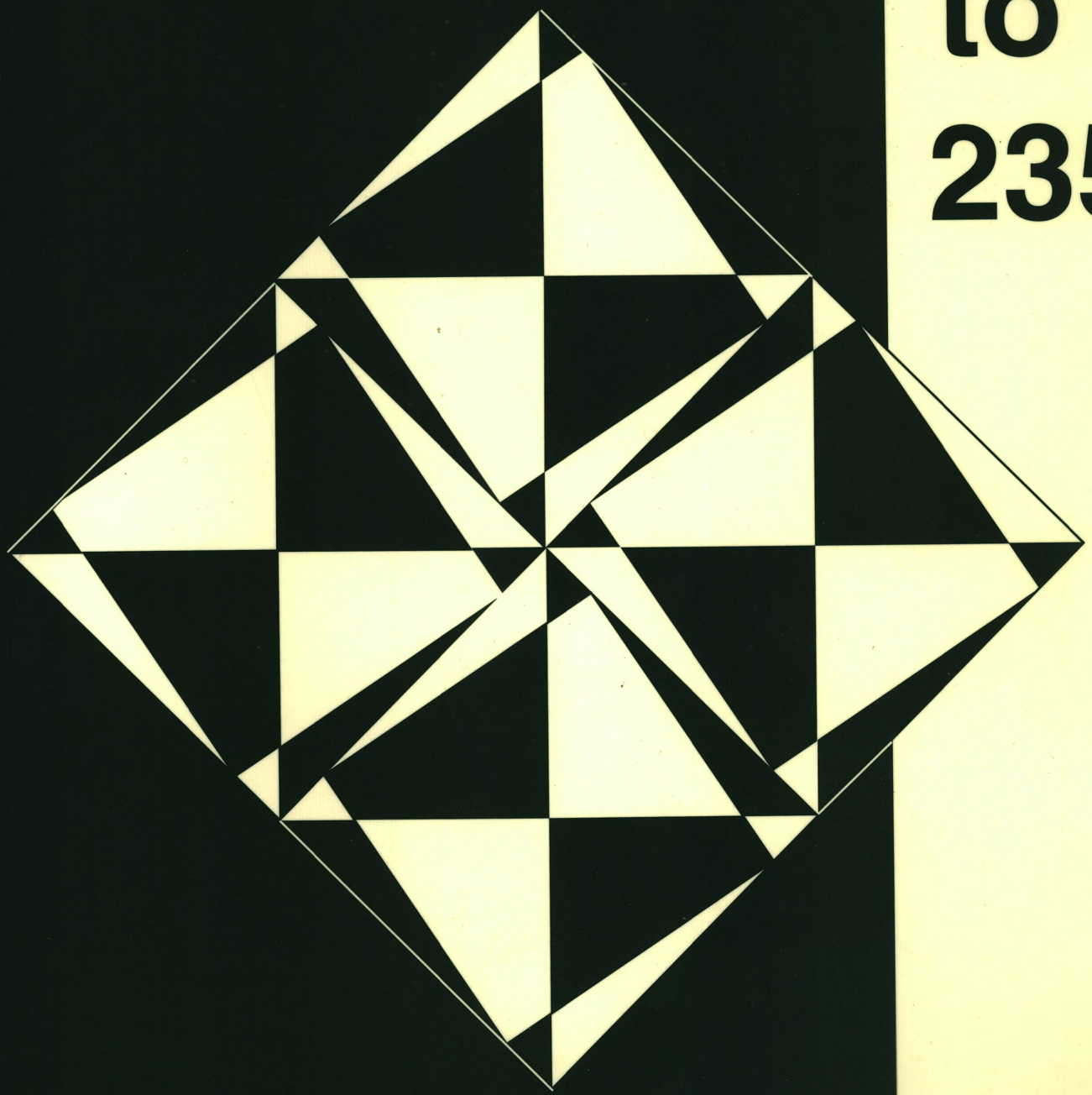
Answers • Answers • Answers

2151 to 2357



SMILE MATHEMATICS

SMILE

ANSWERS

2151 - 2357

Answers

2151 to 2357

SMILE MATHEMATICS

This book contains answers to all SMILE activities between 2151 - 2357, in numerical order.

As well as giving the answers there are also:

- explanations about how solutions have been arrived at
- hints or prompts if you get stuck
- ideas for extending some activities.

Use this book after you have completed each activity, so that you have immediate feedback on your work. You will remember the work more clearly and be able to identify any difficulties or misconceptions more easily. If you have made errors, look through your work again to see if you can spot where you have made an error.

If you then do not understand why your answer is incorrect always seek help from your teacher so that she can help you to clarify any mis-understandings.

You can also use this book while you are working on an activity as it contains hints if you get stuck, or want to know how to continue.

Remember, using the answer book to check your work or to help you if you are stuck is not cheating.

2151 The Root of the Problem

1.

Number (edge length)	Cube (Volume)
1	$1^3 = 1 \times 1 \times 1 = 1$
2	$2^3 = 2 \times 2 \times 2 = 8$
3	$3^3 = 3 \times 3 \times 3 = 27$
4	$4^3 = 4 \times 4 \times 4 = 64$
5	$5^3 = 5 \times 5 \times 5 = 125$
6	$6^3 = 6 \times 6 \times 6 = 216$
7	$7^3 = 7 \times 7 \times 7 = 343$
8	$8^3 = 8 \times 8 \times 8 = 512$
9	$9^3 = 9 \times 9 \times 9 = 729$
10	$10^3 = 10 \times 10 \times 10 = 1000$

2.

Cube Root		Number
$\sqrt[3]{1}$	=	1
$\sqrt[3]{8}$	=	2
$\sqrt[3]{27}$	=	3
$\sqrt[3]{64}$	=	4
$\sqrt[3]{125}$	=	5
$\sqrt[3]{216}$	=	6
$\sqrt[3]{343}$	=	7
$\sqrt[3]{512}$	=	8
$\sqrt[3]{729}$	=	9
$\sqrt[3]{1000}$	=	10

3. a) The volume is $216cm^3$
   b) The volume is $125cm^3$
   c) The volume is $729cm^3$

4. a) Edge length of 3cm
   b) Edge length of 8cm
   c) Edge length of 7cm

continued/

2151 The Root of the Problem (cont)

5. a) The volume of a cube with edge length 7.9cm will be between the volume of a cube with edge length 7cm and the volume of a cube with edge length 8cm. i.e. between $343cm^3$ and $512cm^3$.

   As 7.9 is nearer 8cm, the volume would be nearer to the volume of the 8cm cube, approximately $500cm^3$.

   b) The volume of a cube with edge length 8.5cm will be between 8^3 and 9^3. 8.5 is mid-way between 8 and 9, so an answer between 600 - $640cm^3$ would be reasonable.

   c) The volume of a cube with edge length 3.3cm will be between 3^3 and 4^3. 3.3 is closer to 3 than 4, so an answer between 30 - $40cm^3$ would be reasonable.

6. a) $343 < 370 < 512$
   $7^3 < 370 < 8^3$
   Therefore, $\sqrt[3]{370}$ must lie between 7cm and 8cm.
   A reasonable answer would be between 7.1 - 7.4cm.

   b) $9^3 < 920 < 10^3$
   A reasonable answer would be between 9.5 - 9.9cm.

   c) A reasonable answer would be between 3.1 - 3.5cm.

   To find out the answer use your calculator, using the [y^x] button if it has one.

   7.9^3 → [7] [.] [9] [y^x] [3] [=]

2152 How Likely?

A → 6
B → 10
C → 8
D → 9
E → 7
F → 5
G → 3
H → 4
I → 2
J → 1

2153 £1 Search

There are 17 ways - how many did you find?

2154 Sum Dice

These answers to the puzzle were obtained by a student, who threw the numbers 3, 3, 4, 4, 5 and 6 on the dice. They show one way she managed to make the numbers 1 - 10.

1. $(3-3)+(4-4)+(6-5)$
   $0+0+1 = 1$

2. $(6-5)+[(4-3)\div(4-3)]$
   $1+[1\div1]$
   $1+1 = 2$

3. $(6-5)+(4-3)+(4-3)$
   $1+1+1 = 3$

4. $(4+4-5)+[6\div(3+3)]$
   $3+1 = 4$

5. $(6-3)+(4-3)+(5-4)$
   $3+1+1 = 5$

6. $(6-3)\times(5-3)+(4-4)$
   $3\times2+0 = 6$

7. $(6-3)\times(5-3)+(4-4)$
   $3\times2+1 = 7$

8. $(4+4)+5\times[6-(3+3)]$
   $8+0 = 8$

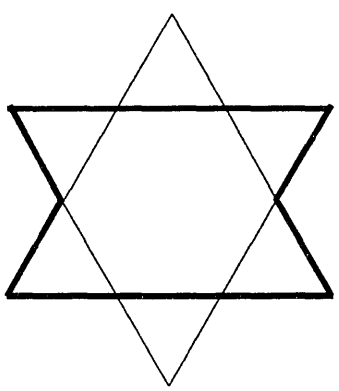
9. $(6+3)+(3\times5)\times(4-4)$
   $9+0 = 9$

10. $(6+4)+(3-3)\times(5+4)$
    $10+0 = 10$

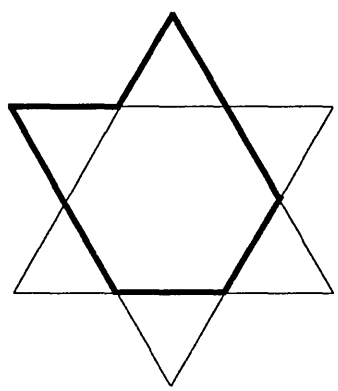
You may have tried using powers, e.g. $3^2 = 3 \times 3 = 9$, as well as +, –, x and ÷.

2155 Visualising

Did your group see these two hexagons?



A hexagon and two triangles



A hexagon and four triangles

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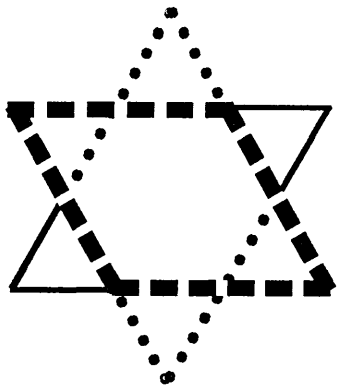
2155 Visualising (cont)

Did you see overlapping shapes?

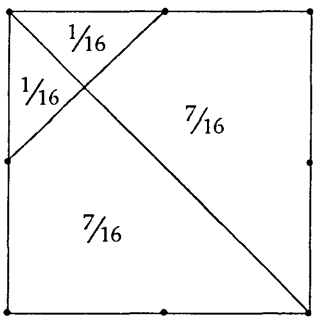


Three overlapping rhombuses

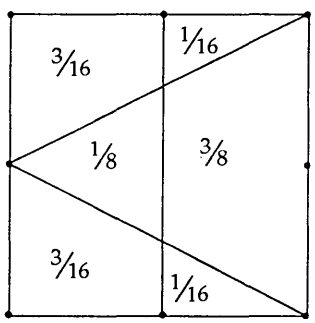
2156 Fraction Squares

1. P = ¼, Q = ⅛, R = (¼ + ¼ + ⅛) = ⅜.

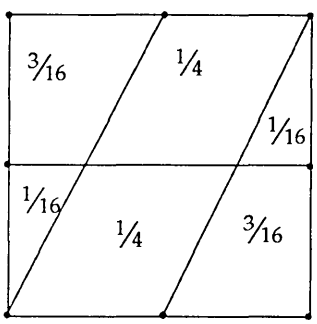
2. a)



b)

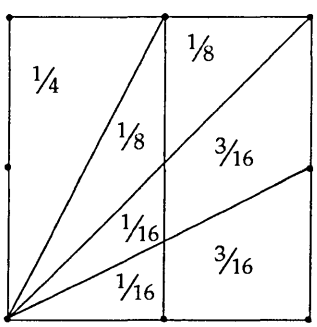


c)

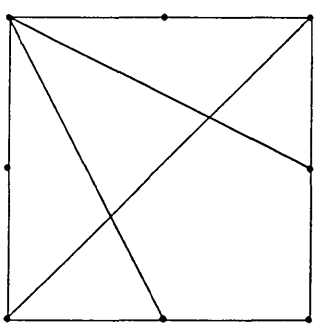


3. Your answers for each of your squares should add up to 1 whole one. Check that they do.

4.



If you have enjoyed this, here is a real challenge!



2157 Some sums for your mind

Questions	Answers	Calculator Answers
$5 - \frac{3}{7}$	Just over 4½	4.571 (to 3 d.p.)
$\frac{5}{7-3}$	One and a quarter	1.25
$7 \div (5-3)$	3 ½	3.5
$\frac{3}{7-5}$	1.5	1.5
$7 - \frac{3}{5}$	About six and a half	6.4
$3 - (7 \div 5)$	A bit more than 1½	1.6
$3 - \frac{5}{7}$	2 and a bit	2.286 (to 3 d.p.)
$\frac{3-5}{7}$	-0. something	-0.286 (to 3 d.p.)
$\frac{3-7}{5}$	$-\frac{4}{5}$	-0.8
$\frac{5-7}{3}$	$-\frac{2}{3}$	-0.667 (to 3 d.p.) or $-0.\dot{6}$
$\frac{7-5}{3}$	$\frac{2}{3}$	0.667 (to 3 d.p.) or $0.\dot{6}$
$\frac{3}{5} - 7$	-6.something	-6.4

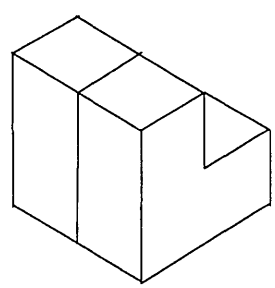
2158 Turning Green

Were you able to sort all the 35 objects into a re-cycling bin? Show your work to your teacher.

2159 Permutating Tricubes

We found 31 different permutations.

Starting with 2 at one end like this there are 5 ways altogether when the two are the same colour, 10 ways when they are not.



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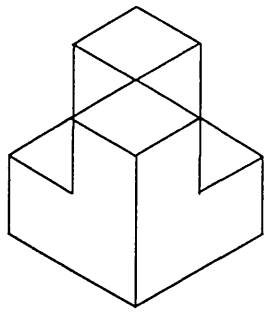
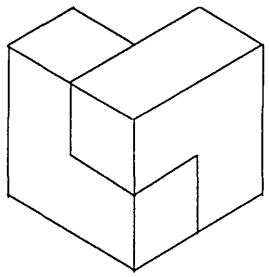
2159 Permutating Tricubes (cont)

When the first two are put together like this, there are 2 ways when the first two are the same colour, 4 ways if they are not.



When the first two are put together like this, there are 4 ways when the first two are the same, 10 ways if they are not.



2160 Folding Fractions

1. $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$ 2. $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$ 3. $\frac{1}{2}$ of $\frac{1}{5} = \frac{1}{10}$ 4. $\frac{1}{2}$ of $\frac{1}{6} = \frac{1}{12}$

5. $\frac{1}{4}$ of $\frac{1}{2} = \frac{1}{8}$ 6. $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$ 7. $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$ 8. $\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{12}$

9. You may have noticed that $\frac{1}{4}$ of $\frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$

   To find a fraction of a fraction, where the numerators (top numbers) are both one, you multiply the numerators together and then multiply the denominators (bottom numbers) together.

10. $\frac{1}{2}$ of $\frac{2}{3} = \frac{2}{6}$ or $\frac{1}{3}$ 11. $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$

12. $\frac{1}{2}$ of $\frac{2}{5} = \frac{2}{10}$ or $\frac{1}{5}$ 13. $\frac{1}{3}$ of $\frac{3}{4} = \frac{3}{12}$ or $\frac{1}{4}$

14. To find fractions of fractions (where numerators can be any number), you multiply the numerators and then multiply the denominators.

15. Another way of saying "**of**" is "**multiply**", so an algorithm for multiplying fractions is to multiply the top numbers and multiply the bottom numbers.

    $\frac{2}{3}$ of $\frac{3}{4} = \frac{6}{12}$ or $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$

    Using your Fraction Ruler, can you find other fractions which are the same as (equivalent to) $\frac{6}{12}$?

2161 Shape Names

A	rectangle	All the angles of this shape are right angles. Not all the sides are equal.
B	scalene triangle	This shape has three sides. None of the angles are equal. It has no right angle.
C	equilateral triangle	This shape has three sides. The angles are all equal.
D	square	All the sides of this shape are equal. All the angles are right angles.
E	right-angled trapezium	This shape has four sides. Two sides are parallel. It has two right angles.
F	right-angled triangle	This shape has three sides. It has one right angle.
G	isosceles triangle	This shape has two equal sides. Two of the three angles are equal.

2162 Angles in Triangles

1. a = 106°

2. x = 67°
   y = 113°

3. b = 38°

4. AB = AC = CD
   i) ∠ACB = 70° ii) ∠BAC = 40° iii) ∠ACD = 110°
   iv) ∠CAD = 35° v) ∠BAD = 75°

5. i) ∠KML = 90° ii) ∠KLM = 54°

6. ∠SPR = 130°

7. i) ∠ADB = 60° ii) ∠BAD = 75° iii) ∠DBC = 30°

2163 Geometry Facts

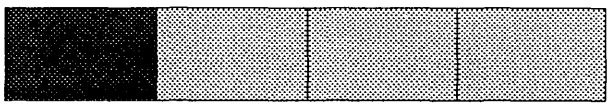
No answers required

2164 Information Displayed

1. a) 90 girls
   b) 90 boys

2. a) 120 can swim
   b) 60 cannot swim

3.

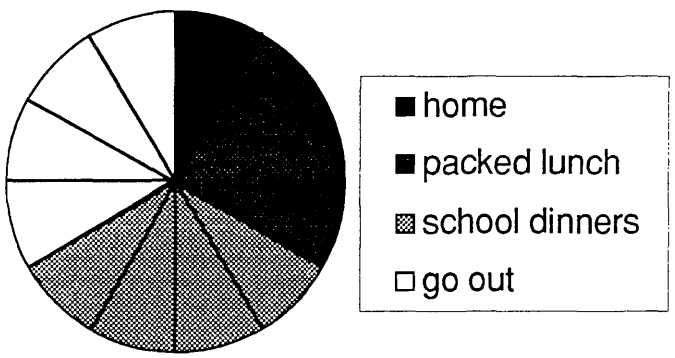


choir not in the choir

4. a) 24 students play netball
   b) 72 students play hockey
   c) 48 students play football

5. a) 36 students belong to the music club
   b) 36 students belong to the homework club
   c) 18 students belong to the drama club
   d) 54 students belong to the maths club

6.



2165 Transform

No answers required.

2166 Matching Equations

Below are examples of each of the methods suggested on the activity. You may have used one method throughout or a variety of methods.

- Choosing 2 points method.

  (2, 0) and (3, 2) are the co-ordinates of two points on graph A.

  Using co-ordinate (2, 0), subsituting $x = 2$ and $y = 0$ into equation 1.

  $4x = 2y - 8$
  $4 \times 2 = 2 \times 0 - 8$
  $8 = 0 - 8$ **This is not right.**

continued/

Using co-ordinate (2, 0) try substituting $x = 2$ and $y = 0$ into equation 2.

$y = 2x - 4$
$0 = 2 \times 2 - 4$
$0 = 4 - 4$ **This works.**

Now using co-ordinate (3, 2), try substituting $x = 3$ and $y = 2$ into equation 2.

$y = 2x - 4$
$2 = 2 \times 3 - 4$
$2 = 6 - 4$ **This works.**

So Equation 2 matches graph A. Equations 7, 8 and 12 also match graph A.

- Choosing the re-arranging method.

$4x = 2y - 8$
Divide both sides by 2.
$2x = y - 4$
Add 4 to both sides.
$2x + 4 = y$
This can be re-written as $y = 2x + 4$ and is the same as equation 5.

Linear equations can all be rearranged into the form $y = mx + c$. The value of m gives the gradient of the line and c gives the intercept (where the graph cuts the y axis).

In the equation y = 2x + 4, the gradient is 2 and the intercept is 4, so equations 1 and 5 match graph B. Equations 3 and 11 also match with graph B.

- Using the MicroSMILE program PLOTTER.

You can either plot points and see the equation of the line, or you can input equations and see them in rearranged form.

Equation 4 matches graph B. Equation 6, 9 and 10 also match graph C.

Regardless of the method used you should have found that:

> Equations 2, 7, 8 and 12 match graph A.
> Equations 1, 3, 5 and 11 match graph B.
> Equations 4, 6, 9 and 10 match graph C.

- Equations which match graph D could be

$y = \frac{1}{2}x + 2$
$2y = x + 4$
$x = 2y - 4$
$2y - x = 4$
$3y = 1\frac{1}{2}x + 6 \ldots$

2167 Range of Area

1. The lower bound of 16 = 15.5, the upper bound of 16 = 16.5.

   a) Smallest possible area = $15.5 \times 15.5 = 240.25\text{cm}^2$
   Largest possible area = $16.5 \times 16.5 = 272.25\text{cm}^2$
   Range of area = $272.25\text{cm}^2 - 240.25\text{cm}^2$
   $= 32\text{cm}^2$

   b) The range of possible areas is
   2 multiplied by 'the length of the side of the square'.

   c) If n = the side of the square measured to a given unit.
   Smallest possible area = $(n - \frac{1}{2})^2 = n^2 - n + \frac{1}{4}$
   Largest possible area = $(n + \frac{1}{2})^2 = n^2 + n + \frac{1}{4}$
   Range of area = $(n^2 + n + \frac{1}{4}) - (n^2 - n + \frac{1}{4})$
   $= n^2 + n + \frac{1}{4} - n^2 + n - \frac{1}{4}$
   $= n + n$
   $= 2n$ square units.

2. **Rectangle**
   a) The range of area when the height and width is measured to the nearest centimetre, is height of rectangle plus the width of the rectangle.

   b) To prove this rule, let h = height and w = width

   Smallest possible area = $(h - \frac{1}{2})(w - \frac{1}{2})$
   Largest possible area = $(h + \frac{1}{2})(w + \frac{1}{2})$
   Range of area = $(h + \frac{1}{2})(w + \frac{1}{2}) - (h - \frac{1}{2})(w - \frac{1}{2})$
   $= (hw + \frac{1}{2}h + \frac{1}{2}w + \frac{1}{4}) - (hw - \frac{1}{2}h - \frac{1}{2}w + \frac{1}{4})$
   $= hw + \frac{1}{2}h + \frac{1}{2}w + \frac{1}{4} - hw + \frac{1}{2}h + \frac{1}{2}w - \frac{1}{4}$
   $= (h + w)\text{cm}^2$

   Circle
   The range of area when the radius is measured to the nearest centimetre is $2\pi r\text{cm}^2$.

   Triangle
   The range of area when the base and height are measured to the nearest centimetre is $\frac{1}{2}(b+h)\text{cm}^2$.

3. We found general rules for squares. If you found rules for other shapes, show these to your teacher.

   a) The range of area of a square, side n, when measured to the nearest half centimetre is $n\text{cm}^2$.
   Can you justify why this is?

   b) The range of area of a square, side n, when measured to the nearest x cm is $2xn\text{cm}^2$.

- The range of volume for a cube measured to the nearest centimetre is $(3n^2 + \frac{1}{4})\text{cm}^3$ where n is the side of the cube measured.

- The range of surface area for a cube measured to the nearest centimetre is $12n\text{cm}^2$.

Similar rules can be obtained for other 3-D shapes, check them with your teacher.

2168 Cube Root Calculator

1.

Edge Length	Cube (Volume)	
4.65	4.65 x 4.65 x 4.65 = 100.54463	too large
4.63	4.63 x 4.63 x 4.63 = 99.252847	too small
4.64	4.64 x 4.64 x 4.64 = 99.8973	too small
4.645	4.645 x 4.645 x 4.645 = 100.221	too large

To save time you can use the x^y button on your calculator.
This is the power button.

4.645 x^y 3 = 100.221

.
.

$(4.642)^3$ = 100.027 = 100 (1 d.p.)
$(4.6416)^3$ = 100.00072 = 100 (2 d.p.)
How many decimal places was your answer correct to?

2. The edge length of a cube with volume 340cm^3 must be between 6cm and 7cm because 6 x 6 x 6 = 216 and 7 x 7 x 7 = 343. It must be nearer to 7.

Edge Length	Cube (Volume)		
6.9	6.9^3	= 6.9 x 6.9 x 6.9 = 328.509	too small
6.95	6.95^3	= 6.95 x 6.95 x 6.95 = 335.702	too small
6.98	6.98^3	= 340.068	too large
6.97	6.97^3	= 338.609	too small
6.975	6.975^3	= 339.338	too small
6.978	6.978^3	= 339.776	too small
6.979	6.979^3	= 339.922	too small
6.9795	6.9795^3	= 339.995	too small
6.9796	6.9796^3	= 340.01	too large
6.97955	6.97955^3	= 340.003	too large
6.97953	6.97953^3	= 340	✔

3. The cube root ($\sqrt[3]{a}$) of a number 'a', is the number, which when you times it by itself and by itself again, gives 'a'.

$$\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$$

2169 Population of Britain: 1880 and 1980

1.

Age	1880	1980
0 – 14	36%	19%
15 – 29	26%	22%
30 – 44	18%	21%
45 – 59	13%	20%
60 – 74	6%	12%
75+	1%	6%
Total	100%	100%

2. The age pyramid shows the percentage of population in each age group, it does not show the actual population.

3. a) The 60 – 74 age group doubled.

   b) The 0 – 14 and the 15 – 29 age groups decreased.

   c) The 30 – 44, the 45 – 59, the 60 – 74 and the 75+ age groups all increased.

4. a) The line graph gives information on the population of the UK from 1840 - 1980.
   It divides the population into three age groups and shows the percentage of the total population in each age group.

   b) The 0 – 14 age group represents people in school.
   The 15 – 59 age group represents the working population.
   The 60+ age group represents retired people.

   c) i) The percentage of population in the 0 – 14 age group is decreasing.
   ii) The percentage of population in the 15 – 59 age group is fairly constant.
   iii) The percentage of population in the 60+ age group is increasing.

   d) Your answer may include factors such as
   - birth control,
   - choice of family size,
   - increase in proportion of people 15+,
   - life expectancy has increased.

   e) Your answer may include factors such as
   - medical advances have lead to higher life expectancy,
   - better health care.

   f) i) The percentage of the population in the 60+ age group will continue to increase, and the percentage of population in the 0 – 14 age group will continue to decrease.

   ii) There will be a higher burden on the workforce to support an increasing 60+ age group in both pensions and health care.

continued/

2169 Population of Britain: 1880 and 1980 (cont)

5. a) In 1880, 38% of the population were aged 30 or over.

   b) In 1980, 41% of the population were under 30.

   c) In 1980, 59% of the population were aged 30 or over.

6. a) Yes, more than 50% is a reasonable estimate.

   - The percentage of the population in the 0 – 14 age group is 36%.
   - The percentage of the population in the 15 – 29 age group is 26%.
   - The mid-value of the 15 - 29 age group is 22.
   - The skew of population suggests that in every age group there would be more people in the youngest half that in the oldest half. So you would expect that the majority would be under 23.

   b) A good estimate of the age that the majority of the population were under in 1980 would be between 36 – 38 years.

7. Your answer should include include factors such as

   - The percentage of the population aged between 0 – 14 and 15 – 29 is decreasing.
   - In 1980 the percentage of the population in the 0 – 14 age group was less than the percentage of the population in the 30 – 44 age group, but in 1880 the percentage of the population in the 0 - 14 age group was double that of the percentage of the population in the 30 - 44 age group.
   - The percentage of the population of working age has remained fairly constant.
   - The percentage of the population in the 75+ age group has increased by 5% due to improved health and medical facilities and improved living standards.

This table shows the changes in the percentage of the population in each age group over the last 100 years.

Age	Change in %
0 – 14	-17%
15 – 29	-4%
30 – 44	+ 3%
45 – 59	+ 3%
60 – 74	+ 6%
75 +	+ 5%

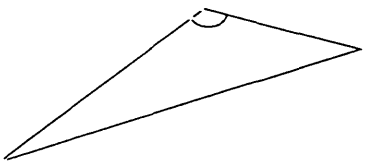
2170 Shape Up

The most useful Attribute cards include
- "4 sides" and
- "One line of symmetry".

The least useful Attribute cards include
- "4 lines of symmetry" and
- "3 lines of symmetry". Why?

For all scalene triangles these Attribute cards are true.
- "All sides are different"
- "All angles are different"
- "No diagonals"
- "No lines of symmetry".

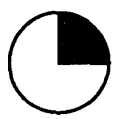
This scalene triangle also has an obtuse angle, so
- "At least one obtuse angle" Attribute card is also true.



The number of cards that you attribute to each shape may vary depending on whether you consider the shapes in general, or the drawings in particular?

Shape	Number of Attribute Card (At least)
Right-angled isosceles triangle	5
Isosceles trapezium	5
Trapezium	5
Scalene triangle	5
Kite	5
Irregular quadrilateral	4
Parallelogram	5
Right-angled scalene triangle	6
Equilateral triangle	6
Square	9
Rectangle	6
Arrowhead	6
Rhombus	9
Isosceles triangle	5

2171 Pie Chart Match

Statement	Pie Chart	Percentage
One third of the world's surface is land. (Atlas)		33.3%
26 out of a pack of 52 playing cards are red.		50%
9 out of 10 eggs for sale in Britain come from battery hens. Source: Compassion in World Farming 1992.		90%
15 in 100 people in the UK are pensioners. Source: Keydata 1991 - 92.		15%
Just under half of households in Inner London have a car. Source: Guardian report on 1991 Census.		46%
Two thirds of the water used in the home is flushed down the toilet. Source: Independent 31/5/92.		66.6%
By 1990, a quarter of the petrol delivered to petrol stations each week was unleaded. Source: Digest of Environment Protection and Water Statistics 1991.		25%
Approximately 70p in each £1 of Health Spending is used for Hospital Services. Source: Regional Trends 1992.		70%
Approximately 4 in 5 households do not have a computer. Source: Keydata 1991 - 92.		80%

2172 Two Down

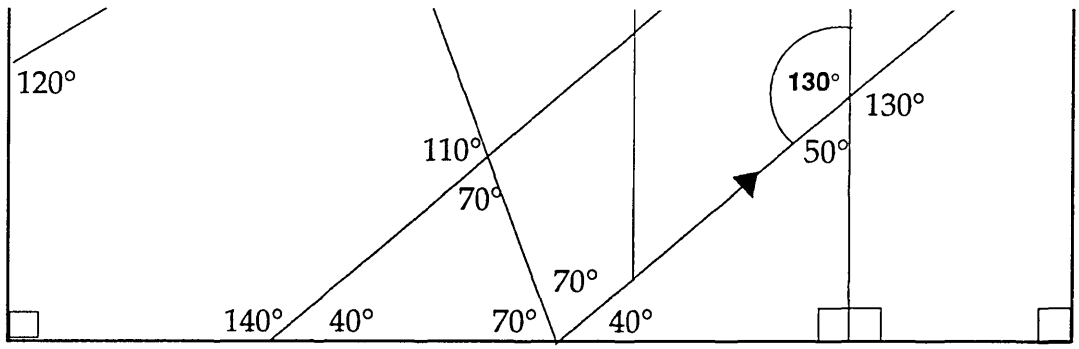
Did you play both games?
Which game is the hardest?

2173 Unmarked Angles

Here are some of the facts that you need to use to calculate all the unmarked angles.

- A rectangle has four right angles.
- The interior angles of a triangle add up to 180°.
- Angles on a straight line add up to 180°.
- Angles around a point add up to 360°.
- Corresponding angles are equal . . .

Here is the part of the worksheet, with some of the angles marked.



2174 The Mode

1. O
   Letter O is the mode.

2. Whichever dice score came up the most frequently is the mode.
   Was it easier to spot the modal dice score from the frequency table or from the pie chart?

3. Test mark 10 is the mode because Kudeza achieved 10 for 6 of her tests.

4. Many possible answers! Do you think the mode would be the same if you took another handful of counters?

5. Many possible answers!
   Do you think you would get the same answer if you did a survey of your whole school? Why?

2175 Grouping Data

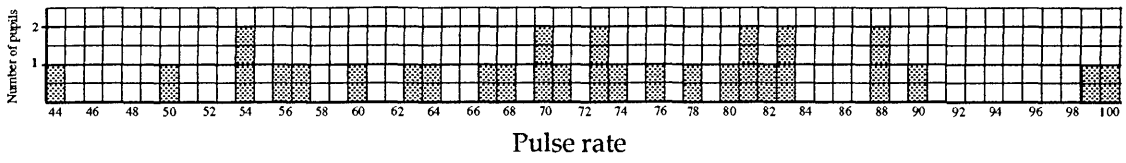
The answers are based upon the survey carried out by Ayten, Lawen and Zoe of form 9HA from Parliament Hill School.

Your results will differ, according to your data. Show your results to your teacher.

1. a)

Pulse rate	44	50	54	56	57	60	63	64	67	68	70	71	73	74	76	78	80	81	82	83	88	90	99	100
Tally	/	/	//	/	/	/	/	/	/	/	//	/	//	/	/	/	/	//	/	//	//	/	/	/
No. of pupils	1	1	2	1	1	1	1	1	1	1	2	1	2	1	1	1	1	2	1	2	2	1	1	1

b) Bar graph to show 9HA's Pulse Rate



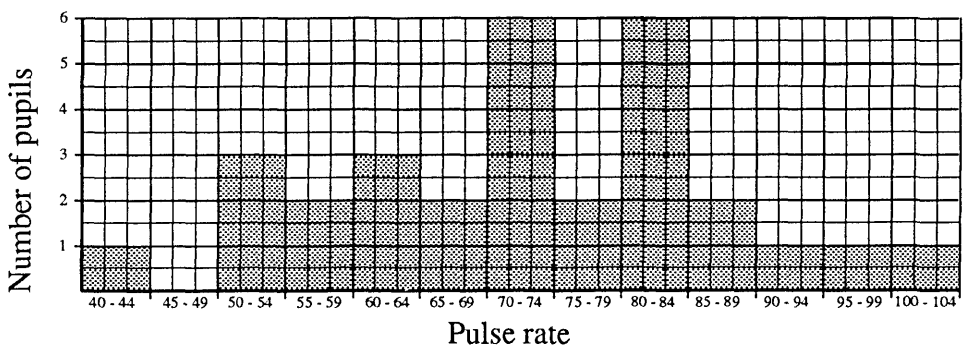
2. a) The range of the data is 56. (100 – 44)

b) The median pulse rate is 73. (The 15th and 16th pulse rate are both 73).

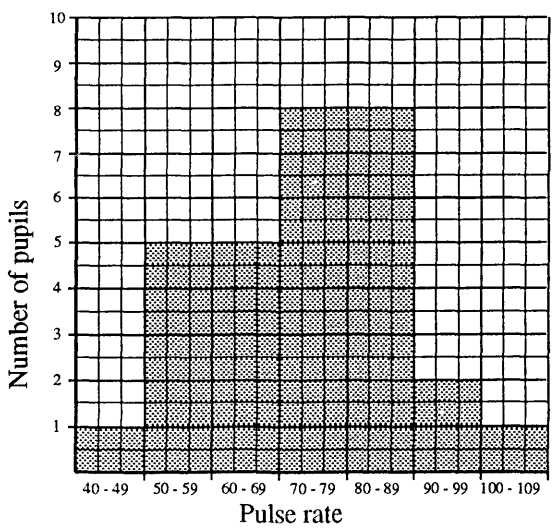
c) There are 6 modal pulse rates, 54, 70, 73, 81, 83 and 88.

d) The arithmetic mean is 72.6 to 1 decimal place. (2177 ÷ 30 = 72.5666666667)

3. a) Bar graph to show 9HA's Pulse Rate (Grouped in 5's)



b) Bar graph to show 9HA's Pulse Rate (Grouped in 10's)



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4. **The range.**
It is impossible to give the accurate range for the data when it is displayed in the graphs in 3a and 3b.

The median.
It is impossible to give the accurate median for the data when it is displayed in the graphs in 3a and 3b.
- From graph 3a, the median is in the 70 – 74 group.
- From graph 3b, the median is in the 70 – 79 group.

The mode.
- From graph 3a, there are two modal groups, 70 – 74 and 80 – 84.
- From graph 3b, there are two modal groups, 70 – 79 and 80 – 89.

The mean.
To find an approximation of the arithmetic mean from grouped data, it is necessary to use the mid-value for each group.

- For data grouped in fives (3a), the mid value of 40 - 44 is 42

⇐ mid-value ⇒

40 41 **42** 43 44

Pulse rates	Mid value	frequency	Mid-value x frequency
40 – 44	42	1	42
45 – 49	47	0	0
50 – 54	52	3	156
55 – 59	57	2	114
60 – 64	62	3	186
65 – 69	67	2	134
70 – 74	72	6	432
75 – 79	77	2	154
80 – 84	82	6	492
85 – 89	87	2	174
90 – 94	92	1	92
95 – 99	97	1	97
100 – 104	102	1	102
	Total	30 pupils	2175

The arithmetic mean = 2175 ÷ 30 = 72.5
The arithmetic mean, for data grouped in 5's is 72.5.

continued/

- For data grouped in tens (3b), the mid value of 40 – 49 is 44.5.

⇐ **mid-value** ⇒

40 41 42 43 44 **44.5** 45 46 47 48 49

Pulse rate	Mid value	frequency	Mid-value x frequency
40 – 49	44.5	1	44.5
50 – 59	54.5	5	272.5
60 – 69	64.5	5	322.5
70 – 79	74.5	8	596
80 – 89	84.5	8	676
90 – 99	94.5	2	189
100 – 109	104.5	1	104.5
	Total	30 pupils	2205

The arithmetic mean = 2205 ÷ 30 = 73.5
The arithmetic mean for data grouped in 10's is 73.5.

5. **Graph showing individual results (1b)**
   - The first graph, giving individual information gives the most accurate details about averages and range.
   - The arithmetic mean can be accurately calculated but takes time. If there had been 300 or 3000 pulse rates collected, it would have been very time consuming to calculate the arithmetic mean.
   - The six modal pulse rates do not give useful information.
   - The median, though accurately found is also time consuming.

Graph showing the data grouped in 5's (3a).
- The graph where the data is grouped in fives, does not show the range, yet allows a quick and accurate method to work out the arithmetic mean.
- The modal groups show a trend, and the median group can be found.

Graph showing data grouped in 10's (3b).
- The graph where the data is grouped in tens is perhaps the most useful for identifying the modal groups. If the data were grouped into 20's, there would be just one modal group, 60 – 79.
- The arithmetic mean can quickly be calculated, though not as accurately as the other two graphs, but probably as accurate as would be necessary for interpreting the results of a survey.

2176 Talking Poster

No answers required.

2177 Population Projections

1. Europe

2. The answer to the nearest million is 272, but an answer between 265 –275 million is acceptable.

3. The answer to the nearest million is 497 – 272 = 225 million, but any answer between 220 – 240 million.

4. a) 537 million, but an answer between 535 – 540 million would be acceptable.
   b) 622 million, but an answer between 615 – 625 million would be acceptable.
   c) 711 – 537 = 164 million, but an answer between 160 – 175 million would be acceptable.

5. a) Approximate populations

North America	360 million
Latin America	885 million
Europe	490 million

   b) The population of Latin America will increase rapidly. It will be almost double its 1988 level by 2040.

   The population of North America will increase slowly.

   The population of Europe will decrease slightly. It will be lower than its 1988 level by 2040.

2178 Volumes

Cuboid	Number of cubes in one layer	Number of layers	Total number of cubes	Volume
A	6	2	12	$12cm^3$
B	6	5	30	$30cm^3$
C	10	3	30	$30cm^3$
D	4	4	16	$16cm^3$
E	14	2	28	$28cm^3$
F	16	3	48	$48cm^3$
G	9	13	117	$117cm^3$
H	8	3	24	$24cm^3$

You may have noticed from your results in the table that

Number of cubes in one layer x Number of layers = Volume of any cuboid

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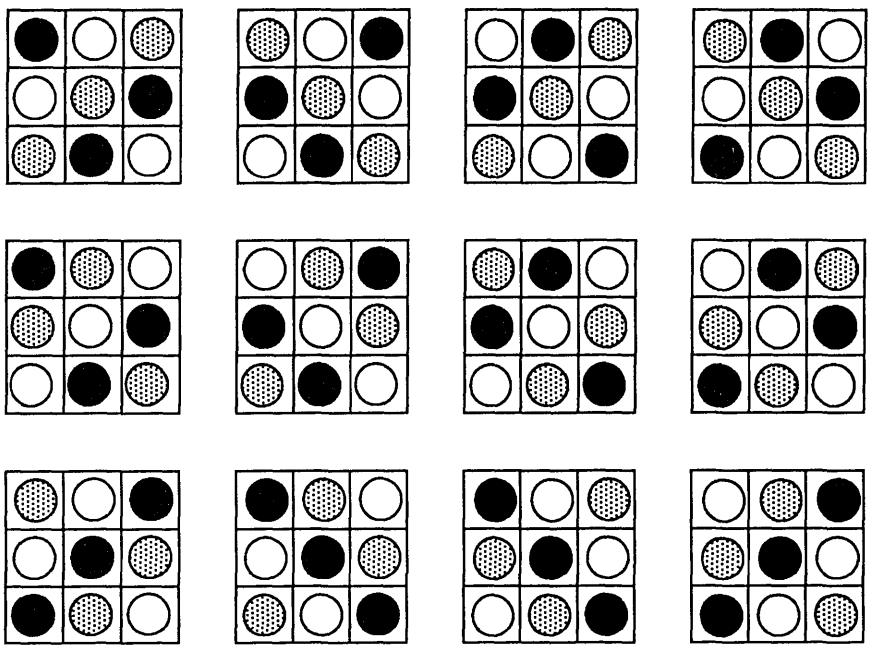
2179 Shakes and Adders

- Were the positive or the negative numbers covered first?
- Were some numbers more difficult to cover than others?
- Did you manage to cover all the numbers between you?

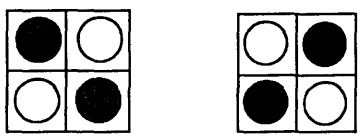
2180 Arranging Counters

There are twelve different arrangements of three coloured counters.

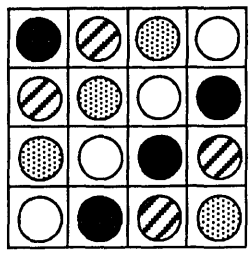


The first four are rotations of each other.
Did you consider them to be the same or different?

With two different coloured counters there are two arrangements which are rotations of each other.



Here is one arrangement with four different coloured counters. How many did you find?

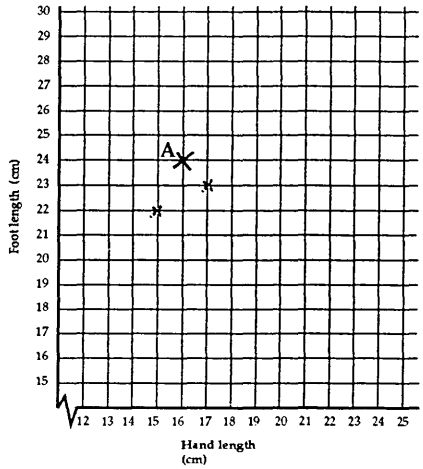


2181 Big Hand . . . Big Foot?

1. Your sample should have included at least 20 people. The more people you included in your survey, the more likely you are to be able to answer the question. Did your sample include people of different ages, male and female, tall and short people . . . ?

2. Does your table show that everyone had larger feet than hands?

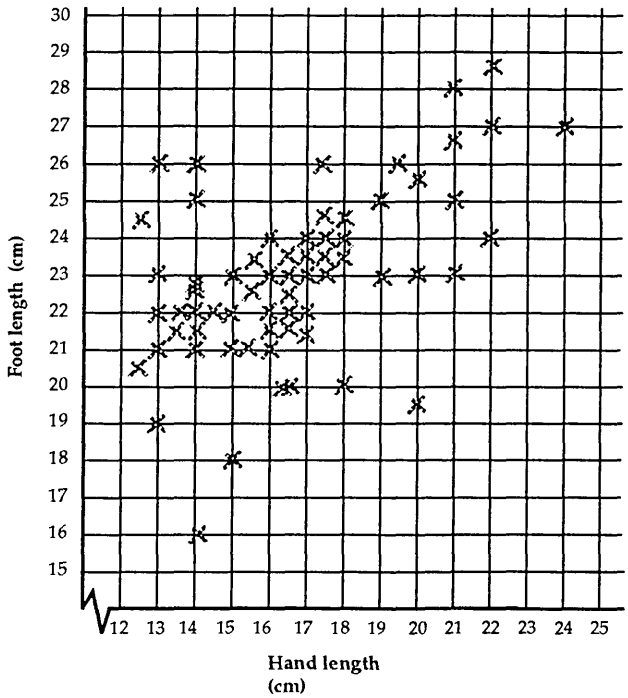
3. This shows the beginning of a scatter graph.



The point 'A' shows a person whose hand measures 16cm and whose foot measures 24cm.

The crooked lines on the axes show that part of the axes are missing, i.e. the hand length axis starts at 12 and the foot length axis starts at 15, not zero.
The crooked line allows you to zoom in on the area of the graph which contains the data. What was the smallest hand length measurement for your sample?

4. Generally the bigger the hand, the bigger the foot. Did your graph show this?

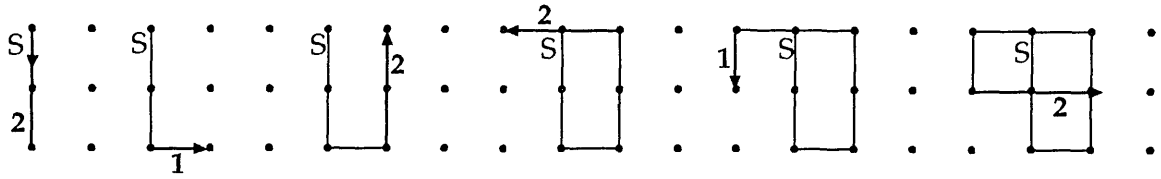


This graph shows that generally the bigger the hand the bigger the foot. The points lie in one direction, i.e. from the bottom left corner to the top right hand corner.

This is an example of **positive correlation**.

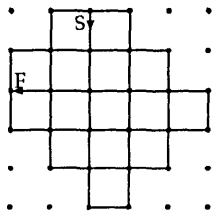
2182 Shongo Networks

The first Shongo Network can be described by the number sequence 2, 1, 2, 2, 1, 2.

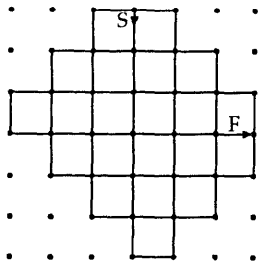


The second network can be described in this way as 3, 1, 3, 2, 2, 3, 1, 3.
The third network can be described as 4, 1, 4, 2, 3, 3, 2, 4, 1, 4.

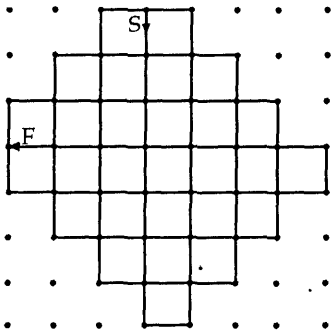
The next 4 patterns are:



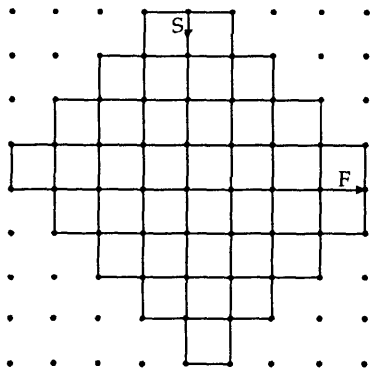
5, 1, 5, 2, 4, 3, 3, 4, 2, 5, 1, 5.



6, 1, 6, 2, 5, 3, 4, 4, 3, 5, 2, 6, 1, 6.



7, 1, 7, 2, 6, 3, 5, 4, 4, 5, 3, 6, 2, 7, 1, 7.

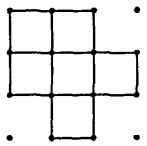


8, 1, 8, 2, 7, 3, 6, 4, 5, 5, 4, 6, 3, 7, 2, 8, 1, 8.

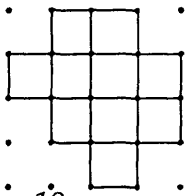
What number sequence describes the next pattern?

Counting the number of squares in each pattern gives another number sequence:

3 squares



6 squares



10 squares

The number of squares in the patterns form the sequence:
3, 6, 10, 15, 21, 28, 36, . . . the triangle numbers.

For lots more interesting ideas on network patterns see:

- 'Mathematics for All', The ESG Project, Salisbury, Wiltshire LEA.
- 'Mathematics from around the World', Phil Dodd, Westgate Community College, Newcastle upon Tyne LEA.

2183 Using standard form

1. a) 8×10^5
   b) 6×10^{-7}
   c) $22.5 \times 10^8 = 2.25 \times 10^9$
   d) $21.3 \times 10^{12} = 2.13 \times 10^{13}$

   This is an example of the key presses needed to check your answers to 1a).

   [2] [x] [1] [0] [y^x] [2] [x] [4] [x] [1] [0] [y^x] [3] [=]

   The answer on the display will probably be 800000
   This is the same as 8×10^5.

   Some calculators will have an [a^b] button rather than a [y^x] button.
   If you are unsure about which button to use, check with the calculator manual or ask your teacher.

   When the answer is too large to be displayed, the calculator will display it in standard form.
   e.g. for 1c) the calculator will probably display the answer as 2.25 09

   This is the same as 2.25×10^9.

If you are unsure how your calculator displays numbers in standard form, discuss your calculator with your teacher.

2. a) 4×10^{12}
   b) 1.1×10^3
   c) $0.7 \times 10^3 = 7 \times 10^2$
   d) 1×10^6
   e) $0.4 \times 10^2 = 4 \times 10$ or 4×10^1

3. a) $40 \times 10^{-7} = 4 \times 10^{-6}$
   b) 1×10^3
   c) $18000 \times 10^{-3} = 1.8 \times 10$
   d) 2.5×10^9
   e) $0.4 \times 10^{-7} = 4 \times 10^{-8}$

4. a) 3.71×10^{-6} to 3 significant figures.
   b) 9.51×10^2 to 3 significant figures.
   c) 1.77×10 to 3 significant figures.
   d) 2.31×10^9 to 3 significant figures.
   e) 4.10×10^{-8} to 3 significant figures.

   - Can you see why $1 \times 10^3 \approx 9.51 \times 10^2$ in part (b)?

   If your estimates were very different to the actual answers check them with your teacher.

continued/

2183 Using standard form (cont)

5. Volume of swimming pool = 25m x 12m x 2.5m
= (25×100)cm x (12×100)cm x (2.5×100)cm
= $(25 \times 10^2 \times 12 \times 10^2 \times 2.5 \times 10^2)$cm^3
= (750×10^6)cm^3

Number of litres required = $(750 \times 10^6) \div (1 \times 10^3) = 750 \times 10^3$
= 7.5×10^5 (in standard form)

6. 2.5 litres is sufficient to cover 24m^2
(2.5×10^3)cm^3 is sufficient to cover (24×10^4)cm^2

$\frac{(2.5 \times 10^3)\text{cm}^3}{(24 \times 10^4)\text{cm}^2}$ = $(0.1041667 \times 10^{-1})$cm thick = $(0.1041667 \times 10^{-1} \times 10^1)$mm
The paint would be (1.0×10^{-1})mm thick correct to 2 sig figs.

2184 Powers of Integers

$1^2 = 1$
$2^2 = 1 + 3$
$3^2 = 1 + 3 + 5$
$4^2 = 1 + 3 + 5 + 7$
⋮
$n^2 = 1 + 3 + 5 \ldots + (2n - 1)$

The pattern generated by cubic numbers expressed as the sum of consecutive odd numbers is:

$1^3 = 1$
$2^3 = 3 + 5$
$3^3 = 7 + 9 + 11$
$4^3 = 13 + 15 + 17 + 19$
$5^3 = 21 + 23 + 25 + 27 + 29$
⋮
$n^3 = (n^2 - n + 1) + \ldots$

You may have noticed that:

- n^3 is the sum of 'n' consecutive odd numbers
- if n is odd the middle term is n^2
  $n^3 = \ldots (n^2 - 4) + (n^2 - 2) + n^2 + (n^2 + 2) + (n^2 + 4) \ldots$
- if n is even
  $n^3 = \ldots (n^2 - 3) + (n^2 - 1) + (n^2 + 1) + (n^2 + 3) \ldots$

Were you able to generate a pattern for quartic numbers expressed as the sum of consecutive odd numbers...

$1^4 = 1$
$2^4 = 7 + 9$
$3^4 = 25 \ldots$

... and generalise for n^4?

Were you able to generalise for n^a and convince someone that your generalisation would always work?

2185 Up the Stairs

- There are 8 different ways of climbing a 5 step staircase by going up in one or two steps at a time.
- In a normal staircase there are 13 steps. There are 377 different ways of climbing up 13 steps in one or two steps at a time.
- Did you try other combinations of steps? Were they realistic sized steps?

It will help to draw a table of your results so that you can look for patterns. You should find a Fibonacci-type sequence. SMILE 2078 may help you.

2186 Missing Pieces

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2	4	6	8	10
12	14	16	18	20
22	24	26	28	30
32	34	36	38	40
42	44	46	48	50

3	6	9	12	15
18	21	24	27	30
33	36	39	42	45
48	51	54	57	60
63	66	69	72	75

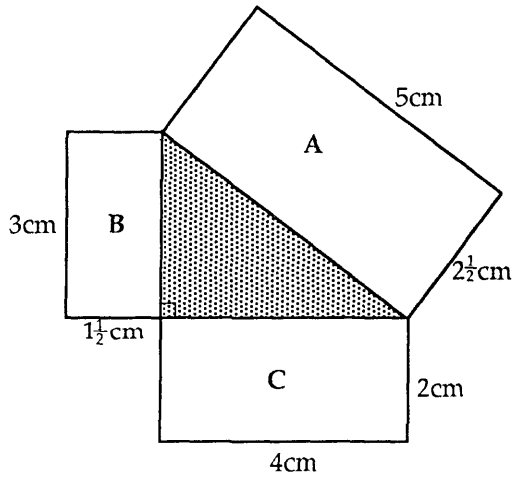
9	18	27	36	45
54	63	72	81	90
99	108	117	126	135
144	153	162	171	180
189	198	207	216	225

6	12	18	24	30
36	42	48	54	60
66	72	78	84	90
96	102	108	114	120
126	132	138	144	150

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Show your own puzzle to your teacher.

- With rectangles instead of squares:



Area A = $5 \times 2\frac{1}{2} = 12\frac{1}{2}cm^2$

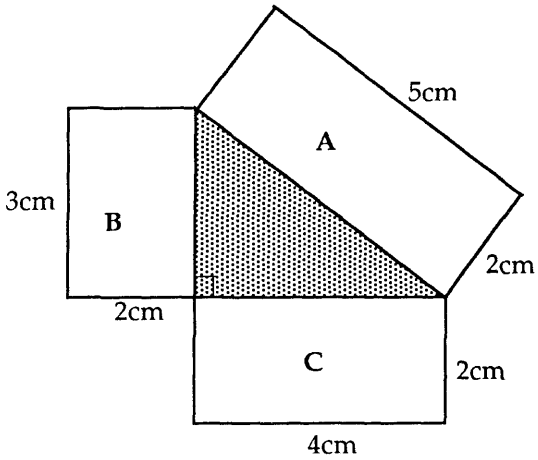
Area B = $3 \times 1\frac{1}{2} = 4\frac{1}{2}cm^2$

Area C = $4 \times 2 = 8cm^2$

} and $4\frac{1}{2} + 8 = 12\frac{1}{2}$

In this case the Pythagoras type rule works.

- All the rectangles are similar.
  The ratio of the long side : short side in each rectangle is the same.
  The ratio of the long side : short side of rectangle A = $5 : 2\frac{1}{2} =$ 2 : 1
  The ratio of the long side : short side of rectangle B = $3 : 1\frac{1}{2} =$ 2 : 1
  The ratio of the long side : short side of rectangle C = 4 : 2 = 2 : 1



Area A = $5 \times 2 = 10cm^2$

Area B = $3 \times 2 = 6cm^2$

Area C = $4 \times 2 = 8cm^2$

} and $6 + 8 \neq 10$

In this case the Pythagoras type rule does not work.

- The rectangles are not similar.
  The ratio of the long side : short side in each rectangle is not the same.
  The ratio of the long side : short side of rectangle A = 5 : 2
  The ratio of the long side : short side of rectangle B = 3 : 2
  The ratio of the long side : short side of rectangle C = 4 : 2

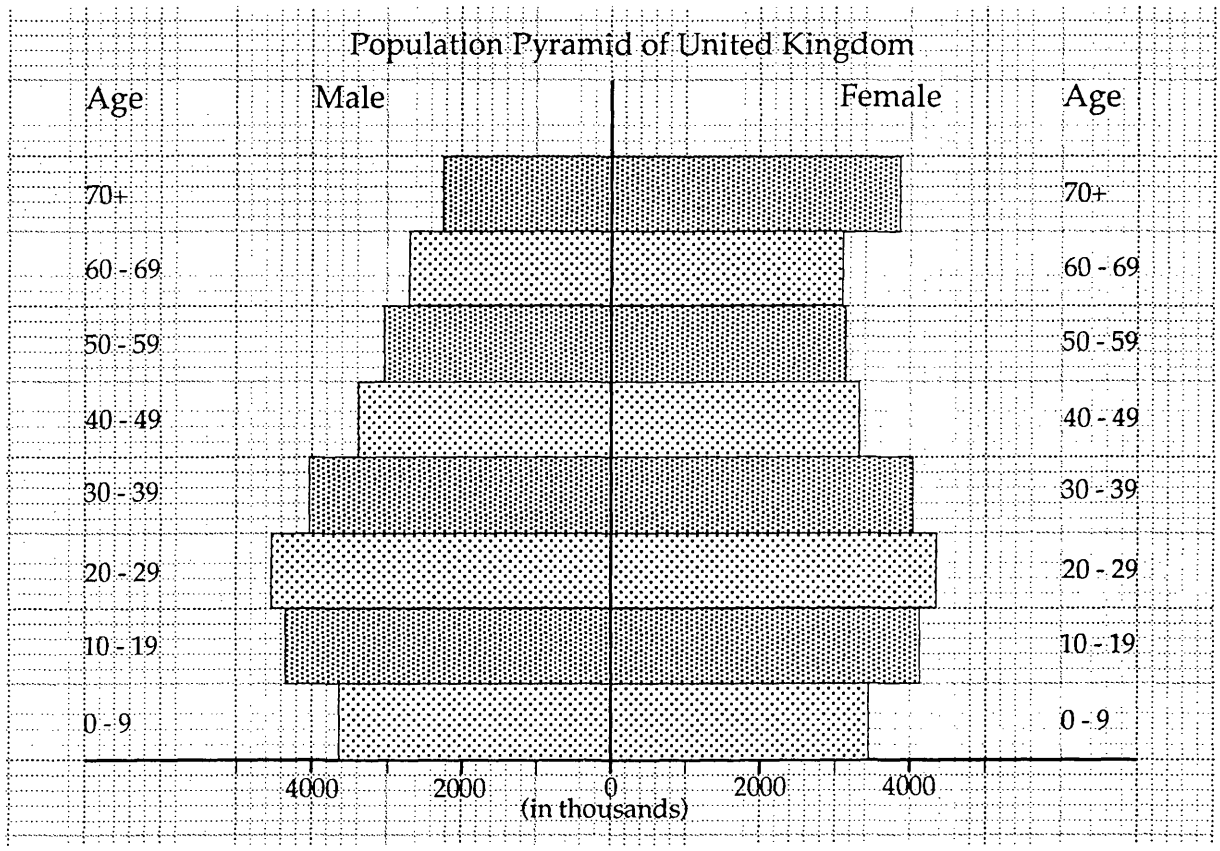
- Pythagoras' rule works when squares are drawn on the sides of a right-angled triangle because the ratio of the sides of any square is the same, 1 : 1, 2 : 2 = 1 : 1, ... so all squares are similar.

- What other shapes did you try . . . semi-circles, triangles, pentagons?
  . . . regular and irregular shapes?

2188 Population Pyramids

1. A = Kuwait
   B = Macau
   C = Greenland
   D = Monaco
   E = Denmark
   F = Algeria

2. Your Population pyramid should look similar to this one.



3. Your statements may differ from the ones below. If you are unsure about yours check them with your teacher.

- **Kuwait**
  Between the ages of 20-69 there are more males than females.
  The largest age group is 0-9, the smallest is 70+.

- **Macau**
  The greatest difference between the numbers of males and females is from 20-39.
  The largest population is in the group 20-29. The smallest is 70+.

- **Greenland**
  The number of males and females are similar in all age groups.
  The largest population is 20-29. The smallest is 70+.

continued/

2188 Population Pyramids (cont)

- **Monaco**
  There are many more females in the 70+ age group.
  The population tends to be older than in a lot of other countries.
  The 0 - 9 age group is the smallest.

- **Denmark**
  Up to 50 there are more males than females.
  The number in each age group remains relatively constant compared with many other countries.

- **Algeria**
  From 30 onwards there are more females than males.
  Going from youngest to oldest, in each age group there are fewer people than in the previous one.

- **United Kingdom**
  In most age groups the number of males and females are similar.
  There are many more females in the 70+ age group.
  The number of people in each age band is fairly equally balanced.

2189 Strange Dice Game

You could record your results in a table like this:

	Player Rolling Dice	Player moving 4
Game 1		
Game 2		
. .		
Total wins		

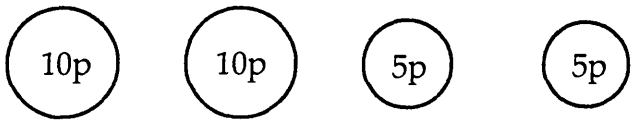
√ Tick for a win.

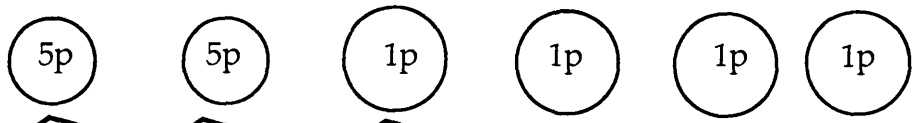
The game is not fair.

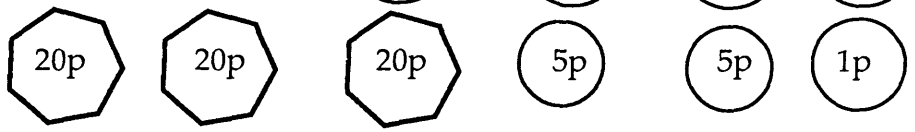
The player moving 4 squares usually wins more games than the player rolling the dice. This is because only 2 numbers on the dice are *bigger* than 4 but 3 numbers are *smaller* than 4.

2190 Twice as many

Here are some possible solutions. You may have found others.

1. 

2. 

3. 

4. Impossible.

5. You will need to get someone else to check your answers.

6. Get someone else to check your answers.
   Did you find any more amounts that were impossible?
   You could try to find amounts that can be made in two different ways so that one set contains:
   - three times as many coins as the other
   - four times as many coins as the other

   .
   .
   .

2191 Calculator Graphs

Using a graphic calculator. Here are some instructions on how to draw a graph for the Texas TI:81, the Sharp EL-9200 and the Casio fx-7000G. If your calculator is different, look in the manual, but these instructions may still be useful. Before you start to draw the graphs you will need to:

- set the range. *Use* (−) *for negative numbers not* (−) *button.*
- clear existing graphs.

- **To graph y = x**

	Texas TI:81	Sharp EL-9200	Casio fx-7000G
Press	Y=	[illegible] [illegible]	Graph
Press	X|T	X|θ|T	ALPHA [illegible]
Display should show	Y1=X Y2= Y3= Y4=	Y1= X	Graph Y=X
Press	GRAPH	[illegible]	EXE

continued/

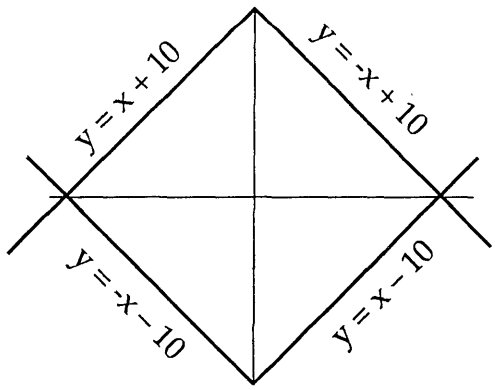
3. The equations used to create the screen are y = x and y = -x.

4. You should have found that any straight line graph:

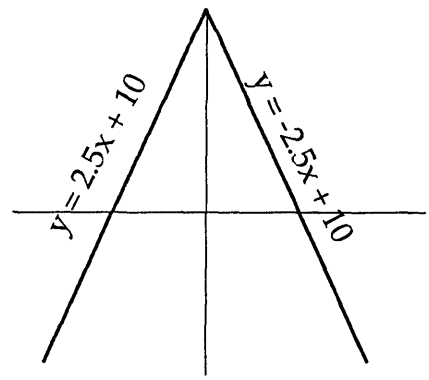
   - of the form y = **m**x such as y = 2x and y = -3x goes through (0, 0). The value of **m** determines the gradient (steepness) and the direction of the line.
     Positive values of **m** go in ⁄ direction.
     Negative values of **m** go in \ direction.
     The graph of y = **m**x has a gradient of **m**.

   - of the form y = x + **c** such as y = x + 10 and y = x – 5 does not go through the origin unless **c** = 0. The value of **c** determines the intercept (the point where the line cuts the y axis). The graph of y = x + **c** cuts the y axis at (0, **c**)

5. a)

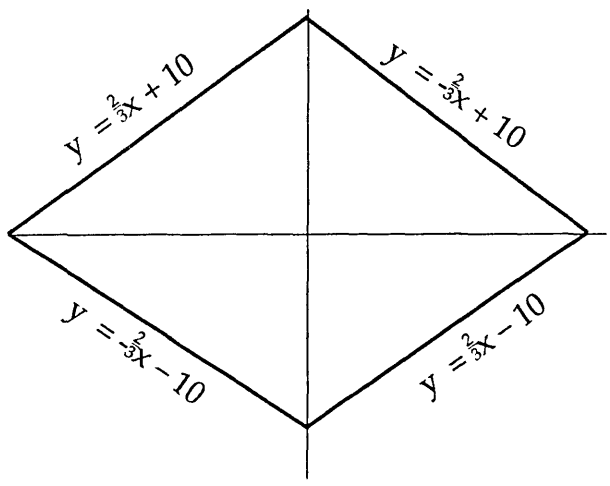


b)



c) The gradient of the four graphs is either $\frac{2}{3}$ or $-\frac{2}{3}$.

The key presses for the graph y = $-\frac{2}{3}$x + 10 on the Texas TI:81 are:



2192 Solving Quadratic Equations

1. a) $x = -4$ b) $x = -1$ c) $x = 5$
   d) $x = 5$ e) $x = -2$ f) $x = 6$
   g) $x = 10$ h) $x = 10$

2. The term in each quadratic equation which does not contain x gives the clue for which "square" to aim for.
   In the general form of a quadratic equation $y = ax^2 + bx + c$, this is the **c** term.

3. Equation b).
   -100 is not a square number, a square number cannot be negative because $(+x)^2 = x^2$ and $(-x)^2 = x^2$.

4. a) In order for the quadratic equation to be a perfect square, the following must be true: $c = k^2$ or $\sqrt{c} = k$

   b) In order for the quadratic equation to be a perfect square, the following must be true: $\frac{b}{2} = k$ or $b = 2k$

   c) In order for the quadratic equation to be a perfect square, the following must be true: $\frac{b^2}{2} = c$ or $b = 2\sqrt{c}$

5. When substituting values for x into the equation you may have found that the equation did not *exactly* equal zero.

   - Using $x = 20.806$ (taking $\sqrt{164}$ correct to 3 decimal places)
     $x^2 - 16x - 100 = -0.006364$

   - Using $x = 20.80624847$ (taking $\sqrt{164}$ correct to 8 decimal places)
     $x^2 - 16x - 100 = -0.000000124$

   Because $\sqrt{164}$ is an irrational number you will never get exactly $x^2 - 16x - 100 = 0$, but it will be close.

6. a) $x = -4 \pm \sqrt{91}$ $x = 5.539$ or $x = -13.539$

   b) $x = 4 \pm \sqrt{91}$ $x = 13.539$ or $x = -5.539$

   c) $x = -5 \pm \sqrt{53}$ $x = 2.280$ or $x = -12.280$

   d) $x = 1/2$

   e) $x = -5 \pm \sqrt{21}$ $x = -9.583$ or $x = -0.417$

   f) $x = -7 \pm \sqrt{115}$ $x = 3.724$ or $x = -17.724$

continued/

7. a) b = -10 c = 4 $x = \frac{+10 \pm \sqrt{84}}{2}$ $x = 9.583$ or $x = 0.417$

b) b = 10 c = 20 $x = \frac{-10 \pm \sqrt{20}}{2}$ $x = -2.764$ or $x = -7.236$

c) b = 10 c = 25 $x = \frac{-10}{2}$ $x = -5$

d) b = -10 c = 25 $x = \frac{10}{2}$ $x = 5$

e) b = 6 c = 19 $x = \frac{-6 \pm \sqrt{-40}}{2}$ No solution

f) b = 10 c = 30 $x = \frac{-10 \pm \sqrt{-20}}{2}$ No solution

8. You should have found that:
   - where there are **two** solutions for x as in 7a) and b), the graphs of these equations cut the x-axis in **two** places
   - where there is only **one** solution for x as in 7c) and d), the graphs of these equations cut the x-axis in **one** place
   - where there are no solutions for x, as in 7e) and f), the graphs of these equations do not cut the x-axis at all.

9. a) 2 solutions b) 2 solutions c) 2 solutions
   d) No solutions e) 2 solutions f) 2 solutions

- In the equation $3x^2 - 10x + 4 = 0$: a = 3 b = -10 c = 4

$$x = \frac{10 \pm \sqrt{100 - 48}}{6}$$

$$x = \frac{10 \pm \sqrt{52}}{6}$$

$$x = \frac{10 + 7.211}{6} \quad \text{or} \quad x = \frac{10 - 7.211}{6}$$

$x = 2.869$ or $x = 0.465$

The graph of the quadratic equation will cut the x-axis in two places.

2193 Number Square Words

				Total in numbers	Total in words
408	59	632	=	1099	One thousand and ninety-nine
342	187	778	=	1307	One thousand three hundred and seven
358	791	499	=	1648	One thousand six hundred and forty-eight
=	=	=			
1108	1037	1909			
One thousand one hundred and eight	One thousand and thirty-seven	One thousand nine hundred and nine			

Total in numbers

Total in words

2194 Tossing Coins

Here is a summary that Julie Kemp, a pupil at Parliament Hill School, Camden wrote, based upon her results and observations. The tables and graphs of her results are on the next page.

From these results, I can see that as the number of tosses increase, the accuracy of probability gets better, e.g.

on the 1st graph (1 - 10 tosses) the results vary from the decimal 0.4 right up to 1.0 creating a very abstract line.

on the 2nd graph (1 - 100 tosses) the results even out a lot more and the line is concentrated around the 0.4 - 0.5 decimal.

on the 3rd graph (1 - 1000 tosses) the line is almost straight, lying around the 0.5 decimal mark.

This tells me that to find an accurate probability, the experiment must be carried out for longer e.g. more tosses in this case.

The theoretical probability of gaining heads by tossing a coin 10 times would be 5 out of 10 or 0.5 (1/2). After carrying out the task a few times I found the experimental probability did not agree.

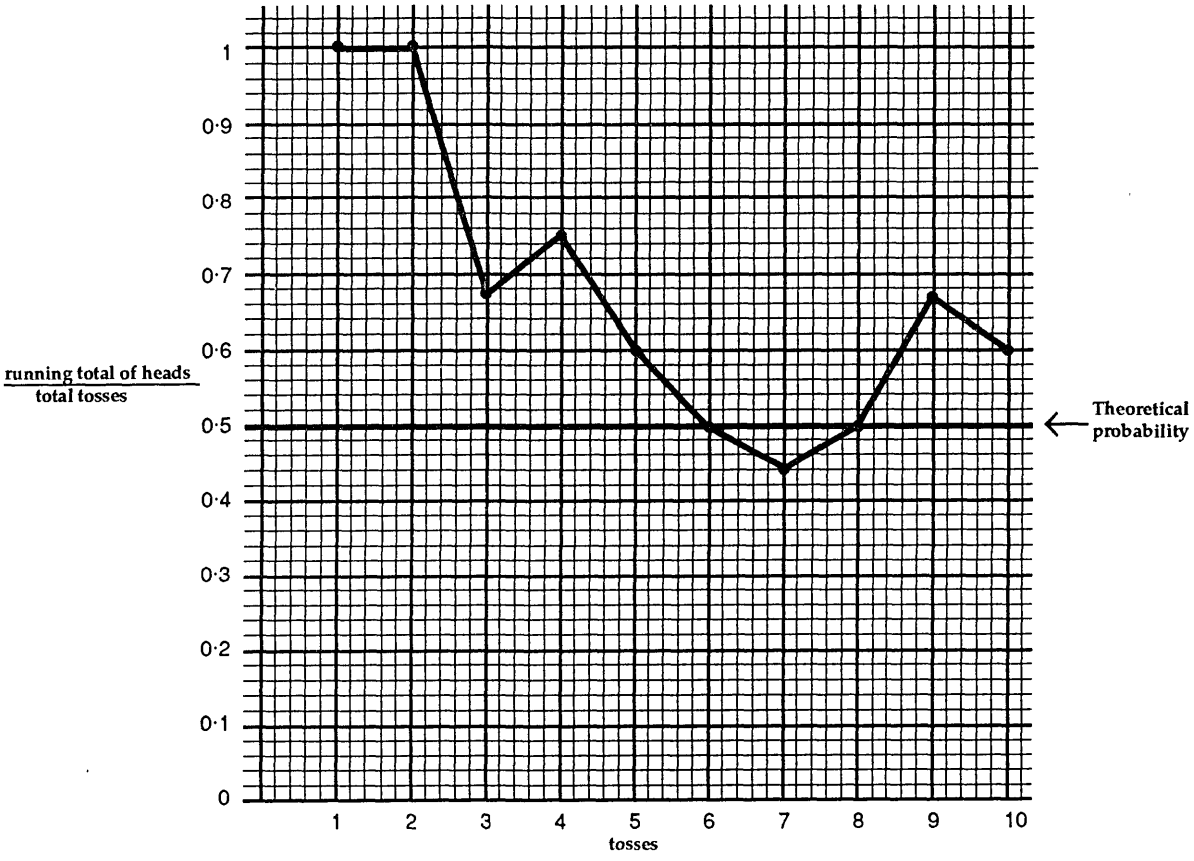
The more times the coin was tossed, the closer the experimental probability became to the theoretical i.e. 0.5 (50 out of 100 and 500 out of 1000).

continued/

Experiment 1

toss	1	2	3	4	5	6	7	8	9	10
heads	1	1	0	1	0	0	0	1	1	1
running total of heads	1	2	2	3	3	3	3	4	5	6
$\frac{\text{running total of heads}}{\text{total tosses}}$	$^1/_1$	$^2/_2$	$^2/_3$	$^3/_4$	$^3/_5$	$^3/_6$	$^3/_7$	$^4/_8$	$^5/_9$	$^6/_{10}$
as a decimal to 2 d. p.	1.00	1.00	0.67	0.75	0.60	0.50	0.57	0.50	0·66	0·60

Julie's results are plotted on the graph below.



Here are some of Julie's comments comparing her results with the ones on the card.

The similarities of the two graphs.

Both 10th tosses read off 0.6 (heads ÷ total)

Both lines are more or less concentrated around the points 0.5 and 0.6 e.g. The probability of getting heads is $\frac{1}{2}$.

The differences of the two graphs.

On the first graph, the first toss starts right at the bottom. In my graph the first toss starts right at the top, at 1.

The results for the first graph range from 0 to 0.67, whereas on my graph the results range from 0.42 to 1.

continued/

2194 Tossing Coins (cont)

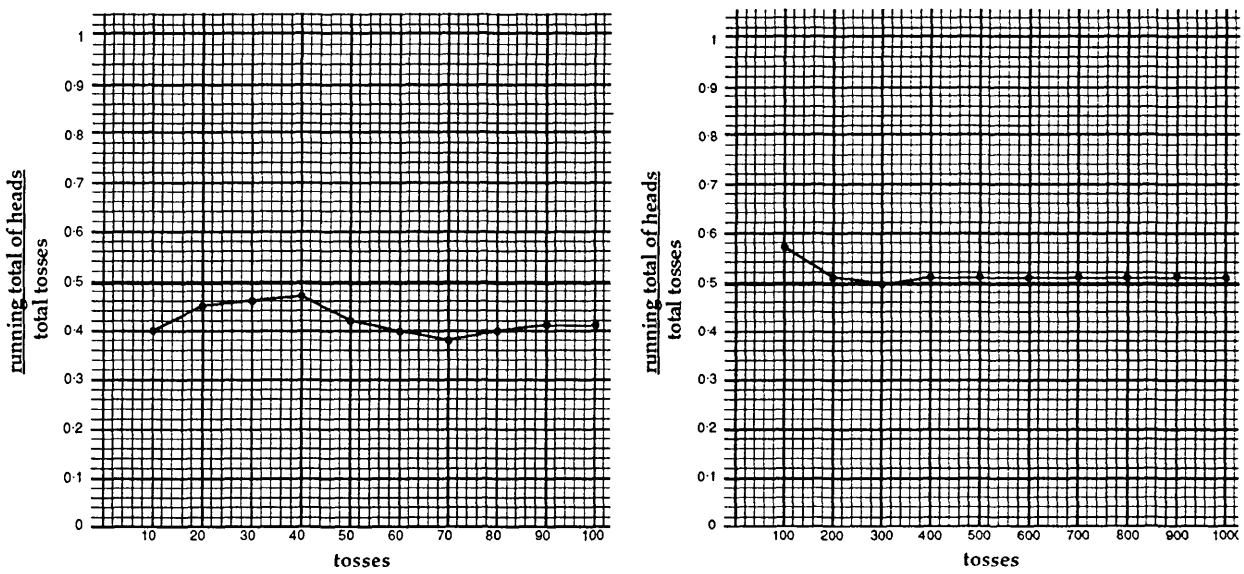
Experiment 2. Results obtained using Pinball

Toss	10	20	30	40	50	60	70	80	90	100
Running total of heads	4	9	14	19	21	24	27	32	37	41
Heads ÷ total	$\frac{4}{10}$	$\frac{9}{20}$	$\frac{14}{30}$	$\frac{19}{40}$	$\frac{21}{50}$	$\frac{24}{60}$	$\frac{27}{70}$	$\frac{32}{80}$	$\frac{37}{90}$	$\frac{41}{100}$
Decimal	0.40	0.45	0.46	0.47	0.42	0.40	0.38	0.40	0.41	0.41

Experiment 3. Results obtained using Pinball

Toss	100	200	300	400	500	600	700	800	900	1000
Running total of heads	57	102	152	204	255	306	356	405	459	507
Heads ÷ total	$\frac{57}{100}$	$\frac{102}{200}$	$\frac{152}{300}$	$\frac{204}{400}$	$\frac{255}{500}$	$\frac{306}{600}$	$\frac{356}{700}$	$\frac{405}{800}$	$\frac{459}{900}$	$\frac{507}{1000}$
Decimal	0.57	0.51	0.50	0.51	0.51	0.51	0.51	0.51	0.51	0.51

These two graphs show Julie's results.



Julie's results show that the more times an experiment is carried out, the more likely it is that the experimental probability will become closer to the theoretical probability. If the experiment was repeated for 10000 or 100000 or 1000000 times, the probability would get closer and closer to 0.5.

2195 The Higher, the Better

The largest number you can get with a 2 digits is 98.
The largest number you can get with 3 digits is 987.
Try playing the game a different way, where the aim of the game is to make the **smallest** number.

2196 Origami Boxes

- The dimensions of the basic box are: height = 5.3cm
  length = 15cm
  width = 10.4cm
  If you have folded the paper carefully and measured the dimensions accurately, you results should be approximately the same. If you are unsure about the measurements of your box, check them with your teacher.

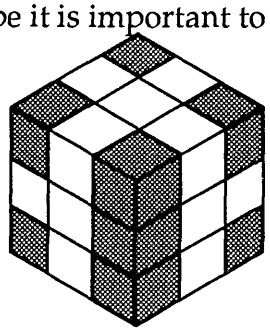
- The volume of the basic box is: height x length x width
  5.3cm x 15cm x 10.4cm = 826.8cm^3.

- By varying the fold lines in Stage 3 of the instructions, it is possible to create a box with a greater volume.
  The largest volume box found by a group of students had height = 4.4cm, length = 18.2cm and width = 12.8cm.
  The volume of this box was 4.4cm x 18.2cm x 12.8cm = 1025cm^3.

  Were you able to find dimensions of a box that had a greater volume?

2197 Blue in the Face

There are many possible answers ranging from 10 - 26 faces.
To maximise the number of blue faces showing on a cube it is important to place the blue cubes:

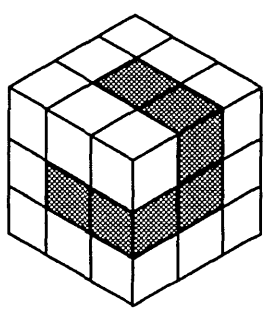


- in each of the eight corners,
  because this allows 3 faces to be seen.
  This gives (8 x 3) faces = 24 faces

- on an edge of the cube.
  This gives (1 x 2) faces = 2 faces

  Maximum total (24 + 2) = 26 faces

To minimise the number of blue faces it is important to place the blue cubes:



- in the centre of the cube,
  so none of the blue cube can be seen.
  This gives (1 x 0) faces = 0 faces

- in the centre of each face.
  This gives (6 x 1) faces = 6 faces

- on an edge of the cube.
  This gives (2 x 2) faces = 4 faces

  Minimum total (0 + 6 + 4) = 10 faces

Were you able to generalise the maximum and minimum number of blue faces for x number of blue cubes in an 'n x n x n' cube? Try to justify your generalisation.
Can the same generalisation be applied to cuboids?

2198 Testing Dice

1.

Sharon		
Number on dice	Tally	Frequency
1	𝍸 ||	7
2	𝍸 |||	8
3	𝍸 |	6
4	𝍸 𝍸	10
5	𝍸 ||||	9
6	𝍸 𝍸	10
	Total	50

2. a) 10 times
   b) 13 times
   c) 9 times

3. a) The mode in Rachida's test was 4.
   b) The modes in Sharon's test were 4 and 6.

4. a) The range was 18. (18 – 0 = 18)
   b) The range was 4. (10 – 6 = 4)

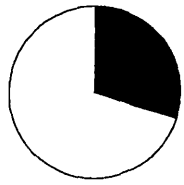
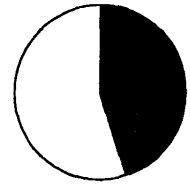
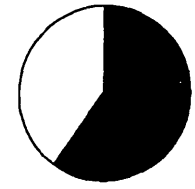
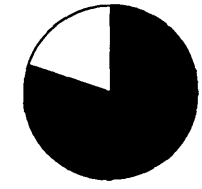
5. Rachida's bar chart is C and her pie chart is i.
   Hannan's bar chart is B and her pie chart is iii.
   Sharon's bar chart is A and her pie chart is ii.

6. You have probably chosen Hannan and Sharon's dice. On a fair dice each number is equally likely to appear. Sharon's bar chart (A) shows the fairest dice as the height of each column is similar and the range is very small.

2199 Percentage Estimation

1. Estimates. If your estimates were within 5 - 10% you have done well.

   Measure.
   a) 20% shaded, 80% unshaded
   b) 90% shaded, 10% unshaded
   c) 25% shaded, 75% unshaded
   d) 60% shaded, 10% shaded with lines, 30% unshaded

2. 30%  45%  60%  80% 

2200 Pie Charts for Breakfast

1 a) The total % for Oatiebran is 100%

b)

	Cornflakes	Oatiebran	Sugar Flakes	Muesli	Choco Pops
% Sugar	8.0	18.0	40.0	26.7	39.0
% Starch	76.0	29.0	**49.0**	39.9	48.0
% Fat	0.7	3.5	0.5	**6.2**	1.0
% Fibre	1.0	24.0	0.6	7.2	1.0
% Other	14.3	25.5	9.9	20.0	**11.0**
Total %	100	**100**	100	**100**	**100**

2. Cornflakes matches pie chart C
Oatiebran matches pie chart D
Sugar Flakes matches pie chart A
Muesli matches pie chart E
Choco Pops matches pie chart B

3. Choco Pops and Sugar Flakes have very similar values for most ingredients. You might have checked which sector is which by measuring the different sectors of the pie chart to check particular values.

4.

	Banana Flakes
% Sugar	20%
% Starch	45%
% Fat	8%
% Fibre	12%
% Other	15%
Total %	100%

2201 Vectors and Squares

The second square can be described by the following vector moves:

$\overrightarrow{AB} = \begin{pmatrix}-1\\3\end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix}-3\\-1\end{pmatrix}$ $\overrightarrow{CD} = \begin{pmatrix}1\\-3\end{pmatrix}$ $\overrightarrow{DA} = \begin{pmatrix}3\\1\end{pmatrix}$

In general the following four vectors will produce a square. $\begin{pmatrix}x\\y\end{pmatrix}$ $\begin{pmatrix}-y\\x\end{pmatrix}$ $\begin{pmatrix}-x\\-y\end{pmatrix}$ $\begin{pmatrix}y\\-x\end{pmatrix}$

Did you create a square when you checked your generalisation?

2202 Visiting Every Point

For 10 points:

Jump sizes that do not visit every point	Jump sizes that do visit every point.
2, 4, 5, 6, 8, 10	1, 3, 7, 9

It will help you to make similar tables for all the different points and jump sizes you investigate.

Hints:

- Investigate the jump sizes that do not visit every points first. You may find it easier to see a pattern here than in those that do visit every point.
- If you find it difficult to describe the patterns that you have found, you could look up the definitions for the mathematical words factor and multiple.

2203 Algebra Match

$a - b - c$	=	$(a - b) - c$	=	$a - (b + c)$
$a + b + c$	=	$(a + b) + c$	=	$a + (b + c)$
abc	=	$(a \times b) \times c$	=	$a \times (b \times c)$
$a - b + c$	=	$(a - b) + c$	=	$a - (b - c)$
$a + b - c$	=	$(a + b) - c$	=	$a + (b - c)$
$ab + ac$	=	$a \times (b + c)$		
$ab - ac$	=	$a \times (b - c)$		
$ac - bc$	=	$(a - b) \times c$		
$ac + bc$	=	$(a + b) \times c$		
$a - bc$	=	$a - (b \times c)$		
$a + bc$	=	$a + (b \times c)$		
$ab + c$	=	$(a \times b) + c$		
$ab - c$	=	$(a \times b) - c$		

continued/

2203 Algebra Match (cont)

$\frac{a}{bc}$	=	$a \div (b \times c)$	=	$(a \div b) \div c$
$\frac{ab}{c}$	=	$(a \times b) \div c$	=	$a \times (b \div c)$
$\frac{ac}{b}$	=	$a \div (b \div c)$	=	$(a \div b) \times c$
$a - \frac{b}{c}$	=	$a - (b \div c)$		
$a + \frac{b}{c}$	=	$a + (b \div c)$		
$\frac{a}{b} - c$	=	$(a \div b) - c$		
$\frac{a}{b} + c$	=	$(a \div b) + c$		
$\frac{a}{b + c}$	=	$a \div (b + c)$		
$\frac{a}{b - c}$	=	$a \div (b - c)$		
$\frac{a + b}{c}$	=	$(a + b) \div c$		
$\frac{a - b}{c}$	=	$(a - b) \div c$		

Hints for substitution.

- Do not use 0 as a value for a, b or c. Can you think why?
- Choose different values for a, b and c.
- It may be best to choose small positive numbers.

2204 Maxagon

The maximum number of sides a polygon can have when drawn on a 4 x 4 grid is 16.

Here are some results recorded in a table.

Size grid	Maximum number of sides.
2 x 2	4
3 x 3	7
4 x 4	16
5 x 5	.
.	
.	
.	

Could you extend the table?
What patterns did you find? Did you find a rule?
Were there any exceptions to the rule?

If you looked at rectangles you may have found some of these results:

No. of dots across

No. of dots down	2	3	4	5	6
2	4				**4**
3		7			
4			16		**24**
5					**30**
6					

Did you find the other answers?
Can you explain the exceptions to your rule?

2205 Making 25p

There are only four ways to make 25p using 5p, 10p and 20p coins.

5p + 5p + 5p + 5p + 5p
5p + 5p + 5p + 10p
5p + 10p + 10p
5p + 20p

Did you use all these ways when you played the game?

2206 Exploring Sine Curves

Using a graphic calculator.

Here are some instructions on how to draw graphs for the Texas TI:81, the Sharp EL-9200 and the Casio fx-7000G. If your calculator is different look in the manual but these instructions may still be useful.

Before you start to draw the graphs you will need to:

- **set the range.**
  The range -360 < x < 360 and -3 < y < 3 will display the graphs similar to the axes on worksheet 2206a.

- **clear exisiting graphs**

To graph y = sin x.

	Texas TI:81	Sharp EL-9200	Casio fx-7000G
Press	Y=	 EQTN	Graph
Press	SIN X|T	SIN X|θ|T	SIN ALPHA +
Display should show	Y1=SINX Y2= Y3= Y4=	Y1= SINX	Graph Y=SINX
Press	GRAPH		EXE

To summarise for effects on the graph y = **a** sin x
- If **a** is positive, the graph is similar to y = sin x, with period 360°.
  If **a** is negative, the graph is a reflection of y = sin x with period 360°.
  The amplitude = | **a** |
  | **a** | means the modulus of a.   i.e. | -a | = a and | a | = a

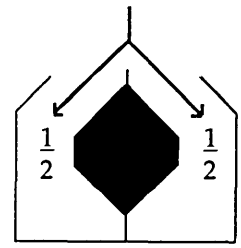
You should have found summaries for:
- y = sin **b**x,
- y = sin x + **c**
- y = sin(x + **d**)

so that you were able to predict what the graph y = 3sin (2x + 60) would look like.

The curve of y = 3sin (2x + 60) has:
- an amplitude of 3
- a period of 180°
- intersections with the x-axis at 60°, 150°, 240°, 330°, -30°, -120°, -210° and -300°.

2207 Pinball Experiment

Experiment 1



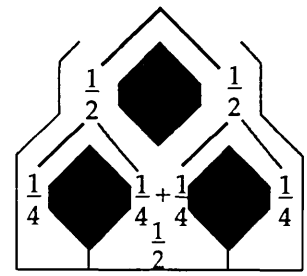
It is equally likely that a ball will go into the left box as into the right box, so when you dropped 40 balls you would expect to get 20 balls in each box.

However, when you experiment you might have these results: 25 15 or 21 19 or 19 21

It is even possible to get 35 5 or even 0 40, but this is very unlikely.

- A sensible prediction when dropping 100 balls would be: 50 50
  How near were your results when you tested your prediction?

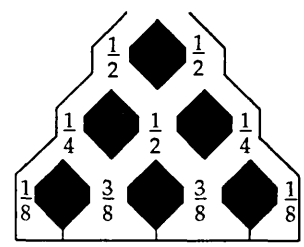
Experiment 2



When you dropped 40 balls you would expect to get
10 balls on the left, 20 in the middle and 10 on the right.
Were your results close? 10 20 10

- A sensible prediction when dropping 100 balls would be: 25 50 25
  This is because it is expected that one half will fall in the middle and one quarter at each end.
  How near were your results when you tested your prediction?

Experiment 3



Can you work out why you get $\frac{3}{8}$ in the middle two boxes?

When you dropped 40 balls you would expect to get 5 15 15 5
Were your results close?

If you drop 200 ball the expected outcome should be near to: 25 75 75 25

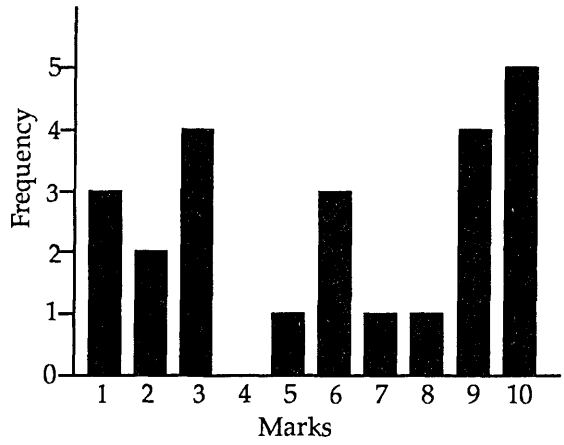
- You might like to consider what the probabilities are when you use 4 levels.
- Can you see a connection with cards 1790 and 0746?

2208 Best Marks

1. **9M results**

Mark	Tally	Frequency
1	| | |	3
2	| |	2
3	| | | |	4
4		0
5	|	1
6	| | |	3
7	|	1
8	|	1
9	| | | |	4
10	𝍸	5

Frequency graph for 9M

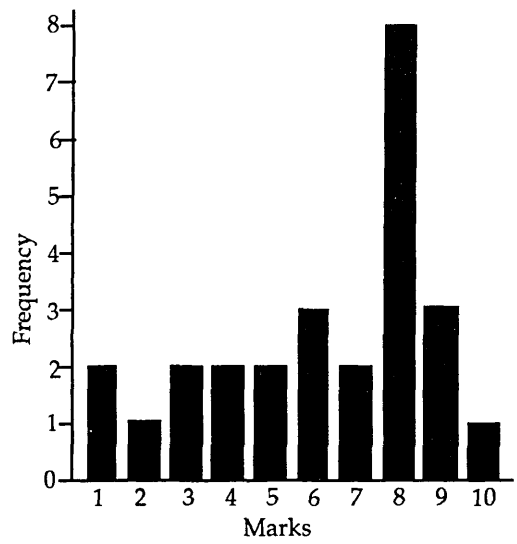


- Mean = $\frac{143}{24}$ = 5.958 to 3 decimal places
- Mode = 10
- Median = 6
- Range = 9

9L results

Mark	Tally	Frequency
1	| | |	2
2	|	1
3	| |	2
4	| |	2
5	| |	2
6	| | |	3
7	| |	2
8	𝍸 | | |	8
9	| | |	3
10	|	1

Frequency graph for 9L



- Mean = $\frac{161}{26}$ = 6.192 to 3 decimal places
- Mode = 8
- Median = 7
- Range = 9

continued/

2208 Best Marks (cont)

2. You could make a case for each class:

- 9L did best because it has the highest mean.
- 9M did best because it has the highest mode.
- 9S did best because more students in 9S scored top marks (9 or 10) than either of the others.

What other factors might play a part in deciding?

- Perhaps one class has improved a lot since Year 8 and so their scores are 'best' because they have made the most effort.
- Perhaps a class has some new students who have not done the work that was tested.

Can you think of any other reasons?

Is the question "*Which class do you think did best in the test?*" a 'fair' question?

2209 Short Orders

1.

	Price code	Eat-in	Take Away
a)	T + S	40p + 95p £1.35	35p + 80p £1.15
b)	C + B + R	£1.50	£1.40
c)	J + T + S + 2R or J + T + S + R + R	£3.55	£3.15

2.

	Price code	Eat-in	Take Away
a)	5C + 3T + 4B	£4.70	£4.30
b)	3J + 2T + 4R + S	£6.85	£6.10
c)		£10.10	£8.65

2210 Handspan

The answers will depend upon your results. Here are Saleha's results.

Name	Left hand span (cm)	Right hand span (cm)
Errol	20	20.5
Ersin	19.1	19.5
Farhan	18.7	19
Georgia	19.1	19.5
Javed	15.5	15
Jeffrey	18.7	18.3
Julian	17	17
Leon	16.8	17
Margaret	17	17.5
Sagal	17.6	18
Saleha	18.2	17.8
Sarah	17.7	18
Vishal	17.5	18
William	22.6	23
Yaqub	20.4	20

- Saleha's left hand span is 18.2cm.
  She found 7 people had a larger left hand span and 7 people had a smaller left hand span.

  Saleha's right hand span is 17.8cm.
  10 people had a larger right hand span and 4 people had a smaller right hand span.

- Saleha's left hand span is larger than her right hand span.

- There was only one person, Julian, who had the same span for both hands.
  10 people had larger right hand spans than left.
  4 people had larger left hand spans than right.

- How did you display your results?
  Were your results similar to Saleha's results?

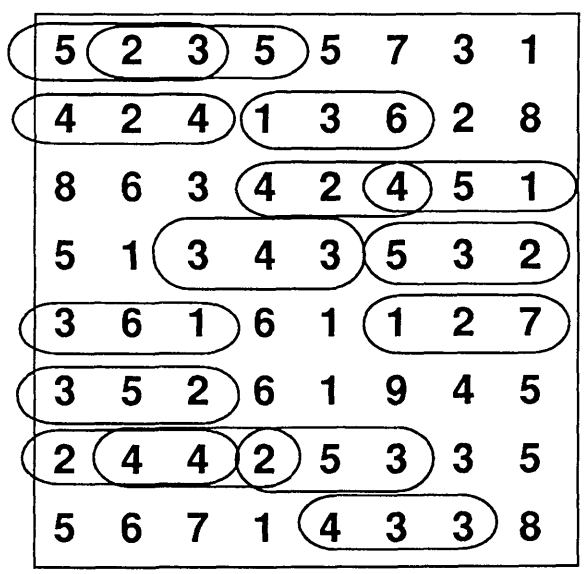
2211 Equivalent Expressions

$2a - a^2$	=	$-a(a - 2)$	=	$a^2 + 2a - 2a^2$
$a(a + 1) + a$	=	$a^2 + 2a$	=	$a(a + 2)$
$^1/_2(6a^2 - 4)$	=	$3a^2 - 2$	=	$3a^2 + 2a - 2 - 2a$
$5(a + 1)$	=	$5a + 5$	=	$^1/_2(10a + 10)$
$4a^2 - 2a$	=	$2a(2a - 1)$	=	$2(2a^2 - a)$
$6a + 3a^2$	=	$3(2a + a^2)$	=	$3a(2 + a)$
$2(a + 3)$	=	$2a + 6$	=	$2(a + 2) + 2$
$\frac{a}{2} + 2$	=	$a - 0.5a + 2$	=	$^1/_2(a + 4)$
$3(a - 7)$	=	$3a - 21$	=	$3(a - 8) + 3$
$0.5(a^2 + 6a)$	=	$a(\frac{a}{2} + 3)$	=	$0.5a^2 + 3a$

- Were the expressions equivalent for *all* values of a?

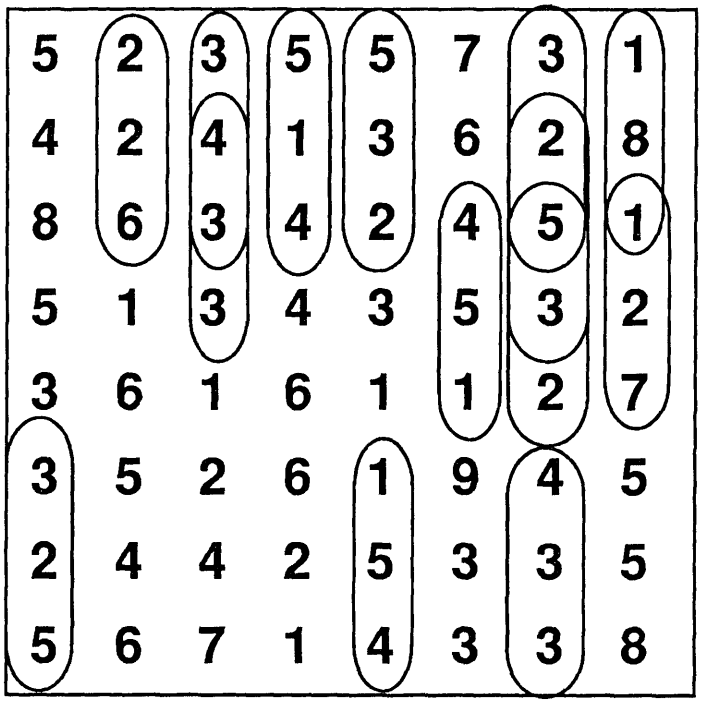
2212 10 Search

There are 15 horizontal ways (↔) of making 10.

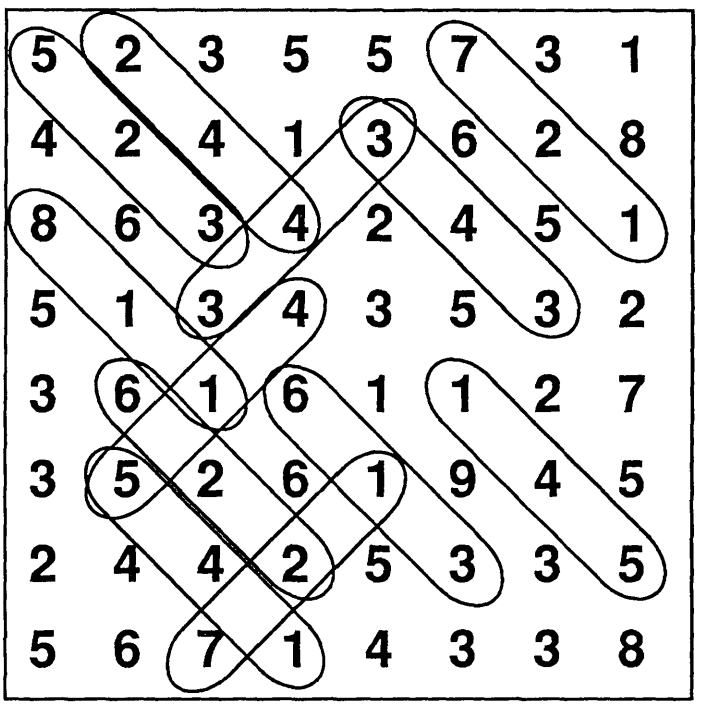


continued/

- 14 vertical ways (↕)

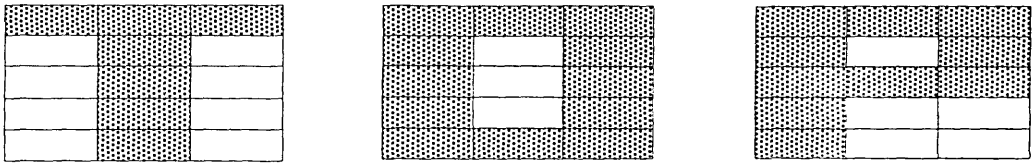


- 12 diagonal ways (↘) or (↗)

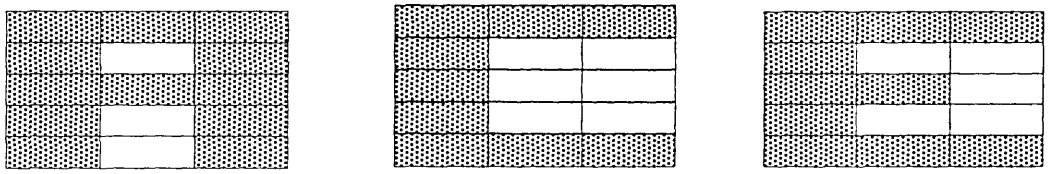


- Altogether there were 41 ways of making 10. (15 + 14 + 12).
- Did you find any other ways?

2213 Sum Message



The word is TOP



The word is ACE

- Show your grid to your teacher.

2214 Shape Sequence

Sequence One

To help you recreate and continue the sequence you could look at:

- the length of each side, (using Pythagoras' Theorem)
- the area of each shape
- the translation of one point (x)
- the angle of rotation about a centre of rotation

Shape	1st	2nd	3rd	4th	5th	6th
Length of each side (cm)	1	$\sqrt{2}$	2	$\sqrt{8} = 2\sqrt{2}$	3	$\sqrt{18} = 3\sqrt{2}$
Area(cm^2)	1.25	2.5	5	10	20	. . .
Translation of (x) as a vector	$\begin{pmatrix}1\\1\end{pmatrix}$	$\begin{pmatrix}2\\0\end{pmatrix}$	$\begin{pmatrix}2\\-2\end{pmatrix}$	$\begin{pmatrix}3\\0\end{pmatrix}$	. . .	

There are many different ways to describe the rules for creating the pattern in terms of a combination of enlargements, rotations and translations. Discuss your description with your teacher.

Sequence Two

What factors did you use to help you continue the pattern. Discuss your description of the sequence with your teacher.

2215 Identicube

$3^3 - 2^3$	=	$3(3 \times 2 \times 1) + 1^3$	$3^3 - 2^3$	=	$3(3 \times 2 \times 1) + 1^3$
$4^3 - 2^3$	=	$3(4 \times 2 \times 2) + 2^3$	$4^3 - 3^3$	=	$3(4 \times 3 \times 1) + 1^3$
$5^3 - 2^3$	=	$3(5 \times 2 \times 3) + 3^3$	$5^3 - 4^3$	=	$3(5 \times 4 \times 1) + 1^3$
$6^3 - 2^3$	=	$3(6 \times 2 \times 4) + 4^3$	$6^3 - 5^3$	=	$3(6 \times 5 \times 1) + 1^3$
.			.		
.			.		
.			.		
$x^3 - 2^3$	=	$3(x \times 2 \times (x-2)) + (x-2)^3$	$x^3 - (x-1)^3$	=	$3(x \times (x-1) \times 1) + 1^3$

Looking at the similarities and differences when y = 2 and y = 1 this identity is found.

$$x^3 - y^3 = 3(x \times y \times (x-y)) + (x-y)^3$$

You may like to expand the right hand side to check that it is equal to the left hand side. Here is the beginning of the expansion.

$$3(x \times y \times (x-y)) + (x-y)^3$$

$$= 3xy(x-y) + (x-y)(x^2 - 2xy + y^2)$$

$$= \ . \ . \ .$$

2216 From Matches to Mappings

1.

Number of triangles (t)	Number of matches (m)
1	3
2	5
3	7
4	9
5	11
6	13

Pattern down:	+ 2
Rule across:	x 2 + 1
Mapping:	$t \rightarrow 2t + 1$
Equation:	$m = 2t + 1$

2.

Number of pentagons (p)	Number of matches (m)
1	5
2	9
3	13
4	17
5	21
6	25

Pattern down:	+ 4
Rule across:	x 4 + 1
Mapping:	$p \rightarrow 4p + 1$
Equation:	$m = 4p + 1$

continued/

2216 From Matches to Mappings

3.

Number of parallelograms (p)	Number of matches (m)
1	4
2	7
3	10
4	13
5	16
6	19

Pattern down:	+ 3
Rule across:	x 3 + 1
Mapping:	$p \rightarrow 3p + 1$
Equation:	$m = 3p + 1$

4.

Number of hexagons (h)	Number of matches (m)
1	6
2	11
3	16
4	21
5	26
6	31

Pattern down:	+ 5
Rule across:	x 5 + 1
Mapping:	$h \rightarrow 5h + 1$
Equation:	$m = 5h + 1$

5.

Number of trapezia (t)	Number of matches (m)
1	5
2	9
3	13
4	17
5	21
6	25

Pattern down:	+ 4
Rule across:	x 4 + 1
Mapping:	$t \rightarrow 4t + 1$
Equation:	$m = 4t + 1$

6.

Number of rectangles (r)	Number of matches (m)
1	10
2	17
3	24
4	31
5	38
6	45

Pattern down:	+ 7
Rule across:	x 7 + 3
Mapping:	$r \rightarrow 7r + 3$
Equation:	$m = 7r + 3$

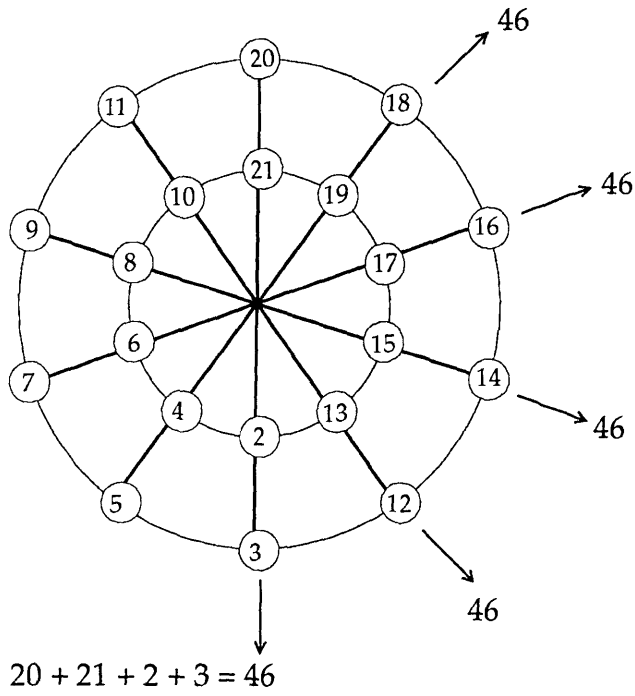
2217 Magic Circles

The magic numbers for the circle are:

172 (the sum of the numbers on the diameter.)

215 (the sum of the numbers on each ring.)

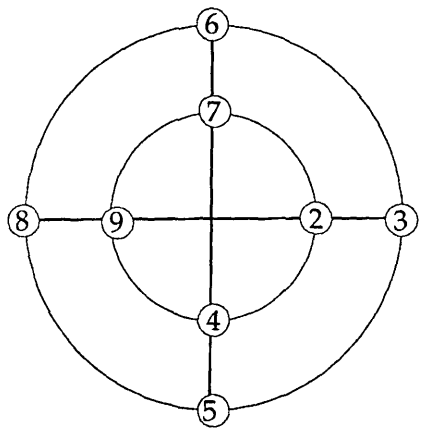
Here is one possible solution for this circle.



20 + 21 + 2 + 3 = 46

The magic numbers for this circle are:
46 from the diameters.
115 from the rings.

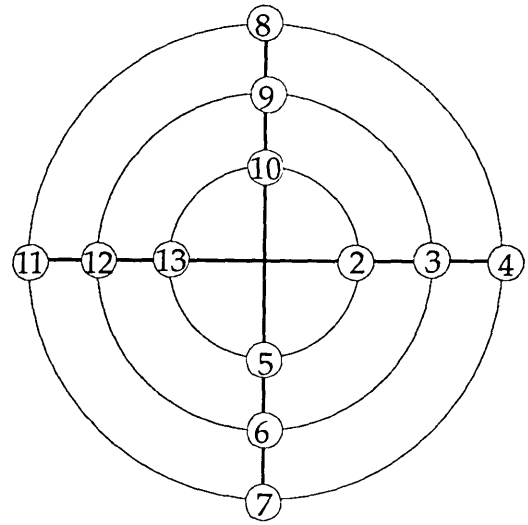
You could start with a simple magic circle:
Two rings and two diameters.



The magic numbers for this circle are:
22 from the diameters.
22 from the rings.

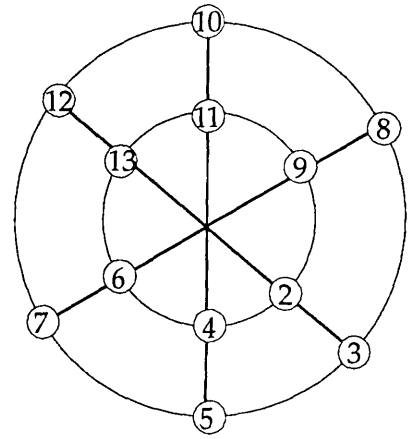
continued/

Then change the number of rings.
Three rings and two diameters.



The magic numbers for this circle are:
45 from the diameters.
30 from the rings.

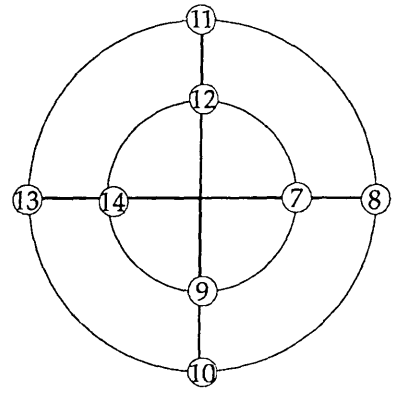
Or change the number of diameters.
Two rings and three diameters.



The magic numbers for this magic circle are:
30 from the diameters
45 from the rings

The magic numbers for the 2 circles above are connected.
Is there a similar connection for:
- Five rings two diameters
- Two rings five diameters?

Or change the starting number.
Two rings, two diameters, starting number 7.



The magic numbers for this magic circle are:
42 from the diameters and
42 from the rings.

The magic numbers are 20 more than when the starting number is 2.
Can you explain why this is?

2218 Origami Dodecahedron

What is the minimum number of colours you need if faces of the same colour are not allowed to touch?

2219 Origami Cube

If you have fitted the six pieces together correctly, you should be able to throw it gently up in the air and catch it, without it falling to pieces. If your cube falls to pieces, try to think of a way of interlocking the pieces so that they all stay together.

No glue or sellotape.

2220 Trig for Any Triangle

Triangle 1.

$$\frac{\sin B}{b} = \frac{\sin 72}{7.0} = 0.136$$

$$\frac{\sin C}{c} = \frac{\sin 56}{6.1} = 0.136$$

Triangle 2.

$$\frac{\sin A}{a} = \frac{\sin 81}{6.9} = 0.143$$

$$\frac{\sin B}{b} = \frac{\sin 63}{6.2} = 0.144$$

$$\frac{\sin C}{c} = \frac{\sin 36}{4.1} = 0.143$$

Triangle 3.

$$\frac{\sin A}{a} = \frac{\sin 80}{10.1} = 0.098$$

$$\frac{\sin B}{b} = \frac{\sin 76}{9.9} = 0.098$$

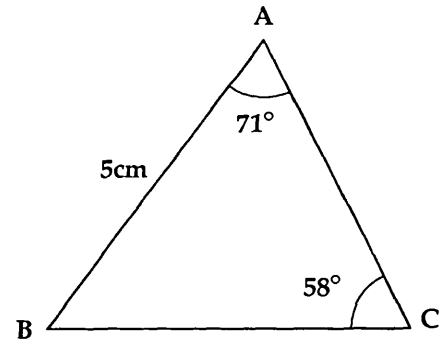
$$\frac{\sin C}{c} = \frac{\sin 24}{4.1} = 0.099$$

The ratio of $\frac{\sin A}{a}$, $\frac{\sin B}{b}$ and $\frac{\sin C}{c}$ for your three obtuse triangles should be almost equal.
The ratios are unlikely to be exact because we can only measure to the nearest millimetre and degree.

continued/

1. $\angle B = 51°$

$$\frac{\sin 71}{a} = \frac{\sin 51}{b} = \frac{\sin 58}{5}$$



To find side a:

$$\frac{\sin 71}{a} = \frac{\sin 58}{5}$$

$$\frac{\sin 71}{a} = 0.169609619$$

$$\sin 71 = 0.169609619a$$

$$\text{side a} = 5.574675429$$

$$= \mathbf{5.575cm}$$

To find side b:

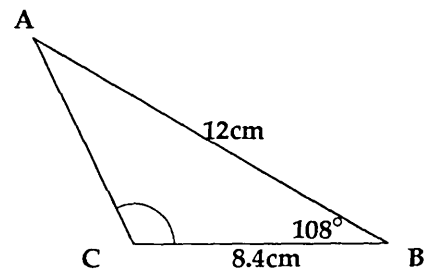
$$\frac{\sin 51}{b} = 0.169609619$$

$$\sin 51 = 0.169609619b$$

$$\text{side b} = 4.581968676$$

$$= \mathbf{4.582cm}$$

2. $$\frac{\sin A}{8.4} = \frac{\sin B}{b} = \frac{\sin 108}{12}$$



$$\frac{\sin A}{8.4} = \frac{\sin 108}{12}$$

$$\frac{\sin A}{8.4} = 0.079254709$$

$$\sin A = 0.665739561$$

$$\angle A = \mathbf{41.739°}$$

so $\angle B = \mathbf{30.261°}$

$$\frac{\sin 30.261}{b} = 0.079254709$$

side b = **6.358cm**

3. $\angle B = 46.434°$
   $\angle C = 63.566°$
   c = 7.909cm

4. $\angle C = 67°$
   c = 9.423cm
   a = 10.237cm

5. $\angle A = 40°$
   a = 3.268cm
   b = 4.991cm

6. $\angle C = 64.558°$
   $\angle A = 43.441°$
   a = 7.158cm

continued/

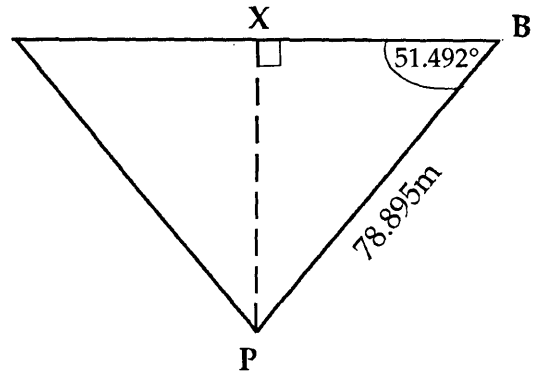
7. a) $\frac{\sin A}{a} = \frac{\sin B}{80} = \frac{\sin 78}{100} = 0.009781476$

$\angle ABP = 51.492°$

so $\angle PAB = 50.508°$

$PB = 78.895m$

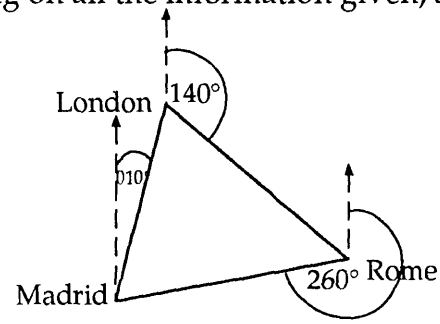
b) Let XP = the width of the river.



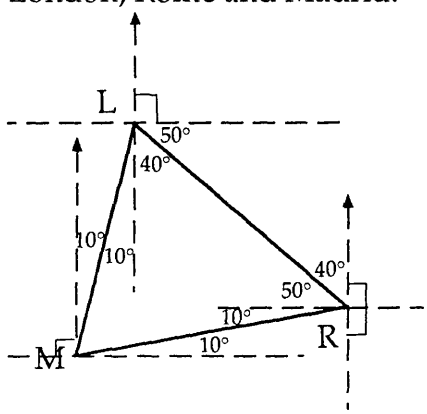
In right-angled triangle XBP: $\sin 51.492° = \frac{XP}{78.895}$

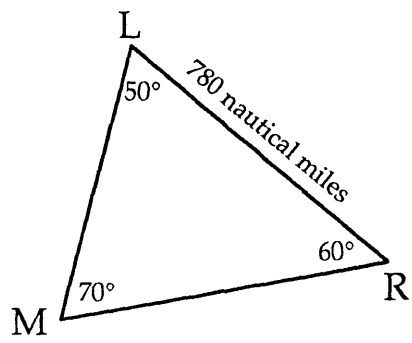
XP = 61.737m

8. When answering a question involving bearings, it is always best to draw a rough sketch first, putting on all the information given, as well as the North lines.



Once a sketch has been drawn, it is possible to work out the angles of the triangle formed by London, Rome and Madrid.





In triangle LRM: $\frac{\sin L}{l} = \frac{\sin R}{r} = \frac{\sin M}{m}$

$\frac{\sin 50}{l} = \frac{\sin 60}{r} = \frac{\sin 70}{m} = 0.001204734$

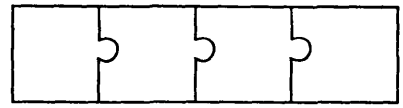
The distance from Madrid to London (r) = 718.852 nautical miles.

The distance from Madrid to Rome (l) = 635.862 nautical miles.

2221 Jigsaws

Your results should have led you to the following conclusions:

- *There are four corner pieces.* **Sometimes true.**
  This statement is true for all rectangles except for those with width 1.



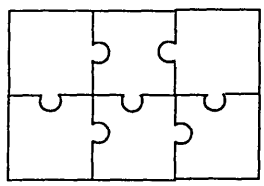
  Rectangles of width 1 are a special case of jigsaw as they have only two corner pieces. You may not have considered these jigsaws in your investigation.

- *For jigsaws with 30 pieces, the maximum number of middle pieces is 20.* **Not true.**
  The jigsaws possible with 30 pieces are:

1 by 30	→	0 middle pieces
2 by 15	→	0 middle pieces
3 by 10	→	8 middle pieces
5 by 6	→	12 middle pieces

  So 12 is the maximum number of middle pieces.

- *The number of edge pieces equals the middle pieces plus the corner pieces.* **Sometimes true.**
  It is true for the example on the card but a counter example is a 6 piece jigsaw arranged in a 3 by 2 rectangle.



  - 2 edge pieces
  - 4 corner pieces
  - 0 middle pieces

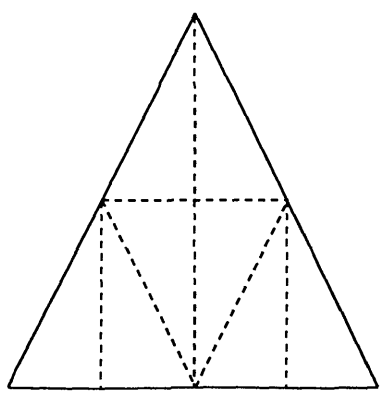
  $2 \neq 0 + 4$

- *For any number of pieces, it is possible to find a jigsaw which has no middle pieces.* **Always true.**
  Jigsaws of width 1 or 2 have no middle pieces.

2222 Equal Areas

You may have used various strategies to solve this problem.

This shows the isosceles triangle divided into eight congruent right-angled triangles.

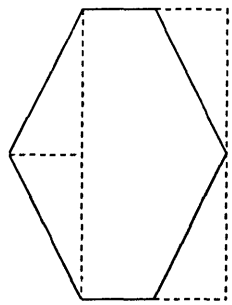


You can divide each of the other shapes into the same eight right-angled triangles.

continued/

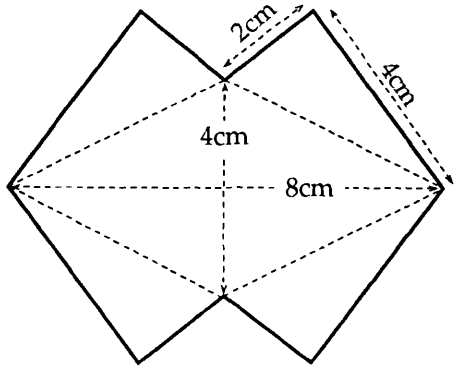
2222 Equal Area? (cont)

This shows the hexagon with two triangles cut off and stuck in different places to make a rectangle.



You can make the same rectangle from each of the other shapes.

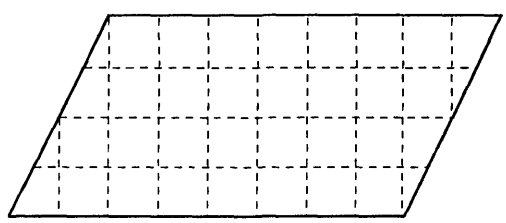
This shows the octagon divided into a rhombus and four right-angled triangles and then the areas found using formulae.



Area of octagon = $\frac{1}{2}(4 \times 8) + 4(\frac{1}{2}(2 \times 4))$
= $16 + 16$
= $32cm^2$

You can find each of the other shapes' areas using fomulae.

This shows the parallelogram on a square grid, the area is 32 squares.



You can find the area of each of the other shapes by putting them on a square grid.

2223 Fractions to Decimal Match

1.

$\frac{1}{2} = 0.5$	$\frac{1}{4} = 0.25$	$\frac{1}{3} = 0.33333333$	$\frac{3}{4} = 0.75$
$\frac{2}{5} = 0.4$	$\frac{2}{9} = 0.22222222$	$\frac{2}{3} = 0.66666666$	$\frac{7}{8} = 0.875$
$\frac{4}{5} = 0.8$	$\frac{3}{10} = 0.3$	$\frac{5}{6} = 0.83333333$	$\frac{1}{10} = 0.1$
$\frac{5}{8} = 0.625$	$\frac{4}{7} = 0.5714286$	$\frac{1}{7} = 0.1428571$	$\frac{5}{9} = 0.55555555$

2. The fractions in order of size are:

$\frac{1}{10}$ $\frac{1}{7}$ $\frac{2}{9}$ $\frac{1}{4}$ $\frac{3}{10}$ $\frac{1}{3}$ $\frac{2}{5}$ $\frac{1}{2}$ $\frac{5}{9}$ $\frac{4}{7}$ $\frac{5}{8}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{5}{6}$ $\frac{7}{8}$

2224 Shajjad's Collection

School Bank
Paying-slip

Monday

Paid in by Shajjad

Coins	Number in pile	Amount £	Amount p
£1	1	1	00
50p	5	2	50
20p	3	0	60
10p	4	0	40
5p	2	0	10
2p	2	0	04
1p	8	0	08
	Total Amount	4	72

School Bank
Paying-slip
Paying-in slip

Tuesday

Paid in by Shajjad

Coins	Number in pile	Amount £	Amount p
£1	0	0	00
50p	4	2	00
20p	1	0	20
10p	3	0	30
5p	3	0	15
2p	2	0	04
1p	5	0	05
	Total Amount	2	74

School Bank
Paying-slip

Wednesday

Paid in by Shajjad

Coins	Number in pile	Amount £	Amount p
£1	0	0	00
50p	2	1	00
20p	5	1	00
10p	0	0	00
5p	2	0	10
2p	5	0	10
1p	4	0	04
	Total Amount	2	24

School Bank
Paying-slip

Thursday

Paid in by Shajjad

Coins	Number in pile	Amount £	Amount p
£1	1	1	00
50p	2	1	00
20p	4	0	80
10p	10	1	00
5p	2	0	10
2p	5	0	10
1p	7	0	07
	Total Amount	4	07

School Bank
Paying-slip
Paying-in slip

Friday

Paid in by Shajjad

Coins	Number in pile	Amount £	Amount p
£1	0	0	00
50p	4	2	00
20p	5	1	00
10p	8	0	80
5p	6	0	30
2p	7	0	14
1p	6	0	06
	Total Amount	4	30

continued/

2224 Shajjad's Collection (cont)

1.

Day	Amount collected
Monday	£4.72
Tuesday	£2.74
Wednesday	£2.24
Thursday	£4.07
Friday	£4.30
Total for one week.	£18.07

2. Monday

3. Wednesday

4. £2.06

5. Monday, Thursday and Friday.

2225 Wildlife Collection

School Bank — **Paying-in slip**

Class 1

Date 10th June

Paid in by Louise

Coins	Number in pile	Amount £	Amount p
£1			
50p			
20p	3		60
10p	5		50
5p	3		15
2p	4		8
1p	3		3
Total Amount		£1	36

School Bank — **Paying-in slip**

Class 2

Date 10th June

Paid in by Louise

Coins	Number in pile	Amount £	Amount p
£1			
50p	1		50
20p	3		60
10p	4		40
5p	2		10
2p	6		12
1p	5		5
Total Amount		£1	77

continued/

2225 Wildlife Collection (cont)

School Bank **Paying-in slip**

Class 3

Date 10th June

Paid in by

Louise

Coins	Number in pile	Amount £	Amount p
£1	5	5	00
50p	4	2	00
20p	3		60
10p	9		90
5p	6		30
2p	4		8
1p	10		10
	Total Amount	£ 8	98

2226 Sum Number Cards

This pack of cards consists of the digits 0 - 9, the operations +, –, x, ÷, the = sign and brackets. If you are unsure about how to use the brackets ask you teacher.

You can use these cards with a number of SMILE activities. You could use them to play games like Countdown.

Alternatively you might like to make up your own game or puzzle using these cards.

2227 5p a line

You can get 5p by placing two 2p's and one 1p in any order in a line.

- Did it matter who went first?
- Did you get more points when you placed the 2p's in the corners of the grid or in the middle position of the grid?
- Can you think of any ways to improve the game?

2228 Vector Match

The 7 sets of equivalent vectors are:

b = $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ = $\vec{RS}$

c = $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ = $\vec{DC}$

d = $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$ = $\vec{GK}$

e = $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ = $\vec{FE}$

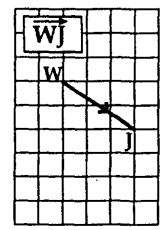
f = $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ = $\vec{PQ}$

g = $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ = $\vec{ML}$

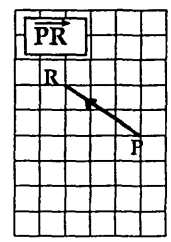
h = $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ = $\vec{BA}$

The three unmatched cards are:

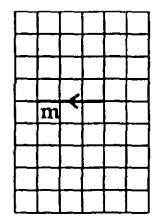
k = $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ = $\vec{WJ}$* =



l* = $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ = $\vec{PR}$* =



m* = $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ = $\vec{VU}$ =



* It does not matter which letters you choose to identify the vectors. Make sure the letters are in the same direction as the vector.

2229 Quadratics and Primes

You can generate primes from the quadratic $x^2 - x + 17$ using a spreadsheet or graphic calculator.

	A	B	
1	x	$x^2 - x + 17$	
2	0	17	⇐ A2^2–A2+17
3	1	17	
4	2	19	⇓
5	3	23	
6	4	29	
7	5	37	Fill Down
⇓	⇓	⇓	⇓
18	16	257	
19	17	289	⇐ Not prime
20	18	323	
21	19	359	
22	20	397	
23	21	437	
24	22	479	

The values of the expression $x^2 - x + 17$ are prime numbers except where $x = 17, 34, 51 \ldots$ In fact whenever x is a multiple of 17 the number generated is not a prime. Can you explain why?

The expression $x^2 - x + 41$ does not produce a prime when $x = 41$ or a multiple of 41.

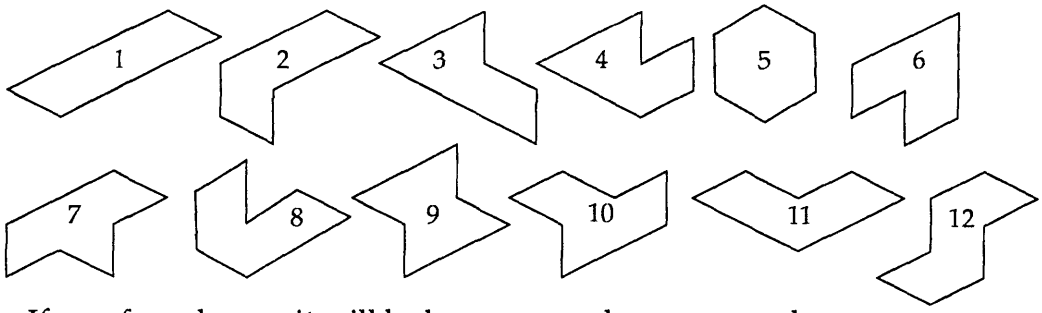
Any expression of the form $x^2 - x + c$ where c is a prime number will generate prime numbers for values of $x < c$. When $x \geq c$ the expression will only generate prime numbers when the value of x is not a multiple of c.

2230 Which has the largest area?

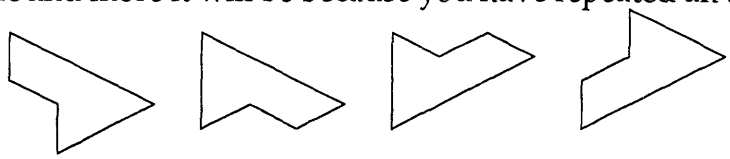
1. b
2. a
3. b
4. b
5. Was the person able to work out which of your two shapes has the largest area?

2231 Hexiamonds

1. These are the 12 different hexiamonds.



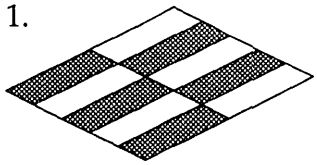
If you found more it will be because you have repeated an arrangement.
e.g.



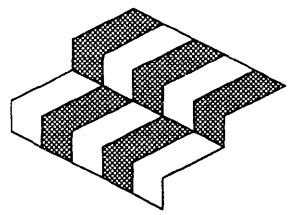
These are all the same shape reflected or rotated. Check you have found all 12 hexiamonds with no repetition.

Here is each hexiamond tessellated.

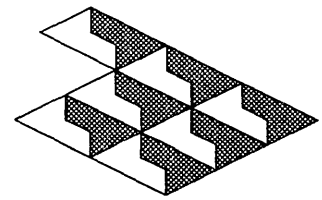
1.



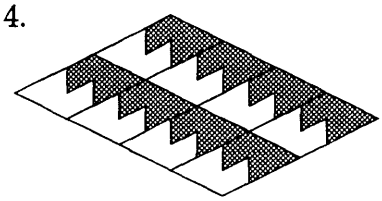
2.



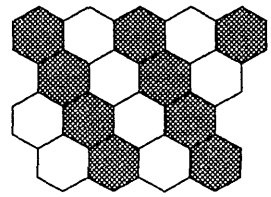
3.



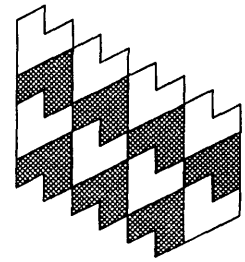
4.



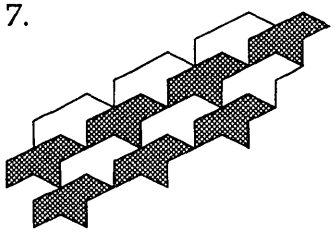
5.



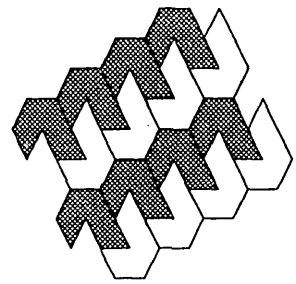
6.



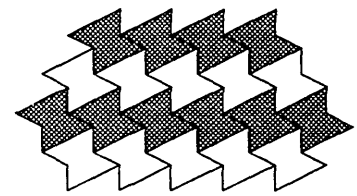
7.



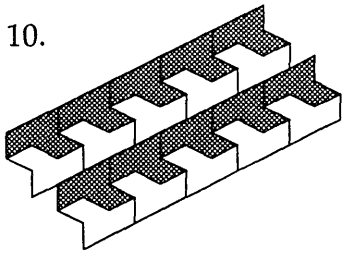
8.



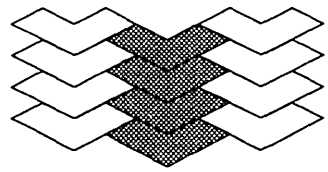
9.



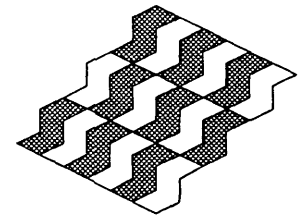
10.



11.



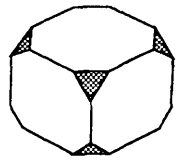
12.



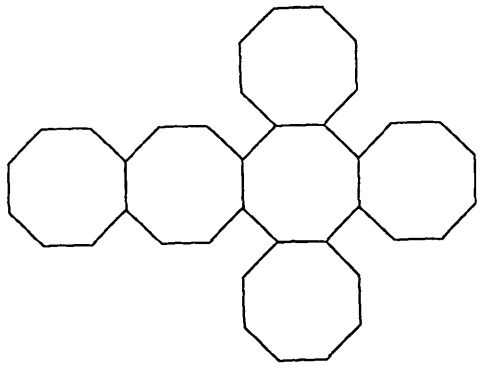
Have you found other ways?

2232 Cut a Cube

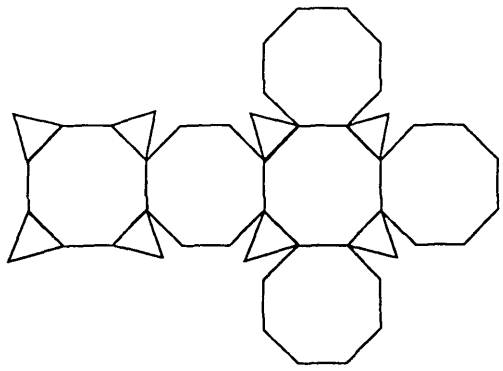
A cube with all its corners cut off should look like this.



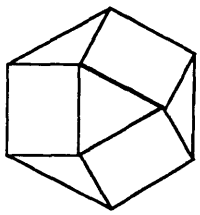
If the cube you imagined was hollow the net could look like this:



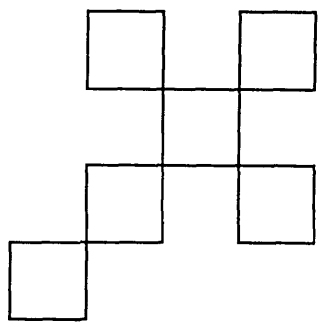
If the cube was solid then the net could appear this way:



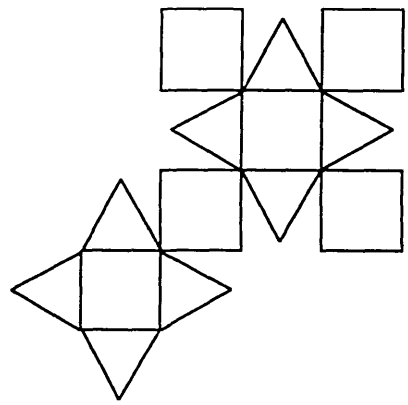
For a cube with all of its original edges removed:



Net for hollow shape



Net for solid shape.



Remember: These are not the only ways to draw the nets. Check that your nets, if different, are genuine nets by making them.

What other solids did you 'cut the corners' off? Show your nets for these solids to your teacher.

2233 Cafe Menu

Flora's Total	£4.45	Change from £10.00	£5.55
Neil's Total	£5.25	Change from £10.00	£4.75
Mai Ling's Total	£4.50	Change from £10.00	£5.50
Saskia's Total	£5.55	Change from £10.00	£4.45
George's Total	£4.70	Change from £10.00	£5.30

2234 Defining Regions

1. $x > -1$
   $y > 2$
   $x + y < 5$

2. $x < 15$
   $y < 10$
   $x + y > 5$

3. $y \leq 13$
   $x + y > 11$
   $y > 2x - 6$

4. $y \geq 0$
   $2x + y < 8$
   $y < 2x$

5. $x + y > 4$
   $x + y < 8$
   $y > x - 2$
   $y < x + 2$

Some of your inequalities maybe in a different form. Rearrange your solutions to check that they are equivalent.

2235 Headlines

1. There are many possible answers. This is what one student wrote:
   "London traffic speed is getting slower."
   Do you agree?

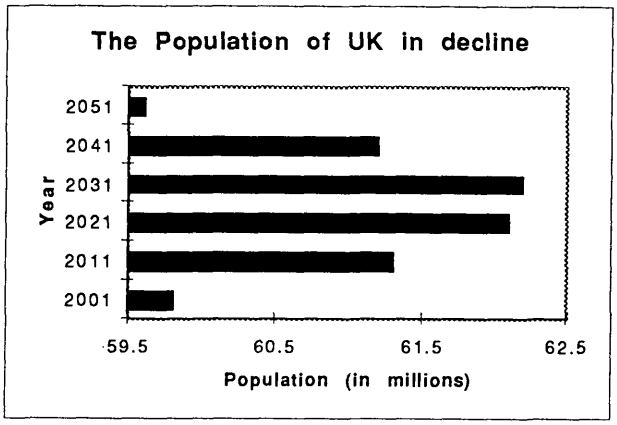
2. There are many possible answers. This is what one student wrote:
   "London traffic stays steady for ten years."
   Do you agree?
   The average speed in London has not changed from 1968 onwards.
   It is just over 10mph.

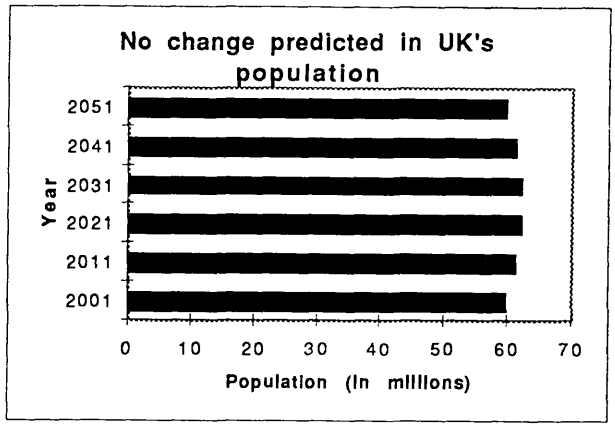
3. The first graph shows speeds from 10mph to 13.5mph, a range of 3.5mph.
   The lowest speed is 10mph. The scale is 1cm = 0.5mph.
   The second graph shows speeds from 0mph to 70mph, a range of 70mph.
   The lowest speed is 0mph. The scale is 1cm = 10mph.

continued/

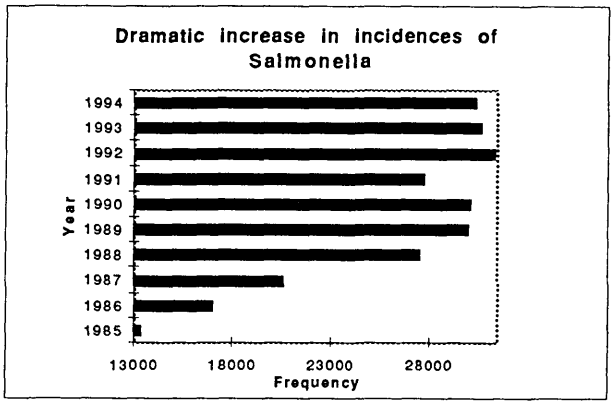
4. You should have drawn **one** of the pairs of graphs below. The graphs have been drawn using MS Excel. If your graphs or headlines are very different, discuss them with your teacher.

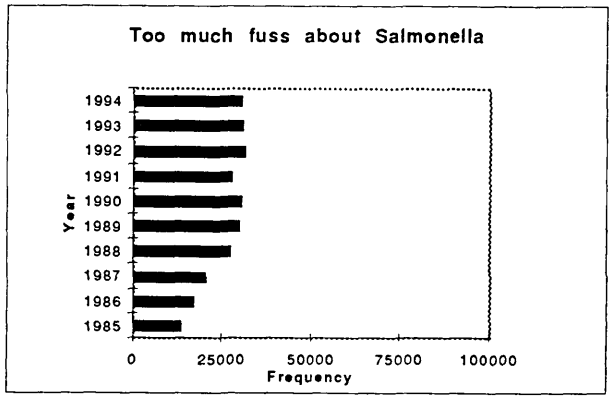
- Predicted population in United Kingdom



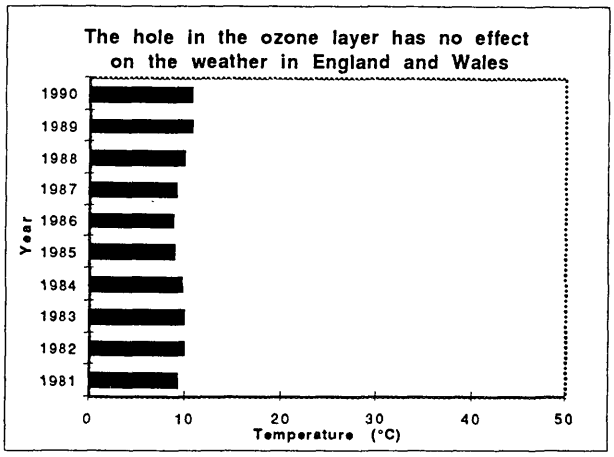


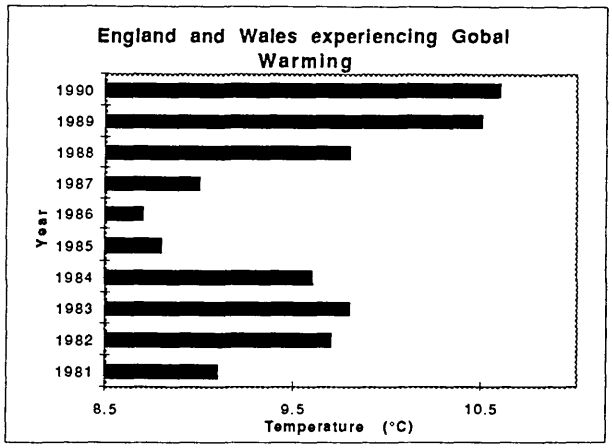
- Incidences of Salmonella in United Kingdom





- General temperature in England and Wales





5. Discuss your opinions about which came first with your teacher.

2236 25% of What?

1. 25% of the land in Liechtenstein is arable
   25% of 160km^2 = 40km^2

2. 25% of 71740km^2 = 17935km^2

3. 25% of 92390km^2 = 23097.5km^2

4.

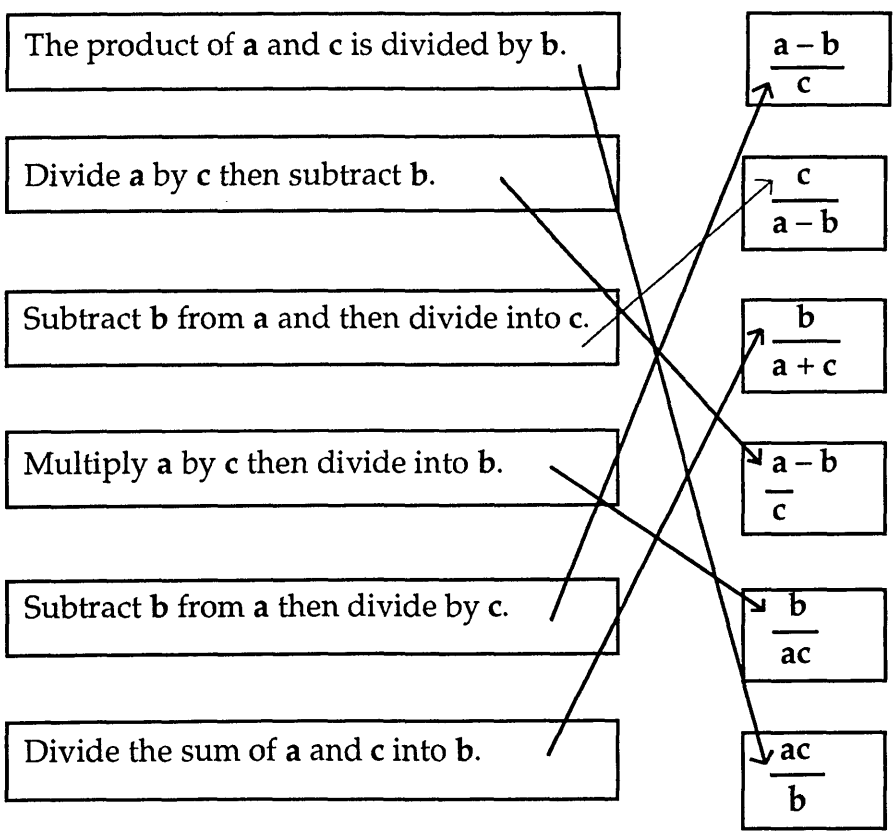
Pie chart	Country	Statement
A	Togo	1
B	Switzerland	2
C	Panama	3

5. The statement is not true.
   Togo and Sierra Leone are not equal in area. A percentage is a proportion of a quantity. 25% of a small quantity will be smaller than 25% of a large quantity.

   Togo's total area is 56790km^2, 25% of 56790km^2 is 14197.5km^2.
   Sierra Leone's total area is 71740km^2, 25% of 71740km^2 is 17935km^2.
   So Togo and Sierra Leone do not have the same amount of arable land.

2237 Words won't fail me!

1.



continued/

2237 Words won't fail me! (cont)

2. $\frac{a+b}{c+d}$

$\frac{cd}{a+b}$

$\frac{a+b}{cd}$

$\frac{a}{c}+\frac{b}{d}$

3. The sum of **a** and **b** is divided by **c**.

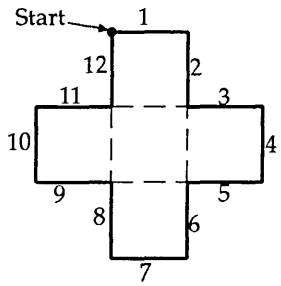
The sum of **c** and **d** is divided by the sum of **a** and **b**.

The product of **c** and **d** is divided by the product of **a** and **b**.

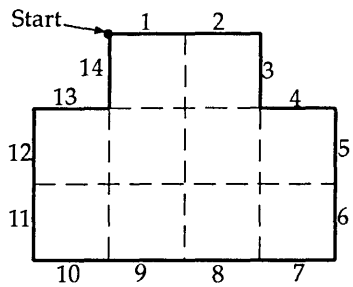
c is divided by the sum of **a** and **b**.

Your statements may vary slightly. If you are unsure whether they are correct, show your statements to your teacher.

2238 What is the perimeter?

1. 

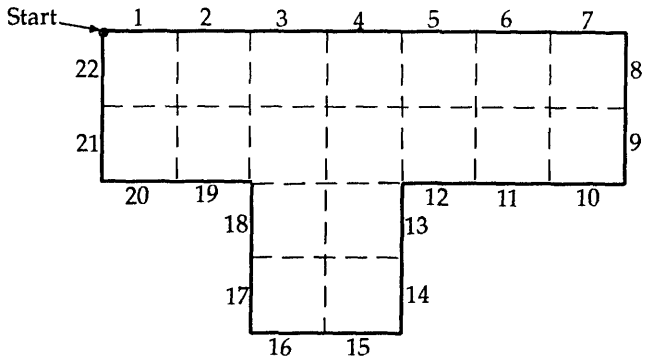
Perimeter = 12cm

2. 

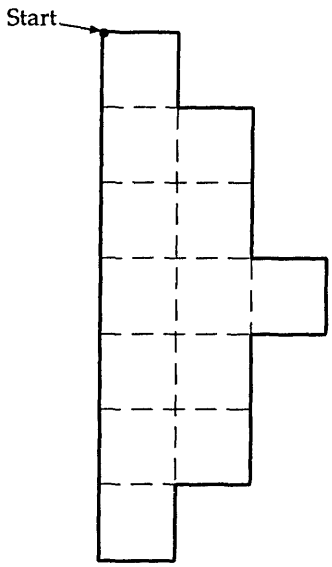
Perimeter = 14cm

continued/

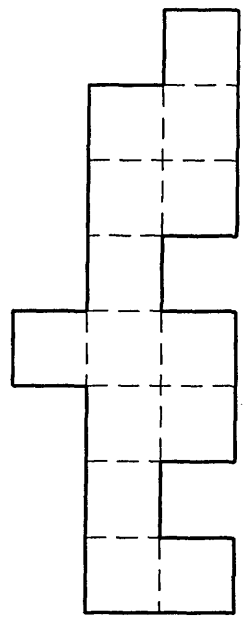
2238 What is the perimeter? (cont)

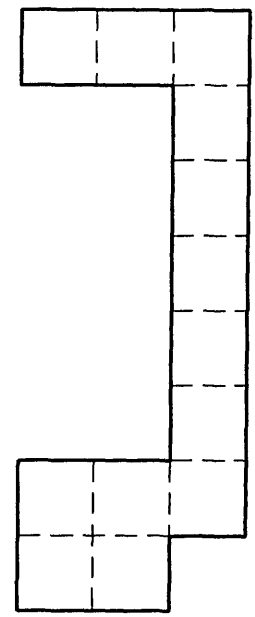
3. 

Perimeter = 22cm

4. 

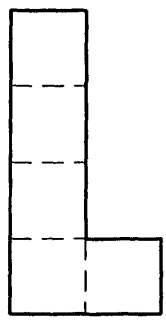
Perimeter = 20cm

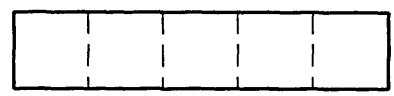
5. 
Perimeter = 26cm

6. 
Perimeter = 26cm

7. Show your two shapes and their perimeters to your teacher.

8. Here are two different shapes with a perimeter of 12cm.





If you have drawn different shapes, show them to your teacher.

2239 Putting in Order

B 1, 2, 3, 5, 7, 8, 9

C 20, 30, 40, 60, 70, 80, 90, 100, 150

D 1, 10, 11, 20, 21, 100, 101, 102, 111

E 0, 1, 2, 5, 10, 25, 52, 55, 255, 522

F 200, 300, 400, 500, 600, 800, 900

G 530, 600, 620, 630, 650, 700, 720

H 5, 50, 500, 5000, 50000

J 44, 54, 55, 545, 554, 555, 5444, 5555

K 1, 99, 100, 101, 396, 400, 1000, 1001

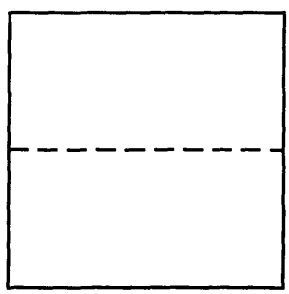
L 22, 25, 89, 225, 324, 346, 387, 516, 812, 824, 978, 1025, 2025, 3046

M 24, 42, 420, 2044, 2404, 2440, 4002, 4022, 4042, 4200, 4242, 4420

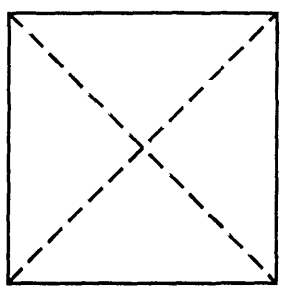
2240 Ask me another . . .

There are different ways of completing the sheet. Your answers may not be exactly the same as these. If you are unsure about your answers, check them with your teacher.

1.



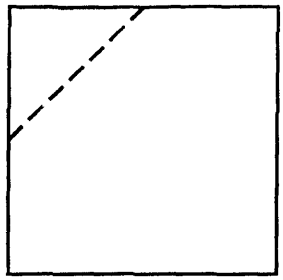
2.



You would get . . .

. . . four congruent isosceles right-angled triangles.

continued/

3. 

You would get . . .

. . . one isosceles right-angled triangle and a pentagon.

4. What would you get . . .

. . . if you drew a line from the middle of the top side to the middle of the right side, and from the middle of the left side to the middle of the bottom side?

5. What would you get . . .

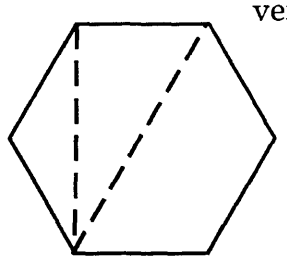
. . . if you drew a line from the bottom left vertex to the top right vertex, and from the middle of the top side to the middle of the bottom side?

You would get . . .

. . . two congruent right-angled triangles and two congruent trapeziums.

6. What would you get . . .

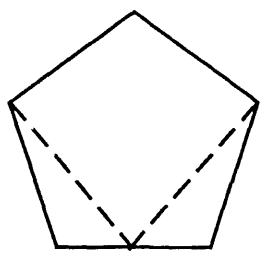
. . . if you drew a line from the top right vertex to the bottom left vertex and from the bottom left vertex to the top left vertex?



You may have a reflection of this drawing, if so ask your teacher to check your wording.

7. What would you get . . .

. . . if you drew a line from the top left vertex to the middle of the bottom side and from the top right vertex to the middle of the bottom side?



You may have a reflection of this drawing, if so ask your teacher to check your wording.

2241 Cuts to Pieces

Experiment 1
Table of results

Number of cuts (c)	0	1	2	3	4	5	6	7	8
Number of pieces (p)	1	2	3	4	5	6	7	8	9

With 50 cuts there would be 51 pieces.

Experiment 2
Table of results with one fold

Number of cuts (c)	0	1	2	3	4	5	6
Number of pieces (p)	1	3	5	7	9	11	13

Pattern

There is a pattern in these results.
The number of pieces is the number of cuts multiplied by 2 and add 1.

Rule

$p = 2c + 1$

Experiment 3
Table of results with two folds

Number of cuts (c)	0	1	2	3	4	5	6
Number of pieces (p)	1	4	7	10	13	16	19

Pattern

There is a pattern in these results.
The number of pieces is the number of cuts multiplied by 3 and add 1.

Rule

$p = 3c + 1$

Experiment 4
Table of results with three folds

Number of cuts (c)	0	1	2	3	4	5	6
Number of pieces (p)	1	5	9	13	17	21	25

Pattern

There is a pattern in these results.
The number of pieces is the number of cuts multiplied by 4 and add 1.

Rule

$p = 4c + 1$

continued/

2241 Cuts to Pieces (cont)

Table of all your rules

Folds	Rule
0	p = c + 1
1	p = 2c + 1
2	p = 3c + 1
3	p = 4c + 1
4	p = 5c + 1
5	p = 6c + 1

You may have been able to find a final pattern and rule from the final table. If so well done!

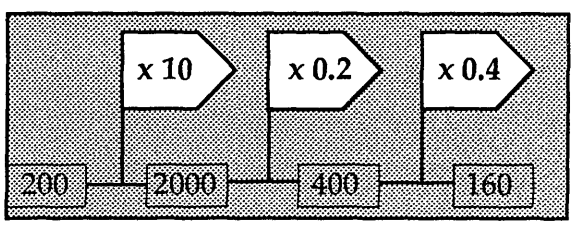
The final pattern

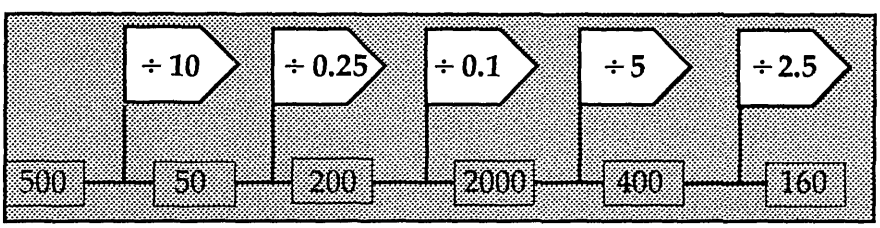
There is a pattern in these results.
The number of pieces is the number of cuts, multiplied by one more than the number of folds, add 1.

The final rule (when f is the number of folds)

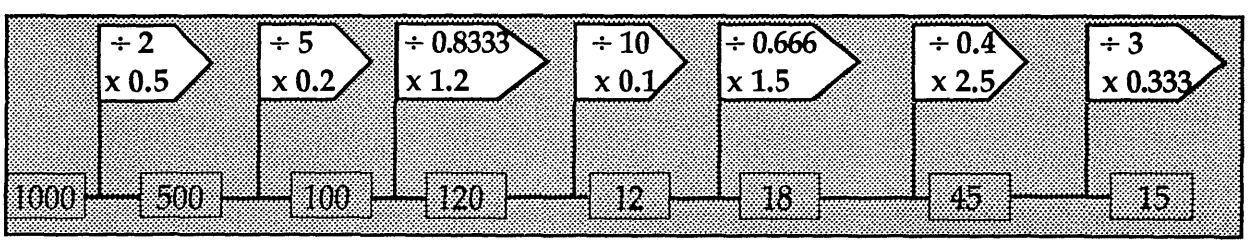
$$p = (f + 1)c + 1$$

2242 Decimal Flags





The following shows the flags with multiplication **and** division. You should only have one answer in each flag.



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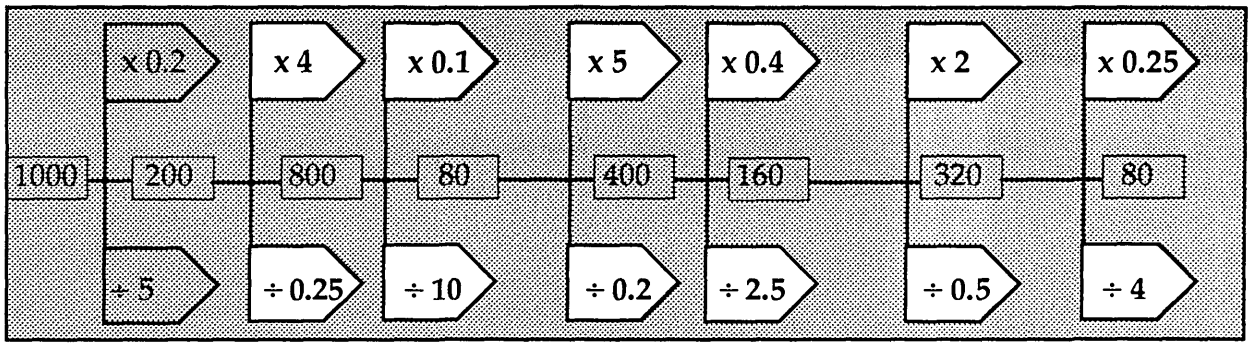


Numbers used for multiplying and dividing are reciprocals of each other.

On your calculator you will find a $\frac{1}{x}$ or x^{-1} button. This is the reciprocal button.

Press 0 . 2 $\frac{1}{x}$ =

The answer should be 5.

The reciprocal of 0.2 is 5.

$0.2^{-1} = \frac{1}{0.2} = 5$

Press 5 $\frac{1}{x}$ =

The answer should be 0.2.

The reciprocal of 5 is 0.2.

$5^{-1} = \frac{1}{5} = 0.2$

The reciprocal of 4 is 0.25.

$4^{-1} = \frac{1}{4} = 0.25$

The reciprocal of 0.1 is 10.

$0.1^{-1} = \frac{1}{0.1} = 10$

The reciprocal of 0.4 is 2.5.

$0.4^{-1} = \frac{1}{0.4} = 2.5$

The reciprocal of 2 is 0.5.

$2^{-1} = \frac{1}{2} = 0.5$

The reciprocal of 0.25 is 4.

$0.25^{-1} = \frac{1}{0.25} = 4$

2243 Who's Rule Okay?

1. • Joe's rule $r = 3(p - 3)$
Rajan's rule is $r = 3p - 3$
Nikki's rule is $r = 2p - 3 + p$

• Joe's rule is incorrect because $3(p - 3) = 3p - 9$
All the other rules can be rearranged to show that they are equal.
$3p - 3 = 3(p - 1) = 2p - 3 + p$
You can check they are the same by substituting some values for p.

• You may have decided that $r = 3p - 3$ looks simplest or you may have decided that $r = 3(p - 1)$ looks simplest.

2. • Joe's rule is $m = 8r - (r - 1)$
Rajan's rule is $m = 7r + 1$
Nikki's rule is $m = 8r - r - 1$

• Nikki's rule is incorrect because $8r - r - 1 = 7r - 1$
The other rules are equal.
$7(r + 1) - 6 = 8r - (r - 1) = 7r + 1$

• $m = 7r + 1$ looks the simplest.

3. • Joe's rule is $m = 3s + 1$
Rajan's rule is $m = 4s - s - 1$
Nikki's rule is $m = 3(s + 1) - 2$

• Rajan's rule is incorrect because $4s - s - 1 = 3s - 1$
The others are equal.
$4s - (s - 1) = 3s + 1 = 3(s + 1) - 2$

• $m = 3s + 1$ is the simplest.

4. If c = *number of circles* and s = *number of sticks*
• Karen's rule is $s = 2c - 2$
Joe's rule is $s = \frac{4c - 2}{2}$
Rajan's rule is $s = \frac{4c}{2} - 2$
Nikki's rule is $s = 2(c - 1)$

• Joe's rule is incorrect because $\frac{4c - 2}{2} = 2c - 1$
The others are equal.
$2c - 2 = \frac{4c}{2} - 2 = 2(c - 1)$

• $s = 2(c - 1)$ or $s = 2c - 2$ are the simplest rules.

• Show your own sequences and different rules to your teacher.

2244 Packing Balls

Throughout this activity:

- take the radius of the tennis ball (sphere) as r
- assume the tennis balls fit *exactly* into the packaging

1. a) The surface area of the tennis ball is $4\pi r^2 = 12.57r^2$

The length of the cube is $2r$.
The surface area of the cube is $6 \times (2r)^2 = 24r^2$

The cube has the greatest surface area. $24r^2 > 12.57r^2$

b) The volume of air inside the cube = (volume of cube – volume of sphere)
Volume of cube = $(2r)^3 = 8r^3$
Volume of sphere = $\frac{4}{3}\pi r^3$
The volume of air inside the cube = $8r^3 - \frac{4}{3}\pi r^3 = (8 - \frac{4}{3}\pi)r^3 = 3.811r^3$
The percentage of the cube containing air = $\frac{3.811r^3}{8r^3} \times 100 = 47.6\%$

2. a) The surface area of the 3 tennis balls is $3 \times 4\pi r^2 = 12\pi r^2$

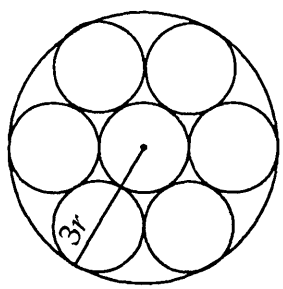
The surface area of the cylinder = (2 x area of base) + (area of rectangle)
= $(2 \times \pi r^2) + (2\pi r \times 6r) = 14\pi r^2$

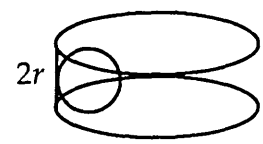
The cylinder has the greatest surface area. $14\pi r^2 > 12\pi r^2$

b) The volume of air inside the cylinder = (volume of cylinder – volume of 3 spheres)
= $(\pi r^2 \times 6r) - (3 \times \frac{4}{3}\pi r^3)$
= $6\pi r^3 - 4\pi r^3 = 2\pi r^3$

The percentage of the cylinder containing air = $\frac{2\pi r^3}{6\pi r^3} \times 100 = 33.3\%$

3.





The volume of air in the cylinder = (volume of cylinder – volume of 7 spheres)
= $(\pi(3r)^2 \times 2r) - (7 \times \frac{4}{3}\pi r^3)$
= $18\pi r^3 - \frac{28\pi r^3}{3} = \frac{26\pi r^3}{3}$

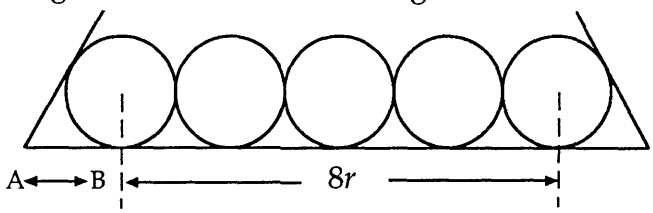
The percentage of the cylinder containing air = $\frac{\frac{26}{3}\pi r^3}{18\pi r^3} = \frac{\frac{26}{3}}{18} = \frac{26}{54} \times 100 = 48.1\%$

continued/

4. The volume of air in the triangular prism = (volume of prism − volume of 15 spheres)

To find the length of the side of the equilateral triangular base.

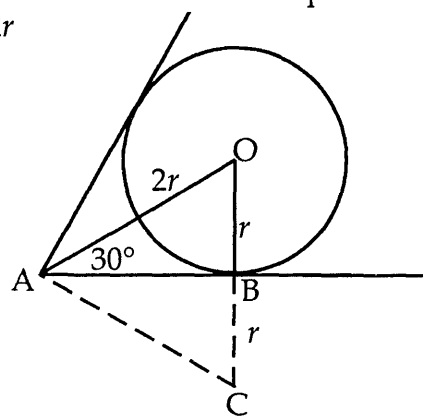
- The length of the side of the triangle *is not* $10r$.



The length is $8r + 2AB$.

AB is the perpendicular bisector of the equilateral triangle AOC.
$AO = OC = CA = 2r$

Using Pythagoras
$AB^2 + OB^2 = AO^2$
$AB^2 + r^2 = (2r)^2$
$AB^2 = 4r^2 - r^2$
$AB^2 = 3r^2$
$AB = \sqrt{3}r$

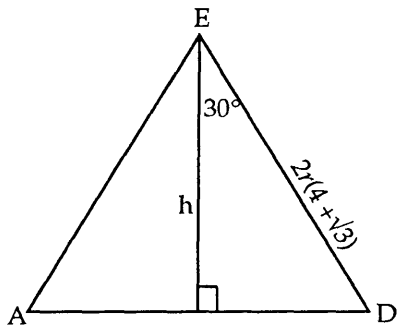


The length of the equilateral triangular base is $8r + 2AB = 8r + 2\sqrt{3}r = 2r(4 + \sqrt{3})$

- To find the height of the equilateral triangular base AED.

$$\cos 30° = \frac{h}{2r(4 + \sqrt{3})}$$

$$h = 2r(4 + \sqrt{3})\cos 30$$



- To find the area of triangular base AED
$\frac{1}{2}$(base x height) $= \dfrac{2r(4 + \sqrt{3}) \times 2r(4 + \sqrt{3})\cos 30}{2}$

- To find the volume of the triangular prism
Area of base x height $= \dfrac{2r(4 + \sqrt{3}) \times 2r(4 + \sqrt{3})\cos 30}{2} \times 2r = 113.818r^3$

The volume of air in the triangular prism = (volume of prism − volume of 15 spheres)
$$= \frac{113.818r^3 - 20\pi r^3}{113.818r^3} = 50.986r^3$$

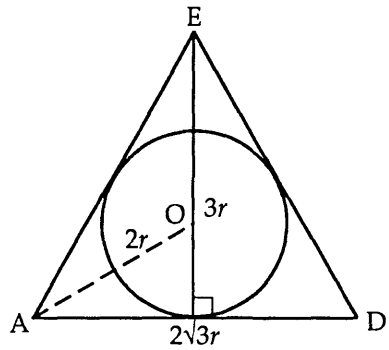
The percentage of air in the triangular prism $= \dfrac{50.986r^3}{113.818r^3} \times 100 = 44.8\%$

continued/

5. a) The surface area of 3 tennis balls is $3 \times 4\pi r^2 = 12\pi r^2 = 37.699r^2$

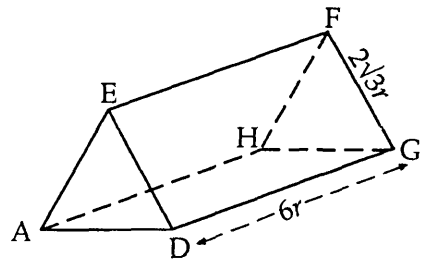
To find total surface area of the equilateral triangular based prism.

- To find the area of the equilateral triangle ADE.



$\frac{1}{2}$(base x height) $= \dfrac{2\sqrt{3}r \times 3r}{2} = 5.196r^2$

- To find the area of one of the rectangular sides DEFG.



Area $= 2\sqrt{3}r \times 6r = 20.785r^2$

Total surface area of equilateral triangular based prism
$= (2 \times \text{ADE}) + (3 \times \text{DEFG})$
$= (2 \times 5.196r^2) + (3 \times 20.785r^2)$
$= 10.392r^2 + 62.354r^2$
$= 72.746r^2$

The prism has the greatest surface area. $72.746r^2 > 37.699r^2$

b) To find the volume of the triangular prism

Area of base x height $= \dfrac{2\sqrt{3}r \times 3r \times 6r}{2}$

$= 31.177r^3$

Voume of 3 spheres $= 4\pi r^3$

The volume of air = (volume of prism – volume of 3 spheres)
$= 31.177r^3 - 4\pi r^3$
$= 18.611r^3$

The percentage of the prism containing air $= \dfrac{18.611r^3}{31.177r^3} \times 100 = 59.693\%$

2245 Rows and Columns

Here are three possible solutions.

1	5	6
9	3	4
2	8	7

1st row = 12
1st column = 12

2nd row = 16
2nd column = 16

3rd row = 17
3rd column = 17

2	1	8
3	4	5
6	7	9

1st row = 11
1st column = 11

2nd row = 12
2nd column = 12

3rd row = 22
3rd column = 22

4	9	2
3	5	7
8	1	6

1st row = 15
1st column = 15

2nd row = 15
2nd column = 15

3rd row = 15
3rd column = 15

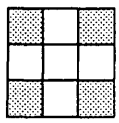
This is a special solution called a 'magic square', because all three rows and columns *and the diagonals* add up to the same number 15!

Did you find a different solution?

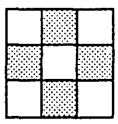
If so, make sure your row and column totals match, and ask your teacher to check it.

Do you notice any pattern in the three solutions above?

For example opposite corners?



opposite middle numbers?



Would you always be able to find solutions with sets of consecutive numbers?
e.g. -4, -3, -2, -1, . . .

2246 Sieve of Eratosthenes

1.-7. Your 100 square should look something like this.

1	(2)	(3)	4	(5)	6	(7)	8	9	10
(11)	12	(13)	14	15	16	(17)	18	(19)	20
21	22	(23)	24	25	26	27	28	(29)	30
(31)	32	33	34	35	36	(37)	38	39	40
(41)	42	(43)	44	45	46	(47)	48	49	50
51	52	(53)	54	55	56	57	58	(59)	60
(61)	62	63	64	65	66	(67)	68	69	70
(71)	72	(73)	74	75	76	77	78	(79)	80
81	82	(83)	84	85	86	87	88	(89)	90
91	92	93	94	95	96	(97)	98	99	100

So the twenty-five prime numbers between 1 and 100 are:

2, 3, 5, 7, 11,

13, 17, 19, 23, 29,

31, 37, 41, 43, 47,

53, 59, 61, 67, 71,

73, 79, 83, 89, 97

- 113, 149 and 173 are prime.

  117 has 6 factors: 1, 3, 9, 13, 39 and 117
  136 has 8 factors: 1, 2, 4, 8, 17, 34, 68 and 136

You might like to check these answers or look for primes above 100 using the MicroSMILE program 'Numbers'.

2247 More than, Less than

1. a) 1, 2, 3, 4, 5, 6, 7, 8, 9
   b) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
   These have a finite number of possibilities, as only the positive integers are considered.
   c) 14, 15, 16, 17, . . .
   d) 9, 10, 11, 12, . . .
   These have an infinite number of possibilities.

2. a) 1, 2, 3, 4, 5, 6, 7, 8
   b) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
   c) 17, 18, 19, 20, . . .
   d) 3, 4, 5, 6, . . .

continued/

2247 More than, Less than (cont)

3. a) 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
   b) 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28
   c) 6, 7, 8, 9, 10, 11, 12
   d) 19, 20, 21, 22, 23, 24, 25, 26, 27, 28
   e) 5
   f) 5, 6, 7, 8
   g) 4, 5, 6, 7
   h) 4, 5, 6, 7

   The answers to g) and h) are the same when only positive integers are used,
   $4 \leq x < 8$ is the same as $3 < x \leq 7$ and $3 < x < 8$ and $4 \leq x \leq 7$

4. a) $1 \leq x \leq 7$ or $1 \leq x < 8$ or $x < 8$
   b) $3 \leq x \leq 10$ or $2 < x < 11$ or $2 < x \leq 10$ or $3 \leq x < 11$
   c) $18 \leq x \leq 26$ or $17 < x < 27$ or $17 < x \leq 26$ or $18 \leq x < 27$
   d) $86 \leq x \leq 89$ or $85 < x < 90$ or $85 < x \leq 89$ or $86 \leq x < 90$

5.

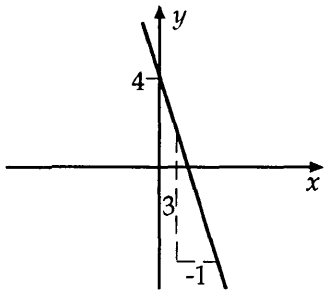
10, 11, 12,	=	$9 < x \leq 12$	= $10 \leq x \leq 12$
9, 10, 11	=	$8 < x \leq 11$	= $9 \leq x < 12$
8, 9, 10, 11	=	$8 \leq x < 12$	= $8 \leq x \leq 11$
9, 10	=	$8 < x < 11$	= $9 \leq x \leq 10$
10	=	$9 < x < 11$	= $9 < x \leq 10$
8, 9, 10	=	$8 \leq x \leq 10$	= $8 \leq x < 11$

2248 Snails' Trails

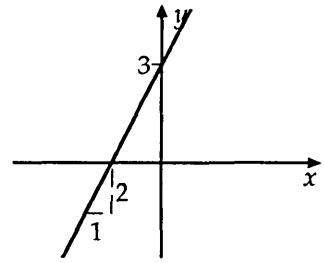
1. 23cm
2. 15cm
3. 2cm
4. 11.5cm or $11\frac{1}{2}$cm
5. 7.5cm or $7\frac{1}{2}$cm
6. 12.5cm or $12\frac{1}{2}$cm
7. 5.5cm or $5\frac{1}{2}$cm
8. 3cm

2249 Gradients and Intercepts

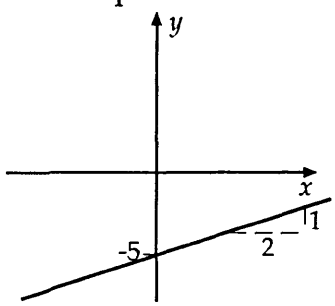
1. a) $y = -3x + 4$
gradient = -3
intercept = 4



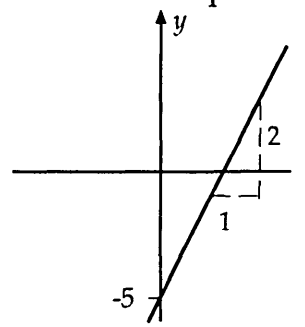
b) $y = 2x + 3$
gradient = 2
intercept = 3



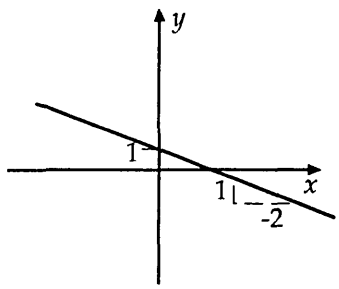
c) $y = \frac{x}{2} - 5$
gradient = $\frac{1}{2}$
intercept = -5



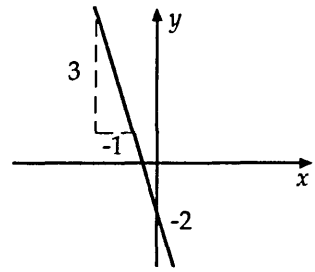
d) $y = 2x - 5$
gradient = 2
intercept = -5



e) $y = -\frac{x}{2} + 1$
gradient = $-\frac{1}{2}$
intercept = 1



f) $y = -3x - 2$
gradient = -3
intercept = -2



2. a) Graph f).
b) Graph b) and d).
c) Any equation where the gradient is $\frac{1}{2}$.
e.g. $y = \frac{x}{2} + 1$ $y = 0.5x - 7$
d) Equation c).
e) Any equation where c = 1.
e.g. $y = 2x + 1$ $y = 5x + 1$
f) Equation e). It has a gradient of $-\frac{1}{2}$.
You should have noticed that the graphs of two equations are perpendicular to each other if the gradient of one is m, the gradient of the other is $-\frac{1}{m}$.
(The negative reciprocal of m)
g) Any equation which has a gradient of -2.
e.g. $y = -2x + 10$ $y = -2x - 4$

continued/

2249 Gradients and Intercepts (cont)

3. $2x = 3 - y$ rearranged gives $y = -2x + 3$
   $y = -2(x + \frac{1}{2})$ rearranged gives $y = -2x - 1$
   $\frac{y + 1}{3} = x$ rearranged gives $y = 3x - 1$

   a) $2x = 3 - y$ and $y = -2(x + \frac{1}{2})$ both have a gradient of -2.
   b) $y = -2(x + \frac{1}{2})$ and $\frac{y + 1}{3} = x$ both have intercept at -1.

2250 A Puzzling Walk

If the first contestant at the back saw 2 green hats, she would know her own colour was red, but she does not. So the shepherd girl **knows** that the hats cannot both be green.

If the second contestant saw a green hat in front she would have known her hat was red, because they cannot both be green because the first contestant would know.

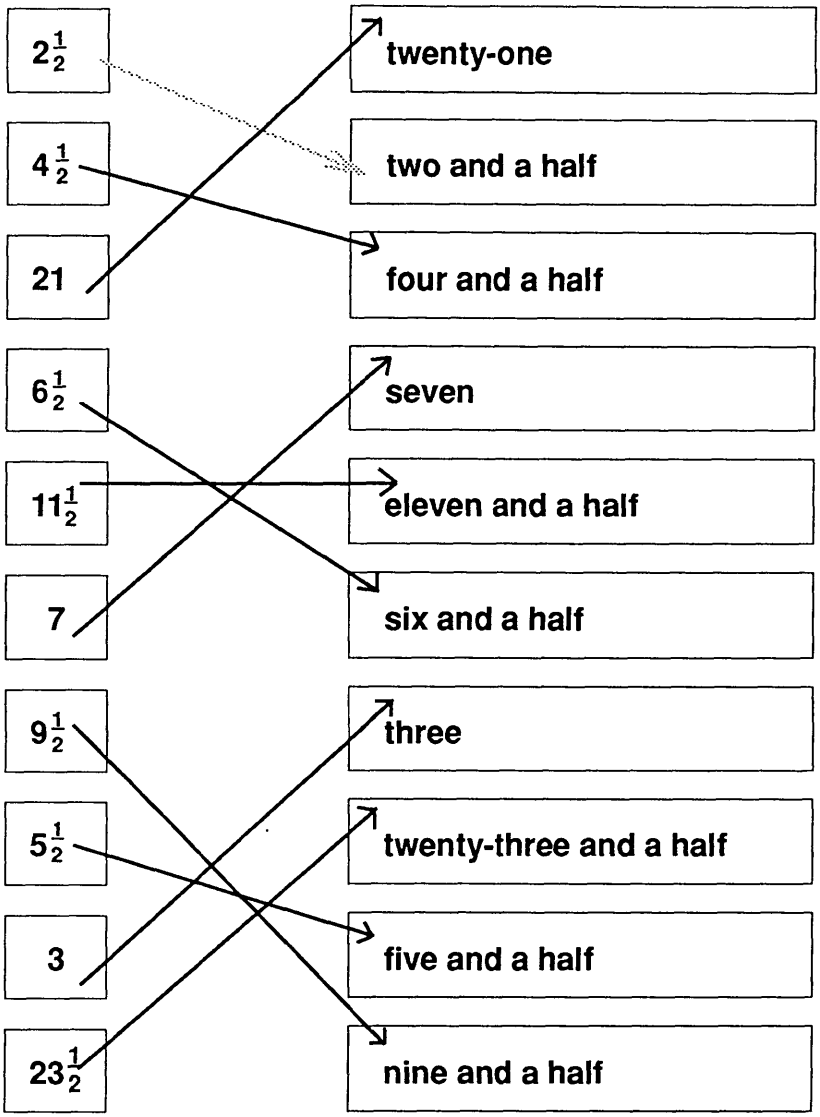
The person in front **cannot** have a green hat, so it must be red.

2251 Put them in their place

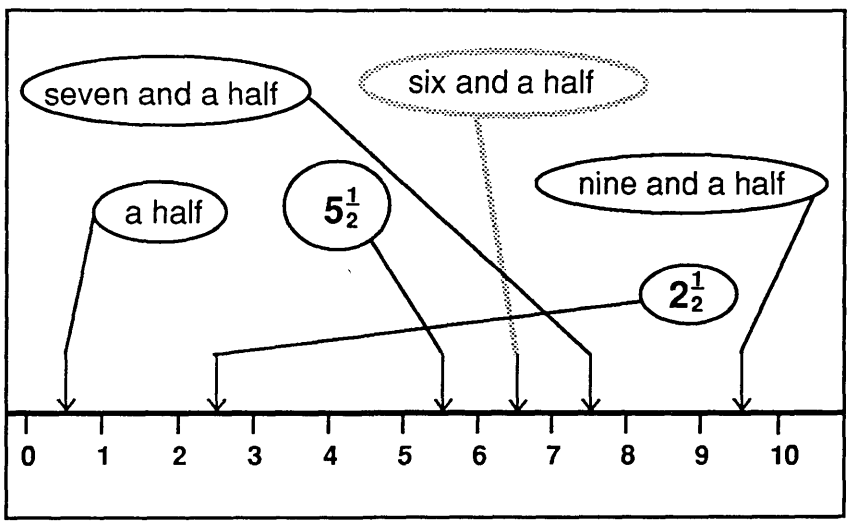
1.	b)	-5,	-2,	-1,	3,	7,	8,	9
	c)	-12,	-7,	-6,	0,	3,	17,	25
	d)	-1000,	-100,	-15,	-5,	8,	300	
2.	b)	0.003,	0.008,	0.037,	0.041,	0.5,	0.57	
	c)	0.17,	0.71,	3.01,	7.01,	7.1,	30.1	
	d)	0.00005,	0.004,	0.036,	0.52,	4.01,	4.1	
3.	b)	$^1/_{10}$,	$^1/_4$,	$^3/_8$,	$^2/_5$,	$^1/_2$,	$^6/_{10}$,	$^9/_8$
	c)	$^1/_5$,	$^1/_3$,	$^4/_{10}$,	$^7/_{10}$,	$^5/_4$,	$1\,^1/_2$,	$^{12}/_7$
	d)	$^1/_9$,	$^5/_8$,	$^8/_{10}$,	$^6/_7$,	$^{12}/_8$,	$2\,^3/_5$,	$^{17}/_2$
4.	b)	1^3,	3,	$\sqrt{12}$,	$\sqrt{36}$,	7,	10,	4^2
	c)	5,	$\sqrt{35}$,	2^3,	3^2 and 9,	11,	6^2	
	d)	$\sqrt[3]{27}$,	2^2,	$\sqrt{64}$,	17,	30,	7^2,	4^3
5.	b)	$\sqrt{0.16}$,	$^1/_2$,	0.51,	$^5/_8$,	$^7/_{10}$,	0.75,	$\sqrt{0.9}$
	c)	-20,	$-^7/_8$,	$\sqrt{10}$,	$^{13}/_4$,	3.6,	$\sqrt{100}$,	3^3
	d)	$-^{10}/_3$,	-3.25,	-3.1,	1.4^2,	2,	$\sqrt[3]{10}$,	$\sqrt{11}$

2252 Something and a half

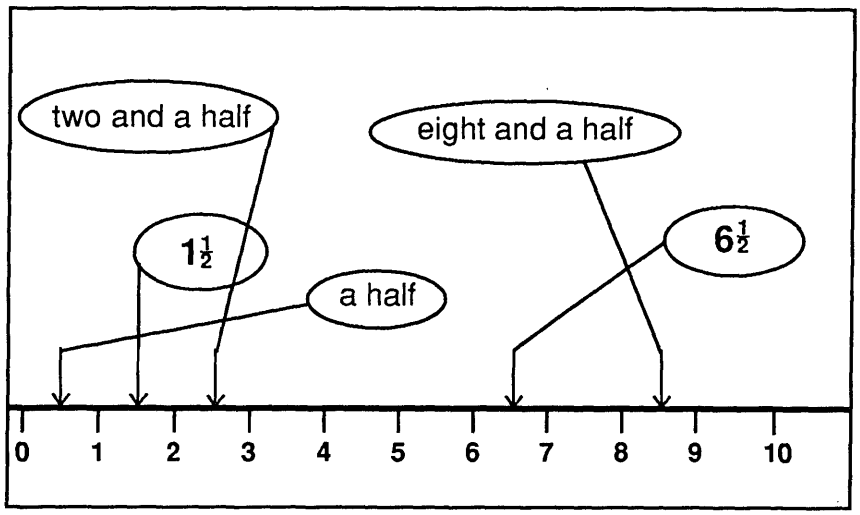
Numbers and words



Halves and the number line



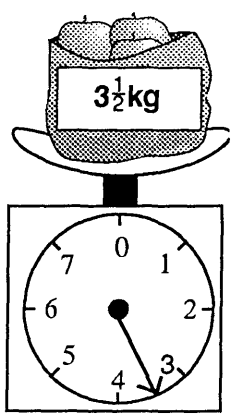
2252 Something and a half (cont)

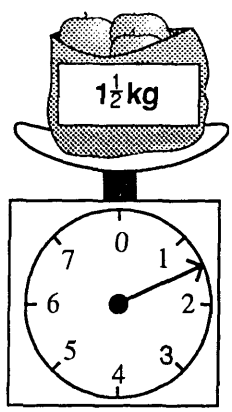


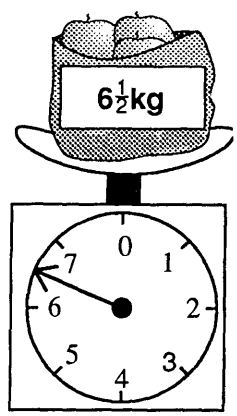
Halves and a ruler

$5\frac{1}{2}$cm
$8\frac{1}{2}$cm
$2\frac{1}{2}$cm

Weights and scales







2253 Solving Inequalities

1. a) $-3 < 9$ b) $-8 < 4$ c) $-1 < 11$
   d) $-8 < 16$ e) $-12 < 0$ f) $-2 < 4$
   g) $-36 < 72$ h $-1 < 2$
   All of these inequalities are true. If you add, subtract, multiply or divide any positive number to each side of an inequality, the inequality remains true.

2. a) $x \leq 12$ b) $x > 13$
   c) $x \leq 5$ d) $7 \geq x$ or $x \leq 7$
   e) $x > 2.5$ f) $x < 24$
   g) $x \geq 9$ h) $10 \leq x$ or $x \geq 10$

continued/

2253 Solving Inequalities (cont)

3. a) $x \leq 3$ b) $x \geq 9$
   c) $1.5 > x$ or $x < 1.5$ d) $6 > x$ or $x < 6$
   e) $x \geq 120$ f) $x \leq 6$
   g) $10.\dot{6} \leq x$ or $x \geq 10.\dot{6}$ h) $x < 3$

4. a) $8 < -16$ This inequality is not true. $8 > -16$
   b) $2 < -4$ This inequality is not true. $2 > -4$
   c) $-7 < 5$ This inequality is true.
   d) $0 < 12$ This inequality is true.
   e) $4 < -8$ This inequality is not true. $4 > -8$
   f) $16 < -32$ This inequality is not true. $16 > -32$
   g) $-14 < -2$ This inequality is true.
   h) $-2 < 10$ This inequality is true.

   If you add or subtract both sides of an inequality by a negative number the inequality remains true.
   If you multiply or divide both sides of an inequality by a negative number, the inequality **must be reversed**.

5. An inequality remains true when you:
   add a negative number to both sides of the inequality
   or
   subtract a negative number from both sides of the inequality.

   The inequality sign must be reversed for the new inequality to remain true when you:
   multiply both sides of the inequality by a negative number
   or
   divide both sides of the inequality by a negative number.

6. a) $x > -0.\dot{6}$ b) $x \leq 7$
   c) $x \leq -6$ d) $x \geq 13$
   e) $x \geq -4$ f) $x < -1.6$
   g) $x < 10$ h) $x > -18$

2254 Calculator Brackets

Here are some possible answers. You may have others.

1 = $(2 - 1) \times (4 - 3)$
2 = $(3 + 1) \div (4 - 2)$
3 = $(4 - 1) \times (3 - 2)$
4 = $(4 - 2) \times (3 - 1)$
5 = $(4 + 1) \times (3 - 2)$
6 = $(3 \times 4) \div (2 \times 1)$
7 = $(4 + 3) \times (2 - 1)$
8 = $2 + 3 + 4 - 1$

continued/

2254 Calculator Brackets (cont)

9	=	2 + 3 + (4 x 1)
10	=	(1 x 2 x 3) + 4
11	=	(4 x 2) + (3 ÷ 1)
12	=	4 x 2 + 3 + 1
13	=	4 x 3 + 2 – 1
14	=	(4 x 3) + (2 x 1)
15	=	(2 + 3) x (4 – 1)
16	=	(4 x 2) x (3 – 1)
17	=	(4 + 2) x 3 – 1
18	=	(4 + 2) x (3 x 1)
19	=	(2 + 3) x 4 – 1
20	=	(2 + 3) x (4 x 1)
21	=	(2 + 3) x 4 + 1
22	=	(4 x 3 – 1) x 2
23	=	4 x 2 x 3 – 1
24	=	4 x 3 x 2 x 1
25	=	(2 + 3) x (4 + 1)

2255 Adding One

When you start with $\frac{50}{100}$ and compare the equivalent decimal, the fractions get larger when you add one to the numerator and denominator.

Starting fraction: $\frac{5}{6}$

To set up a spreadsheet to generate the results more quickly:

- Click in A1 and type **Numerator** ↵
  Click in A2 and type **5** ↵
  Click in A3 and type **=A2+1** ↵
  Highlight A3 to A14 then go to
  Edit menu to Fill Down

- Click in B1 and type **Denominator** ↵
  Click in B2 and type **6** ↵
  Click in B3 and type **=B2+1** ↵
  Highlight B3 to B14 then go to
  Edit menu to Fill Down

- Click in C1 and type **Num/Den** ↵
  Click in C2 and type **A2/B2** ↵
  Highlight C2 to C14 then go to
  Edit menu to Fill Down

	A	B	C
1	Numerator	Denominator	Num/Den
2	5	6	0.8333
3	6	7	0.8571
4	7	8	0.875
5	8	9	0.8889
6	9	10	0.9
7	10	11	0.9091
8	11	12	0.9167
9	12	13	0.9231
10	13	14	0.9286
11	14	15	0.9333
12	15	16	0.9375
13	16	17	0.9412
14	17	18	0.9444

The fraction $\frac{5}{6}$ gets larger.

continued/

The fraction $\frac{5}{6}$ gets smaller until the denominator becomes zero. After subtracting again the equivalent decimal jumps to 2. Then it continues to get smaller, but it will never be less than 1.

Here is a summary of the results for subtracting 1 from the numerator and denominator:

Fraction	Rule		Larger/smaller?
	numerator	denominator	
$\frac{5}{6}$	subtract 1	subtract 1	Gets **smaller** until denominator becomes 0, jumps to 2, then gets **smaller** but never less than 1.
$\frac{3}{2}$	subtract 1	subtract 1	Gets **larger** until denominator becomes 0, jumps to 0.5, then gets **larger** but never more than 1.
$\frac{7}{300}$	subtract 1	subtract 1	Gets **smaller** until denominator becomes 0, jumps to 294, then gets **smaller** but never less than 1.
$\frac{10}{3}$	subtract 1	subtract 1	Gets **larger** until denominator becomes 0, jumps to -6, then gets **larger** but never more than 1.

You should have found that for subtracting 1 from the numerator and denominator:

- any fraction where the numerator is *smaller* than the denominator, the fraction gets *smaller*, then jumps, then continues to get *smaller*, but it will never be less than 1. Can you see why?
- any fraction where the numerator is *larger* than the denominator the fraction gets *larger*, then jumps, then continue to get *larger*, but it will never be more than 1. Can you see why?

By adding a number greater than 1 to the numerator and denominator *or* by subtracting a number greater than 1 to the numerator and denominator, the results will be the same as for adding 1 or subtracting 1, except the spreadsheet will show the pattern in fewer cells.

- What did you find when you subtracted 1 from the numerator and added 1 to the denominator?

Fraction	Rule		Larger/smaller?
	numerator	denominator	
$\frac{5}{6}$	subtract 1	add 1	?
$\frac{3}{2}$	subtract 1	add 1	?
$\frac{7}{300}$	subtract 1	add 1	?
$\frac{10}{3}$	subtract 1	add 1	?

continued/

- To change the starting fraction to $\frac{3}{2}$.

  Change the number in cell A2 to **3**
  Change the number in cell B2 to **2.**
  The spreadsheet will recalculate the equivalent decimal.

8	9	8	1.125
9	10	9	1.1111
10	11	10	1.1
11	12	11	1.0909
12	13	12	1.0833
13	14	13	1.0769
14	15	14	1.0714

The fraction $\frac{3}{2}$ is getting smaller.

Here is a summary of the results:

Fraction	Rule		Larger/smaller?
	numerator	denominator	
$\frac{5}{6}$	add 1	add 1	larger
$\frac{3}{2}$	add 1	add 1	smaller
$\frac{7}{300}$	add 1	add 1	larger
$\frac{10}{3}$	add 1	add 1	smaller

You should have found that:
- The fraction gets larger when the numerator is smaller than the denominator.
- The fraction gets smaller when the numerator is larger than the denominator.

Changing the rule to subtract 1 from the numerator and denominator.
You will need to adapt the first spreadsheet by changing the formulas in cell A3 and B3.

- Click in cell A3 and type **=A2–1**↵
  Highlight A3 to A14 and go to the **Edit** menu and Fill Down.

Click in cell B3 and type **=B2–1**↵
Highlight B3 to B14 and go to the **Edit** menu and Fill Down.

This spreadsheet shows what happens to the starting fraction $\frac{5}{6}$.

getting smaller ⇓

	A	B	C
1	Numerator	Denominator	Num/Den
2	5	6	0.83333333
3	4	5	0.8
4	3	4	0.75
5	2	3	0.66666667
6	1	2	0.5
7	0	1	0
8	- 1	0	#DIV/0!
9	- 2	- 1	2
10	- 3	- 2	1.5
11	- 4	- 3	1.33333333
12	- 5	- 4	1.25
13	- 6	- 5	1.2
14	- 7	- 6	1.16666667

infinity ⇒ jumps to ⇒

⇒ the spreadsheet cannot divide by zero so this message is displayed.

getting smaller ⇓

⇓

never gets less than 1.

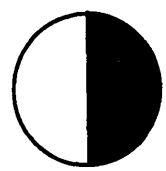
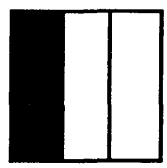
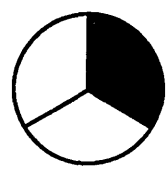
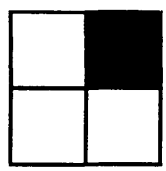
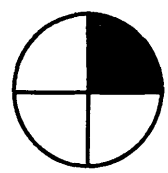
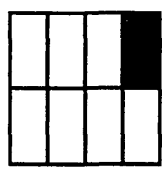
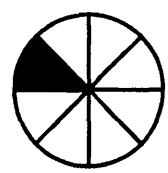
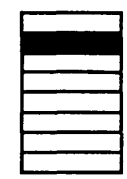
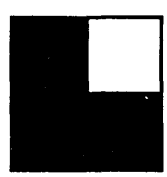
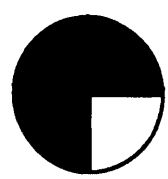
continued/

2255 Adding One (cont)

- What did you find when you added 1 to the numerator and subtracted 1 from the denominator?

Fraction	Rule		Larger/smaller?
	numerator	denominator	
$\frac{5}{6}$	add 1	subtract 1	?
$\frac{3}{2}$	add 1	subtract 1	?
$\frac{7}{300}$	add 1	subtract 1	?
$\frac{10}{3}$	add 1	subtract 1	?

2256 Matching Fractions

Numbers	Square	Circle	Word	Rectangle
$\frac{1}{2}$			**one half**	
$\frac{1}{3}$			**one third**	
$\frac{1}{4}$			**one quarter**	
$\frac{1}{8}$			**one eighth**	
$\frac{3}{4}$			**three quarters**	

2257 Right-angled Triangular Prisms

1. a) The cuboid has a volume of $12cm^3$
   b) The triangular prism has a volume of $6cm^3$, because the volume is half the volume of the cuboid.

2. $13\frac{1}{2}cm^3$

3. $7\frac{1}{2}cm^3$

4. $4\frac{1}{2}cm^3$

5. $6cm^3$

6. $6cm^3$

7. $18cm^3$

2258 Substituting into Formulae

All answers are given correct to 3 decimal places.

1. Using Pythagoras' Theorem formula
   Let the third side be x.
   $x^2 + 10^2 = 14^2$
   $x^2 = 96$
   $x = \sqrt{96}$
   $x = 9.798cm$

2. Using Area of trapezium formula
   a) $^1/_2(6 + 14) \times 8 = 80cm^2$
   b) Let the height be h.
   $^1/_2(6 + 10) \times h = 40$
   $8h = 40$
   $h = 5cm$

3. Using Volume of cylinder formula
   a) $\pi \times 7^2 \times 9 = 1385.442cm^3$
   Using Curved surface area formula
   b) $2 \times \pi \times 3 \times 7 = 131.947cm^2$

4. Using Volume of sphere formula
   a) $^4/_3 \times \pi \times 6^3 = 904.779cm^3$
   Using Surface area of sphere formula
   b) $4 \times \pi \times r^2 = 605$
   $12.566r^2 = 605$
   $r^2 = \frac{605}{12.566}$
   $r = \sqrt{48.144} = 6.939cm$

continued/

5. Using Volume of cone formula

a) $^1/_3 \times \pi \times r^2 \times 6 = 120$

$r^2 = \frac{120}{6.28}$

$r = \sqrt{19.099} = 4.370\text{cm}$

Using Curved surface area of cone formula

b) $\pi \times 5 \times l = 600$

$15.708 \times l = 600$

$l = \frac{600}{15.708} = 38.197\text{cm}$

6. Using Volume of prism formula

$15 \times \text{length} = 105$

$\text{length} = \frac{105}{15} = 7\text{cm}$

7. Using Trigonometry

a) $\tan 20 = \frac{5}{x}$

$x = \frac{5}{\tan 20} = 13.737\text{cm}$

Using Area of triangle formula

b) $\frac{1}{2}(13.737 \times 5) = 34.343\text{cm}^2$

8. Using Quadratic equation

a) $a = 1, \quad b = 3, \quad c = 1$

$x = \frac{-3 \pm \sqrt{(9-4)}}{2}$

$x = \frac{-3 + \sqrt{5}}{2}$ or $x = \frac{-3 - \sqrt{5}}{2}$

$x = -0.382$ or -2.618

b) $a = 2, \quad b = 5, \quad c = -10$

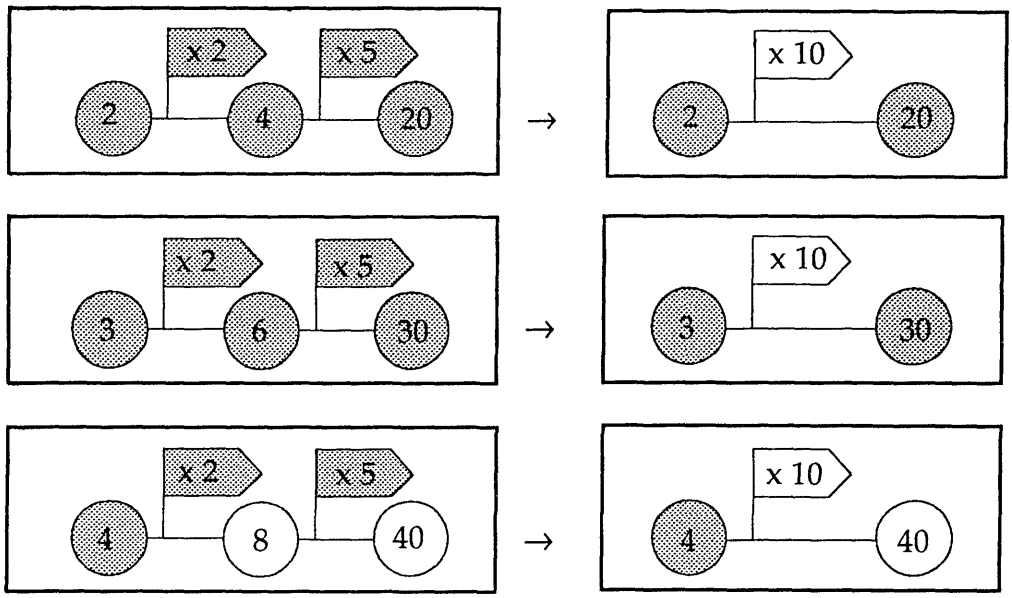
$x = \frac{-5 \pm \sqrt{(25 + 80)}}{4}$

$x = \frac{-5 + \sqrt{105}}{4}$ or $x = \frac{-5 - \sqrt{105}}{4}$

$x = 1.312$ or -3.812

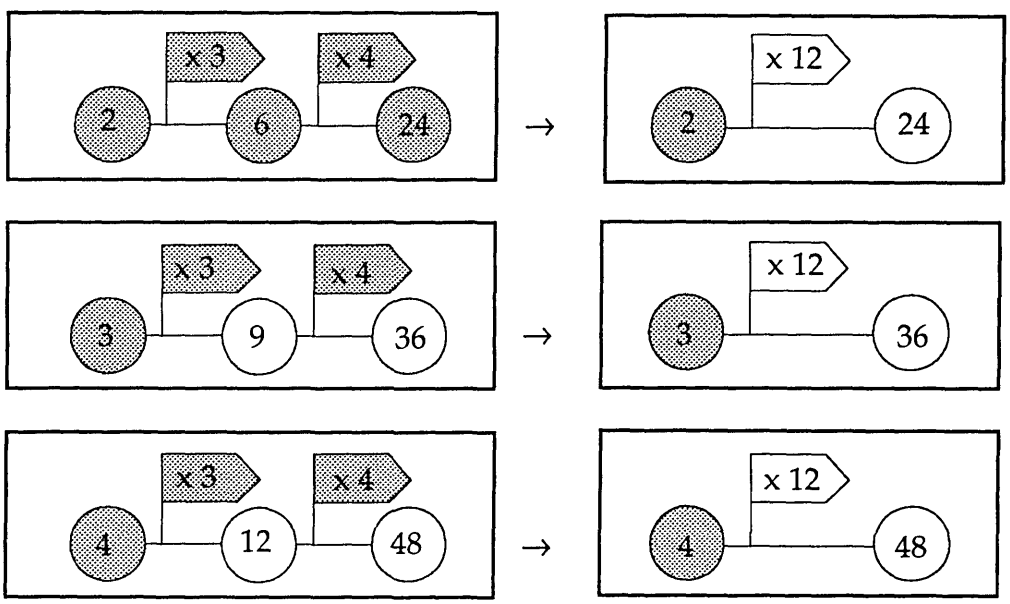
2259 Multiplication Flags

1.



Multiplying by **2** then multiplying by **5** is the same as multiplying by **10**

2.



Multiplying by **3** then multiplying by **4** is the same as multiplying by **12**

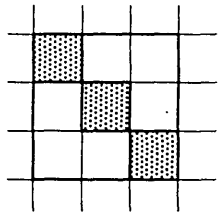
3. Show your completed flag diagrams to your teacher.
Multiplying by **2** then multiplying by **4** is the same as multiplying by **8**

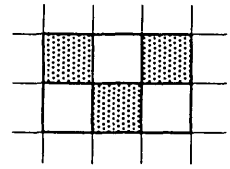
4. Show your examples to your teacher.
Multiplying by **3** then multiplying by **6** is the same as multiplying by **18**
Multiplying by **5** then multiplying by **3** is the same as multiplying by **15**

5. Show your examples and statements to your teacher to check them.

2260 Boxing Areas

There are only two different areas that can be made by joining 3 squares.





- The maximum area is 9cm^2 and the minimum area is 6cm^2

If you looked at the maximum and minimum areas for different numbers of squares, you may find it useful to make a table of your results

Number of squares	Area	
	maximum	minimum
1	1	1
2	4	4
3	9	6
4	16	8
5	25	9
6	36	12
7	49	14
8	.	.
.	.	.
.	.	.
.	.	.

- Can you predict the maximum area for any number of squares?
- Can you predict the minimum area for any number of squares?

You may have decided to look at the number of possible areas you could make with different numbers of squares.

- This is part of the table to show which areas can be made.

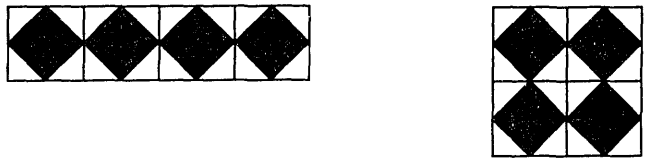
Possible boxed areas

Number of squares	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1	X																															
2				X																												
3						X			X																							
4								X	X			X				X																
5									X	X		X			X	X				X					X							
6												X			X	X		X		X				X	X					X		
7														X	X	X		X		X	X			X	X			X		X		
8															X	X		X		X	X			X	X			X		X		X

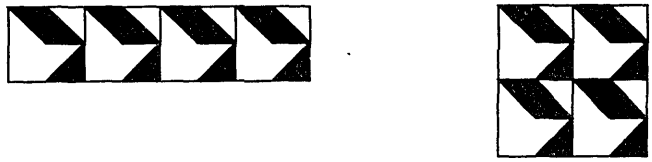
- Which boxed areas were impossible to make?
- What other patterns did you find?

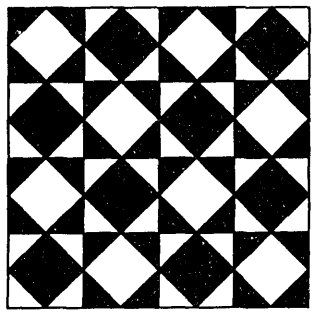
- You may like to investigate triangles joined on isometric paper and the triangles surrounding.

2261 Shape-tiles

1. 

2. 

3. 

4. 

5. Show your own tile to your teacher.

2262 Find the route

Start

2 + 2	3 x 4	11 – 3	12 x 7	7 + 3	8 + 1	5 + 2	3 + 6	9 – 6
1 + 5	10 ÷ 2	9 + 6	7 + 5	6 + 4	11 – 1	8 x 3	2 x 12	3 x 3
11 – 1	20 ÷ 2	5 x 2	2 + 14	5 ÷ 5	4 + 5	6 x 5	9 + 1	12 – 0
3 + 7	11 x 1	26 – 2	20 + 4	6 x 4	4 + 4	7 + 14	5 x 4	10 x 0
1 + 9	15 – 5	9 + 2	18 – 6	12 + 12	8 + 2	10 + 0	4 x 6	23 + 2
3 + 11	12 x 2	13 – 3	5 + 5	6 ÷ 3	7 + 6	8 x 1	3 + 6	7 + 7
5 + 3	7 + 2	9 x 3	3 x 6	10 + 0	10 x 0	9 + 4	4 + 3	3 x 9
4 + 0	10 x 10	8 + 1	4 ÷ 4	28 – 4	3 + 2	2 x 6	13 – 4	18 ÷ 9

End

2263 Spreadsheet Squares

This spreadsheet shows the formulas in cells E1, E3, A5, C5 and E5.

	A	B	C	D	E
1	1	8	3		=A1*B1*C1
2	6	9	7		
3	4	5	2		=A3*B3*C3
4					
5	=A1*A2*A3		=C1*C2*C3		=E1+E3+A5+C5

- The smallest possible total in E5 is 128.
  Here is the spreadsheet

	A	B	C	D	E
1	1	7	4		28
2	8	9	5		
3	3	6	2		36
4					
5	24		40		128

- The largest total in E5 is 830.
  Here is the spreadsheet.

	A	B	C	D	E
1	9	3	6		162
2	5	1	2		
3	8	4	7		224
4					
5	360		84		830

The arrangement to make the smallest possible total needs to have:

- the **largest** number in the centre cell B2 because this number is not multiplied by any other number.
- the **smallest** numbers in the four corner cells A1, A3, C1 and C3 because these are multiplied twice.
  By rearranging the order of the outside numbers you can ensure you get the minimum value in cell E5.

The arrangement to make the largest possible total needs to have:

- the **smallest** number in the centre cell B2.
- the **largest** numbers in A1, A3, C1 and C3.
  By rearranging the order of the outside numbers you can ensure you get the maximum value in cell E5.

Here are the formula for numbers 1 to 16 in a 4 x 4 grid.

	A	B	C	D	E	F
1	1	2	3	4		=A1*B1*C1*D1
2	5	6	7	8		
3	9	10	11	12		
4	13	14	15	16		=A4*B4*C4*D4
5						
6	=A1*A2*A3*A4			=D1*D2*D3*D4		=A6+D6+F1+F4

The smallest possible total that we found was 1382.

- Did you find a smaller total?

The largest possible total that we found was 67012.

- Did you find a larger total?

2264 Plus and Minus Grids

1.

4	2	7	9	6
3	8	2	1	7
3	5	9	10	13
6	3	6	1	5

$2 + 7 = 9$

$3 + 3 = 6$

$7 + 2 = 9$

$9 + 1 = 10$

$6 + 7 = 13$

$6 - 1 = 5$

$8 - 5 = 3$

2.

6	0	9	3	2	1
5	4	8	12	6	15
1	3	11	2	7	8
3	6	0	8	13	7
0	4	4	10	2	4

$4 + 8 = 12$

$2 + 8 = 10$

$6 + 7 = 13$

$0 + 4 = 4$

$6 - 5 = 1$

$3 - 2 = 1$

$15 - 8 = 7$

3. Many possible answers. Did your puzzle work? Do you think it was harder or easier than question 1 and 2?

2265 Rational Numbers

1. a) Infinity because you can always find a higher positive integer.
   b) Infinity because you can always find a higher positive integer or a lower negative integer.
   c) Infinity because there is an infinite number of positive and negative integers and an infinite number of other rational numbers between integers.
   - But are there *more* rational numbers than integers?

2. a) Many possible answers e.g. 2.511, 2.512, ... 2.517 etc.
   b) Many possible answers e.g. $\frac{1}{3}, \frac{5}{12}, \frac{2}{5}, \ldots 0.3, 0.\dot{3}, 0.456, \ldots$
   c) Many possible answers e.g. $\frac{8}{31}, \frac{16}{63}, 0.26, 0.259, \ldots$

continued/

3. Yes, you can always find a rational number between two other rational numbers.
One possible explanation is:
   - A fraction between $\frac{4}{16}$ and $\frac{4}{15}$ could be $\frac{61}{240}$, $\frac{62}{240}$, $\frac{63}{240}$
     since $\frac{4}{16} = \frac{60}{240}$ and $\frac{4}{15} = \frac{64}{240}$

Another is:
   - The mean of two numbers lies between the two numbers, so take the mean of the two fractions.

If your explanation is different discuss it with your teacher.

4. a) $0.\dot{7} = \frac{7}{9}$ b) $1.\dot{3}\dot{4} = \frac{133}{99}$

   c) $0.\dot{2}\dot{6} = \frac{26}{99}$ d) $0.\dot{1}42857\dot{} = \frac{142857}{999999} = \frac{1}{7}$

   e) $0.0\dot{3}\dot{1} = \frac{31}{990}$ Hint: Multiply $0.0\dot{3}\dot{1}$ by 1000 and then by 10.

2266 Irrational Numbers

1. Proof by contradiction that $\sqrt{2}$ is irrational.
Assume $\sqrt{2}$ is rational.

$\sqrt{2} = \frac{p}{q}$ where p and q are integers with no common factors.

$2 = \frac{p^2}{q^2}$

$2q^2 = p^2$ p^2 is a multiple of 2 $\Rightarrow$ p is a multiple of 2.

$2q^2 = (2n)^2$

$2q^2 = 4n^2$

$q^2 = 2n^2$ q^2 is a multiple of 2 $\Rightarrow$ q is a multiple of 2.

p and q are both multiples of 2 which contradicts the initial statement that p and q have no common factors. $\sqrt{2}$ is irrational.

2. The square roots of perfect squares are rational e.g. $\sqrt{4}$, $\sqrt{9}$, $\sqrt{16}$, . . .
All other square roots are irrational.
Similarly the cube roots of perfect cubes are rational $\sqrt[3]{8}$, $\sqrt[3]{27}$, $\sqrt[3]{64}$, . .
All other cube roots are irrational.

3. a) $\sqrt{8}$ irrational

   b) $\sqrt{100} = 10$ rational

   c) $\sqrt[3]{64} = 4$ rational

   d) $\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$ rational

   e) $\sqrt{\frac{2}{25}} = \frac{\sqrt{2}}{\sqrt{25}} = \frac{\sqrt{2}}{5}$ irrational

continued/

4. a) $(\sqrt{5})^2 = 5$ rational

   b) The square of *any* square root is rational, whether the square root is rational or irrational.

5. a) π is irrational, a non-terminating, non-recurring decimal, so 2π must be also.

   b) $(2 + \pi)$ is irrational because π is a non-terminating, non-recurring decimal. If you add 2, it is still a non-terminating, non-recurring decimal.

   c) $(\pi + \sqrt{2})$ is irrational because π is a non-terminating, non-recurring decimal. If you add $\sqrt{2}$, it is still a non-terminating, non-recurring decimal.

6. a) $(2 + \sqrt{2})$ is irrational because $\sqrt{2}$ is a non-terminating, non-recurring decimal. If you add 2, it is still a non-terminating, non-recurring decimal.

   b) $(2 + \sqrt{2})^2 = (2 + \sqrt{2})(2 + \sqrt{2}) = 4 + 4\sqrt{2} + 2 = 6 + 4\sqrt{2}$
   $\sqrt{2}$ is irrational $\Rightarrow 4\sqrt{2}$ is irrational $\Rightarrow 6 + 4\sqrt{2}$ is irrational.

7. a) $(2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$ rational

   b) $(2 + \sqrt{3}) - (2 - \sqrt{3}) = 2\sqrt{3}$ irrational

   c) $(2 + \sqrt{3})(2 - \sqrt{3})$
   $= 4 + 2\sqrt{3} - 2\sqrt{3} - 3 = 1$ rational

8. One pair of examples is $\sqrt{20}$ and $\sqrt{5}$. Both are irrational.
   a) $\sqrt{20} \times \sqrt{5} = \sqrt{100} = 10$ rational

   b) $\sqrt{20} \div \sqrt{5} = \sqrt{4} = 2$ rational
   If you are unsure of your pair of irrational numbers, show them to your teacher.

9. a) $x = +\sqrt{3}$ or $-\sqrt{3}$ irrational

   b) $x^2 = 2$
   $x = +\sqrt{2}$ or $-\sqrt{2}$ irrational

   c) $x^2 = 4$
   $x = \sqrt{4} = +2$ or -2 rational

   d) Using the formula for solving quadratic equations
   $x = \dfrac{-1 \pm \sqrt{(1 + 4)}}{2}$
   $x = \dfrac{-1 + \sqrt{5}}{2}$ or $\dfrac{-1 - \sqrt{5}}{2}$ both solutions are irrational because they contain $\sqrt{5}$

   e) $x = \sqrt[3]{8} = 2$ rational

   f) $x = \sqrt[3]{10}$ irrational

2267 Introducing Ratio

The bracelets show the ratio of red beads to white beads.

Description	Ratio	Ratio in its simplest form	Bracelet*
There are **8 red** beads and **16 white** beads.	The ratio of **red** beads to **white** beads is **8 : 16**	**red : white beads beads** **1 : 2**	
There are **6 red** beads and **18 white** beads.	The ratio of **red** beads to **white** beads is **6 : 18**	**red : white beads beads** **1 : 3**	
There are **4 red** beads and **20 white** beads.	The ratio of **red** beads to **white** beads is **4 : 20**	**red : white beads beads** **1 : 5**	
There are **21 red** beads and **3 white** beads.	The ratio of **red** beads to **white** beads is **21 : 3**	**red : white beads beads** **7 : 1**	
There are **18 red** beads and **6 white** beads.	The ratio of **red** beads to **white** beads is **18 : 6**	**red : white beads beads** **3 : 1**	

* You may have coloured the beads differently. If you are unsure whether these are correct, show your own bracelets to your teacher.

2268 Logo is Amazing

Did your turtle escape from each maze? Show your print outs to your teacher.

You might like to design a maze of your own for someone else to try.

2269 Amazing Logo

Did your turtle escape from the Hampton Court Maze? Show your print out to your teacher.

Have you tried to escape from the real maze at Hampton Court? You might like to design a maze of your own for someone else to try.

2270 Measuring Pencils

Your answers may not be exactly the same. If the difference is less than 2mm mark it correct.

1.

	cm	mm
Joyce	5.5cm	55mm
David	4.1cm	41mm
Colin	6.2cm	62mm
Sally	15.6cm	156mm
Jason	7.3cm	73mm
Jody	3.5cm	35mm
Anila	11.1cm	111mm
Karen	13.9cm	139mm

2. The connection is:
   - if you multiply the centimetre (cm) answer by 10 you get the millimetre (mm) answer.
   - if you divide the millimetre (mm) answer by 10 you get the centimetre (cm) answer.

   So, to change centimetres (cm) into millimetres (mm), multiply by 10. (x 10)
   to change millimetres (mm) into centimetres (cm) divide by 10. (÷ 10)

3.

	cm	mm
Jerome	9cm	**90mm**
Danny	7.5cm	**75mm**
Rosy	**11cm**	110mm
Beth	**12.5cm**	125mm
Nisha	3.9cm	**39mm**
Jamie	**7.2cm**	72mm
Mark	**10.9cm**	109mm
Pat	15.7cm	**157mm**

2271 I've got the power

1. c) 16
   d) 9
   e) 27
   f) 81
   g) 100
   h) 1000
   i) 10000

The x^y key works out powers.

e.g. 5 x^y 2 gives 5^2 = 5 x 5 = 25
2 x^y 5 gives 2^5 = 2 x 2 x 2 x 2 x 2 = 32

continued/

2. a) 343 d) 1024 g) 216
   b) 10000000 e) 27 h) 20736
   c) 32768 f) 625 i) 32768

3. a) 0.5 e) 0.3333333 $= 0.\dot{3}$ i) 0.1
   b) 0.25 f) 0.1111111 $= 0.1\dot{1}$ j) 0.01
   c) 0.125 g) 0.037037 $= 0.\dot{0}3\dot{7}$ k) 0.001
   d) 0.0625 h) 0.0123457 $= 0.\dot{0}12345679\dot{0}$* l) 0.0001
   * Your calculator will probably not have sufficient spaces to show that this is a recurring decimal.
   • x^{-n} means $\frac{1}{x^n}$ the reciprocal of x^n.

4. a) 3.162 e) 6 i) 17.321
   b) 3 f) 8.660 j) 2.236
   c) 4 g) 20 k) 14
   d) 4.472 h) 15 l) 31.623
   • $x^{\frac{1}{2}}$ means the square root of $x \Rightarrow \sqrt{x}$

5. a) 2 e) 7 i) 7.368
   b) 3 f) 7.937 j) 1.710
   h) 4 g) 10 k) 5.809
   d) 5 h) 6 l) 9
   • $x^{\frac{1}{3}}$ means the cube root of $x \Rightarrow \sqrt[3]{x}$

6. a) 4 e) 16 i) 1.6
   b) 2 f) 3 j) 3.5
   c) 10 g) 4.5 k) 4.729
   d) 4 h) 3.6 l) 10
   • $x^{\frac{1}{n}}$ means the nth root of $x \Rightarrow \sqrt[n]{x}$

7. These answers show the key presses on a calculator. Your keys may be different, but check that the key presses that you record give you the same answer.

Key Presses	Calculator display
a) [7] [x^y] [9] [=]	40353607
b) [7] [x^y] [9] [(-)] [=]	2.4781 - 08
c) [7] [x^y] [(] [1] [÷] [9] [)] [=]	1.2413658

8. The [$\sqrt{}$] key gives numbers to the power of $\frac{1}{2}$.
   If you are unsure whether your calculator has this function key, check with your teacher.
   The [$\frac{1}{x}$] key gives numbers to the power of -1.
   If you are unsure whether your calculator has this function key, check with your teacher.

1. a) Any point above the line $y = 4$ will satisfy the inequality $y > 4$,
   e.g. (-1, 5), (0, 5), (1, 5), (2, 6), (3, 6), . . .
   $5 > 4$, $5 > 4$ $5 > 4$ $6 > 4$ $6 > 4$ . . .
   b) Any point below the line $y = 4$ will satisfy the inequality $y < 4$,
   e.g. (-2, 3), (-1, 3), (4, 3) (6, 2), (6, -2) . . .
   $3 < 4$ $3 < 4$ $3 < 4$ $2 < 4$ $-2 < 4$ . . .

2. a) $y > -2$ b) $y > 1$

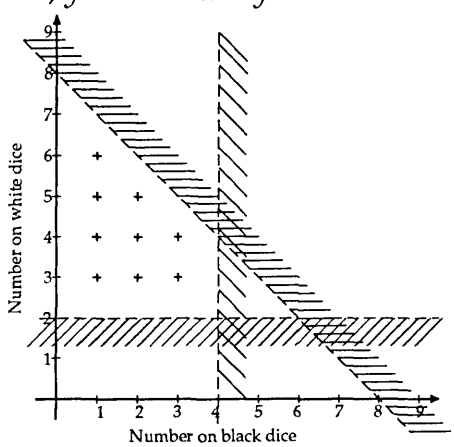
3. Any point in the region will satisfy the two inequalities $y < 4$ and $x < 4$.
   e.g. (-2, 3) because $-2 < 4$ and $3 < 4$
   and (3, 2) because $3 < 4$ and $2 < 4$
   or any pair of co-ordinates (x, y) where $y < 4$ and $x < 4$.

4. a) Any point in the region will satisfy the two inequalities $x > -2$ and $y > -2$.
   e.g. (3, 4) because $3 > -2$ and $4 > -2$
   and (15, 3) because $15 > -2$ and $3 > -2$
   or any pair of co-ordinates (x, y) where $x > -2$ and $y > -2$.
   b) Any point in the region will satisfy the two inequalities $x > -1$ and $y > 1$.
   e.g. (3, 4) because $3 > -1$ and $4 > 1$
   and (15, 3) because $15 > -1$ and $3 > 1$
   or any pair of co-ordinates (x, y) where $x > -1$ and $y > 1$.

5. Any point in the closed region will satisfy the three inequalities.
   e.g. (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)

6. a) $y > -2$, $x > -2$ and $x + y < 1$
   Any pair of co-ordinates (x, y) where $y > -2$, $x > -2$ and $x + y < 1$
   e.g. (0, 0), (1, -1)
   b) $y > 1$, $x > -1$ and $x + y < 5$
   Any pair of co-ordinates (x, y) where $y > 1$, $x > -1$ and $x + y < 5$
   e.g. (1, 2), (0, 4)

7. a) $x < 4$, $y > 2$ and $x + y < 8$
   b)



   c) There are only nine possible solutions, (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), as the dice only have positive values.
   d) $x > 0$

2273 Looping Chains

When your add-on number is 2, you do not need to add on more than 3 times before you can divide.	**always true**

If you start with a multiple of 3, you can divide first time.
e.g. $6 \div 3 \rightarrow 2$

If you start with a number 1 more than a multiple of 3, you can divide the second time.
e.g. $7 + 2 \rightarrow 9 \div 3 \rightarrow 3$

If you start with a number 2 more than a multiple of 3, you can divide the third time
e.g. $8 + 2 \rightarrow 10 + 2 \rightarrow 12 \div 3 \rightarrow 4$

If you start with a number 3 more than a multiple of 3, you get another multiple of 3.

When your starting number is odd and your add-on number is odd, the chain is made of odd numbers.	**not true**

Odd starting number [3]
Odd add-on number [7]
$3 \rightarrow 1 \rightarrow 8$ even number

Every time you add an odd number to an odd number you get an even number, so this is not true.

When your add-on number is a multiple of 3, there is a loop in the chain.	**not true**

Start with a multiple of 3
$9 \rightarrow 3 \rightarrow 1 \rightarrow 4 \rightarrow 7 \rightarrow 10 \rightarrow 13 \rightarrow 16 \rightarrow 19 \ldots$ This chain goes on and on, there is no loop.

Start with another number, which is not in the chain.
$2 \rightarrow 5 \rightarrow 8 \rightarrow 11 \rightarrow 14 \rightarrow 17 \rightarrow 20 \rightarrow 23 \rightarrow 26 \ldots$ This also goes on and on, there is no loop.

Start with another number not in the two chains above.
$6 \rightarrow 2 \rightarrow 5 \rightarrow 8 \ldots$ This joins up with the second chain.

$12 \rightarrow 4 \rightarrow 7 \rightarrow 10 \ldots$ This joins up with the first chain.

$15 \rightarrow 5 \rightarrow 8 \ldots$ This joins up with the second chain and so on.

None of these chains have a loop.

continued/

With the same add-on number, odd starting numbers give longer chains than even starting numbers.	**sometimes true**

With add-on number [4] and odd starting number [7]

$7 \rightarrow 11 \rightarrow 15 \rightarrow 5 \rightarrow 9 \rightarrow 3$ (loops back to 7) — 6 numbers in the chain.

With add-on number [4] and even starting number [8]

$8 \rightarrow 12 \rightarrow 4$ (loops back to 8) — 3 numbers in the chain.

The odd starting number gives a longer chain.

With add-on number [5] and odd starting number [7]

$7 \rightarrow 12 \rightarrow 4 \rightarrow 9 \rightarrow 3 \rightarrow 1 \rightarrow 6 \rightarrow 2$ (loops back to 7) — 8 numbers in the chain.

With add-on number 4 and even starting number

$8 \rightarrow 13 \rightarrow 18 \rightarrow 6 \rightarrow 2 \rightarrow 7 \rightarrow 12 \rightarrow 4 \rightarrow 9 \rightarrow 3 \rightarrow 1$ (loops back to 8) — 11 numbers in the chain.

The even starting number gives a longer chain.

When your starting number is even and your add-on number is odd, there is a loop in the chain.	**sometimes true**

Even starting number [2] and odd add-on number [7]

$2 \rightarrow 9 \rightarrow 3 \rightarrow 1 \rightarrow 8 \rightarrow 15 \rightarrow 5 \rightarrow 12 \rightarrow 4 \rightarrow 11 \rightarrow 18 \rightarrow 6$ (loops back to 2) — This chain has a loop.

Even starting number [2] and odd add-on number [3]

$2 \rightarrow 5 \rightarrow 8 \rightarrow 11 \rightarrow 14 \rightarrow 17 \rightarrow 20 \rightarrow 23 \ldots$ — This chain has no loop.

When your add-on number is 5 there is a loop in the chain.	**always true**

$1 \rightarrow 6 \rightarrow 2 \rightarrow 7 \rightarrow 12 \rightarrow 4 \rightarrow 9 \rightarrow 3$ (loops back to 1) — Loop 1

$5 \rightarrow 10 \rightarrow 15$ (loops back to 5)

$8 \rightarrow 13 \rightarrow 18 \rightarrow 6 \rightarrow 2 \ldots$ — Joins loop 1

$11 \rightarrow 16 \rightarrow 21 \rightarrow 7 \ldots$ — Joins loop 1

$78 \rightarrow 26 \rightarrow 31 \rightarrow 36 \rightarrow 12 \ldots$ — Joins loop 1

$811 \rightarrow 816 \rightarrow 272 \rightarrow 277 \rightarrow 282 \rightarrow 94 \rightarrow 99 \rightarrow 33 \rightarrow 11 \rightarrow 16 \ldots$ Joins loop 1

2274 abc

"Multiply **b** by 2 then subtract from **a**."	→	$a - 2b$
"Subtract **c** from **a** then multiply by **b**."	→	$b(a - c)$
"Add **b** to **a** then divide into **c**."	→	$\frac{c}{a + b}$
"Multiply **a** by **b** then divide by **c**."	→	$\frac{ab}{c}$
"Divide **c** by **b** then multiply by **a**."	→	$\frac{c}{b} \times a$
"Multiply **a** by **c** then divide into **b**."	→	$\frac{b}{ac}$
"Add **b** to **a** then divide by **c**."	→	$\frac{a + b}{c}$
"Multiply **ab** by **c**."	→	abc
"Subtract **c** from **b** then multiply by **a**."	→	$a(b - c)$
"Multiply **a** by 4 then subtract **c**."	=	**4a – c**
Divide a by c then subtract b	=	$\frac{a - b}{c}$

2275 Algebra Problems

Eatmore Crisps

Checking your answer.	$15w$	=	$12(w + 20)$	
Substitute $w = 80$	15×80	=	$12(80 + 20)$	
	1200	=	1200	Therefore $w = 80$

School Dinners

	$9t - 13$	=	$6t + 23$	
Subtract $6t$ from both sides	$3t - 13$	=	23	
Add 13 to both sides	$3t$	=	36	
Divide both sides by 3	t	=	12	

The number of dinner tables is 12.

Did you check your answer?

continued/

Chocolates

$$8x = 10(x-5)$$
$$x = 25$$

The number of chocolates in the original box is 25. Did you check your answer?

How old?

John's age in three years time = $4(x+3)$
So John's age now = $4(x+3)-3$
John's age two years ago = $7(x-2)$
So John's age now = $7(x-2)+2$

$$4(x+3)-3 = 7(x-2)+2$$
$$x = 7$$

Checking your answer

$$4(x+3)-3 = 7(x-2)+2$$
$$4(7+3)-3 = 7(7-2)+2$$
$$37 = 37$$

Therefore son's age now is 7.
John's age now is 37.

Pile of stones

Number of stones in first pile = $7x$
Number of stones in third pile = $x+18$
Number of stones in third pile = $7x-12$

$$x+18 = 7x-12$$
$$x = 5$$

Number of stones in second pile is 5.
Number of stones in first pile is 35.
Number of stones in third pile is 23. Did you check your answer?

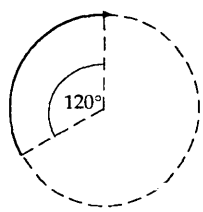
2276 Curvy Tiles in Logo

Most versions of logo use the instructions **arcr** and **arcl**. If the version of logo you are using does not understand the instructions **arcr** and **arcl** you will need to type these procedures. Ask your teacher if you need help.

The two procedures to teach the turtle to understand **arcr** and **arcl** are:

- to arcr :radius :angle
  repeat :angle[fd :radius*pi/360 rt 1]
  end
- to arcl :radius :angle
  repeat :angle[fd :radius*pi/360 lt 1]
  end

1. a) arcr 20 120 draws an arc, which is part of a circle of radius 20, with an angle of 120° at the centre of the circle and turning right.
   b) arcl 20 120 is the same, but turning left.



continued/

2. a) If you change the first number, the radius of the circle, you change the size of the radius.
   b) If you change the second number, the angle at the centre, you change the size of the arc.

3. One way to create a circle could be repeat 3[arcl 20 120].
   Another way is arcl 20 360. You may have found a different way.

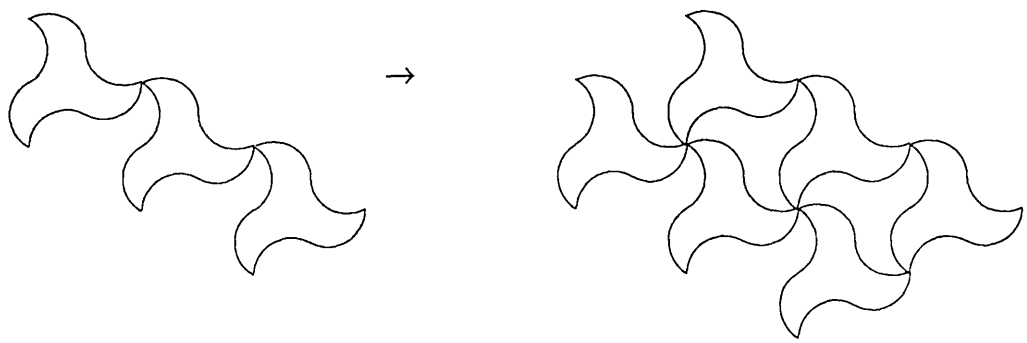
4. This curve is created by
   arcr 20 120 arcl 20 120.

   The tile is based on three of these curves, with a turn after each. The angle of each turn is 120°, because 3 x 120° = 360°.
   One procedure to create the tile is:
   tile
   repeat 3[arcr 20 120 arcl 20 120 rt 120]
   If yours is different speak to your teacher.

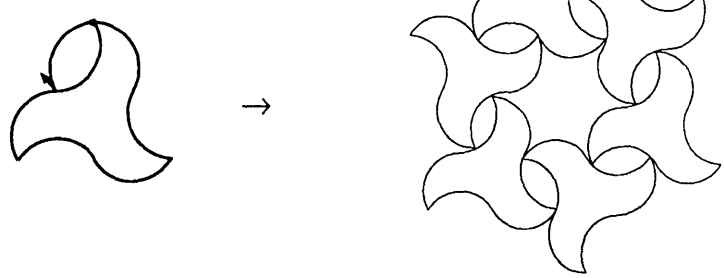
5. The pattern can be created by:
   pattern
   repeat 6[tile rt 60]

Challenge is more difficult.

- The first tessellation requires a procedure which creates a row of the tiles.

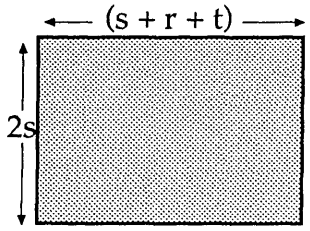


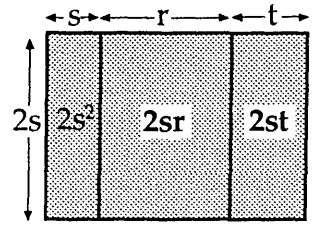
- The second tessellation requires a procedure which creates a circle with the tiles.
  To reach the starting point of the second tile
  arcr 20 120 rt 180
  Use this to create the procedure.



2277 Brackets

1. You should have found that whatever values you gave to a, b and c that
$a(b + c) = ab + ac$

2. 



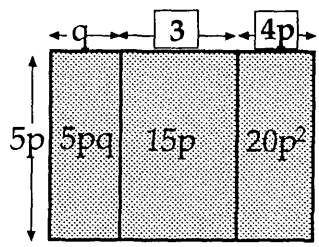
$2s(s + r + t) = 2s^2 + 2sr + 2st$

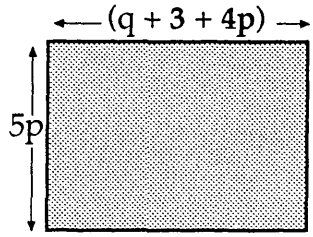
- You should have found that whatever values you gave to s, r and t that
$2s(s + r + t) = 2s^2 + 2sr + 2st$

3. a) $3p + 3q$ b) $5a + 5b + 5c$
c) $x^2 + xy + xz$ d) $2jk + 2jm - 2jn$
e) $s + 2st$ f) $d^2 - d$
g) $2e^2 + 4e$ h) $2fg - 2g^2$

- By substituting suitable values you should have found that the pairs of expressions are equivalent.

4. You should have found that whatever values you gave to a, b and c that
$4a + 8b + 12c = 4(a + 2b + 3c)$

5. 



$5pq + 15p + 20p^2 = 5p(q + 3 + 4p)$

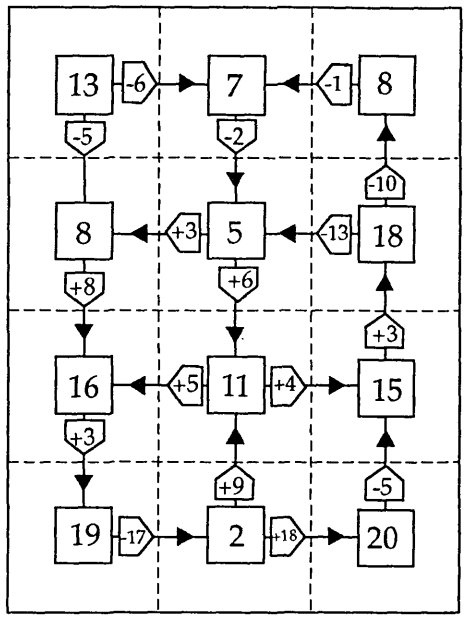
- You should have found that whatever values you gave to p and q that
$5pq + 15p + 20p^2 = 5p(q + 3 + 4p)$

6. a) $5(p + q + r)$ b) $3(m + n - p)$
c) $4(s - 2)$ d) $4(4t - 3s)$
e) $p(2 + 3q - r)$ f) $f(g - 3 + 4f)$
g) $4x(1 + 2y + 3x)$ h) $f(1 + e + 4f)$

- By substituting suitable values you should have found that the pairs of expressions are equivalent.

2278 Mapping Jigsaw

2.



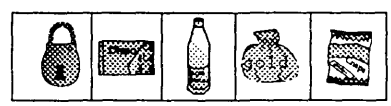
2279 Island Game

Did you play the game sufficient times for each player to win at least once?

2279d Island Game Worksheet

1.

Total number of each object collected.



3	2	3	2	2

2. Your answers may be different, if so check them with your teacher.

North	4
East	5
South	6
West	5
South	2
East	3
South	2
West	4
South	2
West	4

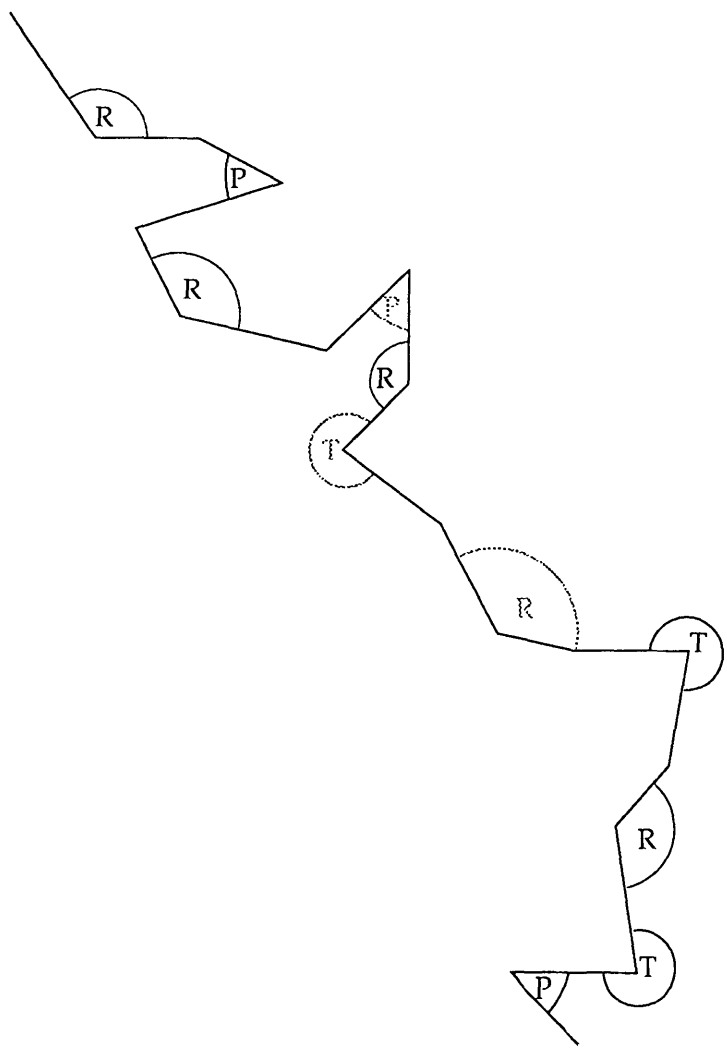
2280 Equal Angles

1.&2. You should have found that all the angles were equal to each other, if not, check your work with your teacher.

3. a), c), d) and e) are all equal angles.

4. $\angle A = \angle G$
$\angle B = \angle L$
$\angle C = \angle E$
$\angle D = \angle H$
$\angle E = \angle C$
$\angle F = \angle K$
$\angle G = \angle A$
$\angle H = \angle D$
$\angle J = \angle M$
$\angle K = \angle F$
$\angle L = \angle B$
$\angle M = \angle J$

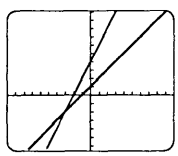
5.



2281 Simultaneous Match

A

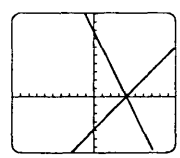
Pairs of simultaneous equations	Calculator screen	Solution	Check
1.	c.	(iv)	$2 = 12 - 10$ √ $2 = 8 - 6$ √
2.	a.	(iii)	$5 = 0 + 5$ √ $5 = 5 - 0$ √
3.	d.	(ii)	$-1 = -4 + 3$ √ $-1 = 8 - 9$ √
4.	b.	(i)	$-2 = 3 - 5$ √ $-2 = -2$ √

B  $x = -3, y = -2$ $-2 = -3 + 1$ √ $-2 = -6 + 4$ √

C $y = -x$, $y = x - 4$ $x = 2,\ y = -2$ $-2 = -2$ √ $-2 = 2 - 4$ √

D There are many possible answers. Here is one pair of simultaneous equations that fits the solution and the calculator screen.

$y = x - 4$
$y = 8 - 2x$

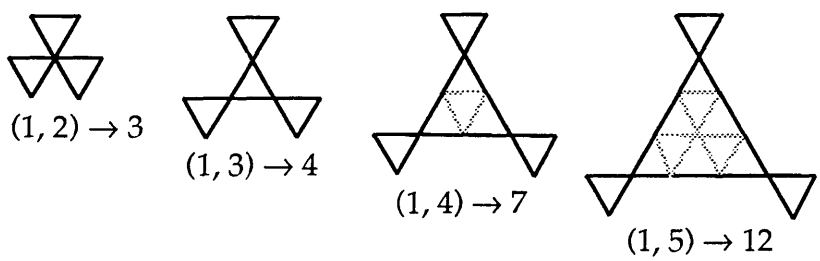


$0 = 4 - 4$ √
$0 = 8 - 8$ √

You should check your answers by substituting the solution into your pair of simultaneous equations.

2282 Springles

This shows the (1, x) set of springles.



continued/

2282 Springles (cont)

Here is a table of results for the set of (1, x) springles.

Springle	inside area (in Δ)	outside area (in Δ)	total area (in Δ)
(1, 1)	0	1	1
(1, 2)	0	3	3
(1, 3)	1	3	4
(1, 4)	4	3	7
(1, 5)	9	3	12
.	.	.	.
.	.	.	.
.	.	.	.

For springles of type (1, x) where $x \geq 2$ the rule is:

- inside area = $(x-2)^2$
- outside area = 3

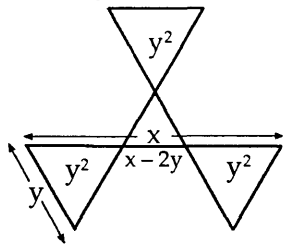
This rule does not work for (1, 1).

Did you find a rule for (2, x) springles? For what values of x did this rule work?

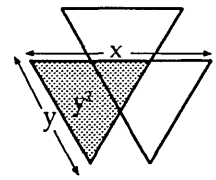
Some springles give overlapping patterns.
The rule for these is more difficult to find.

These diagrams may help you to generalise for any (y, x) springle:

- when $y \geq 2x$



- when $y < 2x$



2283 Jumping

Here are the results of six pupils:

Name	Jump size
Delroy	120cm
Yasmin	160cm
Emma	185cm
Leigh	160cm
Tak Yan	170cm
Colm	180cm

The jump sizes in ascending order are:

120, 160, 160, 170, 180, 185

Were your answers similar? Who jumped the furthest?

2284 BoxN

Were you able to place six numbers correctly in three successive games?

Did you play the game using decimal numbers to 1 decimal place? Were you still able to place six numbers correctly in three successive games?

2285 GuessN

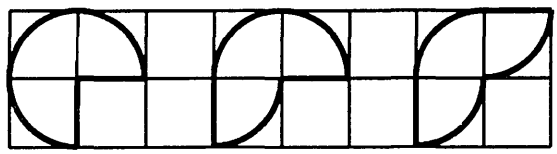
Were you able to find the number in 7 or less guesses?

Did you play the game using decimal numbers to 1 decimal place? Were you still able to find the number in 7 or less guesses?

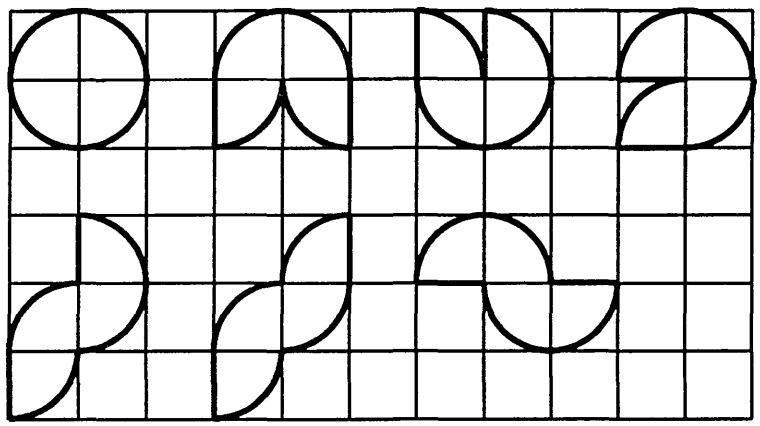
2286 Quadrants and Squares

Sheet A3 - Introducing Quadrants

These are the three different shapes of area $3y$, made from three quadrants. Your shapes may be reflections or rotations of them.



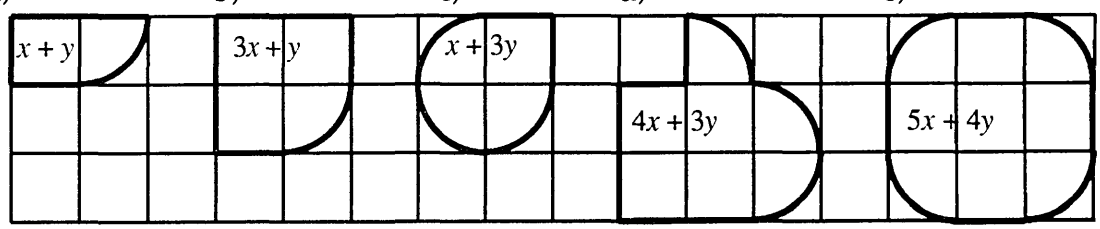
Here are seven different shapes of area $4y$. Did you find any different ones?



Sheet A4 - Squares and Quadrants

Here are some possible answers. You may have found different shapes.

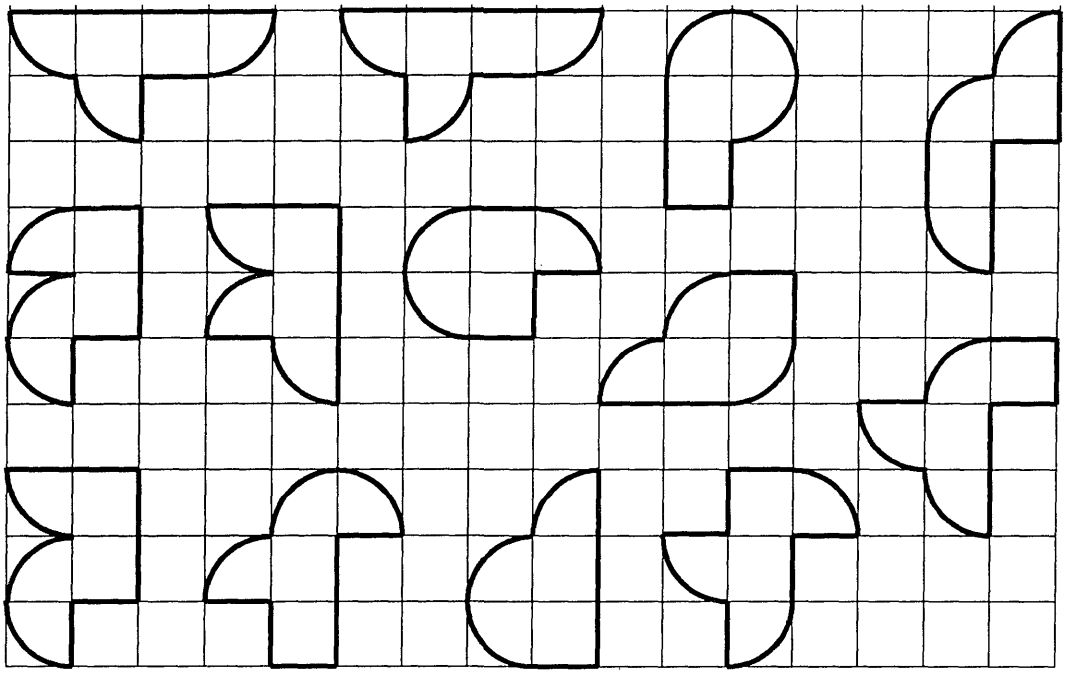
a) b) c) d) e)



2287 Add and Subtract Squares and Quadrants

Sheet A5 - Two Squares and Three Quadrants

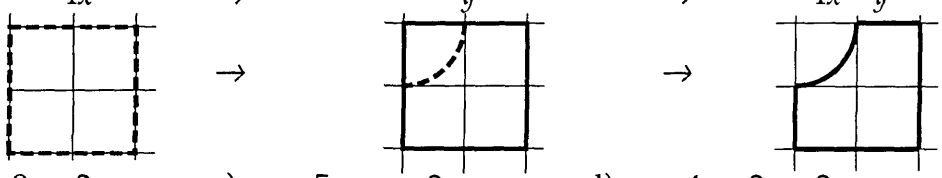
Here are thirteen different shapes. Did you find others? Are you sure they are all different?



Sheet A6 - Subtracting Areas

To explain why shape a) has area $4x - y$, it is useful to think of it as:

- four squares → subtract one quadrant → area is



b) $= 8x - 2y$

c) $= 5x + y - 2y$
$= 5x - y$

d) $= 4x - 2y + 2y$
$= 4x$

e) $= 3x - 2y$

2288 Algebra TakTiles on a Grid

Sheet B1 - Simple TakTiles

Tile A is made from a square and two quadrants.

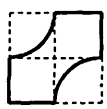
area a = $x + 2y$

Tile B is made from a square and two quadrants.

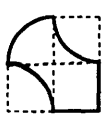
area b = $x + 2y$

Tile C is made from four squares with two quadrants taken away.

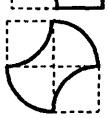
area c = $4x - 2y$



area d = $3x + y - 2y$
= $3x - y$



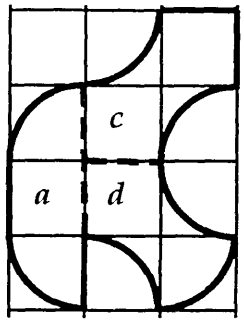
area e = $2x + 2y - 2y$
= $2x$



continued/

Sheet B2 - Finding Areas in Two Ways

Shape 2



- Method 1: Using Geometry

  Area = $8x + 3y - 4y$

  = $8x - y$

- Method 2: Using Algebra

  Area = $a + c + d$

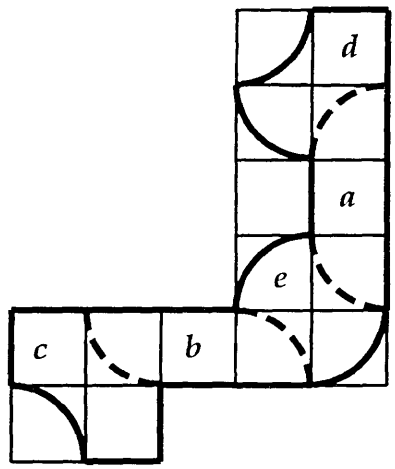
  = $(x + 2y) + (4x - 2y) + (3x - y)$

  = $8x - y$

If your answers were not the same, check your work with your teacher.
Did you find it easier using the algebra method?

Sheet B3 - Larger Areas

Shape 3



- Method 1: Using Geometry

  Area = $11x + 3y - 2y$

  = $11x + y$

- Method 2: Using Algebra

  Area a = $x + 2y$

  b = $x + 2y$

  c = $4x - 2y$

  d = $3x - y$

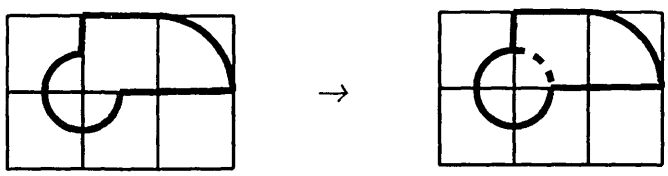
  e = $2x$

  = $11x + y$

If your answers were not the same, check your work with your teacher.
Did you find it easier using the algebra method?

Sheet B4 - The Small Circle

Tile F is made from a square, a quadrant and a circle with a quarter of a circle taken away.



area f = $x + y + y - {}^1/_4y$

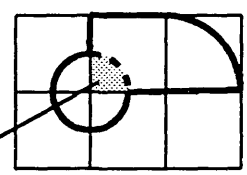
= $x + 1^3/_4y$

area g = $2x + y - {}^1/_4y$

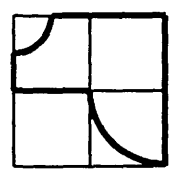
= $2x + {}^3/_4y$

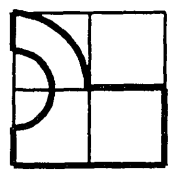
area h = $2x + y - {}^1/_2y$

= $2x + {}^1/_2y$



The quarter of a circle overlaps with the square so it should be subtracted.





continued/

Sheet B5 - More TakTile Shapes

Shape 4

- Method 1: Using Geometry

  Area = $8x + y - 2y - {}^1/_2y$

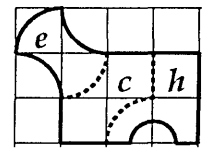
  = $8x - {}^3/_2y$

- Method 2: Using Algebra

  Area = $e + c + h$

  = $(2x) + (4x - 2y) + (2x + {}^1/_2y)$

  = $8x - {}^3/_2y$



Shape 5

- Method 1: Using Geometry

  Area = $6x + 3y - {}^1/_2y$

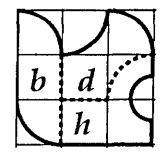
  = $6x + {}^3/_2y$

- Method 2: Using Algebra

  Area = $b + d + h$

  = $(x + 2y) + (3x - y) + (2x + {}^1/_2y)$

  = $6x + {}^3/_2y$



If your answers were not the same, check your work with your teacher.
Did you find it easier using the algebra method?

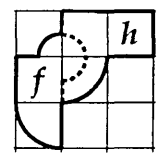
Sheet B6 - Half-Scale Shapes

Shape 6

Area = $f + h$

= $(x + 1^3/_4y) + (2x + {}^1/_2y)$

= $3x + 2^1/_4y$

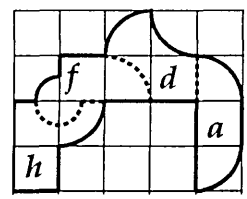


Shape 7

Area = $h + f + d + a$

= $(2x + {}^1/_2y) + (x + 1^3/_4y) + (3x - y) + (x + 2y)$

= $7x + 3^1/_4y$

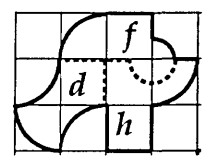


Shape 8

Area = $d + f + h$

= $(3x - y) + (x + 1^3/_4y) + (2x + {}^1/_2y)$

= $6x + 1^1/_4y$

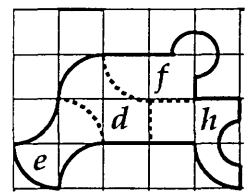


Shape 9

Area = $e + d + f + h$

= $(2x) + (3x - y) + (x + 1^3/_4y) + (2x + {}^1/_2y)$

= $8x + 1^1/_4y$

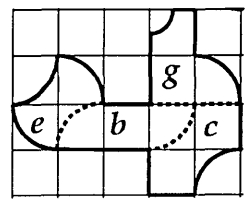


Shape 10

Area = $e + b + g + c$

= $(2x) + (x + 2y) + (2x + {}^3/_4y) + (4x - 2y)$

= $9x + {}^3/_4y$



Sheet C1 - The Easy TakTiles

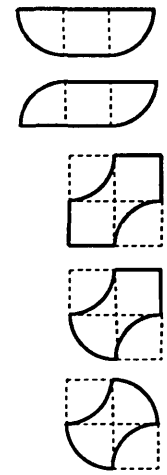
$a = x + 2y$

$b = x + 2y$

$c = 4x - 2y$
$= 2(2x - y)$

$d = 3x + y - 2y$
$= 3x - y$

$e = 2x + 2y - 2y$
$= 2x$

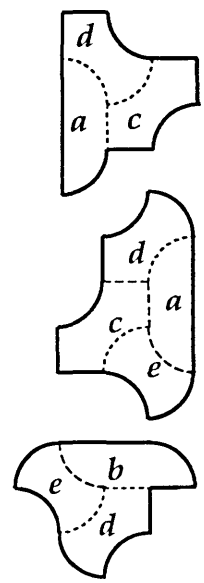


Sheet C2 - Shapes Made of TakTiles

Shape 1 $= a + c + d$
$= (x + 2y) + (4x - 2y) + (3x - y)$
$= 8x - y$

Shape 2 $= d + c + a + e$
$= (3x - y) + (4x - 2y) + (x + 2y) + (2x)$
$= 10x - y$

Shape 3 $= e + b + d$
$= (2x) + (x + 2y) + (3x - y)$
$= 6x + y$



Were you able to explain your answers to your neighbour? If not, try explaining your answers to your teacher.

Sheet C4 - The Other TakTiles

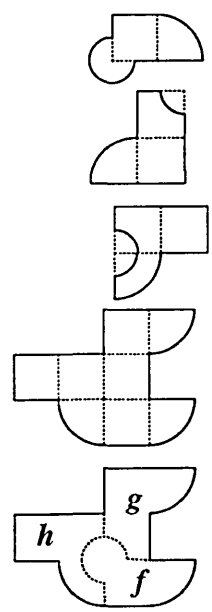
$f = x + y + {}^{3}/_{4}y$
$= x + 1{}^{3}/_{4}y$

$g = 2x + y - {}^{1}/_{4}y$
$= 2x + {}^{3}/_{4}y$

$h = 2x + y - {}^{1}/_{2}y$
$= 2x + {}^{1}/_{2}y$

Shape 4
a) Area $= 5x + 3y$

b) Area $= f + g + h$
$= (x + 1{}^{3}/_{4}y) + (2x + {}^{3}/_{4}y) + (2x + {}^{1}/_{2}y)$
$= 5x + 3y$

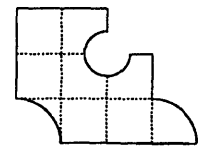


continued/

Shape 5

a) Area = $8x + y - 1^3/_4y$

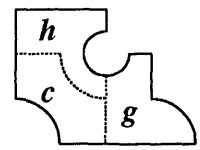
= $8x - {}^3/_4y$



b) Area = $c + g + h$

= $(4x - 2y) + (2x + {}^3/_4y) + (2x + {}^1/_2y)$

= $8x - {}^3/_4y$



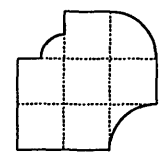
If your answers were not the same, check your work with your teacher.
Did you find it easier using the algebra method?

Sheet C5 - Four More Shapes

Shape 6

a) Area = $7x - y + 1^1/_4y$

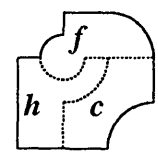
= $7x + {}^1/_4y$



b) Area = $c + f + h$

= $(4x - 2y) + (x + 1^3/_4y) + (2x + {}^1/_2y)$

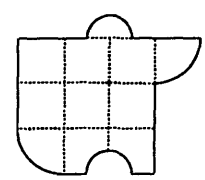
= $7x + {}^1/_4y$



Shape 7

a) Area = $8x + 2^1/_2y - {}^1/_2y$

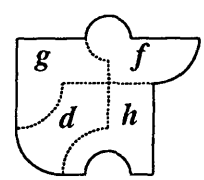
= $8x + 2y$



b) Area = $d + f + g + h$

= $(3x - y) + (x + 1^3/_4y) + (2x + {}^3/_4y) + (2x + {}^1/_2y)$

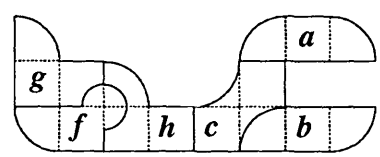
= $8x + 2y$



Shape 8

a) Area = $11x + 6y - y$

= $11x + 5y$



b) Area = $a + b + c + f + g + h$

= $(x + 2y) + (x + 2y) + (4x - 2y) + (x + 1^3/_4y) + (2x + {}^3/_4y) + (2x + {}^1/_2y)$

= $11x + 5y$

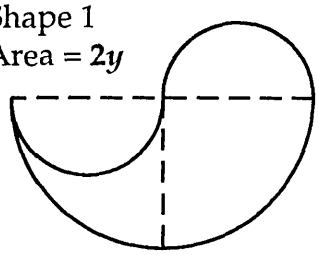
If your answers were not the same, check your work with your teacher.
Did you find it easier using the algebra method?

continued/

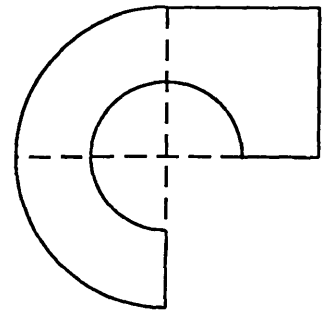
Sheet C6 - Tricky Shapes

The areas of all the shapes are given in terms of x and y.
Check you can understand where each answer comes from. The drawings should help.

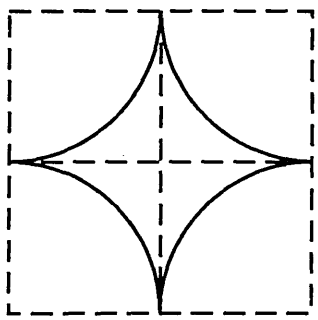
Shape 1
Area = $2y$



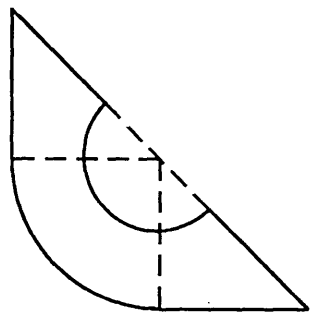
Shape 2
Area = $x + 1\frac{1}{4}y$



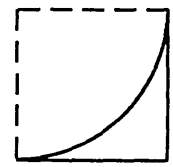
Shape 5
Area = $4x - 4y$



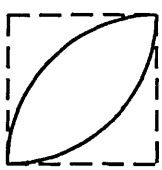
Shape 4
Area = $x + \frac{1}{2}y$



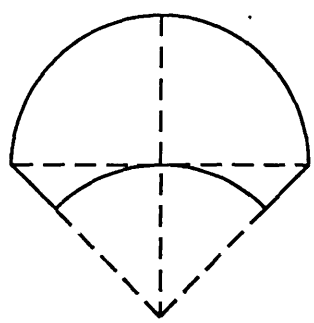
Shape 3
Area = $x - y$



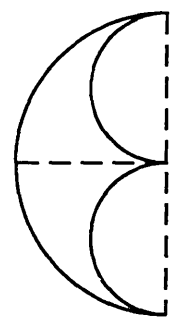
Shape 6
Area = $2y - x$



Shape 7
Area = $x + y$

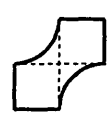


Shape 8
Area = y



Sheet D1 - A New Unit of Area

$c = 2x + 2z$



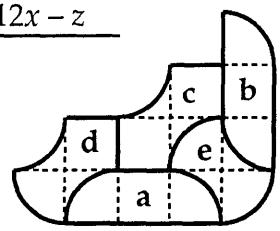
$d = x + y + 2z$
$= 2x + z$

$e = 2y + 2z$

continued/

Sheet D2 - Easy Substitution

(i)		x and y	(ii)		x and z	(iii)		y and z
a	=	$x + 2y$	a	=	$3x - 2z$	a	=	$3y + z$
b	=	$x + 2y$	b	=	$3x - 2z$	b	=	$3y + z$
c	=	$4x - 2y$	c	=	$2x + 2z$	c	=	$2y + 4z$
d	=	$3x - y$	d	=	$2x + z$	d	=	$2y + 3z$
e	=	$2x$	e	=	$2x$	e	=	$2y + 2z$
		$11x + y$			$12x - z$			$12y + 11z$



The area of this shape can be expressed in terms of:

(i)		x and y	(ii)		x and z	(iii)		y and z
area	=	$11x + 3y - y$	area	=	$12x + 2z - 3z$	area	=	$9(y + z) + 3y + 2z$
	=	$11x + y$		=	$12x - z$		=	$12y + 11z$

If your answers were not the same and you could not find your mistake, check your work with your teacher.

Sheet D3 - More Difficult Substitution

In terms of x and z

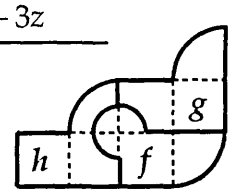
g	=	$2x + {}^{3}/_{4}y$	h	=	$2x + {}^{1}/_{2}y$
	=	$2x + {}^{3}/_{4}(x - z)$		=	$2x + {}^{1}/_{2}(x - z)$
	=	$2{}^{3}/_{4}x - {}^{3}/_{4}z$		=	$2{}^{1}/_{2}x - {}^{1}/_{2}z$

In terms of y and z

g	=	$2x + {}^{3}/_{4}y$	h	=	$2x + {}^{1}/_{2}y$
	=	$2(y + z) + {}^{3}/_{4}y$		=	$2(y + z) + {}^{1}/_{2}y$
	=	$2{}^{3}/_{4}y + 2z$		=	$2{}^{1}/_{2}y + 2z$

Sheet D4 - Checking Your Answer

(i)		x and y	(ii)		x and z	(iii)		y and z
f	=	$x + 1{}^{3}/_{4}y$	f	=	$2{}^{3}/_{4}x - 1{}^{3}/_{4}z$	f	=	$2{}^{3}/_{4}y + z$
g	=	$2x + {}^{3}/_{4}y$	g	=	$2{}^{3}/_{4}x - {}^{3}/_{4}z$	g	=	$2{}^{3}/_{4}y + 2z$
h	=	$2x + {}^{1}/_{2}y$	h	=	$2{}^{1}/_{2}x - {}^{1}/_{2}z$	h	=	$2{}^{1}/_{2}y + 2z$
		$5x + 3y$			$8x - 3z$			$8y + 5z$



The area of this shape equals the area of $f + g + h$, it can be expressed in terms of

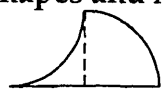
(i)		x and y	(ii)		x and z	(iii)		y and z
area	=	$5x + 3y$	area	=	$8x - 3z$	area	=	$5(y + z) + 3y$
							=	$8y + 5z$

If your answers were not the same and you could not find your mistake, check your work with your teacher.

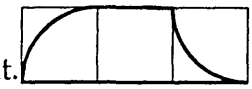
continued/

Sheet D5 - Working with Shapes and Areas

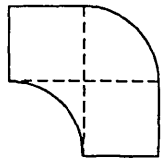
1. Area = $y + z$



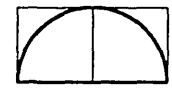
2. One possible shape, your's may be different.



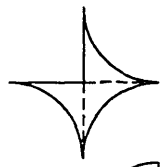
3. Area = $2x + y + z$



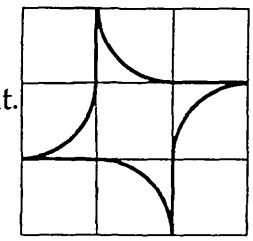
4. One possible shape, your's may be different.



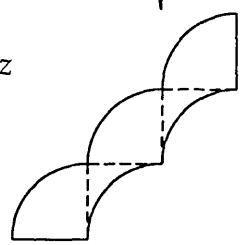
5. Area = $3z$



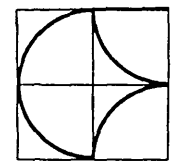
6. One possible shape, your's may be different.



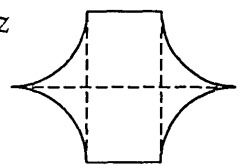
7. Area = $3y + 2z$



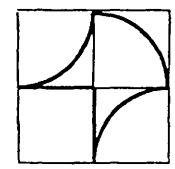
8. One possible shape, your's may be different.



9. Area = $2x + 4z$



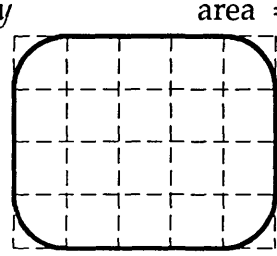
10. One possible shape, your's may be different.

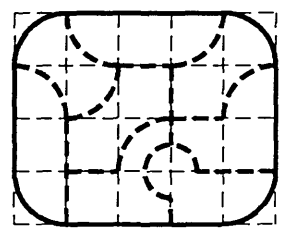


Sheet D6 - The Original TakTile

Total area of panel in terms of:

(i)	x and y	(ii)	x and z	(iii)	y and z
area =	$16x + 4y$	area =	$20x - 4z$	area =	$20y + 16z$





Each tile in terms of:

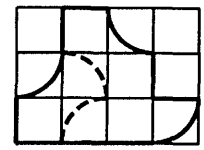
(i)		x and y	(ii)		x and z	(iii)		y and z
a	=	$x + 2y$	a	=	$3x - 2z$	a	=	$3y + z$
b	=	$x + 2y$	b	=	$3x - 2z$	b	=	$3y + z$
c	=	$4x - 2y$	c	=	$2x + 2z$	c	=	$2y + 4z$
d	=	$3x - y$	d	=	$2x + z$	d	=	$2y + 3z$
e	=	$2x$	e	=	$2x$	e	=	$2y + 2z$
f	=	$x + 1^3/_4 y$	f	=	$2^3/_4 x - 1^3/_4 z$	f	=	$2^3/_4 y + z$
g	=	$2x + {}^3/_4 y$	g	=	$2^3/_4 x - {}^3/_4 z$	g	=	$2^3/_4 y + 2z$
h	=	$2x + {}^1/_2 y$	h	=	$2^1/_2 x - {}^1/_2 z$	h	=	$2^1/_2 y + 2z$
		$16x + 4y$			$20x - 4z$			$20y + 16z$

If your answers were not the same, check your work with your teacher.
Did you find it easier using the algebra method?

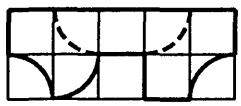
2291 Comparing Areas

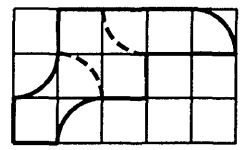
Sheet E1 - Shapes with the same Area

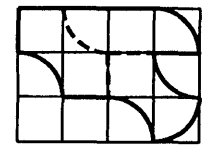
The shape has area = $b + c + d$
= $(x + 2y) + (4x - 2y) + (3x - y)$
= $8x - y$



These shapes have the same area.

a) $a + c + d$ 

b) $b + c + d$ 

c) $b + c + d$ 

You probably found different ones.
If you are unsure about whether your shapes are correct, show them to your teacher.

Sheet E3 - Shapes Using Algebra

Shape P is *any* shape made from tiles *c* and *e*. **p** = $c + e$
= $(4x - 2y) + (2x)$
= $6x - 2y$

Shape Q is *any* shape made from tiles *b* and *c*. **q** = $b + c$
= $(x + 2y) + (4x - 2y)$
= $5x$

Shape R is *any* shape made from tiles *c* and *f*. *r* = $c + f$
= $(4x - 2y) + (x + 1^3/_4 y)$
= $5x - {}^1/_4 y$

Shape S is *any* shape made from tiles *d* and *h*. *s* = $d + h$
= $(3x - y) + (2x + {}^1/_2 y)$
= $5x - {}^1/_2 y$

Shape T is *any* shape made from tiles *f* and *g*. **t** = $f + g$
= $(x + 1^3/_4 y) + (2x + {}^3/_4 y)$
= $3x + 2^1/_2 y$

Shape U is *any* shape made from tiles *b*, *c* and *e*. *u* = $b + c + e$
= $(x + 2y) + (4x - 2y) + (2x)$
= $7x$

Sheet E4 - The Final Task

Shape V is *any* shape made from tiles *a*, *b* and *d*. **v** = $a + b + d$
= $(x + 2y) + (x + 2y) + (3x - y)$
= $5x + 3y$

Shape W is *any* shape made from tiles *f*, *g* and *h*. **w** = $f + g + h$
= $(x + 1^3/_4 y) + (2x + {}^3/_4 y) + (2x + {}^1/_2 y)$
= $5x + 3y$

2292 Towers

After you have played this game, you may have a list of questions which you found difficult to answer, or would like further work on. Show your list to your teacher.

2293 Negative Sequences

1.	-6,	-9,	-12	The rule is **subtract 3.**
2.	-8,	-12,	-16	The rule is **subtract 4.**
3.	2,	4,	6	The rule is **add 2.**
4.	-4,	-9,	-14	The rule is **subtract 5.**
5.	-5,	-9,	-13	The rule is **subtract 4.**
6.	18,	26,	34	The rule is **add 8.**
7.	2,	-4,	-10	The rule is **subtract 6.**

2294 Sum, Product and Difference

Sum

1.	a)	The sum of 2 and 4 is **6.**	2 + 4 = 6
	b)	The sum of 3 and 5 is **8.**	3 + 5 = 8
	c)	The sum of 10 and 2 is **12.**	10 + 2 = 12
	d)	The sum of 9 and 5 is **14.**	9 + 5 = 14
	e)	The sum of 7 and 8 is **15.**	7 + 8 = 15

Product

2.	a)	The product of 2 and 4 is **8.**	2 x 4 = 8
	b)	The product of 3 and 5 is **15.**	3 x 5 = 15
	c)	The product of 10 and 2 is **20.**	10 x 2 = 20
	d)	The product of 9 and 5 is **45.**	9 x 5 = 45
	e)	The product of 7 and 8 is **56.**	7 x 8 = 56

Difference

3.	a)	The difference between 2 and 4 is **2.**	4 – 2 = 2
	b)	The difference between 3 and 5 is **2.**	5 – 3 = 2
	c)	The difference between 10 and 2 is **8.**	10 – 2 = 8
	d)	The difference between 9 and 5 is **4.**	9 – 5 = 4
	e)	The difference between 7 and 8 is **1.**	8 – 7 = 1

Mixed Bag

4.

		Sum	Product	Difference
a)	2 and 4	**2 + 4 = 6**	**2 x 4 = 8**	**4 – 2 = 2**
b)	6 and 3	**6 + 3 = 9**	**6 x 3 = 18**	**6 – 3 = 3**
c)	7 and 9	**7 + 9 = 16**	**7 x 9 = 63**	**9 – 7 = 2**
d)	5 and 1	**5 + 1 = 6**	**5 x 1 = 5**	**5 – 1 = 4**
e)	3 and 11	**3 + 11 = 14**	**3 x 11 = 33**	**11 – 3 = 8**

continued/

2294 Sum, Product and Difference (cont)

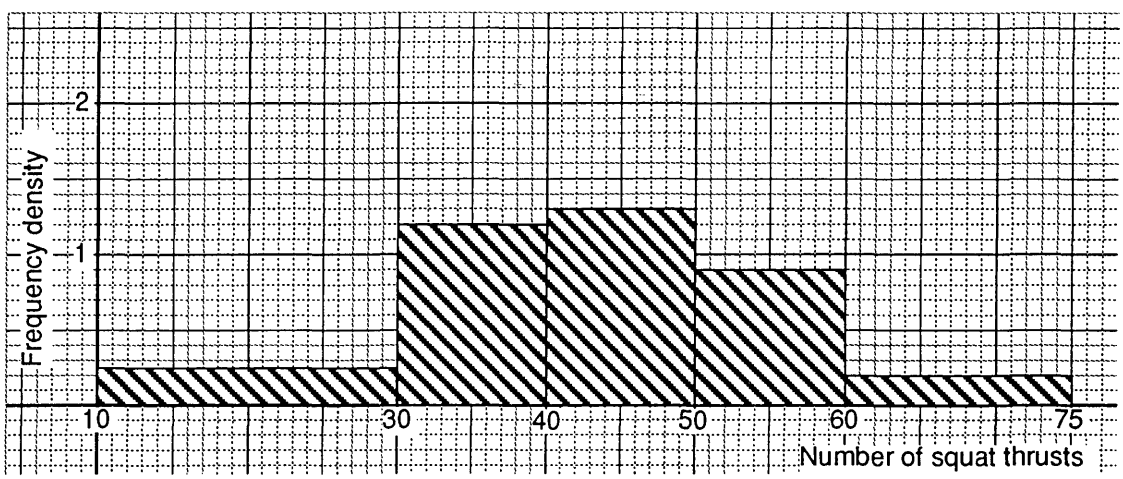
5. a) The sum of 15 and 9 is **24**. $15 + 9 = 24$
   b) The difference between 18 and 6 is **12**. $18 - 6 = 12$
   c) The product of 5 and 9 is **45**. $5 \times 9 = 45$
   d) The product of 8 and 6 is **48**. $8 \times 6 = 48$
   e) The sum of 8 and 6 is **14**. $8 + 6 = 14$

2295 Histograms

1. a)

Number of squat thrusts	Frequency	Class interval	Frequency density
10 - 29	5	20	$5 \div 20 = 0.25$
30 - 39	12	10	$12 \div 10 = 1.2$
40 - 49	13	**10**	$\mathbf{13 \div 10 = 1.3}$
50 - 59	9	**10**	$\mathbf{9 \div 10 = 0.9}$
60 - 74	3	**15**	$\mathbf{3 \div 15 = 0.2}$
75 -	0	**0**	$\mathbf{0 \div 0 = 0}$

b)



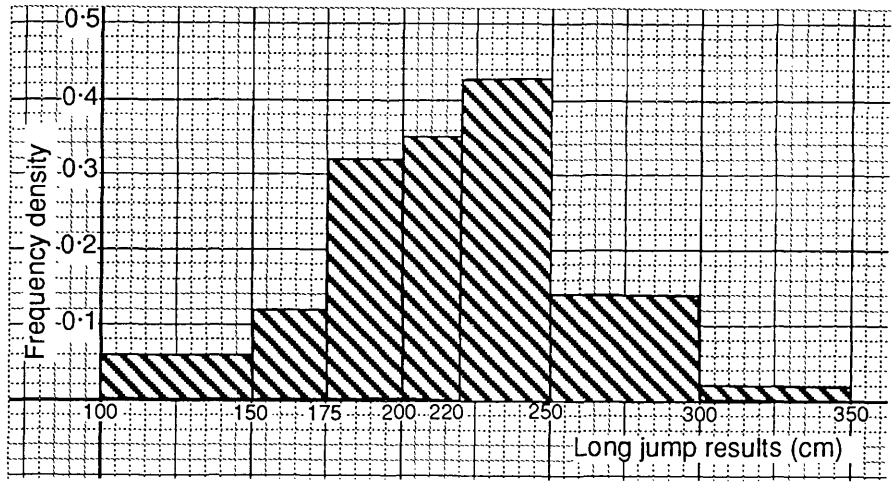
2. a)

Length of jump, (L)cm	Frequency	Class interval	Frequency density
$100 \leq L < 150$	3	50	0.06
$150 \leq L < 175$	3	25	**0.12**
$175 \leq L < 200$	8	25	**0.32**
$200 \leq L < 220$	7	**20**	**0.35**
$220 \leq L < 250$	13	**30**	**0.43̇**
$250 \leq L < 300$	7	**50**	**0.14**
$300 \leq L < 350$	1	**50**	**0.02**

continued/

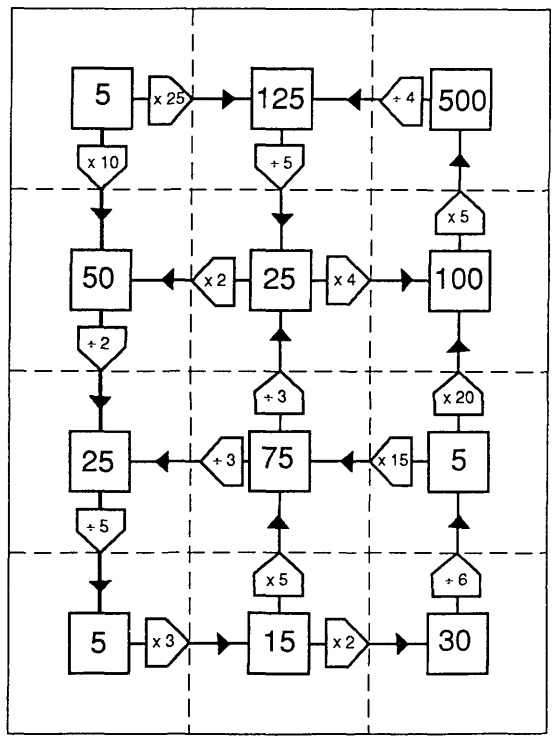
2295 Histograms (cont)

2. b)



3. a) 0.8 x 15 = 12 students
   b) 6 students
   c) 25 students
   d) Answers such as:
   The same number of students can only do between 0 and 14.
   More students at the first school can do between 20 and 25 (13 compared with 10).
   More students at the second school can do between 25 and 44 press-ups, (16 compared with 12).

2296 Mapping Rectangles



2297 Harder Negative Sequences

1. 6, 4, 0, -6, -14, **-24, -36, -50,** -66

   \ / \ / \ / \ /

   – 2 – 4 – 6 – 8

   The rule is **subtract two more each time.**

2. 13, 11, 10, 10, 11, **13, 16, 20,** 25

   \ / \ / \ / \ /

   – 2 – 1 0 **–1**

   The rule is **subtract one less each time.**

3. **4, 5, 7,** The rule is **subtract one less each time.**

4. **-7, -7, -5,** The rule is **subtract two less each time.**

5. **7, 3, -2,** The rule is **add one less each time.**

6. **13, 12, 10,** The rule is **add one less each time.**

7. **-30, -62, -126,** The rule is **subtract twice as many each time.**

8. **-8, -10, -13,** The rule is **add one less each time.**

9. **6, 4, 3,** The rule is **subtract half as many each time.**

10. **-108, -236, -492,** The rule is **subtract twice as many each time.**

2298 Rotating Patterns

Help on using MicroSMILE program Transform.

To create the screen dump, follow these instructions

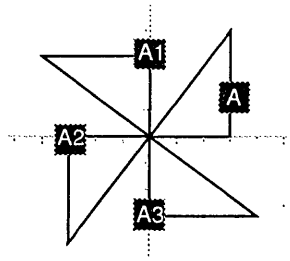
- Select **Shape.**
- Select **Select a Shape.**
- Select the right-angled triangle.
- Enter the starting co-ordinates by typing **0**, press enter and then pressing **0**, press enter.
- Select rOtate.
- Enter shape's label **A** and then press enter.
- Enter centre of rotation (0, 0).
- Enter angle of rotation **90.**

continued/

2298 Rotating Patterns (cont)

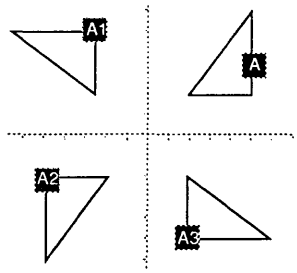
Here is one way to create the patterns using rotation only. You may have created the pattern in different ways.

1.



Starting shape	Angle of rotation	Centre of rotation	New shape
A	90°	(0, 0)	A1
A1	90°	(0, 0)	A2
A2	90°	(0, 0)	A3

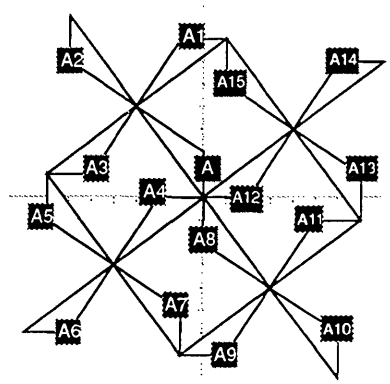
2. This screen was created using the same starting shape, but with one vertex placed at (2, 2). You may have created the pattern in different ways.



Starting shape	Angle of rotation	Centre of rotation	New shape
A	90°	(0, 0)	A1
A	180°	(0, 0)	A2
A	270°	(0, 0)	A3

3. This screen was created using the flag shape with the base placed at (0, 0). The rotations are all about (0, 0) and are multiples of 45°.

4. There are many possible ways of making this pattern.



This is the start of one method.

Starting shape	Angle of rotation	Centre of rotation	New shape
A	90°	(-3, 4)	A1
A	180°	(-3, 4)	A2
A	270°	(-3, 4)	A3
A	90°	(0, 0)	A4
A4	90°	(-4, -3)	A5
A4	180°	(-4, -3)	A6

5. Show your own rotating pattern to your teacher. You may like to make a display of your pattern.

2299 Enlarging Areas

Help on using MicroSMILE program Transform.

To recreate the screen dump, follow these instructions.

- Select **Shape**.
- Select **Select a Shape**.
- Select the **L** shape.
- Enter the starting co-ordinates by typing **2**, press enter and then typing **1**, press enter.
- Select **Enlarge.**
- Enter shape's label **A** and press enter.
- Enter centre of enlargement **(0, 0)**.
- Enter scale factor **2**.
- Enlarge axes to accommodate shapes? **Yes**
- Select **Axes**, select **Show Grid**.

- When each shape is enlarged by scale factor 2, the area becomes 4 times as large. The ratio

  area of starting shape : area of enlarged shape
  1 : 4

- You should have found that the new area can be found by multiplying the original area by the square of the scale factor of enlargement.

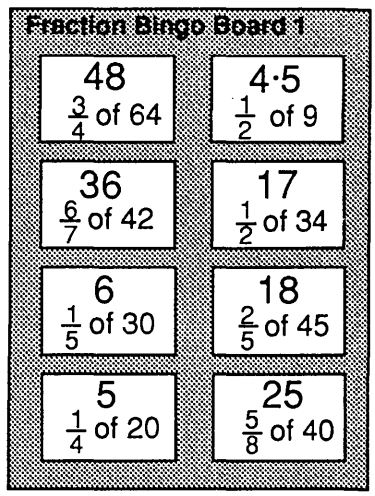
e.g.

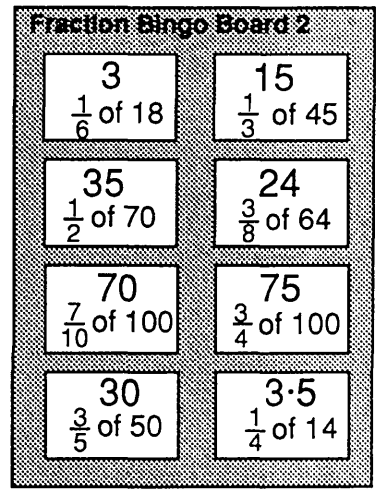
Scale factor of enlargement	Square of scale factor	Area of starting shape	Area of enlarged shape
3	$3^2 = 9$	2	18 (2 x 9)
3	$3^2 = 9$	5	45 (5 x 9)
3	$3^2 = 9$	10	90 (10 x 9)
.	.	.	.
.	.	.	.
.	.	.	.
-4	$-4^2 = 16$	2	32 (2 x 16)
-4	$-4^2 = 16$	5	80 (5 x 16)
-4	$-4^2 = 16$	10	160 (10 x 16)
.	.	.	.
.	.	.	.
.	.	.	.
$\frac{1}{2}$	$\frac{12}{2} = \frac{1}{4}$	2	$\frac{1}{2}$ $(2 \times \frac{1}{4})$
$\frac{1}{2}$	$\frac{12}{2} = \frac{1}{4}$	5	$1\frac{1}{4}$ $(5 \times \frac{1}{4})$
$\frac{1}{2}$	$\frac{12}{2} = \frac{1}{4}$	10	$2\frac{1}{2}$ $(10 \times \frac{1}{4})$
.	.	.	.
.	.	.	.
.	.	.	.
x	$x = x^2$	a	ax^2 $(a \times x^2)$

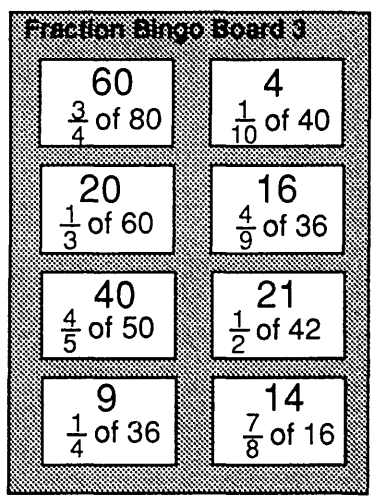
You should have found that this rule will work for any scale factor.

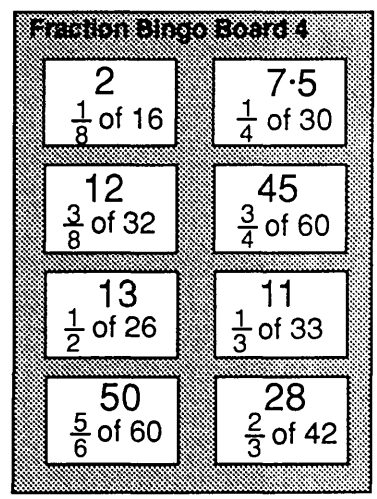
2300 Fraction Bingo

When you have finished playing, the boards should look like this:









2301 Simultaneous Equations from Graphs

1. a) (0, 0) b) (0, -3)

   c) (-1.5, -1.5) d) (-3, -6)

   e) You should have checked each of your solutions by substituting values for x and y, back into the original equations.

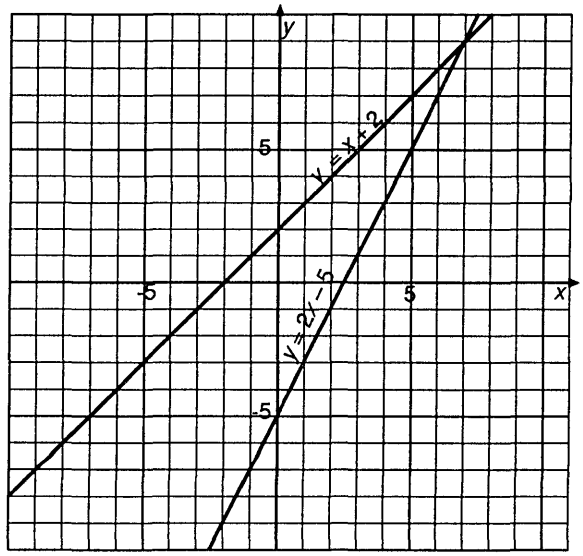
2. a) $\left.\begin{array}{l} x + y = 3 \\ y = x - 3 \end{array}\right\}$ b) $\left.\begin{array}{l} y + x = -3 \\ y = -2x + 1 \end{array}\right\}$

   c) $\left.\begin{array}{l} x + y = 3 \\ y = -2x + 1 \end{array}\right\}$ d) $\left.\begin{array}{l} y = x \\ x + y = 3 \end{array}\right\}$

continued/

3. The two lines of the equations $y = x$ and $y = x - 3$ are parallel, therefore there is no solution.

4.



The two lines of the equations $y = x + 2$ and $y = 2x - 5$ intersect at the point (7, 9)

$x = 7$, $y = 9$ is the solution to the simultaneous equations,

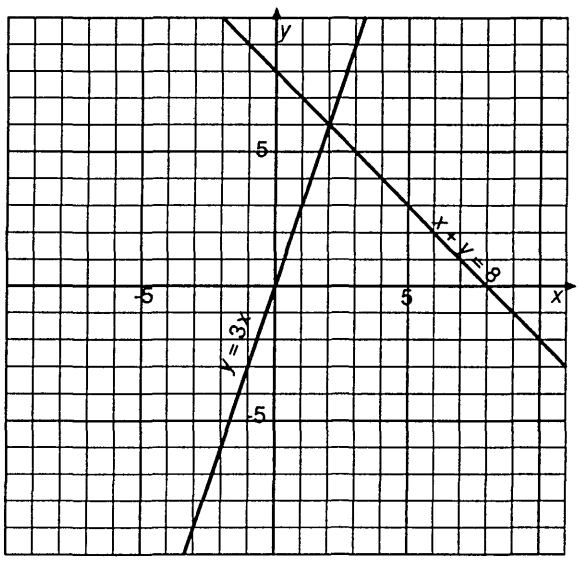
$$\left.\begin{aligned} y &= x + 2 \\ y &= 2x - 5 \end{aligned}\right\}$$

Check by substituting $x = 7$, $y = 9$

$y = x + 2$
$9 = 7 + 2$

$y = 2x - 5$
$9 = (2 \times 7) - 5$
$9 = 14 - 5$

5.



The two lines of the equations $y = 3x$ and $x + y = 8$ intersect at the point (2, 6)

$x = 2$, $y = 6$ is the solution to the simultaneous equations,

$$\left.\begin{aligned} y &= 3x \\ x + y &= 8 \end{aligned}\right\}$$

Check by substituting $x = 2$, $y = 6$

$y = 3x$
$6 = 3 \times 2$

$x + y = 8$
$2 + 6 = 8$

2302 Bearings

1. 390m

2.

	Bearing of B from A	A to B	
		Distance on diagram	Distance represented
a)	028°	5.6cm	560m
b)	252°	3.4cm	340m
c)	063°	4.9cm	490m
d)	205°	4.0cm	400m
e)	020°	9.5cm	950m
f)	300°	4.5cm	450m

Your answers may be slightly different. If they are very different, show your answers to your teacher.

2303 Hundred Fit

The Blue Puzzle

73	74	75	76	77	78	79	80	81	82
72	43	44	45	46	47	48	49	50	83
71	42	21	22	23	24	25	26	51	84
70	41	20	7	8	9	10	27	52	85
69	40	19	6	1 Start	2	11	28	53	86
68	39	18	5	4	3	12	29	54	87
67	38	17	16	15	14	13	30	55	88
66	37	36	35	34	33	32	31	56	89
65	64	63	62	61	60	59	58	57	90
100	99	98	97	96	95	94	93	92	91

The pattern made by the numbers is a spiral pattern.

The White Puzzle

46	37	29	22	16	11	7	4	2	1
56	47	38	30	23	17	12	8	5	3
65	57	48	39	31	24	18	13	9	6
73	66	58	49	40	32	25	19	14	10
80	74	67	59	50	41	33	26	20	15
86	81	75	68	60	51	42	34	27	21
91	87	82	76	69	61	52	43	35	28
95	92	88	83	77	70	62	53	44	36
98	96	93	89	84	78	71	63	54	45
100	99	97	94	90	85	79	72	64	55

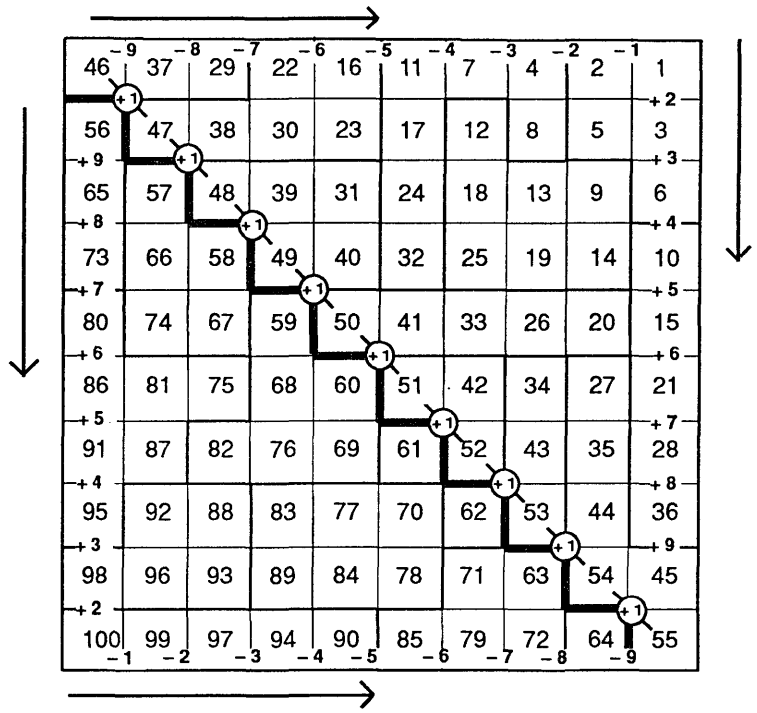
The numbers 1 - 100 have been arranged from the top right-hand corner to the bottom left-hand corner.

continued/

2303 Hundred Fit (cont)

The White Puzzle (cont)

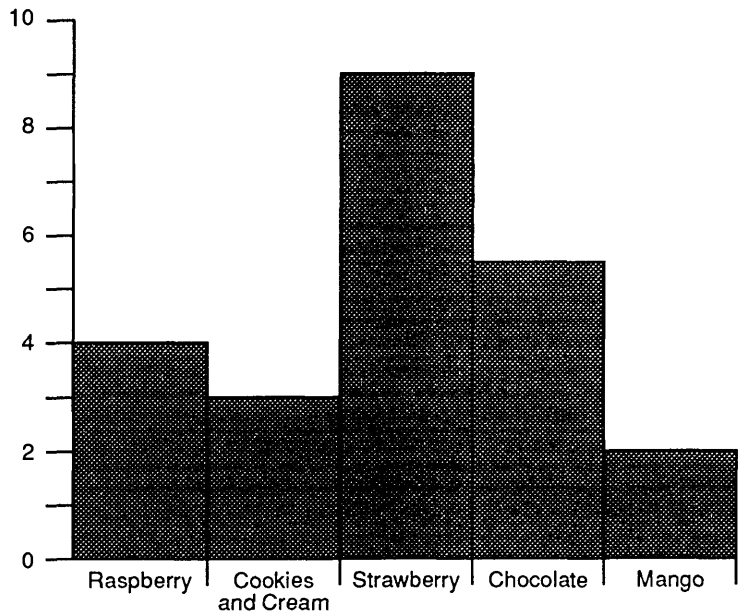
Here are other patterns that you might have found.



2304 Favourite Ice-Cream

1. Chocolate was the mode in Geeta's class.

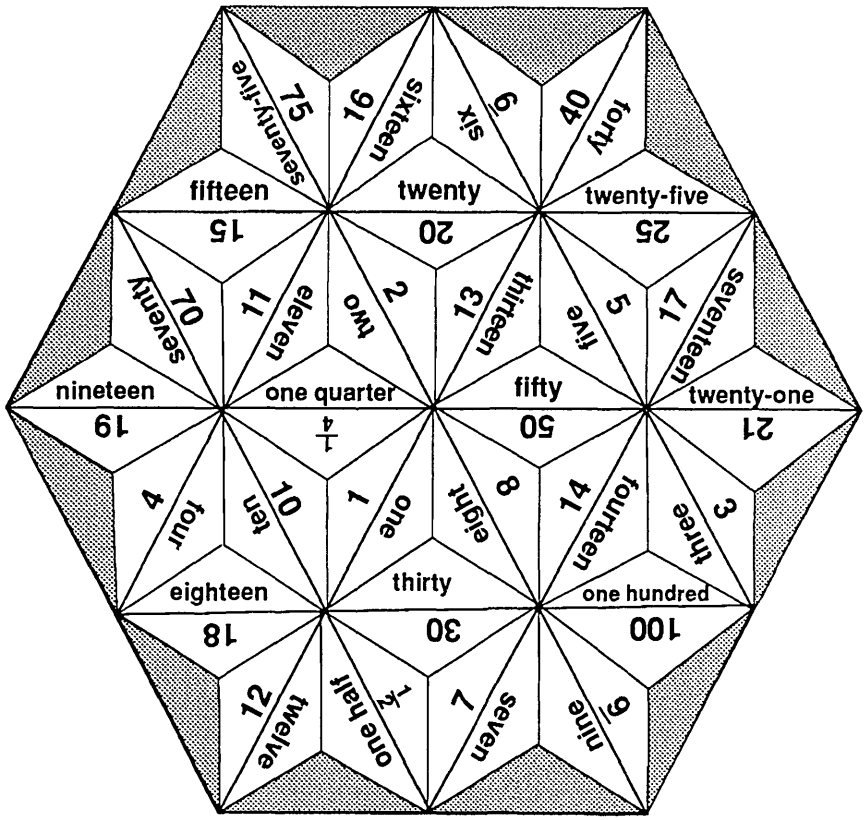
2. Here are the results of a survey that Jane carried out in her class.



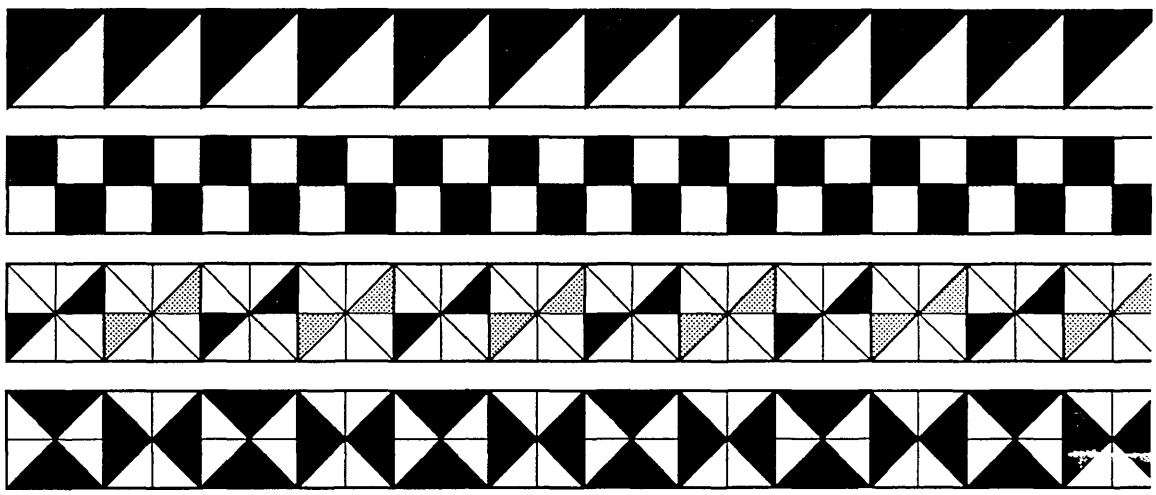
The mode is strawberry.

3. Looking at her results, Jane decided to buy 2 tubs of Strawberry, 1 tub of Chocolate and 1 tub of Raspberry.
In your survey, did you decide to buy 4 different flavours?
Show your results to your teacher and tell her why you made your decision.

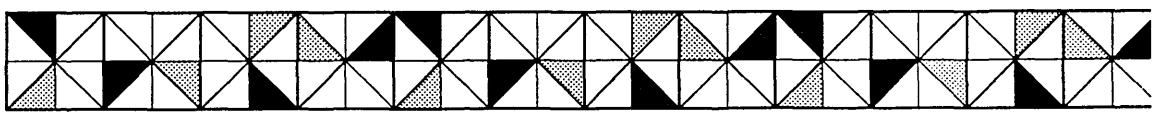
2305 Hexagon Puzzle



2306 Patterns on a line



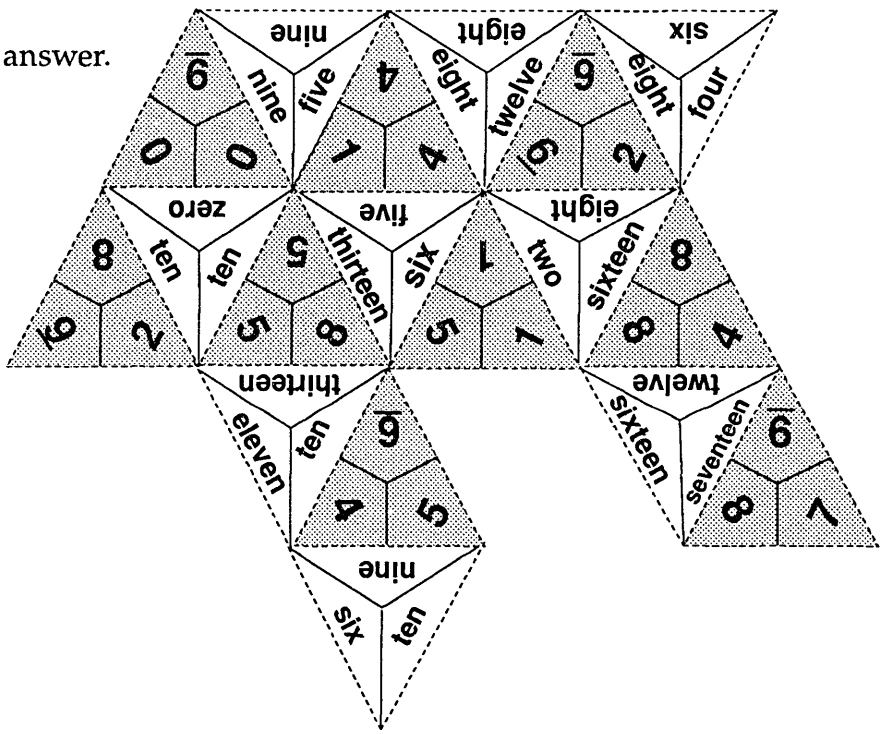
There are many possible patterns you could have created. Here is a possible pattern using two colours. You may like to make a display of your own patterns.



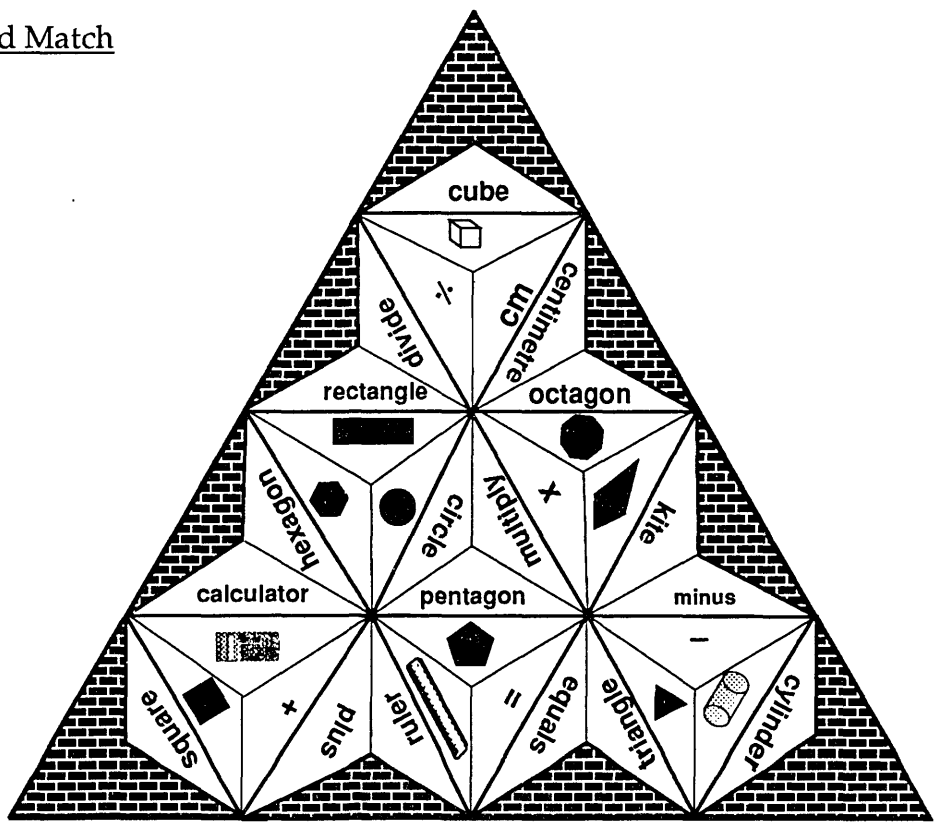
2307 Triangle Sums Game

If you are lucky it is possible to use all the pieces and make a rhombus shape. Because there is more than one way of making some numbers you will not always get this shape.
It is possible to use all the pieces and end up with different shapes. Do not worry about 'holes', it can still be done.

Here is part of a possible answer.



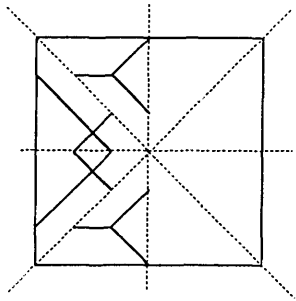
2308 Word Match



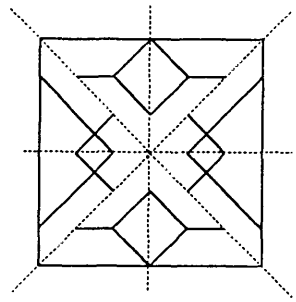
2309 Rangoli Patterns

Rangoli Patterns are used by many Hindu and Sikh families to decorate their homes for important festivals. Some of these patterns are based on a square grid of dots and use reflections to create symmetrical patterns.

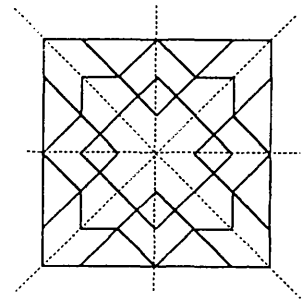
7. The lines reflected in the . . .



. . . horizontal line . . .

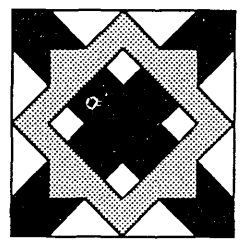


the vertical line . . .

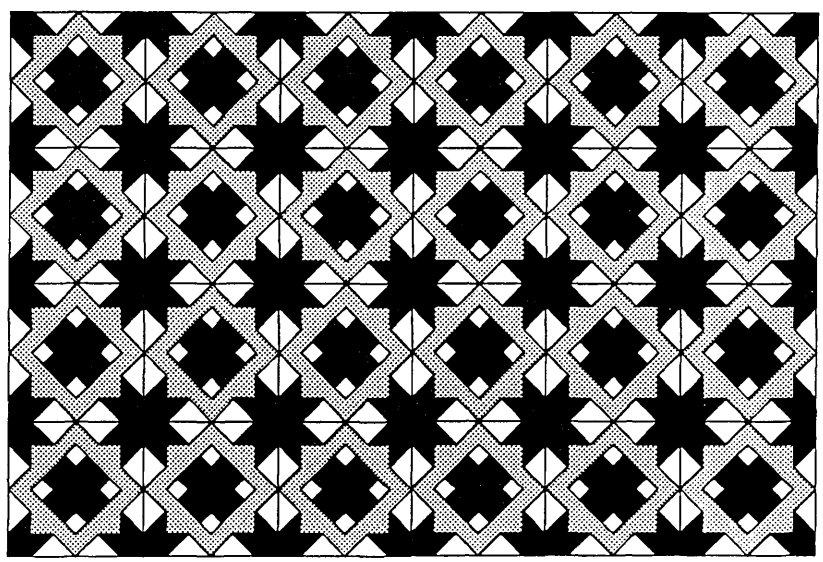


the diagonal lines.

Here is one way to colour the completed Rangoli pattern once the lines of symmetry have been removed. You may have coloured the pattern differently.



8. This shows the repeated pattern.

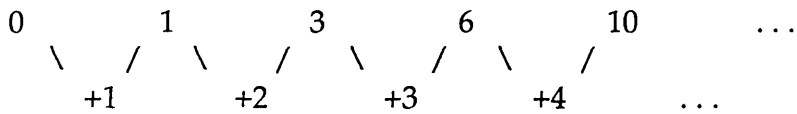


2310 Sequences Jigsaw

This is the solution to the puzzle.

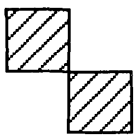
0	1	3	6	10	15	21	28	36	45
1	2	4	7	11	16	22	29	37	46
3	4	6	9	13	18	24	31	39	48
6	7	9	12	16	21	27	34	42	51
10	11	13	16	20	25	31	38	46	55
15	16	18	21	25	30	36	43	51	60
21	22	24	27	31	36	42	49	57	66
28	29	31	34	38	43	49	56	64	73
36	37	39	42	46	51	57	64	72	81
45	46	48	51	55	60	66	73	81	90

You probably noticed that the numbers across and the numbers going down follow the rule, 'you add one more each time to the difference of the previous two numbers'.
e.g. Look at the first row.

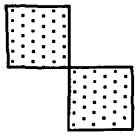


In fact the numbers in the first row are the **triangle** numbers.

There are some other patterns too.



The numbers shaded with diagonal lines are 2 times the triangle numbers.
e.g. 0 1 3 6 10
↓ ↓ ↓ ↓ ↓
x 2 x 2 x 2 x 2 x 2



The numbers on the diagonal shaded with dots are **square** numbers.
e.g. 1 4 9 16 25 ...

2311 Start with 60°

1.& 2. Check your constructions by measuring the angles using an angle indicator.

Hints for the Challenge.

- Half of 60° = 30°
  Half of 30° = 15°

  How could this be used to construct an angle of 75°?

2312 Number Challenge

Hints

- The fact that "one of the numbers is one more than a multiple of ten" combined with the knowledge that "two of the numbers are even" implies that all the numbers on this grid are possible solutions.

1	2	4	6	8	10
11	12	14	16	18	20
21	22	24	26	28	30
31	32	34	36	38	40
41	42	44	46	48	50
51	52	54	56	58	60
61	62	64	66	68	70
71	72	74	76	78	80
81	82	84	86	88	90
91	92	94	96	98	100

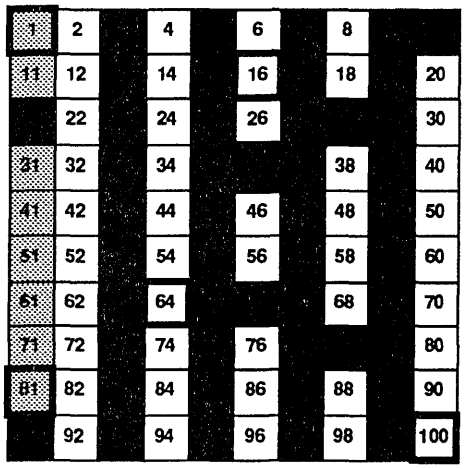
- One number is the square root of one of the numbers implies that the other number is a square number.

1	2	4	6	8	10
11	12	14	16	18	20
21	22	24	26	28	30
31	32	34	36	38	40
41	42	44	46	48	50
51	52	54	56	58	60
61	62	64	66	68	70
71	72	74	76	78	80
81	82	84	86	88	90
91	92	94	96	98	100

continued/

2312 Number Challenge (cont)

- None of the numbers are triangle numbers.



You need to check your solution so that all the conditions are satisfied.

2313 Turning the Cards

Here are some possible answers to each of the games. If your answers are very different, show them to your teacher.

Game 2

a) Predict that the next card is **higher**.
b) Because there are four higher but only two lower cards left.

Game 3

a) Predict that the next card is **lower**.
b) Because there are three lower but only two higher cards left.

Game 4

a) Predict that the next card is **higher**.
b) Because there is only the 6 card left, so it is **certain** to be higher.

Game 5

a) Cannot predict.
b) Because there are three **higher** and three lower cards left.

Game 6

a) Predict that the next card is **lower**.
b) Because all the cards are lower than 9.
You can be **certain** about this prediction.

2314 Describing Sequences

A	Description	Sequence					
2.	Add five	4,	9,	14,	**19,**	**24,**	**29**
3.	Subtract four	20,	16,	12,	**8,**	**4,**	**0**
4.	Add one more each time	2,	3,	5,	**8,**	**12,**	**17**
5.	Divide by two	16,	8,	4,	**2,**	**1,**	**$\frac{1}{2}$**
6.	Multiply by three	2,	6,	18,	**54,**	**162,**	**486**
7.	Subtract one less each time	50,	41,	33,	**26,**	**20,**	**15**

B	Sequence						Description
1.	5,	9,	13,	17,	**21,**	**25,**	Add four
2.	81,	27,	9,	3,	**1,**	**$\frac{1}{3}$**	Divide by 3
3.	3,	11,	18,	24,	**29,**	**33**	Add one less each time
4.	42,	36,	30,	24,	**18,**	**12**	Subtract 6
5.	3,	6,	12,	24,	**48,**	**96**	Multiply by 2
6.	40,	38,	35,	31,	**26,**	**20**	Subtract one more each time

---

2315 With a ruler

1. The line is **6cm** long.

2. a) The line is **3cm** long.
   b) The line is **7cm** long.
   c) The line is **4cm** long.

3. Show your lines to your teacher.

4. a) 6cm

   This line is twice as long. It is 12cm long. (6cm x 2 = 12cm)

   b) 2cm

   This line is twice as long. It is 4cm long. (2cm x 2 = 4cm)

   c) 4cm

   This line is twice as long. It is 8cm long. (4cm x 2 = 8cm)

continued/

2315 With a ruler (cont)

5. a) 6cm + 4cm = 10cm
   b) 3cm + 6cm = 9cm
   c) 7cm + 4cm = 11cm

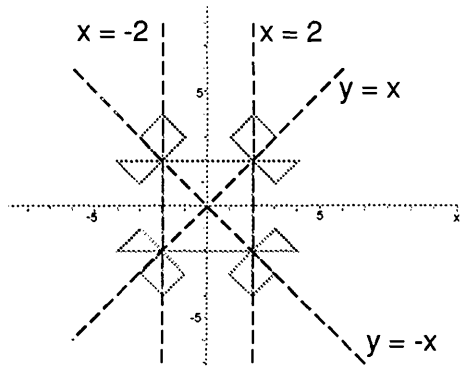
6. Show your zig-zag lines to your teacher.

2316 Reflecting Shapes

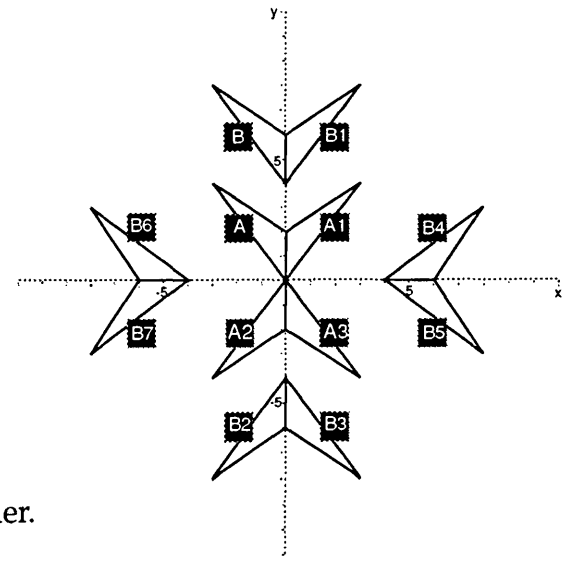
1. a) Help on using MicroSMILE program Transform to recreate the screen.
   - Select **Shape**
   - Select **Select a Shape**
   - Select **a flag**
   - Enter the starting co-ordinates by typing **2** and pressing enter and then typing **0** and pressing enter.
   - Select **Reflect**.
   - Select **Reflect** in **Line $y = 0$**.
   - Enter shape's label **A** and then press enter.

   b) From **A** to **A2**
   - Select **Line of reflection $y = x$**.

2. There are many ways to recreate the screen using reflections in the lines:
   $y = x$,
   $y = -x$,
   $x = 2$ and
   $x = -2$.



3. a) Here is one way to recreate the first pattern using Reflect only.
   There are several other ways.
   - Put shape in position at (0, 0)
     Label **A**
     Reflect in line $x = 0$ to give **A1**.
   - Put shape in position at (0, 4)
     Label **B**.
     Reflect in line $x = 0$ to give **B1**.
   - Reflect these 4 shapes in **$y = 0$**.
   - Try reflecting in $y = x$.



   b) Show your own pattern to your teacher.

2317 Reflecting Flags

1. a) Help on using MicroSMILE program Transform to recreate the screen.
   - Select **Shape**
   - Select **Select a Shape**
   - Select **a flag**
   - Enter the starting co-ordinates by typing **2** and press enter and then typing **0** and press enter.
   - Select **Reflect.**
   - Select **Line parallel to y axis.**
   - Enter the value for x, **-2** press enter.
   - Enter shape's label **A** press enter.

   b) From **A** to **A2**
   - Select **Reflect**
   - Select **Line parallel to x axis.**
   - Reflect the shape in the **line y = -1** press enter.
   - Enter shape's label **A** press enter.

2. To create each of the five screens, follow the first four steps in 1. a) to place the starting flag, labelled **A**.

   a) From **A** to **A1.**
   - Select **Line of reflection y = 0.**

   b) From **A** to **A1**
   - Select **Line of reflection y = -x.**

   c) There are several ways to create this screen. Here is one way.
   From **A** to **A1**
   - Select **Line of reflection x = 4.**

   From **A** to **A2**
   - Select **Line of reflection y = -x.**

   From **A1** to **A3**
   - Select **Line of reflection y = -x.**

   or

   From **A2** to **A3**
   - Select **Line of reflection y = -4**

   d) From **A** to **A1**
   - Select **Line of reflection y = x.**

   From **A** to **A2**
   - Select **Line of reflection y = -x.**

   From **A1** to **A3**
   - Select **Line of reflection y = -x.**

continued/

2317 Reflecting Flags (cont)

2. e) There are several ways to create this screen. Here is one way.

From **A** to **A1**
- Select **Line of reflection** $x = 2$.

From **A** to **A2**
- Select **Line of reflection** $y = 5$.

From **A2** to **A3**
- Select **Line of reflection** $x = 8$.

From **A3** to **A4**
- Select **Line of reflection** $y = 0$.

From **A4** to **A5**
- Select **Line of reflection** $y = 8$.

From **A** to **A6**
- Select **Line of reflection** $y = -x$

From **A6** to **A7**
- Select **Line of reflection** $y = -2$.

From **A6** to **A8**
- Select **Line of reflection** $y = -5$.

From **A8** to **A9**
- Select **Line of reflection** $y = -8$.

From **A9** to **A10**
- Select **Line of reflection** $y = 0$.

From **A10** to **A11**
- Select **Line of reflection** $y = 8$.

2318 A Mean Challenge!

1. 11

2. 16kg

3. 13 years

4. 10 pears

5. At least 90%

6. 85kg

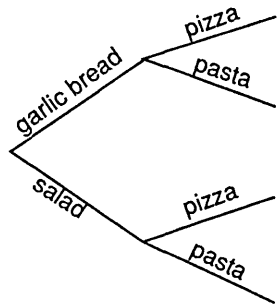
7. 8, 8, 3, 7, 9 *or* 10, 10, 1, 5, 9 *or* 10, 10, 3, 5, 7

2319 Pizza or Pasta

1.

	Starter	Main Course
1.	Garlic Bread	Pizza
2.	Garlic Bread	**Pasta**
3.	**Salad**	**Pizza**
4.	**Salad**	**Pasta**

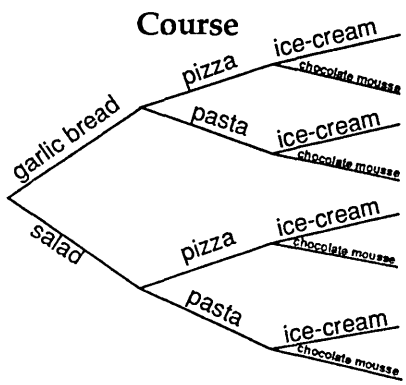
2.



Meal

- garlic bread, pizza
- garlic bread, pasta
- **salad, pizza**
- **salad, pasta**

3.

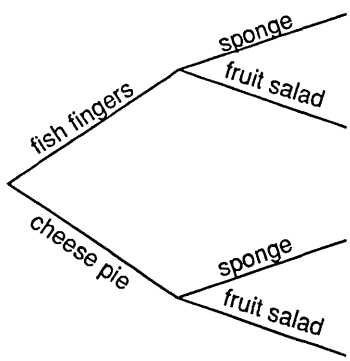


Meal

- garlic bread, pizza, ice-cream
- **garlic bread, pizza, chocolate mousse**
- **garlic bread, pasta, ice-cream**
- **garlic bread, pasta, chocolate mousse**
- **salad, pizza, ice-cream**
- **salad, pizza, chocolate mousse**
- **salad, pasta, ice-cream**
- **salad, pasta, chocolate mousse**

4. Jay can choose 8 different meals.

5. a)



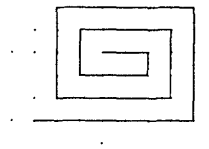
Meal

- fish fingers, sponge
- fish fingers, fruit salad
- cheese pie, sponge
- cheese pie, fruit salad

b) There are 4 different meals.

2320 Patterns in Spirals

1. c) 3, 2, 4, 3, 5, 4, 6, **5, 7, 6, ...**



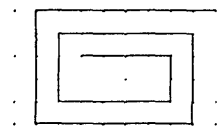
You can check your predictions by continuing the sequence.

2.

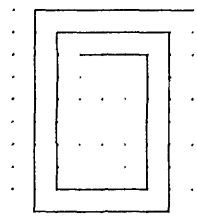
3.

Sequence (i)	matches with	Description	B or A
Sequence (ii)	matches with	Description	A or B
Sequence (iii)	matches with	Description	D
Sequence (iv)	matches with	Description	C

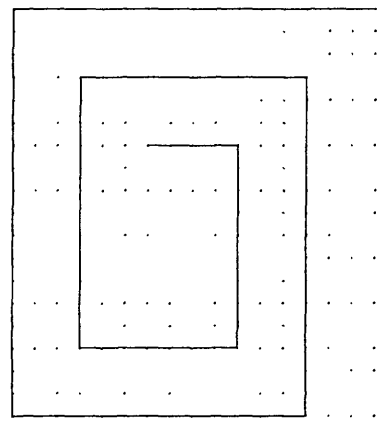
4. This spiral matches with Sequence (i) and Description B or A.



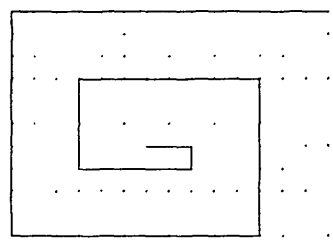
This spiral matches with Sequence (ii) and Description A or B.



This spiral matches with Sequence (iii) and Description D.



This spiral matches with Sequence (iv) and Description C.

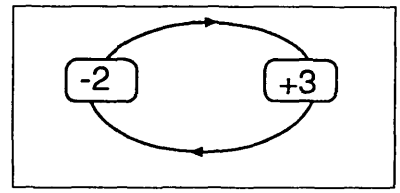


continued/

2320 Patterns in Spirals (cont)

5. The sequence is: 4, 2, 5, 3, 6, 4, . . .

The description is:



2321 The Algebra Game

Here is the start of a game.

Algebra Rule	d	Working	Move	Total Score
d + 2	4	d + 2 = 4 + 2 = 6	6	6
2d	3	2d = (2 x 3) = 6	6	12
2(d + 3)	1	2(d + 3) = 2(1 + 3) = 8	8	20
d – 5	2	d – 5 = 2 – 5 = -3	-3	17
5 + 3d	2	5 + 3d = 5 + (3 x 2) = 11	11	28
3(d – 6)	4	3(d – 6) = 3 x (4 – 6) = -6	-6	22
4(d + 2)	6	4(d + 2) = 4 x (6 + 2) = 32	32	54

Make sure you show all your working. Get the rest of your group to check your answers.

2322 The Algebra Game 2

These are the possible ways in which the game could have started.

If you had thrown	Place you should have landed.
1	2(d – 3)
2	-(-d)
3	-2 + d
4	-d + 7
5	(d – 4)(d + 1)
6	-(d – 2)

Make sure you show all your working. Get the rest of your group to check your answers.

2323 Statistical Investigations Help Book

No answers required.

2324 Reckonings

No answers required.

2325 Grouped Data, Reviewed

Mean from grouped data
Year 8 grouped in class intervals of 10.

1. a)

Number of SMILE activities completed	Mid-Value	Frequency	Mid-value x Frequency
41 - 50	45.5	6	273
51 - 60	55.5	24	1332
61 - 70	65.5	18	**1179**
71 - 80	**75.5**	14	**1057**
81 - 90	**85.5**	11	**940.5**
91 - 100	**95.5**	29	**2769.5**
101 - 110	**105.5**	27	**2848.5**
111 - 120	**115.5**	27	**3118.5**
121 - 130	**125.5**	18	**2259**
131 - 140	**135.5**	5	**677.5**
Total		179	**16454.5**

b) $\frac{16454.5}{179} = 91.92 =$ 92 to the nearest whole activity.

An estimate for the mean number of SMILE activities completed is 92.

Year 8 grouped in class intervals of 20.

c)

Number of SMILE activities completed	Mid-Value	Frequency	Mid-value x Frequency
41 - 60	50.5	30	1515
61 - 80	70.5	32	**2256**
81 - 100	**90.5**	40	**3620**
101 - 120	**110.5**	54	**5967**
121 - 140	**130.5**	23	**3001.5**
Total		179	**16359.5**

continued/

1. d) $\frac{16359.5}{179} = 91.39 = 91$

An estimate for the mean number of activities completed is 91.

e) The estimate of the mean from the data grouped in class intervals of 10 is different to the estimate of the mean from the data grouped in class intervals of 20, but not very different.

One reason for this is that the class intervals do not contain the same frequency. Hence the Mid-value x frequency total will be different.

Modal Group from grouped data
Year 8 grouped in class intervals of 20.

2.

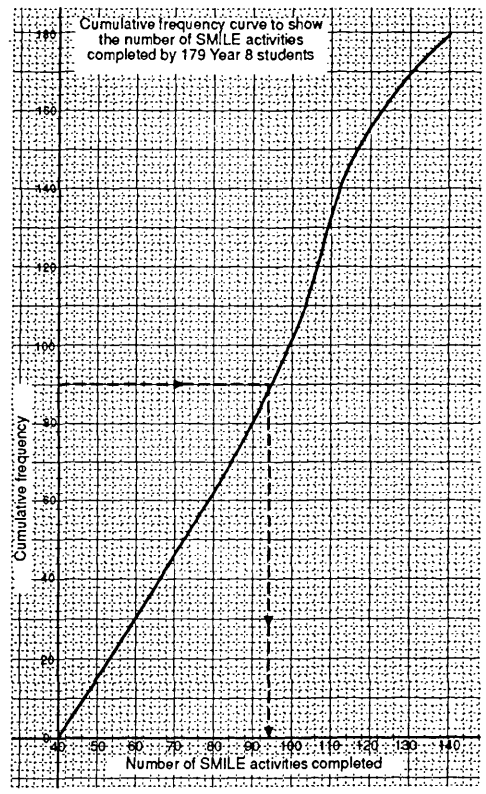
Number of SMILE	Frequency
41 - 60	30
61 - 80	32
81 - 100	40
101 - 120	54
121 - 140	23
Total	179

The modal group is 101 - 120.

Median from grouped data
Year 8 grouped in class intervals of 20.

3. a)

Number of SMILE activities completed	Frequency	Cumulative Frequency
41 - 60	30	30
61 - 80	32	62
81 - 100	40	102
101 - 120	54	156
121 - 140	23	179
Total		179



b) The estimated median number of SMILE activities is not the same when the data is grouped in 20's.

The estimated median is 94.

continued/

Year 9

4. a)

Number of SMILE activities completed	Mid-Value	Frequency	Mid-value x Frequency
51 - 60	55.5	5	277.5
61 - 70	65.5	12	786
71 - 80	75.5	21	1585.5
81 - 90	85.5	28	2394
91 - 100	95.5	32	3056
101 - 110	105.5	40	4220
111 - 120	115.5	25	2887.5
121 - 130	125.5	17	2133.5
131 - 140	135.5	4	542
141 - 150	145.5	1	145.5
Total		185	18027.5

$\frac{18027.5}{185} = 97.45 = 97$

The estimate for the mean number of SMILE activities is 97.

b)

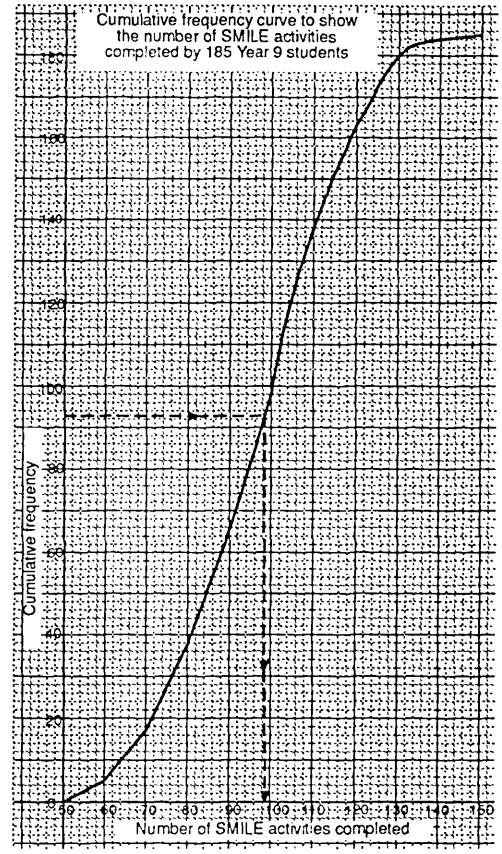
Number of SMILE activities completed	Frequency
51 - 60	5
61 - 70	12
71 - 80	21
81 - 90	28
91 - 100	32
101 - 110	40
111 - 120	25
121 - 130	17
131 - 140	4
141 - 150	1
Total	185

The modal group is 101 - 110.

continued/

4. c)

Number of SMILE activities completed	Frequency	Cumulative Frequency
51 - 60	5	5
61 - 70	12	17
71 - 80	21	38
81 - 90	28	66
91 - 100	32	98
101 - 110	40	138
111 - 120	25	163
121 - 130	17	180
131 - 140	4	184
141 - 150	1	185
Total		185



The estimated mean number of SMILE activities is 98.

5. Comparing data grouped in 10's

	Estimated mean	Estimated modal group	Estimated median
Year 8	92	91 - 100	98
Year 9	97	101 - 110	98

Although the estimated median is the same for both years, Year 9 has a higher estimated mean and modal group. Year 9 has worked harder than Year 8.

2326 Hanoi

Once you have solved the original puzzle in the minimum number of moves, go on to set your own puzzle. Using **Option/List of moves** will help you develop a strategy. Using **Option/Table of results** will help you make a generalisation.

2327 Hats

Once you have solved the original puzzle in the minimum number of moves, go on to set your own puzzle. Using **Option/List of moves** will help you develop a strategy. Using **Option/Table of results** will help you make a generalisation.

2328 Quadratic Rules

1. i) Pretty's rule $g = b(b-2) + b$
   Molly's rule $g = b(b-1)$
   ii) Jo's rule $g = b^2 - b$
   Pretty's rule $g = b(b-2) + b$
   $g = b^2 - 2b + b$
   $g = b^2 - b$
   Molly's rule $g = b(b-1)$
   $g = b^2 - b$

2. i) Jo's rule $m = 2(s^2 + s)$
   Pretty's rule $m = 2s^2 + 2s$
   ii) Molly's rule The number of matches equals the side of the square plus 1, all multiplied by 2 times the side of the square.
   iii) Jo's rule $m = 2(s^2 + s)$
   $m = 2s^2 + 2s$
   Pretty's rule $m = 2s^2 + 2s$
   Molly's rule $m = 2s(s + 1)$
   $m = 2s^2 + 2s$

3. i) Jo's rule The number of dots equals the side of the square add one, all squared, then minus 4.
   Molly's rule The number of dots is equal to the side of the square plus 2, multiplied by the side of the square then minus 3.
   ii) Jo's rule $d = (s + 1)^2 - 4$
   Molly's rule $d = s(s + 2) - 3$
   iii) Pretty's rule The number of dots is equal to the side of the square squared plus 2 times the side of the square then minus 3.
   iv) Jo's rule $d = (s + 1)^2 - 4$
   $d = s^2 + 2s + 1 - 4$
   $d = s^2 + 2s - 3$
   Molly's rule $d = s(s + 2) - 3$
   $d = s^2 + 2s - 3$
   Pretty's rule $d = s^2 + 2s - 3$

4. i) Molly's rule The number of black squares equals the side length squared minus 4 times the side length minus two.
   Pretty's rule The number of black squares equals the side length squared, minus 4 times the side length, then add 8.
   ii) Molly's rule $b = s^2 - 4(s - 2)$
   Pretty's rule $b = s^2 - 4s + 8$
   iii) Jo's rule The number of black squares equals the side length minus 2 all squared, then add 4.
   iv) Molly's rule $b = s^2 - 4(s - 2)$
   $b = s^2 - 4s + 8$
   Pretty's rule $b = s^2 - 4s + 8$
   Jo's rule $b = (s - 2)^2 + 4$
   $b = s^2 - 4s + 4 + 4$
   $b = s^2 - 4s + 8$

2329 The Median

1. The median height is 156cm.

2. The median height is 157cm.

3. Many possible answers. (The median length is the length of the pen in the middle.)

4. 15°C, 15°C, **17°C,** 18°C, 19°C
The median temperature is 17°C.

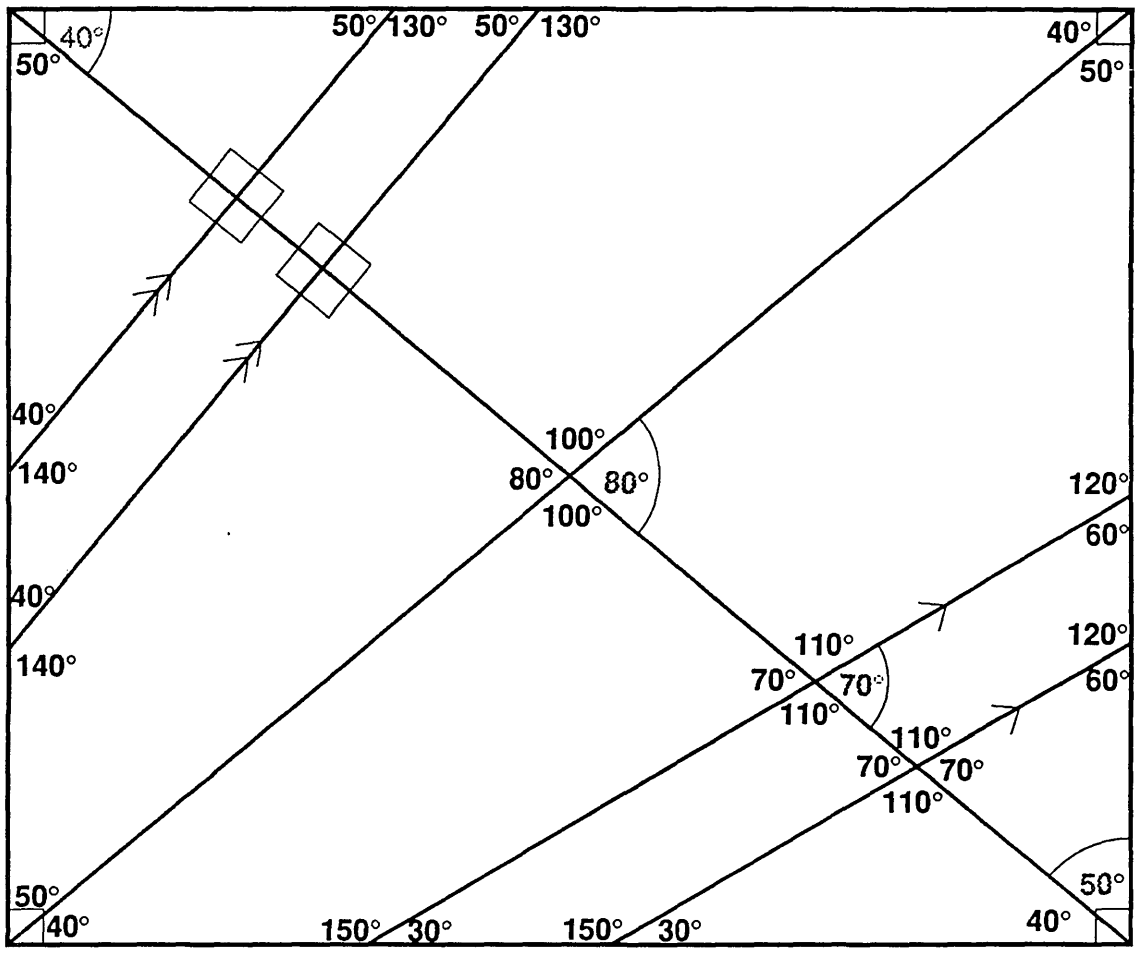
5. 12°C, 15°C, 15°C, **17°C,** 18°C, 19°C, 25°C
The median temperature is still 17°C.
Here is one explanation. If you have a different one, show it to your teacher.

The median has not changed because one temperature was higher than 17°C and one temperature was lower than 17°C, so 17°C is still the middle temperature.

6. Show your results to your teacher.

2330 Missing Angles

1.



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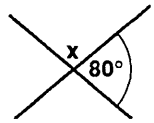
2330 Missing Angles (cont)

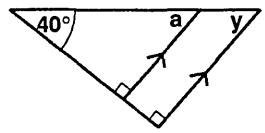
2. Here are some possible solutions:

- Angle **x**



x + 80° = 180° (Angles on a straight line.)
x = 100°

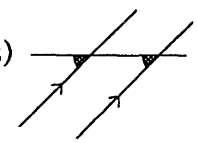
- Angle **y**



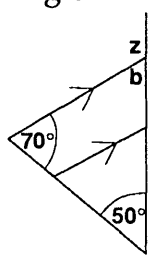
First find the angle marked **a**.

40° + 90° + **a** = 180° (Angles in a triangle.)
a = 50°

y = **a** (Corresponding angles)
y = 50°



- Angle **z**



First find the angle marked **b**.

70° + 50° + **b** = 180° (Angles in a triangle.)
b = 60°

z + 60° = 180° (Angles on a straight line.)
z = 120°

If you have used a different method to find angle **x**, angle **y** or angle **z**, show your teacher.

2331 Half-time Scores

- There are 12 possible half-time score when the final score is 3 - 2.

0 - 0	1 - 0	2 - 0	3 - 0
0 - 1	1 - 1	2 - 1	3 - 1
0 - 2	1 - 2	2 - 2	3 - 2

- Here is one way to show your results.

Goals scored by 2nd team

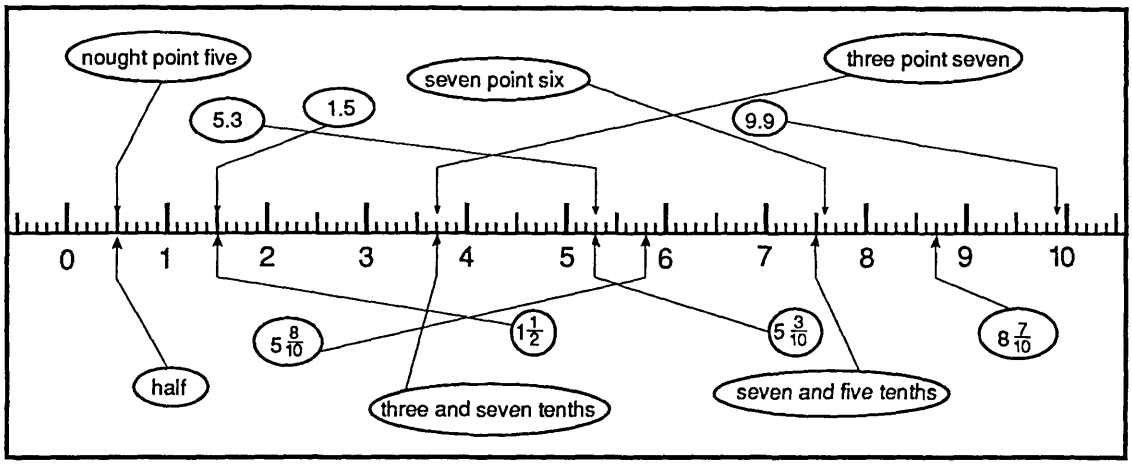
Goals scored by 1st team

	0	1	2	3	4
0	1				
1					
2				12	
3			12		
4					

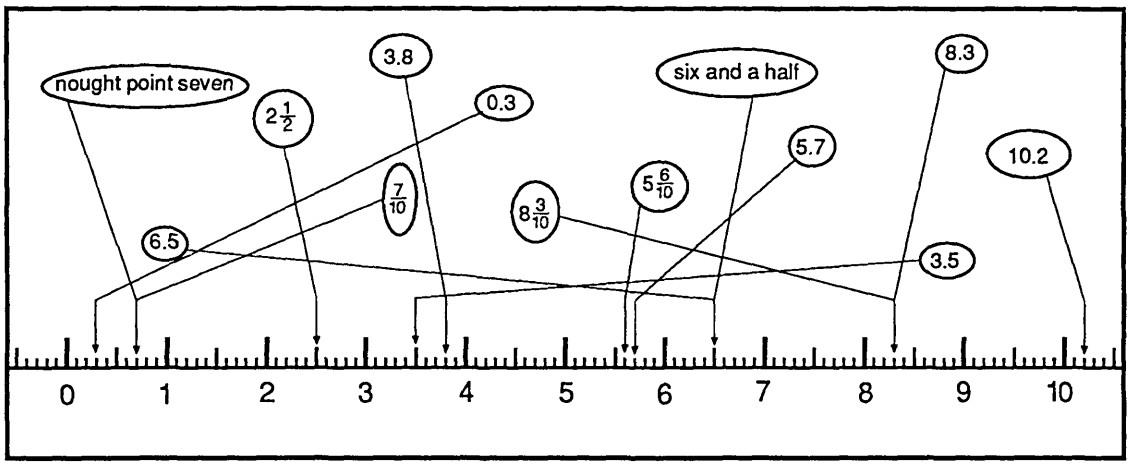
- Can you predict how many possible half-time scores there would be for a full time score of 14 - 12, **without writing them all down?**

2332 Decimals on a number line

1.



2.



2333 Quiz Times

Moses		David		Aisha		Jay	
5 x 4 = 20	✔	5 x 6 = 30	✔	7 x 4 = 25	✘	5 x 5 = 25	✔
5 x 3 = 15	✔	2 x 8 = 16	✔	4 x 3 = 12	✔	3 x 7 = 21	✔
7 x 5 = 30	✘	4 x 8 = 32	✔	8 x 5 = 40	✔	6 x 9 = 54	✔
2 x 8 = 16	✔	9 x 3 = 27	✔	6 x 2 = 12	✔	4 x 12 = 48	✔
4 x 7 = 22	✘	4 x 4 = 16	✔	9 x 4 = 36	✔	8 x 3 = 24	✔
5 x 9 = 44	✘	6 x 7 = 42	✔	5 x 7 = 35	✔	10 x 7 = 70	✔
10 x 6 = 59	✘	9 x 4 = 36	✔	6 x 11 = 63	✘	8 x 8 = 64	✔
3 x 6 = 18	✔	7 x 3 = 21	✔	7 x 8 = 56	✔	2 x 11 = 22	✔
6 x 6 = 36	✔	9 x 9 = 64	✘	3 x 6 = 21	✘	4 x 9 = 36	✔
4 x 12 = 46	✘	11 x 4 = 44	✔	6 x 0 = 6	✘	0 x 11 = 0	✔

continued/

2333 Quiz Times (cont)

Moses's mark was 5/10.

Moses's corrections
7 x 5 = 35
4 x 7 = 28
5 x 9 = 45
10 x 6 = 60
4 x 12 = 48

David's mark was 9/10.

David's corrections: 9 x 9 = 81

Aisha's mark was 6/10.

Aisha's corrections:
7 x 4 = 28
6 x 11 = 66
3 x 6 = 18
6 x 0 = 0

Jay's mark was 10/10.

2334 Beat the code

1.

Musical symbol	Name of note	Number of beats
	Brieve	8
	Semi-brieve	4
	Minim	2
	Crochet	1
	Quaver	$\frac{1}{2}$
	Semi-quaver	$\frac{1}{4}$
	Demi-semi-quaver	$\frac{1}{8}$

2. a) 2 =  = 2 beats

b)  +  +  =  = 4 beats

c)  +  =  = 1 beat

d) + =  = 4 beats

e) + 2 =  = 1 beat

f) + 4 =  = 1 beat

continued/

2334 Beat the code (cont)

2. g) 2♪ + 3♩ = [𝅝] = [4] beats

h) 4𝅘𝅥𝅯 + 2♪ = [𝅗𝅥] = [2] beats

i) 𝅝 + 𝅗𝅥 + ♩ + 2♪ + 4𝅘𝅥𝅯 = [𝅜] = [8] beats

3. a) ♩ + [♩] = 𝅗𝅥 = 2 beats

b) [𝅗𝅥] + 4♪ = 𝅝 = 4 beats

c) 2𝅘𝅥𝅰 + [𝅘𝅥𝅯] = ♪ = ½ beat

d) 𝅝 + 4 [♩] = 𝅜 = 8 beats

e) 𝅗𝅥 + 2 [♩] = 𝅝 = 4 beats

4. a) 𝅗𝅥. + ♩ = [𝅝] = [4] beats

b) 𝅗𝅥. + 2♪ = [𝅝] = [4] beats

c) ♪. + 𝅘𝅥𝅯 = [♩] = [1] beat

d) ♪. + ♪. = [♩.] = [1½] beats

e) 𝅗𝅥. + 4♪. = [𝅝.] = [6] beats

2335 Using Decimals

1. Length of safetypin = **5.3**cm

2. Length of banana = **8.2**cm

3. Length of scissors = **6.2**cm

4. Length of leaf = **3.7**cm

5. 7.2cm
6. 3.6cm
7. 5.9cm
8. 8.3cm

Your answers may vary according to how accurately you have measured. If you are unsure, check with you teacher.

continued/

2335 Using Decimals (cont)

9. 1.2kg

10. 0.6kg

11. 4.8kg

2336 Comparing ratio

1. 3 : 2 is 15 : 10 expressed in its simplest form.
   The largest number that you can divide 15 and 10 by is 5.

$$\begin{array}{c} 15 : 10 \\ 15 \div \mathbf{5} : \ 10 \div \mathbf{5} \\ 3 \ : \ 2 \end{array}$$

2. David's class.

My name is David. There are **27** pupils in my class.	There are **12** girls and **15** boys in my class.	The ratio of girls to boys is **12 : 15**	The ratio of girls to boys in my class in its simplest form is **4 : 5**	There are **4** girls to every **5** boys.

Ali's class

My name is Ali. There are **30** pupils in my class.	There are **10** girls and **20** boys in my class.	The ratio of girls to boys is **10 : 20**	The ratio of girls to boys in my class in its simplest form is **1 : 2**	There is **1** girl to every **2** boys.

Buki's class.

My name is Buki. There are **25** pupils in my class.	There are **10** girls and **15** boys in my class.	The ratio of girls to boys is **10 : 15**	The ratio of girls to boys in my class in its simplest form is **2 : 3**	There are **2** girls to every **3** boys.

Stella's class.

My name is Stella. There are **28** pupils in my class.	There are **20** girls and **8** boys in my class.	The ratio of girls to boys is **20 : 8**	The ratio of girls to boys in my class in its simplest form is **5 : 2**	There are **5** girls to every **2** boys.

Jo's class.

My name is Jo. There are **21** pupils in my class.	There are **14** girls and **7** boys in my class.	The ratio of girls to boys is **14 : 7**	The ratio of girls to boys in my class in its simplest form is **2 : 1**	There are **2** girls to every boy.

continued/

2336 Comparing ratio (cont)

3. a) 16 students, 6 girls and 10 boys because:

$$6 : 10$$
$$6 \div 2 : 10 \div 2$$
$$3 : 5$$

b) 24 students, 9 girls and 15 boys or
32 students, 12 girls and 20 boys or
40 students, ...

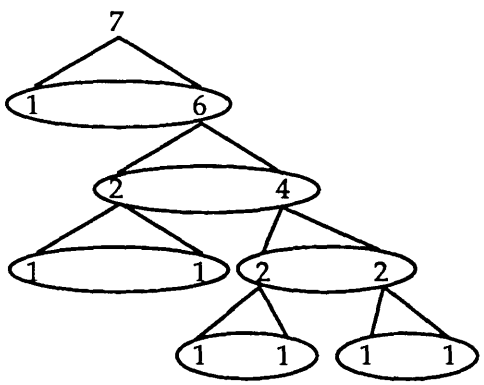
4. a) 10 : 25 written in its simplest form is 2 : 5

b)

	No. of people	Eggs	Butter	Tomatoes	Mushrooms	Cheese
Jasmine	2	4	10g	**2**	**6**	**50g**
Jason	**5**	**10**	25g	5	15	125g

Jason's omelette will feed 5 people.

2337 Splitting numbers



1 x 6 = 6

2 x 4 = 8

1 x 1 = 1
2 x 2 = 4

1 x 1 = 1
1 x 1 = 1

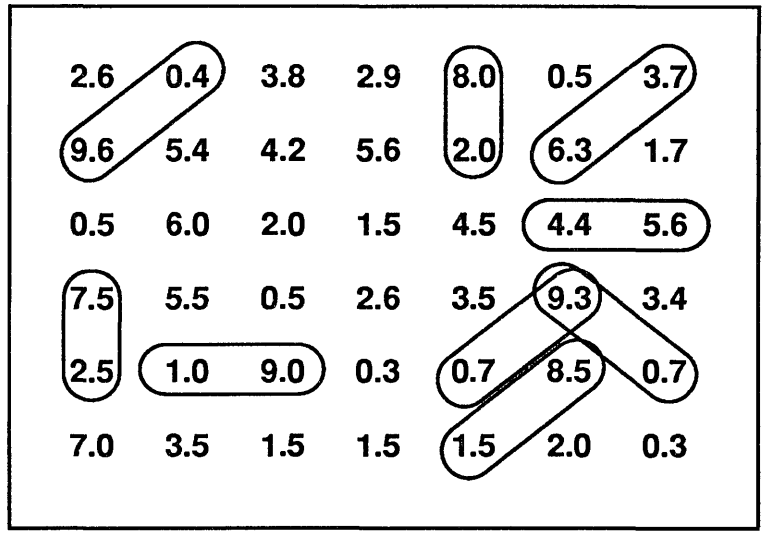
Starting number:	7
Splits:	6
Products:	6, 8, 1, 4, 1, 1.
Sum:	21

When you start with the number 7 you should notice that the sum is always 21.
Is the number of splits always equal to 6?

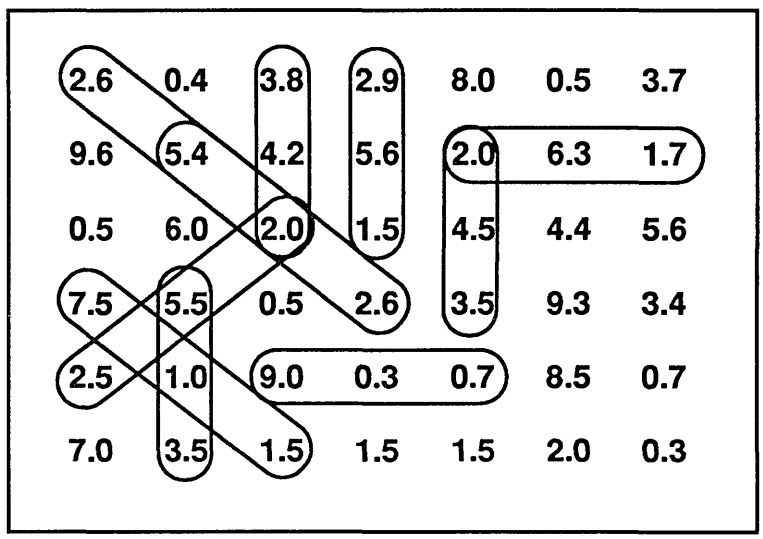
- There are rules connecting the starting number and the number of splits and the sum. Can you find the rules?
- Can you justify your results?

2338 Decimal Search

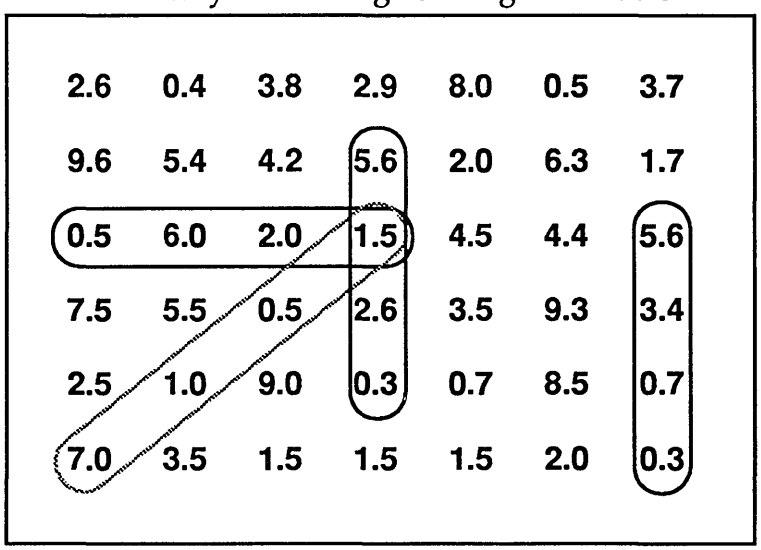
1. **2 numbers.** There are 9 ways of making 10 using 2 numbers.



3 numbers. There are 10 ways of making 10 using 3 numbers.

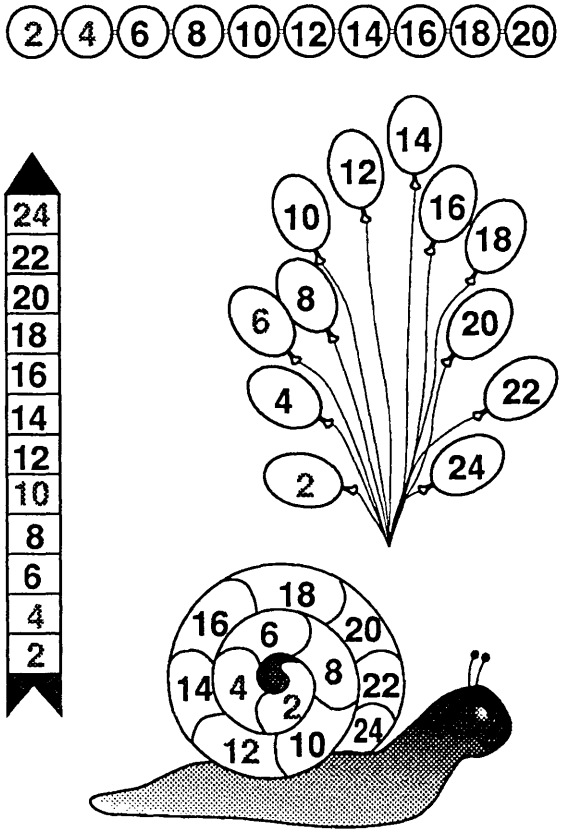


4 numbers. There are 4 ways of making 10 using 4 numbers.

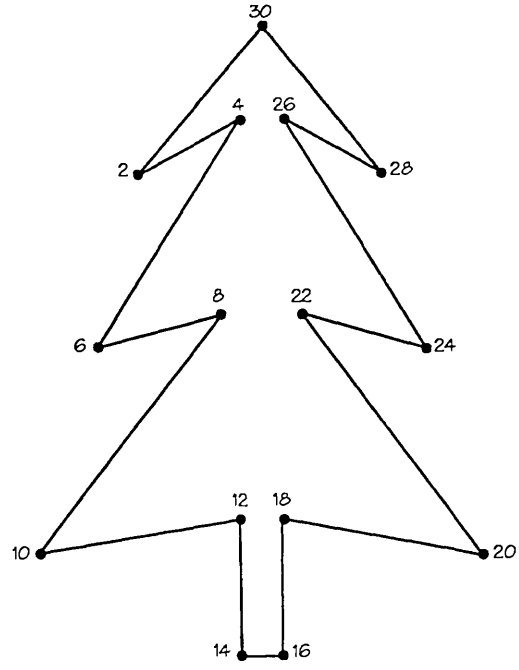


There is 1 way of making 10 using 5 numbers. Did you find it?

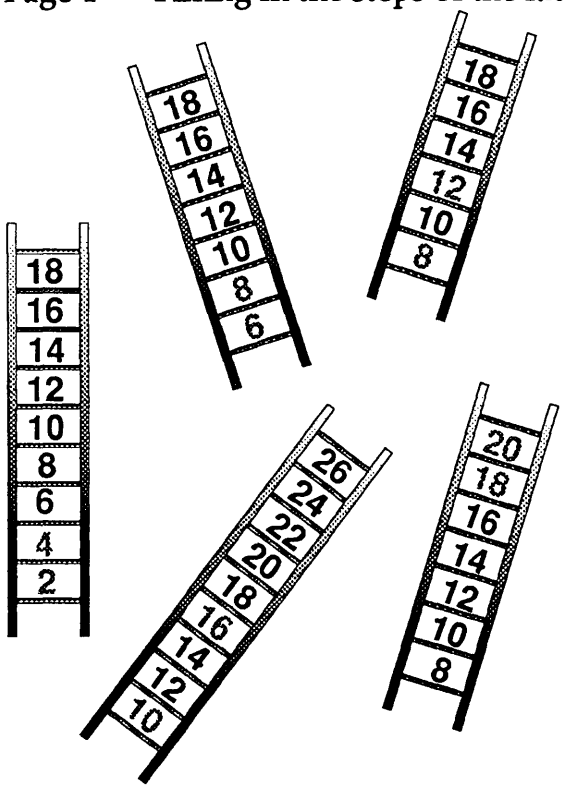
Page 2 **Jumping in 2's pattern**



Page 3 **Joining multiples of 2.**



Page 4 **Filling in the steps of the ladder.**



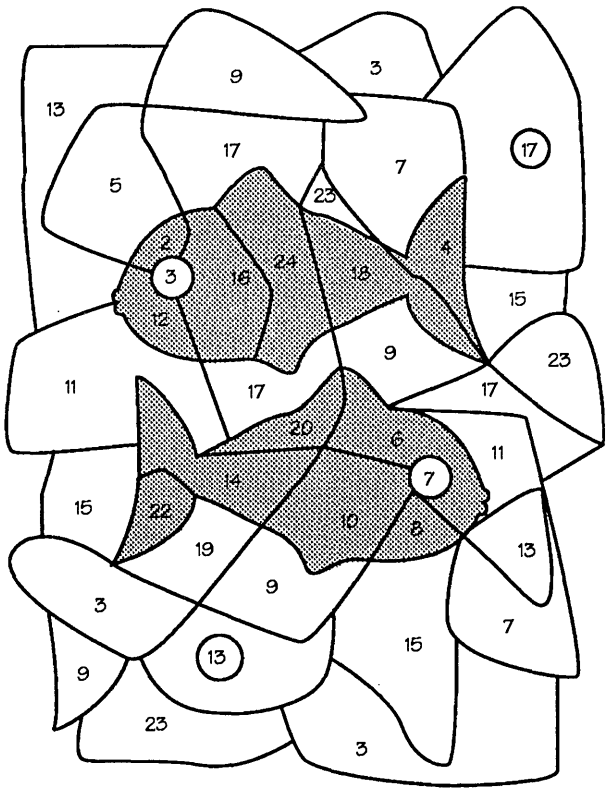
Page 5 **The 2 times table**

2 x 1 = 2	2 x 7 = 14
2 x 2 = 4	2 x 8 = 16
2 x 3 = 6	2 x 9 = 18
2 x 4 = 8	2 x 10 = 20
2 x 5 = 10	2 x 11 = 22
2 x 6 = 12	2 x 12 = 24

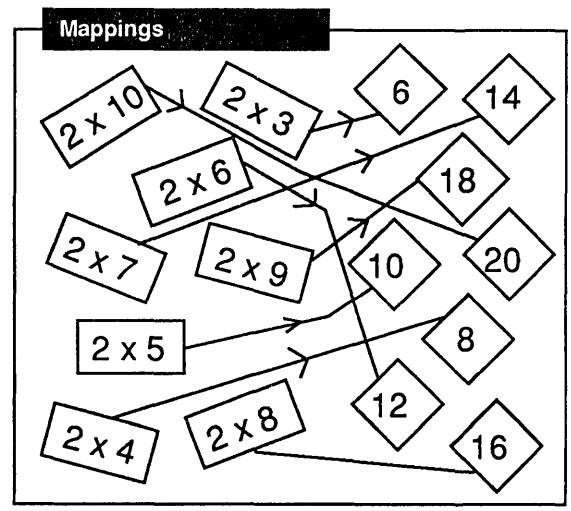
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

continued/

Page 6 **Shading multiples of 2.**



Page 7 **Mappings**



Mark the test paper

1.	2 x 6 = 12 ✔	6.	2 x 8 = 20 ✘
2.	2 x 7 = 16 ✘	7.	2 x 4 = 8 ✔
3.	2 x 5 = 10 ✔	8.	2 x 9 = 18 ✔
4.	2 x 3 = 6 ✔	9.	2 x 2 = 6 ✘
5.	2 x 10 = 16 ✘	10.	2 x 11 = 22 ✔

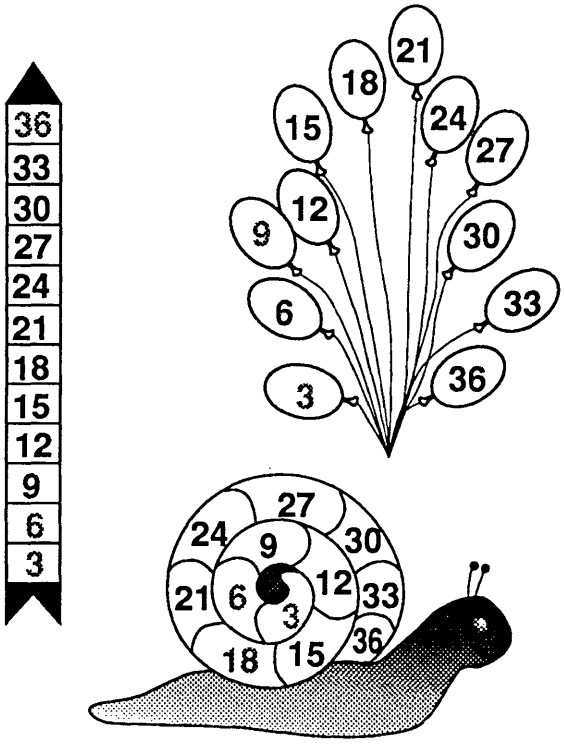
Page 8 **The test.**

Here are the answers to the test, but you should make sure that you know your 2 times table **before** you ask your teacher to test you on it.

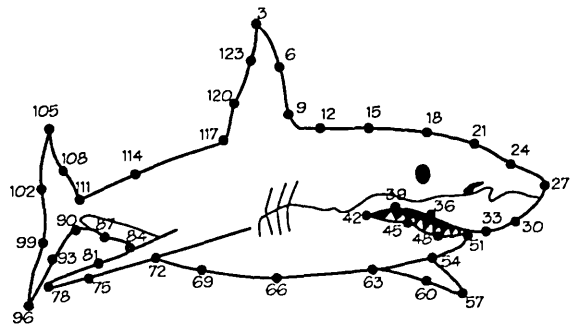
2 x 9 = **18**	2 x 4 = **8**
2 x 6 = **12**	2 x 1 = **2**
2 x 2 = **4**	2 x 7 = **14**
2 x 3 = **6**	2 x 10 = **20**
2 x 5 = **10**	2 x 8 = **16**

Page 2 **Jumping in 3's pattern**

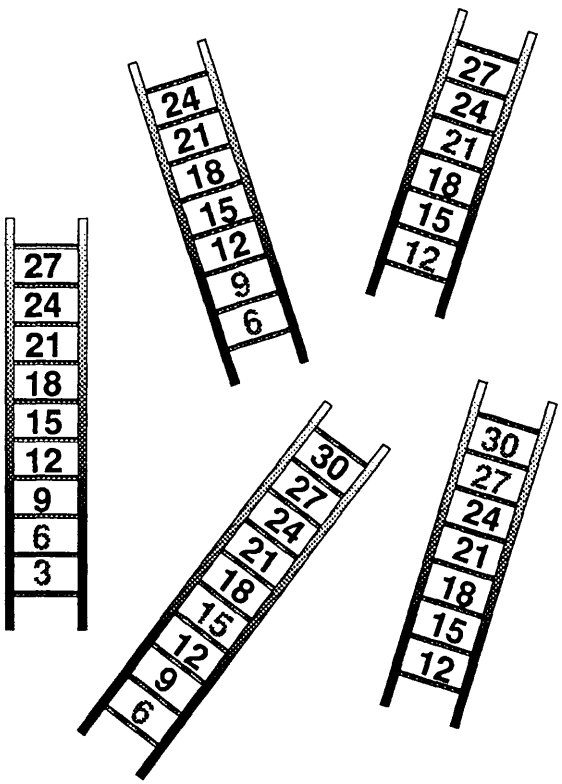
3 6 9 12 15 18 21 24 27 30



Page 3 **Joining multiples of 3.**



Page 4 **Filling in the steps of the ladder.**



Page 5 **The 3 times table**

3 x 1 = 3	3 x 7 = 21
3 x 2 = 6	3 x 8 = 24
3 x 3 = 9	3 x 9 = 27
3 x 4 = 12	3 x 10 = 30
3 x 5 = 15	3 x 11 = 33
3 x 6 = 18	3 x 12 = 36

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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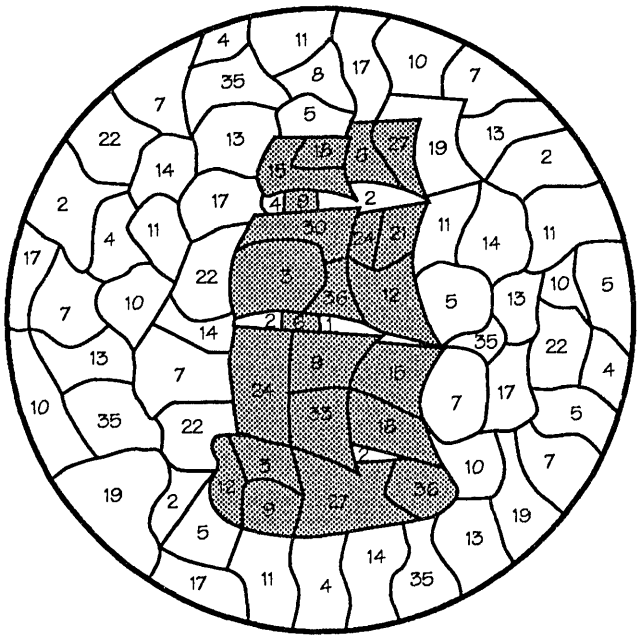
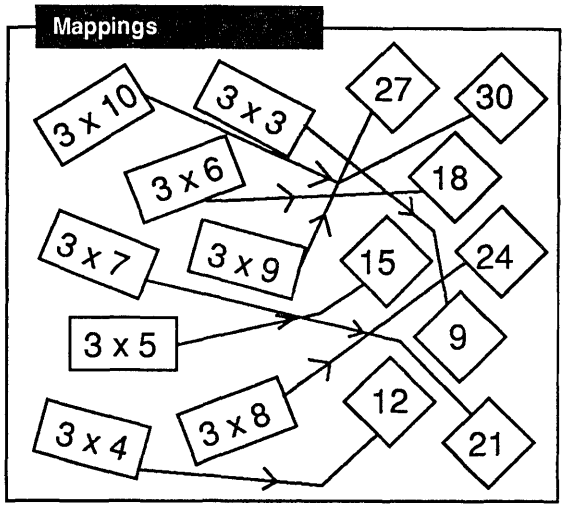
Page 6 **Shading multiples of 3.**



Page 7 **Mappings**



Mark the test paper

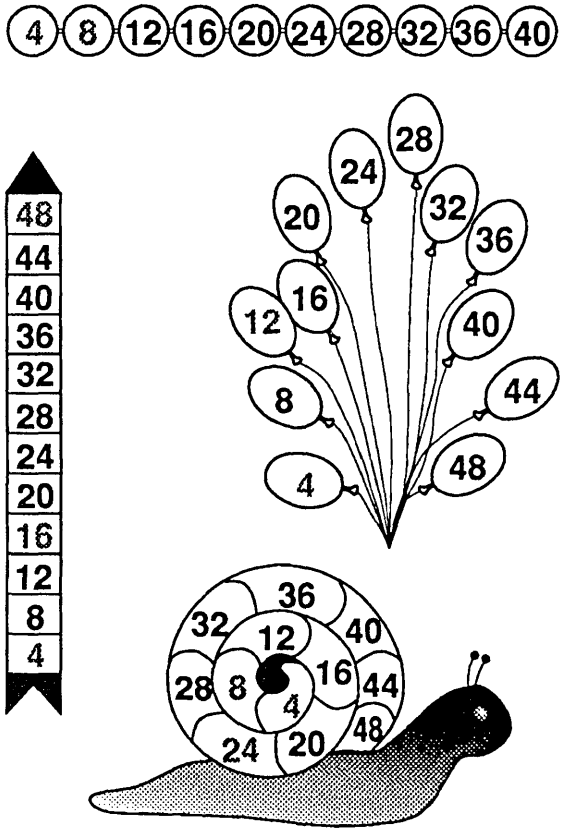
1.	3 x 6 = 18 ✔	6.	3 x 8 = 18 ✘
2.	3 x 7 = 23 ✘	7.	3 x 4 = 12 ✔
3.	3 x 5 = 15 ✔	8.	3 x 9 = 27 ✔
4.	3 x 3 = 6 ✘	9.	3 x 2 = 6 ✔
5.	3 x 10 = 30 ✔	10.	3 x 12 = 36 ✔

Page 8 **The test.**

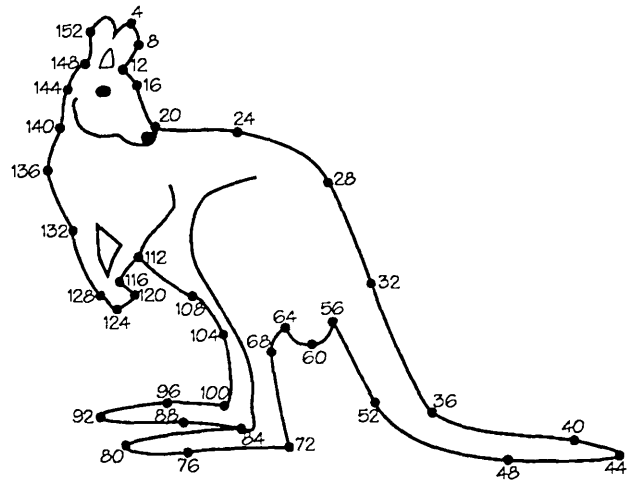
Here are the answers to the test, but you should make sure that you know your 3 times table **before** you ask your teacher to test you on it.

3 x 9 = **27**	3 x 4 = **12**
3 x 6 = **18**	3 x 1 = **3**
3 x 2 = **6**	3 x 7 = **21**
3 x 3 = **9**	3 x 10 = **30**
3 x 5 = **15**	3 x 8 = **24**

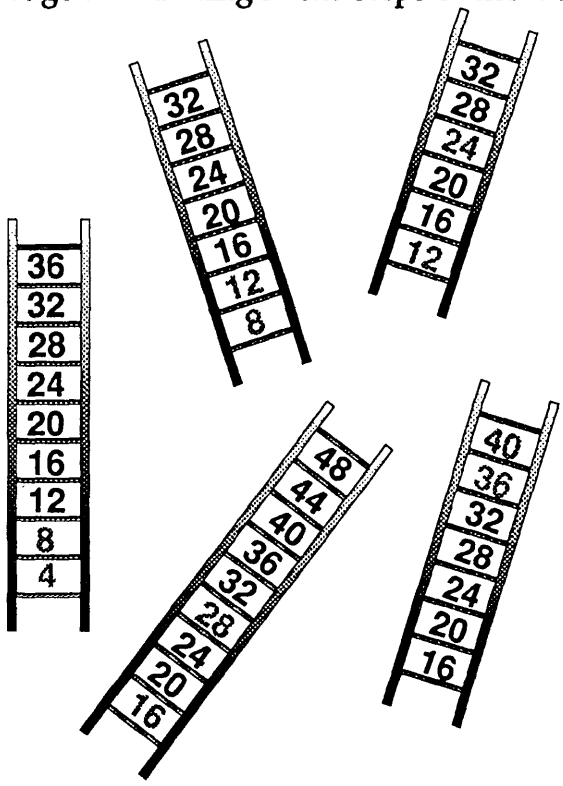
Page 2 **Jumping in 4's pattern**



Page 3 **Joining multiples of 4.**



Page 4 **Filling in the steps of the ladder.**



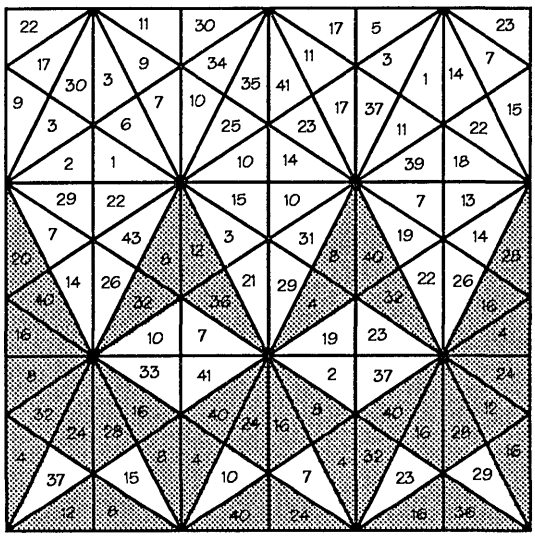
Page 5 **The 4 times table**

4 x 1 = 4	4 x 7 = 28
4 x 2 = 8	4 x 8 = 32
4 x 3 = 12	4 x 9 = 36
4 x 4 = 16	4 x 10 = 40
4 x 5 = 20	4 x 11 = 44
4 x 6 = 24	4 x 12 = 48

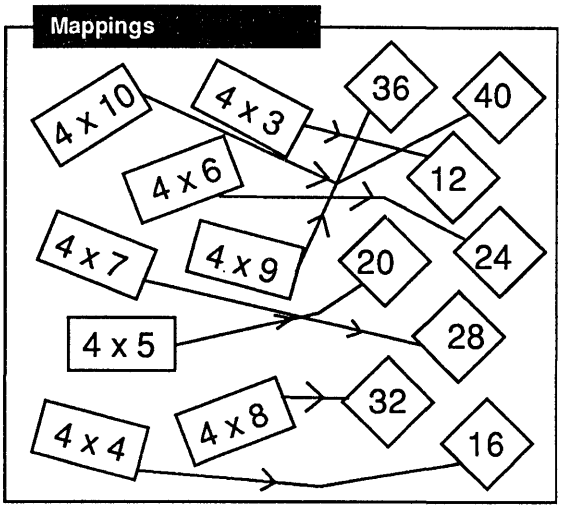
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

continued/

Page 6 Shading multiples of 4.



Page 7 Mappings



Mark the test paper

1.	4 x 6 = 24 ✔	6.	4 x 8 = 32 ✔
2.	4 x 7 = 26 ✘	7.	4 x 4 = 16 ✔
3.	4 x 5 = 20 ✔	8.	4 x 9 = 36 ✔
4.	4 x 3 = 12 ✔	9.	4 x 2 = 8 ✔
5.	4 x 10 = 40 ✔	10.	4 x 1 = 4 ✔

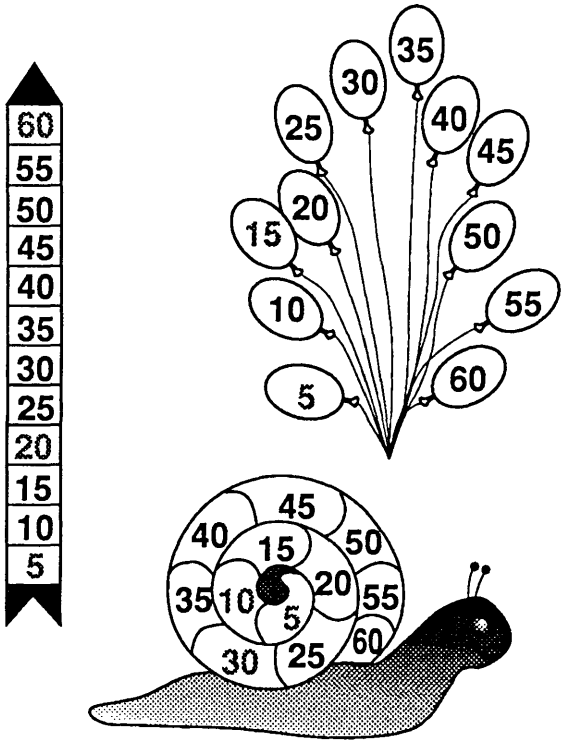
Page 8 The test.

Here are the answers to the test, but you should make sure that you know your 4 times table **before** you ask your teacher to test you on it.

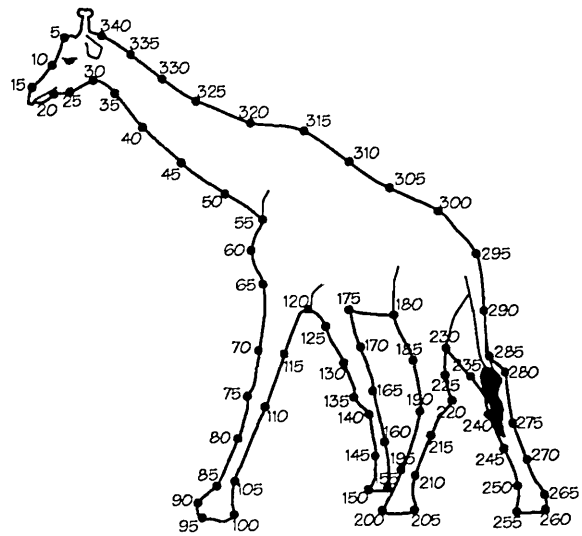
4 x 9 = **36**	4 x 4 = **16**
4 x 6 = **24**	4 x 1 = **4**
4 x 2 = **8**	4 x 7 = **28**
4 x 3 = **12**	4 x 10 = **40**
4 x 5 = **20**	4 x 8 = **32**

Page 2 **Jumping in 5's pattern**

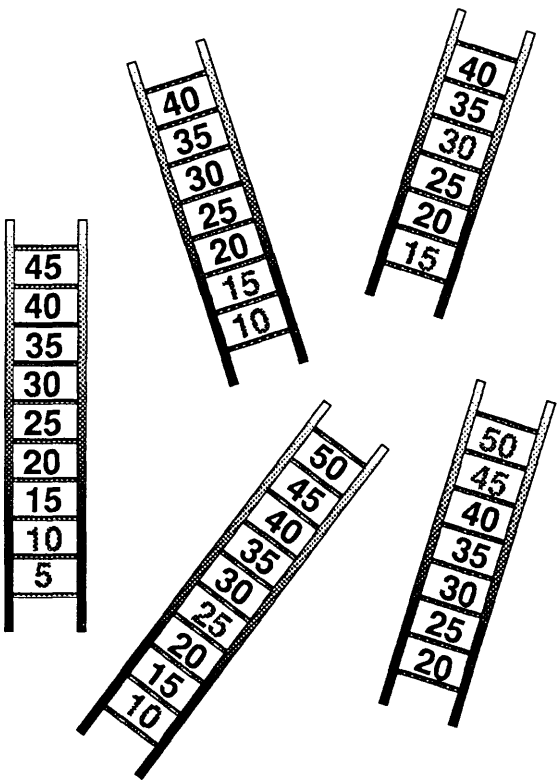
5 10 15 20 25 30 35 40 45 50



Page 3 **Joining multiples of 5.**



Page 4 **Filling in the steps of the ladder.**



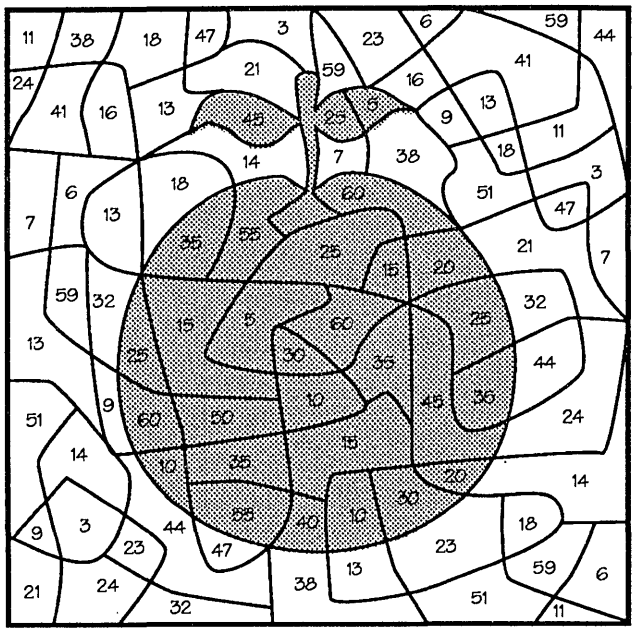
Page 5 **The 5 times table**

5 x 1 = 5	5 x 7 = **35**
5 x 2 = 10	5 x 8 = **40**
5 x 3 = **15**	5 x 9 = **45**
5 x 4 = **20**	5 x 10 = **50**
5 x 5 = **25**	5 x 11 = **55**
5 x 6 = **30**	5 x 12 = **60**

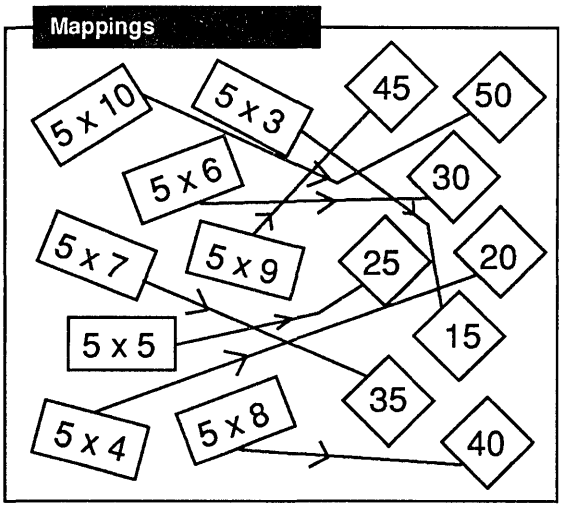
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

continued/

Page 6 **Shading multiples of 5.**



Page 7 **Mappings**



Mark the test paper

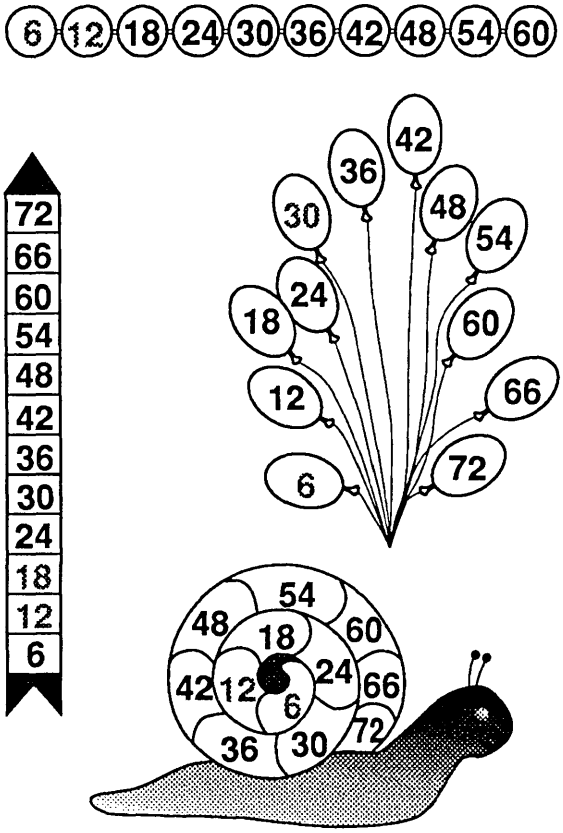
1.	5 x 6 = 30 ✔	6.	5 x 8 = 45 ✘
2.	5 x 7 = 35 ✔	7.	5 x 4 = 20 ✔
3.	5 x 5 = 25 ✔	8.	5 x 9 = 40 ✘
4.	5 x 3 = 15 ✔	9.	5 x 2 = 10 ✔
5.	5 x 10 = 50 ✔	10.	5 x 11 = 55 ✔

Page 8 **The test.**

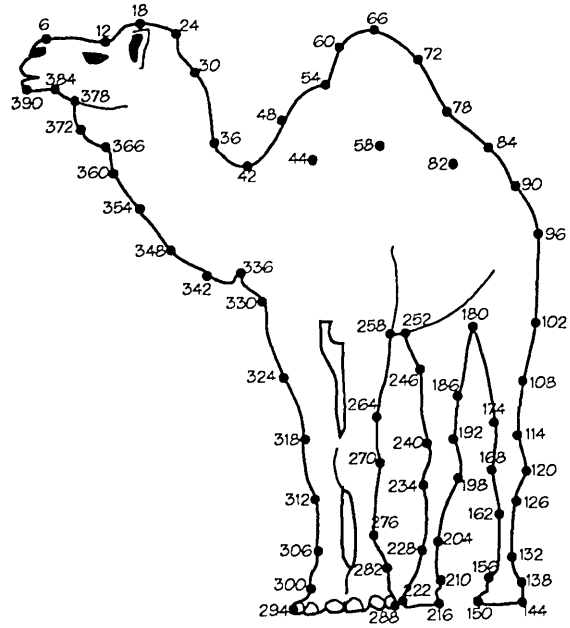
Here are the answers to the test, but you should make sure that you know your 5 times table **before** you ask your teacher to test you on it.

5 x 9 = **45**	5 x 4 = **20**
5 x 6 = **30**	5 x 1 = **5**
5 x 2 = **10**	5 x 7 = **35**
5 x 3 = **15**	5 x 10 = **50**
5 x 5 = **25**	5 x 8 = **40**

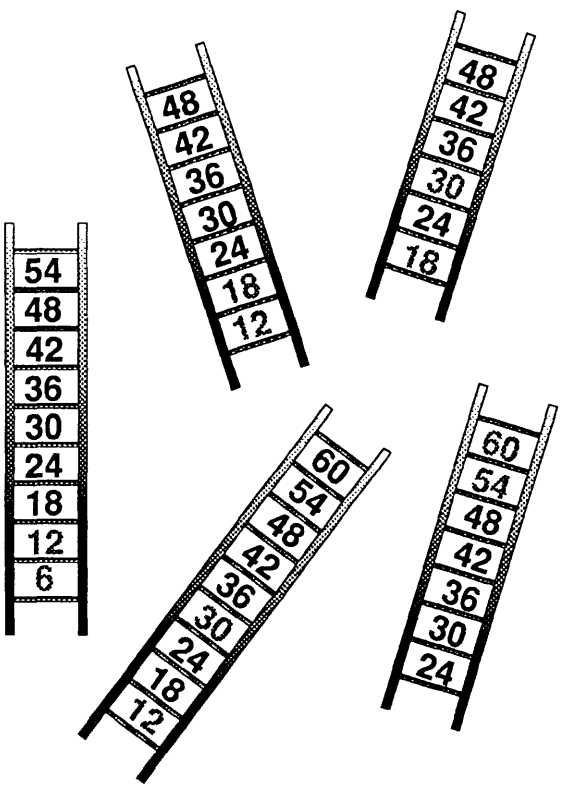
Page 2 **Jumping in 6's pattern**



Page 3 **Joining multiples of 6.**



Page 4 **Filling in the steps of the ladder.**



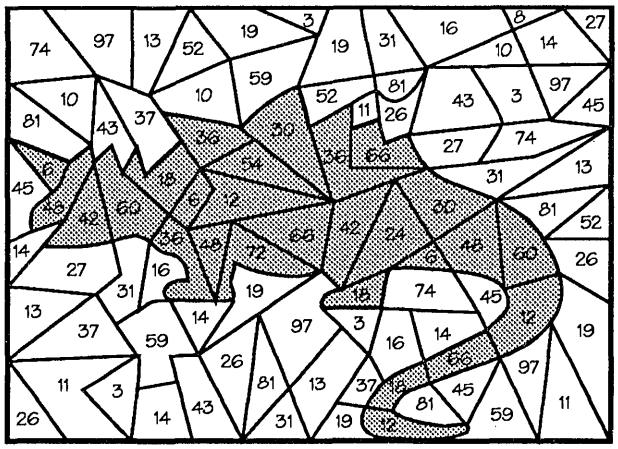
Page 5 **The 6 times table**

6 x 1 = 6	6 x 7 = 42
6 x 2 = 12	6 x 8 = 48
6 x 3 = 18	6 x 9 = 54
6 x 4 = 24	6 x 10 = 60
6 x 5 = 30	6 x 11 = 66
6 x 6 = 36	6 x 12 = 72

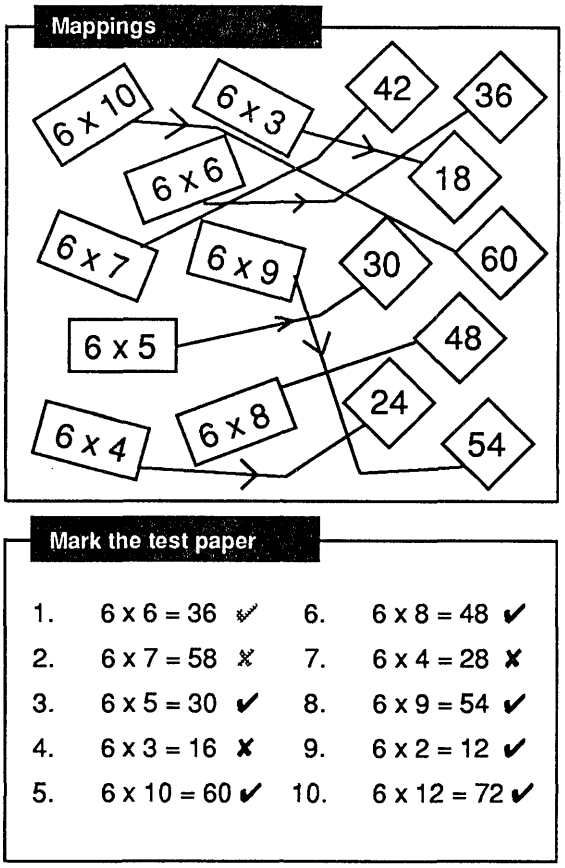
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

continued/

Page 6 **Shading multiples of 6.**



Page 7 **Mappings**



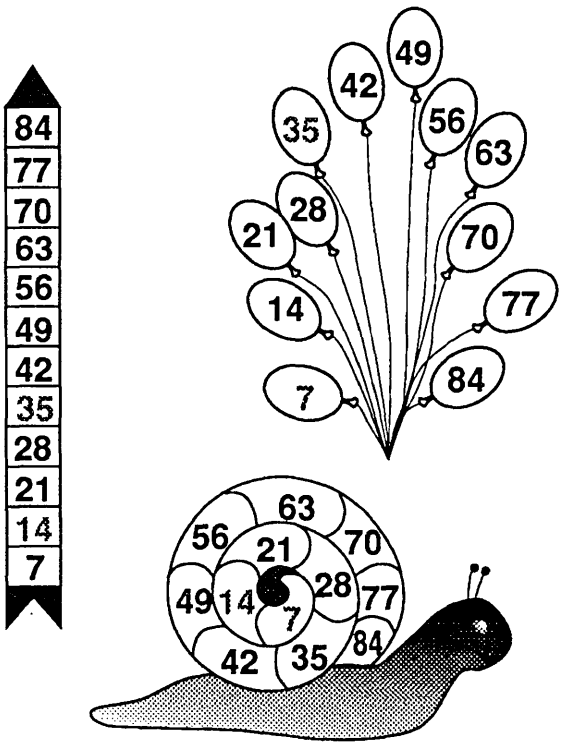
Page 8 **The test.**

Here are the answers to the test, but you should make sure that you know your 6 times table **before** you ask your teacher to test you on it.

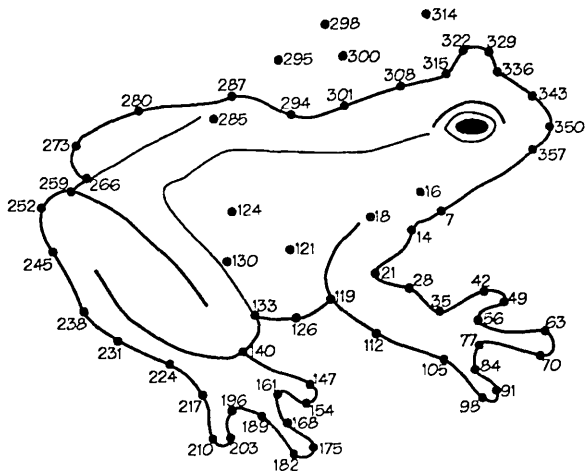
6 x 9 = **54**	6 x 4 = **24**
6 x 6 = **36**	6 x 1 = **6**
6 x 2 = **12**	6 x 7 = **42**
6 x 3 = **18**	6 x 10 = **60**
6 x 5 = **30**	6 x 8 = **48**

Page 2 **Jumping in 7's pattern**

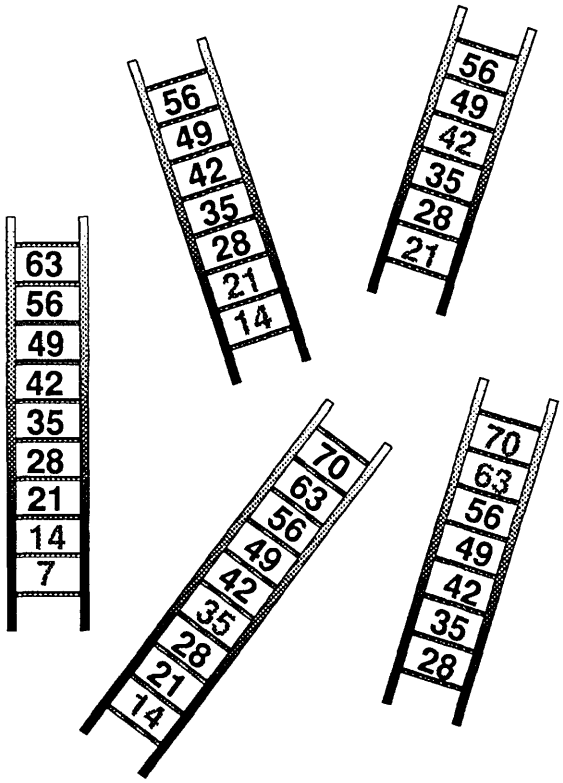
7 14 21 28 35 42 49 56 63 70



Page 3 **Joining multiples of 7.**



Page 4 **Filling in the steps of the ladder.**

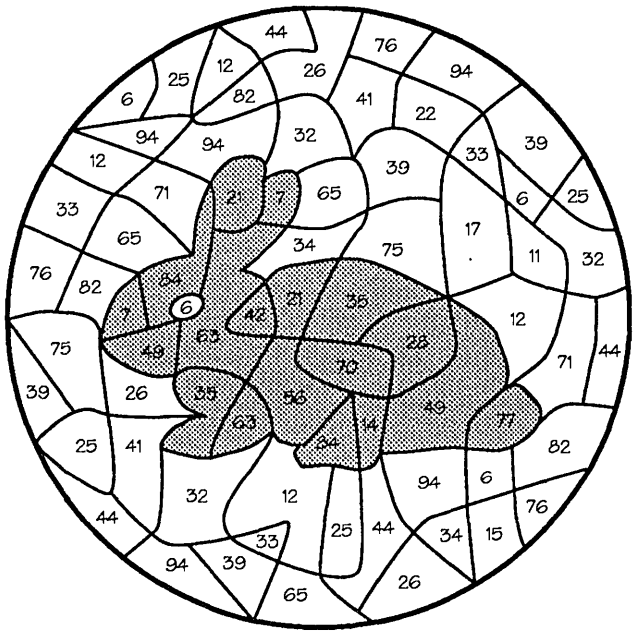


Page 5 **The 7 times table**

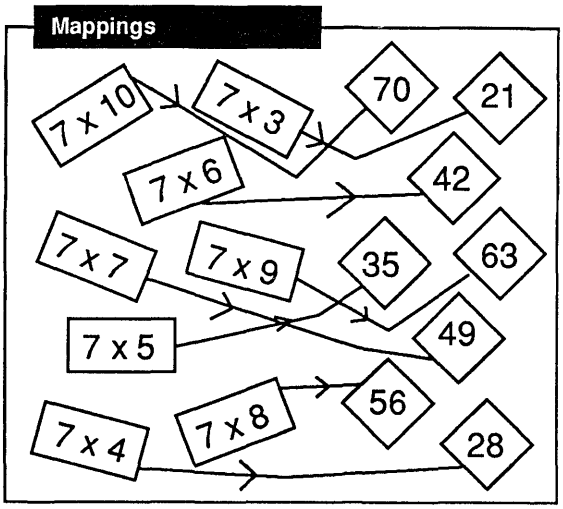
7 x 1 = 7
7 x 2 = 14
7 x 3 = 21
7 x 4 = 28
7 x 5 = 35
7 x 6 = 42
7 x 7 = 49
7 x 8 = 56
7 x 9 = 63
7 x 10 = 70
7 x 11 = 77
7 x 12 = 84

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Page 6 **Shading multiples of 7.**



Page 7 **Mappings**



Mark the test paper

1.	7 x 7 = 49 ✔	6.	7 x 8 = 56 ✔
2.	7 x 6 = 44 ✘	7.	7 x 4 = 26 ✘
3.	7 x 5 = 35 ✔	8.	7 x 9 = 63 ✔
4.	7 x 3 = 21 ✔	9.	7 x 2 = 14 ✔
5.	7 x 10 = 70 ✔	10.	7 x 12 = 84 ✔

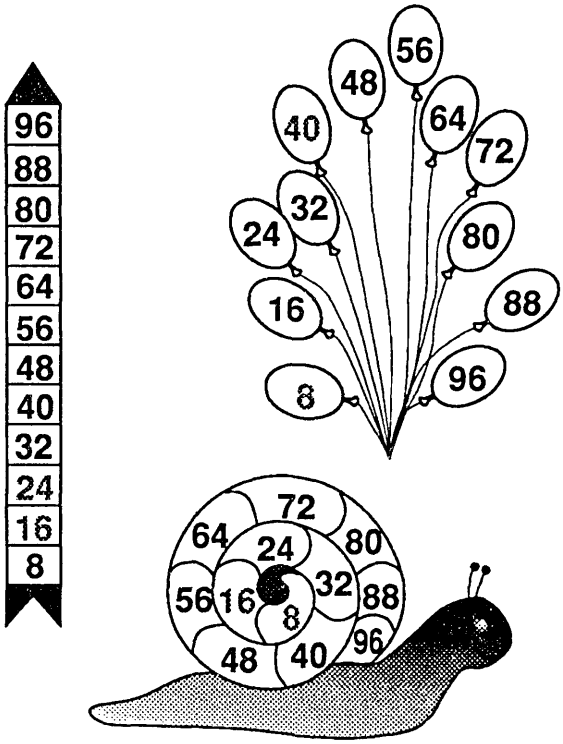
Page 8 **The test.**

Here are the answers to the test, but you should make sure that you know your 7 times table **before** you ask your teacher to test you on it.

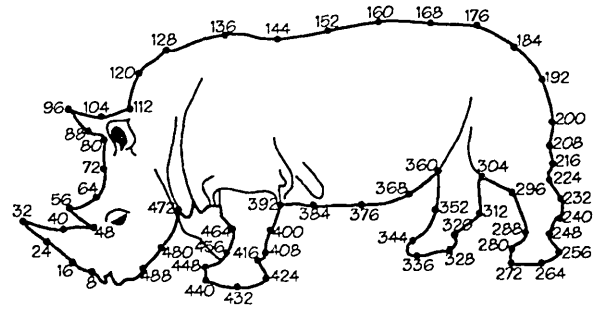
7 x 9 = **63**	7 x 4 = **28**
7 x 6 = **42**	7 x 1 = **7**
7 x 2 = **14**	7 x 7 = **49**
7 x 3 = **21**	7 x 10 = **70**
7 x 5 = **35**	7 x 8 = **56**

Page 2 **Jumping in 8's pattern**

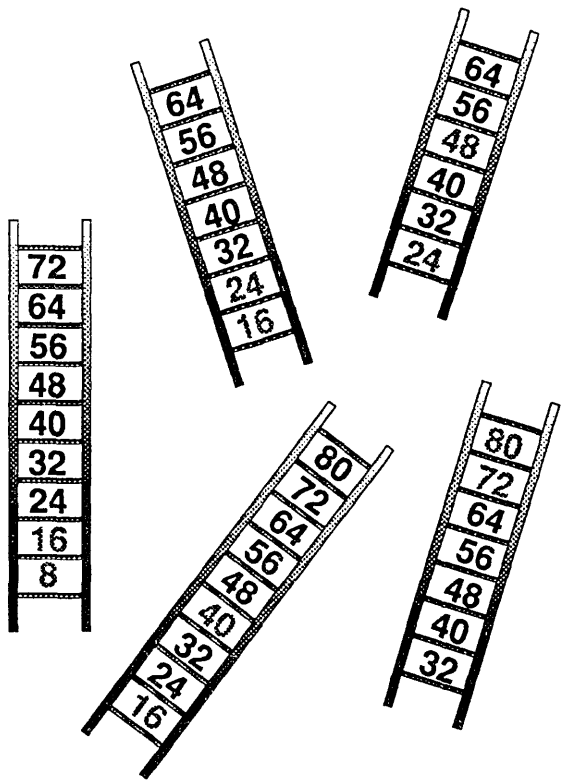
8 16 24 32 40 48 56 64 72 80



Page 3 **Joining multiples of 8.**



Page 4 **Filling in the steps of the ladder.**



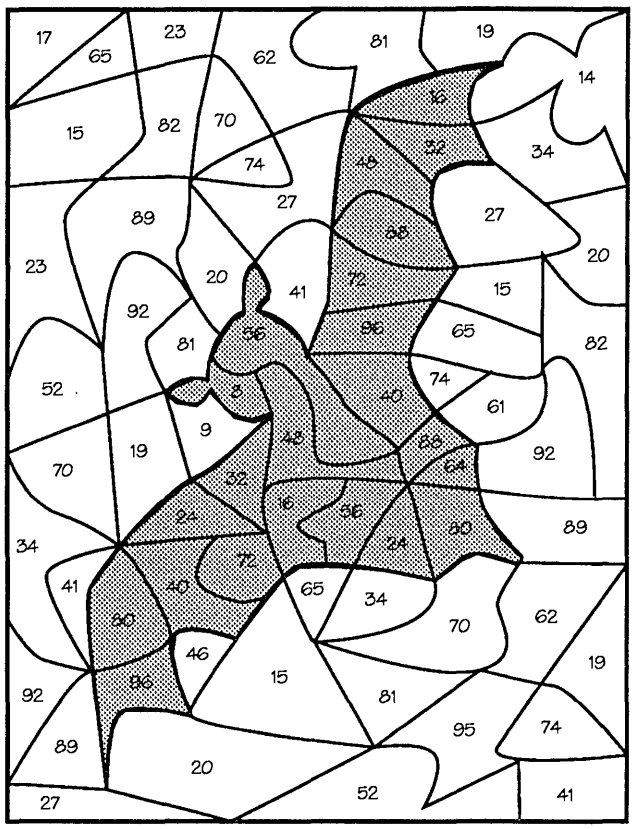
Page 5 **The 8 times table**

8 x 1 = 8	8 x 7 = 56
8 x 2 = 16	8 x 8 = 64
8 x 3 = 24	8 x 9 = 72
8 x 4 = 32	8 x 10 = 80
8 x 5 = 40	8 x 11 = 88
8 x 6 = 48	8 x 12 = 96

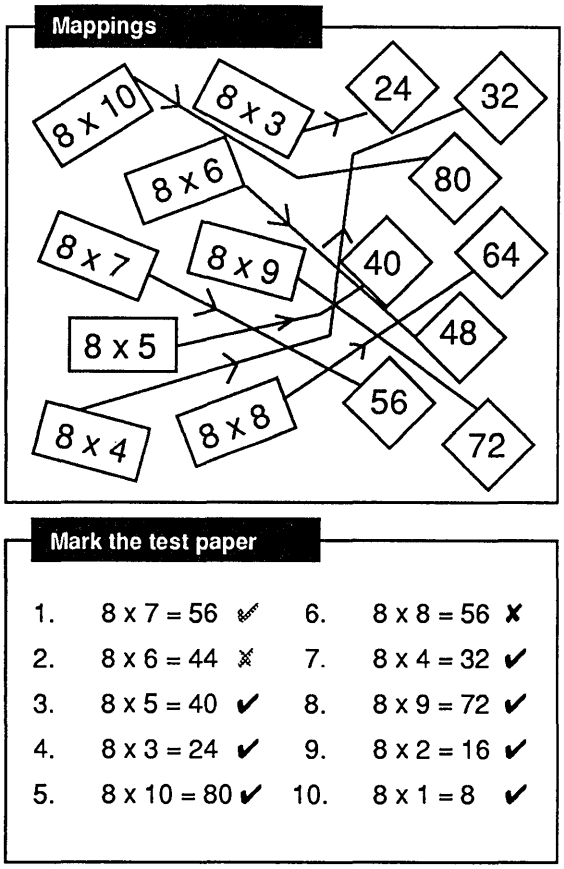
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

continued/

Page 6 **Shading multiples of 8.**



Page 7 **Mappings**



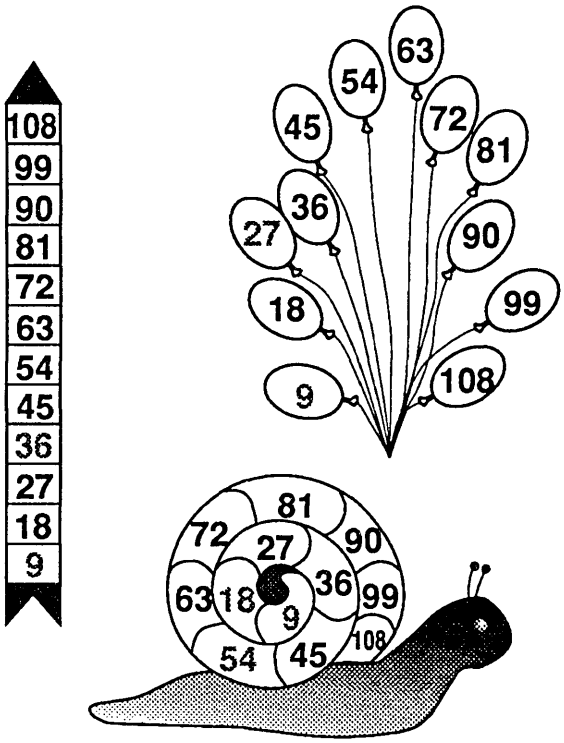
Page 8 **The test.**

Here are the answers to the test, but you should make sure that you know your 8 times table **before** you ask your teacher to test you on it.

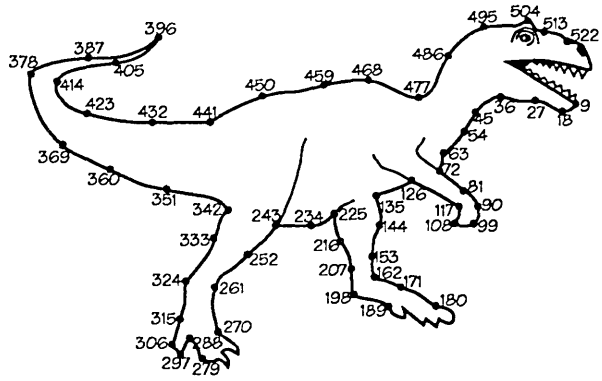
8 x 9 = **72**	8 x 4 = **32**
8 x 6 = **48**	8 x 1 = **8**
8 x 2 = **16**	8 x 7 = **56**
8 x 3 = **24**	8 x 10 = **80**
8 x 5 = **40**	8 x 8 = **64**

Page 2 **Jumping in 9's pattern**

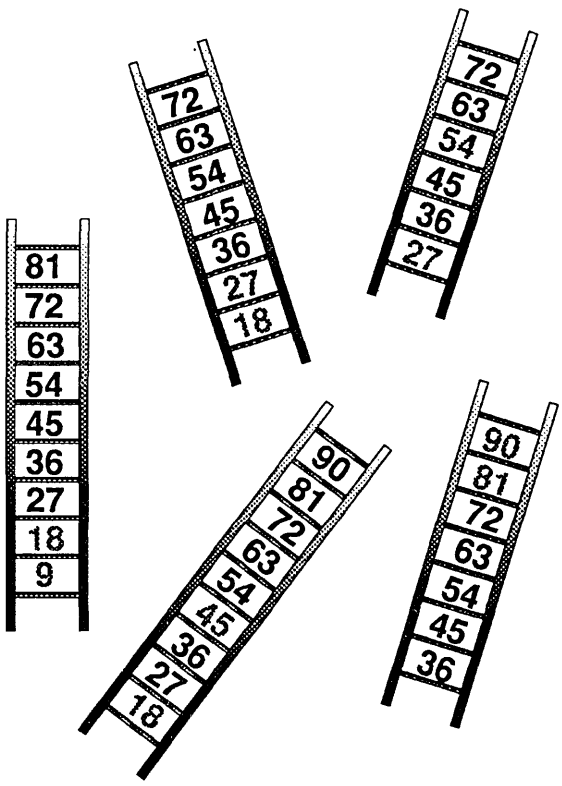
(9) (18) (27) (36) (45) (54) (63) (72) (81) (90)



Page 3 **Joining multiples of 9.**



Page 4 **Filling in the steps of the ladder.**



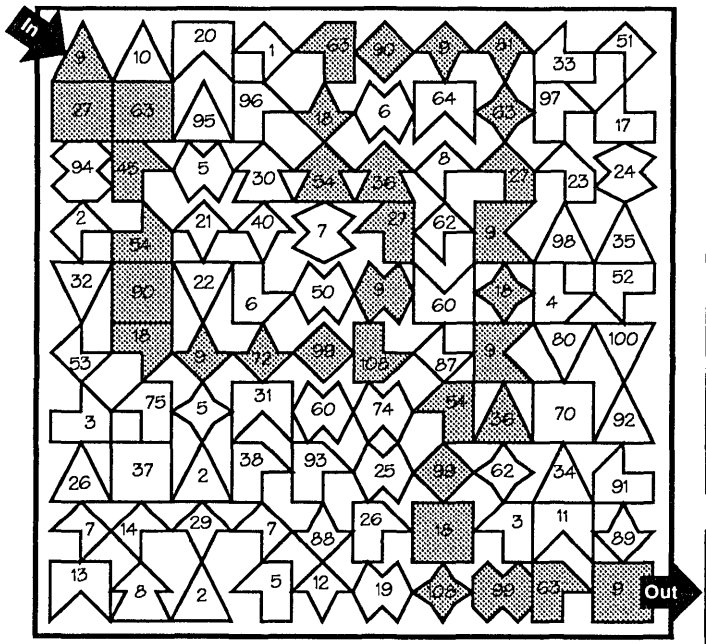
Page 5 **The 9 times table**

9 x 1 = 9	9 x 7 = 63
9 x 2 = 18	9 x 8 = 72
9 x 3 = 27	9 x 9 = 81
9 x 4 = 36	9 x 10 = 90
9 x 5 = 45	9 x 11 = 99
9 x 6 = 54	9 x 12 = 108

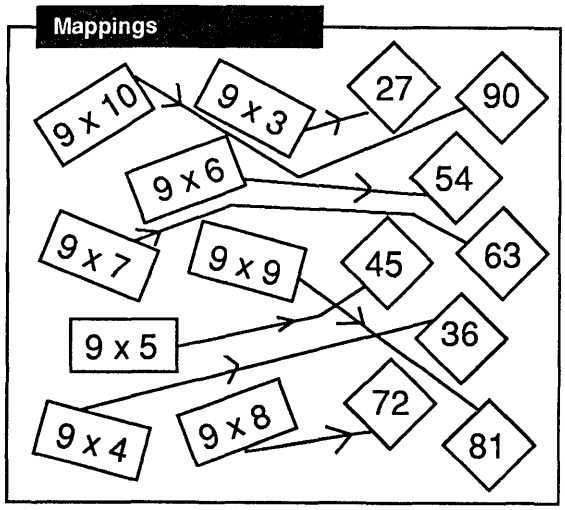
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

continued/

Page 6 **Shading multiples of 9.**



Page 7 **Mappings**



Mark the test paper

1.	9 x 7 = 63 ✓	6.	9 x 8 = 72 ✓
2.	9 x 6 = 44 ✗	7.	9 x 4 = 32 ✗
3.	9 x 5 = 45 ✓	8.	9 x 9 = 81 ✓
4.	9 x 3 = 28 ✗	9.	9 x 2 = 18 ✓
5.	9 x 10 = 90 ✓	10.	9 x 11 = 99 ✓

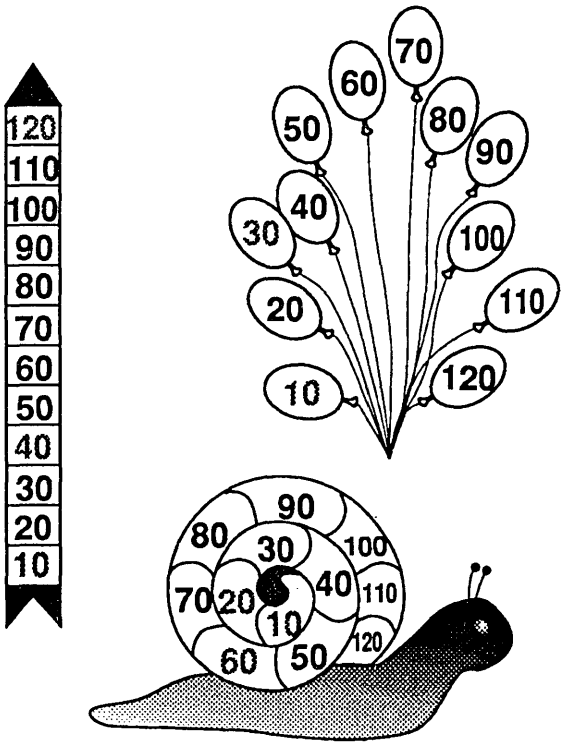
Page 8 **The test.**

Here are the answers to the test, but you should make sure that you know your 9 times table **before** you ask your teacher to test you on it.

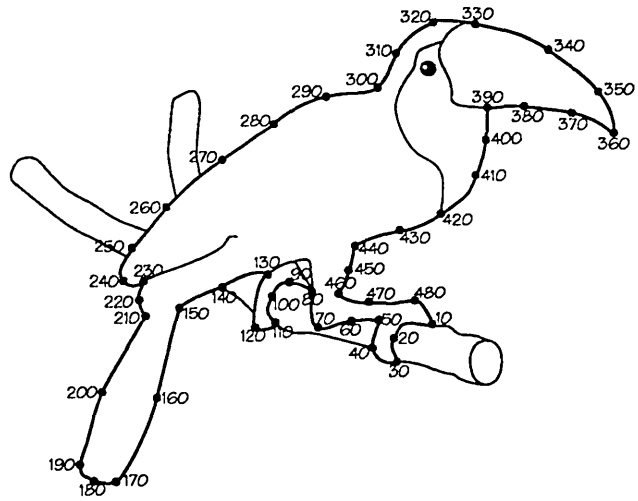
9 x 9 = **81**	9 x 4 = **36**
9 x 6 = **54**	9 x 1 = **9**
9 x 2 = **18**	9 x 7 = **63**
9 x 3 = **27**	9 x 10 = **90**
9 x 5 = **45**	9 x 8 = **72**

Page 2 **Jumping in 10's pattern**

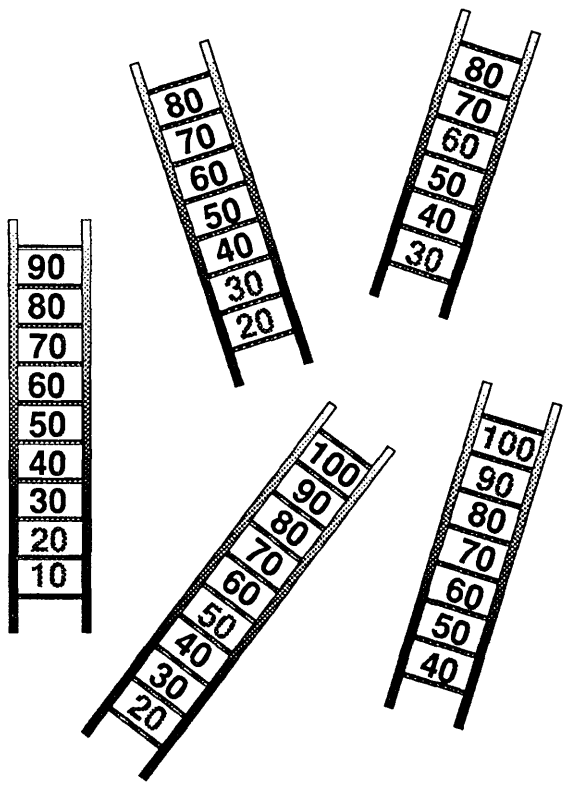
10 20 30 40 50 60 70 80 90 100



Page 3 **Joining multiples of 10.**



Page 4 **Filling in the steps of the ladder.**



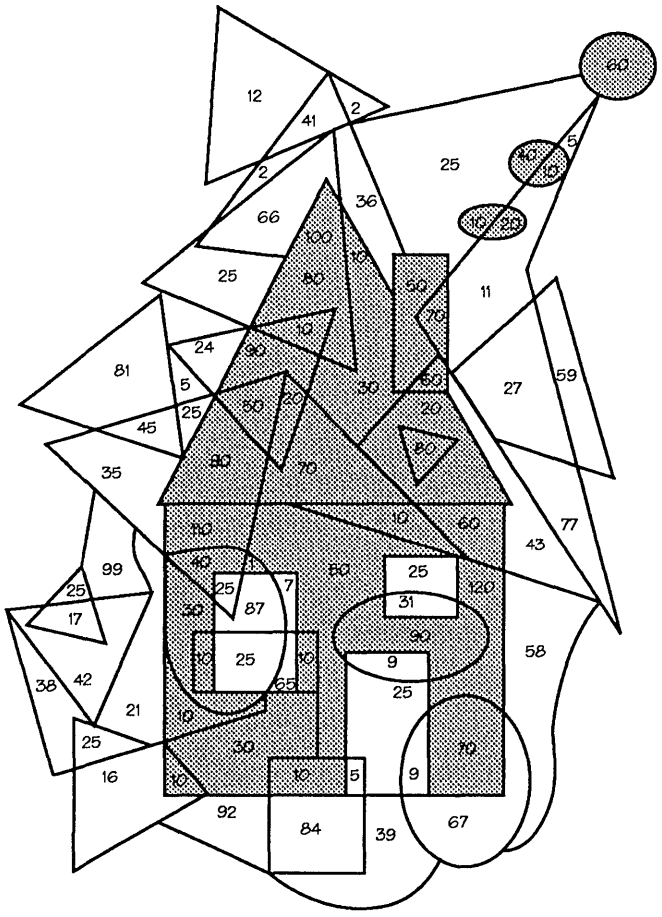
Page 5 **The 10 times table**

10 x 1 = 10	10 x 7 = 70
10 x 2 = 20	10 x 8 = 80
10 x 3 = 30	10 x 9 = 90
10 x 4 = 40	10 x 10 = 100
10 x 5 = 50	10 x 11 = 110
10 x 6 = 60	10 x 12 = 120

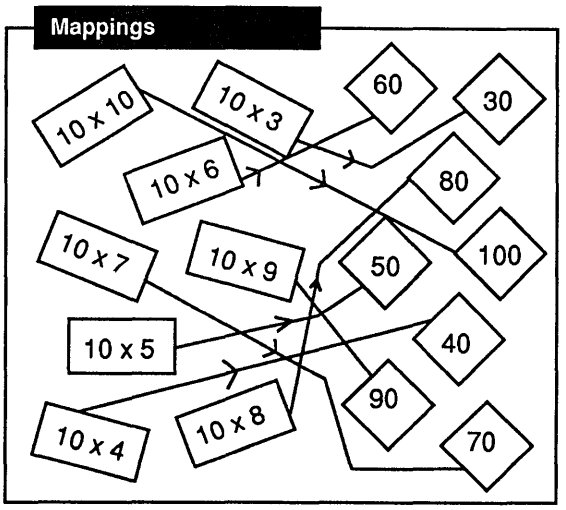
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

continued/

Page 6 **Shading multiples of 10.**



Page 7 **Mappings**



Mark the test paper

1.	10 x 6 = 60 ✓	6.	10 x 8 = 80 ✓
2.	10 x 7 = 77 ✗	7.	10 x 4 = 44 ✗
3.	10 x 5 = 55 ✗	8.	10 x 9 = 90 ✓
4.	10 x 3 = 30 ✓	9.	10 x 2 = 20 ✓
5.	10 x 10 = 100 ✓	10.	10 x 12 = 120 ✓

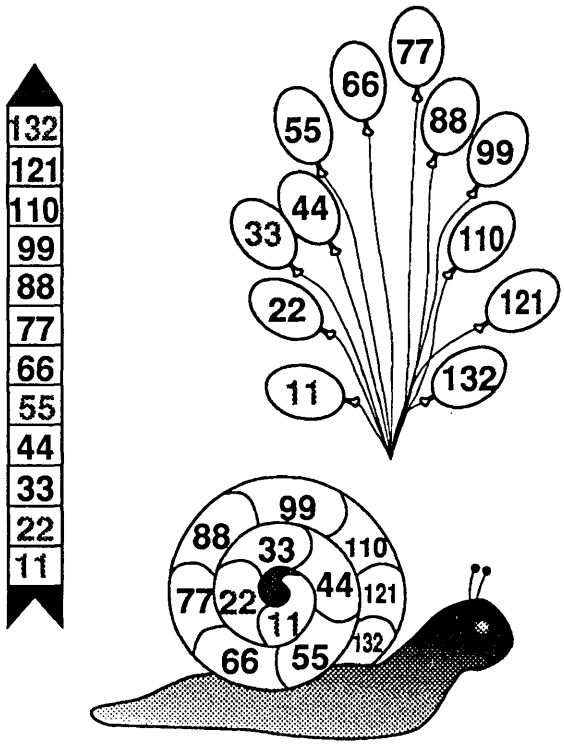
Page 8 **The test.**

Here are the answers to the test, but you should make sure that you know your 10 times table **before** you ask your teacher to test you on it.

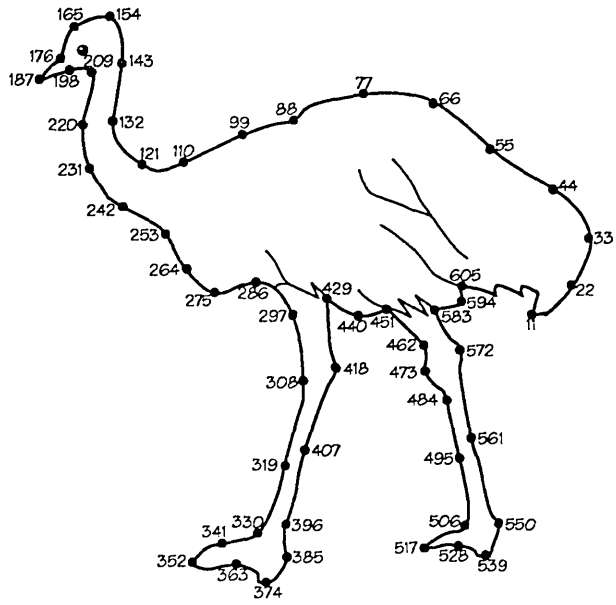
10 x 9 = **90**	10 x 4 = **40**
10 x 6 = **60**	10 x 1 = **10**
10 x 2 = **20**	10 x 7 = **70**
10 x 3 = **30**	10 x 10 = **100**
10 x 5 = **50**	10 x 8 = **80**

Page 2 **Jumping in 11's pattern**

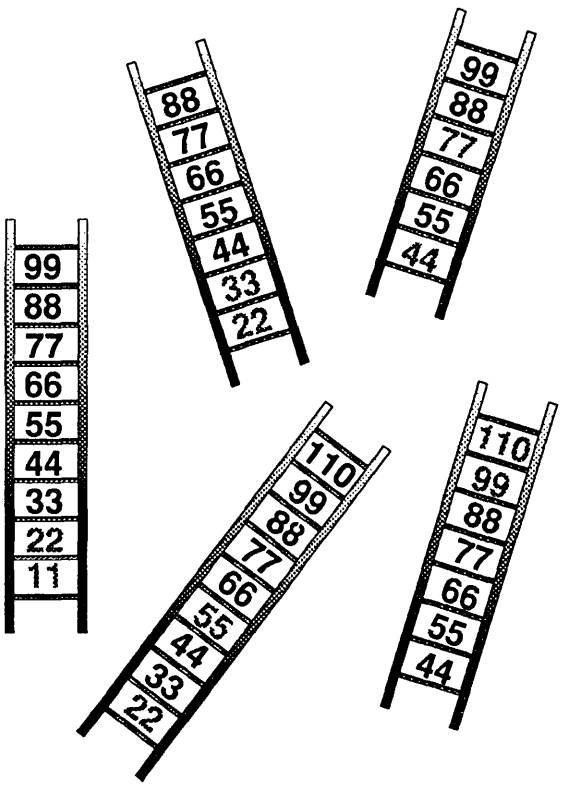
11 22 33 44 55 66 77 88 99 110



Page 3 **Joining multiples of 11.**



Page 4 **Filling in the steps of the ladder.**



Page 5 **The 11 times table**

11 x 1 = 11
11 x 2 = 22
11 x 3 = 33
11 x 4 = 44
11 x 5 = 55
11 x 6 = 66
11 x 7 = 77
11 x 8 = 88
11 x 9 = 99
11 x 10 = 110
11 x 11 = 121
11 x 12 = 132

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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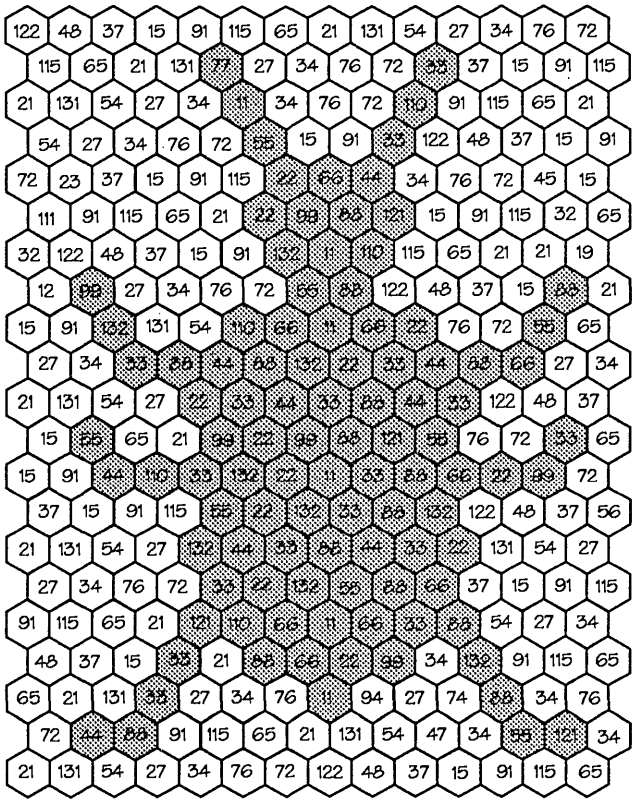
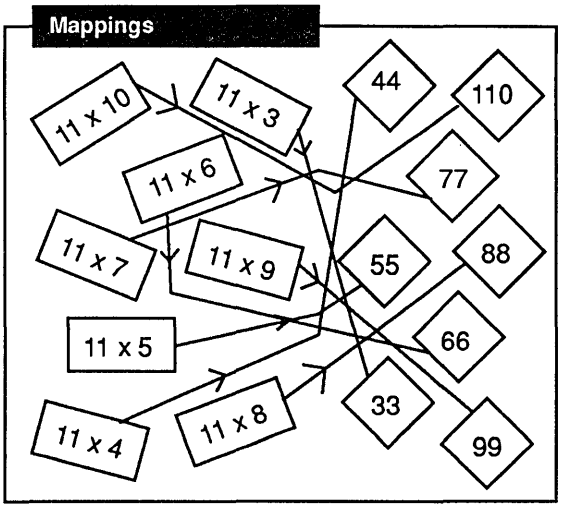
Page 6 Shading multiples of 11.



Page 7 Mappings



Mark the test paper

1. 11 x 6 = 66 ✓
2. 11 x 7 = 87 ✗
3. 11 x 5 = 55 ✓
4. 11 x 3 = 33 ✓
5. 11 x 10 = 111 ✗
6. 11 x 8 = 88 ✓
7. 11 x 4 = 41 ✗
8. 11 x 9 = 99 ✓
9. 11 x 2 = 20 ✗
10. 11 x 1 = 11 ✓

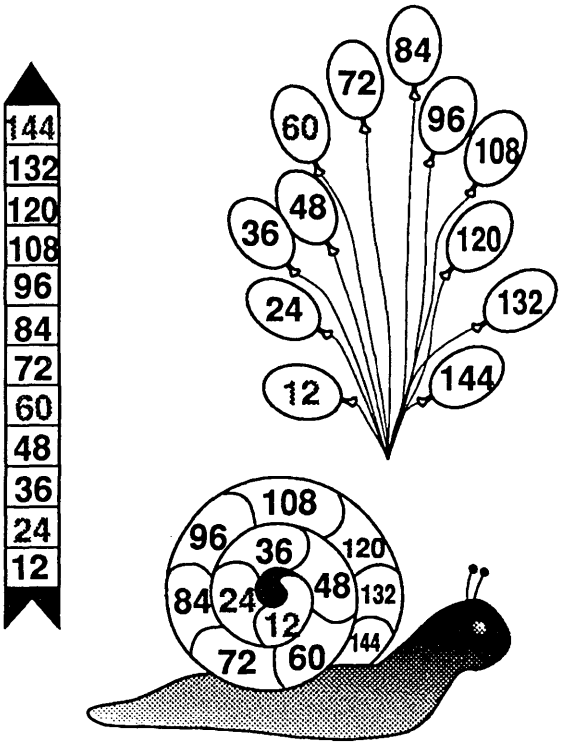
Page 8 The test.

Here are the answers to the test, but you should make sure that you know your 11 times table **before** you ask your teacher to test you on it.

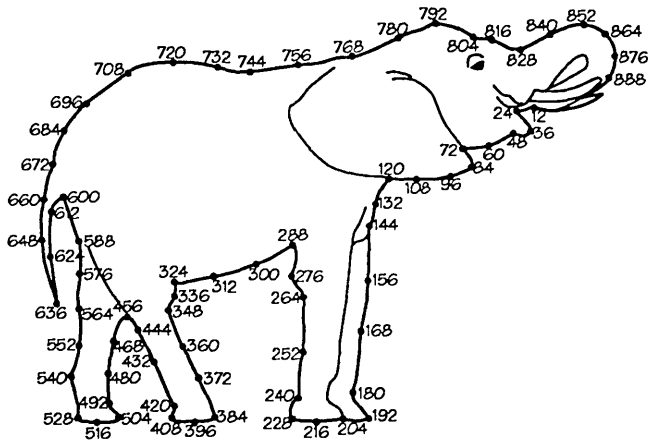
11 x 9 = **99**	11 x 4 = **44**
11 x 6 = **66**	11 x 1 = **11**
11 x 2 = **22**	11 x 7 = **77**
11 x 3 = **33**	11 x 10 = **110**
11 x 5 = **55**	11 x 8 = **88**

Page 2 **Jumping in 12's pattern**

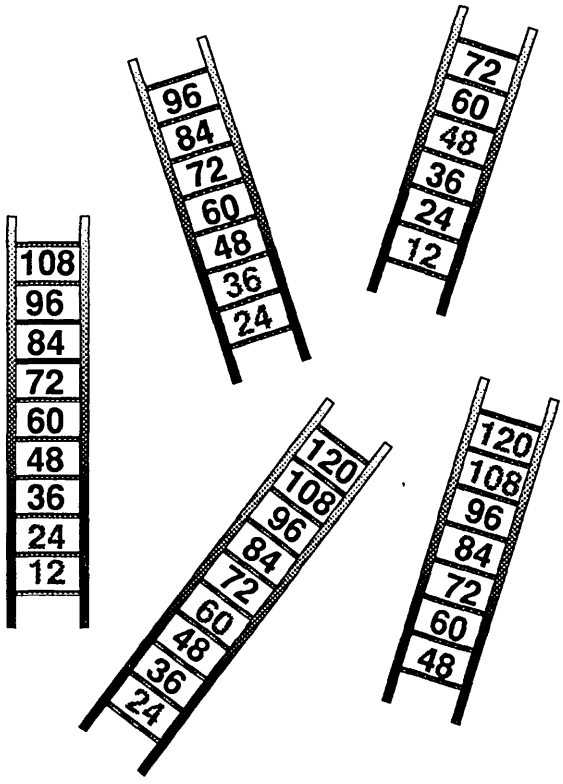
12 24 36 48 60 72 84 96 108 120



Page 3 **Joining multiples of 12.**



Page 4 **Filling in the steps of the ladder.**



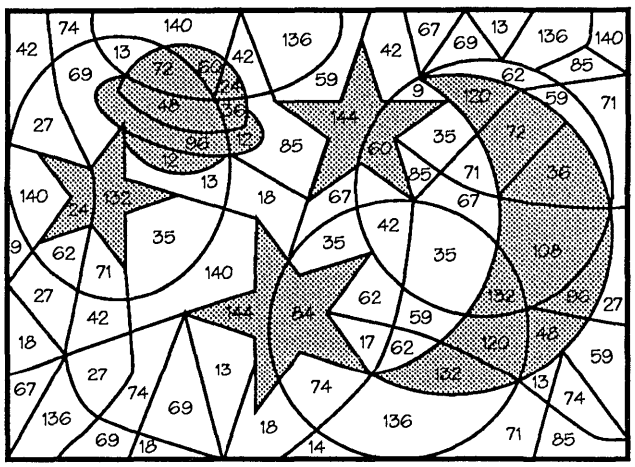
Page 5 **The 12 times table**

12 x 1 = 12 12 x 7 = 84
12 x 2 = 24 12 x 8 = 96
12 x 3 = 36 12 x 9 = 108
12 x 4 = 48 12 x 10 = 120
12 x 5 = 60 12 x 11 = 132
12 x 6 = 72 12 x 12 = 144

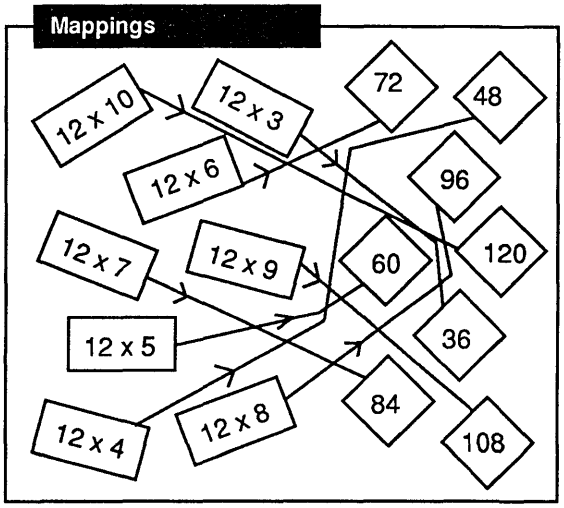
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

continued/

Page 6 Shading multiples of 12.



Page 7 Mappings



Mark the test paper

1.	12 x 6 = 72 ✓	6.	12 x 8 = 96 ✓
2.	12 x 7 = 86 ✗	7.	12 x 4 = 48 ✓
3.	12 x 11 = 132 ✓	8.	12 x 9 = 96 ✗
4.	12 x 3 = 32 ✗	9.	12 x 2 = 24 ✓
5.	12 x 10 = 120 ✓	10.	12 x 5 = 50 ✗

Page 8 The test.

Here are the answers to the test, but you should make sure that you know your 12 times table **before** you ask your teacher to test you on it.

12 x 9 = **108**	12 x 4 = **48**
12 x 6 = **72**	12 x 1 = **12**
12 x 2 = **24**	12 x 7 = **84**
12 x 3 = **36**	12 x 10 = **120**
12 x 5 = **60**	12 x 8 = **96**

2350 End of Level 3 Review

The answers to all the questions are on pages 20 - 23. If there are any questions which you feel you require further help on, make a note of these and the suggestions at the bottom of the page and talk to your teacher.

2351 End of Level 4 Review

The answers to all the questions are on pages 32 - 38. If there are any questions which you feel you require further help on, make a note of these and the suggestions at the bottom of the page and talk to your teacher.

2352 End of Level 5 Review

The answers to all the questions are on pages 33 - 39. If there are any questions which you feel you require further help on, make a note of these and the suggestions at the bottom of the page and talk to your teacher.

2353 End of Level 6 Review

The answers to all the questions are on pages 47 - 61. If there are any questions which you feel you require further help on, make a note of these and the suggestions at the bottom of the page and talk to your teacher.

2354 End of Level 7 Review

The answers to all the questions are on pages 36 - 49. If there are any questions which you feel you require further help on, make a note of these and the suggestions at the bottom of the page and talk to your teacher.

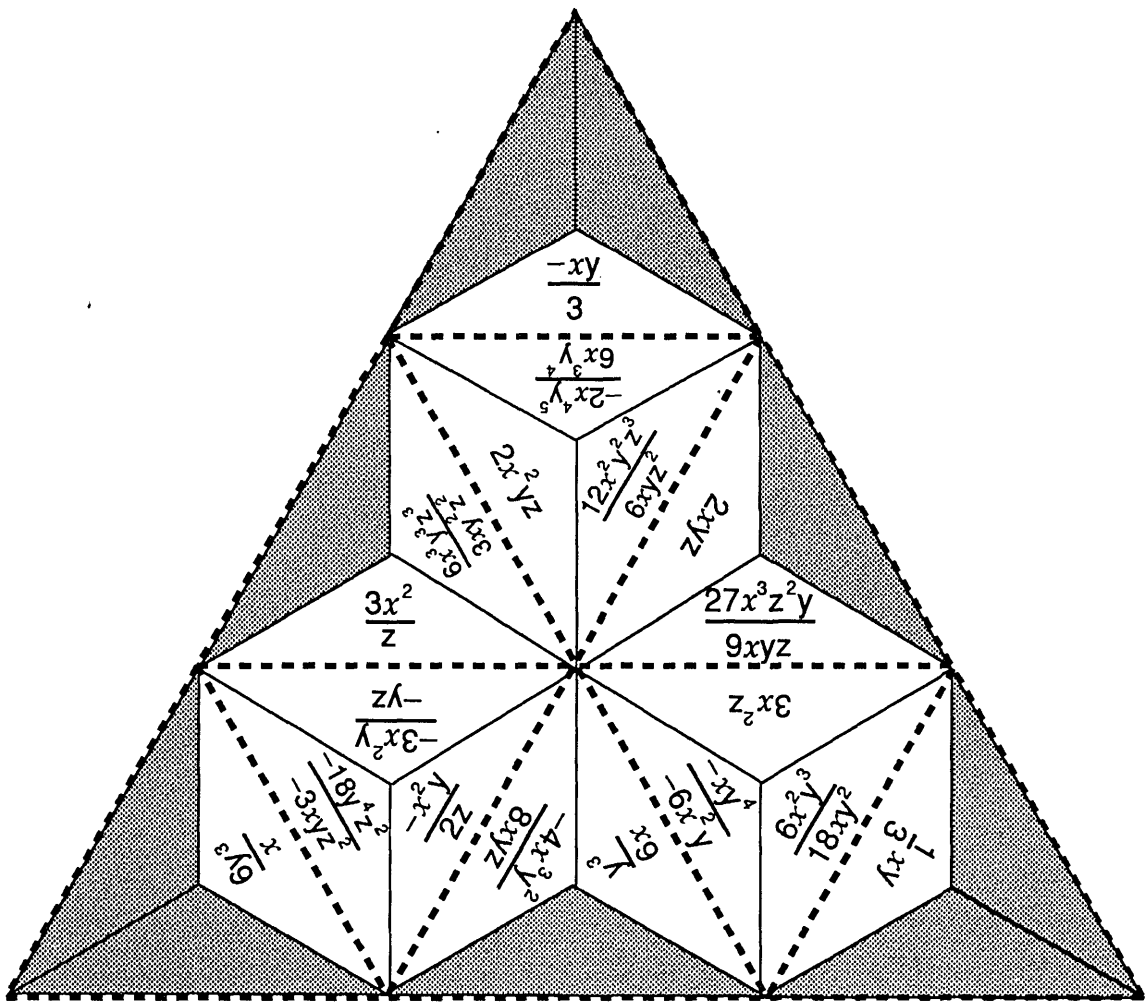
2355 End of Level 8 Review

The answers to all the questions are on pages 26 - 32. If there are any questions which you feel you require further help on, make a note of these and the suggestions at the bottom of the page and talk to your teacher.

2356 End of Exceptional Performance Review

The answers to all the questions are on pages 44 - 61. If there are any questions which you feel you require further help on, make a note of these and the suggestions at the bottom of the page and talk to your teacher.

2357 Matching Algebraic Expressions



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Answers