SMILE WORKCARDS

Properties of Number Pack Two

Contents

AT2 PNO LEVEL 5

Smile **0308**

Prime Numbers

A **prime** number is a number that has only two factors, 1 and itself.

1 x 5 = 5

The factors of 5 are 1 and 5
The number 5 has only **two** factors.
So 5 is a **prime** num ber.

2 x 4 = 8

The factors of 8 are 1, 2, 4 and 8
The number 8 has **four** factors.
So 8 is not a **prime** number.

1 x 8 = 8

1) Which of the following numbers is prime?
   a) 6
   b) 12
   c) 7
   d) 16

2) List all the prime numbers less than 30.

3) Write a definition of a **prime** number in your book.

Multiple Patterns

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

1. Take a hundred square and shade **all** the multiples of 2.

   This makes a **column** pattern.

2. Take another hundred square. This time shade the multiples of 3. *Do you get a column pattern?*

3. Take some more hundred squares and find out which other multiples make column patterns.

Turn over

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
			22	23	24

4. Take a square with 6 columns *(from Smile Worksheet 0330A).*

5. Shade the multiples of 2. *Do you get a column pattern?*

6. Find out which other multiples make column patterns.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21

7. Are there any multiples which make column patterns using a square with 7 columns?

8. Which multiples give column patterns on the 10 square?
   Which multiples give column patterns on the 6 square?
   Which multiples give column patterns on the 7 square?

Can you predict which multiples would give a column pattern on a 12 square?

What is the general rule?

You might find it helpful to use the micro program called MULTIPLE (0330B)

Multiple Patterns

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

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36	37	38	39	40	41	42
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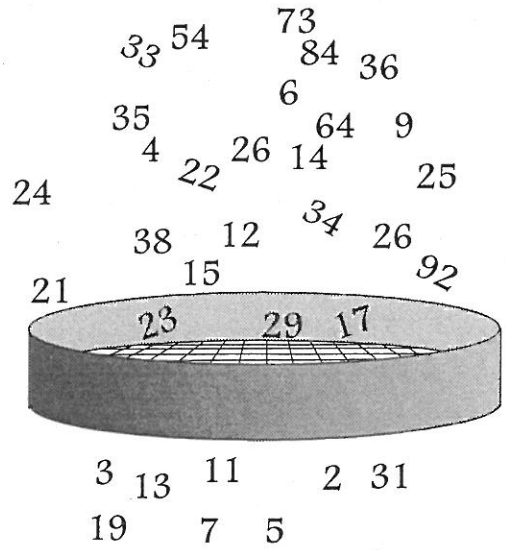
Sieve of Eratosthenes

You will need a *100 square.*



One of the first people to try to find
all the prime numbers was Eratosthenes,
some two thousand years ago.

He was a Greek mathematician, astronomer, athlete,
historian and poet who was
born about 276BC and died about 194BC.

Eratosthenes devised a mathematical sieve;
any numbers
that fell through his sieve
were prime.

Turn over
to find out how to use the
Sieve of Eratosthenes
to find all the primes from
1 to 100.

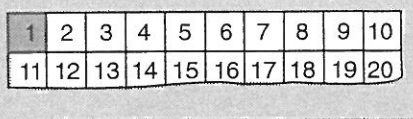
Reminder

A **prime number**	A number with exactly two factors; **1** and the number itself. (**1** is not a prime number because it has only one factor.)
A **factor**	A whole number that divides into another number without any remainder.
A **multiple**	A multiple of a number *n* is k x *n* when k is a whole number. Some multiples of 4 are 4, 8, 12 . . .

Take a 100 square. Follow this method to find all the prime numbers between 1 and 100.

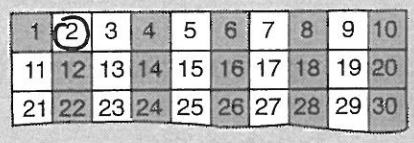
1

1 is **not** a prime number.
It has only one factor.
- Shade it in.



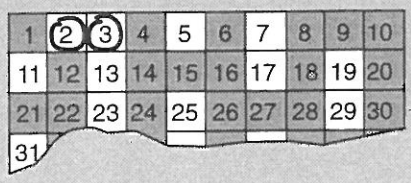
2

2 is the first prime number. It has two factors, 1 and 2.
- Circle **2**.
- Other *multiples of 2* are not prime, shade them in.



3

3 is the next prime number. It has two factors, 1 and 3.
- Circle **3**.
- Other *multiples of 3* are not prime, shade them in.

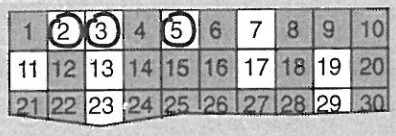


4

4 is already shaded.
- Explain why. Other *multiples of 4* should also be shaded. Check that they are.

5

5 is the next unshaded number. It is prime.
- Circle **5**.
- Other *multiples of 5* are not prime, shade them in.

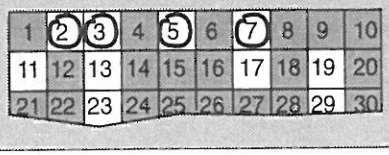


6

All the *multiples of 6* are already shaded in.
- Explain why.

7

The next unshaded number is **7**. It is prime.
- Circle **7**.
- Other *multiples of 7* are not prime, shade them in.



Continue using this method until all the numbers on the 100 square are either circled or shaded.

The **circled** numbers are the **prime numbers** between 1 and 100.

There should be twenty-five.

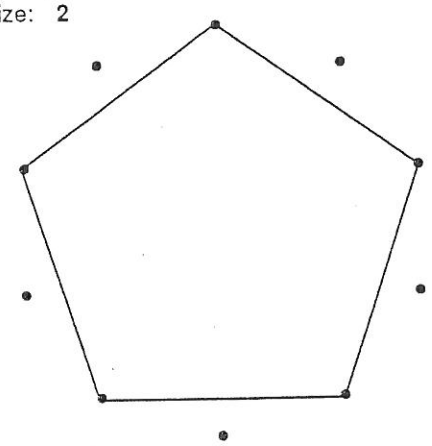
- Find out which of these numbers are prime using your knowledge of factors and multiples.

113	117	136	149	173

You will need MicroSMILE program CIRCLE.

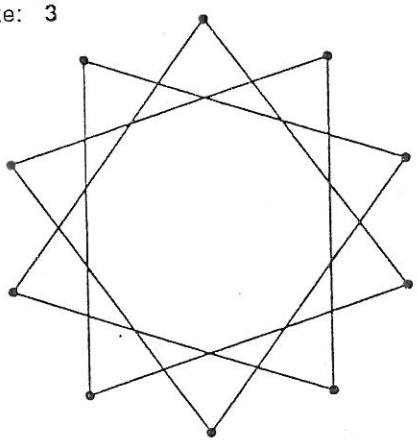
Visiting every point

Points: 10
Jump size: 2



With **10** points and jump size **2**, the line *does not* visit every point.

Points: 10
Jump size: 3



With **10** points and jump size **3**, the line *does* visit every point.

Keep 10 points.
Try other jump sizes.

- Which jump sizes *do not* visit every point?
- Which jump sizes *do* visit every point?

Record your results.

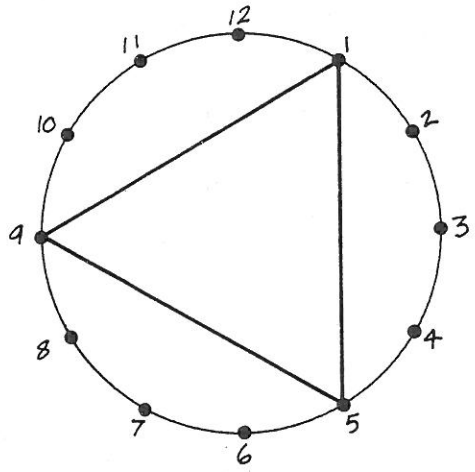
Start with different numbers of points and investigate different jump sizes.

Can you predict:

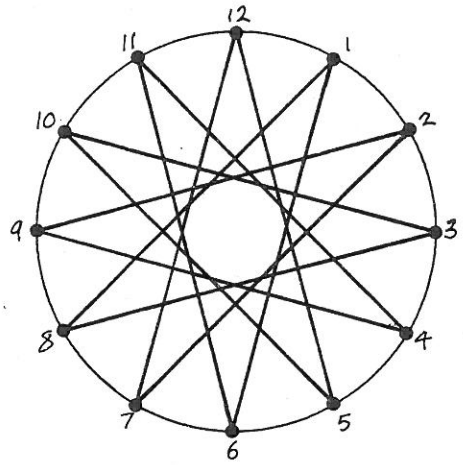
- Which patterns *will not* visit every point?
- Which patterns *will* visit every point?

Did you find a number of points where the line visited every point, whatever the jump size?
What is special about such numbers of points?

Point circles

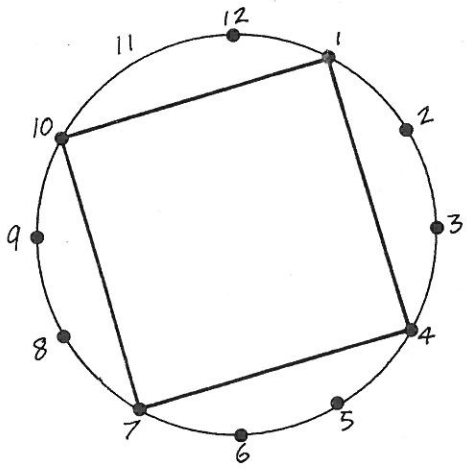


Start at 1 on a clock-face. Add 4, and continue until you return to the 1.
The result is a triangle. 3 points are visited.



Start at 1 on the same clock-face. Add 5.

12 points are visited.



Start at 1. Add 9.

The result is a square. 4 points are visited.

Investigate add 1, add 2, add 3, etc. on a clock-face.
Investigate different rules on other circles, e.g. an 8-point circle.

Write about your results and try to explain any patterns.

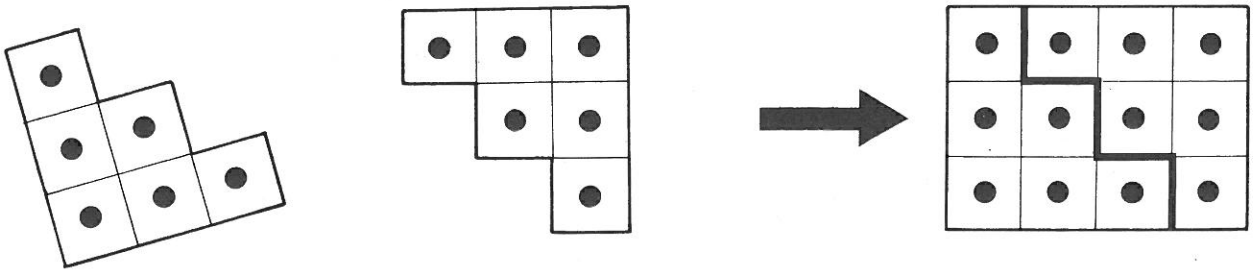
You might like to test out any theories with the micro program called Circle (1702)

Triangle Numbers 2

Smile **0221**

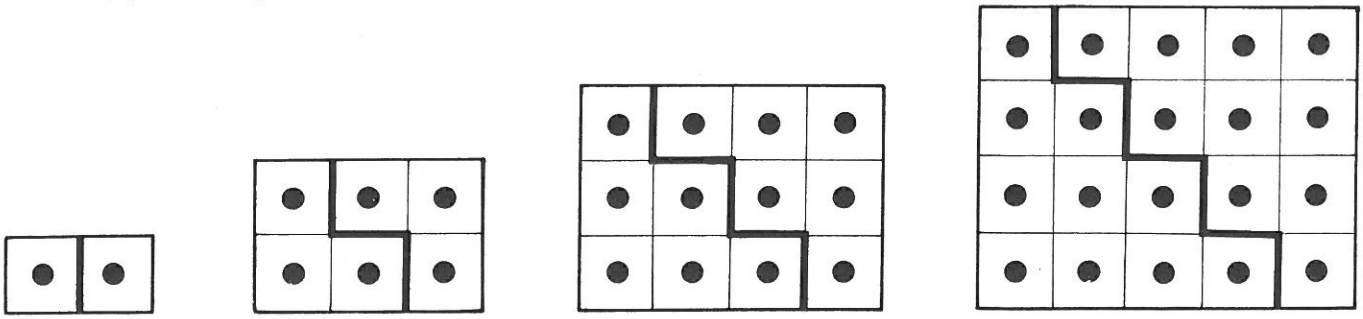
When you add a triangle number to itself you get a rectangle number.

For example, with the 3rd triangle number:



2 × 3rd triangle number = 3 × 4 = 12
So, 3rd triangle number = $\frac{1}{2}$ (3×4) = 6

1. Copy this sequence and draw the next two patterns.



2. Use your drawings to complete this table.
   (a) 1st triangle number is $\frac{1}{2}$ (1×2) = 1
   (b) 2nd triangle number is $\frac{1}{2}$ (2×3) = 3
   (c) 3rd triangle number is $\frac{1}{2}$ (3×4) = 6
   (d) 4th triangle number is
   (e) 5th triangle number is
   (f) What is the 10th triangle number?

Do your answers agree with those you got for Triangle Numbers 1?

Black and Red Triangle Patterns

1 black triangle.
0 red triangles

Total 1 triangle

3 black triangles.
1 red triangle.

Total 4 triangles.

How many black triangles?
How many red triangles?

What is the total?

Draw more triangle patterns.

What number patterns can you find?

You will need a calculator.

The Factor Game

A game for 2 players.

Player 1:	Enter a 4 digit number

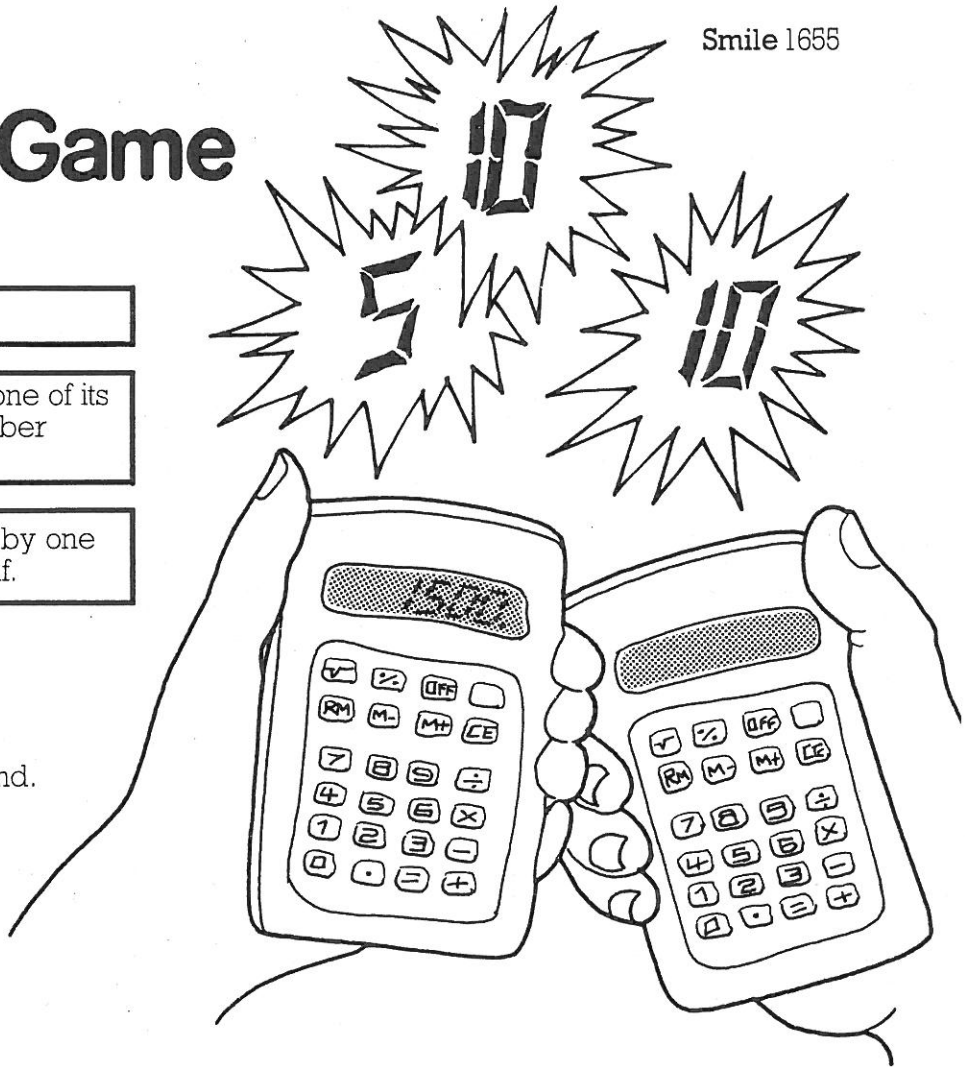
Player 2:	Divide the number by one of its factors, except the number itself.

Player 1:	Divide the new answer by one of its factors except itself.

And so on.

Score a point for each factor you find.

Play the game several times.



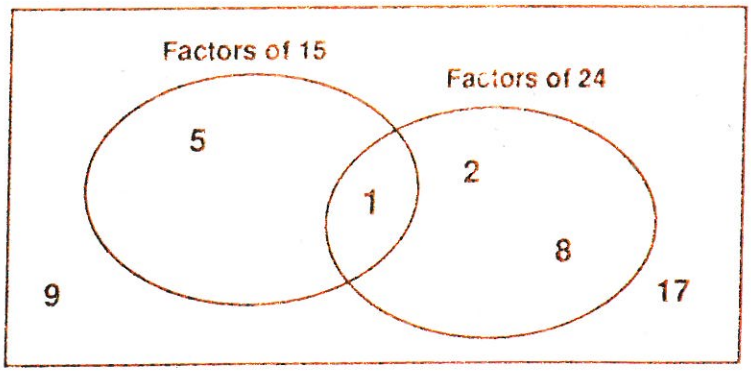
Common Factors

1) Copy and complete : {factors of 15} = { ■, ■, ■, ■ }

{factors of 24} = { ■, ■, ■, ■, ■, ■, ■, ■ }

2) Copy and complete this Venn diagram, using all the numbers from 1 to 24.

{1 - 24}



3) Which numbers are factors of **both** 15 and 24?

Write {common factors of 15 and 24} = { ■, ■ }

Turn over

4) Write {factors of 20} = { ■, ■, ■, ■, ■, ■ }

{factors of 30} = { ■, ■, ■, ■, ■, ■, ■, ■ }

{factors of 16} = { ■, ■, ■, ■, ■ }

{factors of 32} = { ■, ■, ■, ■, ■, ■ }

{factors of 10} = { ■, ■, ■, ■ }

{factors of 15} = { ■, ■, ■, ■ }

{factors of 18} = { ■, ■, ■, ■, ■, ■ }

5) Draw new Venn diagrams to show:

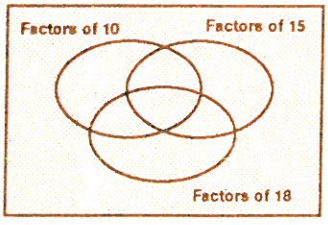
(a) The common factors of 20 and 30.

(b) The common factors of 16 and 32.

(c) The common factors of 10, 15 and 18.

The Venn diagram for part (c) is shown for you to copy and complete.

{1 - 18}



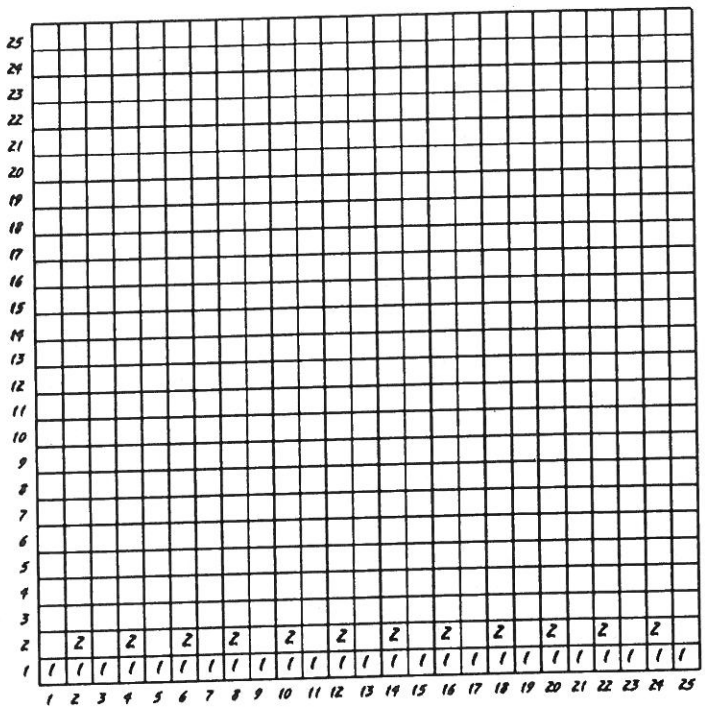
Write {common factors of 20 and 30} = { ■, ■, ■, ■ }

{common factors of 16 and 32} = { ■, ■, ■, ■, ■ }

{common factors of 10, 15 and 18} = { ■ }

Factor Finder

Smile **0311**



In row 1 there is a 1 above every multiple of 1 (every number).
In row 2 there is a 2 above every multiple of 2.

Copy the table and fill in the numbers along each row.

1) Which numbers are written in column 6?
   What is special about these numbers?

2) Where can you find all the factors of 8?

3) Where are all the factors of 24?

4) How can you find the common factors of 15 and 20?
   What are they?

5) Write down all the numbers with exactly 2 factors.
   These numbers are called prime numbers.

6) What can you say about their factors?

Find some more patterns on your table and write about them.

ODDS AND EVENS TABLES

1) Write out the sentences below using the correct words.
You might find it helpful to do some additions with odd and even numbers first.

(a) An **even** number added to an **even** number {always / sometimes / never} makes an {odd / even} number.

(b) An **odd** number added to an **odd** number {always / sometimes / never} makes an {odd / even} number.

(c) An **odd** number added to an **even** number {always / sometimes / never} makes an {odd / even} number.

(d) An **even** number added to an **odd** number {always / sometimes / never} makes an {odd / even} number.

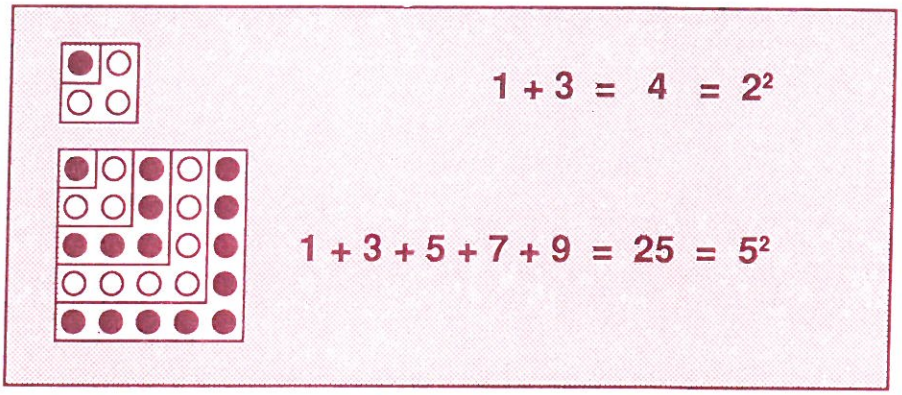
2) Use these properties of addition to complete an operation table:

Second number

First number +	ODD	EVEN
ODD	*Even*	*Odd*
EVEN		

3) *Addition, subtraction, multiplication and division are all operations.*
Can you make operation tables for the others?

Summing the Odds



Investigate some more square number patterns like this.

Explain how you can work out the sum of the first 25 **odd** numbers.

Can you make this into a general rule?