SMILE WORKCARDS

Angle Properties Pack Two

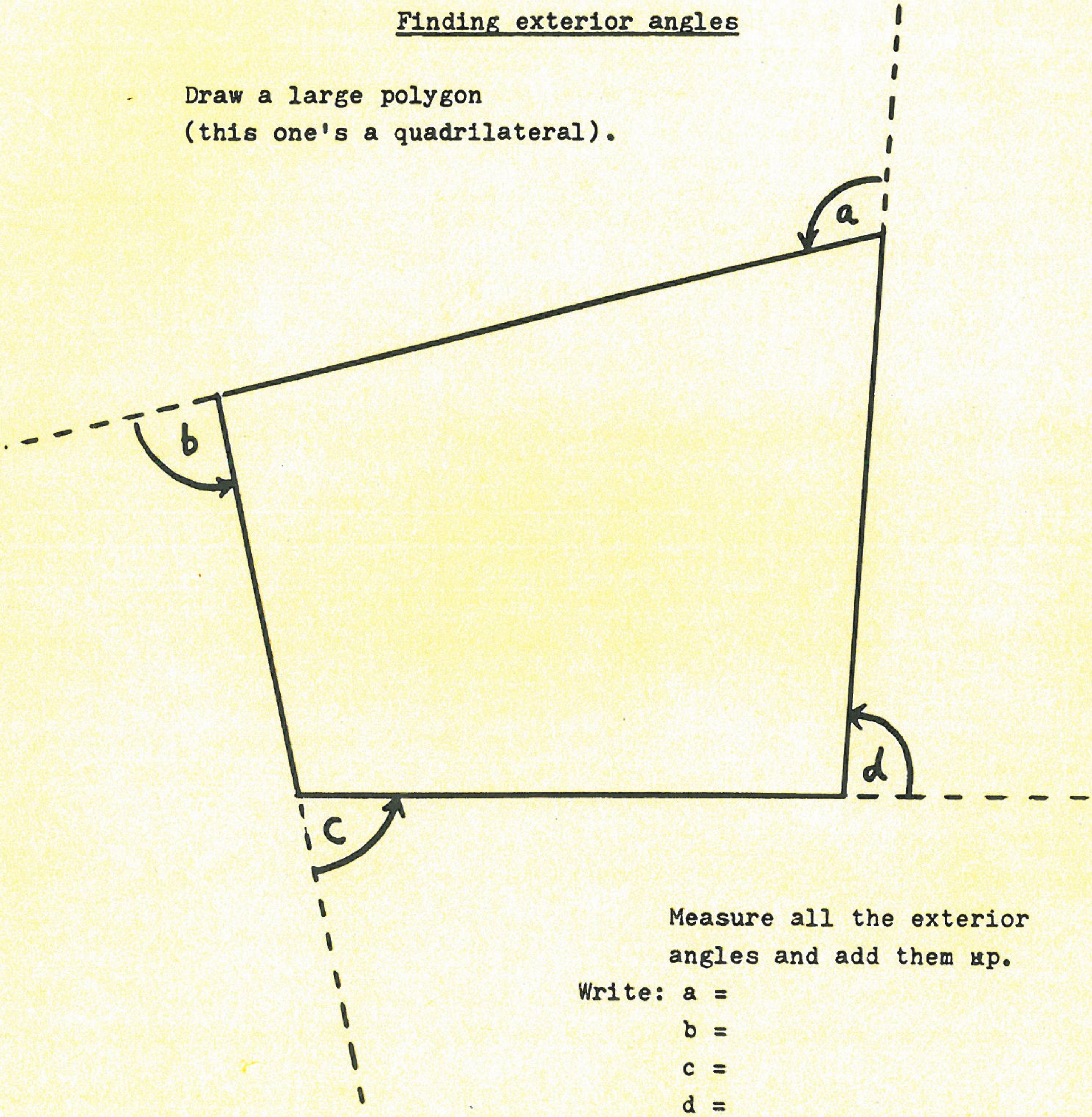
Contents

smile

0269

Finding exterior angles

Draw a large polygon
(this one's a quadrilateral).



Measure all the exterior angles and add them up.

Write: a =
b =
c =
d =

sum =

Repeat this with 5 more polygons (not just quadrilaterals).

Notice anything?

Turn over

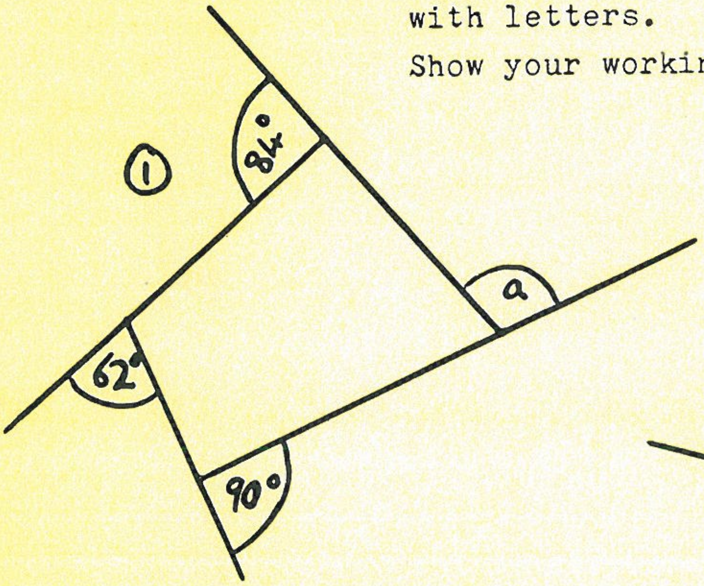
You should find that:

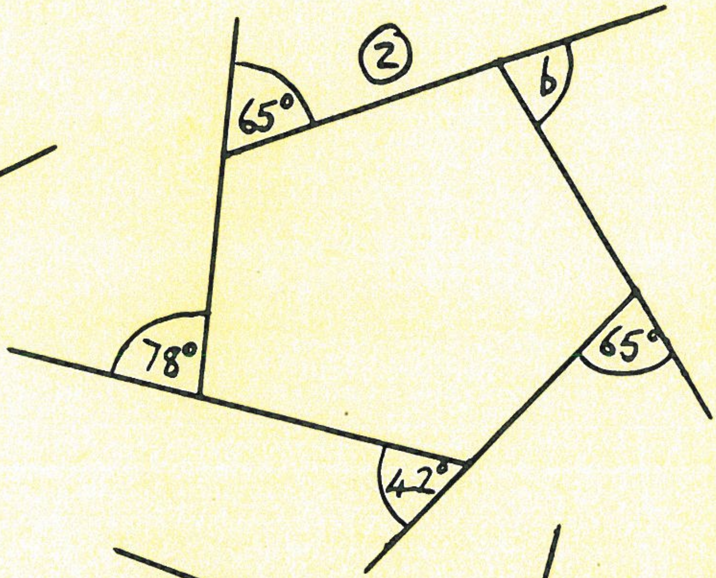
> the exterior angles of any polygon always add up to 360°.

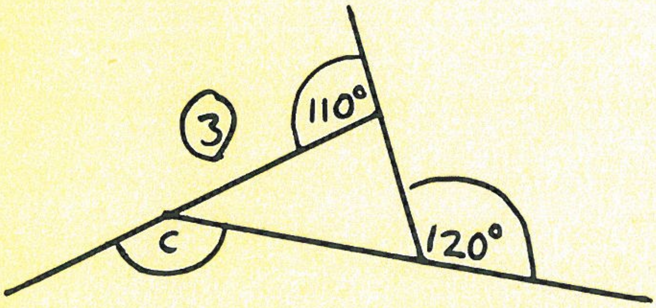
In the questions below, you must

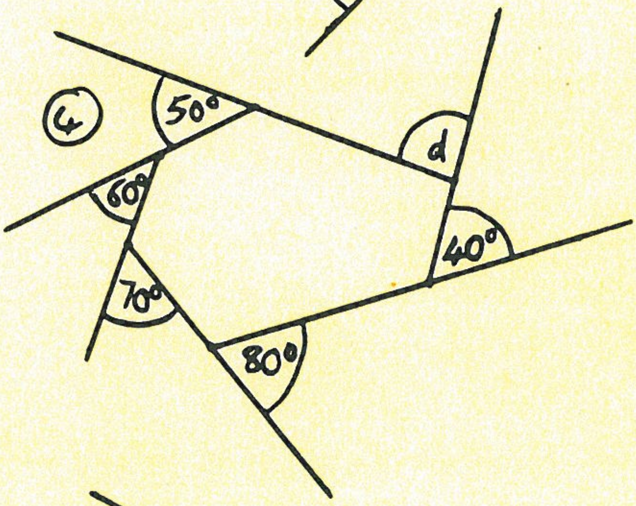
(a) sketch the polygon and mark the angles given.

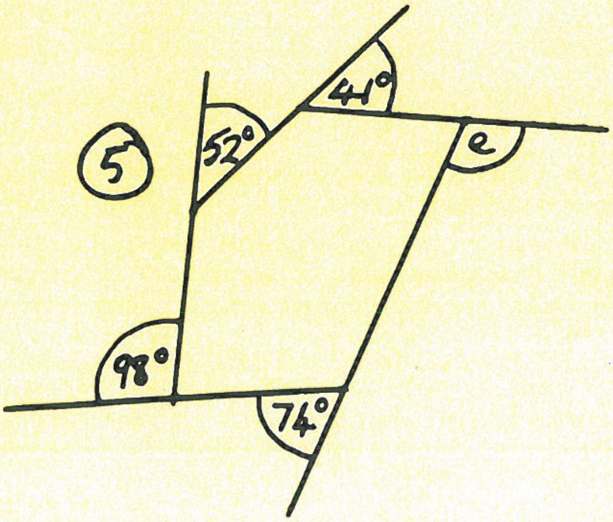
(b) Calculate (no protractors) the angles marked with letters. Show your working.

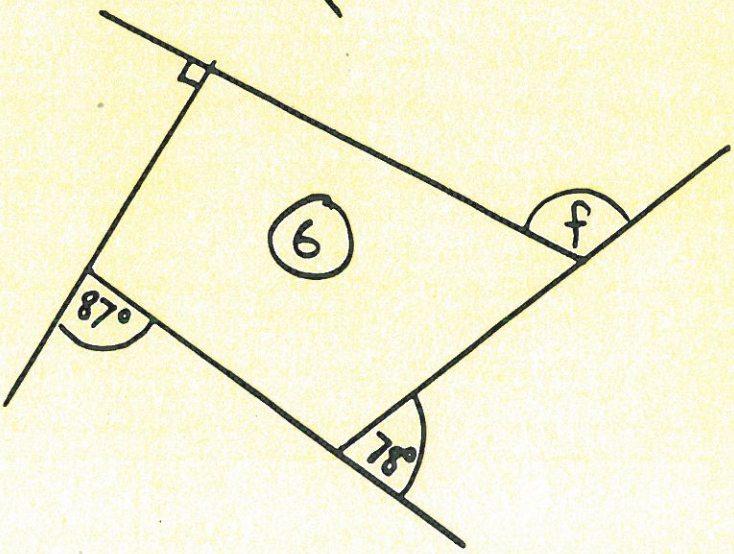








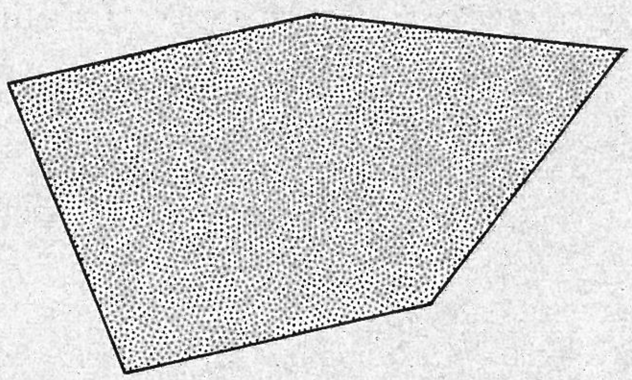


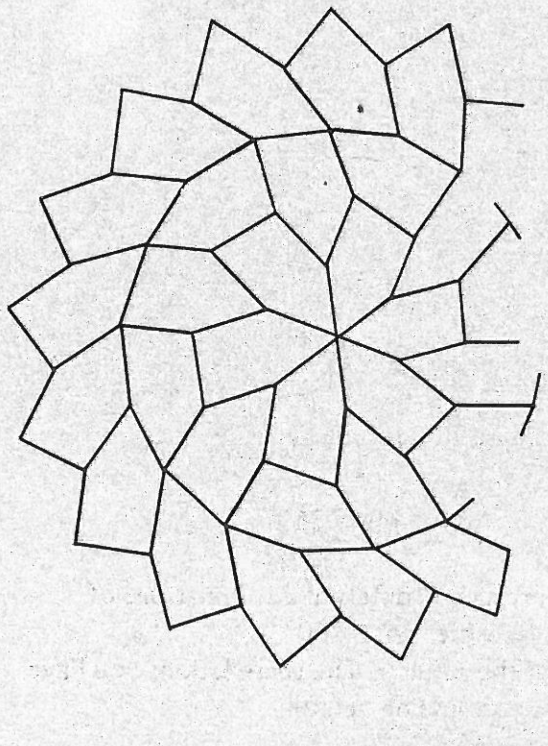


You will need several copies of worksheet 1419A, printed on different colours.

Versa-tiles

This is an equilateral pentagon : it is a pentagon because it has 5 sides; it is equilateral because they are all the same length.





1. Use the equilateral pentagons from worksheet 1419A to make this pattern. *Use worksheets of different coloured paper to make the pattern more interesting.*

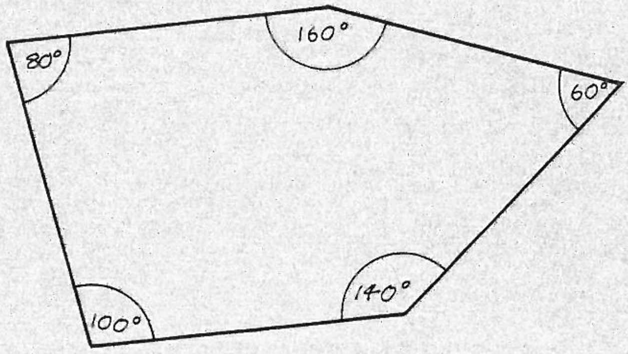
2. Can you extend the pattern?

3. Which other tessellations can you make from this shape? You can either stick the shapes onto paper or use one shape to draw round.

Turn over

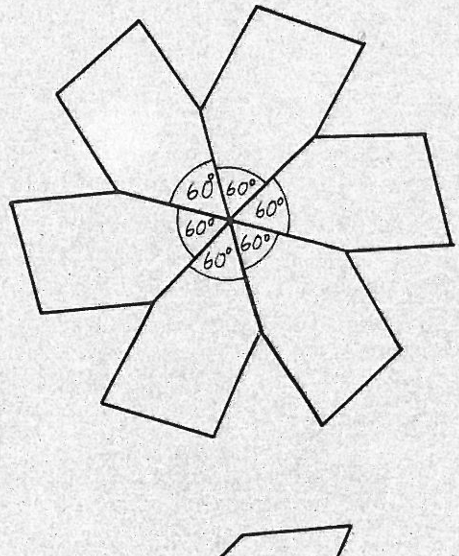
The shape is not regular; it has no symmetry and yet many different tessellations are possible.

This is because the angles fit together to make 360° in many different ways.



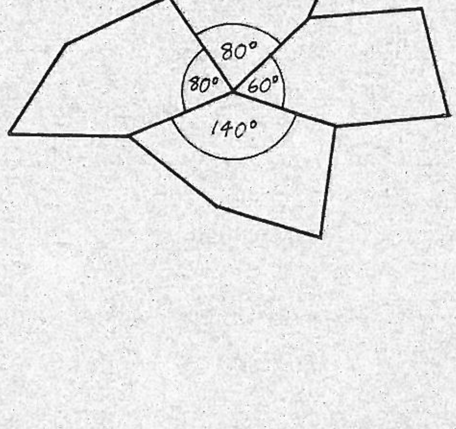
This is the centre of the pattern in question (1). The angles at the centre are all 60°.

60° + 60° + 60° + 60° + 60° + 60° = 360°



This is a different part of the same pattern.

140° + 80° + 80° + 60° = 360°

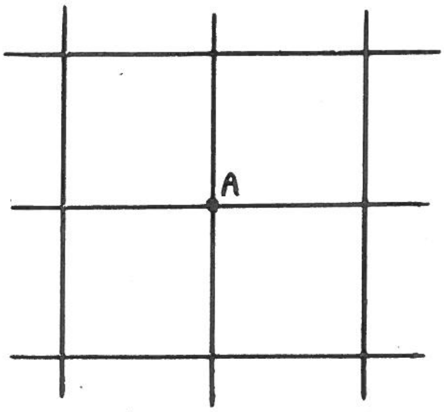


4. There are 11 different combinations of angles which total 360°.
   Find the other 9. The tessellations you have made should be helpful.

5. Can you use the angle combinations to find any tessellations which you had missed before?

smile
0284

Angles from tessellations



Here is part of a tessellation of squares.

(1) How many angles are there at A?

(2) What fraction of a complete turn in each angle?

A complete turn is 360°, so.....

(3) What is the angle at each corner of a square?

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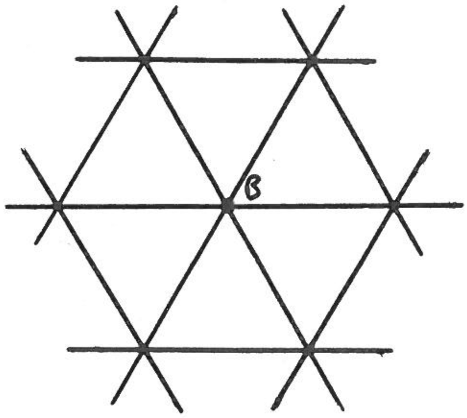
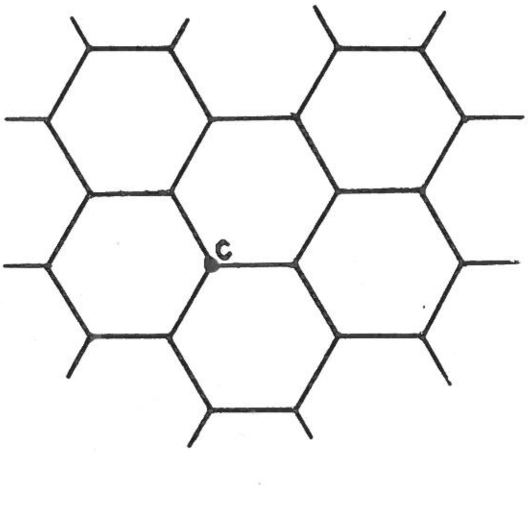
(4) How many angles are there at B?

(5) What is the angle at each corner of an equilateral triangle?

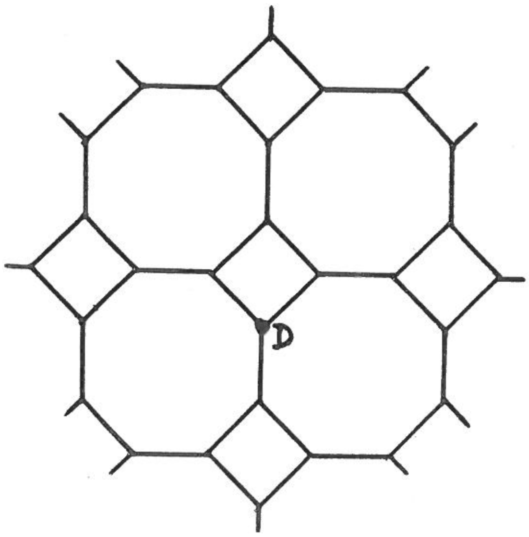




(6) What is the angle at each corner of a regular hexagon ?

(7) Here is part of a tessellation of squares and regular octagons.

Work out the angle at the corner of a regular octagon.

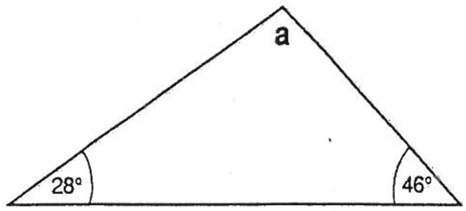


Angles and Triangles

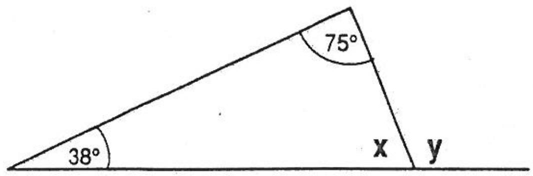
You will need *Geometry Facts* Smile **2163**.

These drawings are not to scale.

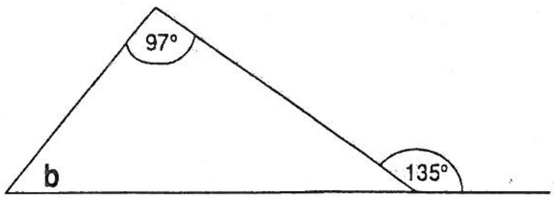
1. Calculate in degrees the size of the angle marked '**a**'.



2. Calculate the size of the angles marked '**x**' and '**y**'.

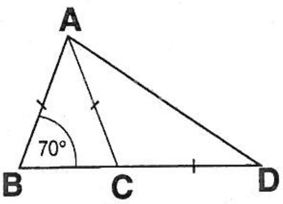


3. Calculate the size of the angle marked '**b**'.



Turn over

4. In this diagram what can you say about **AB**, **AC** and **CD**?



Use you answer to help you find the size of :

(i) ∠ **ACB** (ii) ∠ **BAC** (iii) ∠ **ACD**

(iv) ∠ **CAD** (v) ∠ **BAD**

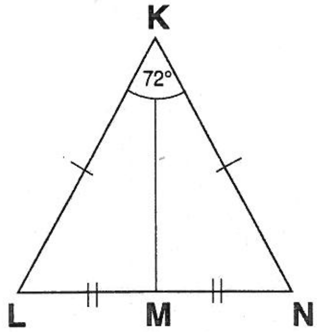
5. In this diagram

LM = **MN**

KL = **KN**

and also

∠ **LKN** = 72°

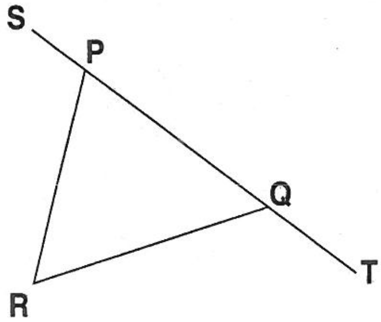


Find the size of (i) ∠ **KML** (ii) ∠ **KLM**

6.

∠ **TQR** = 110°

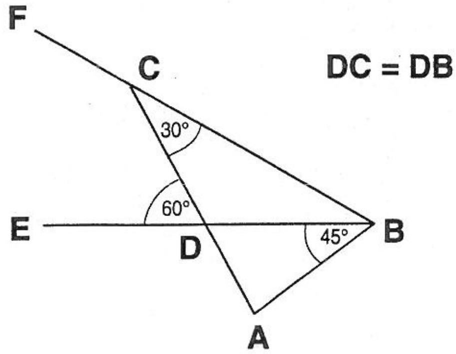
∠ **PRQ** = 60°



Find the size of ∠ **SPR**

7.

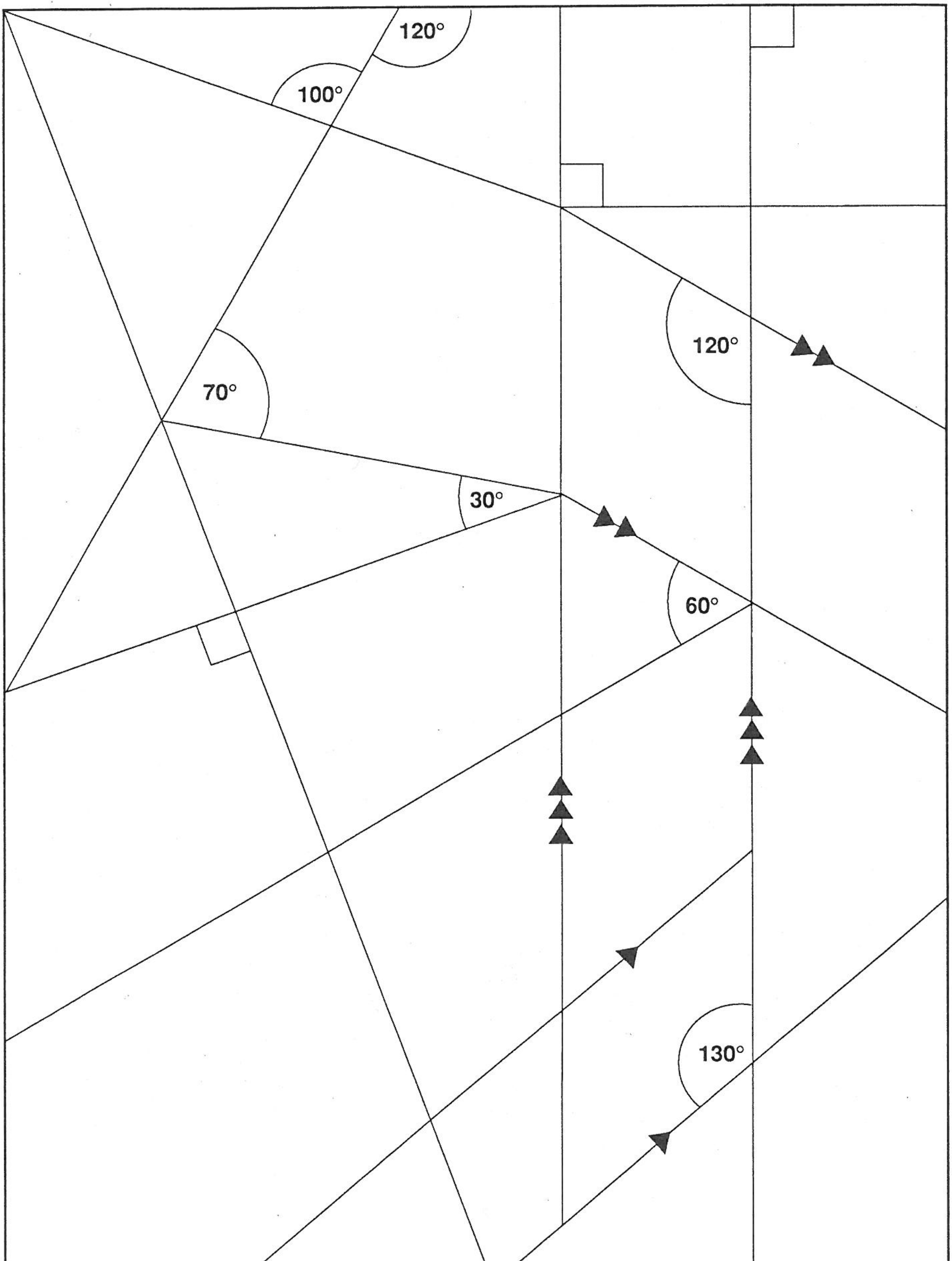
DC = **DB**



Find i) ∠ **ADB** ii) ∠ **BAD** iii) ∠ **DBC**

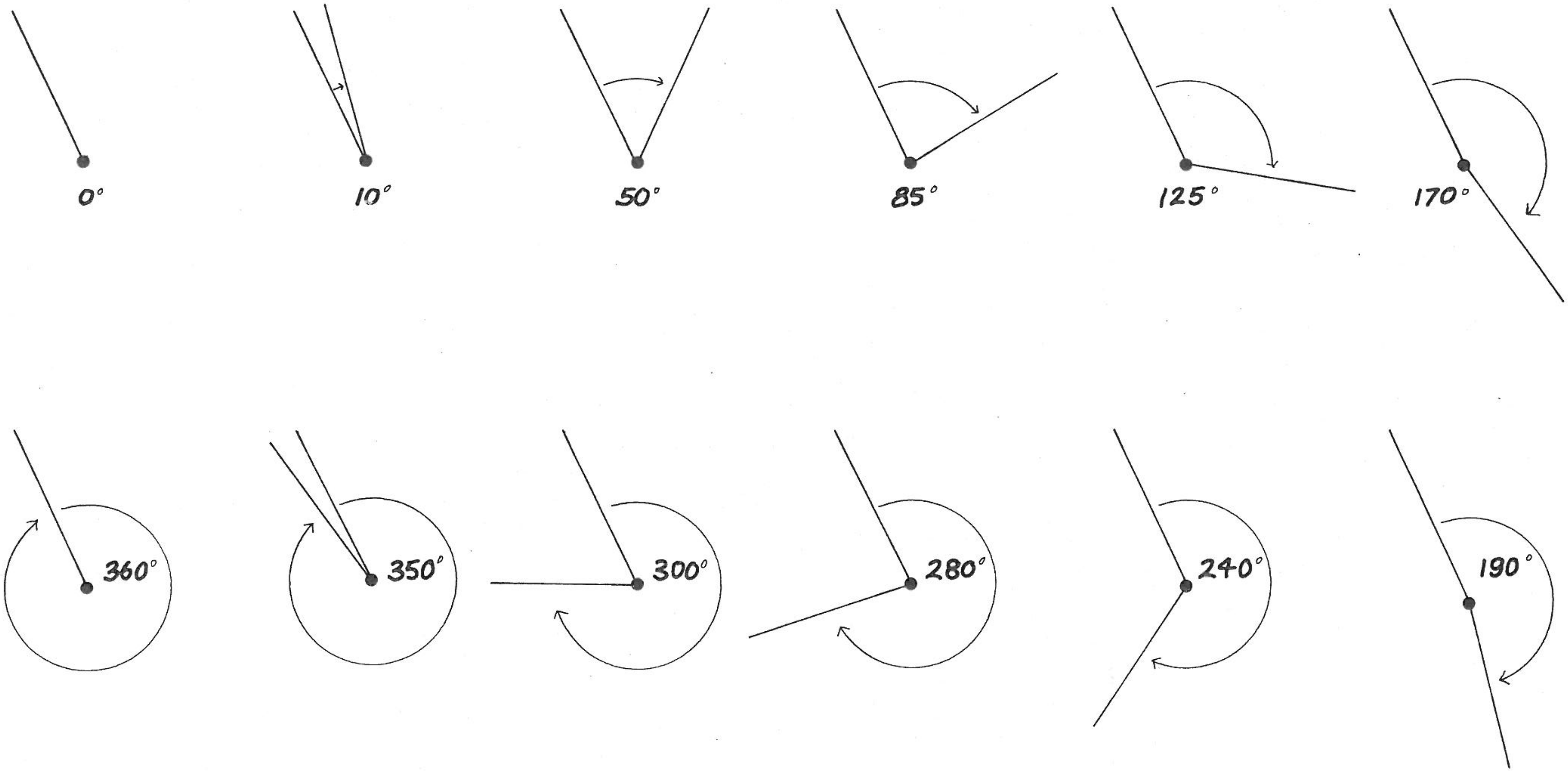
UNMARKED ANGLES

Work out the unmarked angles inside this rectangle.
(Do **not** use an angle indicator.)

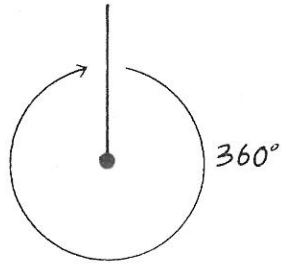


You may want to use *Geometry Facts* (Smile **2163**).

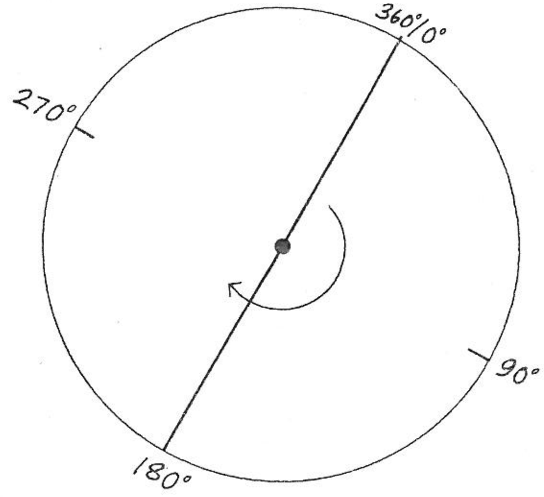
Angle 4 Review

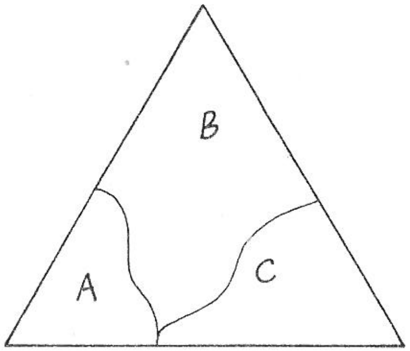


One complete turn ➡ 360°

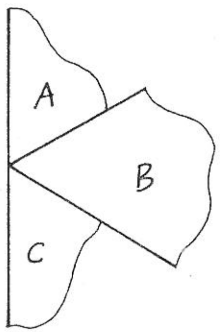


180° ➡ half turn ➡ straight line

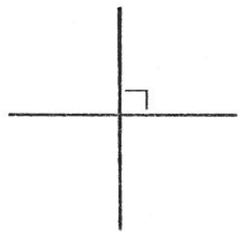




➡

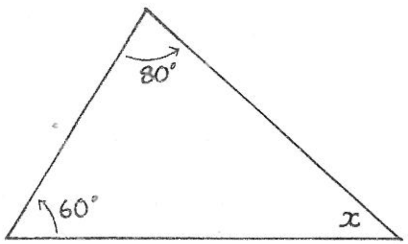


➡ $A + B + C = 180°$

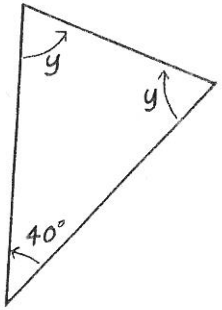


$4 \times 90° = 360°$

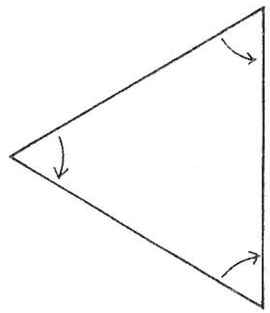
Solve problems 1-10 without measuring.



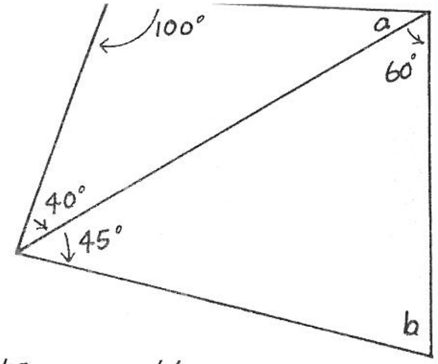
1) $x + 60° + 80° = 180°$
$\therefore x =$ ■



2) In this isosceles triangle
$y =$ ■

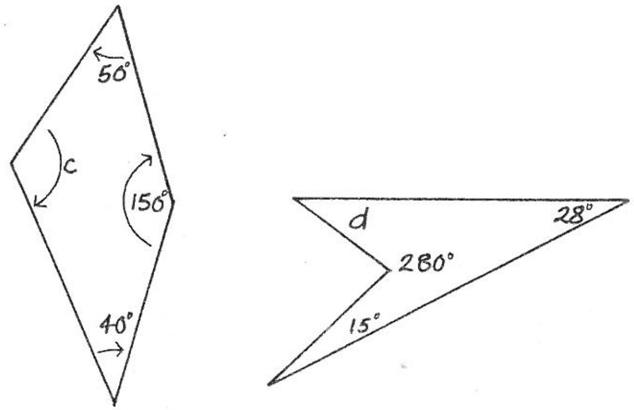


3) In an equilateral triangle each angle is ■

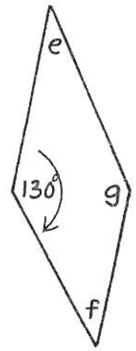


4) Calculate a and b

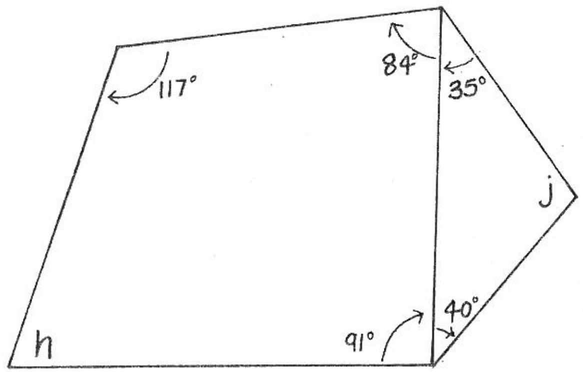
5) Calculate the total sum of the angles of the quatrilateral



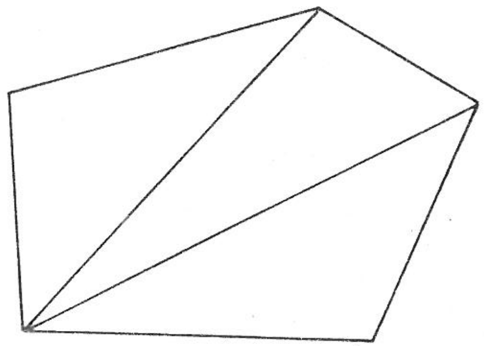
6) Calculate c and d



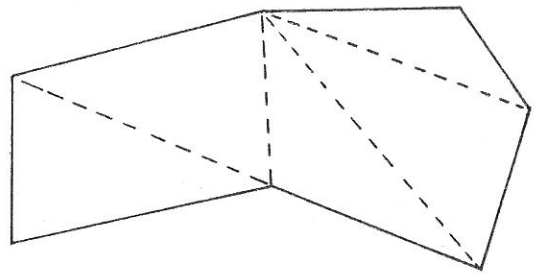
7) Find the missing angles in this rhombus



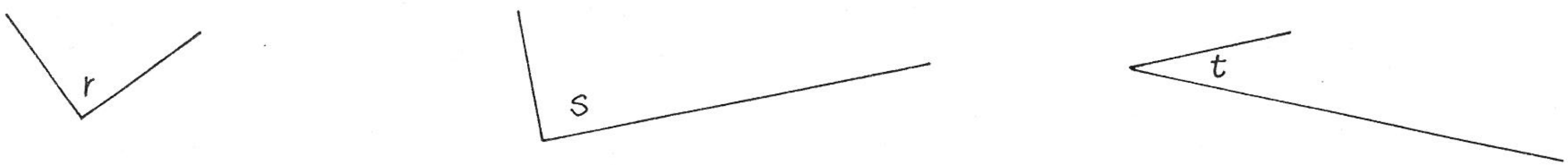
8) Calculate h and j to find the angle sum of a pentagon



9) Use this diagram to prove that your answer to (8) is correct

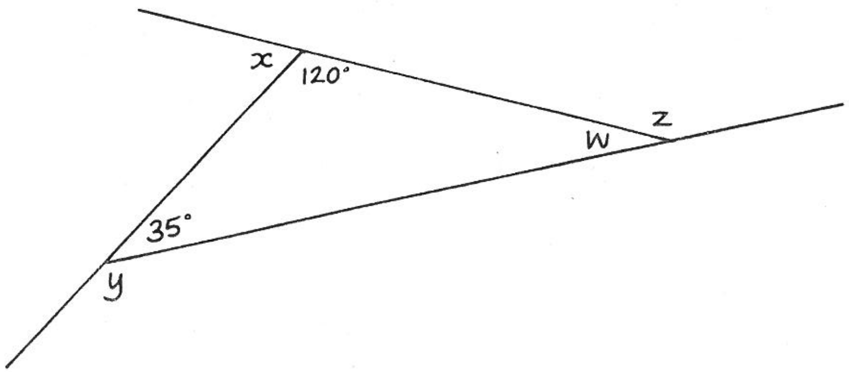


10) Calculate the angle sum of this septagon

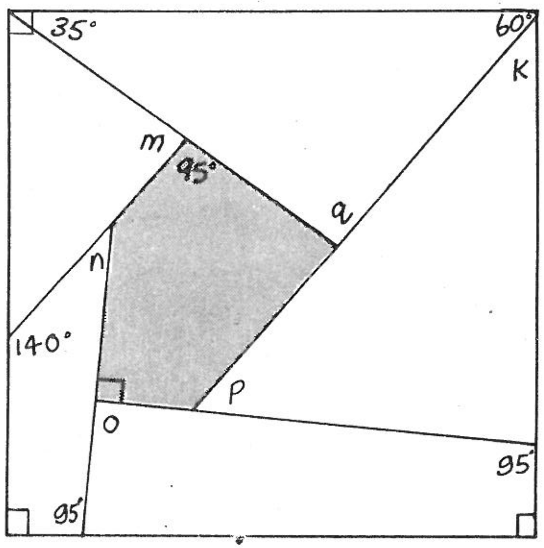


11) Which two angles are the same ?

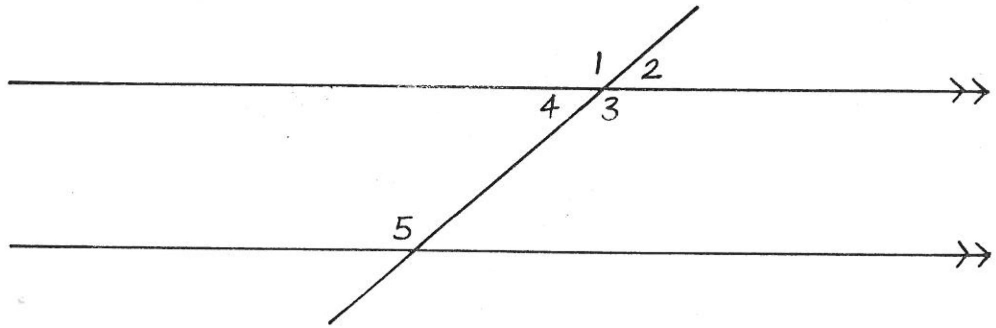
Turn over



12) a) $w =$ ■
b) $x =$ ■
c) $x + y + z =$ ■



13) Find all the missing angles and complete:
a) $k =$ ■°
b) $m + n + o + p + q =$ ■



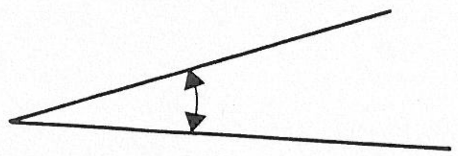
14) Which three angles are the same?
Are the other two angles equal?

15) Write briefly how you would recognise:
a right angle
an acute angle
an obtuse angle

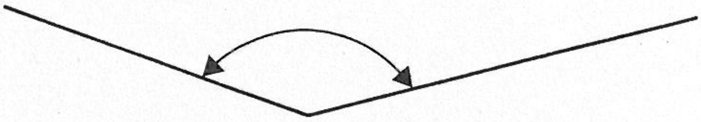
16) What is the difference between an isosceles triangle and an equilateral triangle?

You will need geostrips and split pins

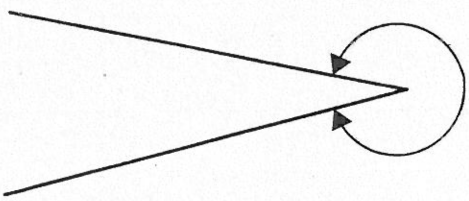
Acute/Obtuse



An ACUTE angle is **smaller** than a right-angle (90°).
Make an acute angle and draw it.



An OBTUSE angle is **bigger** than a right-angle (but not as big as a straight line). *Make an obtuse angle and draw it.*



A REFLEX angle is **bigger** than a straight line (180°).
Make a reflex angle and draw it.

Can you make these?

Draw them or write 'impossible'.

1) A triangle with 3 acute angles

2) A triangle with 1 obtuse angle

3) A triangle with 2 obtuse angles

4) A triangle with 2 right angles

5) A quadrilateral with 2 obtuse angles

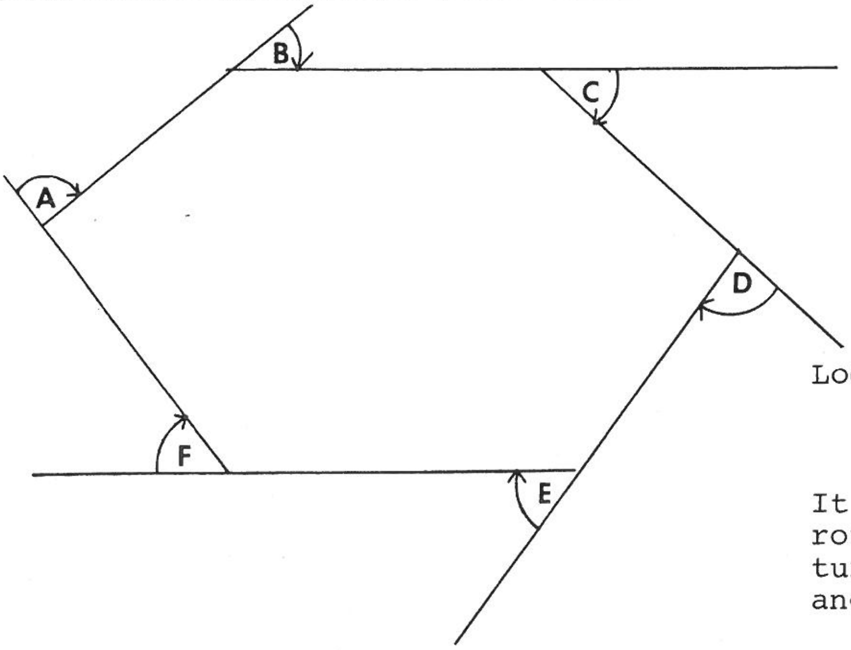
6) A quadrilateral with 1 reflex angle

7) A quadrilateral with only 1 obtuse angle

8) A quadrilateral with only 3 right-angles

0800 Smile

Polygons interior angles



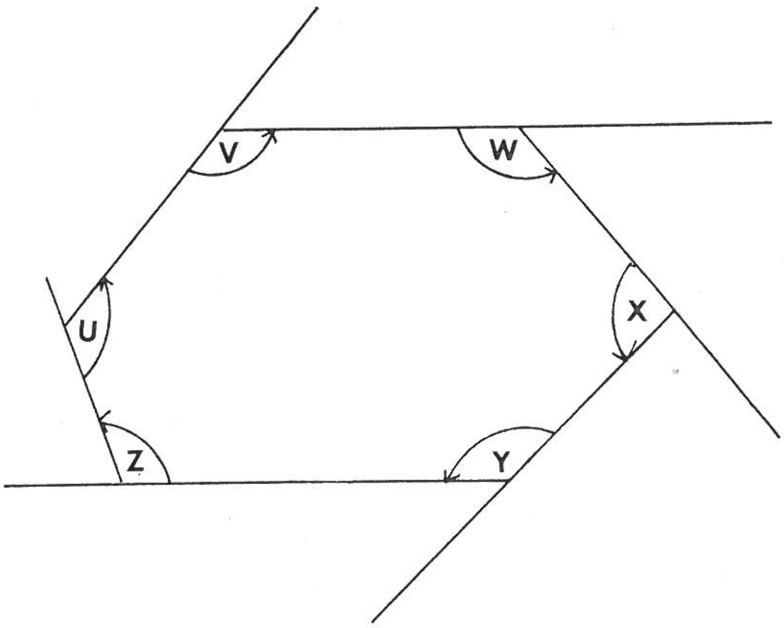
Look back at card 0799.

It says that if you walk round a polygon, you will turn through all the <u>exterior</u> angles ....

A + B + C + D + E + F = 360°

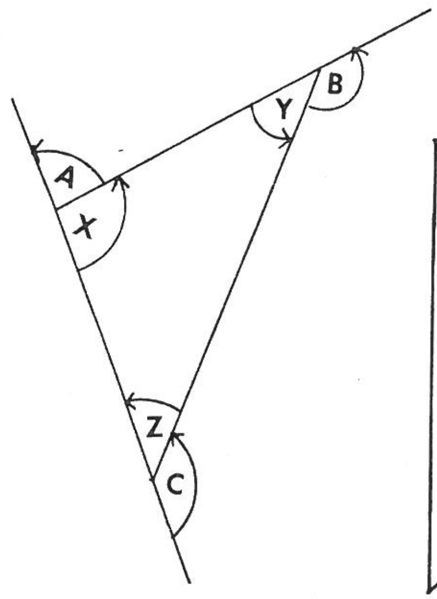
.... so the sum of the exterior angles of a polygon is 360°

This card is about the sum of the <u>interior</u> angles of a polygon.



U + V + W + X + Y + Z = ?

Start with a triangle



We want to know

X + Y + Z

We know

A + X = ■

B + Y = ■

C + Z = ■

Adjacent angles add up to ■

Adding

A+X+B+Y+C+Z

= ■

*

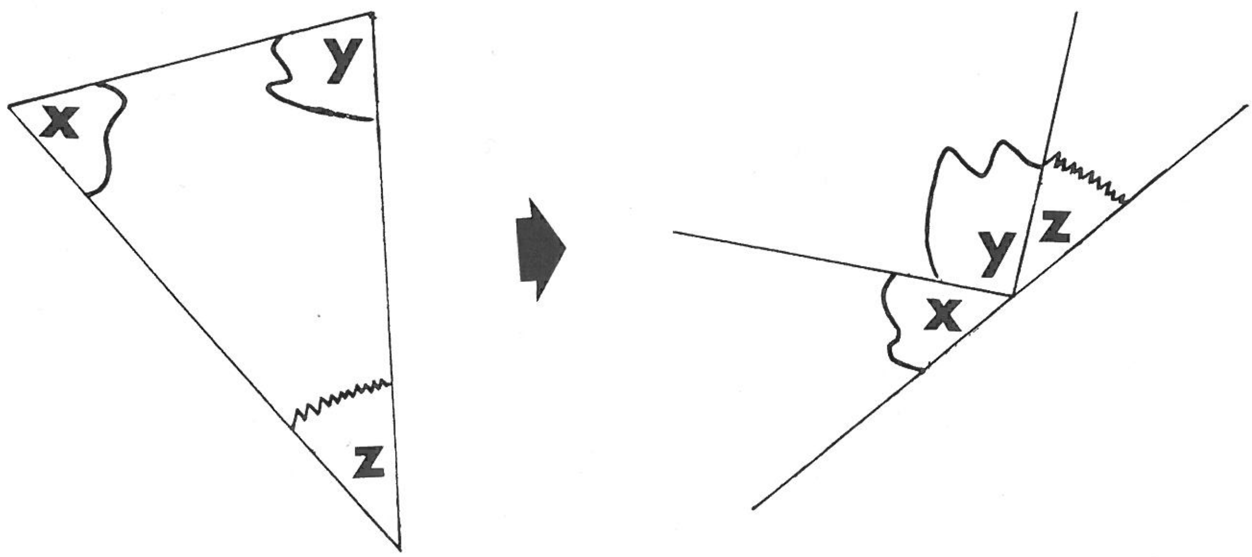
We know

A + B + C = ■

The exterior angles of a triangle add up to ■

You will come to same conclusion by putting the three corners together (card 0159).

Which method is best?



Subtracting ≋ from

*

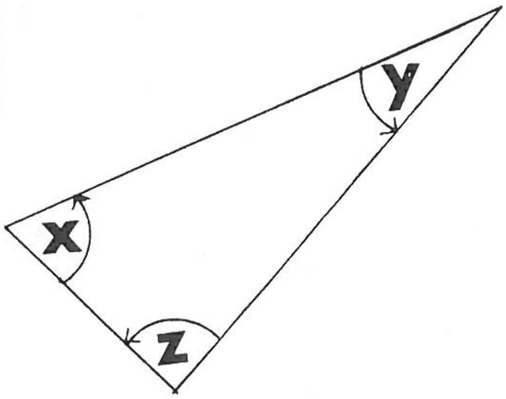
X + Y + Z = ■

The interior angles of a triangle add up to ■

You should have found that: the interior angles of a triangle add up to 180°.
If not, check again or use the answer book.

Why?

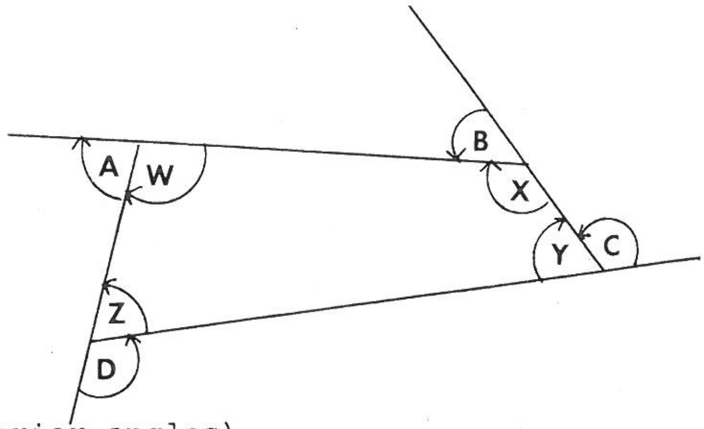
You could measure X, Y and Z (card 0235).



X + Y + Z = ■

Is the total exactly 180°?

(1) In a quadrilateral there are 4 interior angles and 4 exterior angles.



Copy and complete:

a) A + B + C + D = ■ (exterior angles)

b)

	A + W	= ■
	B + X	= ■
	C + Y	= ■
	D + Z	= ■
A + B + C + D + W + X + Y + Z		= ■

c) So W + X + Y + Z = ■

d) The interior angles of a quadrilateral total ■

---

(2) Work out the sum of the interior angles of:

(a) a pentagon ?

(b) an octagon ?

(3) Copy and complete:

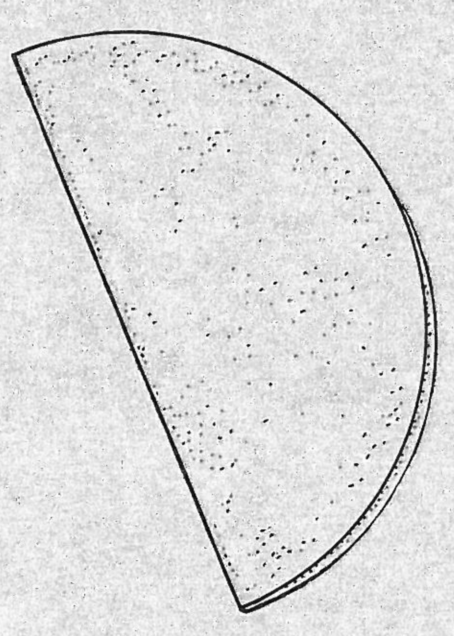
Polygon	Number of sides	Sum of interior angles (in degrees)
Triangle	3	(3 x 180) - 360 = 180
	4	(4 x 180) - 360 = 360
Pentagon	5	(5 x 180) - 360 =
	6	
	8	
Decagon		
-	22	
-	n	

You will need filter papers.

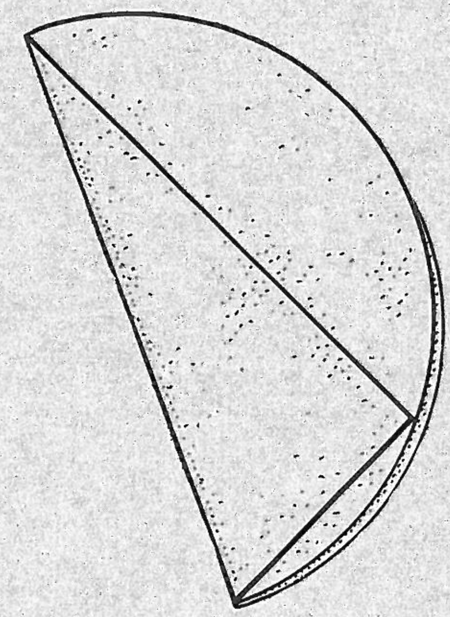
Smile **1935**

Angles in semi-circles

Fold a filter paper in half.



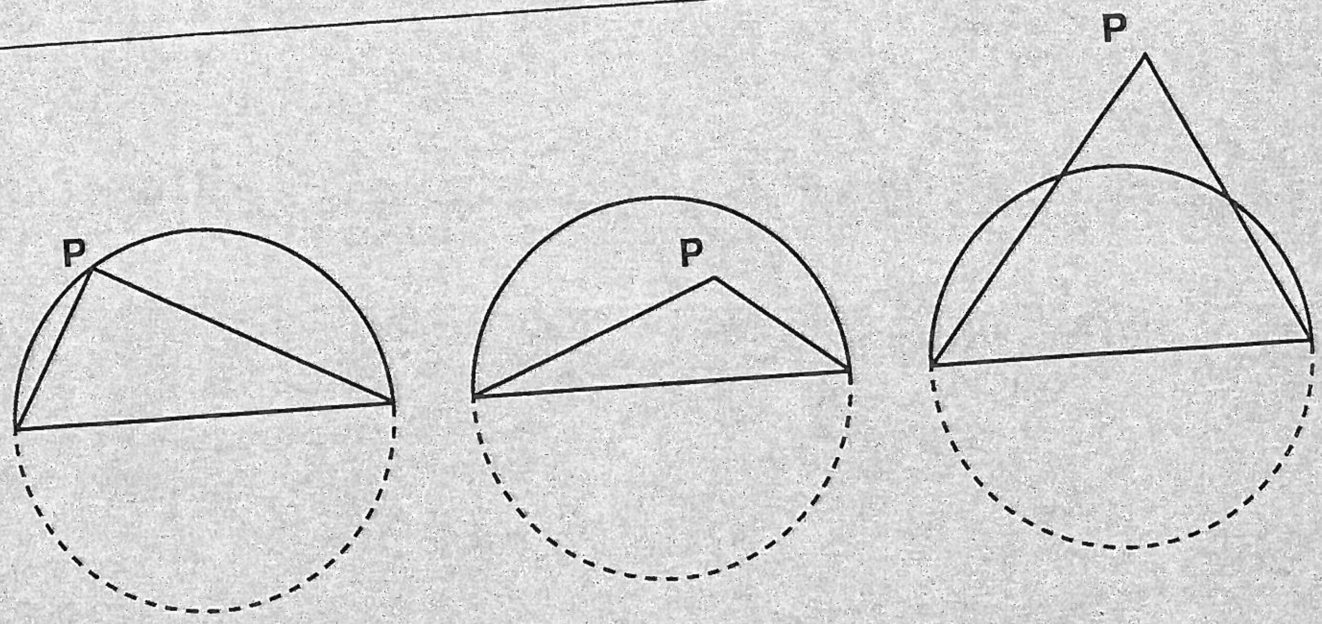
Draw different triangles using the fold as the base.



Investigate the angles of your triangles.

Does the size of the circle matter?

Turn over



What happens to the angles if P is inside the circle ?

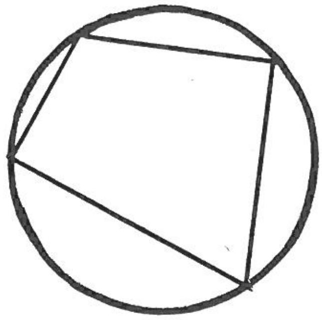
. . . outside the circle?

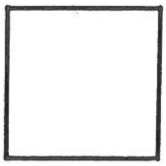
You will need: 2 paper circles, squared paper, compasses

The Cyclic Quadrilateral

Draw a circle. Mark any FOUR points on the circumference. Join them to make a quadrilateral.

When a circle can be drawn through all four vertices of a quadrilateral we have a CYCLIC QUADRILATERAL.





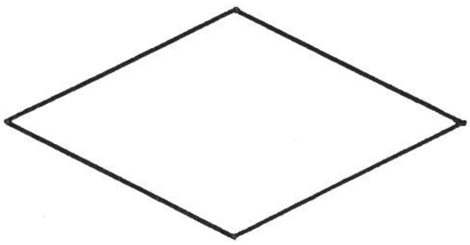
On squared paper draw any square and any rectangle

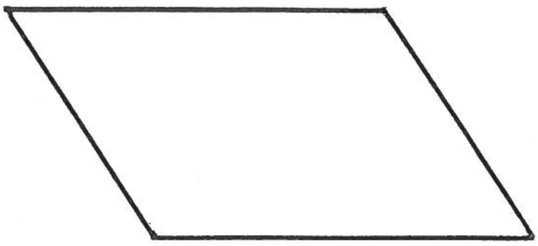
(1) Is it possible to draw a circle that passes through all four vertices of the SQUARE? If it is , draw it.

(2) Is it possible to draw a circle that passes through all four vertices of the RECTANGLE? If it is, draw it.

(3) SQUARES and RECTANGLES are (always / sometimes / never) cyclic quadrilateral.

Copy the sentence out using the correct word.

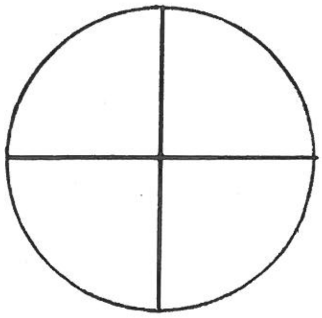




On squared paper draw any RHOMBUS and any PARALLELOGRAM.

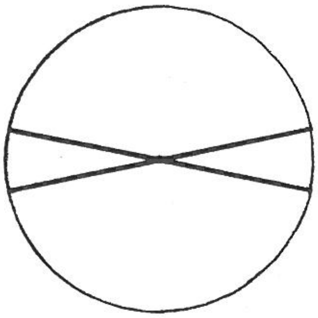
(4) Is it possible to draw a circle through all four vertices of the rhombus or the parallelogram?

Turn over



Fold a paper circle into quarters as shown in the diagram.
Pencil over the folds.
With the folds as diagonals, draw a CYCLIC RHOMBUS.

(5) What is the usual name for this shape?



Fold another circle with two diameters as shown in this diagram.
Pencil over the folds.
Complete your CYCLIC PARALLELOGRAM.

(6) What is the usual name for this shape?

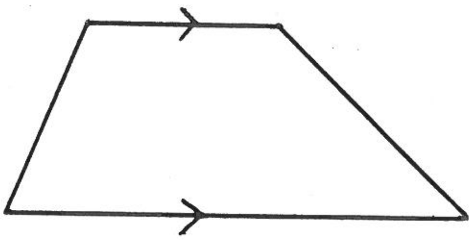
(7) Copy this sentence using the correct word:-

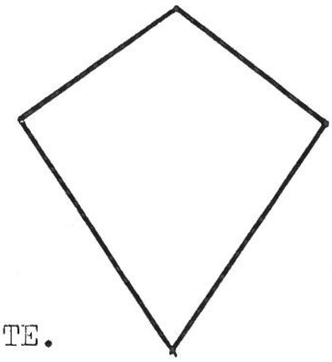
RHOMBUSES AND PARALLELOGRAMS are (always / sometimes / never) cyclic quadrilaterals.

Copy and complete:-

(8) A cyclic rhombus is a [redacted]

(9) A cyclic parallelogram is a [redacted]





On squared paper draw any TRAPEZIUM and any KITE.

(10) Can you draw a circle through all four vertices of the trapezium or the kite?
On squared paper draw two circles. In one circle draw a CYCLIC TRAPEZIUM and in the other a CYCLIC KITE.

(11) What can you say about the sides and angles of this trapezium?

(12) Copy this sentence using the correct word:-

TRAPEZIUMS and KITES are (always / sometimes / never) cyclic quadrilaterals.

0731

You will need: angle indicator, graph paper

REGULAR POLYGONS
or
'The V and A'

n = number of sides
V = angle at each vertex
A = angle at centre

IS THERE ANY CONNECTION BETWEEN V AND A?

(1) **MEASURE** angles V and A for the different regular polygons on pages 2 and 3.

(2) **RECORD YOUR RESULTS**

in a table:

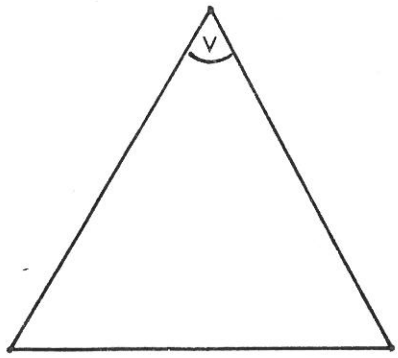
Polygon	n	V	A
Triangle	3	60°	120°
Square	4	90°	

(3) **PLOT A GRAPH** of your results. Plot V on one axis against A on the other.

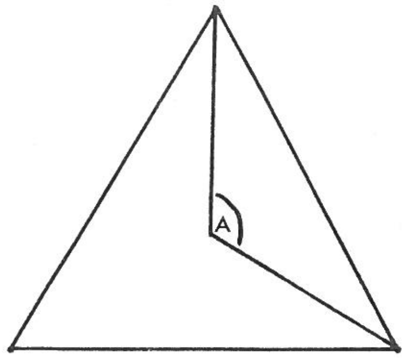
(4) Why should you NOT join the points on this graph?

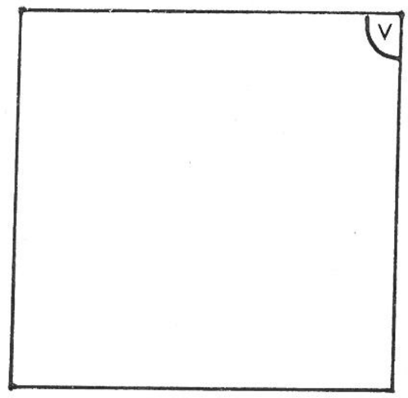
(5) What does your table and/or graph suggest about a connection between V and A?

Now turn to page 4.

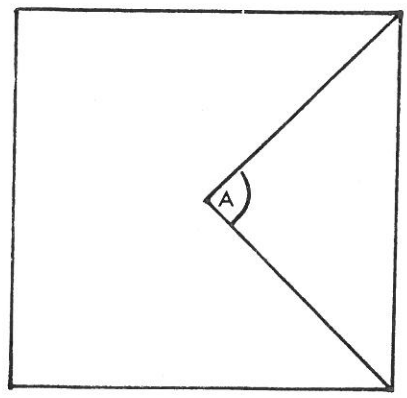


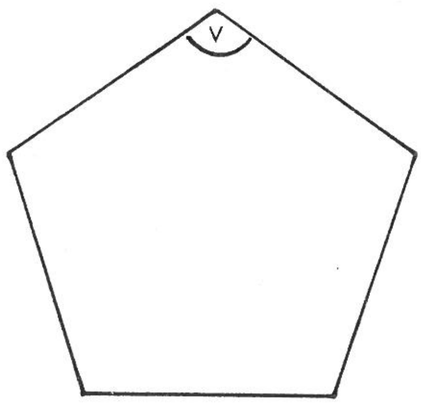
EQUILATERAL
TRIANGLE



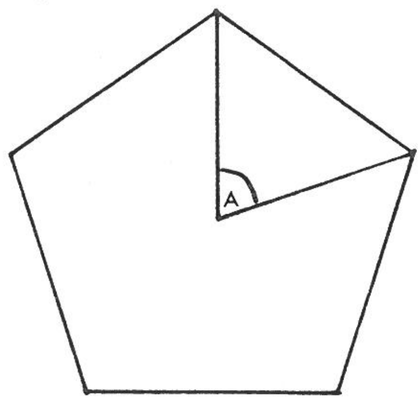


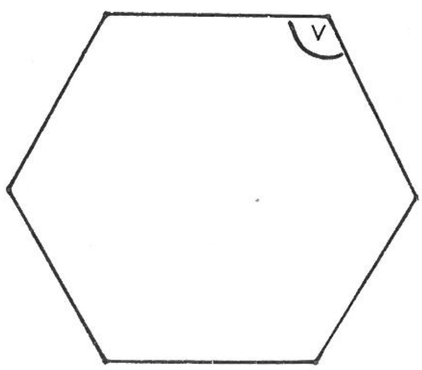
REGULAR
QUADRILATERAL
(SQUARE)

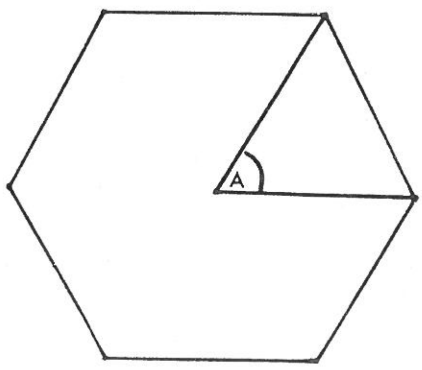


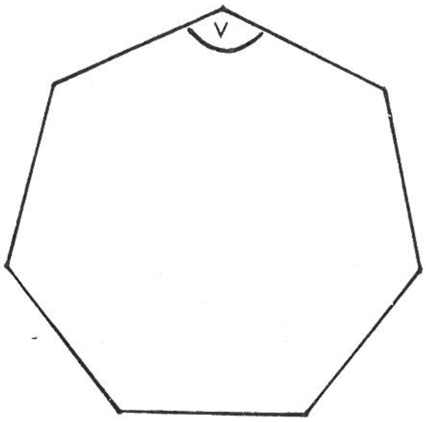


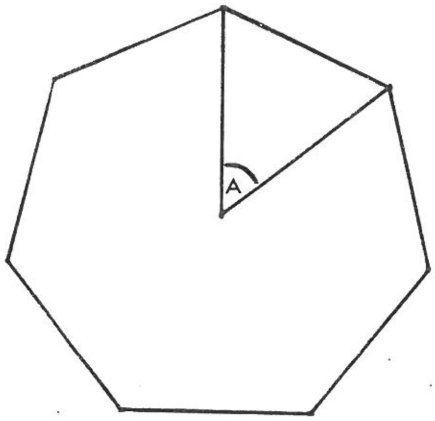
REGULAR
PENTAGON

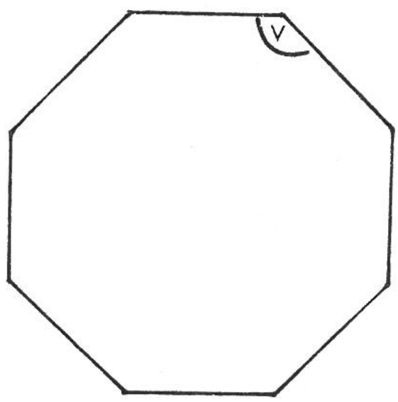


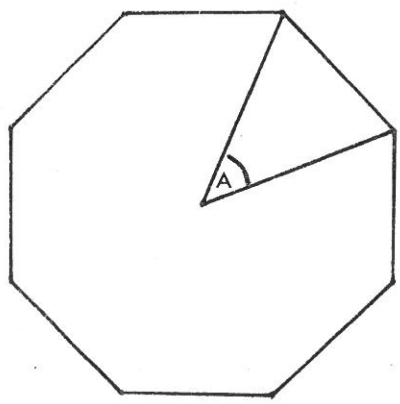


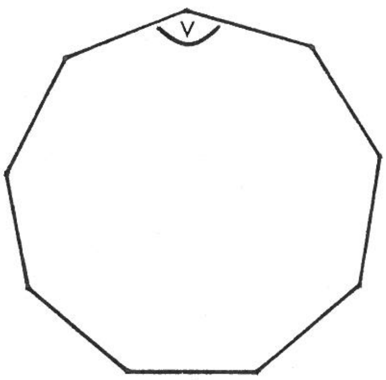


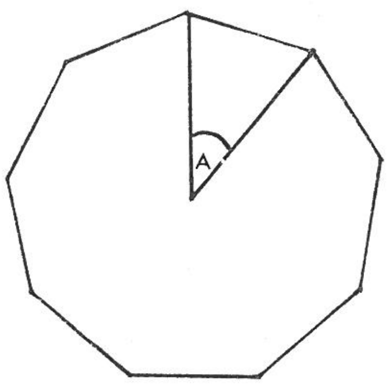

V

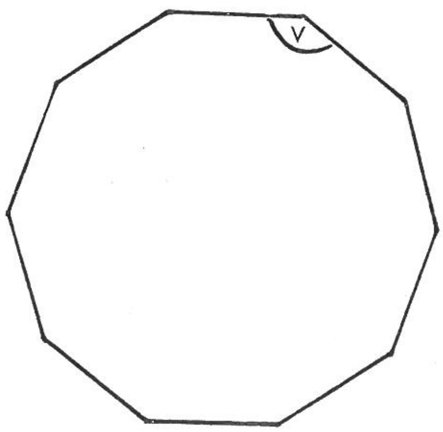

A

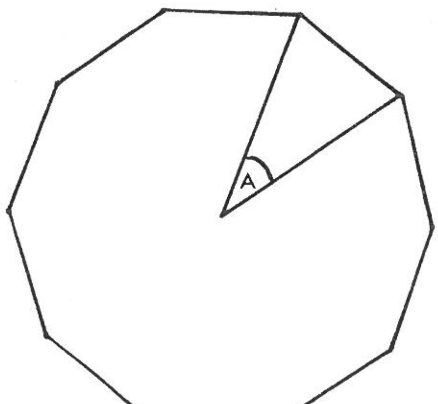

V


A

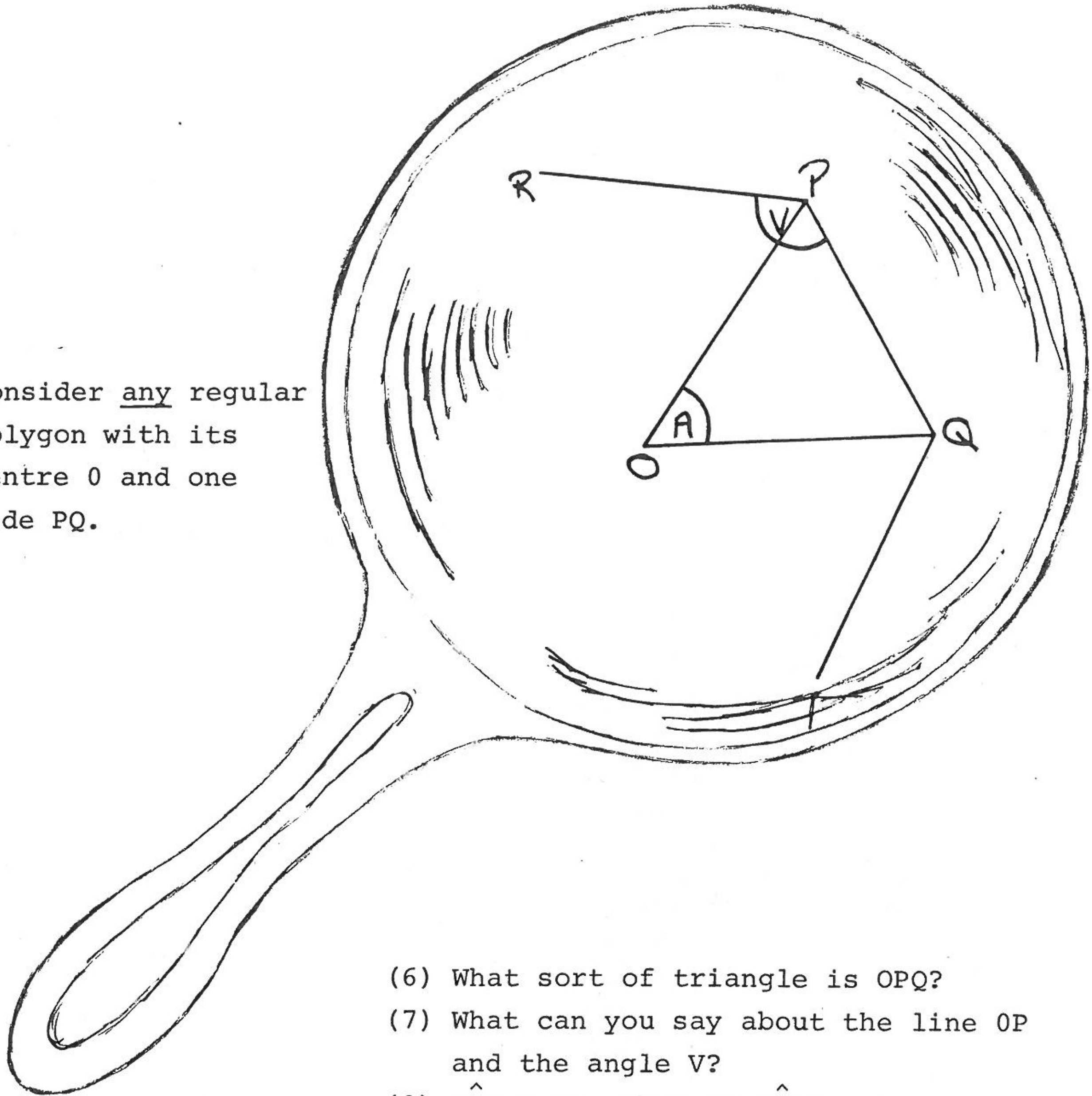

V


A


V


A

Consider <u>any</u> regular polygon with its centre 0 and one side PQ.



(6) What sort of triangle is OPQ?

(7) What can you say about the line OP and the angle V?

(8) $R\hat{P}Q = V$; what is $O\hat{P}Q$?

(9) What is $P\hat{Q}0$?

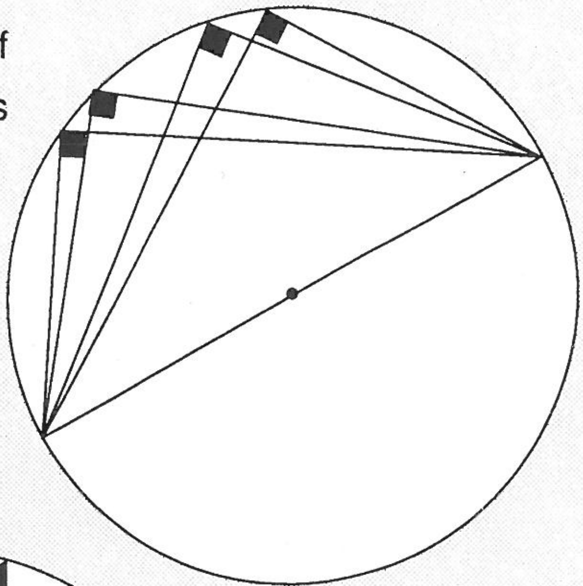
(10) Use the angle sum of a triangle to write an equation connecting V and A.

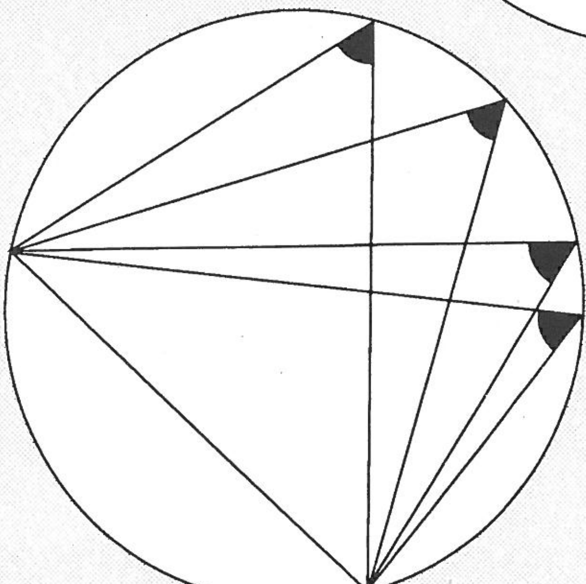
(11) Your work from page 1 should have <u>suggested</u> a connection between V and A. If your answers to the questions on this page are correct, you have now <u>proved</u> the connection.

What do you think is the difference between these two pieces of work?

Angles in Circles

Calculate the sizes of all the marked angles on the twelve circles inside this card.

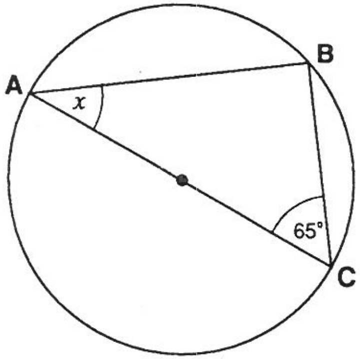




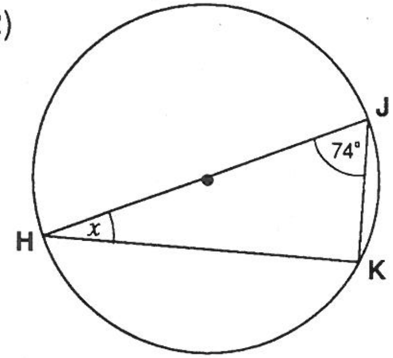
There are some facts about circles on the back of this card.

These are not drawn accurately.
Show all your working.

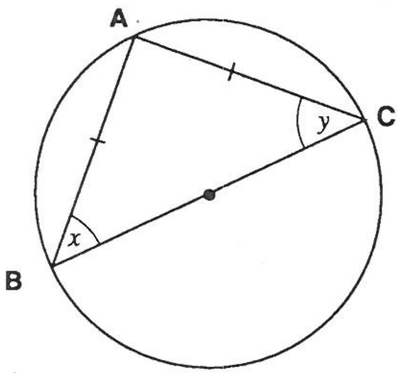
1)



2)

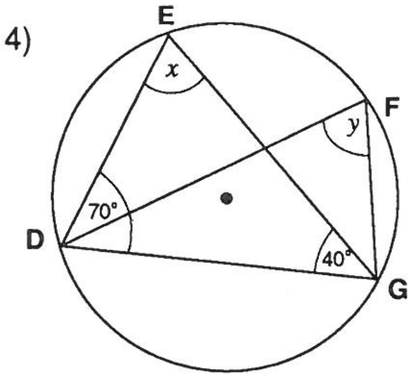


3)

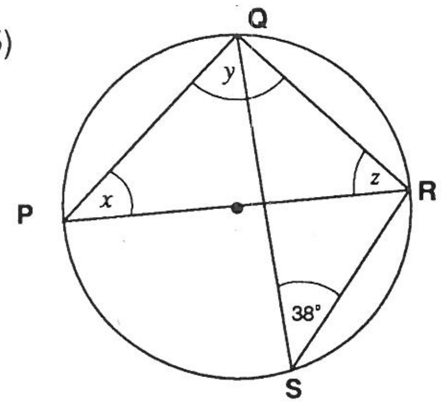


AB = AC

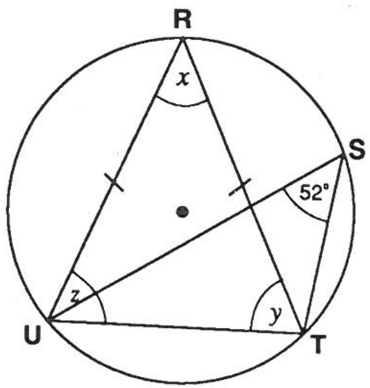
4)



5)

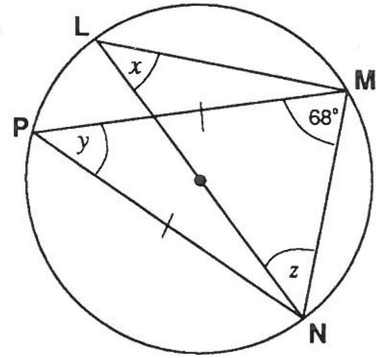


6)



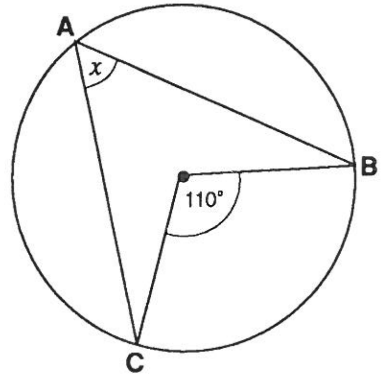
RU = RT

7)

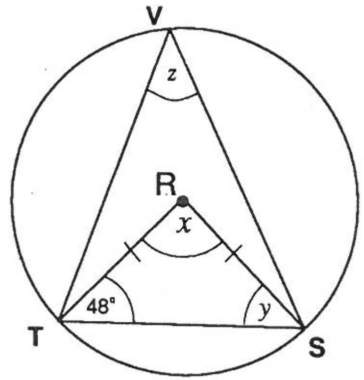


MP = NP

8)

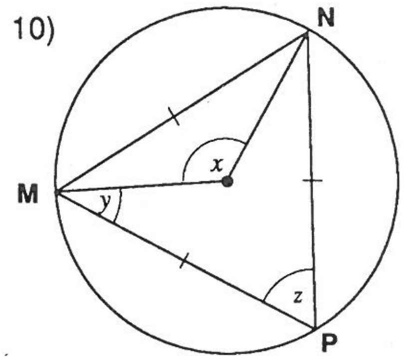


9)



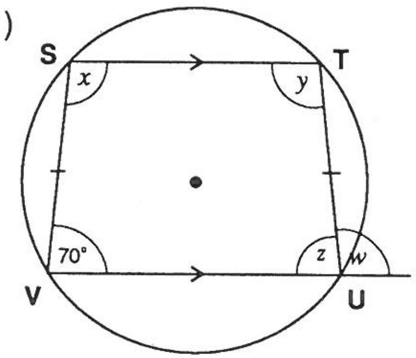
RT = RS

10)



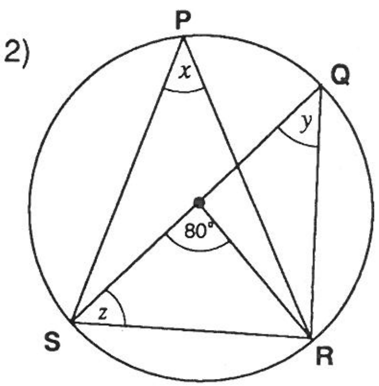
MN = NP = PM

11)



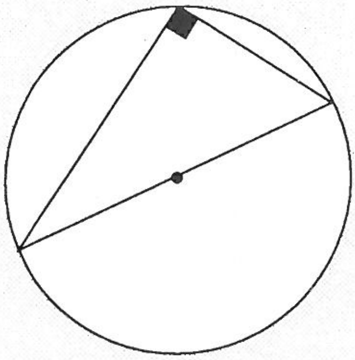
SV = TU

12)

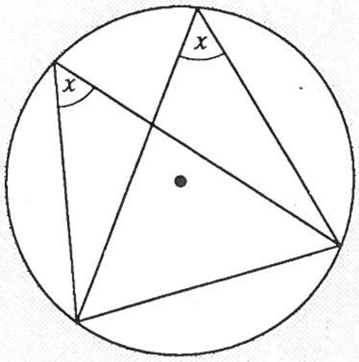


Angles in Circles

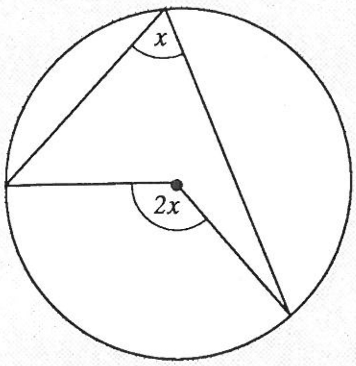
The angle at the circumference of a semi-circle is 90°.



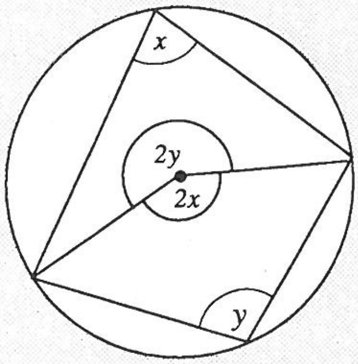
The angles at any two points of the circumference standing on the same arc are equal.



The angle at the centre is twice the angle at any point on the circumference standing on the same arc.



The sum of the opposite angles of a cyclic quadrilateral is 180°.



$x + y = 180°$