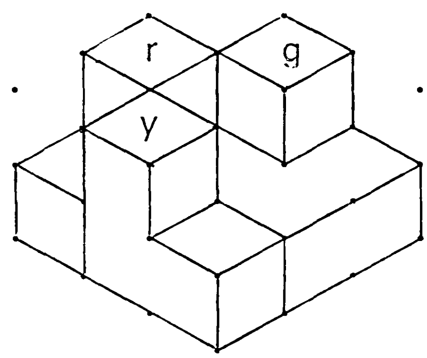
Sheet D8





Sheet D10

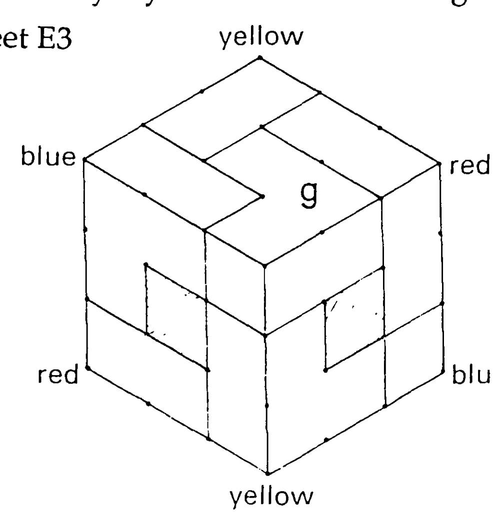


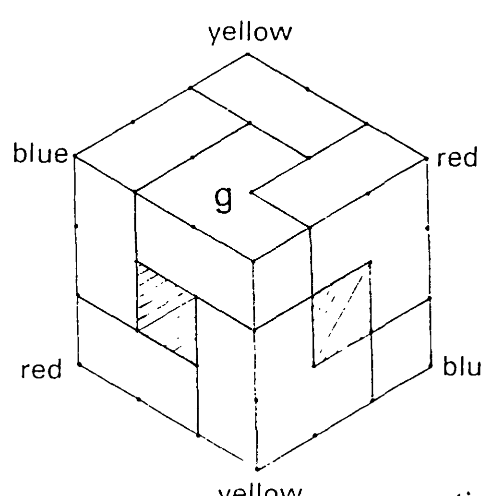


2077 Making a 3 x 3 x 3 cube

r = red y = yellow b = blue g = green

Sheet E3





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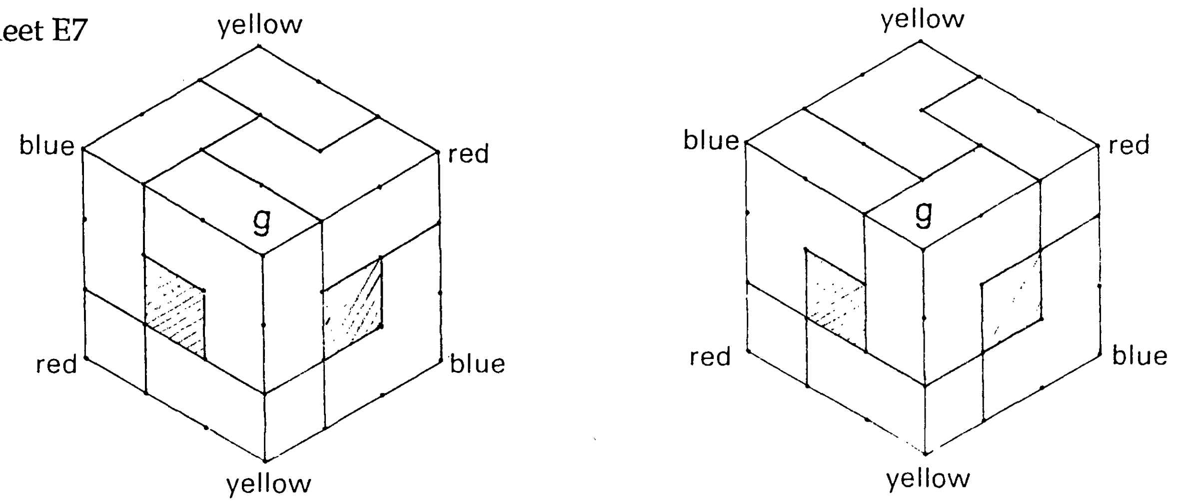
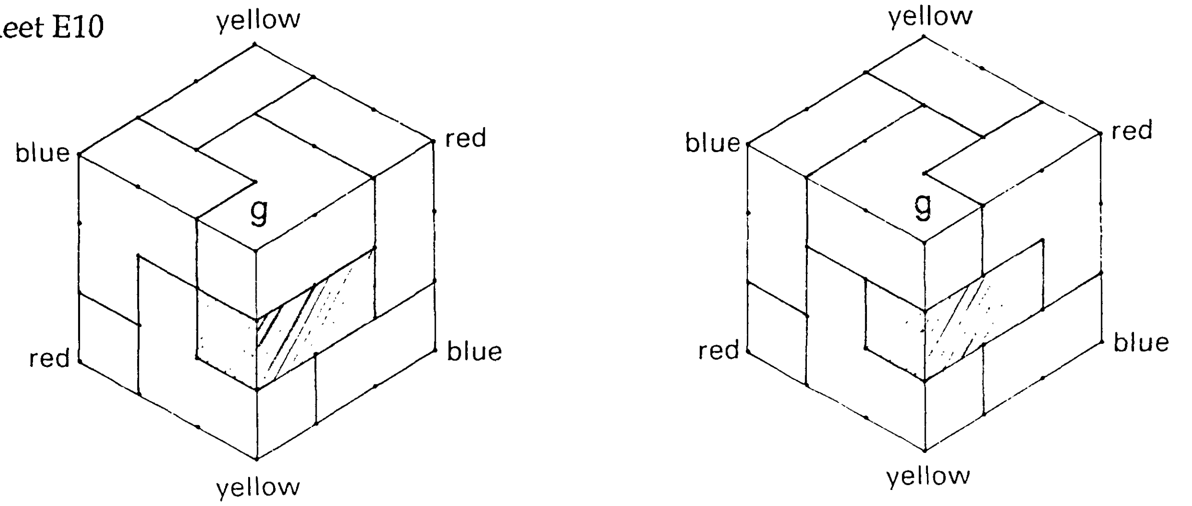
2077 Making a 3 x 3 x 3 cube (cont)

Sheet E7



Sheet E10



2078 Fibonacci-type Sequences

The missing numbers in the Fibonacci - type sequences are :-

12	20	32
22	30	

One way of investigating 'end' numbers is to choose your starting number and look at the different Fibonacci type sequences that can be made.

8,	9,	17,	26,	43,	69,	112 . . .
8,	10,	18,	28,	46,	74,	120 . . .
8,	11,	19,	30,	49,	79,	128 . . .
8,	12 . . .					

* You might like to look at the differences between these sequences.

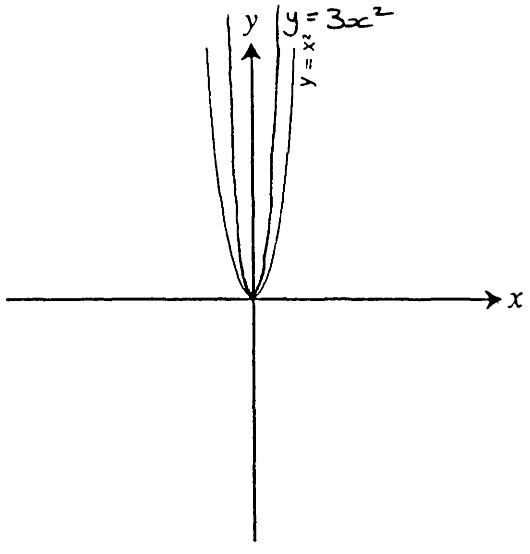
* You might like to look at the link between the first and the last number.

* What happens when a sequence starts off. . .

a,	a,	a + a,	. . .
a,	2a,	a + 2a,	. . .
a,	a+1,	a + a+1,	. . .
a,	a+2,	a + a+2,	. . . ?

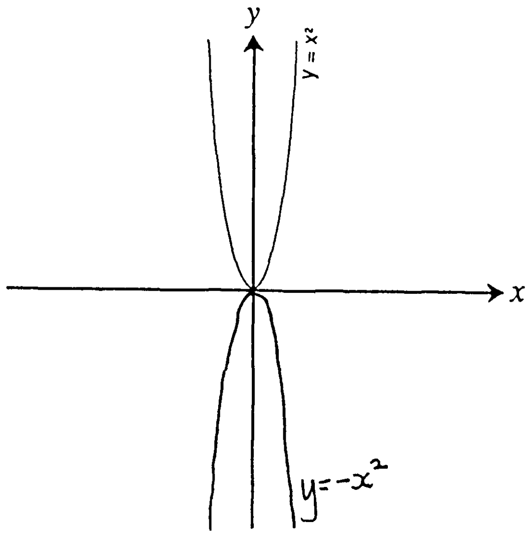
2079 A Sketchy Activity

1. a)



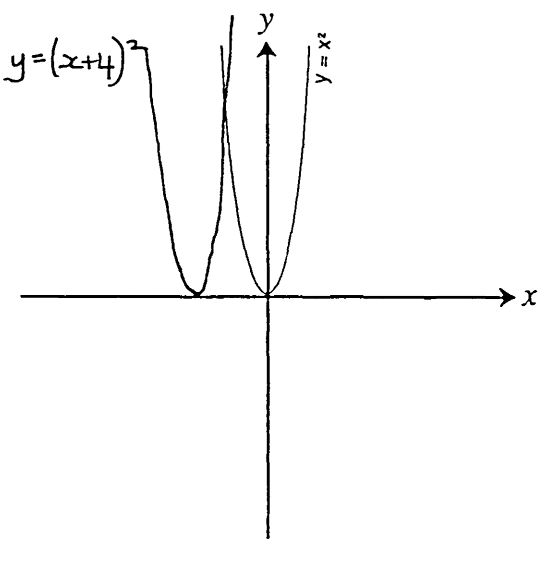
Gradient steeper,
minimum value at (0,0)
y axis is line of symmetry

b)



Gradient is the same,
maximum value at (0,0)
y axis is line of symmetry
identical shape as $y=x^2$ but
reflected in the x axis.

c)

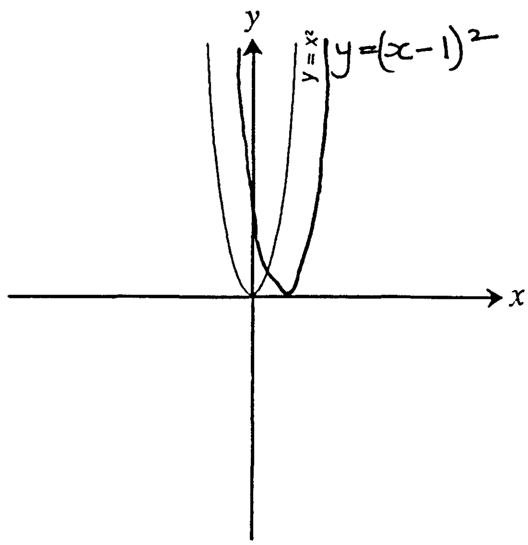


Gradient is the same,
minimum value at (-4,0)
x = -4 is line of symmetry,
identical shape as $y=x^2$ but
translated by -4 along the x axis.

continued/

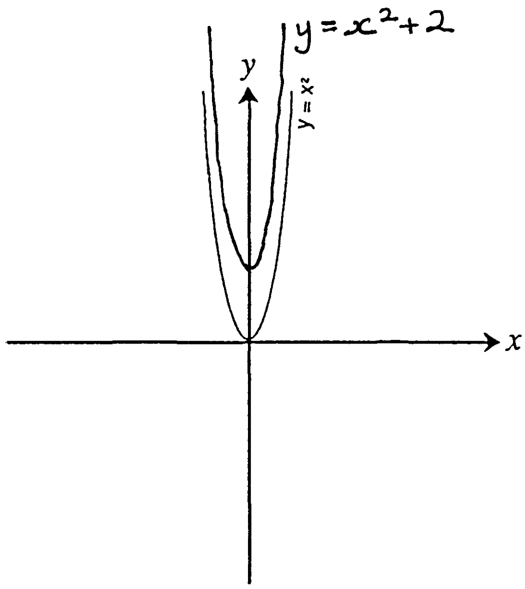
2079 A Sketchy Activity(cont)

d)



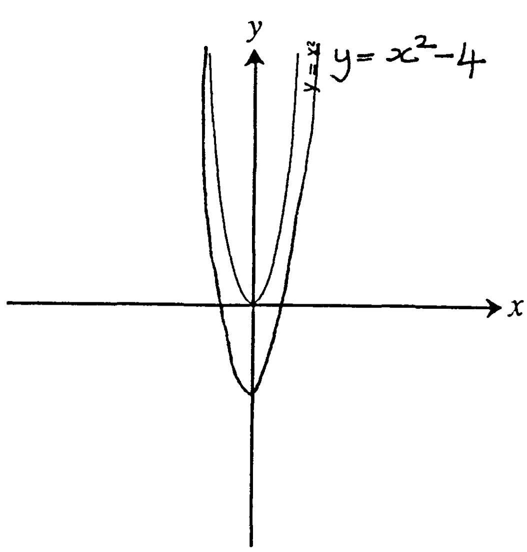
Gradient is the same,
minimum value at (+1,0)
x = +1 is line of symmetry,
identical shape as $y = x^2$ but
translated by +1 along the x axis.

e)



Gradient is the same,
minimum value at (0,2)
y axis is the line of symmetry,
identical shape as $y = x^2$ but
translated by +2 up the y axis.

f)



Gradient is the same,
minimum value at (0,-4)
y axis is the line of symmetry,
identical shape as $y = x^2$ but
translated by -4 down the y axis.

continued/

2079 A Sketchy Activity (cont)

2. All the graphs of the form $y = ax^2$ are symmetrical about the y axis. Different values of 'a' change the gradient for the same values of x. As 'a' increases the gradient increases and as 'a' decreases the gradient decreases.

   All the graphs of the form $y = (x+b)^2$ are the same shape as $y = x^2$ but they touch the x axis at different places. The minimum or maximum value (turning point) will be (-b,0).

   All the graphs of the form $y = x^2 + c$ are the same shape as $y = x^2$ but they cut the y axis at different places. The minimum or maximum value (turning point) will be (0,c).

3. Discuss your predictions with your teacher, then check that your predictions are correct using MicroSMILE program Quad.

   You might like to extend this activity by investigating the effects of a, b and c on other graphs using a function graph plotter or graphic calculator.

   $y = x^3$
   $y = x$
   $y = \sin x$ are some examples.

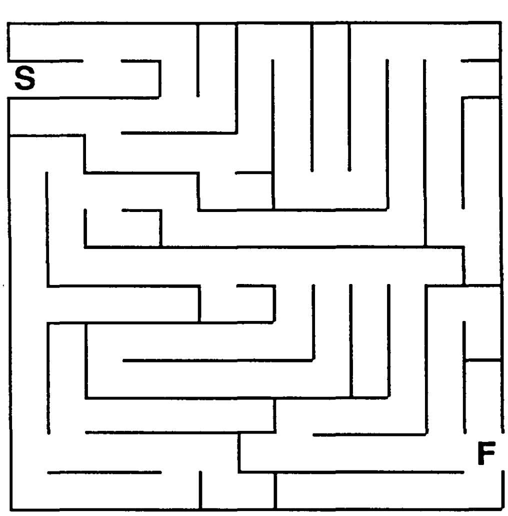
2080 Symmetrical Tiles

Many possible answers.
Did you design a symmetrical pattern with more than one axis of symmetry?
You may like to display your symmetrical tiling patterns.

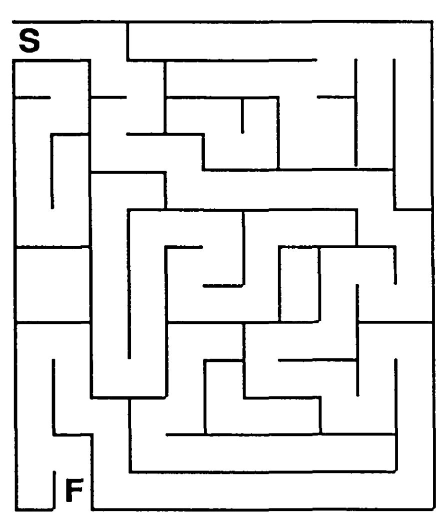
2081 Inventing Mazes

Here are examples of pupils' work from Daneford School.

Delwar



Rafiqul



2082 Opposite Adjacent & Hypotenuse

The answers in the first three columns of each table will depend upon the size you have drawn your triangles. Your answers in the last three columns of the table should be roughly the same.

40° Angle

$\frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{\text{adjacent}}{\text{hypotenuse}}$	$\frac{\text{opposite}}{\text{adjacent}}$
0.6	0.8	0.8
0.6	0.8	0.8
0.6	0.8	0.8

20° Angle

$\frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{\text{adjacent}}{\text{hypotenuse}}$	$\frac{\text{opposite}}{\text{adjacent}}$
0.3	0.9	0.4
0.3	0.9	0.4
0.3	0.9	0.4

60° Angle

$\frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{\text{adjacent}}{\text{hypotenuse}}$	$\frac{\text{opposite}}{\text{adjacent}}$
0.9	0.5	1.7
0.9	0.5	1.7
0.9	0.5	1.7

If your answers differ significantly, discuss your work with your teacher.

$\cos 32° = \frac{a}{1}$
$1\cos 32° = a$
$= 0.848048096$
$= 0.848\text{cm}$

$\tan 65° = \frac{b}{1}$
$1\tan 65° = b$
$= 2.144506921$
$= 2.145\text{cm}$

$\sin 42 = \frac{10}{c}$
$c\sin 42 = 10$
$= 14.9447655$
$= 14.945\text{cm}$

It is important that you show all your working out in a similar way.

d = 5.642cm   e = 4.275cm   f = 14.220cm   x = 12.833cm

2083 All about Circles

1. It is a line of symmetry. It divides the circle in 2 equal parts. This line is called the **diameter.**

continued/

2083 All about Circles (cont)

2. One method could be to fold the circle into quarters. The centre of the circle is where the fold lines cross.

3. There are an infinite number of lines of symmetry of a circle. The line of symmetry of a circle is a diameter. (Look in your mathematical dictionary if you are not sure what infinite means).

4. A circle has rotational symmetry because it can be rotated about its centre by different amounts and yet still look the same.

5. The fold line is a diameter again. The line joining the 2 points is called a chord. These statements will be true for any pair of points.

6. There are many ways. Here is one method.

   - Take a paper circle.
   - Draw a chord
   - Bisect the chord using a compass (look at SMILE 0211)
   - Draw another chord and bisect it.
   - The centre of the circle is the point where the bisecting lines cross.

   If your method is different convince your teacher that it works.

2084 Polygon Areas

Area of A = $6\frac{1}{2}$ cm^2 Area of B = $5\frac{1}{2}$ cm^2 Area of C = 12cm^2

Area of D = 9cm^2 Area of E = $8\frac{1}{2}$ cm^2 Area of F = 6cm^2

2085 Scale Maps

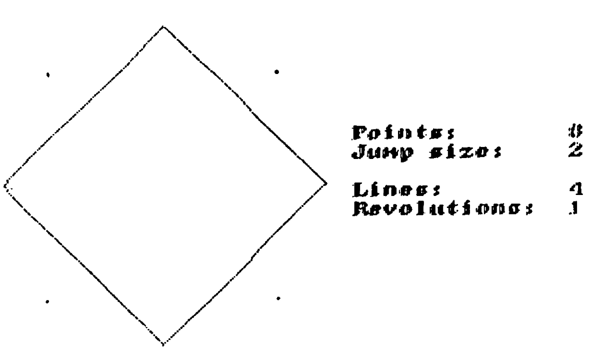
Your solution can be checked by considering the problem as a transformation problem. The smaller map has been enlarged and then rotated. The common point is the point which is both the centre of enlargement and the centre of rotation.

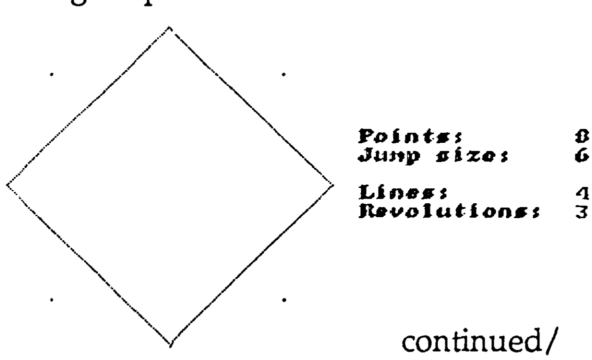
Check that the point you have found satisfies these 2 conditions.

Try this again with the smaller map in some different positions.

2086 Circles to Polygons

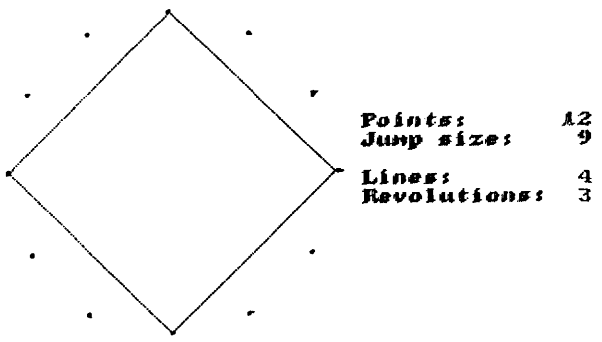
These screen dumps show four possible ways of drawing a square.

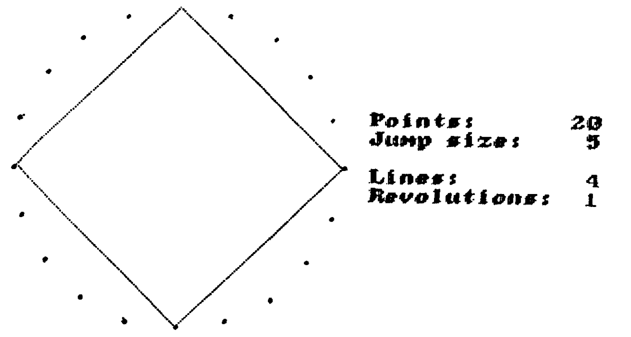




continued/

2086 Circles to Polygons (cont)





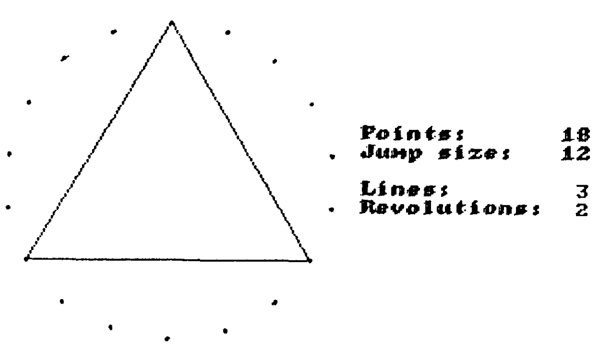
Other possible results could be :

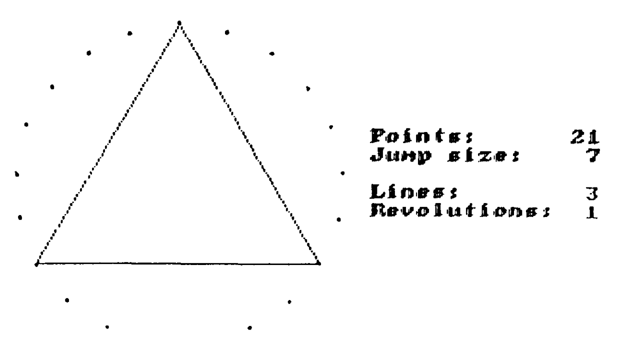
points	jumps
4	1
8	2
12	3
16	4
20	5
.	.
.	.
.	.

or

points	jumps
4	3
8	6
12	9
16	12
20	15
.	.
.	.
.	.

These screen dumps show two possible ways of getting a triangle.





Other possible results could be

points	jumps
3	1
6	2
9	3
12	4
15	5
.	.
.	.
.	.

or

points	jumps
3	2
6	4
9	6
12	8
15	10
.	.
.	.
.	.

continued/

2086 Circles to Polygons (cont)

For pentagons your result should be one of the following

points	jumps
5	1
10	2
15	3
20	4
25	5
.	.
.	.
.	.

or

points	jumps
5	4
10	8
15	12
20	16
25	20
.	.
.	.
.	.

For a hexagon

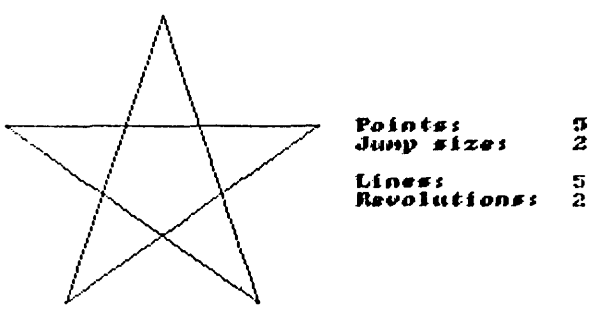
points	jumps
6	1
12	2
18	3
24	4
.	.
.	.
.	.

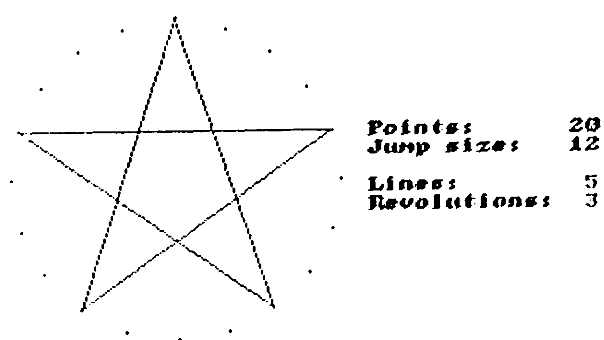
or

points	jumps
6	5
12	10
18	15
24	20
.	.
.	.
.	.

What other polygons did you draw?

Here are two possible ways of drawing a pentagram.





other possible results could be

points	jumps
5	2
10	4
15	6
20	8
.	.
.	.
.	.

or

points	jumps
5	3
10	6
15	9
20	12
.	.
.	.
.	.

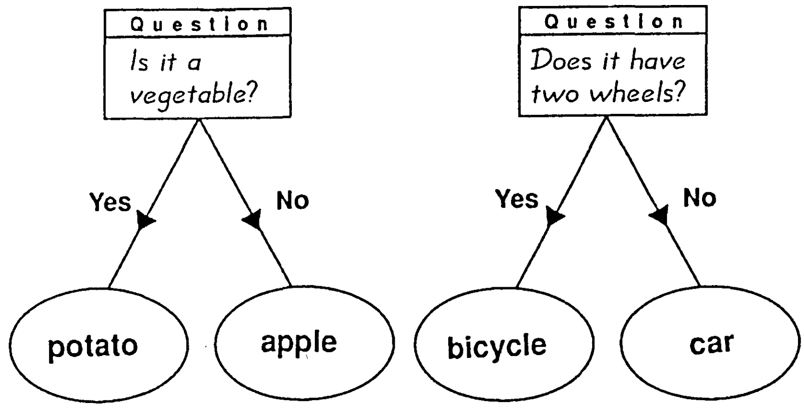
Did you draw stars with other numbers of points?

2087 Without looking

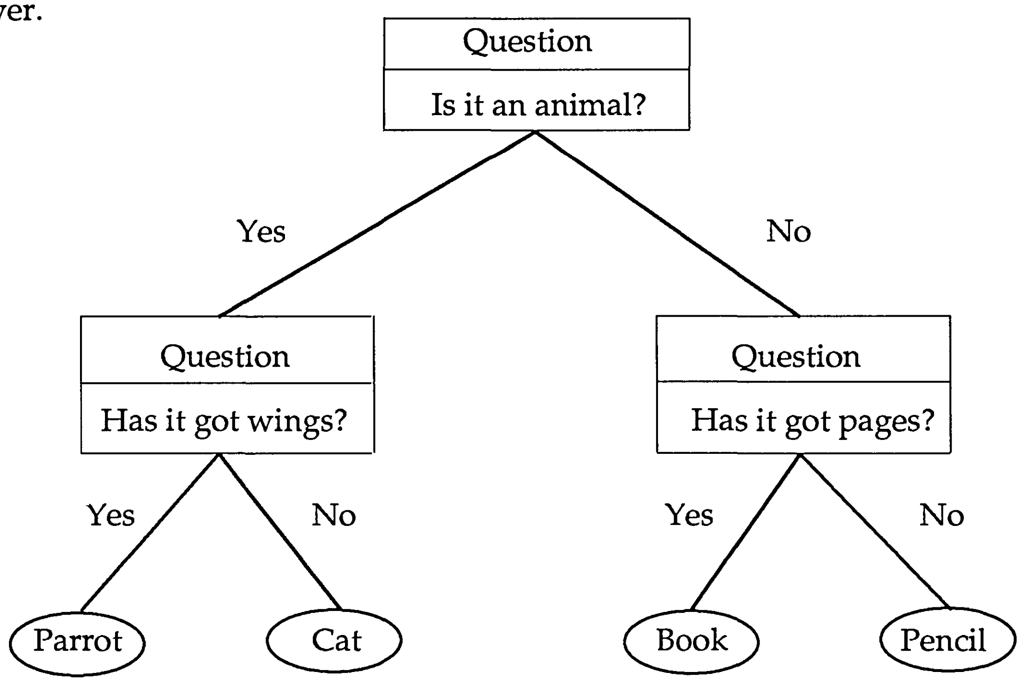
There are 166 possible shapes to make with 6 cubes, so we could not draw them all. Get your partner to check your isometric drawing.

2088 What's the Difference?

1.



2. When this card was written Victoria Clarke, from Waverley School came up with this answer.



You may have used different questions.

To check the rest of the card, find someone else who has done it and compare your work.

2089 Oxford Street

A. Regents Park
Bond Street
Marble Arch
Tottenham
Court Road

B. Tottenham
Court Road
Regents Park
Piccadilly Circus

C. Piccadilly Circus
Tottenham
Court Road
Bond Street
Marble Arch

D. Bond Street
Marble Arch
Piccadilly Circus
Regents Park

2090 Black and Red Triangle Patterns

Including the number of layers of triangles in your table of results will help you to see the number patterns.

Number of Layers	Number of Black Triangles	Number of Red Triangles	Total
1	1	0	1
2	3	1	4
3	6	3	9
4	10	6	16
5	15	10	25
6	21	15	36
.	.	.	.
.	.	.	.
.	.	.	.
10	?	?	?
.	.	.	.
.	.	.	.
.	.	.	.
100	?	?	?
	These are the triangle numbers.		What are these?

2091 Right Angled or not?

Area of A = $9cm^2$ Area of B = $25cm^2$ Area of C = $23.04cm^2$
Area of D = $16cm^2$ Area of E = $1.96cm^2$ Area of F = $4cm^2$
Area of G = $36cm^2$

Squares A, B and D will fit exactly around a right-angled triangle, so will squares C, E and B. This can be checked by using Pythagoras' Theorem.

There are only two **exact** solutions for the size of square H.
Square H can either be 5.2cm x 5.2cm then C, F and H surround a right-angled triangle, or square H can be 3.6cm x 3.6cm, then D, G and H surround a right-angled triangle.

You can find the areas of many squares that would fit with 2 squares from A, B, C, D, E, F, and G to make a right-angled triangle, but you could not calculate the **exact** length of the sides of the square.

For example, square A and square E would fit with a square of area $10.96cm^2$.
$\sqrt{10.96} = 3.310589071.......$
It is impossible to draw an **exact** line of this square.

2092 What's Recurring?

Fractions represented as [1 ; 0] are $\frac{1}{2}$

$\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$

$\frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}$

continued/

2092 What's Recurring? (cont)

Equivalent fractions give same code, so fractions should be written in the lowest terms.
For example $\frac{7}{21} = \frac{1}{3}$

Fractions represented as [2 ; 0] are $\frac{1}{4}, \frac{3}{4}$

$\frac{1}{20}, \frac{3}{20}, \ldots$

$\frac{1}{25}, \ldots$

$\frac{1}{50}, \ldots$

What about [3 ; 0]?
[4 ; 0]?

Can you generalise for [a ; 0]?

Fractions represented as [0 ; 1] are $\frac{1}{3}, \frac{2}{3}$

$\frac{1}{9}, \frac{2}{9}, \ldots$

[0 ; 2] are $\frac{1}{11}, \frac{2}{11}, \ldots$

$\frac{1}{33}, \ldots$

What about [0 ; 3]?
[0 ; 6]?

Any others fractions of the form [0 ; b]?

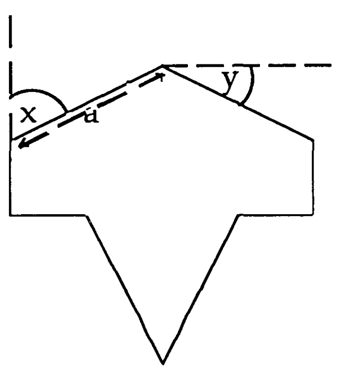
What is special about the denominators of a fraction represented as [0; b]?
What is special about the possible values for b?

Fractions represented as [1 ; 1] include $\frac{1}{6}, \frac{1}{15}, \frac{1}{18}, \frac{1}{30}, \frac{1}{90}$.

Find other rules and connections between fractions and the codes they are represented by.

2093 Islamic Patterns in Logo

First build a Logo program to draw this tile.

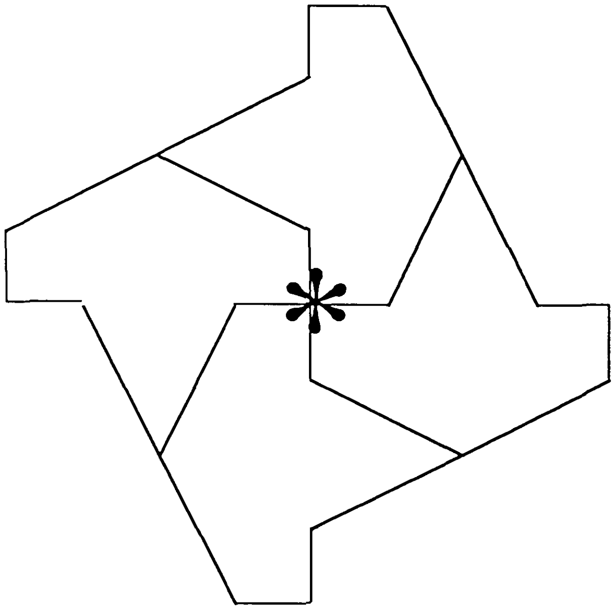


Using Pythagoras' Thereom
$a = \sqrt{5} = 2.34$ (2 decimal places)
Using trigonometry
$x = 63.43°$
$y = 26.57°$

continued/

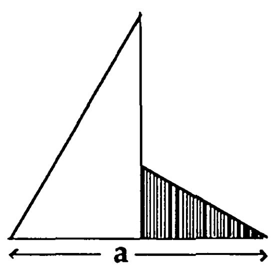
2093 Islamic Patterns in Logo (cont)

Next build a program to produce a unit. This is one produced by rotating the original tile about *

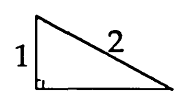


Then you can tessellate your unit to fill the screen.

For the second tessellation, this is the starting tile.



Using Pythagoras' Thereom with this triangle



you can calculate **a**.

2094 Squares

You can make the square from a circle with 4 points and a jump size of 1. These will be 4 lines and 1 revolution.

This table shows other ways to make a square.

points	jump size
4	1
4	3
8	2
8	6
12	3
12	9
.	.
.	.
.	.

What do you notice about the numbers of points?

2095 Squares, Cubes and Roots

1.

1	6		3
	4	9	
5			1
	1		2
1	3	3	1

2. Show your own puzzle to your teacher.

2096 Fraction Playing Cards

Write about the rules of the game you played.

2097 Fraction Families

Which fraction 'families' did you collect?

2098 4 - in - a - Line

One way of approaching this investigation is to keep one of the two variables, rows or columns, constant. By working systematically it is possible to produce results which generate patterns to predict other outcomes.

These results were obtained by looking at n x 4 boards.

Board Size Row x Column	Row Wins	Column Wins	Diagonal Wins	Total Wins
1 x 4	1	0	0	1
2 x 4	2	0	0	2
3 x 4	3	0	0	3
4 x 4	4	4	2	10
5 x 4	5	8	4	17
6 x 4	6	12	6	24
7 x 4	7	16	8	31
⋮	⋮	⋮	⋮	⋮
R x 4	R	4R – 12	2R – 6	7R – 18

continued/

2098 4 - in - a - Line (cont)

What happens when the board size changes

1 x 5	1 x 6
2 x 5	2 x 6
3 x 5. . . ?	3 x 6 . . . ?

Can you find a general rule for any size board?

Can you find a general rule for 'n' - in - a - line?

2099 Naksha

Make a display of your designs.

2100 Putting it to the test

1. The statement is false.

	1	2	3	4	5	6
1	✗	✗	✗	✗	✗	✓
2	✗	✗	✗	✗	✗	✓
3	✗	✗	✗	✗	✗	✓
4	✗	✗	✗	✗	✗	✓
5	✗	✗	✗	✗	✗	✓
6	✓	✓	✓	✓	✓	✓

You would expect to see a 6, 11 times out of 36.
You would expect not to see a 6, 25 times out of 36.

If you said 'true', try the experiment again.

2. The statement is true. If you have more dice then you have more chances of getting a 6.

3. This statement is false.

With two dice.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

You would expect to get a total of 8 or above 15 times out of 36 throws.
i.e. $\frac{15}{36}$

continued/

2100 Putting it to the test (cont)

With three dice the possible totals are

3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

(median marked between 10 and 11)

The median total is 10.5.
You would expect half the totals to be below 10.5 and half above 10.5. But it is only possible to have whole number totals. So the probability of scoring 11 or above is $\frac{1}{2}$.

Since $\frac{1}{2} = \frac{18}{36}$ which is bigger than $\frac{15}{36}$ then the statement is false.

4. The statement is false.
Each number on the dice is equally likely.
The probability of throwing a 6 is $\frac{1}{6}$ which is the same as the probability of throwing a 1, or a 2 , or. . .

2101 Logiblock Sets

This activity is a mixture of luck and skill. Luck is involved when choosing the first logiblock, but from then on skill is involved when selecting other logiblocks.

Did the choice of attribute cards affect the minimum number of logiblocks that had to be placed?

2102 Is it luck . . . or is it skill?

1. a) You might have thought of a game like Snakes and Ladders or Bingo or . . .

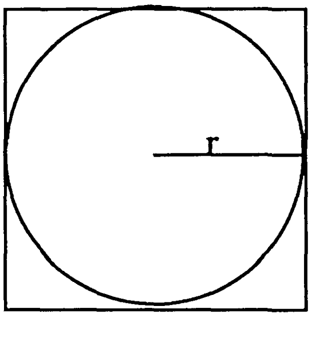
b) You might have thought of something like chess or swimming. . .

2. Many possible answers.

If you made a display of your group's work, did others in your class agree?

2103 Circle Packing

1. Using the π buttton
Your answers may vary if you took $\pi = 3.14$.

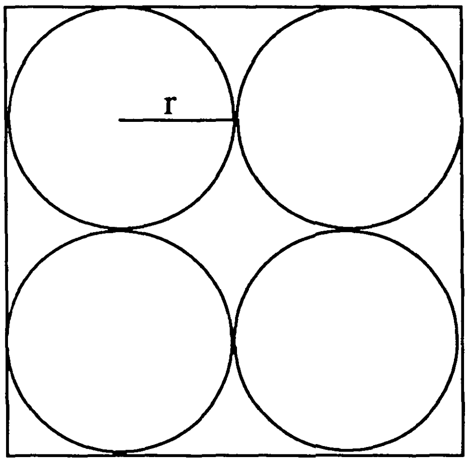


Taking the radius as r.
Area of square $= 2r \times 2r = 4r^2$
Area of circle $= \pi r^2$
Shaded area $= 4r^2 - \pi r^2$
$= (4 - \pi) r^2$
$= 0.858r^2$

$\frac{\text{Shaded area}}{\text{Whole square}} = \frac{0.858r^2}{4r^2}$
$= 0.2146$
$= 21.460\%$

continued/

2103 Circle Packing (cont)



Area of first circle	$= \pi r^2$
Area of four circles	$= 4 \times \pi r^2$
Area of square	$= 4r \times 4r = 16r^2$
Shaded area	$= 16r^2 - (\pi r^2 \times 4)$
	$= (16 - 4\pi)r^2$
	$= 3.433r^2$

$$\frac{\text{Shaded area}}{\text{Whole square}} = \frac{3.433r^2}{16r^2} = 0.2146 = 21.460\%$$

You should find that for circles of the same size arranged in any square, the answer is 21.460%

2. An arrangement has been found using circles which are all the same size so that the wastage is as low as 9.3%. Did you find it?

   What about if you used circles of different sizes?

2104 Averaging Out

Using a spreadsheet or writing a program is essential for this investigation. Hints for using Excel and writing a program in Basic or for the Graphic Calculator are given below.

Spreadsheet (Excel)

	What to do	**How to do it**
1.	Enter 4 into the first cell (A1)	Type (4) (↵)
2.	Enter 10 into the second cell (A2)	Click on A2 Type (10) (↵)
3.	Enter a formula to calculate $\frac{4 + 10}{2}$	Click on A3 Type (=) to enter a formula Type (() Click on A1 Type (+) Click on A2 Type ()) (/) (2) (↵)

This will appear on the top part of the screen

	A	B	C	D	E
1	4				
2	10				
3	7				

4.	Copy this formula down the spreadsheet.	Click on A3 Edit menu to Copy. Highlight 100 cells from A4. Edit menu to Paste.

You can change the first two numbers by clicking on A1 and A2 and entering in different numbers.
This is easily adapted for 3 or more numbers.

continued/

Using BASIC

The program for BBC BASIC is

```
10    INPUT X
20    INPUT Y
25    FOR A=1 TO 100
30    Z=(X+Y)/2
40    PRINT Z
50    X=Y
60    Y=Z
70    GOTO 30
```

This is easily adapted to $\frac{A + B + C}{3}$ or $\frac{A + B + C + D}{4}$ etc . . .

Using a graphic calculator

A program for the Texas Instrument T1–81 Graphic Calculator.

```
:    Input A
:    Input B
:    Lbl 1
:    (A+B)/2 → C
:    Disp C
:    B → A
:    C → B
:    Pause
:    Go to 1
:    End
```

Keep pressing Enter when running the program.

Starting with two numbers

Starting with 4 and 10 a spreadsheet generated the sequence

	A
1	4
2	10
3	7
4	8.5
5	7.75
6	8.125
7	7.9375
8	8.03125
9	7.984375
10	8.0078125
11	7.99609375
12	8.00195313
13	7.99902344
14	8.00048828
15	7.99975586
16	8.00012207
17	7.99993896
18	8.00003052
19	7.99998474
20	8.00000763

The numbers are getting closer and closer to 8. This is called a <u>**limit**</u>.

Investigating other starting numbers.

Starting numbers			Limit
1,	1,	. . .	
1,	2,	. . .	The limit goes up in $0.\dot{6}$ or $\frac{2}{3}$
1,	3,	. . .	
.	.	.	
.	.	.	
.	.	.	

continued/

2104 Averaging Out (cont)

6, 1, ... What is the limit for these numbers?
6, 2, ...

What happens when the two starting numbers are the same?
For each sequence can you predict the limit?
Is there a general rule for calculating the limit?

Starting with three numbers

Using a spreadsheet, 3, 18, 5, generates this sequence which has a limit of 9.

	A
1	3
2	18
3	5
4	8.66666667
5	10.5555556
6	8.07407407
7	9.09876543
8	9.24279835
9	8.80521262
10	9.04892547
11	9.03231215
12	8.96215008
13	9.01446256
14	9.00297493
15	8.99319586
16	9.00354445
17	8.99990508
18	8.9988818
19	9.00077711
20	8.99985466

For each sequence can you predict the limit?
Is there a general rule for calculating the limit with three numbers?
Is there a general rule for calculating the limit with 'n' numbers?

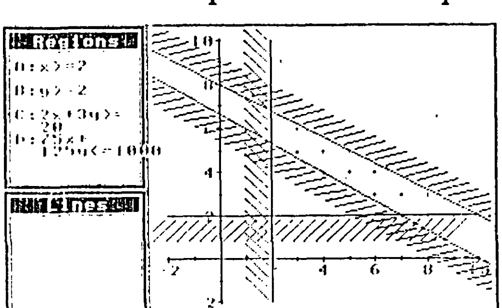
2105 Equal Fraction Pairs

Make a list of the equal fraction pairs you collected.

2106 Party Solutions

1 b) You have to have at least 2 bottles of cola.
c) Orangeade is in two litre bottles (2x) and cola in three litre bottles (3y) and the combined number of litres must be at least 20.
d) Orangeade costs 75p (75x) and cola costs 125p (125y), the combined cost must be less then 1000p.

2& 3 a) This screen dump shows the 4 inequalities and the possible solutions.



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<u>2106 Party Solutions (cont)</u>

b&c) This table shows the fifteen combinations which satisfies all the conditions.

Bottles of Orangeade(x)	Bottles of Cola (y)	No. of Litres	Cost
2	6	(2x2)+(6x3) = 22	(2x75)+(6x125) = 900p = £9.00
3	5	(3x2)+(5x3) = 21	(3x75)+(5x125) = 850p = £8.50
3	6	(3x2)+(6x3) = 24	(3x75)+(6x125) = 975p = £9.75
4	4	(4x2)+(4x3) = 20	(4x75)+(4x125) = 800p = £8.00
4	5	(4x2)+(5x3) = 23	(4x75)+(5x125) = 925p = £9.25
5	4	(5x2)+(4x3) = 22	(5x75)+(4x125) = 875p = £8.75
5	5	(5x2)+(5x3) = 25	(5x75)+(5x125) = 1000p = £10.00
6	3	(6x2)+(3x3) = 21	(6x75)+(3x125) = 825p = £8.25
6	4	(6x2)+(4x3) = 24	(6x75)+(4x125) = 925p = ~~£9.25~~ £9.50
7	2	(7x2)+(2x3) = 20	(7x75)+(2x125) = 775p = £7.75
7	3	(7x2)+(3x3) = 23	(7x75)+(3x125) = 900p = £9.00
8	2	(8x2)+(2x3) = 22	(8x75)+(2x125) = 850p = £8.50
8	3	(8x2)+(3x3) = 25	(8x75)+(3x125) = 975p = £9.75
9	2	(9x2)+(2x3) = 24	(9x75)+(2x125) = 925p = £9.25
10	2	(10x2)+(2x3) = 26	(10x75)+(2x125) =1000p = £10.00

d) The combination which gives the most drink is 10 bottles of orangeade and 2 bottles of cola.

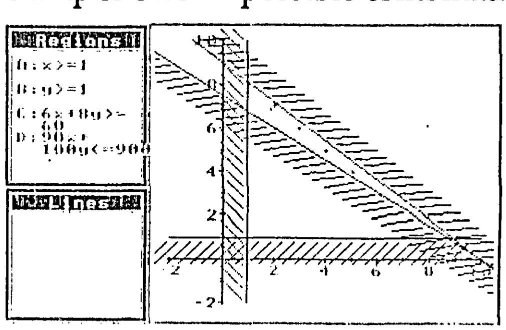
e) The combinations which use up all of the £10 are 5 bottles of orangeade and 5 bottles of cola and 10 bottles of orangeade and 2 bottles of cola.

f) These 2 points lie on the line $75x + 125y = 1000$.

4. Let x = the number of Bumper Pack of Crisps
   y = the number of Hoola Hoops.

$$x \geq 1$$
$$y \geq 1$$
$$6x + 8y \geq 60$$
$$90x + 100y \leq 900$$

5. The screen dump shows all possible combinations.



continued/

2106 Party Solutions (cont)

6. a) 1 Bumper Pack of Crisps and 8 Economy Packs of Hoola Hoops.
   b) 6 Bumper Packs of Crisps and 3 Economy Packs of Hoola Hoops.
   c) 5 Bumper Packs of Crisps and 4 Economy Packs of Hoola Hoops.

2107 Oxfam Collection

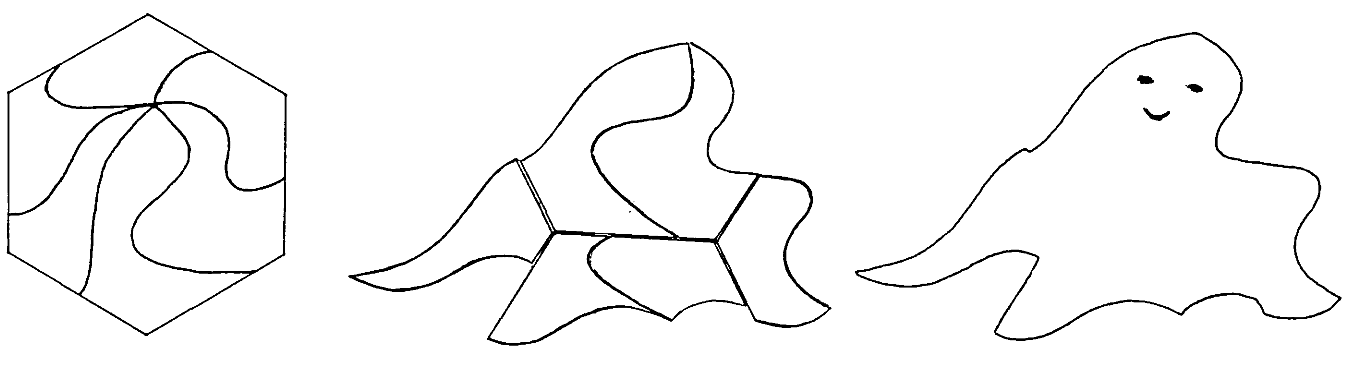
Sam's total –	£11.15	Shelene's total –	£10.85
Kieu's total –	£12.20	Dean's total –	£12.40

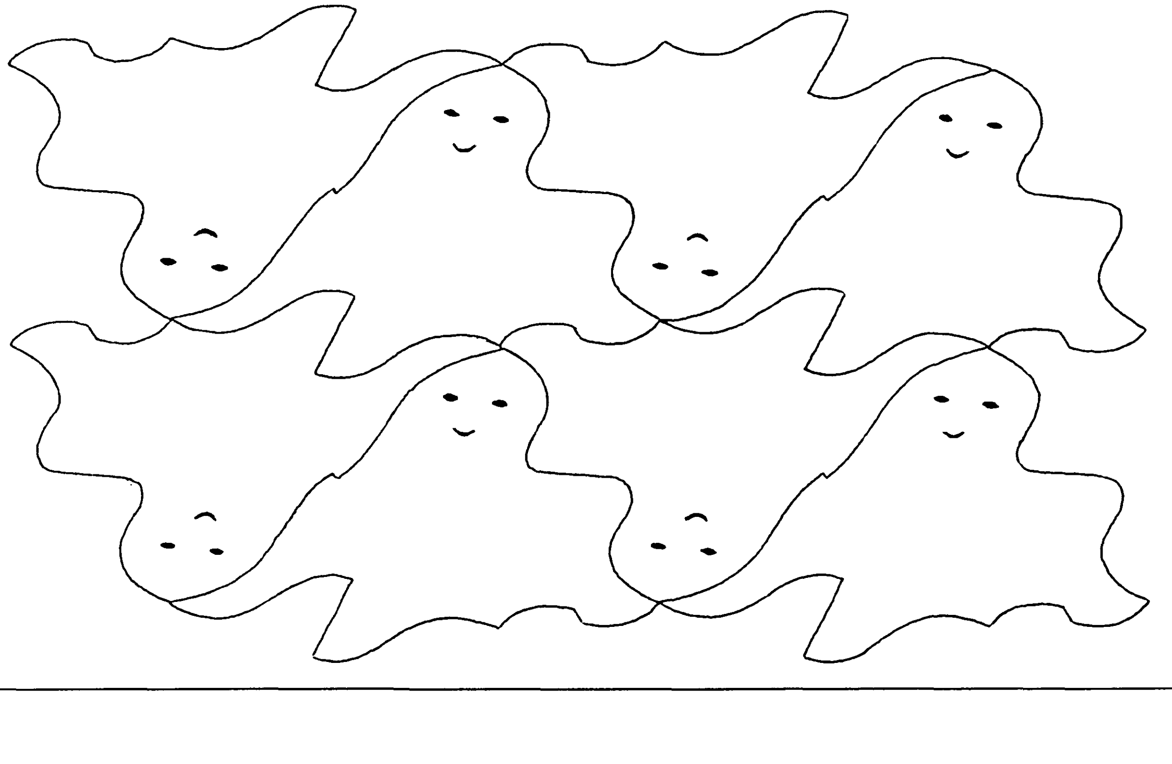
2108 Wiggly Tessellation

If you move 2 of the adjacent pieces from one side to the other, the shape will tessellate. If you move them randomly, the shape might not tessellate.

We could not make a wiggly tessellation from a triangle. Can you?

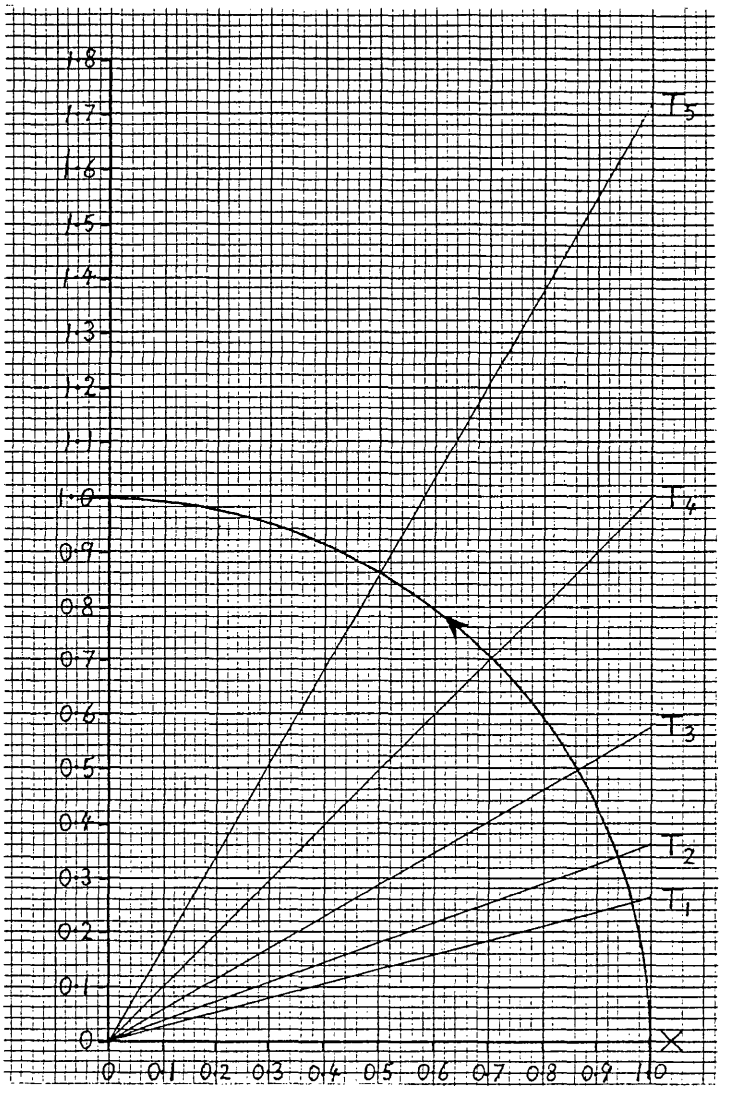
This tessellation is made from a hexagon.
Notice that the wiggly lines all meet at the same point.





2109 Another Trig Line

1.



d) Distance of XT
15° → 0.26
20° → 0.36
30° → 0.58
45° → 1.0
60° → 1.72

2. As the angle gets closer to 90° the distance XT gets longer and longer. When the angle is 90° you can not measure the distance XT.

3. tan35 = 0.7002075 = 0.700 to 3 decimal places.

4. 15° → tan15 = 0.2679492 = 0.268
20° → tan20 = 0.3639702 = 0.364
30° → tan30 = 0.5773503 = 0.577
45° → tan45 = 1
60° → tan60 = 1.7320508 = 1.732

5. a) 0.510 b) 1.327 c) 1.150
d) 1.664 e) 0.754 f) 0.344

6. a) 1.963 b) 0.404

7. 0.728

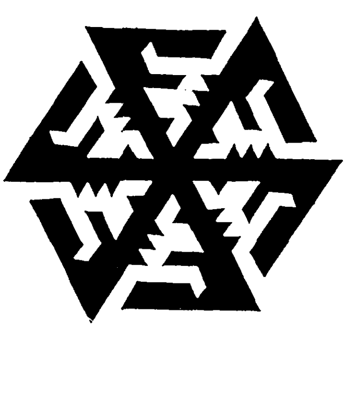
8. a) 1.154 b) 8.391 c) 9.602
d) 10.898 e) 15.012 f) 4.898

2110 Number Sort

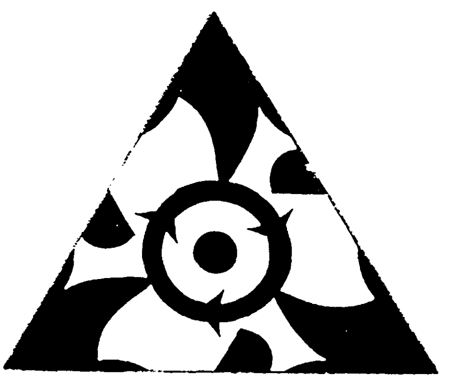
0 – 9	5 six 8
10 – 19	12 15 16 seventeen 18
20 – 29	21 23 28 twenty nine
30 – 39	30 33 thirty five
40 – 49	forty two 43 45
50 – 59	fifty 54 57
60 – 69	60 sixty four 66
70 – 79	seventy one 72
80 – 89	81 82 eighty eight
90 – 99	ninety three 99

2111 Rotational Symmetry Jigsaws

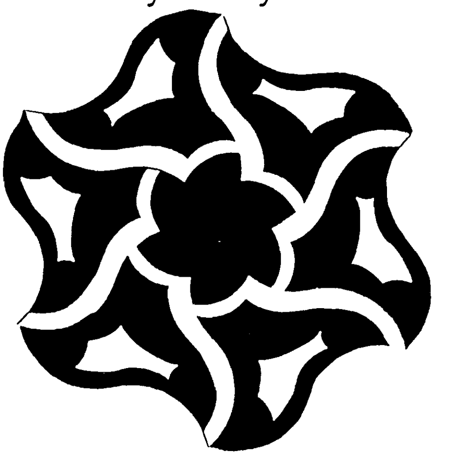
This is one pattern that can be made from Jigsaw a.
It has rotational symmetry of order 6.



This is one pattern that can be made from Jigsaw b.
It has rotational symmetry of order 3.



This is one pattern that can be made from Jigsaw c.
It has rotational symmetry of order 6.



2112 Imaginings

The answers to many of the activities will depend upon your imagination. In all of the activities it is best for you and other members of your group or class to discuss your imaginings.

2113 Mystery

After completing each calculation, turn your calculator upside-down to read where the next clue is.

2114 2 Puzzles

Magic Square
One of the 16's should be a 14, which one?

100 Up
Try 37 + 73 – 7 – 3

2115 Missing digit

	35 x 57 = 1995	13 x 59 = 767
	131 x 23 = 3013	36 x 47 = 1692
	56 x 72 = 4032	59 x 17 = 1003
	48 x 47 = 2256	14 x 29 = 406
or	48 x 57 = 2736	
	Cannot be done	73 x 57 = 4161
	37 x 43 = 1591	

2116 Operations

This shows one way of making each number in set B from one number in set A. If your answers are different, check them with your teacher.

Set A	Operation	Set B
18	÷3	6
30	–15	15
81	÷3	27
24	x4	96
2	+17	19
15	÷3	5
17	+17	34
78	+17	95
19	–15	4

2117 Rumour

How did you display your results?

2118 Ticket Sales

Five tickets in the stalls and three in the balcony.

2119 Patterns

Were you able to find a general rule for every pattern? The calculator is useful for working out the answers for the first few in each sequence, but using a spreadsheet might be useful so that you can check your rules.

2120 Productive

With the first five digits, the largest product is 431 x 52.

What is the largest product with 1, 2, 3, 4, 5 and 6?

2121 Hot & Cold

California has the hottest record temperature.

11 USA States	max temp C°	min temp C°	difference	range
Alabama	44	–33	44 – (–33)	77
Alaska	37	–62	37 – (–62)	99
Arizona	53	–40	53 – (–40)	93
Arkansas	49	–34	49 – (–34)	83
California	57	–43	57 – (–43)	100
Colorado	48	–51	48 – (–51)	99
Connecticut	41	–36	41 – (–36)	77
Delaware	43	–27	43 – (–27)	70
District of Columbia	41	–26	41 – (–26)	67
Florida	43	–19	43 – (–19)	62
Hawaii	38	–10	38 – (–10)	48

California has the biggest temperature range.

2122 Target 200

You could score 200 with 8 arrows, **4** x 11, **2** x 23, **1** x 37, **1** x 73.

- Could you score 200 using less arrows?
- What other totals can you make?

2123 Missing Signs

17 **x** 17 **+** 17 = 306	38 **–** 47 **+** 58 = 49
(47 **+** 53) **x** 10 = 1000	27 **x** (5 **x** 5) = 675
34 **+** (37 **x** 18) = 700	(437 **x** 2) **+** 126 = 1000
91 **x** 7 **÷** 13 = 49	47 **+** 23 **+** 27 **+** 15 = 112
768 **÷** (43 **–** 37) = 128	1116 **–** (23 **x** 47) = 35

2124 Date of Birth

- Were you able to work out the day on which you were born?
- Can you find a way to explain why it work?

2125 Escape

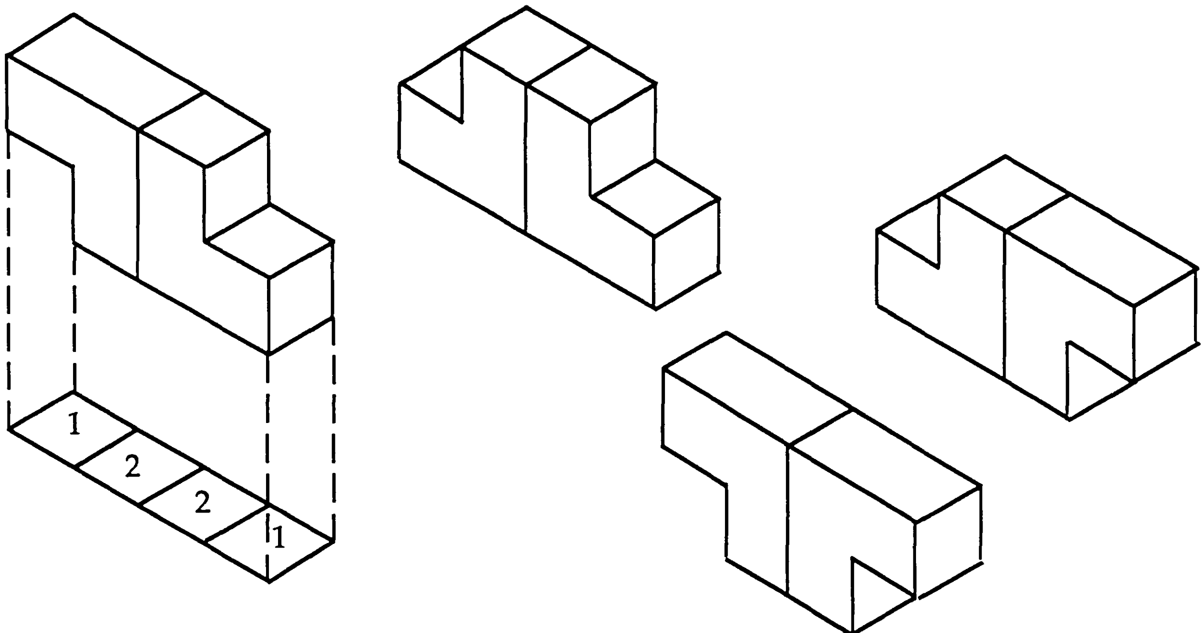
- Were there any numbers that could not escape?
- Did you change the escape rules so that no numbers could escape?

2126 Problems

1. About 100 years!
2. 100kg, about the same weight as the largest man you have ever seen.
3. At least 1hour 15minutes.
4. Non-stop at about 4mph, more than 7 years!
5. At least once around the world.
6. Nearly 16000 million.
7. You would have a party every 3 years 2 months.

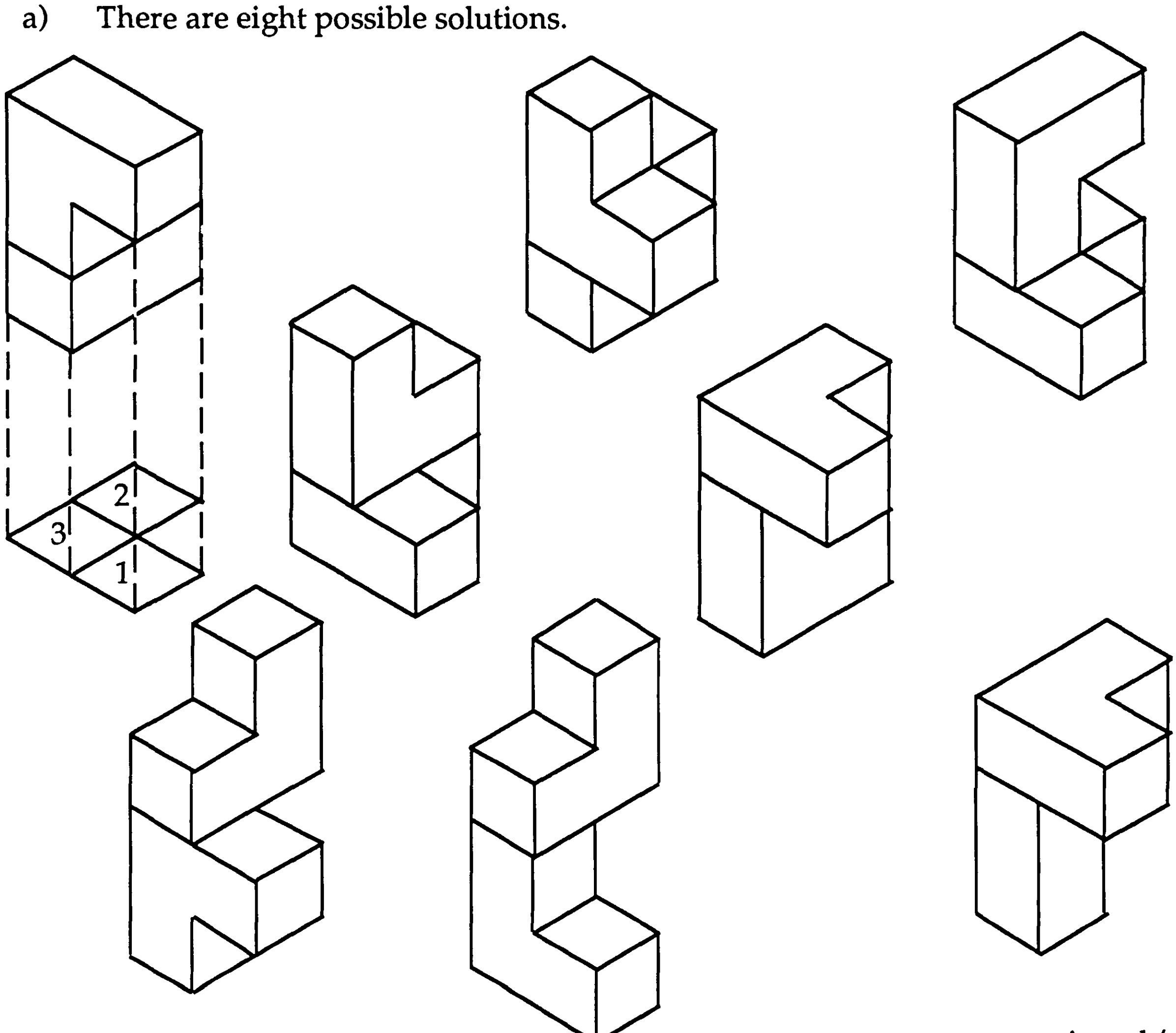
2127 Tricubes Codes

1. There are four different solids.



2. There are many possible coded plans, but your code should have numbers that add up to 6. Why?

3. a) There are eight possible solutions.



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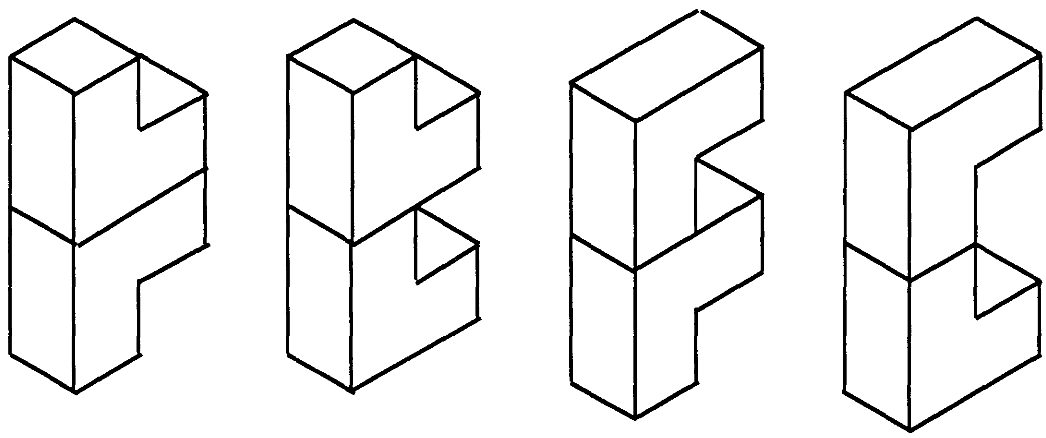
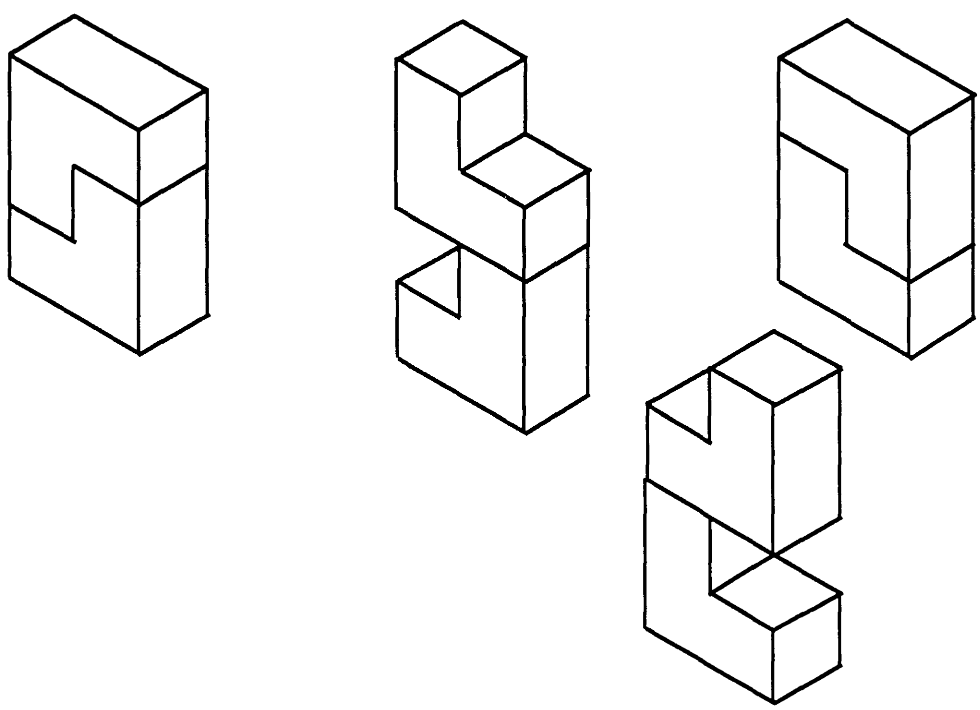
2127 Tricubes Codes (cont)

b) There are four possible solutions.

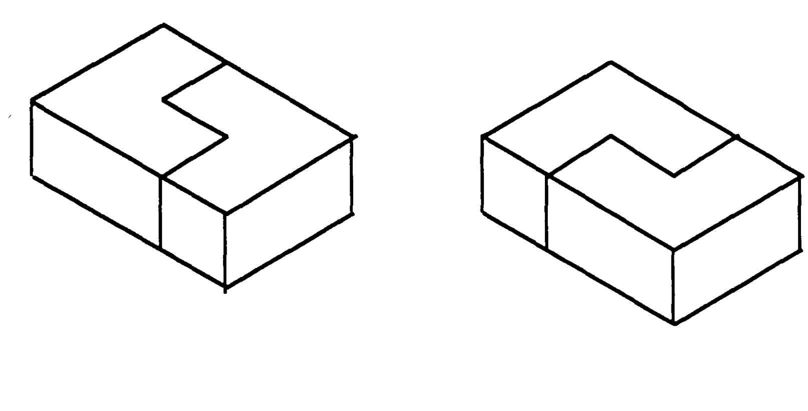


c) Impossible.

d) There are four possible solutions.

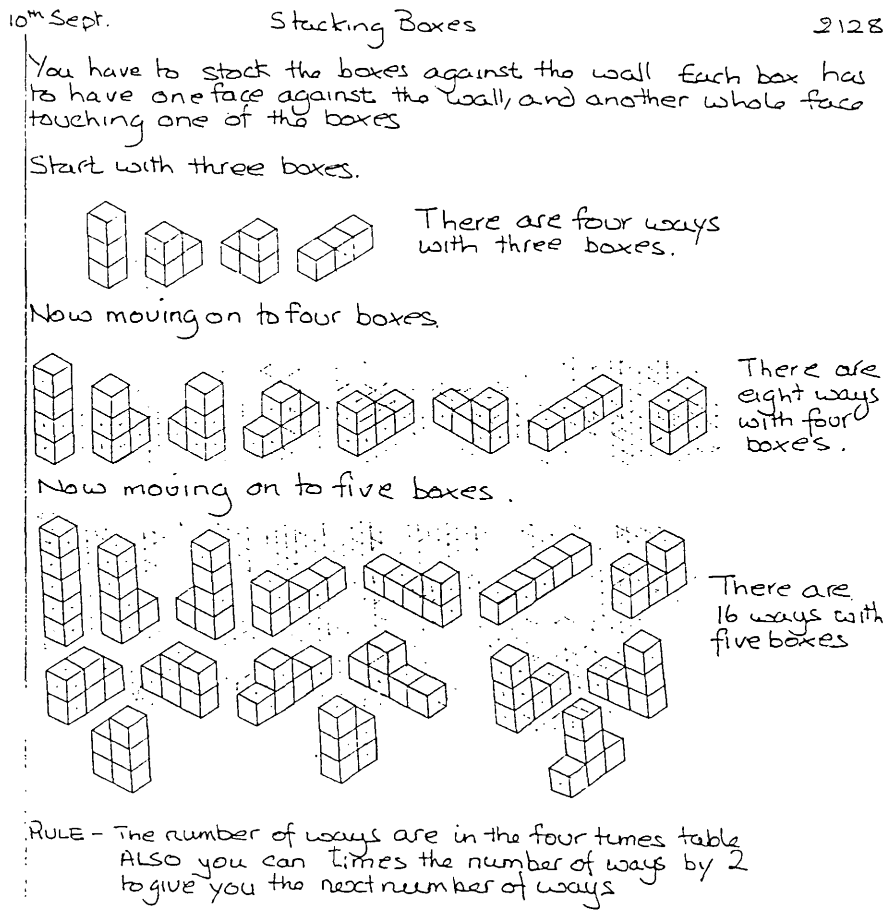


e) There are two possible solutions.



2128 Stacking

This is part of Linda's work (Waverley School 1992)



Try drawing a table for your results.
Can you predict how many ways there are of stacking 20 boxes?

2129 Tens and Fives

1.

3	⟶	30
5	⟶	50
17	⟶	170
234	⟶	2340
698	⟶	6980
57	⟶	570

2. Get someone to check your answers.

3. 200, 340, 2350.

4. Get someone to check your answers.

continued/

2129 Tens and Fives (cont)

5. Any number in the ten times table will end with a nought.

6.

6	→	30
7	→	35
13	→	65
324	→	1620
798	→	3990
25	→	125

7. Get someone to check your answers.

8. 300, 750, 1365, 6555.

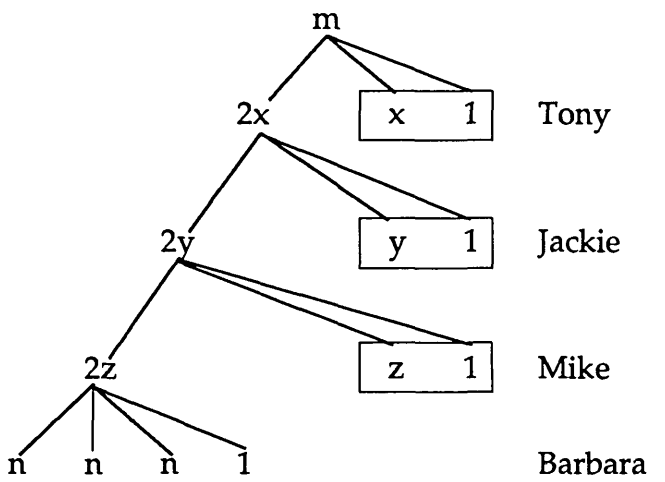
9. Get someone else to check your answers.

10. Any number in the five times table will end with a five or a nought.

2130 A Disappearing Act

The algebraic solution to this problem is a bit tricky.
Here is the beginning of the solution.

m is the original number
of marbles



n is the number of marbles in each of the final 3 groups.

$2z = 3n + 1$

$2y = \frac{3}{2}(3n + 1) + 1$ i.e $\frac{3}{2}(2z) + 1$

$2x = \frac{3}{2}\,[\frac{3}{2}(3n + 1) + 1] + 1$ i.e. $\frac{3}{2}(2y) + 1$

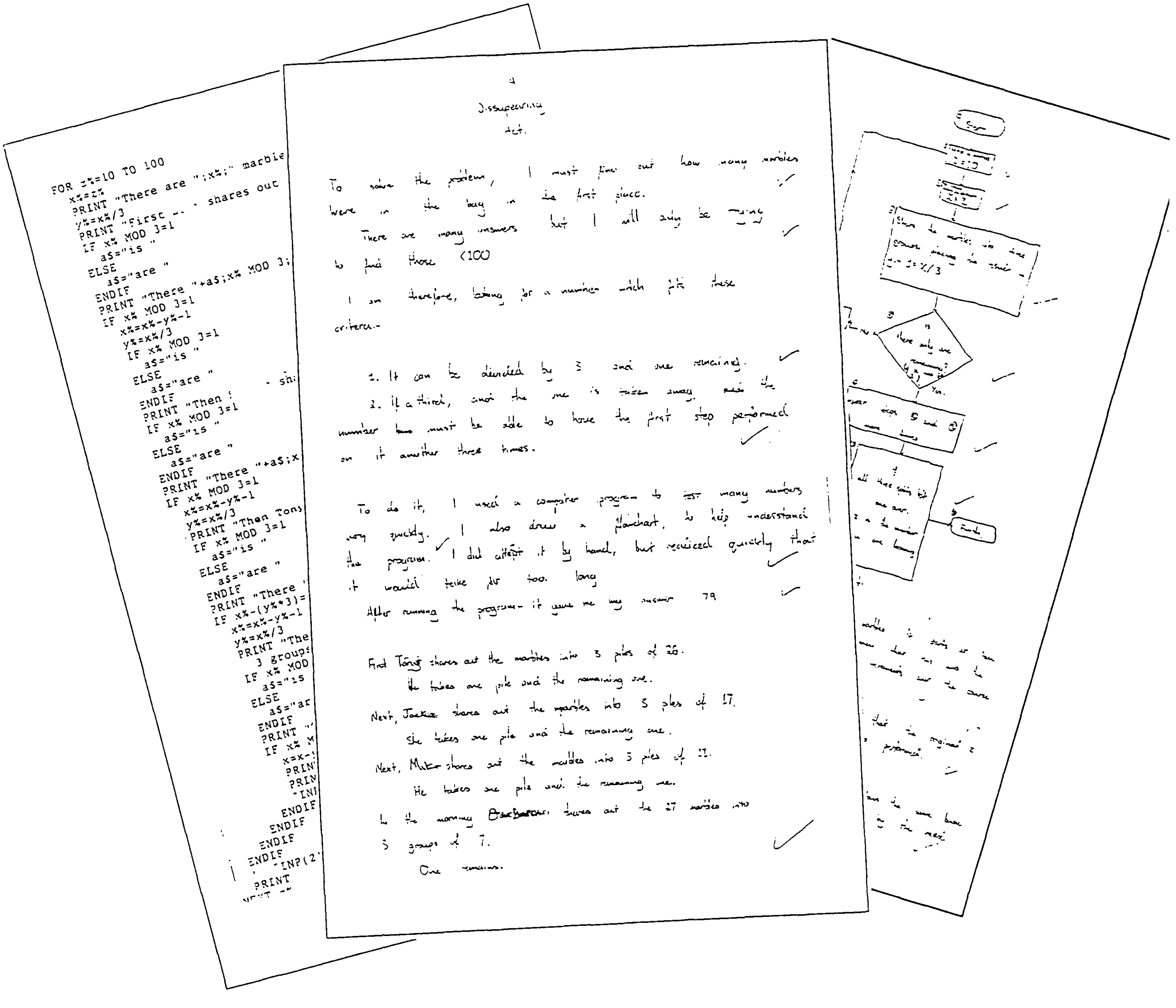
$m = \frac{3}{2}\,(\frac{3}{2}\,[\frac{3}{2}(3n+1) + 1] + 1) + 1$

continued/

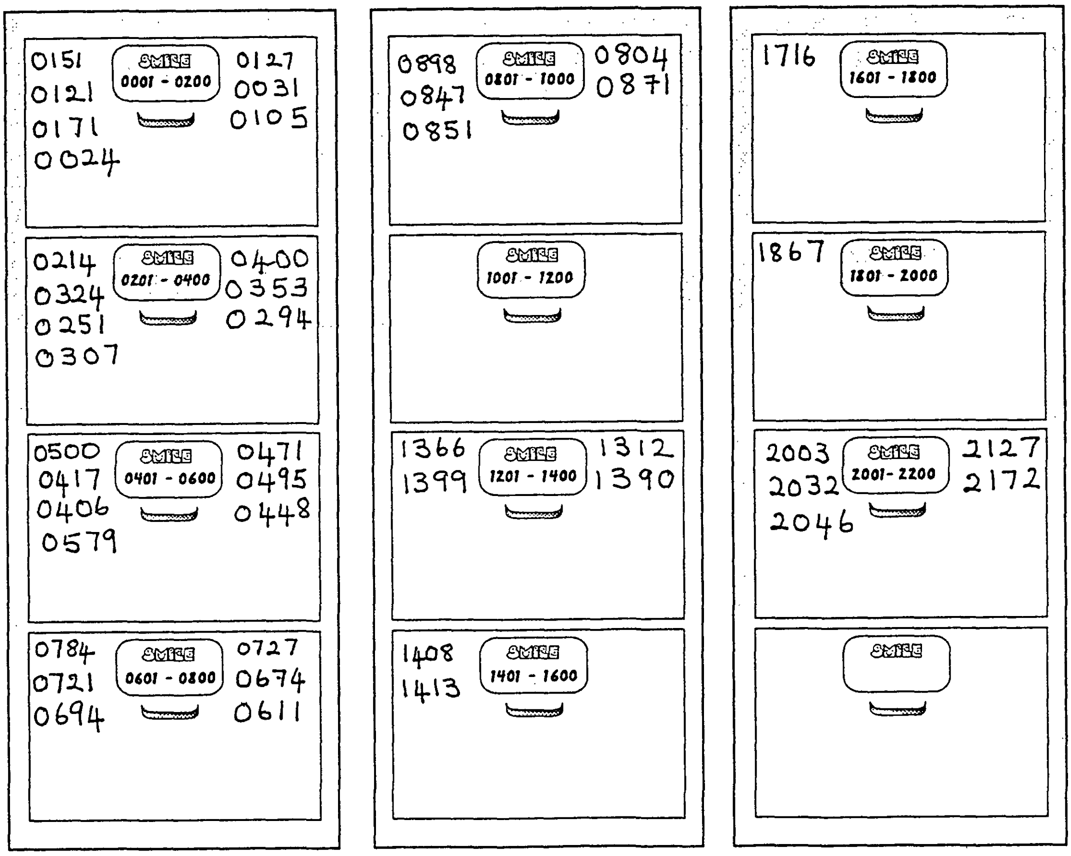
2130 A Disappearing Act (cont)

A simpler method of solution would be to use a "trial and improvement" technique. A spreadsheet might help produce data more quickly, or you may choose to write a program using a computer or graphic calculator.

In 1992, Ben Thomas of George Greens School wrote a computer program to solve this problem. Here is his report.



2131 Filing Cards



2132 Cutting Corners

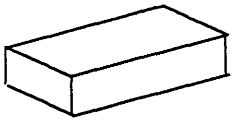
You can approach this investigation in a number of ways. Here are three possible ways.

1. Group together different types of solids: Prisms, Pyramids, etc

2. For any cut look at the number of Faces, Vertices and Edges created and lost, e.g. on a cuboid cutting a single corner creates
   3 new Edges,
   1 new Face and
   3 new Vertices, but loses 1 Vertex.

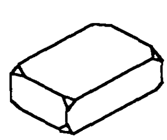
   So after cutting each corner the following Edges, Faces and Vertices are created
   8 x 3 = 24 new Edges
   8 x 1 = 8 new Faces
   8 x (3 – 1)= 8 x 2 = 16 new Vertices

3. Start with a cuboid.



F = 6
V = 8
E = 12

Cut each corner.



F = 14
V = 24
E = 36

Cut each corner again . . . and again.

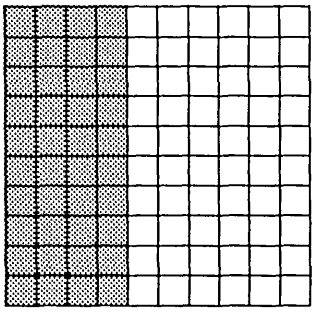
? ?

Will the angle of the cut make a difference?
How about the size of the cut?

2133 Out of 100

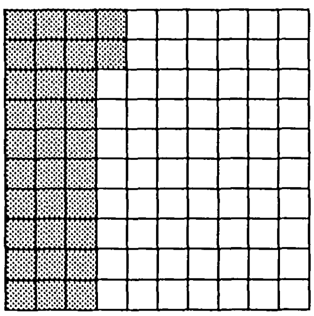
1. $\frac{30}{100} = 30\%$

2. $\frac{45}{100} = 45\%$

3. 

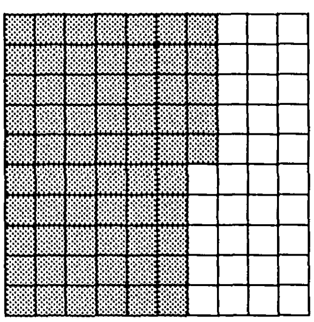
$\frac{40}{100} = 40\%$

4. $\frac{25}{100} = 25\%$

5. 

$\frac{32}{100} = 32\%$

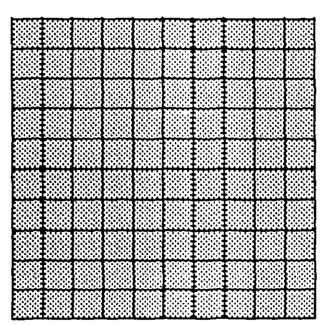
6. $\frac{72}{100} = 72\%$

7. 

$\frac{65}{100} = 65\%$

8. $\frac{78}{100} = 78\%$

9. $\frac{15}{100} = 15\%$

10. 

$\frac{100}{100} = 100\%$

2134 Similar Rectangles?

2. One set of similar rectangles contains rectangles C, F and H. The other set contains D, E and G.

3.

Rectangle	Long Side	Short Side	Ratio Long Side : Short Side
C	6	4	6 : 4 = 3 : 2
F	9	6	9 : 6 = 3 : 2
H	3	2	3 : 2

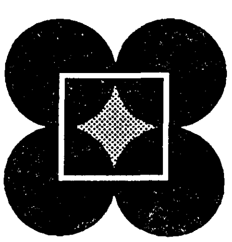
Rectangle	Long Side	Short Side	Ratio Long Side : Short Side
D	8	6	8 : 6 = 4 : 3
E	12	9	12 : 9 = 4 : 3
G	4	3	4 : 3

4. When you cut out your two rectangles, the diagonals of one of your rectangles should coincide with the diagonals of rectangles C, F and H. The other rectangle's diagonal will coincide with the diagonals of rectangles D, E and G.

Another check to see whether your rectangles are similar, is to look at the ratio of the Long Side : Short Side. The ratio for one of your rectangles should be 3 : 2 in its lowest terms. The ratio for your other rectangle should be 4 : 3 in its lowest terms.

2135 Grey Areas

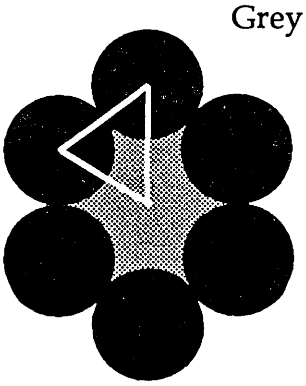
The grey area for 4 circles.



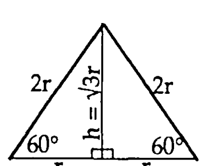
Grey area = (Area of square) – $(4 \times \frac{1}{4}$ area of circle)

$$= (2r)^2 - 4 \times \tfrac{1}{4}(\pi r^2)$$
$$= 4r^2 - \pi r^2$$
$$= r^2(4 - \pi)$$

The grey area for 6 circles.



Grey area = (Area of 6 equilateral triangles) – $(6 \times 2 \times \frac{1}{6}$ area of circle)



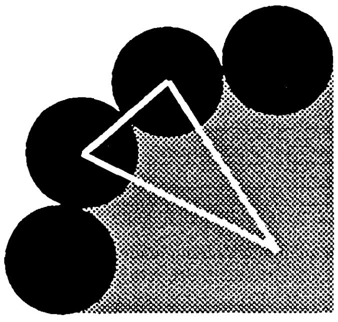
$$= 6[\tfrac{1}{2}(2r \times \sqrt{3}r)] - 12 \times \tfrac{1}{6}\pi r^2$$
$$= 6(\sqrt{3}r^2) - 2\pi r^2$$
$$= 2r^2(3\sqrt{3} - \pi)$$

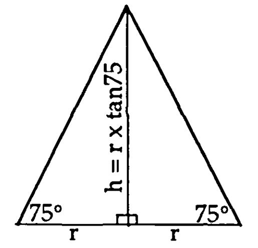
continued/

2135 Grey Areas (cont)

The grey area for 12 circles.

Grey area = (Area of 12 isosceles triangles) – 12 (2 x $\frac{5}{24}$ area of circle)





= $12(\frac{1}{2} \times 2r \times r\tan75) - 12(\frac{5}{12} \times \pi r^2)$
= $12r^2\tan75 - 5\pi r^2$
= $r^2(12\tan75 - 5\pi)$

The pattern on the outside of this activity is made up of "4 circles", "6 circles" and "12 circles" patterns.
There are:-

3 complete,	5 x ½ and 2 x ¼	"12 circles" pattern	= 6 x "12 circles" pattern.
9 complete,	6 x ½	" 6 circles" pattern	= 12 x" 6 circles" pattern.
14 complete,	5 x ½ and 2 x ¼	"4 circles" pattern	= 17 x "4 circles" pattern.

The amount of grey = $6r^2(12\tan75 - 5\pi) + 24r^2(3\sqrt{3} - \pi) + 17\,r^2(4 - \pi)$

If you decide to investigate the grey area for "n circles" pattern, you may wish to look back at your answers for the "4 circles" and "6 circles" and rewrite them in a similar form to the "12 circles" pattern.

2136 What could x be?

1.

Guess x	x^2	3x	$x^2 - 3x$	= 18
-1	1	-3	$1 - {}^-3 = 4$	too small
-2	4	-6	$4 - {}^-6 = 10$	too small
-3	9	-9	$9 - {}^-9 = 18$	✔

The two solutions to $x^2 - 3x = 18$ are x = 6 or x = -3.

2. x = -2, x = 5

3. x = 2, x = -4

4. x = 1, x = 7

5. x = -3, x = -10

6. x = -4,

7. x = -1, x = 4

8. x = 1.5, x = -2.5

continued/

2136 What could x be ? (cont)

9. $x = \frac{1}{3}$, $x = -4$

10. $x = +4.583$, $x = -4.583$ correct to 3 decimal places.

2137 Using Sine and Cosine 1

1. The hypotenuse is 5.
   The opposite is 5 $\sin\theta°$.
   The adjacent is 5 $\cos\theta°$.

2. The hypotenuse is 4.5.
   The opposite is 4.5 $\sin\theta°$.
   The adjacent is 4.5 $\cos\theta°$.

$a = 4 \sin 33° \approx 2.179$
$b = 4 \cos 33° \approx 3.355$
$c = 10 \sin 65° \approx 9.063$
$d = 10 \cos 65° \approx 4.226$
$e = 84 \sin 67.5° \approx 77.606$
$f = 84 \cos 67.5° \approx 32.145$
$g = 7.7 \sin 10° \approx 1.337$
$h = 7.7 \cos 10° \approx 7.583$
$i = 3 \sin 18° \approx 0.927$
$j = 3 \cos 18° \approx 2.853$
$k = 8 \sin 70° \approx 7.518$
$m = 8 \cos 70° \approx 2.736$

$m \times \cos 37° = 8$

$$m = \frac{8}{\cos 37°}$$

$m \approx 10.017$
$p = 10.017 \sin 37° \approx 6.028$

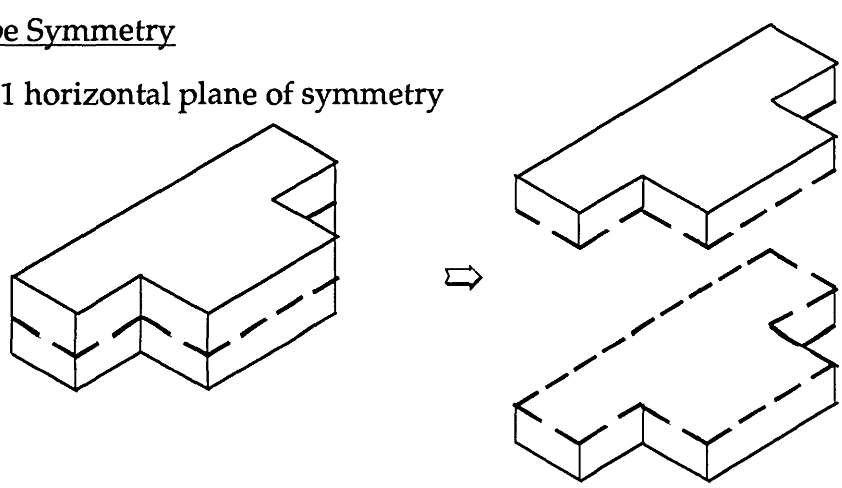
2138 Which Hand works the Hardest?

Which letter occured the most?

How did you display your results?

2139 Tricube Symmetry

1. It has 1 horizontal plane of symmetry



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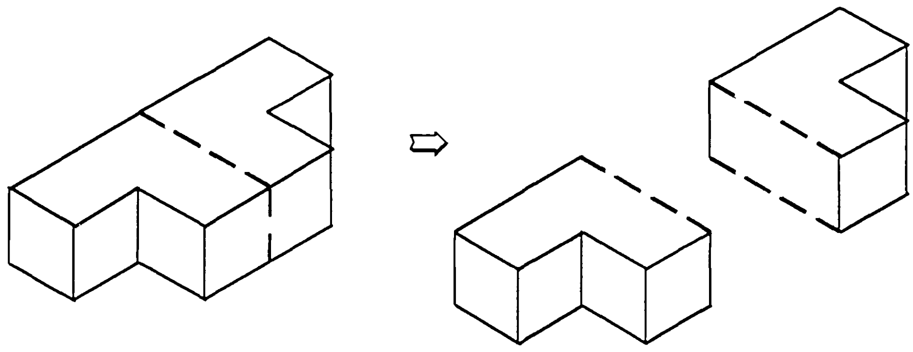
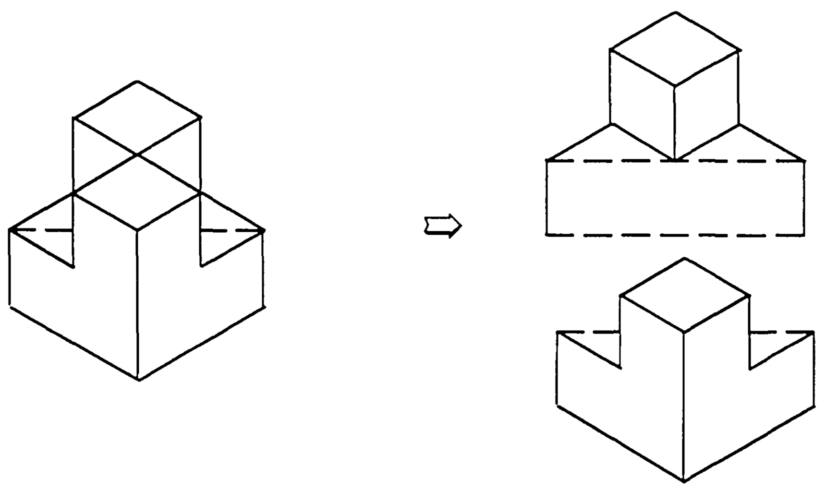
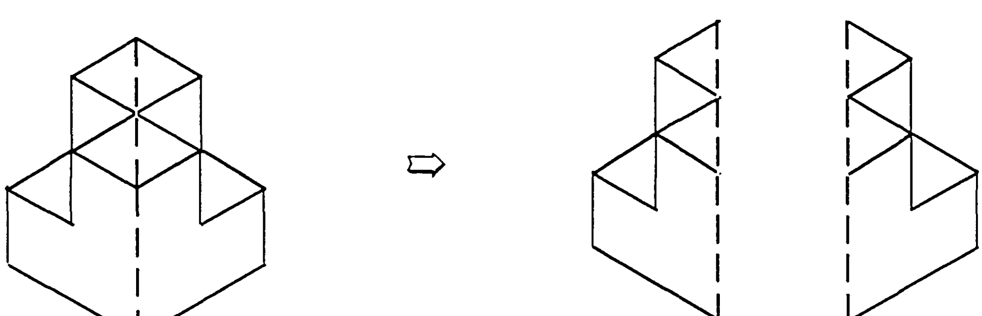
2139 Tricube Symmetry (cont)

It has 1 vertical plane of symmetry

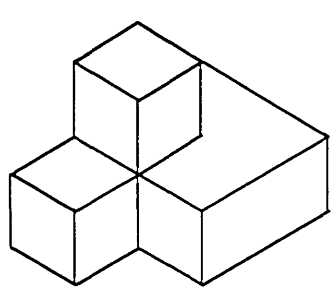


2. This shape has 2 planes of symmetry





This is an example of a shape with no planes of symmetry.



Did you find a shape with 3 planes of symmetry?

There are many possible solutions, one of ours using 4 tricubes had 5 planes of symmetry and using 9 tricubes we made 9 planes of symmetry.

2140 Quadratic Solutions

All answers are given to 1 decimal place, as using a graphical method does not allow you to read off a more accurate answer.

1.	$x^2 - 3x + 2 = 12$	has two solutions	$x = 5$	or	$x = -2$
	$x^2 - 3x + 2 = 0$	has two solutions	$x = 2$	or	$x = 1$
	$x^2 - 3x + 2 = 8$	has two solutions	$x = 4.4$	or	$x = -1.4$
	$x^2 - 3x + 2 = 3x - 3$	has two solutions	$x = 1$	or	$x = 5$
	$x^2 - 3x + 2 = 3 - x$	has two solution	$x = 2.4$	or	$x = -0.4$

2.	a)	$4x^2 + 8x + 4 = 0$	has one solutions	$x = -1$		
	b)	$4x^2 + 8x + 4 = 8$	has two solutions	$x = -2.4$	or	$x = 0.4$
	c)	$4x^2 + 8x + 4 = -5$	has no solutions.			
	d)	$4x^2 + 8x + 4 = 4$	has two solutions	$x = -2$	or	$x = 0$
	e)	$4x^2 + 8x + 4 = 2x + 10$	has two solutions	$x = -2.2$	or	$x = 0.7$
	f)	$4x^2 + 8x + 4 = 2x - 3$	has no solutions.			
	g)	$4x^2 + 8x + 4 = x^2 + x + 5$	has two solutions	$x = -2.5$	or	$x = 0.1$
	h)	$4x^2 + 8x + 4 = -x^2 + x - 5$	has no solutions.			

The solutions given for Question 3 and 4 are all horizontal lines. If your answers have included graphs of the form $y = mx + c$, or $y = ax^2 + bx + c$, check them with a graphic calculator, or with your teacher.

3. The graph of $y = 3x^2 + 12x + 12$ has a minimum value when $x = -2$ and $y = 0$.

   a) As long as the value of $y > 0$, it will cross $y = 3x^2 + 12x + 12$ in two places. Any straight line graph, such as $y = 1$, will have two points of intersection.

   b) The straight line graph which touches the minimum value will only have one point of intersection. The graph $y = 0$ will only have one solution.

   c) As long as the value of $y < 0$, it will not intersect $y = 3x^2 + 12x +12$. Any straight line graph such as $y = -3$ will have no solution.

4. The graph of $y = -x^2 - 2x + 2$ has a maximum value when $x = -1$ and $y = 3$.

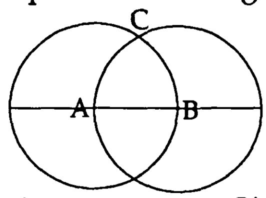
   a) As long as the value of $y < 3$, it will cross $y = -x^2 - 2x + 2$ in two places. Any straight line graph, such as $y = -2$, will have two points of intersection.

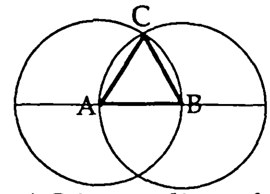
   b) The straight line graph which touches the maximum value will only have one point of intersection. The graph $y = 3$ will only have one solution.

   c) As long as the value of $y > 3$, it will not intersect $y = -x^2 - 2x + 2$. Any straight line graph such as $y = +4$ will have no solutions.

<u>2141 Constructive Designs</u>

a) Equilateral Triangle





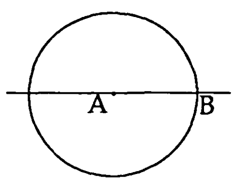
Circle 1
Centre A
Radius AB

Circle 2
Centre B
Radius BA

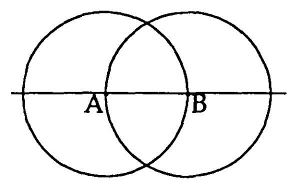
AC is a radius of Circle 1
BC is radius of Circle 2
AB is a radius of both circles.

Because both circles have the same radius AB, AB = AC = BC. Therefore ABC is an equilateral triangle.

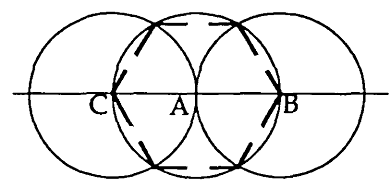
b) Hexagon



Draw a circle, radius AB. Extend AB in both directions.

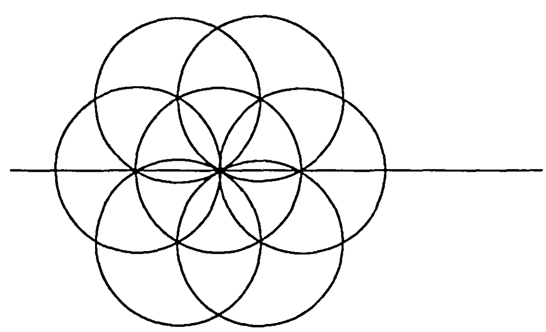


Draw a circle centre B, radius BA.



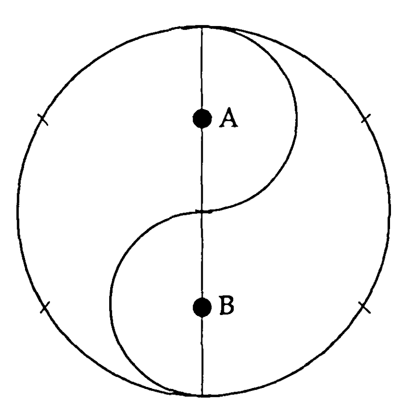
Draw another circle centre C, radius CA. Join up the intersections as marked to form a hexagon.

You may wish to create this floral pattern by drawing more circles.



c) Yin Yang symbol

This one is rather different because the two small circles cannot be accurately constructed.
It can be constructed in this way.

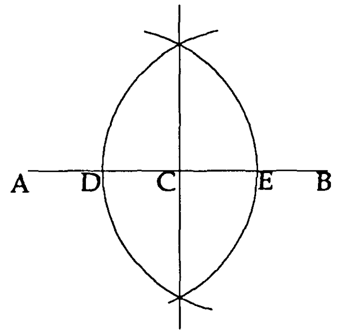


- Draw a large circle.
- Draw a vertical diameter.
- Bisect the radii to find A and B using perpendicular bisectors.
- Using A and B as centres, draw two semicircles.
- Draw two small circles using centres A and B.

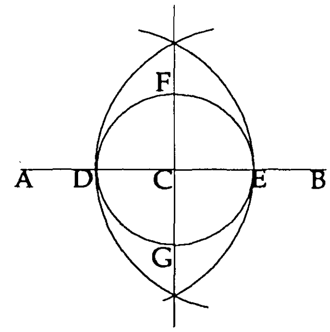
continued/

d) Square

Draw line AB

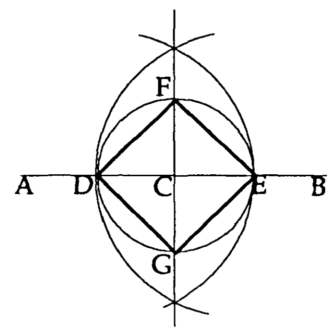


Construct the perpendicular bisector of AB and label C, D, and E as shown.



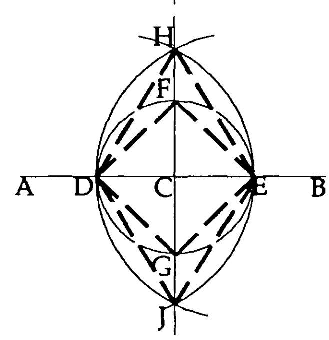
Draw a circle, centre C, radius CE (or CD). Label F and G.

Join DFEG to make a square.

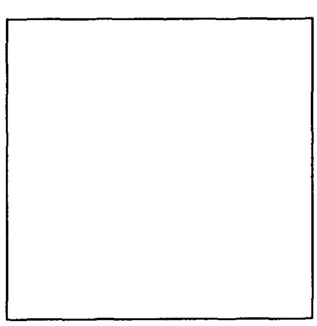


To make a kite or a rhombus with this construction, label H and J as shown.

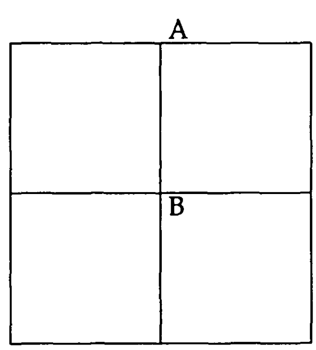
HDGE or FEJD make a kite and HDJE makes a rhombus.



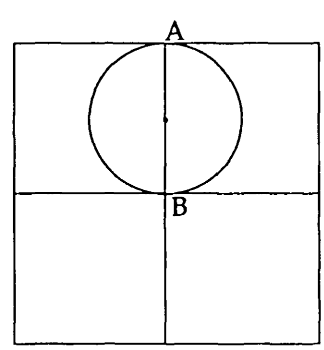
e) Half and half



Draw a square



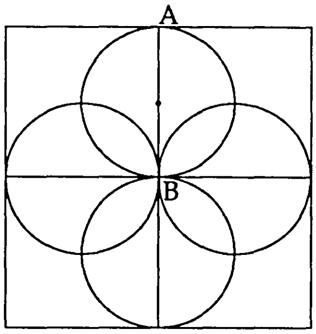
Divide it into 4 smaller squares. Label points A, and B.



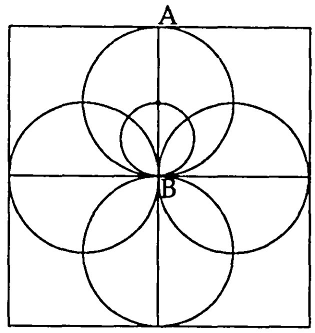
Draw a circle with centre ½ way along AB.

continued/

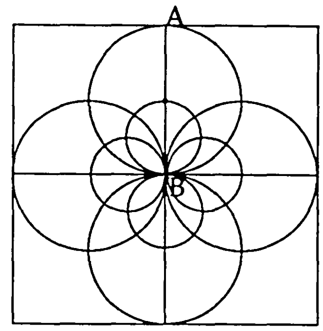
2141 Constructive Designs (cont)



Repeat this on all 4 sides.



Draw a circle with centre ¼ way along the line AB.

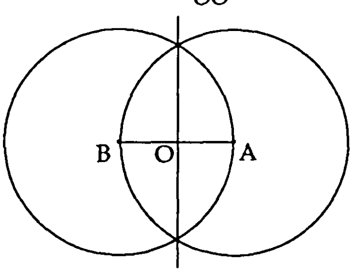


Repeat on all 4 sides.

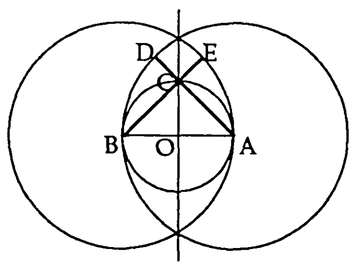
Shade in the appropriate regions.

It is half black and half white. To prove this, use the vertical or horizontal line of symmetry. Check that each shaded region has a corresponding unshaded region.

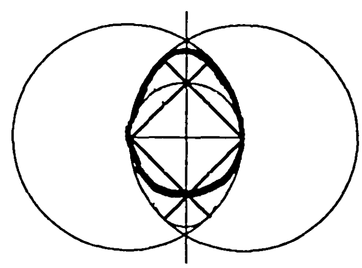
f) Moss' Egg



Draw a line AB. Draw a circle, centre A, radius AB. With centre B and radius BA, draw another circle. Use the intersection of the arcs to construct the perpendicular bisector of AB.



Draw circle, centre O, radius OA. Join A to C and extend to meet the large circle at D. Similarly join B to C to meet circle at E.

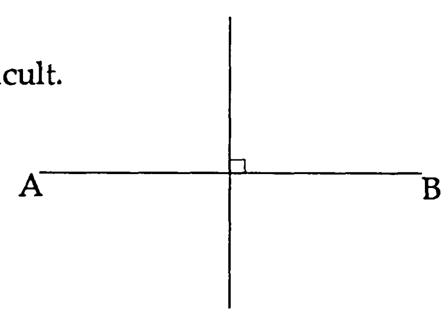


Draw an arc, centre C, radius CD, from D to E. This is Moss' Egg. To draw the ellipse, repeat these steps at the bottom of the circle.

g) Pentagon
h) Decagon

These two are more difficult.

For both of them start with the horizontal line AB and draw the perpendicular bisector.



If you would like to explore other constructions, Robert Dixon's book Mathographics (ISBN 0631 148272) has lots of ideas.

2142 Making Circles

2. Here are some results.

Circumference	Diameter	C ÷ D
20 cm	6.5cm	3.076923
18cm	6cm	3
16cm	5cm	3.2
24cm	7.5cm	3.2

The answers to C ÷ D are 3 to 1 significant figure (1 s.f.).

- An 8-figure calculator displays π = 3.1415927 to 8 significant figures.
- A 10-figure calculator displays π = 3.141592654 to 10 significant figures.

3. Answers using π = 3.14. | Answers using the π button.

a) π x D = 3.14 x 3.2cm
= 10.048cm
= 10.0cm (3 s.f.)

π x D = π x 3.2cm
= 10.053096cm
= 10.1cm (3 s.f.)

	Answers using π = 3.14		Answers using the π button
b)	3.14 x 5.1cm = 16.014cm	= 16.0cm (3 s.f.)	16.022123cm = 16.0cm (3 s.f.)
c)	3.14 x 4.0cm = 12.56 cm	= 12.6cm (3 s.f.)	12.566371cm = 12.6cm (3 s.f.)
d)	3.14 x 6.3cm = 19.782cm	= 19.8cm (3 s.f.)	19.792034cm = 19.8cm (3 s.f.)
e)	3.14 x 4.6cm = 14.444cm	= 14.4cm (3 s.f.)	14.451326cm = 14.5cm (3.s.f.)

2143 Percentages of Money

Original amount	**50%**	**25%**	**75%**	**100%**
60p	30p	15p	45p	60p
80p	40p	**20p**	**60p**	**80p**
£16	**£8**	**£4**	**£12**	**£16**
84p	**42p**	21p	**63p**	**84p**
£1	**£0.50 or 50p**	**£0.25 or 25p**	**£0.75 or 75p**	**£1 100p**
£128	**£64**	**£32**	**£96**	**£128**
£5	**£2.50**	£1.25	**£3.75**	**£5**
£1.80	**£0.90 or 90p**	**£0.45 or 45p**	**£1.35**	£1.80
£6.20	**£ 3.10**	**£1.55**	**£4.65**	**£6.20**
40p	20p	**10p**	**30p**	**40p**
£5.60	**£2.80**	£1.40	**£4.20**	**£5.60**

continued/

2143 Percentages of Money (cont)

25% of £2.40 is 60p or £0.60

75% of 96p is 72p

50% of £3.80 is £1.90

25% of £26 is £6.50

75% of £5.40 is £4.05

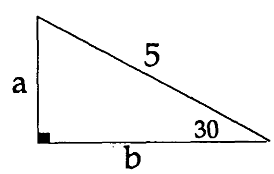
100% of £7.57 is £7.57

- When you used your calculator to check your answers it will have shown £0.50 as 0.5 and £2.50 as 2.5, etc. Do you know why? If not ask your teacher.
- The % button works differently on different calculators. If you do not understand how yours works, ask your teacher or look in the manual.

2144 Using Sine and Cosine 2

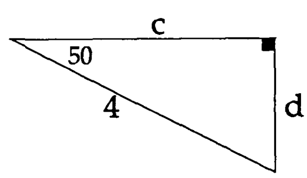
If your answers differ slightly from the ones below, it may be because you have rounded off too early. In your calculations, you should not use any rounded values. You should round off to 3 decimal places (3 d.p.) **only** at the very end of your calculations.

It helps to label the sides.
All answers are given correct to 3 decimal places, where appropriate.

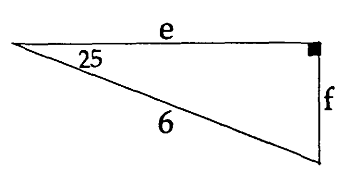
1.


a = 5 sin 30
= 2.5

b = 5 cos 30
= 4.330127
= 4.33

2.


c = 2.571
d = 3.064

3.


e = 5.438
f = 2.536

4. Having found r = 8.1213462
The third side = r cos 38
= 8.1213462 x 0.7880108
= 6.3997081
= 6.4

5. $r = \dfrac{7}{\sin 40}$
= 10.890067
= 10.890
Third side = r cos 40
= 10.890067 x 0.7660444
= 8.3422752
= 8.342

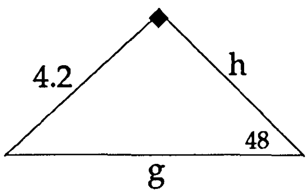
continued/

2144 Using Sine and Cosine 2 (cont)

6. $r = \dfrac{7.5}{\sin 20}$
$= 21.928533$
$= 21.929$

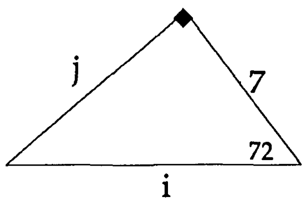
Third side $= r \cos 20$
$= 21.928533 \times 0.9396926$
$= 20.606081$
$= 20.606$

7.



g = 5.652
h = 3.782

8.

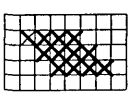


i = 22.652
j = 21.544

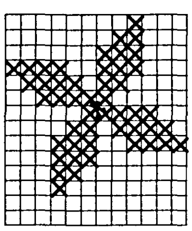
2145 Cross Stitch

Below is one description of how the motifs were transformed to create each of the patterns. Discuss with your teacher if your description is different.

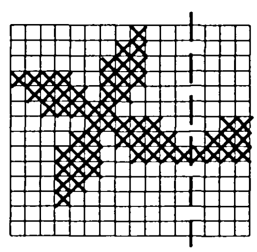
The motif...



is rotated, about * through 90°, 180° and 270°

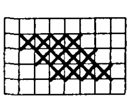


then reflected in the line...

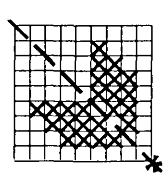


then the whole shape is translated by the vector $\begin{pmatrix}24\\0\end{pmatrix}$

The motif ...

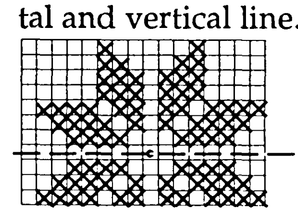


is reflected in this diagonal ...

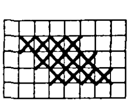


then rotated through 90° clockwise about * . . .

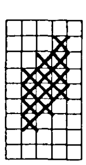
then the whole shape is reflected in the horizon tal and vertical line.



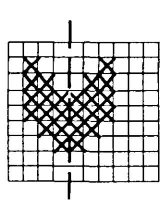
The motif ...



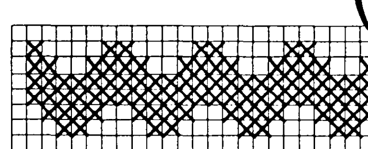
is first rotated by 90°...



then reflected...



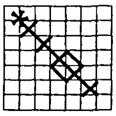
then the whole shape is repeatedly translated by the vector $\begin{pmatrix}6\\0\end{pmatrix}$



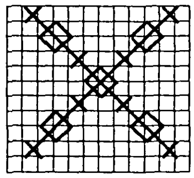
continued/

2145 Cross Stitch (cont)

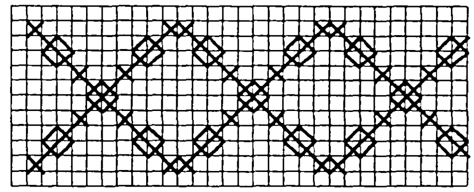
This motif



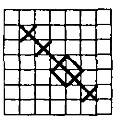
is rotated through 90° 180° and 270° about *.



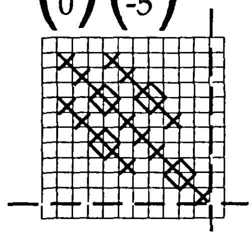
The whole shape is then repeatedly reflected.



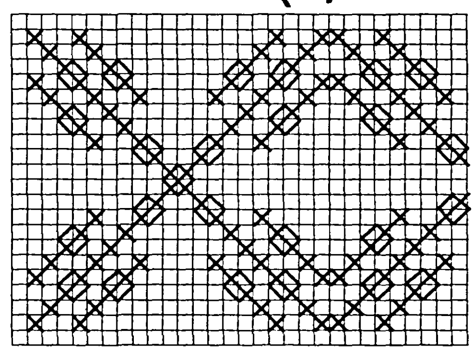
This motif . . .



is used to create a new motif by using vectors $\begin{pmatrix}3\\0\end{pmatrix}\begin{pmatrix}5\\-5\end{pmatrix}$ and $\begin{pmatrix}0\\-3\end{pmatrix}$.



The new motif is then reflected in the horizontal and vertical lines, and then repeatedly translated by the vector $\begin{pmatrix}20\\0\end{pmatrix}$.

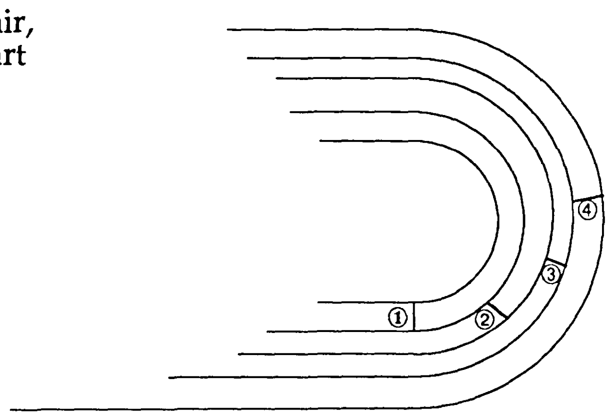


2146 It's not fair!

1

Lane Number	Distance around the track. (Your answers should be between)
1	390 – 410m
2	420 – 440m
3	450 – 480m
4	480 - 510m

2. To make the race fair, each girl should start in a different place. That way they will all run the same distance.



3. The width is 5m. This is not realistic, it is much too wide. Why do you think the worksheet was drawn like this?

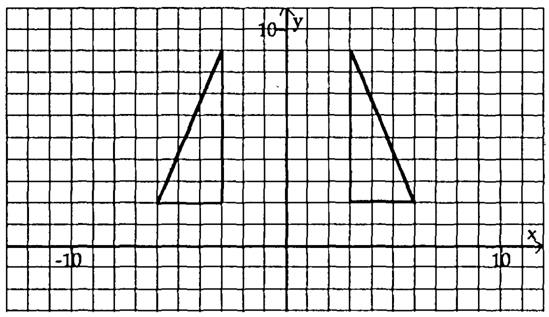
2147 Odd Animal

You should have drawn a camel.

2148 Transforming Triangles

1. The mapping $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ y \end{pmatrix}$ simple means you change the sign of the x co-ordinate.

   The three vertices of the triangle become (-3, 2), (-6, 2) and (-3, 9).
   The following diagram shows the transformation.



The transformation is a reflection in the y axis or the line x = 0.

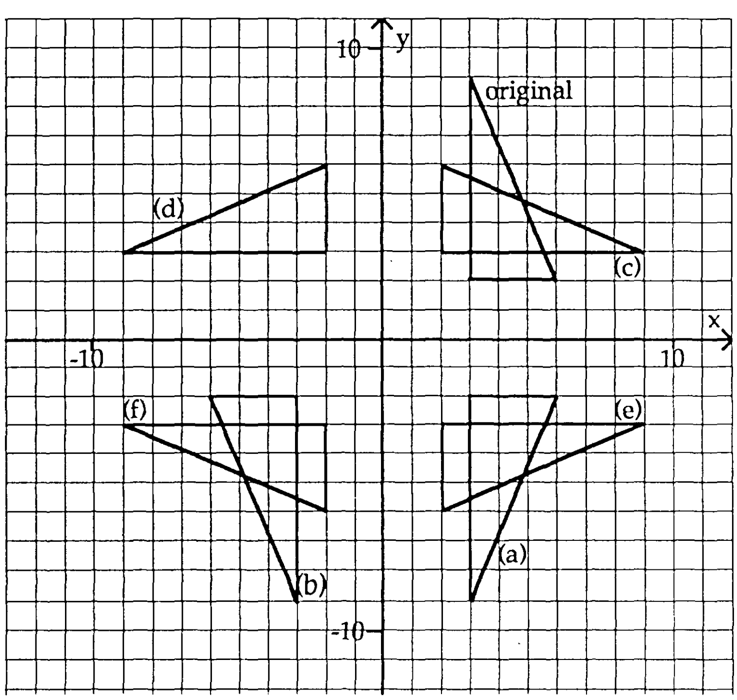
The mapping which returns the transformed triangle to its original position is called the inverse.
The points of the new triangle can now be expressed as $\begin{pmatrix} x \\ y \end{pmatrix}$.

The inverse mapping must change the points (-3, 2) (-6, 2) (-3, 9) into (3, 2) (6, 2) and (3, 9).

The mapping $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ y \end{pmatrix}$ does this.

When the mapping and its inverse are the same it is called a self inverse mapping.

2.



continued/

2148 Transforming Triangles (cont)

Transformation	Description of Mapping	Inverse
(a)	Reflection in x axis or line y = 0	$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ -y \end{pmatrix}$
(b)	Rotation of 180° about the point (0, 0)	$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \end{pmatrix}$
(c)	Reflection in the line y = x	$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y \\ x \end{pmatrix}$
(d)	Anticlockwise rotation of 90° about the point (0, 0)	$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y \\ -x \end{pmatrix}$
(e)	Clockwise rotation of 90° about the point (0, 0)	$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ x \end{pmatrix}$
(f)	Reflection in the line y = -x	$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ -x \end{pmatrix}$

2149 Circle Coverage

The calculator display for π shows π as 3.1415927.
This is the approximate value for π correct to 7 decimal places.

1.

Radius	Area using π = 3.142	Area using π button on calculator corrected to 3 decimal places
2cm	12.568cm^2	12.566cm^2
3cm	28.278cm^2	28.274cm^2
4cm	50.272cm^2	50.265cm^2

2.

5cm	78.55cm^2	78.540cm^2
6cm	113.112cm^2	113.097cm^2
7cm	153.958cm^2	153.938cm^2
8cm	201.088cm^2	201.062cm^2
9cm	254.502cm^2	254.469cm^2

3. Doubling the radius does not double the area. When the radius is doubled, the area is four times larger.

4. Area of a circle with radius 18cm = 1018.008cm^2 (using π = 3.142), or
= 1017.876cm^2 (using the π button, correct to 3 d.p.).

Another way to find the area is to use the area of the circle radius 9 cm.
When the radius is doubled, the area is four times larger.
Area of circle with radius 9cm = 254.502cm^2 (using π = 3.142)
Area of circle with radius 18cm = 254.502cm^2 x 4
= 1018.008cm^2

5. Area of circle with radius 0.5cm = 0.786cm^2 (using π = 3.14), or
= 0.785cm^2 (using the π button, correct to 3.d.p.).

2150 Pizza Paradise

1. Your answers will depend upon how large your appetite is.

2. The table of results has been created by using the formula for the area of a circle.

$$\text{Area of a circle} = \pi r^2$$

Diameter	Radius	Area using $\pi = 3.142$.	Area using the π button corrected to 3 decimal places
7 inches	3.5 inches	38.49 inches^2	38.485 inches^2
10 inches	5 inches	78.55 inches^2	78.540 inches^2
12 inches	6 inches	113.112 inches^2	113.097 inches^2

3. The medium pizza is approximately 2 times larger than the small pizza. The large pizza is approximately 3 times larger than the small pizza.

4. If your estimates were very different to those given in the answer to Question 1, and you still think that they are correct, please discuss them with your teacher.

5. No. Look at what happens when a medium pizza's diameter is doubled.

Pizza	Diameter	Radius	Area using $\pi = 3.142$
Medium	10 inches ↘	5 inches ↘	78.55inches^2 ↘
	x 2	x 2	x 4
	20 inches ↙	10 inches ↙	314.2 inches^2 ↙

Doubling the diameter has made the area of the pizza 4 times larger.

6. A small pizza serves 2 people.
A large pizza is 3 times larger, so a large pizza serves 6 people (3 x 2).
For 40 people you would need 40 ÷ 6 pizzas = 6.66666667 pizzas!
This answer is **not** a sensible answer as you cannot buy .66666667 of a pizza.
To feed 40 people, you would need to buy **7 pizzas**.

7. The small pizza serves 2 people, so the festival pizza will have the same area as 50 small pizzas.

Area of small pizza = 38.49 inches^2
Area of festival pizza = 38.49 inches^2 x 50 = 1924.5 inches^2

$$\text{Area of circle} = \pi r^2$$
$$1924.5 = \pi r^2$$
$$\frac{1924.5}{\pi} = r^2$$

Using $\pi = 3.14$	Using the π button
$612.8980892 = r^2$	$612.587376 = r^2$
$\sqrt{612.8980892} = r$	$\sqrt{612.587376} = r$
$24.75677865 = r$	$24.75050254 = r$
49.5135573 = diameter	49.50100508 = diameter

Approximate diameter of festival pizza to the nearest inch = 50 inches.

2151 The Root of the Problem

1.

Number (edge length)	Cube (Volume)
1	$1^3 = 1 \times 1 \times 1 = 1$
2	$2^3 = 2 \times 2 \times 2 = 8$
3	$3^3 = 3 \times 3 \times 3 = 27$
4	$4^3 = 4 \times 4 \times 4 = 64$
5	$5^3 = 5 \times 5 \times 5 = 125$
6	$6^3 = 6 \times 6 \times 6 = 216$
7	$7^3 = 7 \times 7 \times 7 = 343$
8	$8^3 = 8 \times 8 \times 8 = 512$
9	$9^3 = 9 \times 9 \times 9 = 729$
10	$10^3 = 10 \times 10 \times 10 = 1000$

2.

Cube Root		Number
$\sqrt[3]{1}$	=	1
$\sqrt[3]{8}$	=	2
$\sqrt[3]{27}$	=	3
$\sqrt[3]{64}$	=	4
$\sqrt[3]{125}$	=	5
$\sqrt[3]{216}$	=	6
$\sqrt[3]{343}$	=	7
$\sqrt[3]{512}$	=	8
$\sqrt[3]{729}$	=	9
$\sqrt[3]{1000}$	=	10

3. a) The volume is $216cm^3$
   b) The volume is $125cm^3$
   c) The volume is $729cm^3$

4. a) Edge length of 3cm
   b) Edge length of 8cm
   c) Edge length of 7cm

continued/

2151 The Root of the Problem (cont)

5. a) The volume of a cube with edge length 7.9cm will be between the volume of a cube with edge length 7cm and the volume of a cube with edge length 8cm. i.e. between $343cm^3$ and $512cm^3$.

   As 7.9 is nearer 8cm, the volume would be nearer to the volume of the 8cm cube, approximately $500cm^3$.

   b) The volume of a cube with edge length 8.5cm will be between 8^3 and 9^3. 8.5 is mid-way between 8 and 9, so an answer between 600 - $640cm^3$ would be reasonable.

   c) The volume of a cube with edge length 3.3cm will be between 3^3 and 4^3. 3.3 is closer to 3 than 4, so an answer between 30 - $40cm^3$ would be reasonable.

6. a) $343 < 370 < 512$
   $7^3 < 370 < 8^3$
   Therefore, $\sqrt[3]{370}$ must lie between 7cm and 8cm.
   A reasonable answer would be between 7.1 - 7.4cm.

   b) $9^3 < 920 < 10^3$
   A reasonable answer would be between 9.5 - 9.9cm.

   c) A reasonable answer would be between 3.1 - 3.5cm.

To find out the answer use your calculator, using the [y^x] button if it has one.

7.9^3 → [7] [.] [9] [y^x] [3] [=]

2152 How Likely?

A → 6
B → 10
C → 8
D → 9
E → 7
F → 5
G → 3
H → 4
I → 2
J → 1

2153 £1 Search

There are 17 ways - how many did you find?

2154 Sum Dice

These answers to the puzzle were obtained by a student, who threw the numbers 3, 3, 4, 4, 5 and 6 on the dice. They show one way she managed to make the numbers 1 - 10.

1. $(3 - 3) + (4 - 4) + (6 - 5)$
   $0 + 0 + 1 = 1$

2. $(6 - 5) + [(4 - 3) \div (4 - 3)]$
   $1 + [\,1 \div 1\,]$
   $1 + 1 = 2$

3. $(6 - 5) + (4 - 3) + (4 - 3)$
   $1 + 1 + 1 = 3$

4. $(4 + 4 - 5) + [6 \div (3 + 3)]$
   $3 + 1 = 4$

5. $(6 - 3) + (4 - 3) + (5 - 4)$
   $3 + 1 + 1 = 5$

6. $(6 - 3) \times (5 - 3) + (4 - 4)$
   $3 \times 2 + 0 = 6$

7. $(6 - 3) \times (5 - 3) + (4 - 4)$
   $3 \times 2 + 1 = 7$

8. $(4 + 4) + 5 \times [6 - (3 + 3)]$
   $8 + 0 = 8$

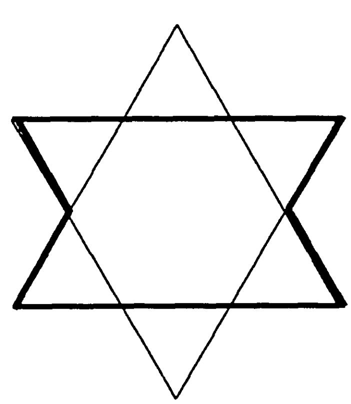
9. $(6 + 3) + (3 \times 5) \times (4 - 4)$
   $9 + 0 = 9$

10. $(6 + 4) + (3 - 3) \times (5 + 4)$
    $10 + 0 = 10$

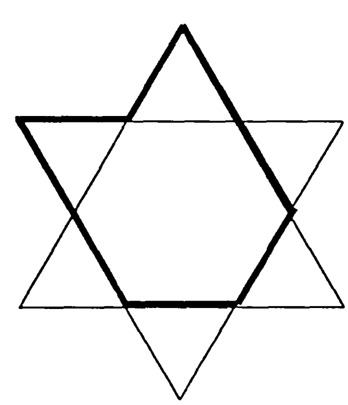
You may have tried using powers, e.g. $3^2 = 3 \times 3 = 9$, as well as +, –, x and ÷.

2155 Visualising

Did your group see these two hexagons?



A hexagon and two triangles

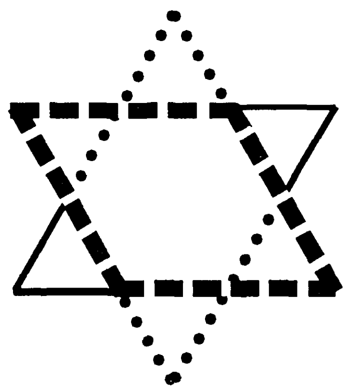


A hexagon and four triangles

continued/

2155 Visualising (cont)

Did you see overlapping shapes?

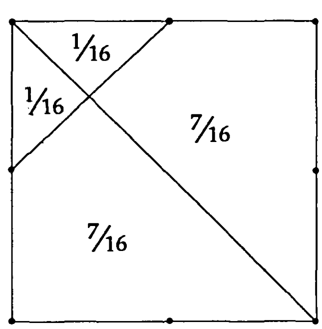


Three overlapping rhombuses

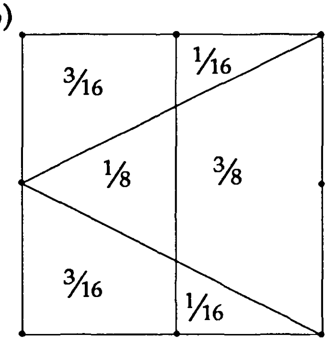
2156 Fraction Squares

1. P = ¼, Q = ⅛, R = (¼ + ¼ + ⅛) = ⅜.

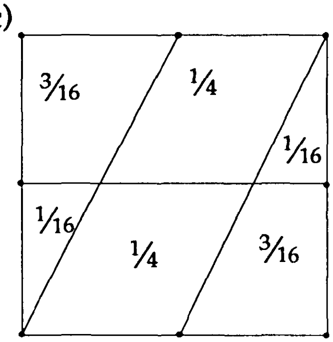
2. a)



b)

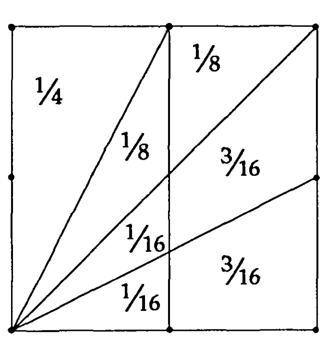


c)

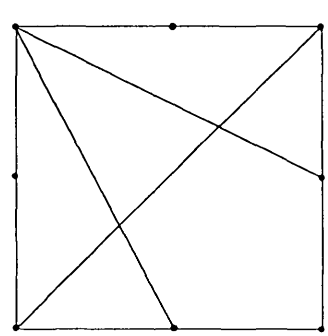


3. Your answers for each of your squares should add up to 1 whole one. Check that they do.

4.



If you have enjoyed this, here is a real challenge!



2157 Some sums for your mind

Questions	Answers	Calculator Answers
$5 - \frac{3}{7}$	Just over 4½	4.571 (to 3 d.p.)
$\frac{5}{7-3}$	One and a quarter	1.25
$7 \div (5-3)$	3 ½	3.5
$\frac{3}{7-5}$	1.5	1.5
$7 - \frac{3}{5}$	About six and a half	6.4
$3 - (7 \div 5)$	A bit more than 1½	1.6
$3 - \frac{5}{7}$	2 and a bit	2.286 (to 3 d.p.)
$\frac{3-5}{7}$	-0. something	-0.286 (to 3 d.p.)
$\frac{3-7}{5}$	$-\frac{4}{5}$	-0.8
$\frac{5-7}{3}$	$-\frac{2}{3}$	-0.667 (to 3 d.p.) or $-0.\dot{6}$
$\frac{7-5}{3}$	$\frac{2}{3}$	0.667 (to 3 d.p.) or $0.\dot{6}$
$\frac{3}{5} - 7$	-6.something	-6.4

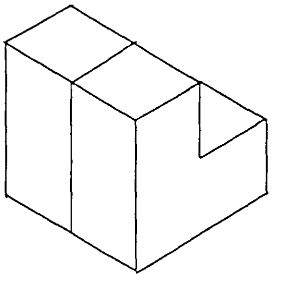
2158 Turning Green

Were you able to sort all the 35 objects into a re-cycling bin? Show your work to your teacher.

2159 Permutating Tricubes

We found 31 different permutations.

Starting with 2 at one end like this there are 5 ways altogether when the two are the same colour, 10 ways when they are not.



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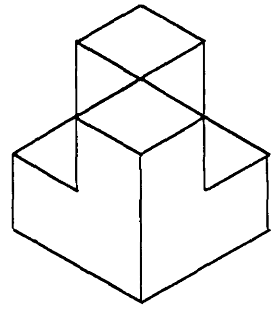
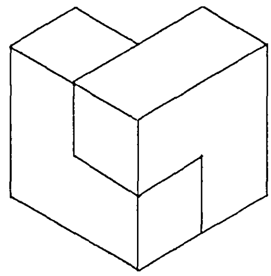
2159 Permutating Tricubes (cont)

When the first two are put together like this, there are 2 ways when the first two are the same colour, 4 ways if they are not.





When the first two are put together like this, there are 4 ways when the first two are the same, 10 ways if they are not.

2160 Folding Fractions

1. $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$ 2. $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$ 3. $\frac{1}{2}$ of $\frac{1}{5} = \frac{1}{10}$ 4. $\frac{1}{2}$ of $\frac{1}{6} = \frac{1}{12}$

5. $\frac{1}{4}$ of $\frac{1}{2} = \frac{1}{8}$ 6. $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$ 7. $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$ 8. $\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{12}$

9. You may have noticed that $\frac{1}{4}$ of $\frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$

To find a fraction of a fraction, where the numerators (top numbers) are both one, you multiply the numerators together and then multiply the denominators (bottom numbers) together.

10. $\frac{1}{2}$ of $\frac{2}{3} = \frac{2}{6}$ or $\frac{1}{3}$

11. $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$

12. $\frac{1}{2}$ of $\frac{2}{5} = \frac{2}{10}$ or $\frac{1}{5}$

13. $\frac{1}{3}$ of $\frac{3}{4} = \frac{3}{12}$ or $\frac{1}{4}$

14. To find fractions of fractions (where numerators can be any number), you multiply the numerators and then multiply the denominators.

15. Another way of saying "**of**" is "**multiply**", so an algorithm for multiplying fractions is to multiply the top numbers and multiply the bottom numbers.

$\frac{2}{3}$ of $\frac{3}{4} = \frac{6}{12}$ or $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$

Using your Fraction Ruler, can you find other fractions which are the same as (equivalent to) $\frac{6}{12}$?

2161 Shape Names

A	rectangle	All the angles of this shape are right angles. Not all the sides are equal.
B	scalene triangle	This shape has three sides. None of the angles are equal. It has no right angle.
C	equilateral triangle	This shape has three sides. The angles are all equal.
D	square	All the sides of this shape are equal. All the angles are right angles.
E	right-angled trapezium	This shape has four sides. Two sides are parallel. It has two right angles.
F	right-angled triangle	This shape has three sides. It has one right angle.
G	isosceles triangle	This shape has two equal sides. Two of the three angles are equal.

2162 Angles in Triangles

1. $a = 106°$

2. $x = 67°$
   $y = 113°$

3. $b = 38°$

4. $AB = AC = CD$
   i) $\angle ACB = 70°$ ii) $\angle BAC = 40°$ iii) $\angle ACD = 110°$
   iv) $\angle CAD = 35°$ v) $\angle BAD = 75°$

5. i) $\angle KML = 90°$ ii) $\angle KLM = 54°$

6. $\angle SPR = 130°$

7. i) $\angle ADB = 60°$ ii) $\angle BAD = 75°$ iii) $\angle DBC = 30°$

2163 Geometry Facts

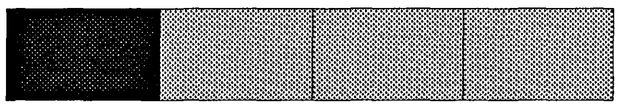
No answers required

2164 Information Displayed

1. a) 90 girls
   b) 90 boys

2. a) 120 can swim
   b) 60 cannot swim

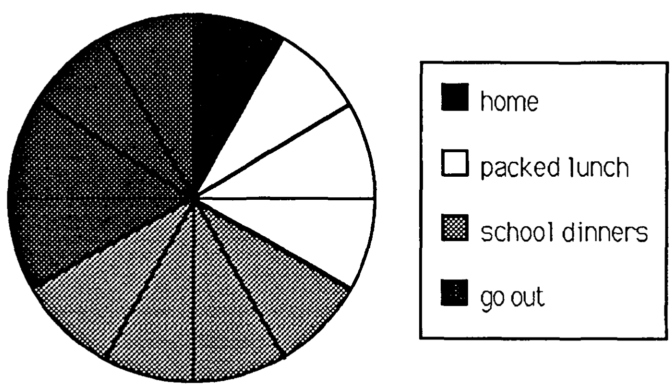
3.



4. a) 24 students play netball
   b) 72 students play hockey
   c) 48 students play football

5. a) 36 students belong to the music club
   b) 36 students belong to the homework club
   c) 18 students belong to the drama club
   d) 54 students belong to the maths club

6)



2165 Transform

No answers required.

2166 Matching Equations

Below are examples of each of the methods suggested on the activity. You may have used one method throughout or a variety of methods.

- Choosing 2 points method.

  (2, 0) and (3, 2) are the co-ordinates of two points on graph A.

  Using co-ordinate (2, 0), subsituting $x = 2$ and $y = 0$ into equation 1.

$$4x = 2y - 8$$
$$4 \times 2 = 2 \times 0 - 8$$
$$8 = 0 - 8$$

This is not right.

continued/

Using co-ordinate (2, 0) try substituting $x = 2$ and $y = 0$ into equation 2.

$y = 2x - 4$
$0 = 2 \times 2 - 4$
$0 = 4 - 4$ **This works.**

Now using co-ordinate (3, 2), try substituting $x = 3$ and $y = 2$ into equation 2.

$y = 2x - 4$
$2 = 2 \times 3 - 4$
$2 = 6 - 4$ **This works.**

So Equation 2 matches graph A. Equations 7, 8 and 12 also match graph A.

- Choosing the re-arranging method.

$4x = 2y - 8$
Divide both sides by 2.
$2x = y - 4$
Add 4 to both sides.
$2x + 4 = y$
This can be re-written as $y = 2x + 4$ and is the same as equation 5.

Linear equations can all be rearranged into the form $y = mx + c$. The value of m gives the gradient of the line and c gives the intercept (where the graph cuts the y axis).

In the equation y = 2x + 4, the gradient is 2 and the intercept is 4, so equations 1 and 5 match graph B. Equations 3 and 11 also match with graph B.

- Using the MicroSMILE program PLOTTER.

You can either plot points and see the equation of the line, or you can input equations and see them in rearranged form.

Equation 4 matches graph B. Equation 6, 9 and 10 also match graph C.

Regardless of the method used you should have found that:

Equations 2, 7, 8 and 12 match graph A. Equations 1, 3, 5 and 11 match graph B. Equations 4, 6, 9 and 10 match graph C.

- Equations which match graph D could be

$y = ½x + 2$
$2y = x + 4$
$x = 2y - 4$
$2y - x = 4$
$3y = 1½x + 6 \ldots$

2167 Range of Area

1. The lower bound of 16 = 15.5, the upper bound of 16 = 16.5.

   a) Smallest possible area $= 15.5 \times 15.5 = 240.25cm^2$
   Largest possible area $= 16.5 \times 16.5 = 272.25cm^2$
   Range of area $= 272.25cm^2 - 240.25cm^2$
   $= 32cm^2$

   b) The range of possible areas is
   2 multiplied by 'the length of the side of the square'.

   c) If n = the side of the square measured to a given unit.
   Smallest possible area $= (n - \frac{1}{2})^2 = n^2 - n + \frac{1}{4}$
   Largest possible area $= (n + \frac{1}{2})^2 = n^2 + n + \frac{1}{4}$
   Range of area $= (n^2 + n + \frac{1}{4}) - (n^2 - n + \frac{1}{4})$
   $= n^2 + n + \frac{1}{4} - n^2 + n - \frac{1}{4}$
   $= n + n$
   $= 2n$ square units.

2. **Rectangle**

   a) The range of area when the height and width is measured to the nearest centimetre, is height of rectangle plus the width of the rectangle.

   b) To prove this rule, let h = height and w = width

   Smallest possible area $= (h - \frac{1}{2})(w - \frac{1}{2})$
   Largest possible area $= (h + \frac{1}{2})(w + \frac{1}{2})$
   Range of area $= (h + \frac{1}{2})(w + \frac{1}{2}) - (h - \frac{1}{2})(w - \frac{1}{2})$
   $= (hw + \frac{1}{2}h + \frac{1}{2}w + \frac{1}{4}) - (hw - \frac{1}{2}h - \frac{1}{2}w + \frac{1}{4})$
   $= hw + \frac{1}{2}h + \frac{1}{2}w + \frac{1}{4} - hw + \frac{1}{2}h + \frac{1}{2}w - \frac{1}{4}$
   $= (h + w)cm^2$

   Circle
   The range of area when the radius is measured to the nearest centimetre is $2\pi r cm^2$.

   Triangle
   The range of area when the base and height are measured to the nearest centimetre is $\frac{1}{2}(b+h)cm^2$.

3. We found general rules for squares. If you found rules for other shapes, show these to your teacher.

   a) The range of area of a square, side n, when measured to the nearest half centimetre is $n cm^2$.
   Can you justify why this is?

   b) The range of area of a square, side n, when measured to the nearest x cm is $2xn cm^2$.

- The range of volume for a cube measured to the nearest centimetre is $(3n^2 + \frac{1}{4})cm^3$ where n is the side of the cube measured.

- The range of surface area for a cube measured to the nearest centimetre is $12n cm^2$.

Similar rules can be obtained for other 3-D shapes, check them with your teacher.

2168 Cube Root Calculator

1.

Edge Length	Cube (Volume)		
4.65	4.65 x 4.65 x 4.65	= 100.54463	too large
4.63	4.63 x 4.63 x 4.63	= 99.252847	too small
4.64	4.64 x 4.64 x 4.64	= 99.8973	too small
4.645	4.645 x 4.645 x 4.645	= 100.221	too large

To save time you can use the x^y button on your calculator.
This is the power button.

4.645 x^y 3 = 100.221

.

.

$(4.642)^3$ = 100.027 = 100 (1 d.p.)
$(4.6416)^3$ = 100.00072 = 100 (2 d.p.)

How many decimal places was your answer correct to?

2. The edge length of a cube with volume $340cm^3$ must be between 6cm and 7cm because 6 x 6 x 6 = 216 and 7 x 7 x 7 = 343. It must be nearer to 7.

Edge Length	Cube (Volume)		
6.9	6.9^3	= 6.9 x 6.9 x 6.9 = 328.509	too small
6.95	6.95^3	= 6.95 x 6.95 x 6.95 = 335.702	too small
6.98	6.98^3	= 340.068	too large
6.97	6.97^3	= 338.609	too small
6.975	6.975^3	= 339.338	too small
6.978	6.978^3	= 339.776	too small
6.979	6.979^3	= 339.922	too small
6.9795	6.9795^3	= 339.995	too small
6.9796	6.9796^3	= 340.01	too large
6.97955	6.97955^3	= 340.003	too large
6.97953	6.97953^3	= 340	✔

3. The cube root ($\sqrt[3]{a}$) of a number 'a', is the number, which when you times it by itself and by itself again, gives 'a'.

$$\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$$

2169 Population of Britain: 1880 and 1980

1.

Age	1880	1980
0 – 14	36%	19%
15 – 29	26%	22%
30 – 44	18%	21%
45 – 59	13%	20%
60 – 74	6%	12%
75+	1%	6%
Total	100%	100%

2. The age pyramid shows the percentage of population in each age group, it does not show the actual population.

3. a) The 60 – 74 age group doubled.

   b) The 0 – 14 and the 15 – 29 age groups decreased.

   c) The 30 – 44, the 45 – 59, the 60 – 74 and the 75+ age groups all increased.

4. a) The line graph gives information on the population of the UK from 1840 - 1980.
   It divides the population into three age groups and shows the percentage of the total population in each age group.

   b) The 0 – 14 age group represents people in school.
   The 15 – 59 age group represents the working population.
   The 60+ age group represents retired people.

   c) i) The percentage of population in the 0 – 14 age group is decreasing.
   ii) The percentage of population in the 15 – 59 age group is fairly constant.
   iii) The percentage of population in the 60+ age group is increasing.

   d) Your answer may include factors such as
   - birth control,
   - choice of family size,
   - increase in proportion of people 15+,
   - life expectancy has increased.

   e) Your answer may include factors such as
   - medical advances have lead to higher life expectancy,
   - better health care.

   f) i) The percentage of the population in the 60+ age group will continue to increase, and the percentage of population in the 0 – 14 age group will continue to decrease.

   ii) There will be a higher burden on the workforce to support an increasing 60+ age group in both pensions and health care.

continued/

5. a) In 1880, 38% of the population were aged 30 or over.

   b) In 1980, 41% of the population were under 30.

   c) In 1980, 59% of the population were aged 30 or over.

6. a) Yes, more than 50% is a reasonable estimate.

   - The percentage of the population in the 0 – 14 age group is 36%.
   - The percentage of the population in the 15 – 29 age group is 26%.
   - The mid-value of the 15 - 29 age group is 22.
   - The skew of population suggests that in every age group there would be more people in the youngest half that in the oldest half. So you would expect that the majority would be under 23.

   b) A good estimate of the age that the majority of the population were under in 1980 would be between 36 – 38 years.

7. Your answer should include include factors such as

   - The percentage of the population aged between 0 – 14 and 15 – 29 is decreasing.
   - In 1980 the percentage of the population in the 0 – 14 age group was less than the percentage of the population in the 30 – 44 age group, but in 1880 the percentage of the population in the 0 - 14 age group was double that of the percentage of the population in the 30 - 44 age group.
   - The percentage of the population of working age has remained fairly constant.
   - The percentage of the population in the 75+ age group has increased by 5% due to improved health and medical facilities and improved living standards.

This table shows the changes in the percentage of the population in each age group over the last 100 years.

Age	Change in %
0 – 14	-17%
15 – 29	-4%
30 – 44	+ 3%
45 – 59	+ 3%
60 – 74	+ 6%
75 +	+ 5%

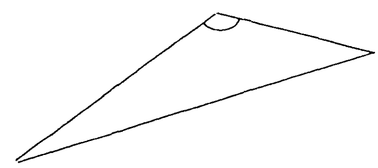
2170 Shape Up

The most useful Attribute cards include
- "4 sides" and
- "One line of symmetry".

The least useful Attribute cards include
- "4 lines of symmetry" and
- "3 lines of symmetry". Why?

For all scalene triangles these Attribute cards are true.
- "All sides are different"
- "All angles are different"
- "No diagonals"
- "No lines of symmetry".

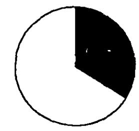
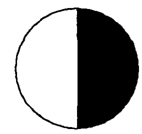
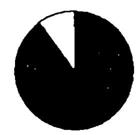
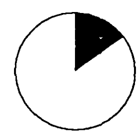
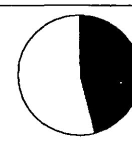
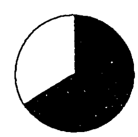
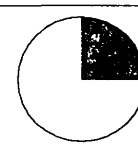
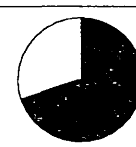
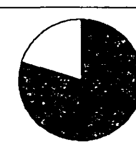
This scalene triangle also has an obtuse angle, so
- "At least one obtuse angle" Attribute card is also true.



The number of cards that you attribute to each shape may vary depending on whether you consider the shapes in general, or the drawings in particular?

Shape	Number of Attribute Card (At least)
Right-angled isosceles triangle	5
Isosceles trapezium	5
Trapezium	5
Scalene triangle	5
Kite	5
Irregular quadrilateral	4
Parallelogram	5
Right-angled scalene triangle	6
Equilateral triangle	6
Square	9
Rectangle	6
Arrowhead	6
Rhombus	9
Isosceles triangle	5

2171 Pie Chart Match

Statement	Pie Chart	Percentage
One third of the world's surface is land. (Atlas)		$33.\dot{3}\%$
26 out of a pack of 52 playing cards are red.		50%
9 out of 10 eggs for sale in Britain come from battery hens. Source: Compassion in World Farming 1992.		90%
15 in 100 people in the UK are pensioners. Source: Keydata 1991 - 92.		15%
Just under half of households in Inner London have a car. Source: Guardian report on 1991 Census.		46%
Two thirds of the water used in the home is flushed down the toilet. Source: Independent 31/5/92.		$66.\dot{6}\%$
By 1990, a quarter of the petrol delivered to petrol stations each week was unleaded. Source: Digest of Environment Protection and Water Statistics 1991.		25%
Approximately 70p in each £1 of Health Spending is used for Hospital Services. Source: Regional Trends 1992.		70%
Approximately 4 in 5 households do not have a computer. Source: Keydata 1991 - 92.		80%

2172 Two Down

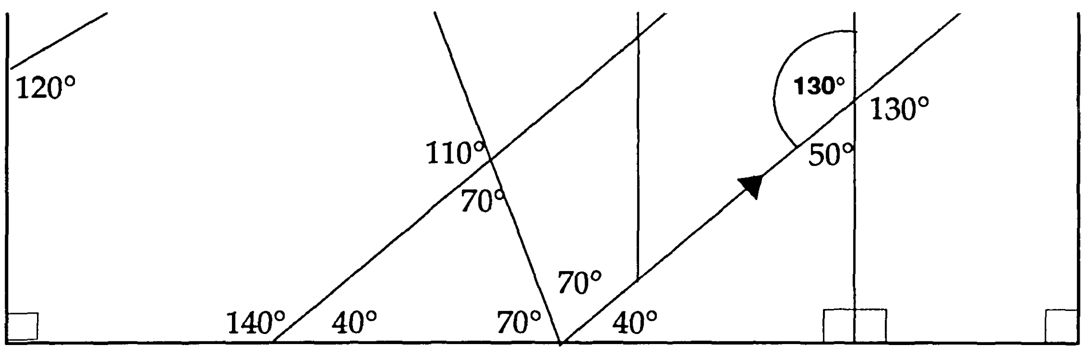
Did you play both games?
Which game is the hardest?

2173 Unmarked Angles

Here are some of the facts that you need to use to calculate all the unmarked angles.

- A rectangle has four right angles.
- The interior angles of a triangle add up to 180°.
- Angles on a straight line add up to 180°.
- Angles around a point add up to 360°.
- Corresponding angles are equal . . .

Here is the part of the worksheet, with some of the angles marked.



2174 The Mode

1. O
   Letter O is the mode.

2. Whichever dice score came up the most frequently is the mode.
   Was it easier to spot the modal dice score from the frequency table or from the pie chart?

3. Test mark 10 is the mode because Kudeza achieved 10 for 6 of her tests.

4. Many possible answers! Do you think the mode would be the same if you took another handful of counters?

5. Many possible answers!
   Do you think you would get the same answer if you did a survey of your whole school? Why?

2175 Grouping Data

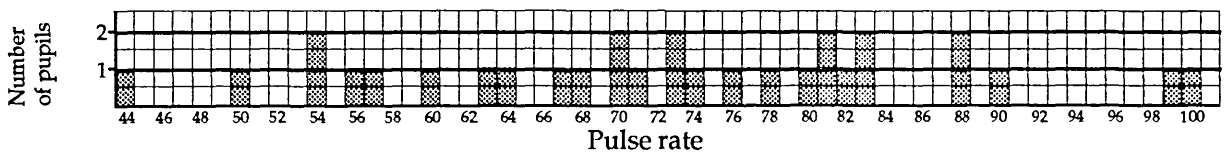
The answers are based upon the survey carried out by Ayten, Lawen and Zoe of form 9HA from Parliament Hill School.

Your results will differ, according to your data. Show your results to your teacher.

1. a)

Pulse rate	44	50	54	56	57	60	63	64	67	68	70	71	73	74	76	78	80	81	82	83	88	90	99	100
Tally	/	/	//	/	/	/	/	/	/	/	//	/	//	/	/	/	/	//	/	//	//	/	/	/
No. of pupils	1	1	2	1	1	1	1	1	1	1	2	1	2	1	1	1	1	2	1	2	2	1	1	1

b) Bar Graph to show 9HA's Pulse Rate



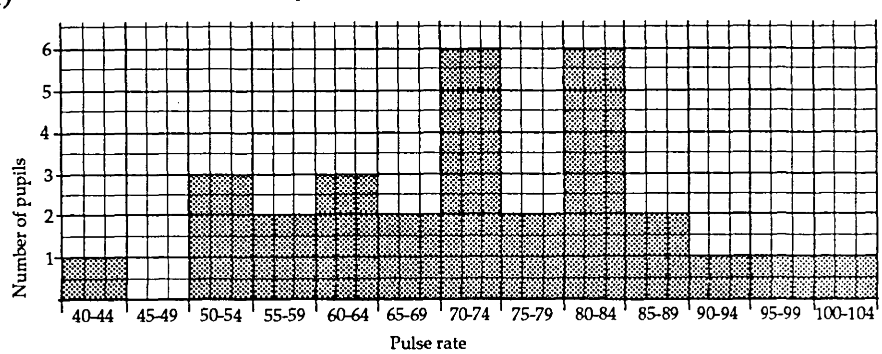
2. a) The range of the data is 56. (100 – 44)

b) The median pulse rate is 73. (The 15th and 16th pulse rate are both 73)

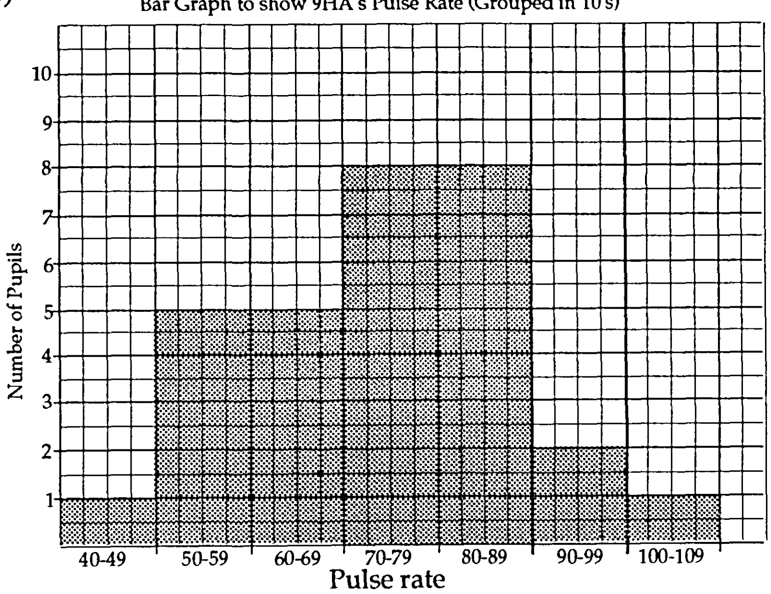
c) There are 6 modal pulse rates, 54, 70, 73, 81, 83 and 88.

d) The arithmetic mean is 72.6 to 1 decimal place. (2177 ÷ 30 = 72.5666666667)

3. a) Bar Graph to show 9HA's Pulse Rate (Grouped in 5's)



b) Bar Graph to show 9HA's Pulse Rate (Grouped in 10's)



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4. **The range.**
It is impossible to give the accurate range for the data when it is displayed in the graphs in 3a and 3b.

The median.
It is impossible to give the accurate median for the data when it is displayed in the graphs in 3a and 3b.
- From graph 3a, the median is in the 70 – 74 group.
- From graph 3b, the median is in the 70 – 79 group.

The mode.
- From graph 3a, there are two modal groups, 70 – 74 and 80 – 84.
- From graph 3b, there are two modal groups, 70 – 79 and 80 – 89.

The mean.
To find an approximation of the arithmetic mean from grouped data, it is necessary to use the mid-value for each group.

- For data grouped in fives (3a), the mid value of 40 - 44 is 42

$\Leftarrow$ **mid-value** $\Rightarrow$
40 41 **42** 43 44

Pulse rates	Mid value	frequency	Mid-value x frequency
40 – 44	42	1	42
45 – 49	47	0	0
50 – 54	52	3	156
55 – 59	57	2	114
60 – 64	62	3	186
65 – 69	67	2	134
70 – 74	72	6	432
75 – 79	77	2	154
80 – 84	82	6	492
85 – 89	87	2	174
90 – 94	92	1	92
95 – 99	97	1	97
100 – 104	102	1	102
	Total	30 pupils	2175

The arithmetic mean = 2175 ÷ 30 = 72.5
The arithmetic mean, for data grouped in 5's is 72.5.

continued/

- For data grouped in tens (3b), the mid value of 40 – 49 is 44.5.

⇐ **mid-value** ⇒

40 41 42 43 44 **44.5** 45 46 47 48 49

Pulse rate	Mid value	frequency	Mid-value x frequency
40 – 49	44.5	1	44.5
50 – 59	54.5	5	272.5
60 – 69	64.5	5	322.5
70 – 79	74.5	8	596
80 – 89	84.5	8	676
90 – 99	94.5	2	189
100 – 109	104.5	1	104.5
	Total	30 pupils	2205

The arithmetic mean = 2205 ÷ 30 = 73.5
The arithmetic mean for data grouped in 10's is 73.5.

5. **Graph showing individual results (1b)**
   - The first graph, giving individual information gives the most accurate details about averages and range.
   - The arithmetic mean can be accurately calculated but takes time. If there had been 300 or 3000 pulse rates collected, it would have been very time consuming to calculate the arithmetic mean.
   - The six modal pulse rates do not give useful information.
   - The median, though accurately found is also time consuming.

   Graph showing the data grouped in 5's (3a).
   - The graph where the data is grouped in fives, does not show the range, yet allows a quick and accurate method to work out the arithmetic mean.
   - The modal groups show a trend, and the median group can be found.

   Graph showing data grouped in 10's (3b).
   - The graph where the data is grouped in tens is perhaps the most useful for identifying the modal groups. If the data were grouped into 20's, there would be just one modal group, 60 – 79.
   - The arithmetic mean can quickly be calculated, though not as accurately as the other two graphs, but probably as accurate as would be necessary for interpreting the results of a survey.

2176 Talking Poster

No answers required.

2177 Population Projections

1. Europe

2. The answer to the nearest million is 272, but an answer between 265 –275 million is acceptable.

3. The answer to the nearest million is 497 – 272 = 225 million, but any answer between 220 – 240 million.

4. a) 537 million, but an answer between 535 – 540 million would be acceptable.
   b) 622 million, but an answer between 615 – 625 million would be acceptable.
   c) 711 – 537 = 164 million, but an answer between 160 – 175 million would be acceptable.

5. a) Approximate populations

North America	360 million
Latin America	885 million
Europe	490 million

   b) The population of Latin America will increase rapidly. It will be almost double its 1988 level by 2040.

   The population of North America will increase slowly.

   The population of Europe will decrease slightly. It will be lower than its 1988 level by 2040.

2178 Volumes

Cuboid	Number of cubes in one layer	Number of layers	Total number of cubes	Volume
A	6	2	12	$12cm^3$
B	6	5	30	$30cm^3$
C	10	3	30	$30cm^3$
D	4	4	16	$16cm^3$
E	14	2	28	$28cm^3$
F	16	3	48	$48cm^3$
G	9	13	117	$117cm^3$
H	8	3	24	$24cm^3$

You may have noticed from your results in the table that

Number of cubes in one layer x Number of layers = Volume of any cuboid

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Front cover illustration:

Grace Chisholm Young (1868-1944) was born in London.

Grace was a brilliant mathematician and was the first person to gain high enough marks for a first class degree in any subject at both Oxford and Cambridge. Grace then went to Göttingen University in Germany to continue her studies in mathematics and was the first women to gain a doctorate.

As well as her outstanding work in mathematics, Grace was also very concerned with educational issues. She educated her six children and wrote a book to help her youngest child to learn biology.

Grace believed that students should be encouraged to find out things for themselves. She was keen that they should learn to make 3-D models to get a feel for geometry so that they would be able to visualise ideas later on.

SMILE 1895 Flat Patterns contains further information on the life and work of Grace Chisholm Young.

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