Smile 0824

THE GOLDEN RECTANGLE

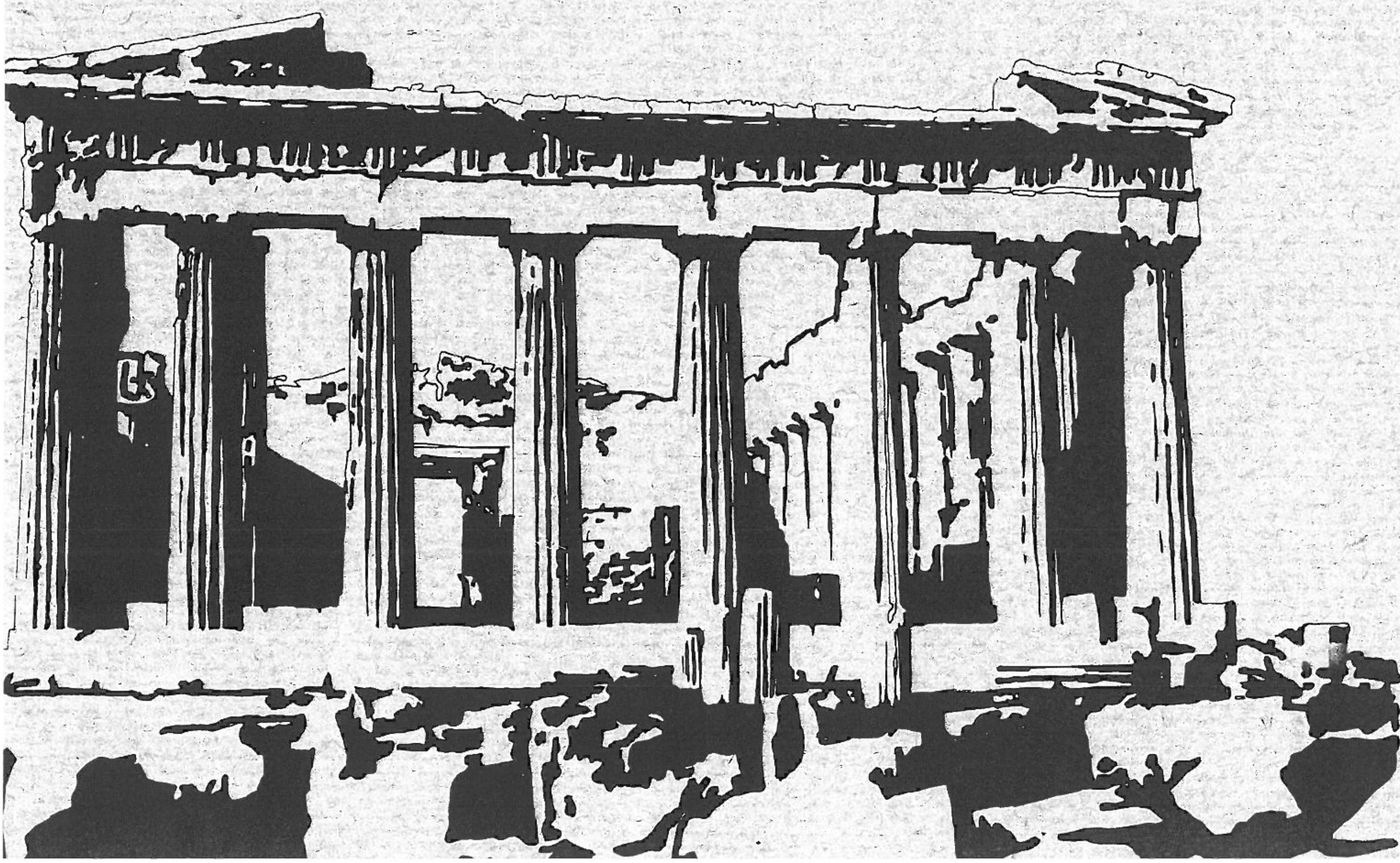
Contents
Workcards 0824 a, b, c, d
Booklets 0824 e, f, g
Reading list 0824k

You will need:
Worksheet 0824h (3 copies)
Worksheet 0824j

This work should be shared between a group of 2 or 3.

You must all start with card 0824A. When you have worked through this and understood it, you may choose what else to do.....

.....**remember** it is much more important to do two or three of the other cards thoroughly than it is to do everything in a hurry.



THE GOLDEN RECTANGLE

You will need a calculator

The large rectangle on this card is a **golden rectangle**. It is not the size which makes it golden, but the shape. The long side **(b)** and the short side **(a)** are in the right proportion – the **golden proportion**. The ratio is called the **golden ratio**.

1. Measure **a** and **b** and calculate the **golden ratio, b/a.**

In a **golden rectangle** the following equation is always true:

$$\frac{b}{a} - \frac{a}{b} = 1$$

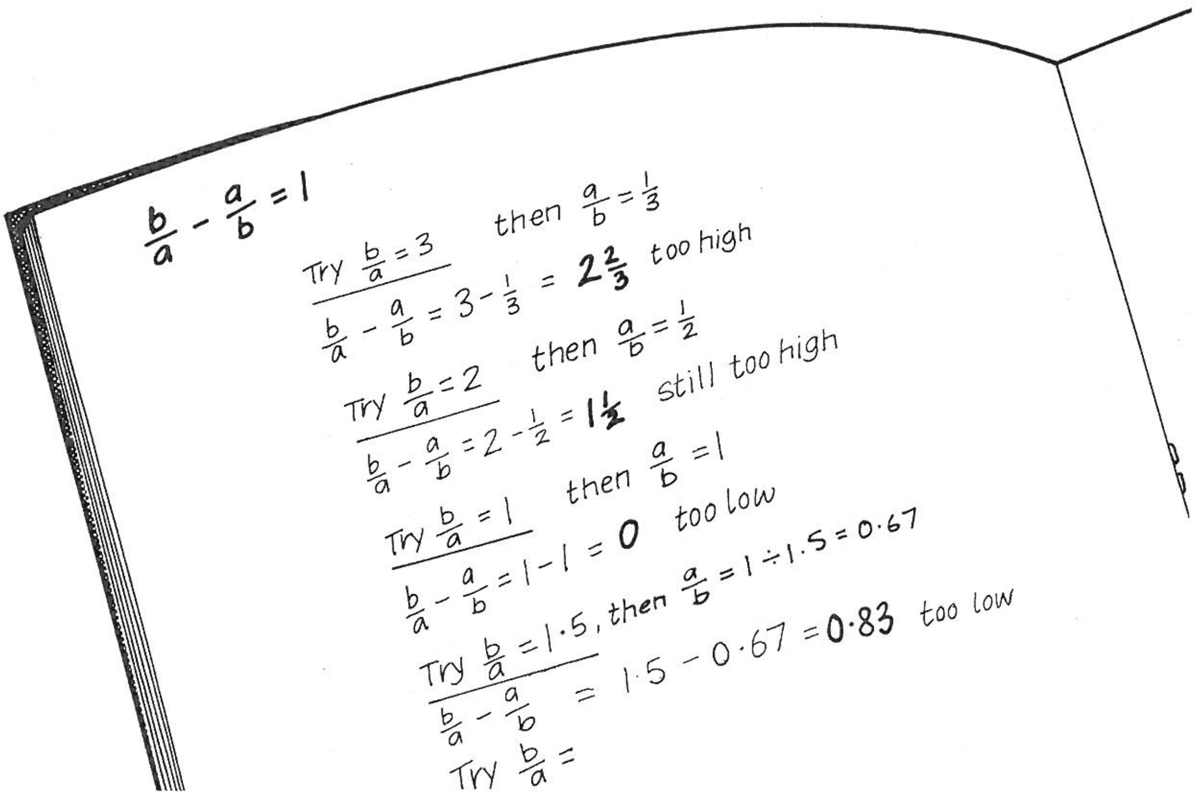
(There is an explanation to follow through on the back of the card)

2. Check that this equation is true for the values of **a** and **b** which you measured.

3. Use a calculator and a method of trial and error to find a more accurate value for the **golden ratio, b/a**. Look at the note-book below if you have not met this method before.

4. Use your results to draw an accurate **golden rectangle** in your book.

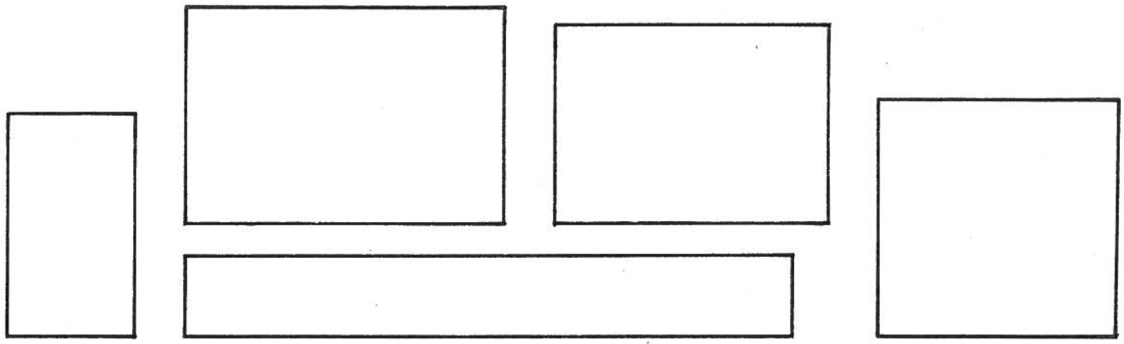
b



Two Rectangle Surveys

It is said that the golden rectangle has a more attractive shape than all other rectangles because it has the most pleasing proportions.

What do you think?

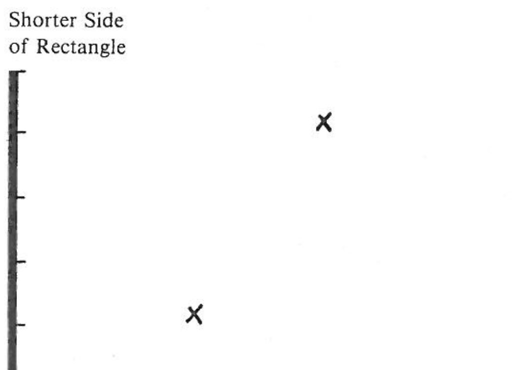


Survey 1

If the **golden rectangle** is the most pleasing one it will probably be used frequently to aid the sale of packets of washing powder, cereals and so on.

Measure the sides of as many rectangles as you can find and calculate the ratio, $\frac{\textbf{longer side}}{\textbf{shorter side}}$ for each one.

Plot your results on a scattergram.



Survey 2

Devise your own survey to find out the proportions of the rectangle which most people find the most pleasing.

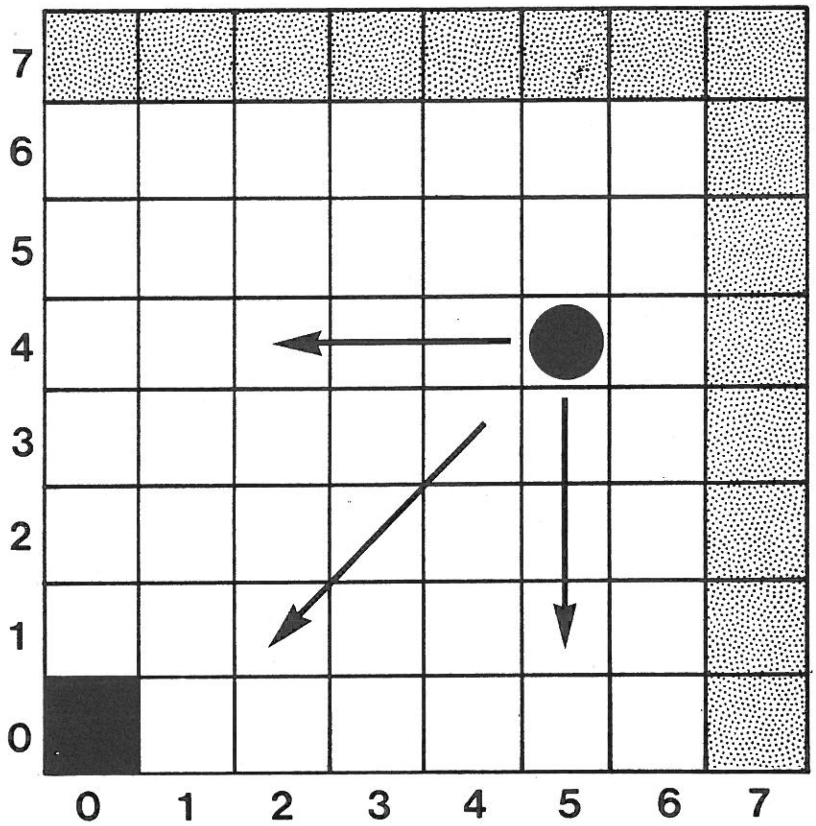
Results

Present the results of your surveys as reports using diagrams/short paragraphs/conclusions.

Cornering the Queen

A game for 2 players.

You will need an 8 × 8 board and a counter.

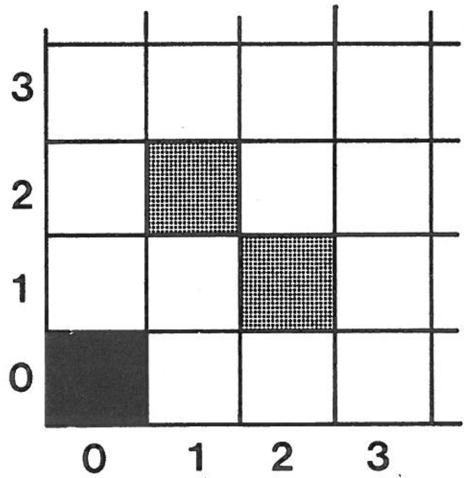


Rules

The first player puts the counter on any shaded square.
The players then take turns to move the counter any number of squares west, south, or south-west.
The player who moves onto the black square (0, 0) is the winner.

1) Play a few times.
2) If a player lands on (2, 1) or (1, 2) he **must** win. Why?
3) Where are the other winning squares?
4) Where are the safe starting squares?
5) If the same game were played on a very large board, ...?

Investigate the safe squares.



Is it Golden?

You will need worksheet 0824 J and a calculator.

A **golden rectangle** has a special property:

If the square is cut off, the remaining shape is another **golden rectangle.**

On worksheet 0824J, one of the rectangles is a **golden rectangle** — the other is not.
Can you find out which one is the golden rectangle?

1 For each of the rectangles A and B:

a) find the ratio, $\frac{\textbf{longer side}}{\textbf{shorter side}}$, for the complete rectangle.

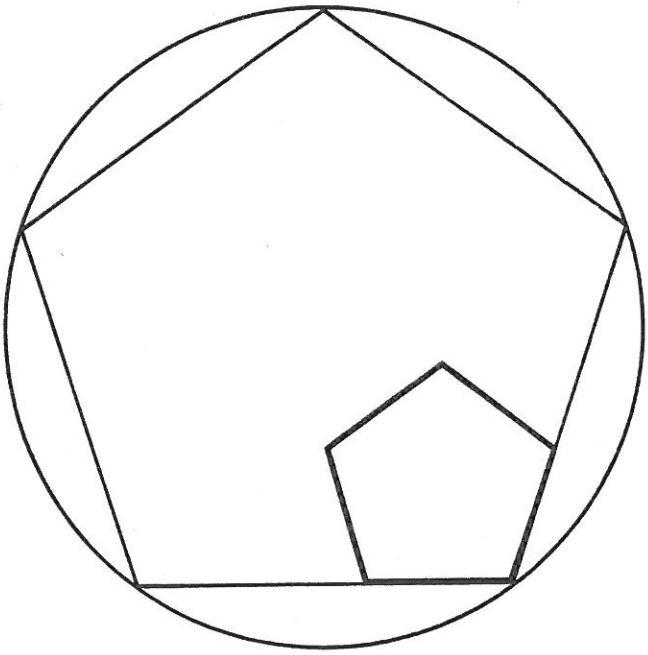
b) cut off the square.

c) find the ratio, $\frac{\textbf{longer side}}{\textbf{shorter side}}$, for the remaining rectangle.

2 In which rectangle does the ratio, $\frac{\textbf{longer side}}{\textbf{shorter side}}$, remain the same when the square is cut off?

3 You now have 2 squares and 2 smaller rectangles.
What will happen if you repeat (1) and (2) for these rectangles?
If you are not sure, do it and find out.

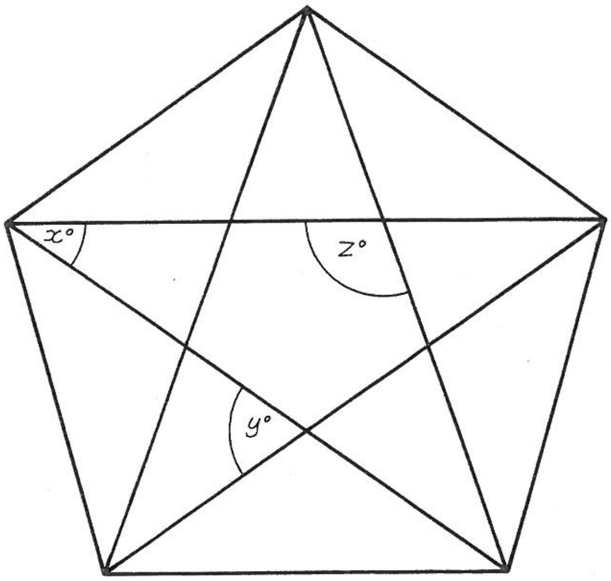
Dissection Puzzle



Take a copy of worksheet 0824 H and cut out the ten triangles and the pentagon.

Can you rearrange the pieces so that the small pentagon is in a corner of the large one? (You can use another worksheet to put the pieces on).

Angles



We want to find the connections between the angles x, y and z.

1 Take a copy of worksheet 0824, and label all the angles x, y or z (You can use the pieces from the dissection puzzle to help you).

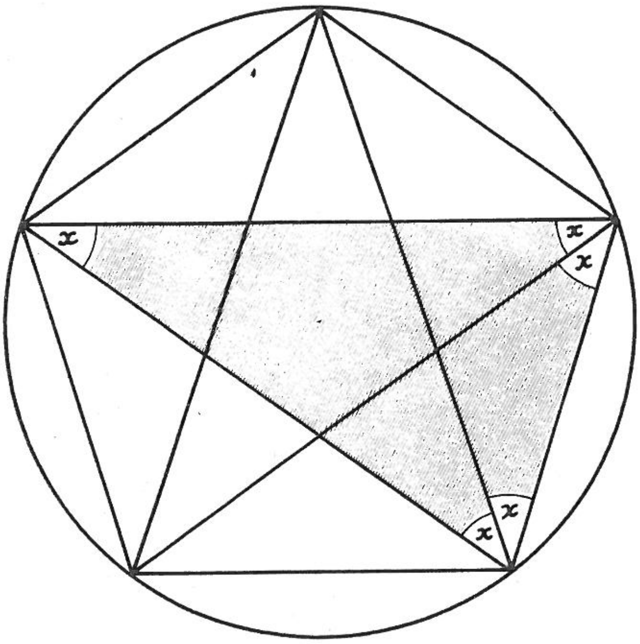
2 Copy and complete using x, y or z.

$x + y =$ ■

$2x =$ ■

$3x =$ ■

Angles



Look at the triangle in the diagram and then copy and complete:

3 The angles marked add up to ■ x

4 The angles of a triangle add up to ■°

5 Write an equation in terms of x

■ = ■

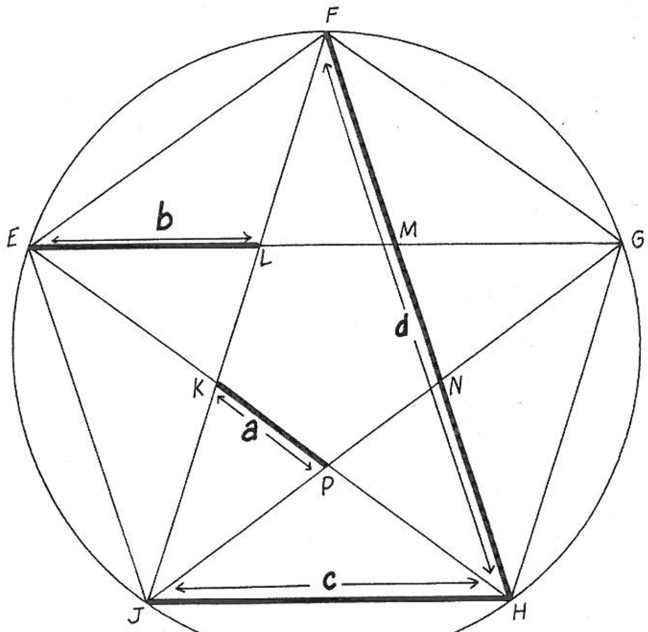
so x = ■

6 y = ■° (See question (2))
z = ■°

Check your answers for x, y and z.

7 Find another triangle and use it to find another equation. Check the equation using your answers to (5) and (6).

Lengths



There are many connections between the lengths a, b, c and d.

8 Complete this list (your dissection pieces will help):

a	b	c	d
KP	EL	JH	FH
MN	PH	MH	

9 Copy and complete using a, b, c or d.

a + b = ■ a + 2b = ■

d − c = ■ 2c − d = ■

10 There are many more equations like these. How many can you find?

Golden Ratio

11 Very carefully, measure the lengths of a, b, c and d, correct to two significant figures. Copy and complete: a = ■cm

b = ■cm

c = ■cm

d = ■cm

12 Use a calculator to work out the following ratios correct to two significant figures.

$\frac{d}{c}$ = ■

$\frac{c}{b}$ = ■

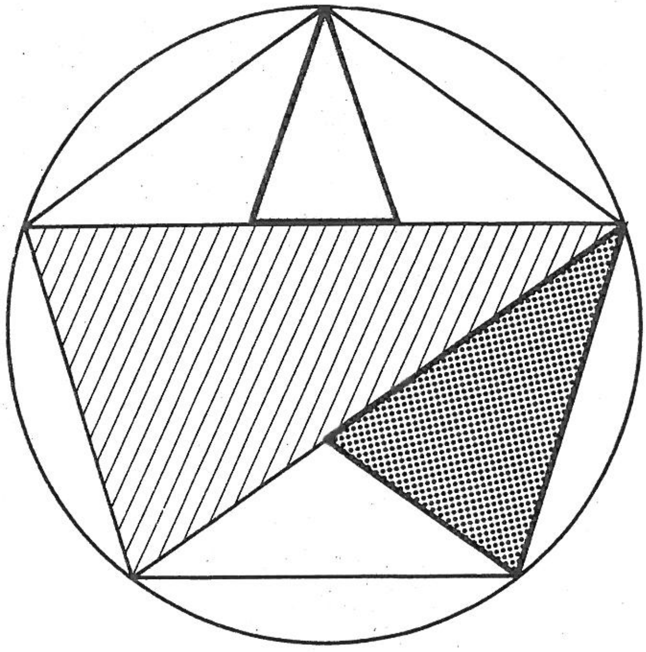
$\frac{b}{a}$ = ■

13 Why do you think the answers to (12) are all approximately the same?

14 Why do you think the answers are all approximately equal to the **golden ratio**?

These last two questions are difficult but the work on the last two pages will help.

Similar Triangles



15 Compare the angles of the three triangles shown above and write down anything you notice.

16 Why are the triangles similar?

17 Why are these equations true?

$$\frac{d}{c} = \frac{c}{b} = \frac{b}{a}$$

18 Copy and complete:
From the work on the previous two pages we know that:

$$\frac{c}{b} = \frac{b}{\blacksquare}$$

From the work on page 5 we know that

$$c = a + \blacksquare$$

Substituting this value of c in the first equation gives

$$\frac{\blacksquare + \blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}$$

19 Look at card 0824 a
Can you see why $\frac{b}{a}, \frac{c}{b}, \frac{d}{c}$, are

all equal to the **golden ratio**?

Pythagoras was a member of a society which used the pentagram as its secret sign. Find out more about this.

You will need squared paper and a calculator.

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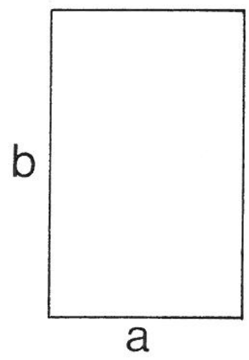
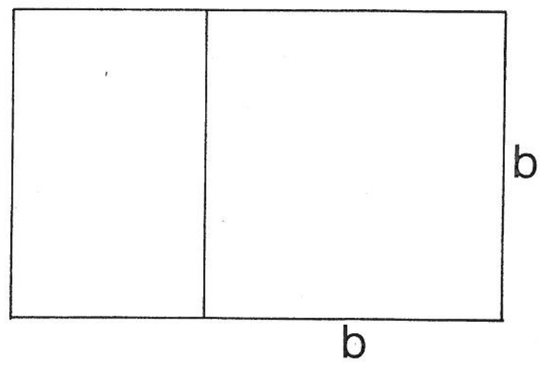
FIBONACCI SEQUENCE

The definition of a **golden rectangle** is as follows:

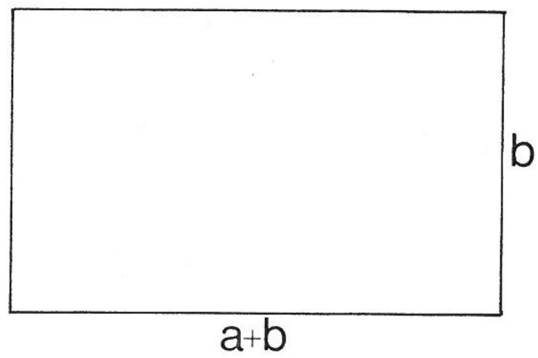
If you start with
a **golden rectangle** . . .



. . . and add a square
onto the longer side,



the new rectangle has
the same proportions
as the original one.



Follow through the working below to see why the equation

$$\frac{b}{a} - \frac{a}{b} = 1$$

is always true for **golden rectangles.**

The ratio, $\dfrac{\textbf{longer side}}{\textbf{shorter side}}$ is the same for both the original rectangle and the new rectangle above (because they are both **golden rectangles**).

So $\dfrac{b}{a} = \dfrac{a+b}{b}$

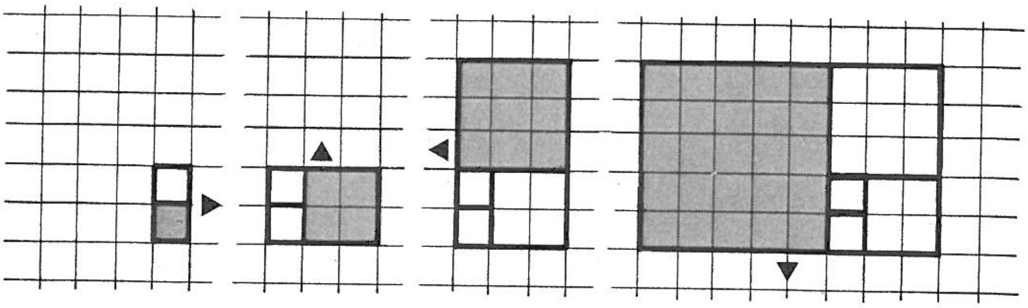
$= \dfrac{a}{b} + \dfrac{b}{b}$

$= \dfrac{a}{b} + 1$

So $\dfrac{b}{a} - \dfrac{a}{b} = 1$

Start near the centre of your paper with 2 squares the same size.

Add on squares to fit the longer side, one at a time.......



....until no more will fit on the paper
Make sure your turn anti-clockwise every time.

1 If the first two squares are 1 × 1 squares, the next is 2 × 2 and the next is....
How does the sequence continue? Give the sizes of the first eight squares.

2 Measure the sides of your final rectangle and calculate their ratio.

Any comments?

1, 1, 2, 3, 5, 8, 13, -, -, -, -

This is called the **Fibonacci Sequence.**

3 How is the **Fibonacci Sequence** made?

4 What are the next three terms (after 13)?

5 Look at the ratio of successive terms.....

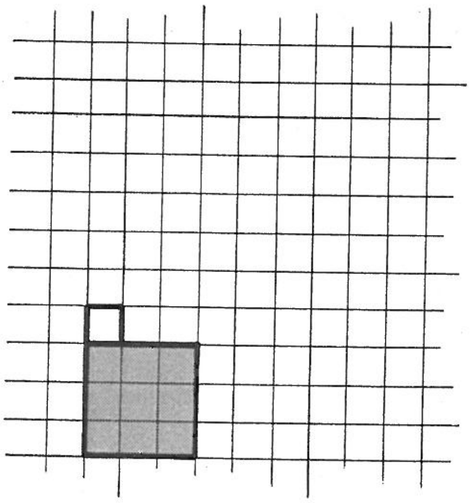
$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}$, _, _, _

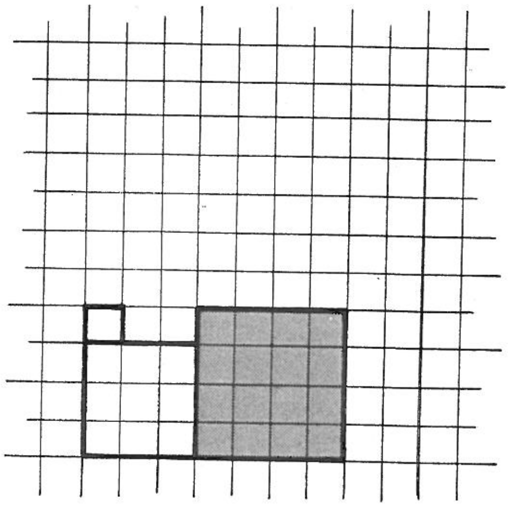
Use a calculator to find the first eight terms in this sequence.

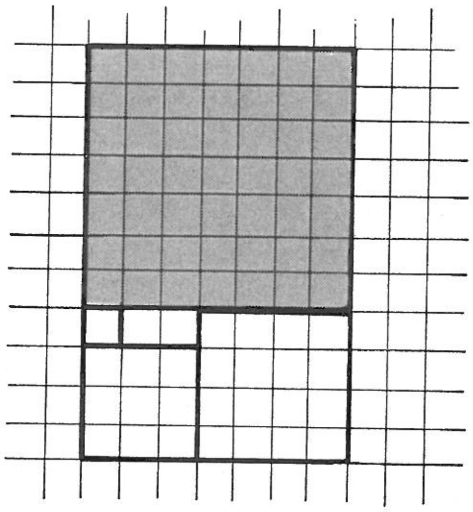
6 What is the connection between questions (3) and (4) and question (1)?

7 What is the connection between question (5) and question (2)?

What happens if you start with 2 squares which are not the same size?







1, 3, 4, 7, 11,.....

8 How does this Fibonacci-type sequence continue? Calculate the ratios of successive terms.

9 If you kept on drawing squares like this, what would the shape of the rectangle become?

10 Investigate the questions above for other Fibonacci-type sequences

e.g. 2, 7, 9, 16, 25,.........

You should have noticed that in **any** Fibonacci-type sequence, the ratio of successive terms gets closer and closer to the **golden ratio**...

...... but **why** does this happen?

.... a, b, c, ...

Suppose a, b and c are three successive terms.

If successive ratios get closer and closer, it is approximately true that:

$$\frac{b}{a} = \frac{c}{\blacksquare}$$

If the sequence is a Fibonacci – type sequence

$$c = \blacksquare + \blacksquare$$

Substituting this value of c in the first equation gives

$$\frac{b}{a} = \frac{\blacksquare + \blacksquare}{\blacksquare}$$

11 Look at card 0824 a
Can you explain the connection between the **Fibonacci Sequence** and the **golden ratio?**

Leonardo Fibonacci

Leonardo Fibonacci was born about 1175 in Pisa, which was a commercial centre of Italy. His father was a merchant, which probably accounted for Leonardo's early interest in arithmetic. Trips to Egypt, Sicily, Greece and Syria brought him in contact with Eastern and Arabic mathematics and Fibonacci became thoroughly convinced of the practical superiority of the Hindu-Arabic methods of calculation. In 1202 he published his famous work *Liber abaci.* This book strongly advocated the Hindu-Arabic notation and did much to encourage the introduction of these numerals into Europe.
Fibonacci published two other important books: *Practica geometricae* in 1220 and *Liber quadratorum* in 1225.

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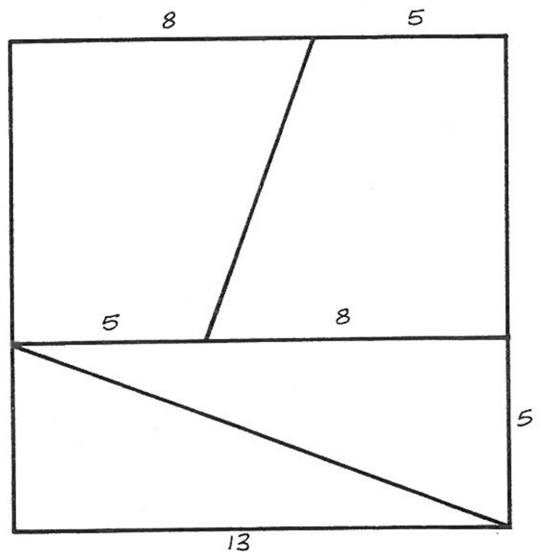
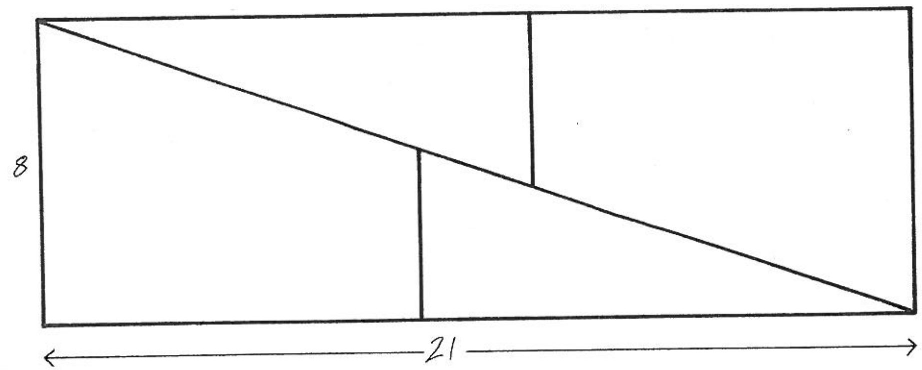
A Dissection Problem



1 Draw this square, cut along the lines, and rearrange the 4 shapes to make a rectangle.

You probably made this rectangle?..............

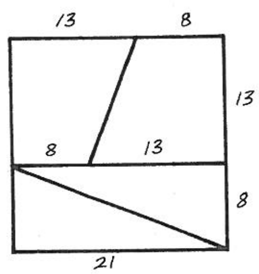


..... but something is wrong.

2 What are the areas of the original square and the rectangle above? So what is wrong?

3 Repeat the dissection for a 21 × 21 square.

What is the area of the rectangle?



By how much do the areas of square and rectangle differ?

4 Repeat again for an 8 by 8 square.

There is a connection between these dissection problems and the Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21........

5 Can you find this connection?

6 In each case the area of the square is 1 square unit more or less than the area of the rectangle.

Give the sizes of 3 more squares where this happens and show the dimensions of each dissection on a diagram.

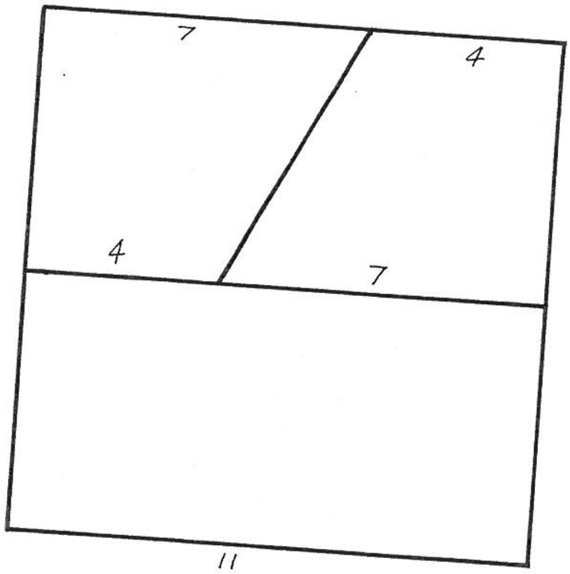
A Fibonacci-type sequence can be constructed from any two starting numbers.

eg. 1, 3, 4, 7, 11, 18, 29.....

A dissection, similar to the previous ones, can again be designed using 3 consecutive numbers from the sequence.

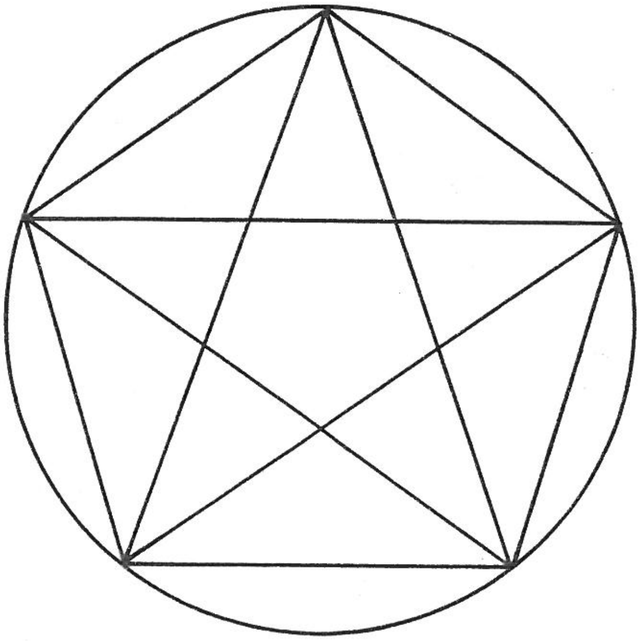
The area of this square is 121.........

but the area of the rectangle will **not** be 120 or 122.



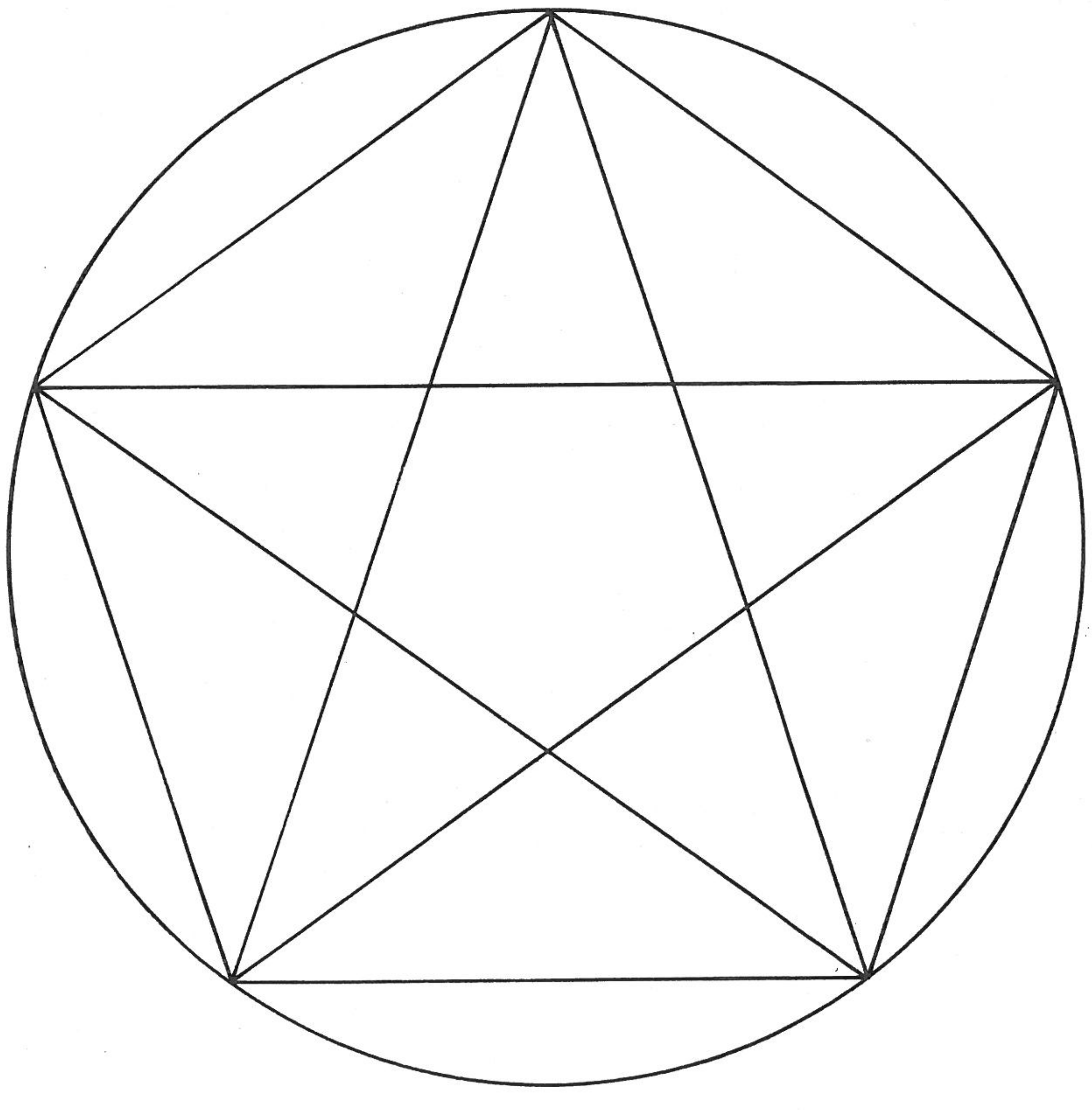
7 Investigate this for different Fibonacci-type sequences.

The Pentagram



You will need 3 copies of Worksheet 0824h , calculator and angle indicator.

PENTAGRAM WORKSHEET



Rectangle Worksheet

Rectangle A

Rectangle B

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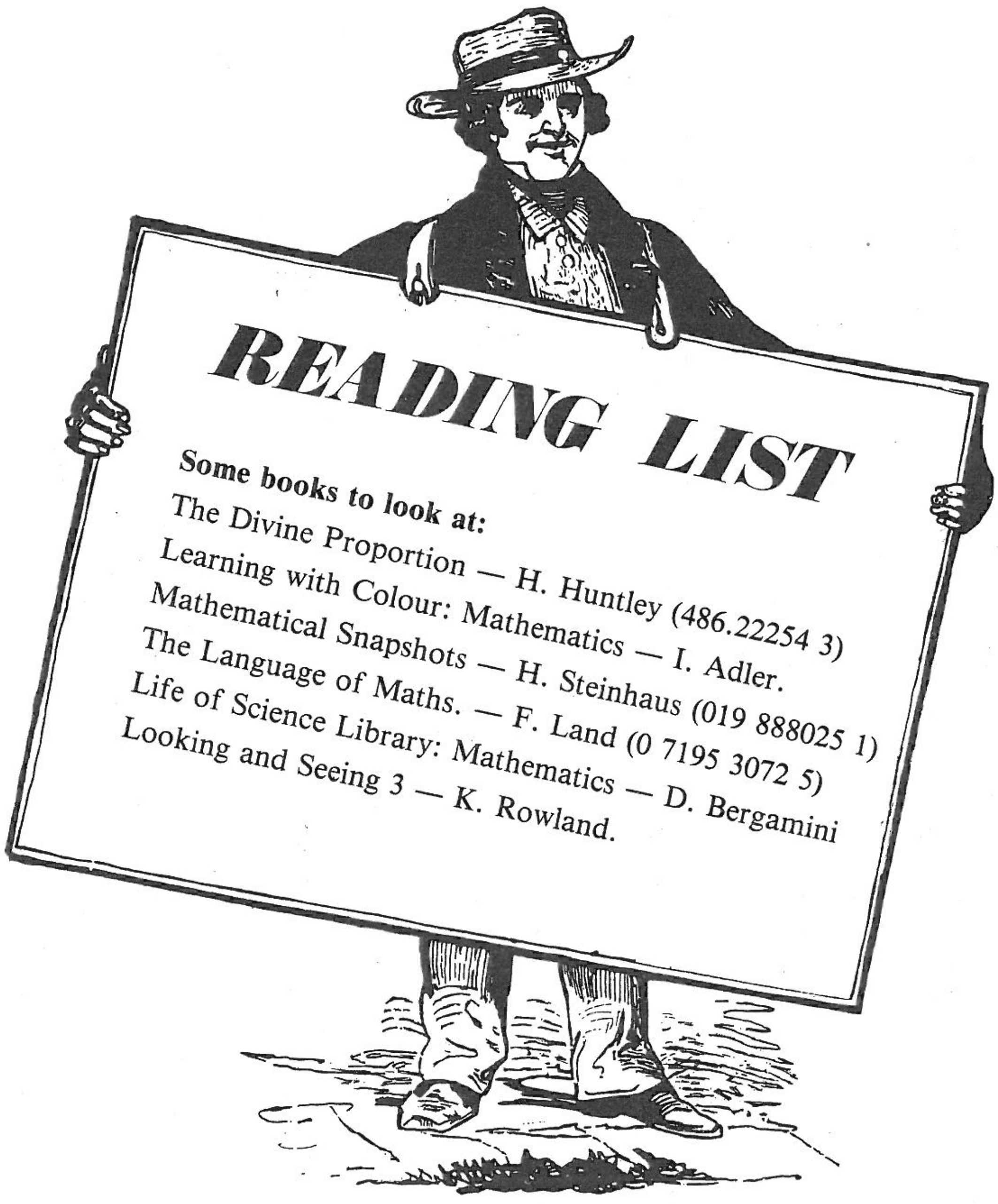
READING LIST
Some books to look at:
The Divine Proportion — H. Huntley (486.22254 3)
Learning with Colour: Mathematics — I. Adler.
Mathematical Snapshots — H. Steinhaus (019 888025 1)
The Language of Maths. — F. Land (0 7195 3072 5)
Life of Science Library: Mathematics — D. Bergamini
Looking and Seeing 3 — K. Rowland.