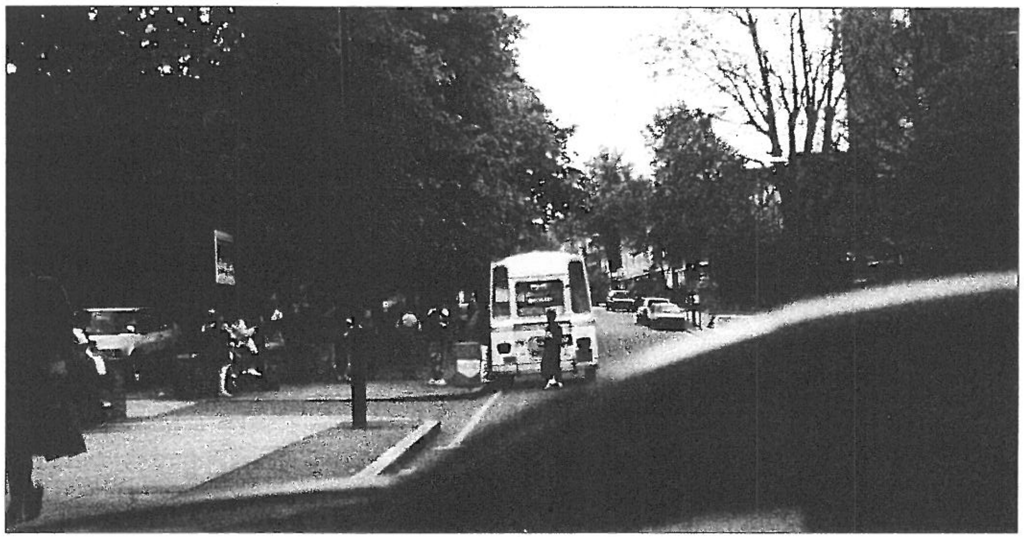
SMILE WORKCARDS

Using Graphs Pack Two

Contents

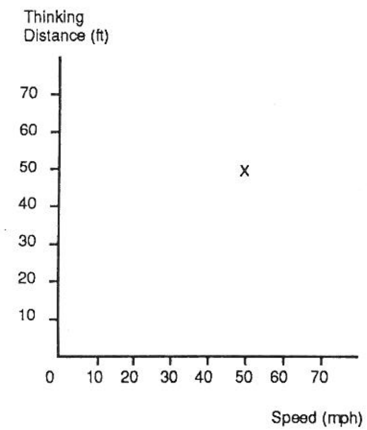
Thinking and Braking

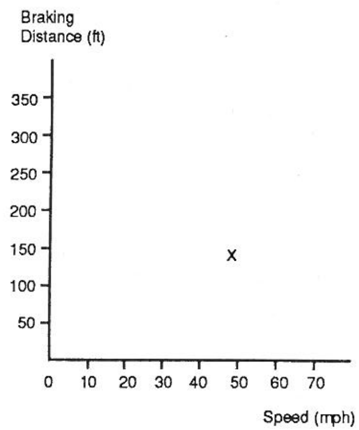
You will need a copy of the Highway Code and graph paper.



Look at the back cover of the Highway Code. It shows the shortest stopping distances.

The information from the table on page 14 of the Highway Code could be shown on graphs.





Use graph paper to plot:

- thinking distance against speed
- braking distance against speed

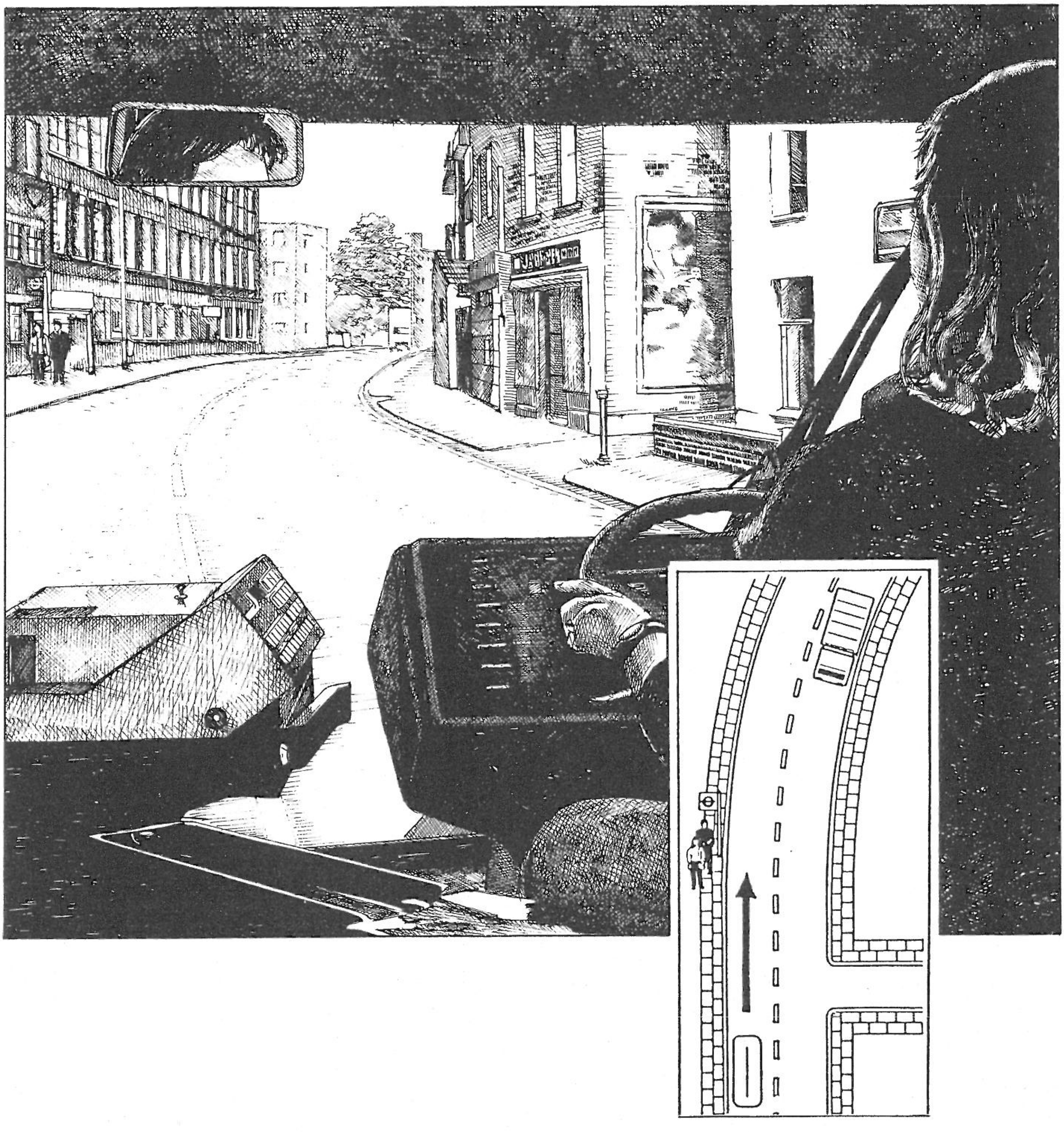
If the driver was tired, which of the two graphs would be different?

On the same axes draw another line to show how it might look.

What would happen if the car had worn brakes?
Show this on the graph of braking distance against speed.

The 'smoothing out' principle

An activity for two people.

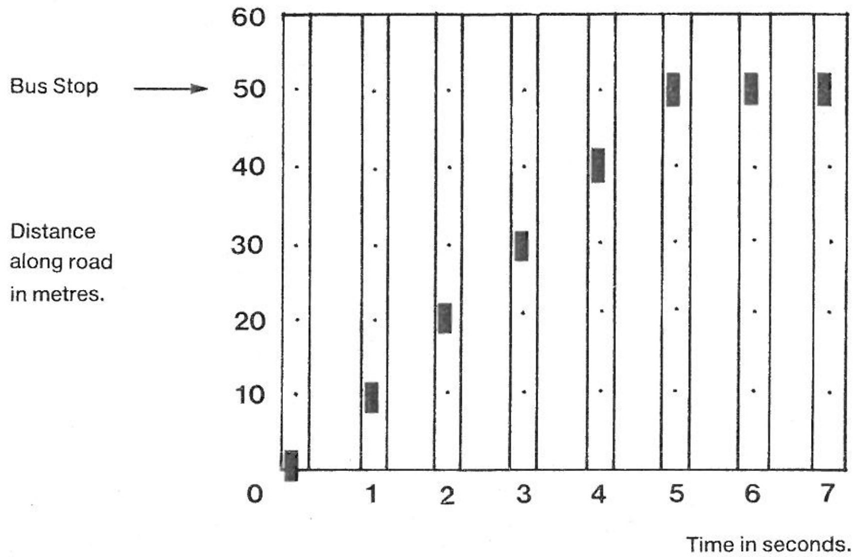


A bus is travelling at 10 metres per second towards a bus stop 50 metres away.

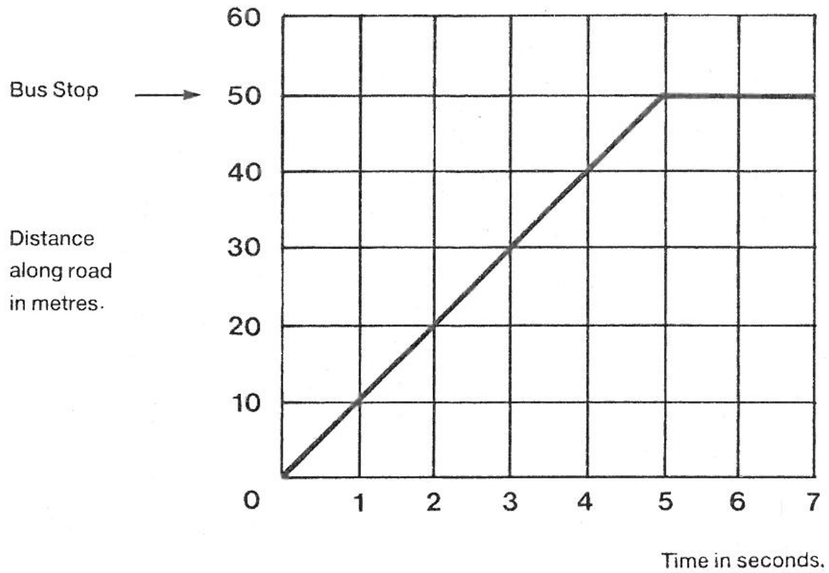
What do you think the distance-time graph will look like for the bus?

Your graph may have looked like one of the following:

A 'One second photograph' graph.



A 'Cine film' graph.



During the first 5 seconds, the bus is travelling at a constant speed of 10 m/s.

Then, on arrival at the bus stop, it *suddenly* stops dead in it's tracks!

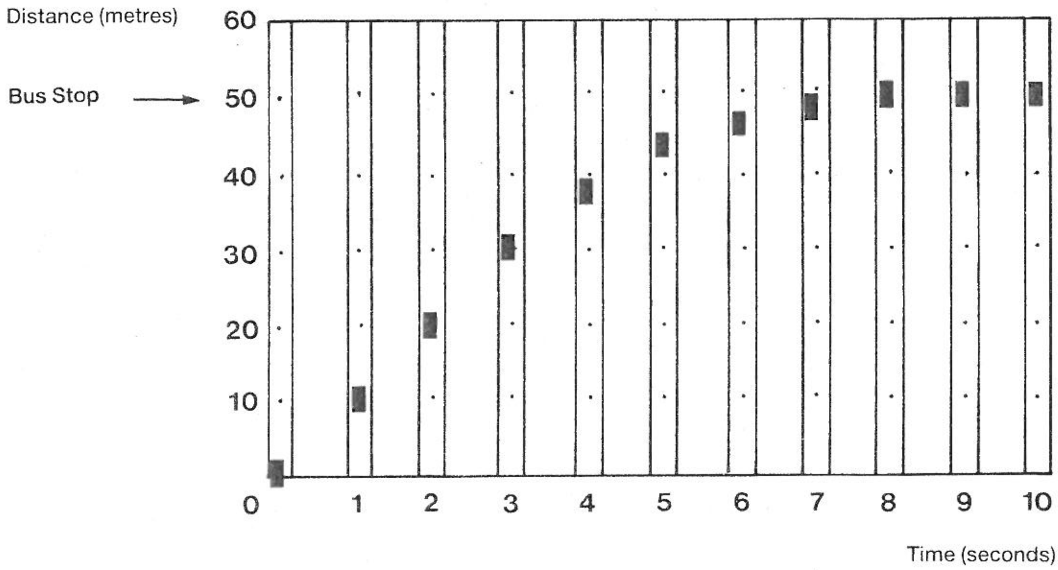
This is shown by the way in which the graph makes a sudden turn.

What would happen if a real bus behaved in this manner?

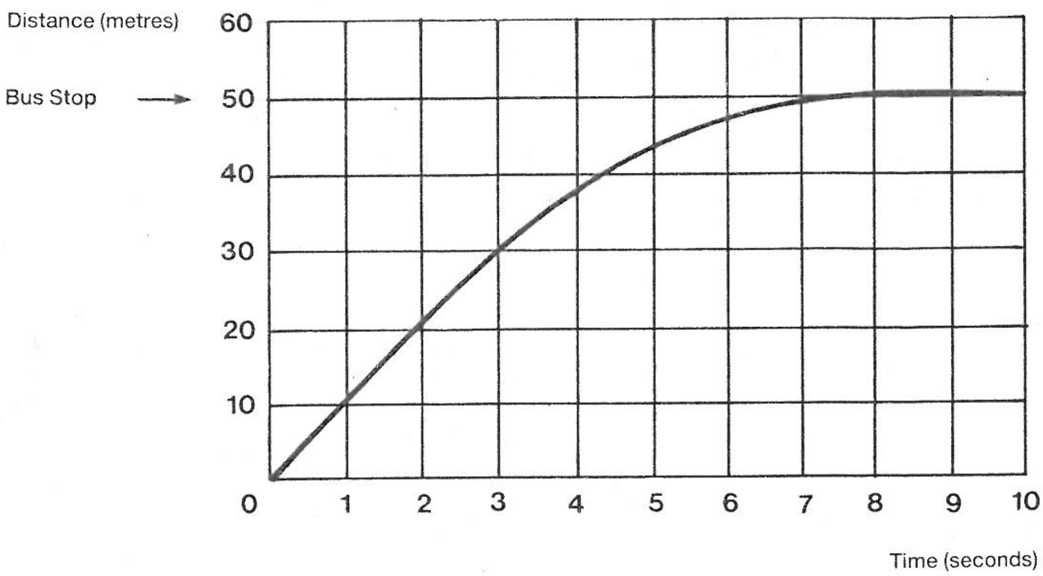
In the real world, a bus begins to slow down *before* it reaches the stop, and *gradually* comes to a halt.

The following graphs show this.

'One-second photographs' of a real bus.



'Cine film' graph of a real bus.



1) a. When does the driver begin to brake?

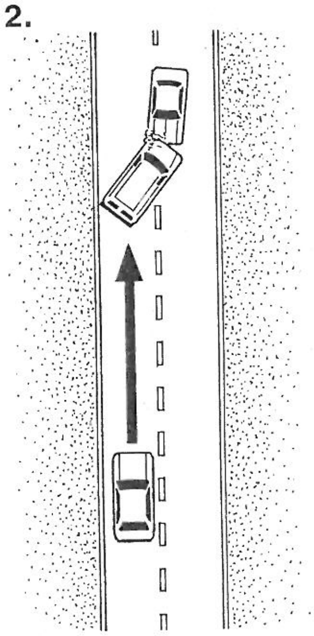
b. When does the bus actually stop?

c. For how long is the bus decelerating? (slowing down)

Notice that the above graphs are smooth curves. This is because the bus changes speed gradually.

For each of the situations drawn below, sketch a distance-time graph which will describe the events of the next few seconds . . .

2.



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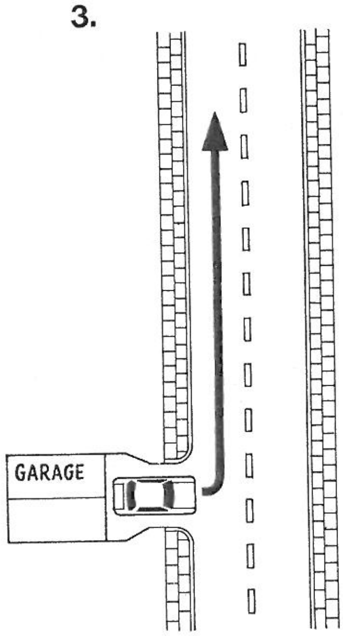

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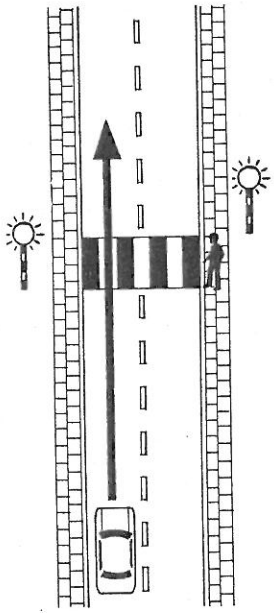

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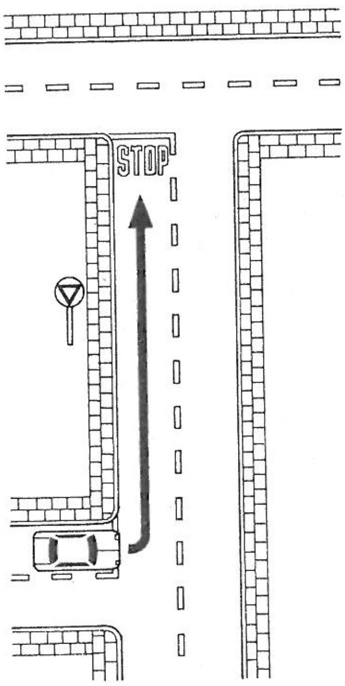

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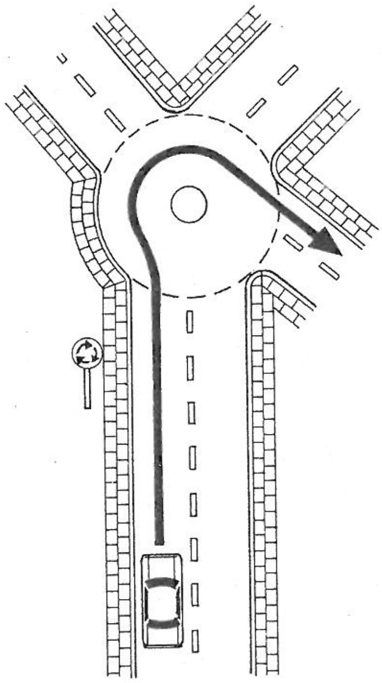

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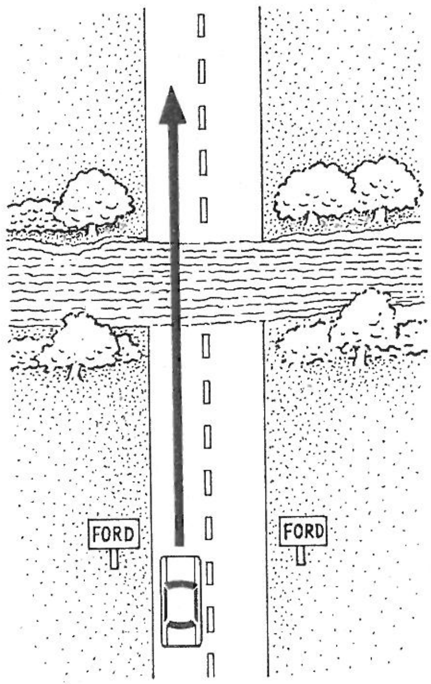
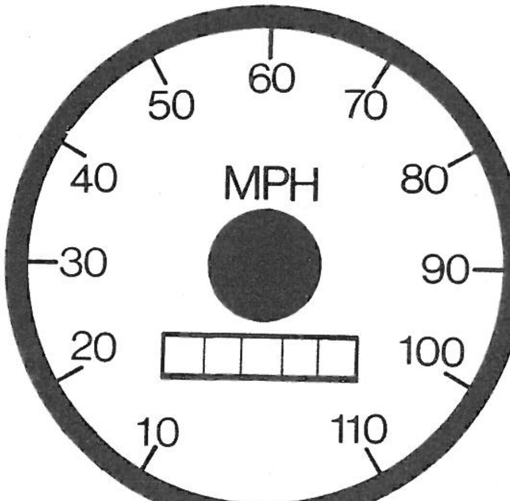


This activity is taken, with kind permission, from [illegible], a collection of teaching materials by Malcolm Swan ISBN 0906126 11 8

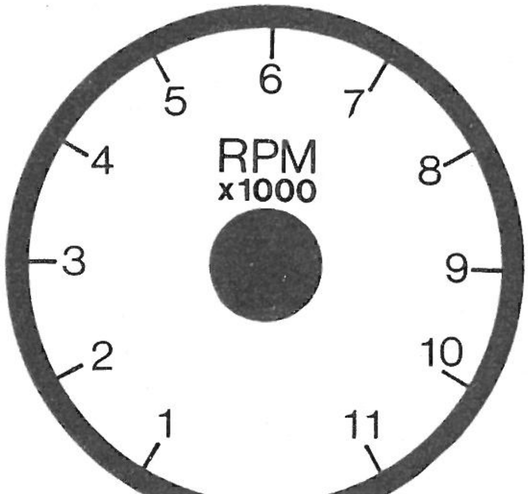
Motorcycle Ratios

This activity is about the various speeds of a motorbike engine.

An engine speed (in revolutions per minute – rpm) does not always produce the same vehicle speed. It depends upon which gear is being used. The bottom gear is engaged when the motorbike is starting off, and the top gear is engaged when travelling on the open road. A bike may have as many as six different gears.



Speedometer



Rev counter
(measures engine speed)

Complete these two lists:

In sixth gear *(top gear)*:

6000 rpm corresponds to 60 mph
4000 rpm corresponds to 40 mph
5000 rpm corresponds to 50 mph
8000 rpm corresponds to ■ mph
■ rpm corresponds to 35 mph
6500 rpm corresponds to ■ mph

In first gear *(bottom gear)* the engine has to work harder so:

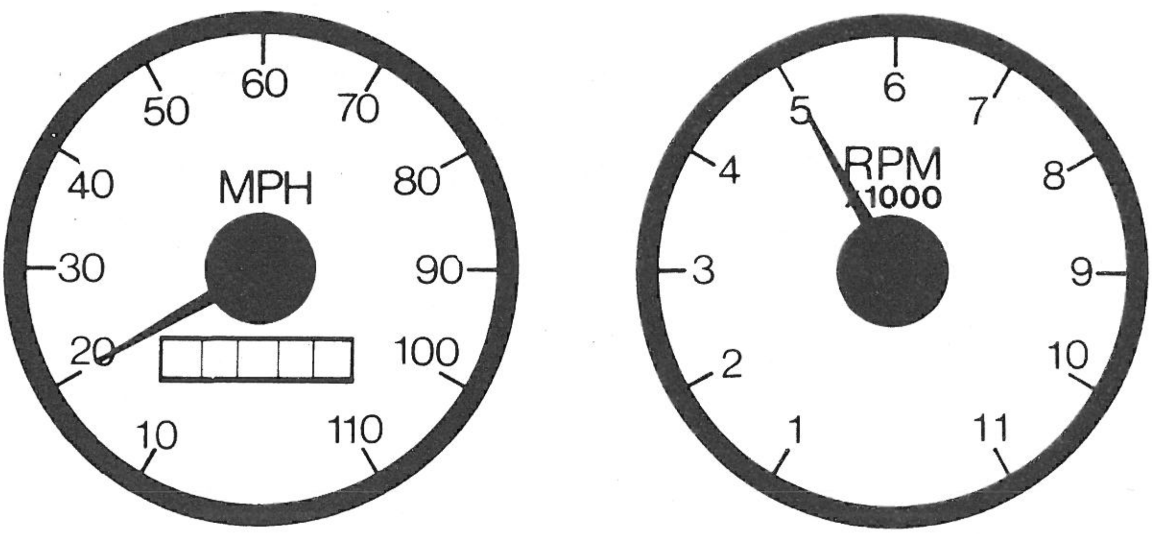
6000 rpm corresponds to 20 mph
■ rpm corresponds to 10 mph
1500 rpm corresponds to ■ mph
■ rpm corresponds to 12½ mph

Running the engine at more than 9500 rpm damages it.

What is the top speed of the bike in sixth gear?

What is the top speed in first gear?

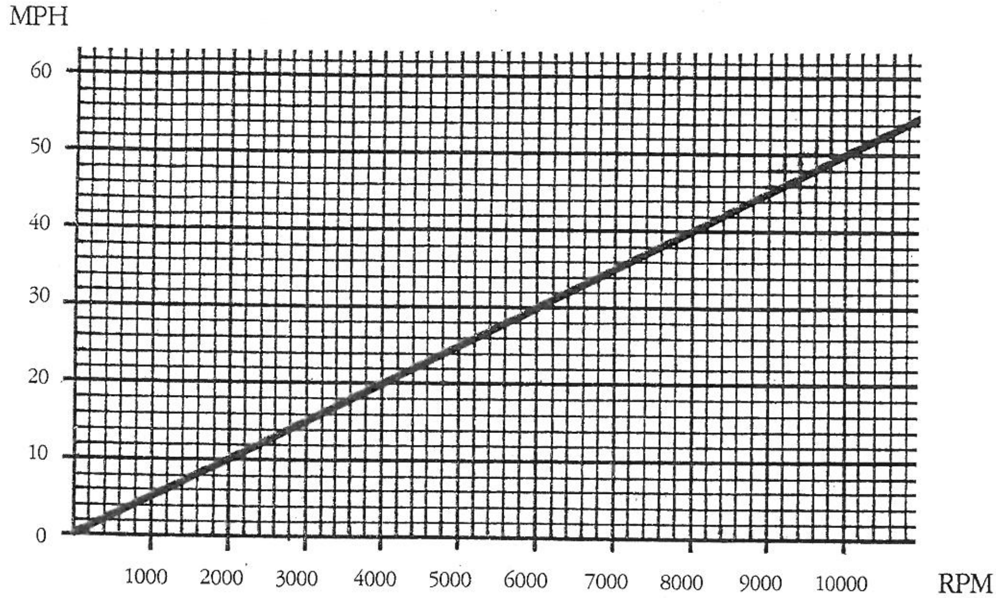
In second gear:



The diagram above shows the correspondence of mph to rpm in second gear.

What is the top speed in second gear?

In third gear:



The graph above shows the correspondence of mph to rpm in third gear.

What is the top speed in third gear?

In fourth gear:

In this gear 37½ mph corresponds to 6000 rpm.
Draw a graph for speed against revs in fourth gear.
Use the graph to find top speed in this gear.

On the same graph:

Add a new line to your graph for each of the other gears.

At 30 miles per hour:

Work out the engine speeds at 30 mph:

(a) In first gear
(b) In second gear
(c) In third gear
(d) In fourth gear
(e) In sixth gear

Estimate the engine speed at 30 mph in fifth gear.

The overlap between gears:

The engine will not run smoothly below 2000 rpm. This means, for example, that in sixth (top) gear the top speed is 95 mph and the lowest speed is 20 mph.

Find the top and lowest speeds for first, second, third, and fourth gears.

Show this information on a graph.

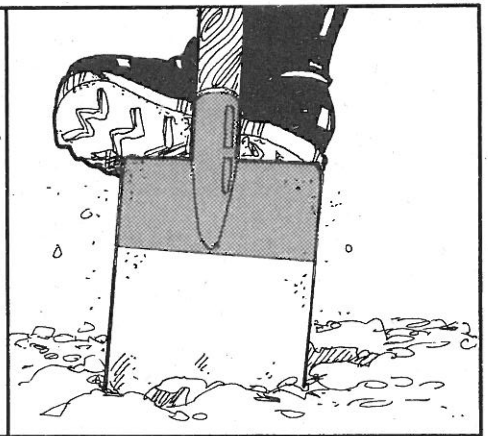
In order to accelerate quickly it is necessary to keep the revs as high as possible.

At what speeds would you change gears in order to get the maximum acceleration?

Smile 1774

Modelling with Graphs

An activity for a group of 3 or more.



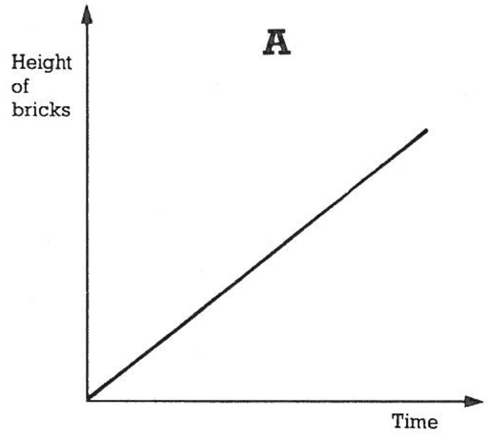
In sections A,B and C a situation is described in words and this is followed by six graphs.
For each situation you are asked to

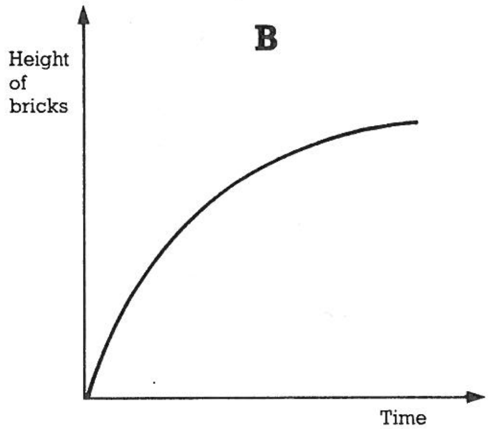
1. Choose the graph which shows the situation most realistically.
2. Explain your choice in words.
3. If you do not think any of the graphs are realistic draw your own and explain it.

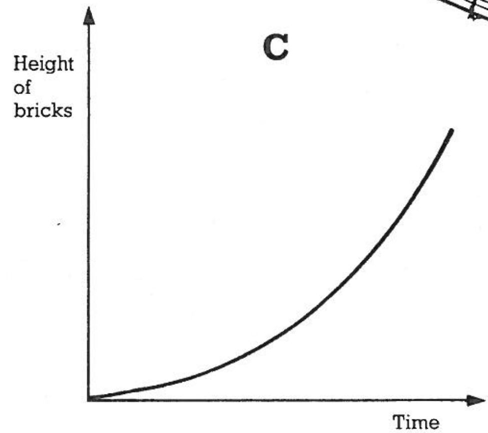
This activity is based upon suggestions from *The Language of Graphs*, a collection of teaching materials by Malcolm Swan (ISBN 0906126118).

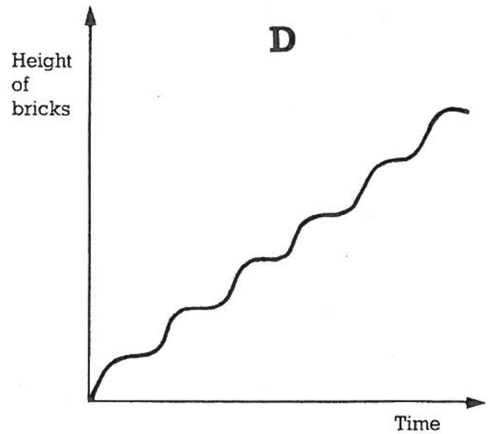
Raising bricks

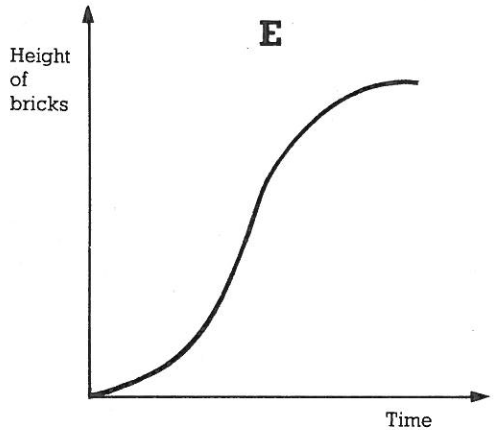
Every morning the bricklayer has to hoist the bricks to the top of the building.

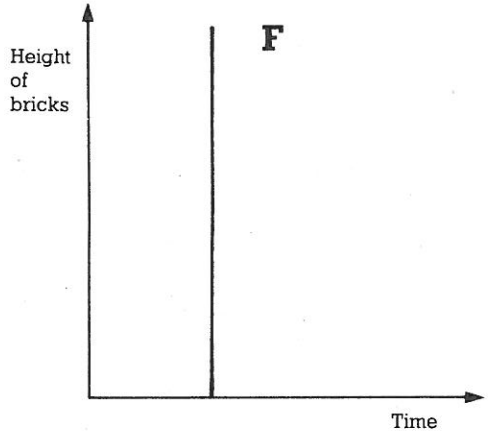


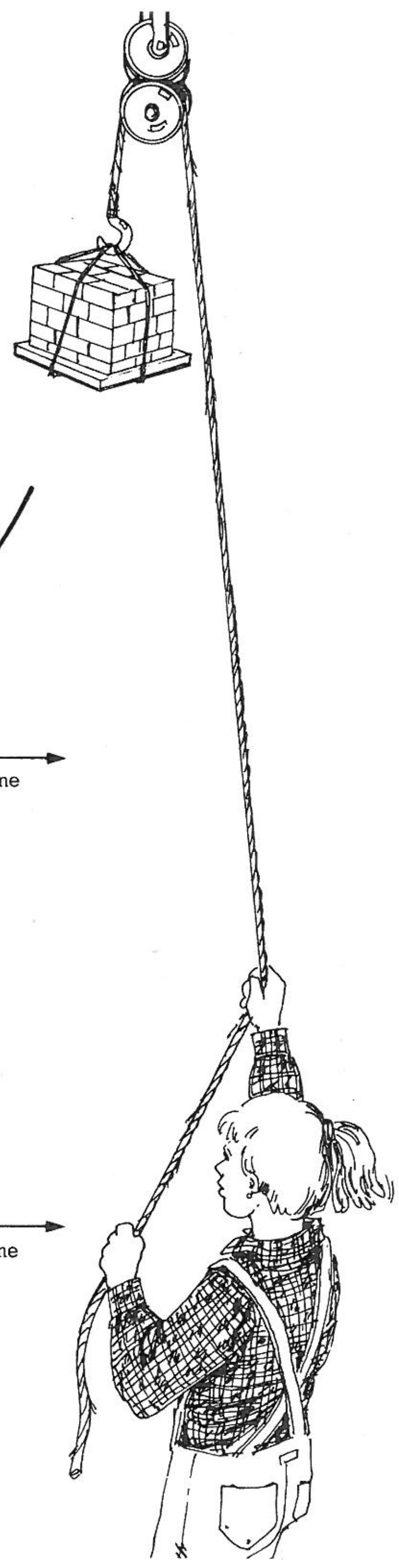






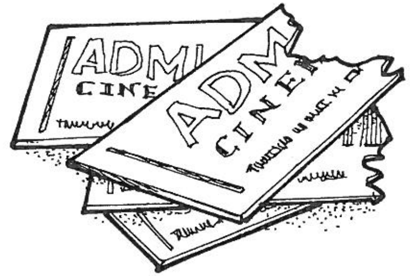


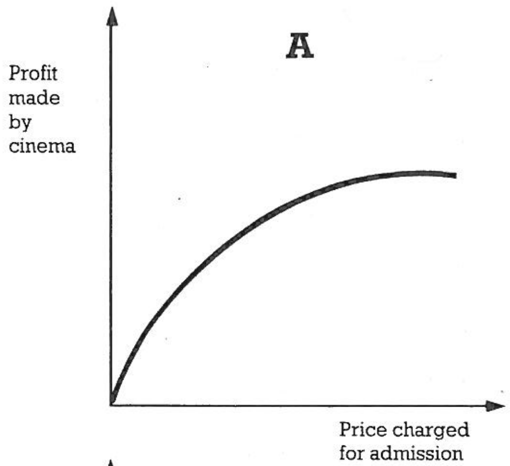


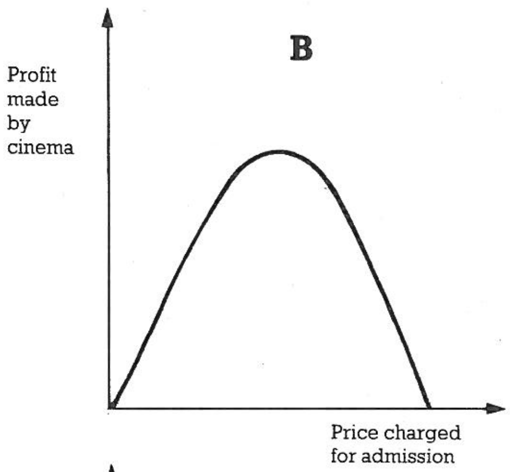


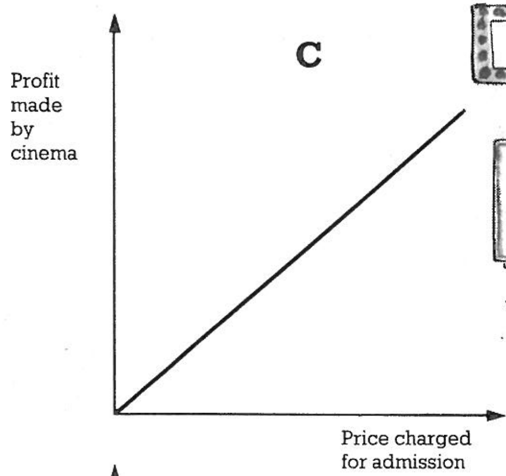
Cinema admission prices

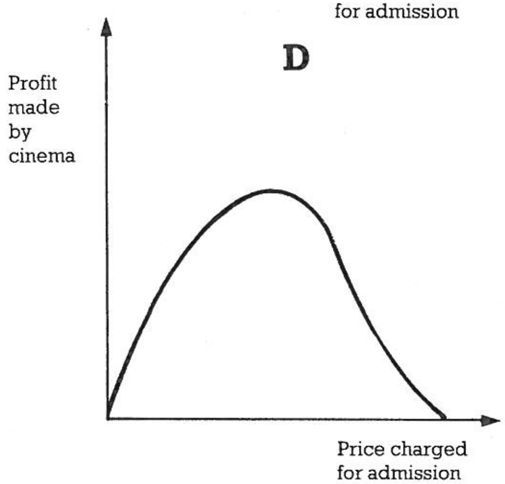
The manager of a certain cinema wishes to know what effect changing the prices of admission will have on profits.

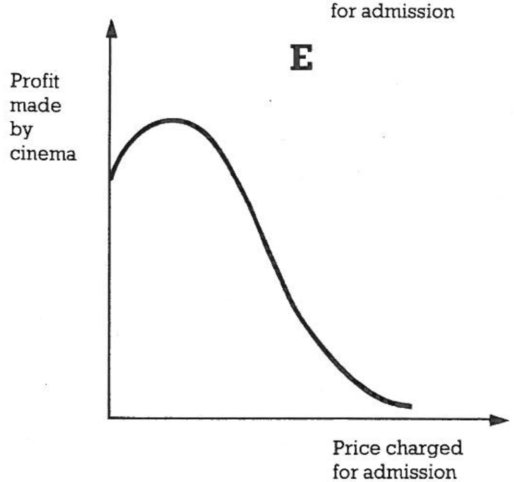


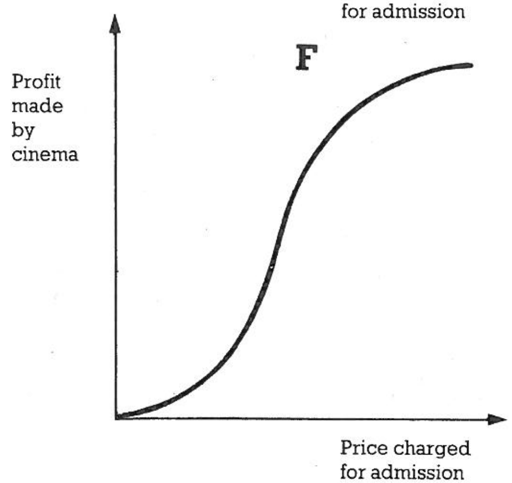






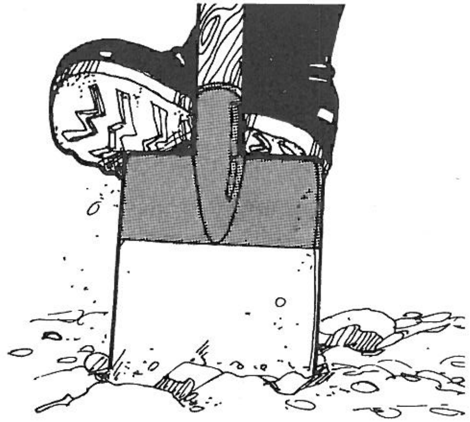


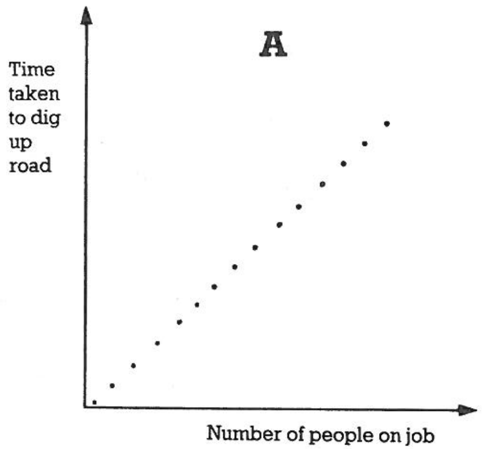


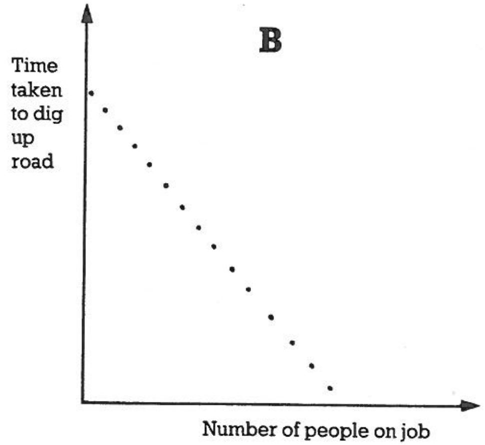


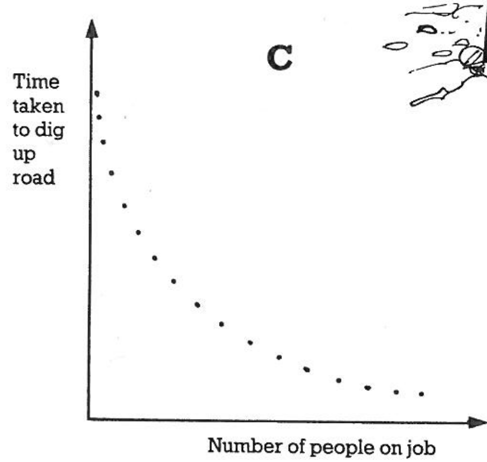
Digging up the road

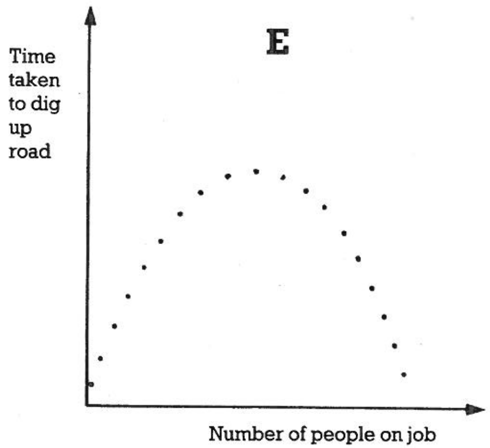
A number of people are required to dig up a stretch of road.

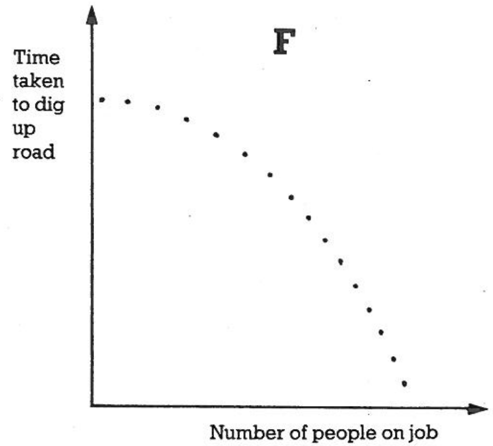












Suggestions for modelling by graphs

Choose at least three of the following situations, decide what the variables are and draw an appropriate graph.

1 The value of a car depends upon its age.

2 Your height varies with your age.

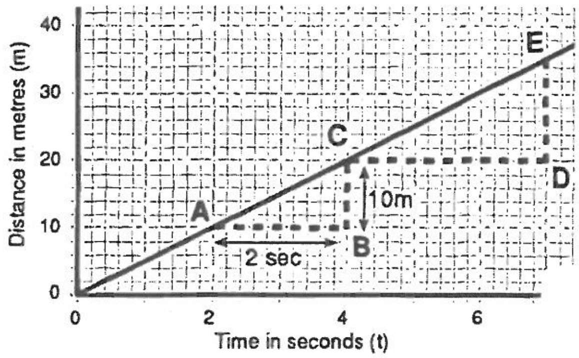
3 The amount you learn depends upon the difficulty of your homework.

4 Enjoyment of a cup of coffee varies with its temperature.

5 The amount of daylight we get depends upon the time of year.

6 The number of people in a school building varies during the day.

A runner is training for the London marathon. This graph shows her progress over 7 seconds.



Triangle **ABC** can be used to calculate the gradient which gives her speed between t = 2 and t = 4.

Her speed of running $= \frac{10}{2}$

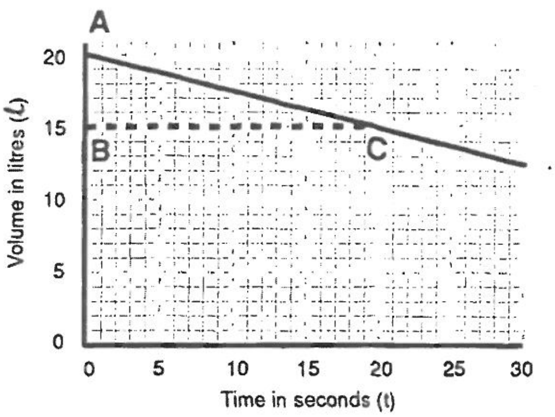
$=$ **5 metres per second.**

This can be written as **5m/s** or **5ms⁻¹**.

☐ Use triangle **CDE** to calculate her speed between t = 4 and t = 7.

The **gradient** is 5 between any two points on this line. This shows that she was running at a **constant speed** of $5ms^{-1}$.

Water is escaping through a hole in a water tank. This graph shows the volume of water in the tank over a period of 30 seconds.



Triangle **ABC** can be used to calculate the rate of water loss between t = 0 and t = 20.

The rate of water loss $= \frac{5}{20}$

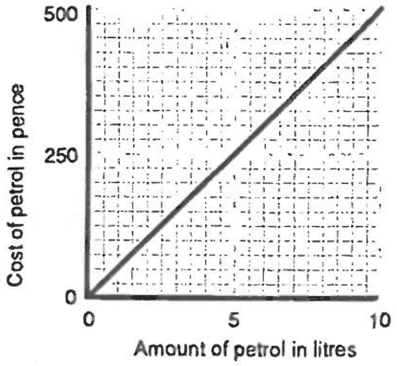
$=$ **0·25 litres per second**

$=$ **0·25ℓs⁻¹**

The graph shows that the tank is losing water at a **constant rate** of **0·25ℓs⁻¹**

1. In each of the following graphs, describe:
   - what the graph shows.
   - what information the gradient gives.

a)



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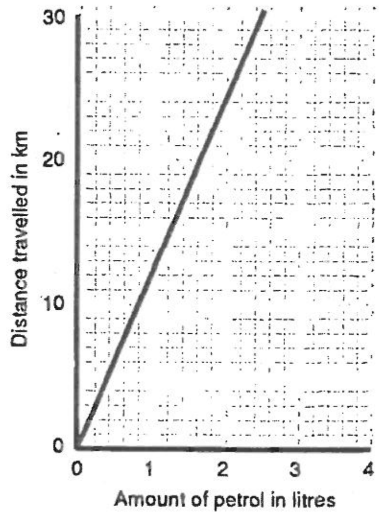

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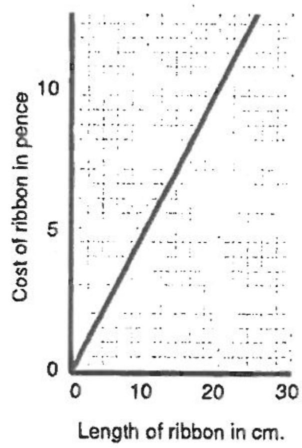
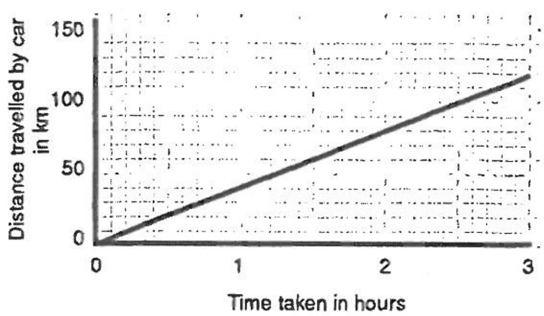
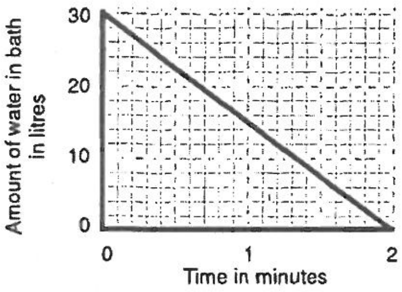


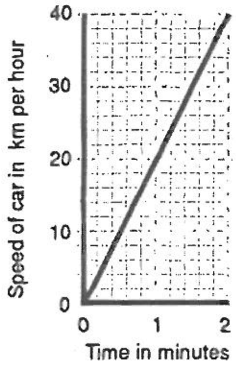
d)



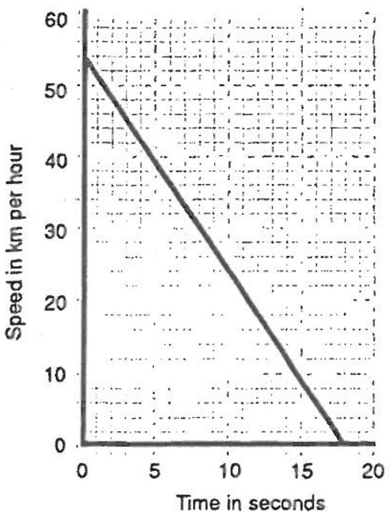
e)



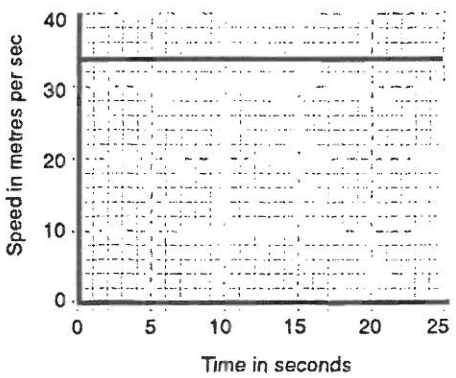
f)



g)



h)



2. In each of the following draw graphs to show each situation.

a) A girl running to catch a bus at 6 metres per second for 30 metres.

b) A lorry travelling along a motorway at 80 kilometres per hour for two hours.

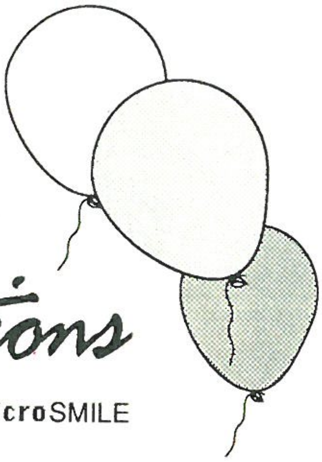
c) A child's paddling pool being filled at the rate of 0.4 litres per second for 6 seconds.

d) A bucket contains one litre of water. For the next 12 seconds water is poured in at a rate of $^1/_2$ litre per second.

e) A bath contains 30 litres of water. The water is run out at a rate of 2 litres per second until the bath is empty.

f) A motorbike has stopped at traffic lights. The lights change and it starts to move. Its speed increases by 10km per hour each second. Show how its speed changes in the first 5 seconds.

Party Solutions



You will need the **Micro**SMILE program **Regions**.

At an end of term class party, **£10** is set aside for the drinks.

> A **2** litre bottle of orangeade costs **75p** and
> a **3** litre bottle of cola costs **£1.25**.
> *(Paper cups are provided free.)*

There are **20** pupils attending and there must be at least **1** litre of drink per pupil.

To give everyone a choice there must be at least **2** bottles of each drink available.

You need to calculate how many bottles you could buy.

Let x = number of bottles of orangeade
and y = number of bottles of cola.

1. Explain how these inequalities are connected to the problem. *The first one has been done for you.*

   a) $x \geqslant 2$ *(You have to have at least 2 bottles of orangeade)*

   b) $y \geqslant 2$

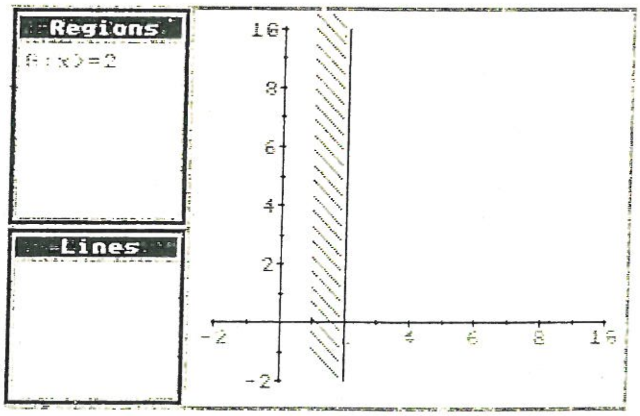
   c) $2x + 3y \geqslant 20$

   d) $75x + 125y \leqslant 1000$

The **Micro**SMILE program **Regions** uses linear programming to solve problems like these.

2. Load **Regions** and then input the 4 inequalities from above.

   (For $\geqslant$ use $>$ followed by $=$)



3. a) Press **P** for Points to find the possible combinations.

   b) Record the combinations of bottles which satisfy all of the conditions. (There are 15 solutions.)

   c) Calculate the quantity of drink and the cost for each solution.

You may like to use a spreadsheet.

Bottles of orangeade (x)	Bottles of cola (y)	No. of litres	Cost
2	6	4 + 18 = 22	£1.50 + £7.50 = £9.00

   d) Which combination gives the most drink?

   e) Which combinations use up all of the £10?

   f) Where do these points lie?

Turn over

At the same party

£9 has been set aside for snacks.

It is decided to buy Bumper Packs of Crisps, which contain **6** bags, costing **90p** and Economy Packs of Hoola Hoops, which contain 8 bags and cost **£1**.

At least **1** Bumper Pack of Crisps and **1** Economy Pack of Hoola Hoops must be bought.

There must be at least **3** bags for each pupil.

4. Write down the inequalities expressing these conditions.

5. Find all possible combinations that satisfy these conditions. (Use **Regions**.)

6. a) Which gives the most bags in total?

   b) Which is the cheapest?

   c) Which gives the most equal numbers of bags of Hoola Hoops and Crisps?

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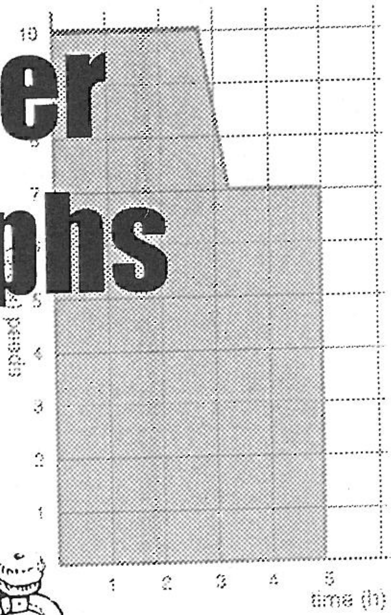
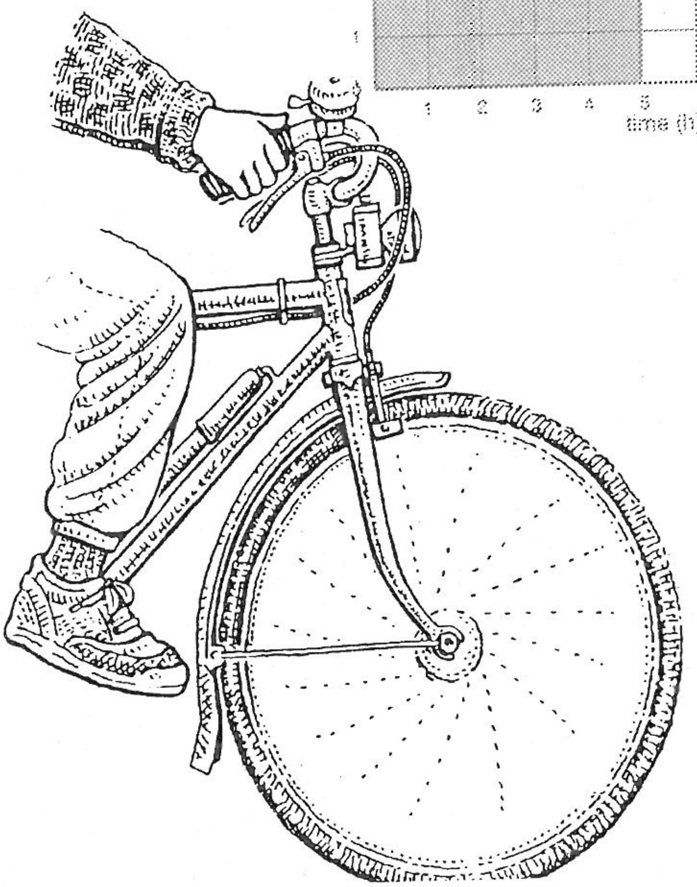
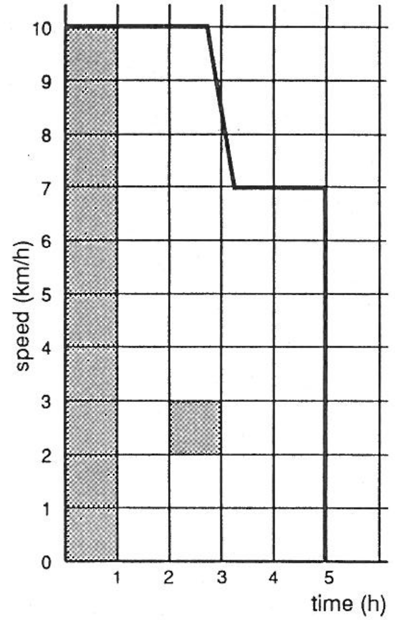
Areas under graphs





This **speed-time graph** shows the journey of a cyclist.



She travels at a speed of 10 kilometres per hour (10km/h or $10kmh^{-1}$) for $2^3/_4$ hours then slows down for $^1/_2$ hour to a speed of 7 kmh^{-1} which she maintains for another $1^3/_4$ hours.

Over the first hour she travelled at a speed of $10kmh^{-1}$.

$10kmh^{-1}$ x 1h = 10km.

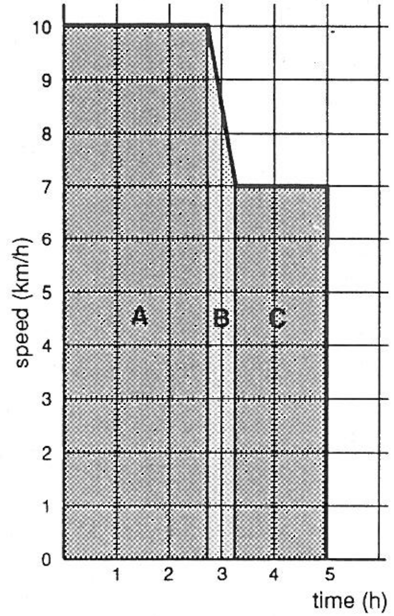
She covered a distance of 10km in the first hour.

The shaded rectangle represents the 10km the cyclist travelled in the first hour.

The shaded square represents $1kmh^{-1}$ x 1h, a distance of 1km.

The **area** under a **speed-time** graph represents **distance** travelled.

To find the total distance travelled by the cyclist calculate the total area under the graph.



Rectangle **A** = $10kmh^{-1}$ x 2.75h
= 27.5km

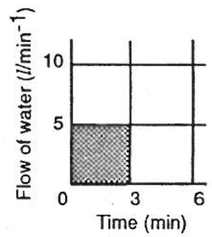
Trapezium **B** = $\frac{(10 + 7)}{2}kmh^{-1}$ x 0.5h
= 4.25km

Rectangle **C** = $7kmh^{-1}$ x 1.75h
= 12.25km

The total distance travelled by the cyclist
= (27.5 + 4.25 + 12.25)
= 44km

1. Copy and complete the information shown by these graphs.

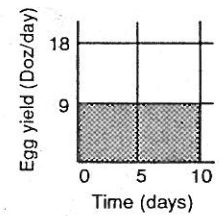
a) This graph shows flow of water, in litres per minute ($lmin^{-1}$) over time, in minutes (min).



The shaded area represents

$5lmin^{-1}$ x 3min = ▒▒ litres.

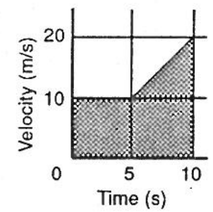
b) This graph shows egg yield, in dozens per day over time, in days.



The shaded area represents

9 dozen per day x 10 days = ▒▒

c) This graph shows velocity in metres per second (ms^{-1}) over time in seconds (s).

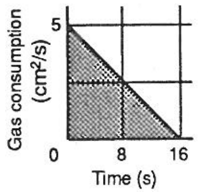


The shaded area represents

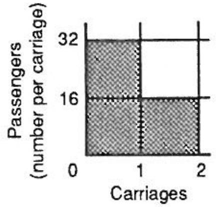
▒▒ + ▒▒ = 125 ▒▒

Write similar information for these graphs.

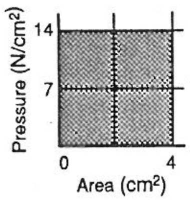
d)



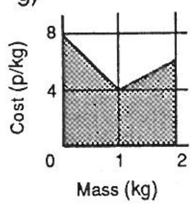
e)



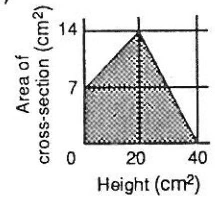
f)



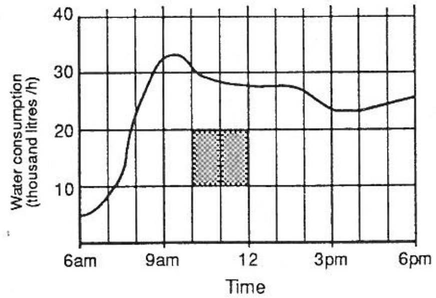
g)



h)



2. This graph represents the rate at which water flows from a reservoir in a 12 hour period.



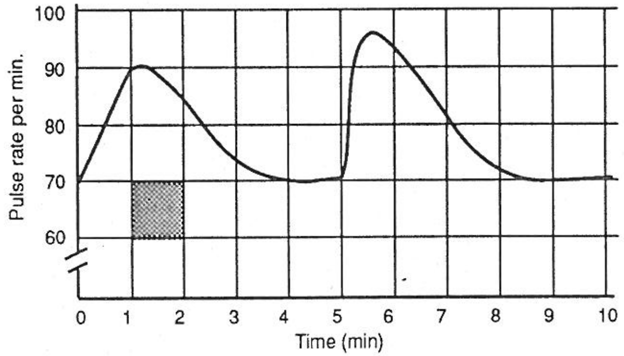
a) What volume of water is represented by the shaded area?
Find the approximate area under the graph to calculate the volume of water used during the 12 hours.

b) Use the graph to decide if more water is used from 6am - 9am than 3pm - 6pm. Explain your answer.

c) If the water flows into the reservoir at a steady rate of 28 thousand litres per hour during the 12 hour period, will the water be lower or higher at 6pm than at 6am? Explain your answer.

3. This graph shows a person's pulse rate over a ten minute period.

Notice that the vertical scale starts at 60.



a) What does the shaded square represent?

b) Estimate the average pulse rate for the first $3\frac{1}{2}$ minutes.

c) What was the normal resting pulse rate?

Estimate the average pulse rate for the 10 minutes shown.

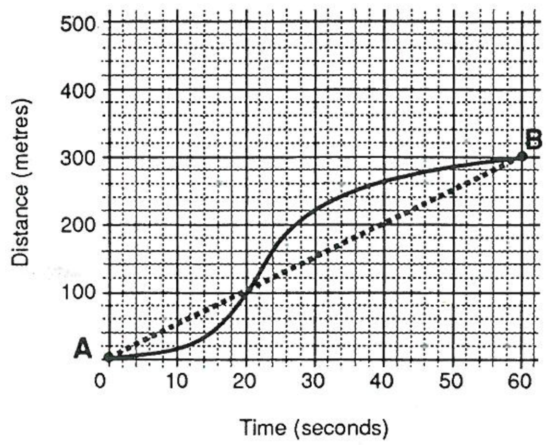
Velocity from Distance-Time Graphs

- The rate at which distance is travelled is called *speed.*
- *Velocity* is a measure of speed with the direction of motion specified.
- If one direction is regarded as *positive velocity*, a speed in the opposite direction has *negative velocity.*

This graph describes the journey of a cyclist from **A** to **B**.

The graph can be used to find her *average velocity* from **A** to **B**.

This is found from the gradient of the chord **AB**.

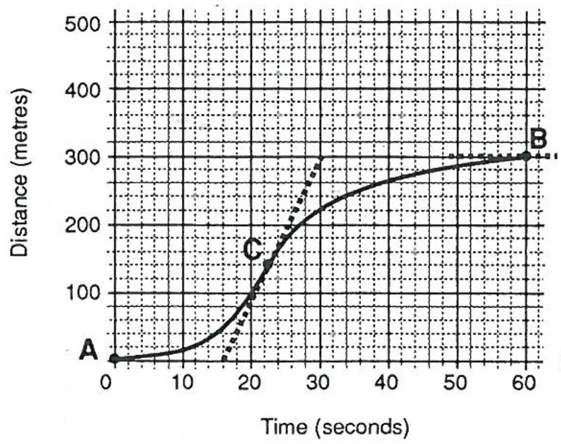


$$\text{Average Velocity} = \text{gradient of chord} = \frac{\text{increase in distance}}{\text{increase in time}} = \frac{300-0}{60-0} = 5\text{m/s}$$

The graph can be used to estimate the cyclist's *maximum velocity* between **A** and **B**. This is found from the gradient of the **tangent to the curve** (drawn by eye) at the steepest point, **C**.

At point **C**, $t = 30$,

$$\text{maximum velocity} = \text{gradient of tangent} = \frac{300-0}{30-16} = \frac{300}{14} = 21.4\text{m/s}$$



The cyclist stops at 60 seconds. The gradient of the tangent to the curve at **B** is 0.

1. Sketch a possible distance-time graph of a car journey assuming that the car travels at constant velocity except when it is held up by traffic lights.

- What is the gradient of the tangent to the curve when the car stops at traffic lights?

2. Draw an accurate distance-time graph of this 3-stage journey:
- a steady velocity of 15m/s for 3 minutes
- a $^1/_2$ minute stop
- a steady velocity of 10m/s for 4 minutes.

- Use the graph to find the average velocity for the journey.
- Check your answers by calculation.

Turn over

3. This table shows the distance of a car from home at 15 minute intervals.

Time	10.00	10.15	10.30	10.45	11.00	11.15	11.30	11.45	12.00
Distance (km)	0	10	20	33	58	80	98	116	120

- Draw a distance-time graph.

a) Calculate the average velocity for the whole journey.

b) Calculate the average velocity between 10.30 and 11.30.

c) Draw a chord on the graph to estimate the average speed from when the car was 30km from the start to when it was 90km from the start.

d) Draw a tangent at the steepest point of the curve and use it to estimate the maximum velocity.

4. This table shows the time a train passed kilometre posts on a journey.

Distance (km)	0	10	20	30	40	45	50	60	70	80
Time (hours)	0	0.17	0.26	0.32	0.42	0.53	0.75	0.86	0.91	1.0

- Draw a distance-time graph and use it to estimate:

a) the average velocity for the first half hour of the journey

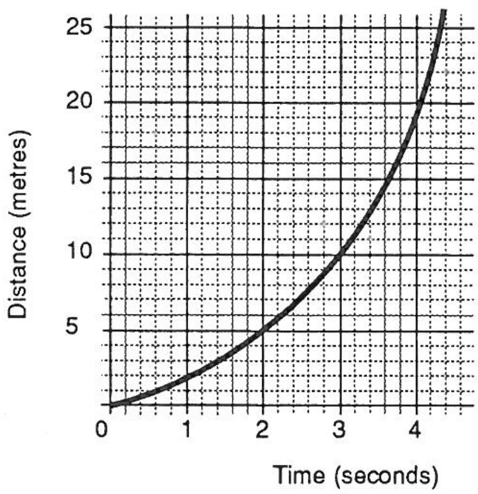
b) the velocity when the train was 40km from the start

c) the maximum velocity.

5. Trace this graph and draw suitable tangents to estimate:

a) the velocity at 4 seconds

b) the time when the velocity is 5m/s.



Distance, Velocity and Acceleration

Velocity is a measure of speed when the direction of motion is specified.
Velocity is a measure of the rate of change of distance with time.

$$\textbf{Velocity} = \frac{\text{change in distance}}{\text{change in time}}$$

Acceleration is a measure of the rate of change of velocity with time.

$$\textbf{Acceleration} = \frac{\text{change in velocity}}{\text{change in time}}$$

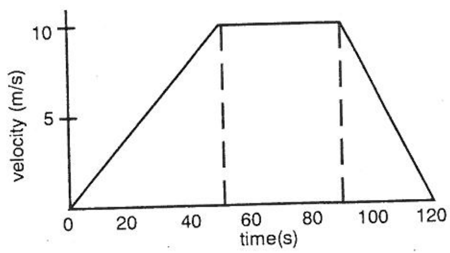
A negative acceleration is called deceleration.

Section A Finding acceleration from a velocity-time graph

This velocity-time graph represents a two minute cycle ride.

The cyclist's velocity increases steadily for the first 50 seconds, from 0 to 10 m/s.

She then travels at a constant velocity for 40 seconds.



Then she slows down until she stops.

The gradient of a velocity-time graph is a measure of acceleration.

To find the acceleration of the cyclist between . . .

0 and 50 seconds . . . The gradient of the graph for the range $0 \leq t \leq 50$

$$= \frac{10 - 0}{50 - 0}$$

$$= 0{\cdot}2$$

The velocity is increasing by 0.2 m/s every second.
The **acceleration** is 0.2 metres per second per second, or 0.2 m/s^2 or 0.2 ms^{-2}.

50 and 90 seconds . . . The gradient of the graph for the range $50 \leq t \leq 90$

$$= \frac{10 - 10}{90 - 50}$$

$$= 0$$

The cyclist has 0 **acceleration**, she is travelling at constant velocity.

90 and 120 seconds. The gradient of the graph for the range $90 \leq t \leq 120$

$$= \frac{0 - 10}{120 - 90}$$

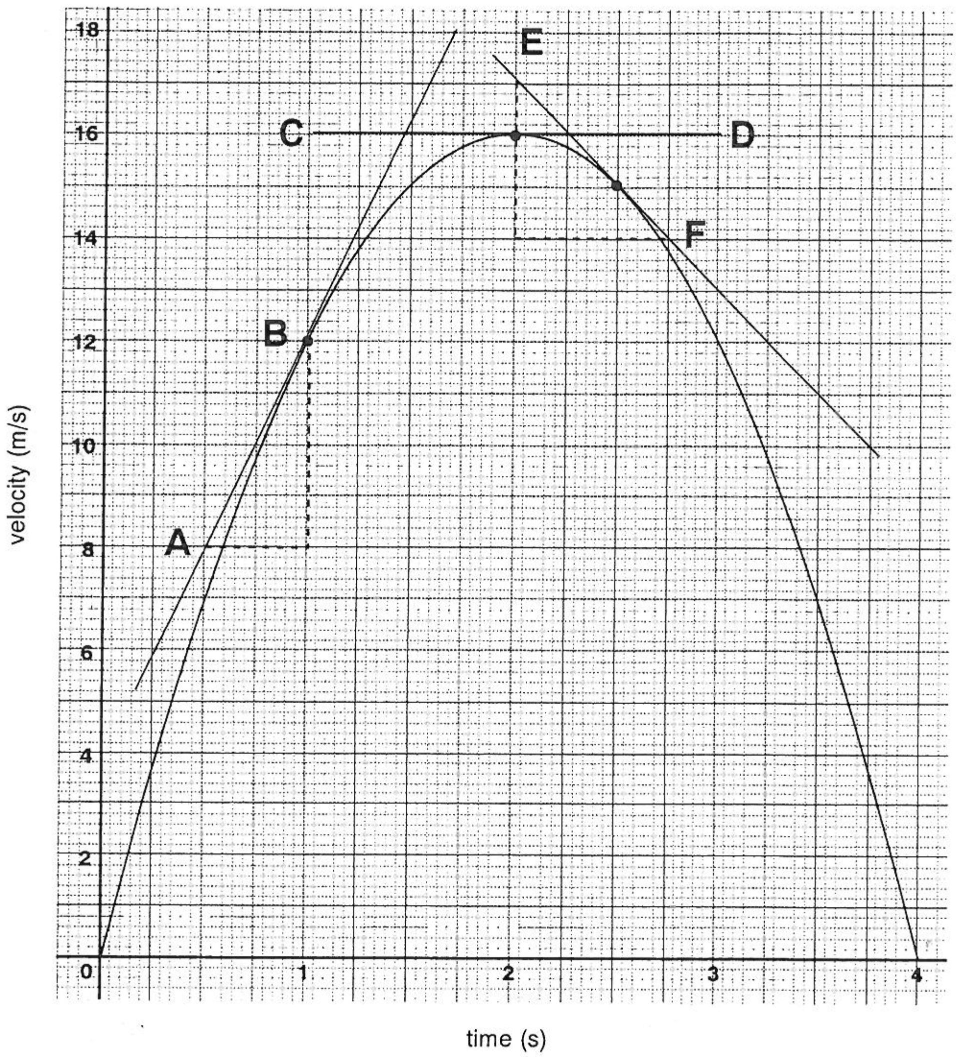
$$= \frac{-10}{30}$$

$= -0{\cdot}33$ to 2 dps.

The velocity is decreasing by 0·33 m/s^2.
The deceleration is 0·33 m/s^2.
The **acceleration** is -0·33 m/s^2.

This velocity-time graph represents the journey of an object with velocity **v**m/s, at time **t** seconds given by the equation.

$$v = 16t - 4t^2$$



The graph is a curve. To estimate acceleration a tangent to the curve is drawn by eye and the gradient calculated.

- At **t = 1 second** the acceleration is estimated from the gradient of the tangent **AB**.

  $$\textbf{Acceleration} \approx \frac{12 - 8}{1 - 0{\cdot}5}$$

  $$\approx \mathbf{8m/s^2}$$

- At **t = 2 seconds** the gradient of the tangent to the curve **CD** is zero.

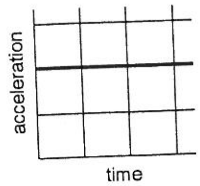
  The acceleration is zero and the velocity is constant.

- At **t = 2·5 seconds** the acceleration is estimated from the gradient of the tangent **EF**.

  $$\textbf{Acceleration} \approx \frac{17 - 14}{2 - 2{\cdot}75}$$

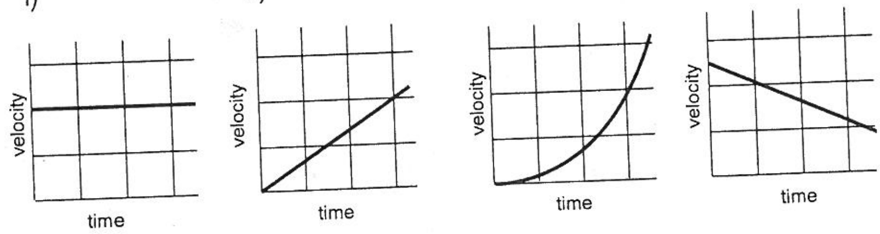
  $$\approx \mathbf{-4m/s^2}$$

1. This acceleration-time graph represents a journey with constant acceleration.



Which of these velocity-time graphs could represent the same journey?

i) ii) iii) iv)

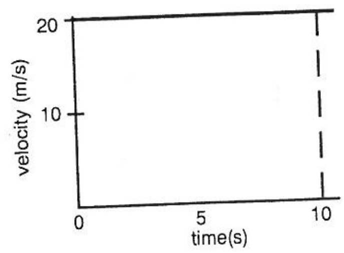


2. A cyclist's journey after **t** seconds has a velocity of **v**m/s where **v** = $\frac{1}{5}$**t**^{2}.
   a) Sketch a graph of her velocity against time for the first 5 seconds.
   b) Draw tangents at suitable points to find the acceleration at:
      i) t = 2
      ii) t = 4.
   c) Find the approximate time when her acceleration was 1m/s^2.

Section B — Finding distance from velocity-time graphs and velocity from acceleration-time graphs.

This velocity-time graph shows a journey of constant velocity of 20m/s for 10 seconds.

In 10 seconds the distance covered is 200m, this is represented by the area under the graph.



The area under a velocity-time graph is a measure of distance.

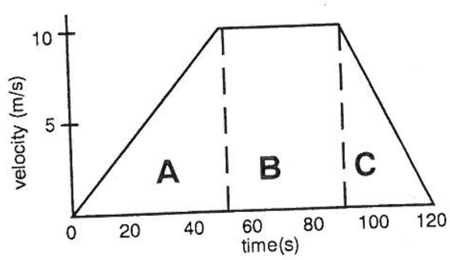
This velocity-time graph represents a two minute cycle ride.
To find the distance travelled by the cyclist find the area under the graph.

Triangle **A** = $\frac{1}{2}(50 \times 10)$
= 250

Rectangle **B** = 40 x 10
= 400

Triangle **C** = $\frac{1}{2}(30 \times 10)$
= 150

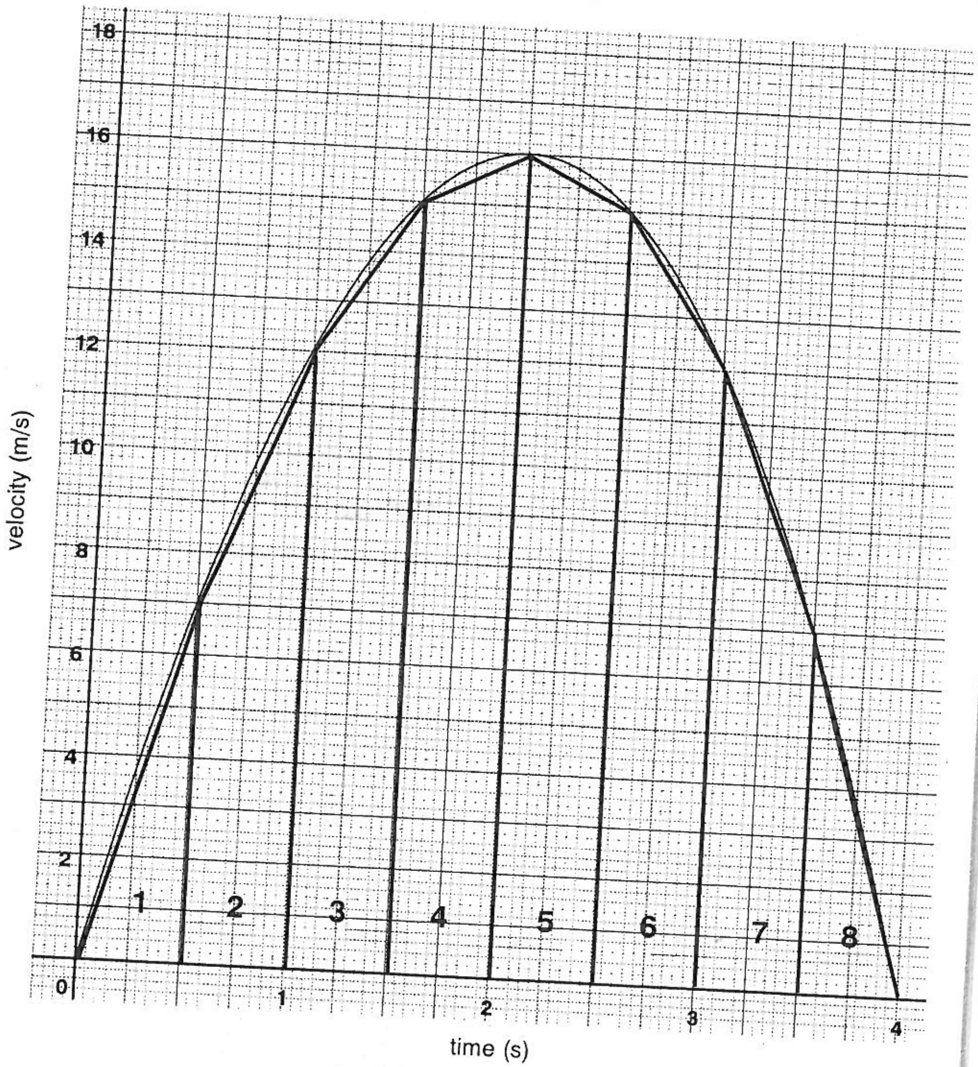
Total = 800



The area is 800 units, the cyclist travelled 800 metres.

This velocity-time graph represents the journey of an object with velocity **v**m/s, at time **t** seconds given by the equation.

$$v = 16t - 4t^2$$



As it is a curve it is more difficult to calculate the distance travelled. Methods that can be used are:

- estimation
- counting squares
- finding the areas of strips which will approximate closely to trapezia.

Area of trapezium **1** $= \frac{1}{2}(0 + 7) \times 0{\cdot}5$

Area of trapezium **2** $= \frac{1}{2}(7 + 12) \times 0{\cdot}5$

Area of trapezium **3** $= \frac{1}{2}(12 + 15) \times 0{\cdot}5$

Area of trapezium **4** $= \frac{1}{2}(15 + 16) \times 0{\cdot}5$

Area of trapezium **5** $= \frac{1}{2}(16 + 15) \times 0{\cdot}5$

Area of trapezium **6** $= \frac{1}{2}(15 + 12) \times 0{\cdot}5$

Area of trapezium **7** $= \frac{1}{2}(12 + 7) \times 0{\cdot}5$

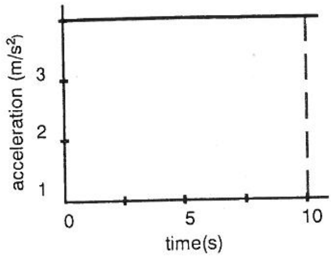
Area of trapezium **8** $= \frac{1}{2}(7 + 0) \times 0{\cdot}5$

Total area $= 42$

The distance travelled is *approximately* **42m**.

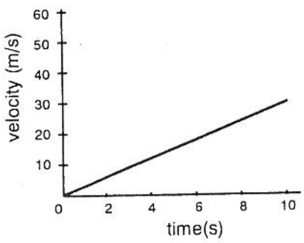
This acceleration-time graph shows a journey of constant acceleration of $3 m/s^2$ for 10 seconds.

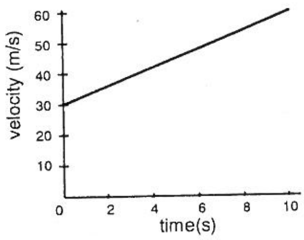
In 10 seconds the velocity has increased by 30m/s this is represented by the area under the graph.

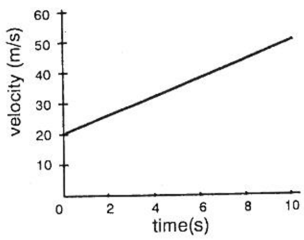


The area under a acceleration-time graph is a measure of velocity.

The velocity-time graphs below each show an increase in velocity of 30m/s during a 10 second interval.



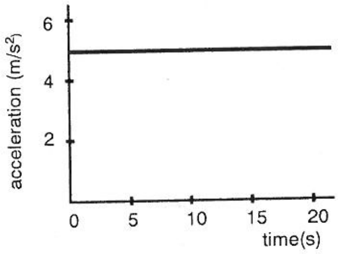


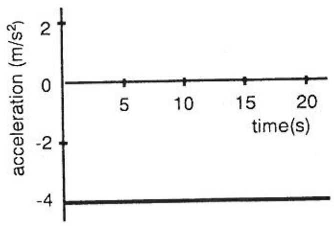


They all show acceleration of $3 m/s^2$.

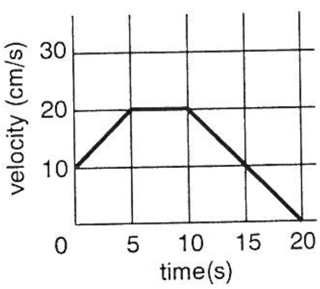
Any one of them could represent the same journey as the velocity-time graph above.

1. Sketch at least two possible velocity-time graphs which correspond to these two acceleration-time graphs.





2. This velocity-time graph represents a journey.



Which of these statements are true?

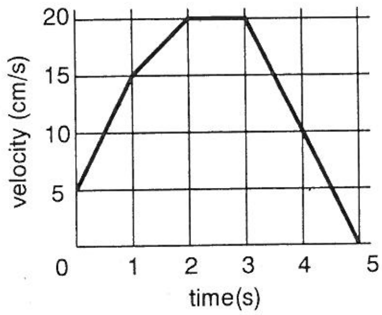
a) The initial velocity is 10cm/s.

b) The total distance covered is 225cm.

c) The acceleration for $0 < t < 5$ is $2 cm/s^2$.

d) The acceleration for $10 < t < 20$ is $1 cm/s^2$.

3. In this velocity-time graph:



a) Describe the acceleration during the first 2 seconds of the journey.

b) What is happening from t = 2 to t = 3?

c) What is the acceleration and velocity at time t = 4?

d) Calculate the distance travelled between t = 3 and t = 5.

4. The acceleration, **a**cm/s^2, of an object is represented by the equation $\mathbf{a = -t^2 + 4t + 6}$, where t is time in seconds.

a) Sketch a graph of the acceleration of the object from t = 0 to t = 4.

b) Use the trapezium method to estimate the velocity of the object at t = 1, t = 2, t = 3, and t = 4.

c) Use these values of velocity to sketch a velocity-time graph of the object for the first 4 seconds.

d) Estimate the total distance travelled by the object.