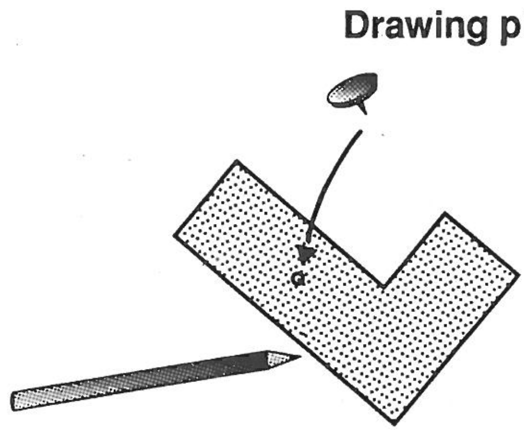
SMILE WORKCARDS

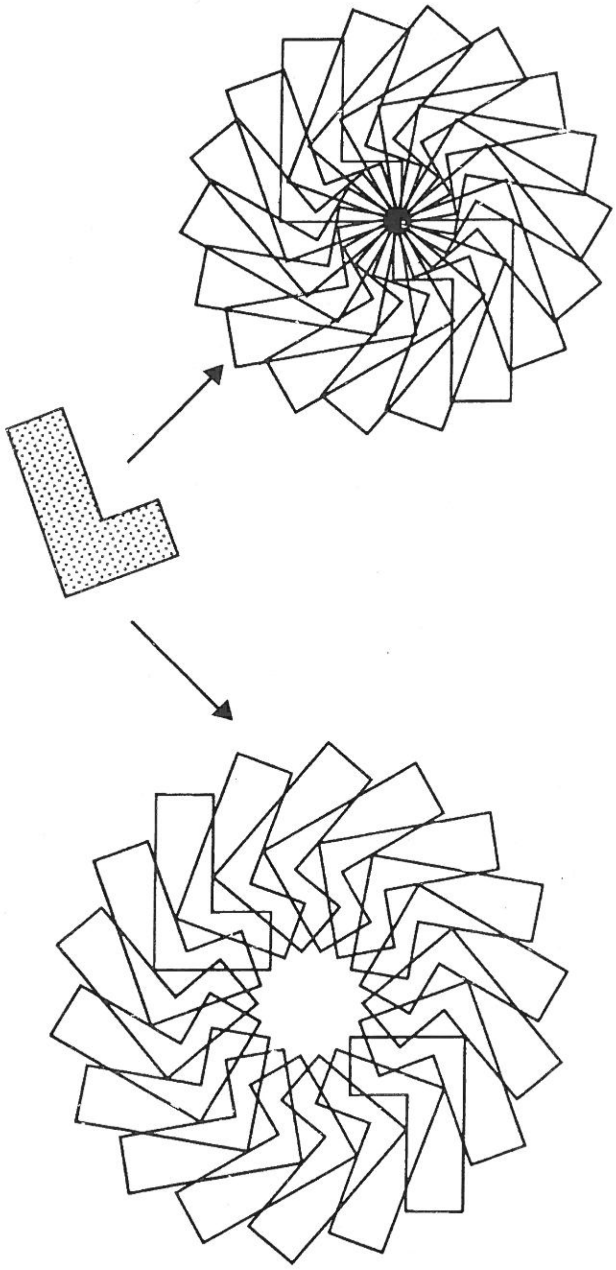
Rotation

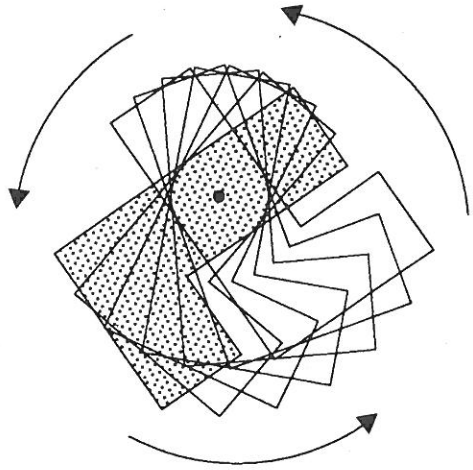
Contents

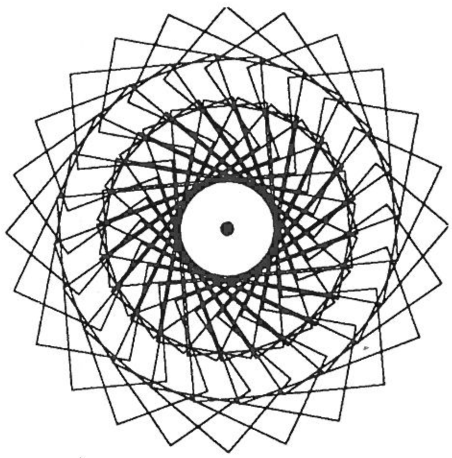
You will need :
scissors, cardboard, drawing pin.

Turning Patterns









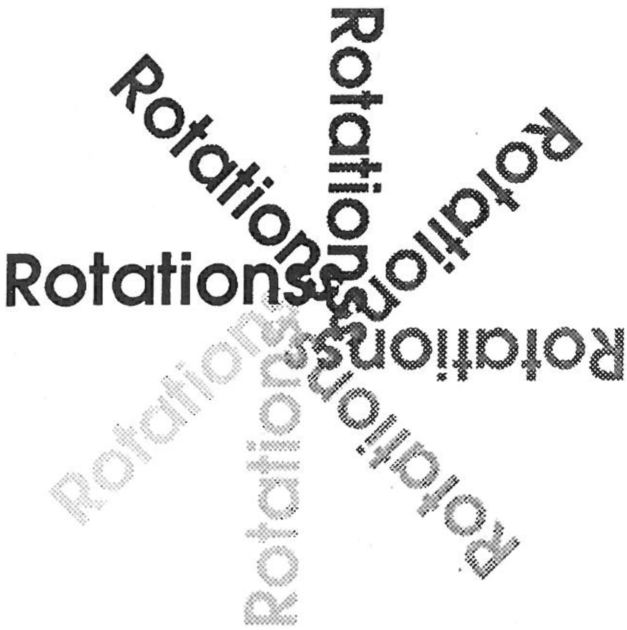
These patterns can be made by turning and drawing around an shape.

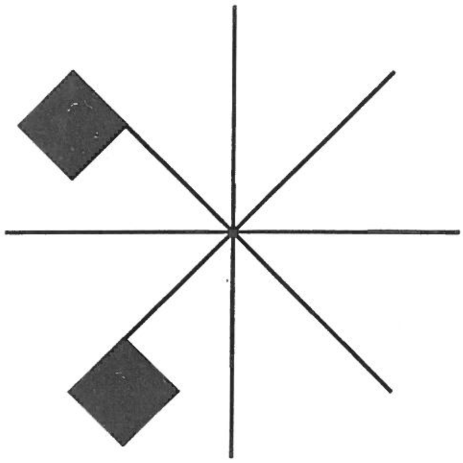
Change the position of the drawing pin to make different patterns.

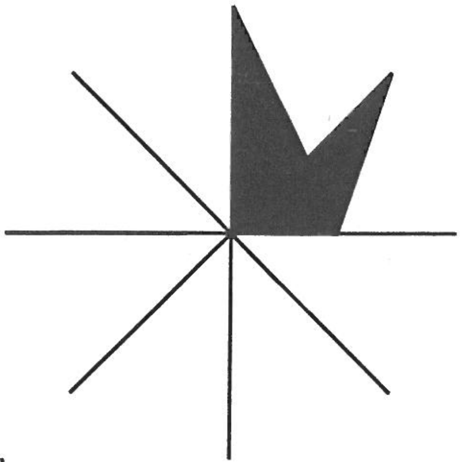
Try turning a different shape.

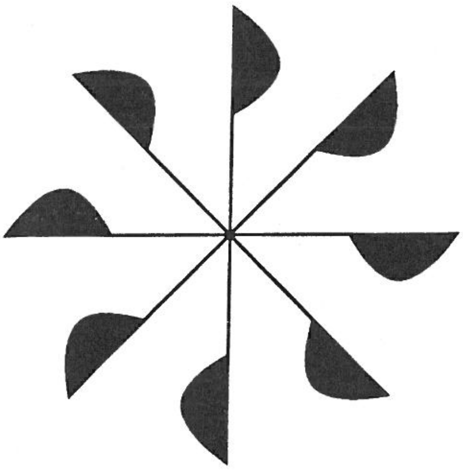
Turn over

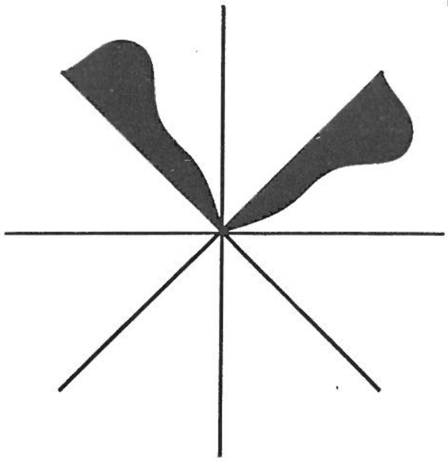
You will need tracing paper.



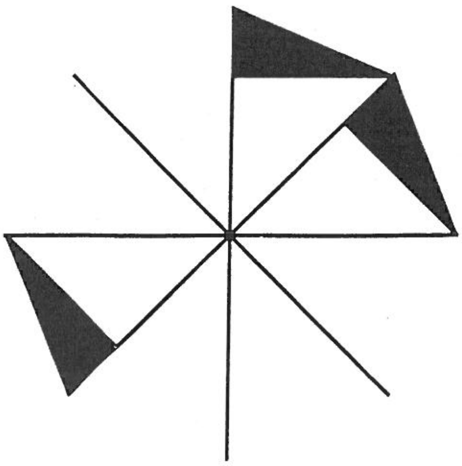


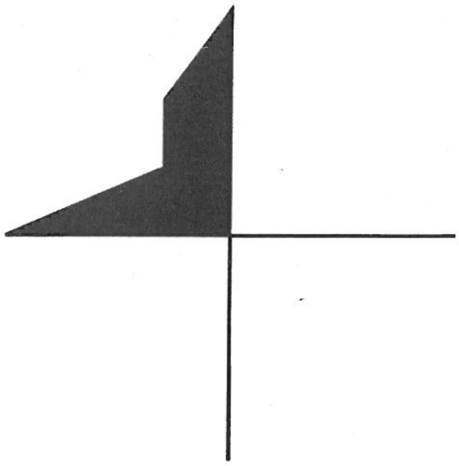






Copy and complete the following patterns.



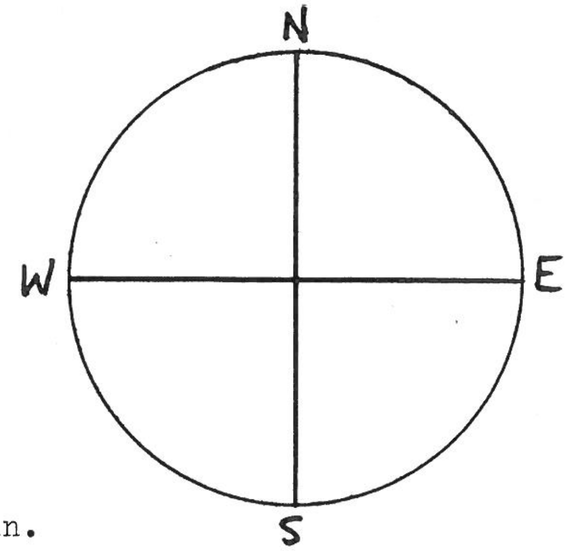


Now make up some more of your own.
You may like to use **LOGO.**

Turn over

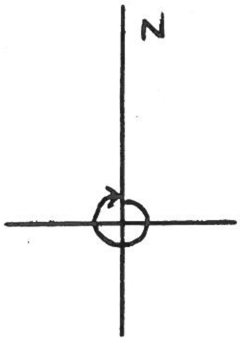
You will need: compasses

Angles: the compass



Draw a circle
in your book.

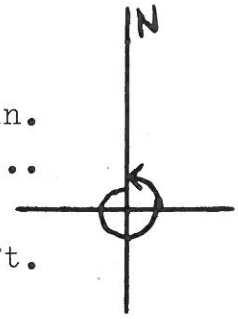
Mark on N, S, E and W.



Stand by your book.
Face NORTH.
Turn RIGHT until you
are facing north again.

This is one whole turn
to the right.

Face north again.
Turn LEFT and....
make one whole
turn to the left.

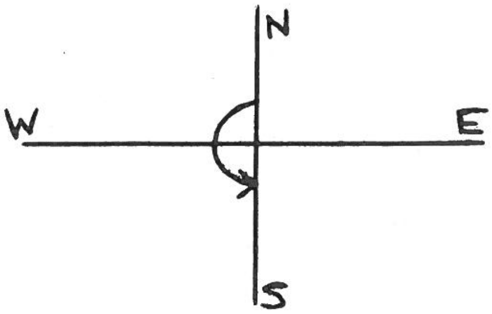


Copy and complete this table:-

	START	TURN which way	TURN how much	END
(a)	NORTH	left	½ turn	SOUTH
(b)	SOUTH	left	½ turn	
(c)	WEST	right	¼ turn	
(d)	NORTH	right	¾ turn	
(e)	EAST	left	2 turns	
(f)	NORTH	right	1½ turns	
(g)	WEST	right	¾ turn	
(h)	NORTH	left		WEST
(i)	EAST	left		WEST
(j)	WEST	right		WEST
(k)	SOUTH	right		EAST
(l)	WEST	right		South

(a) has been done for you.

Here is a diagram to show the answer another way:



Draw a diagram like this for each row of your table.

Compass Game

Each player needs a counter or centicube.

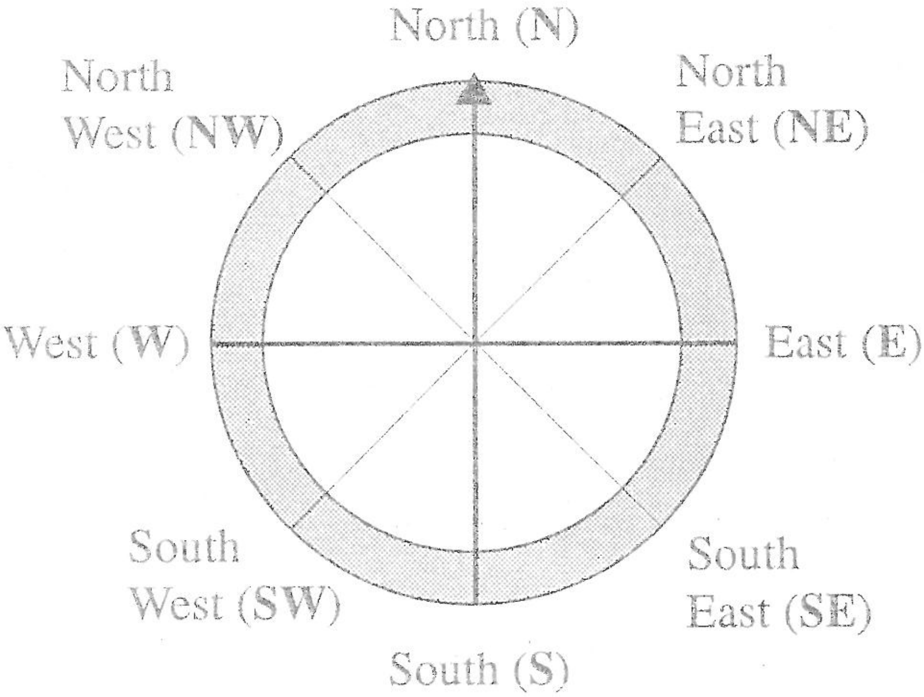
You will need a dice marked

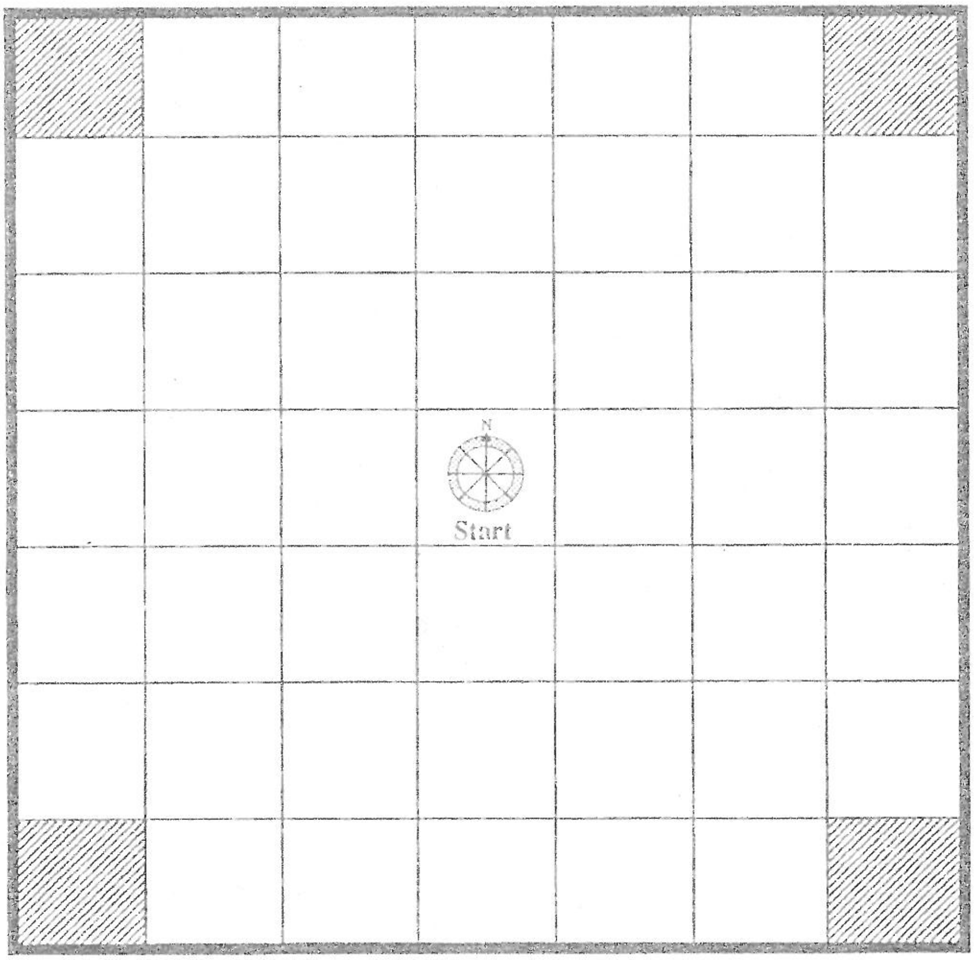
N / S	E / W	NE / SW	NW / SE	NW / NE	SW / SE

Take turns to throw the dice.

Choose one of the directions shown.
Move **one** square in that direction.

The winner is the first person to reach a shaded square.


North (N)
North West (NW)
North East (NE)
West (W)
East (E)
South West (SW)
South East (SE)
South (S)

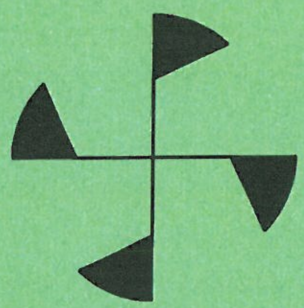

N
Start

You will need copies of Smile Worksheets **2111a**, **2111b**, **2111c**.

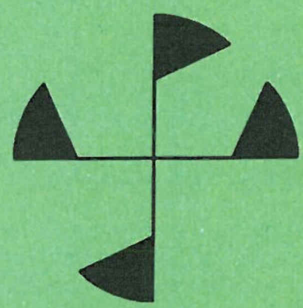
Smile **2111**

Rotational Symmetry Jigsaws

This pattern has rotational symmetry.



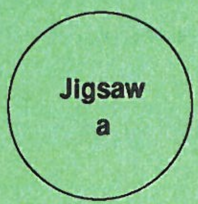
This pattern does not have rotational symmetry.



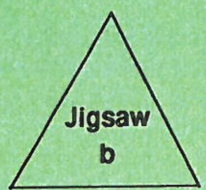
Can you see why?

Turn over

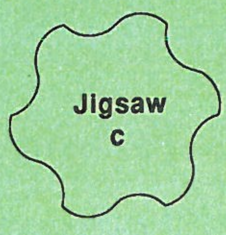
Cut out the pieces from the worksheets and find these rotational symmetry patterns.



The jigsaw on worksheet **2111a** can make 2 different rotational symmetry patterns.



The jigsaw on worksheet **2111b** can make 3 different rotational symmetry patterns.



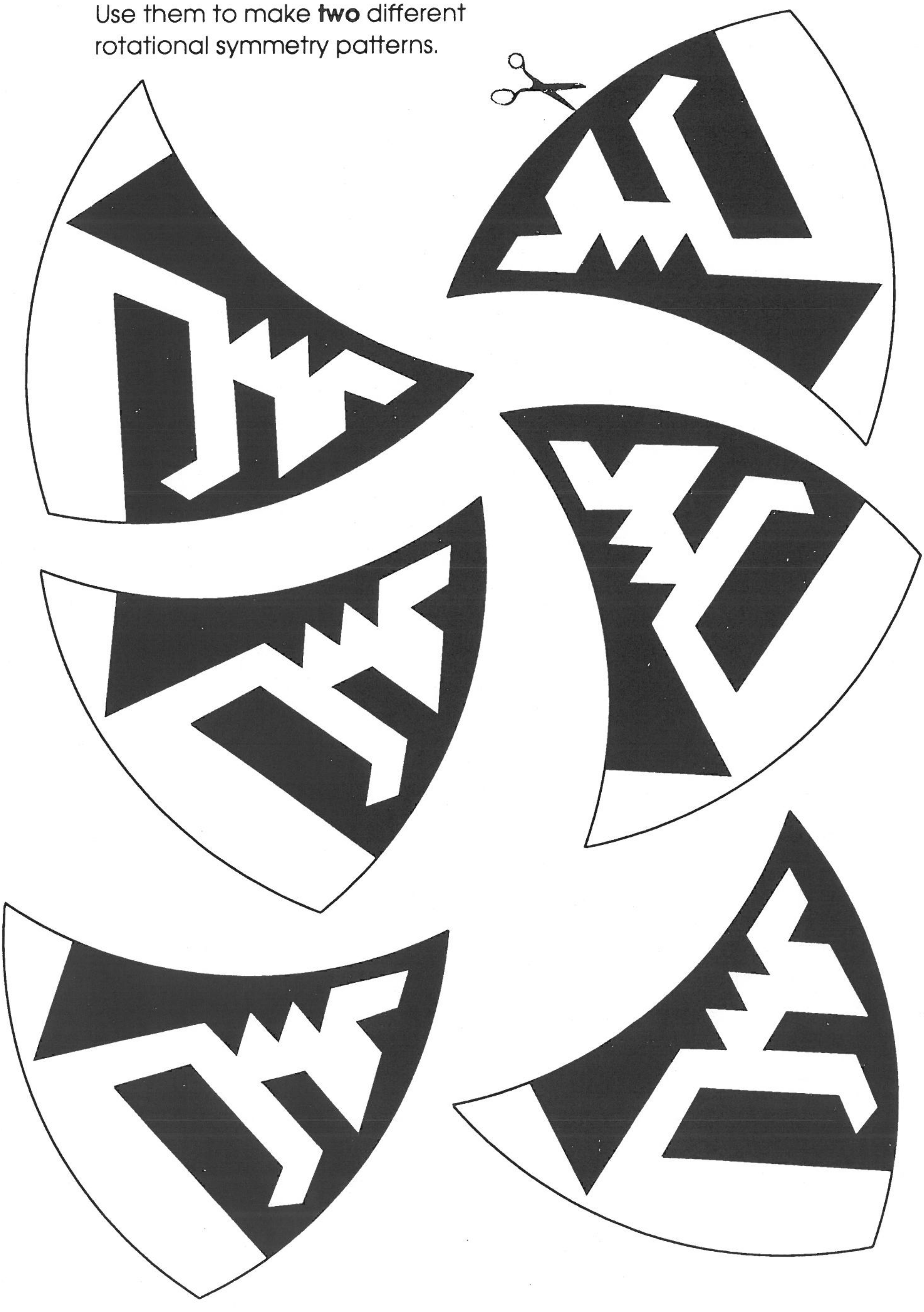
The jigsaw on worksheet **2111c** can make many rotational symmetry patterns.

How many can you make?

You may like to display your results.

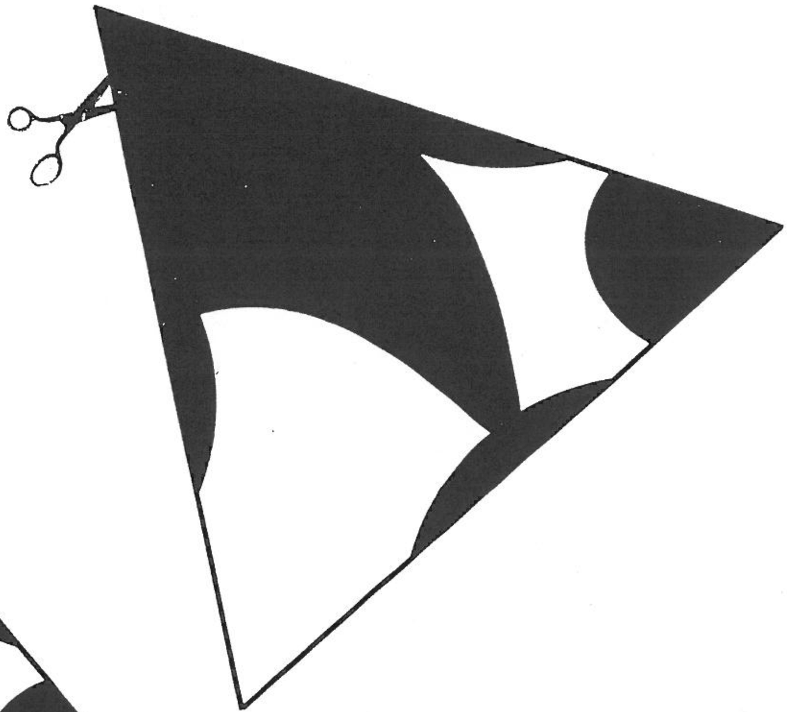
Cut out these 6 shapes.

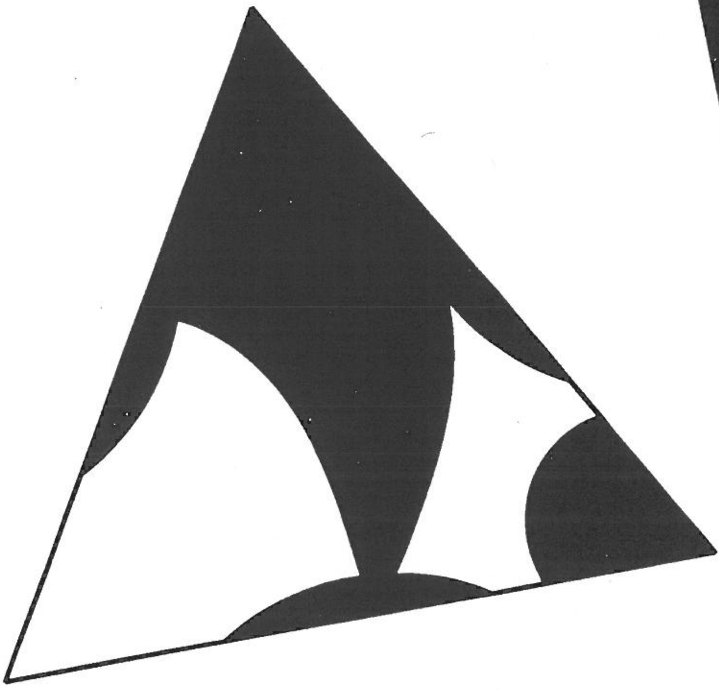
Use them to make **two** different rotational symmetry patterns.

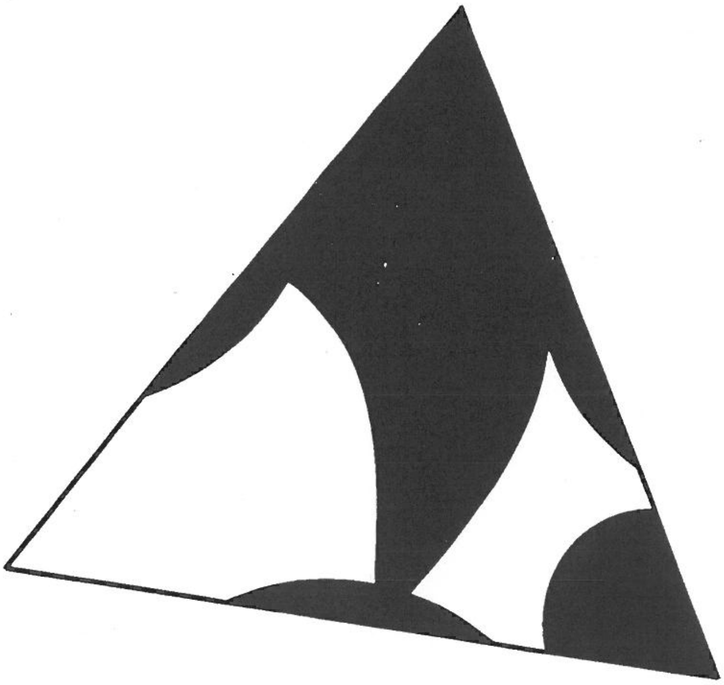


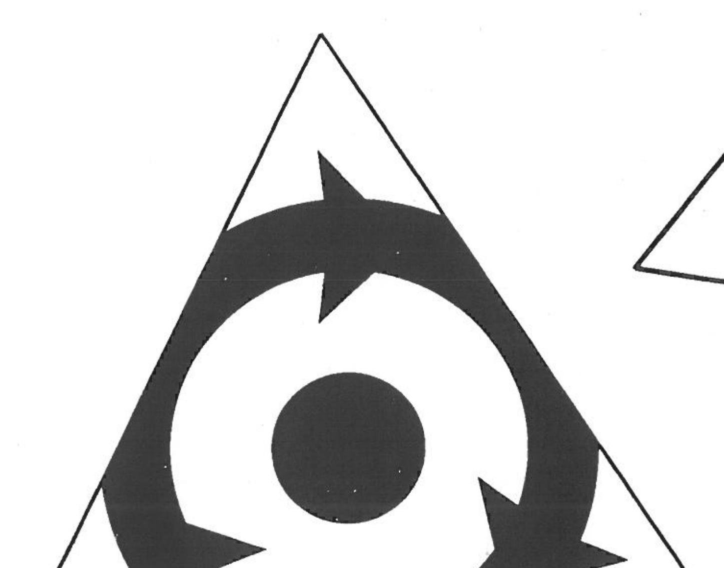
Cut out these 4 triangles.

Use them to make **three** different rotational symmetry patterns.



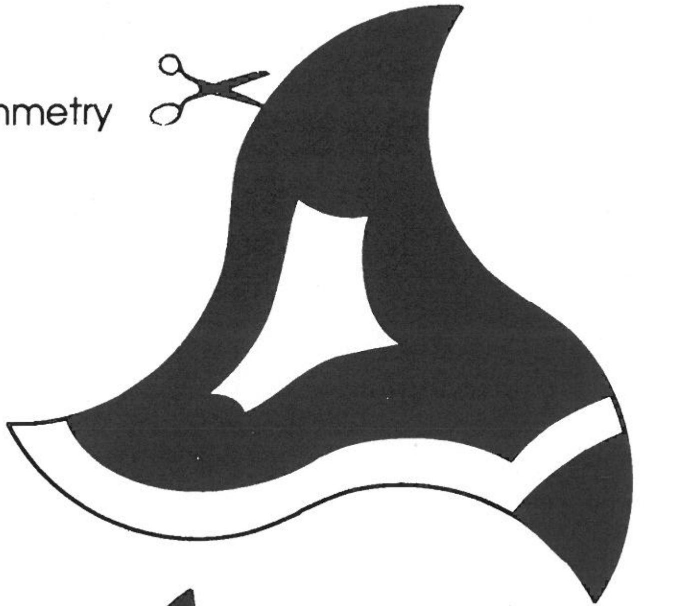


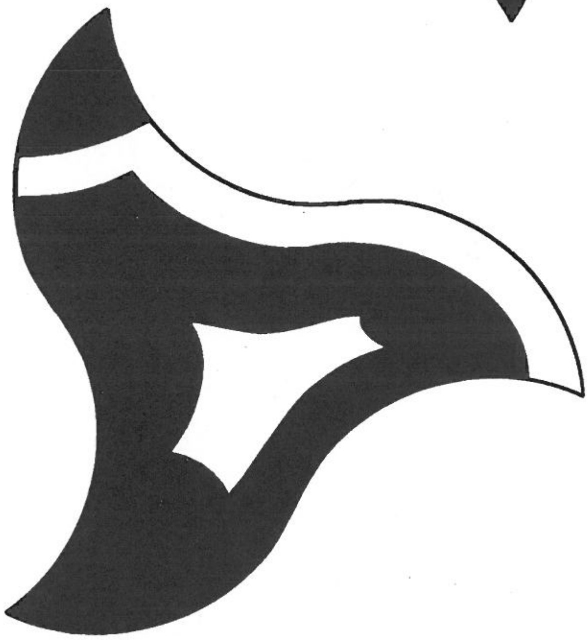


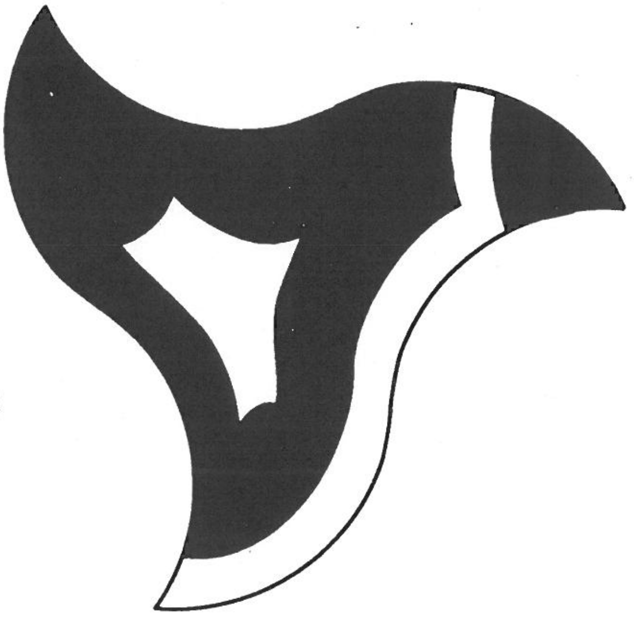


Cut out these 6 shapes.

How many different rotational symmetry patterns can you make?



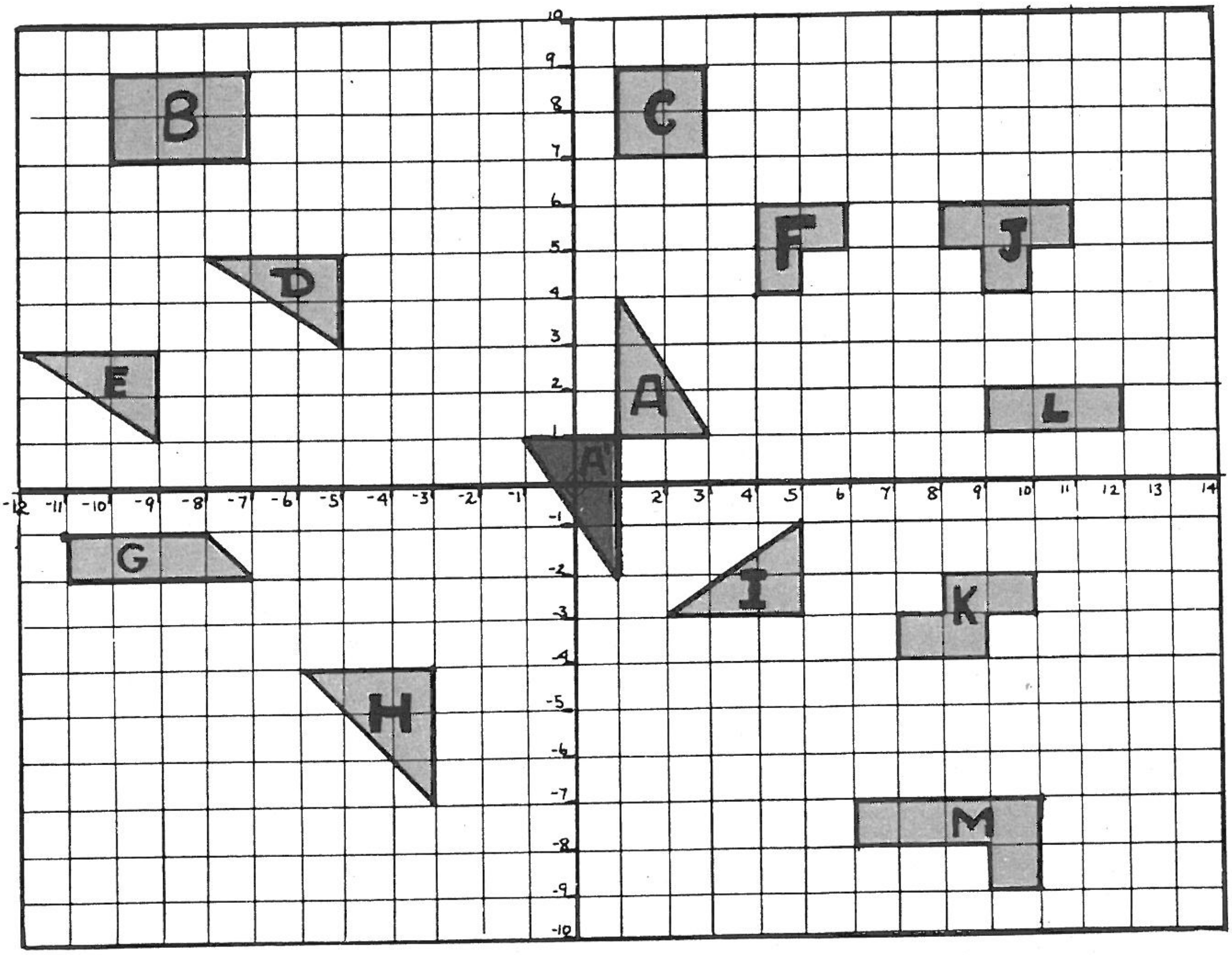




0730

ROTATION WORKSHEET

Rotate the shapes as instructed below.
Tracing paper might help.



A half turn anti-clockwise about (1,1).(This has been done for you)

B quarter turn clockwise about (-7,7)

C quarter turn anti-clockwise about (3,9)

D quarter turn clockwise about (-5,3)

E quarter turn anti-clockwise about (-12,3)

F half turn clockwise about (5,4)

G quarter turn anti-clockwise about (-7,-2)

H three quarter turn anti-clockwise about (-3,-4)

I quarter turn clockwise about (2,-3)

J half turn about (11,6)

K quarter turn anti-clockwise about (7,-4)

L half turn about (11,1)

M half turn about (10,-7)

Centre of Rotation

You will need tracing paper.

Trace one of the  shapes

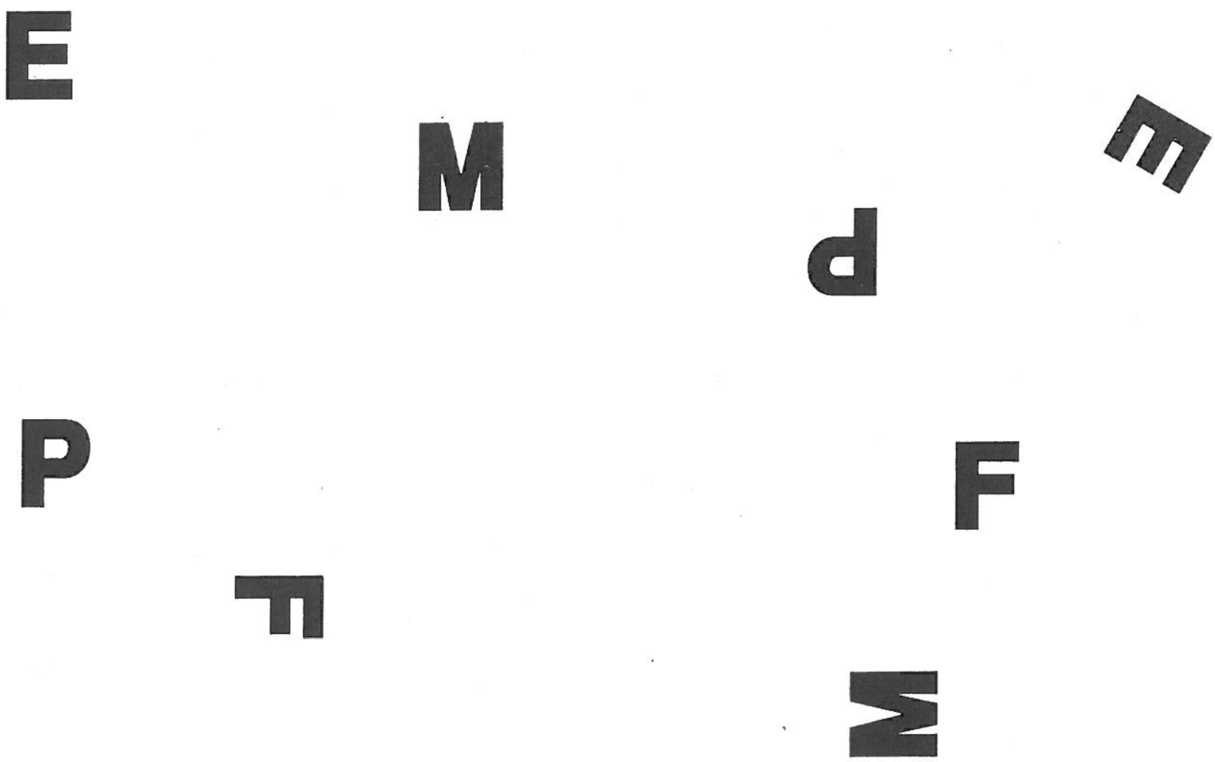
Put your pencil point on **x** and rotate the tracing paper.

One  shape can be rotated

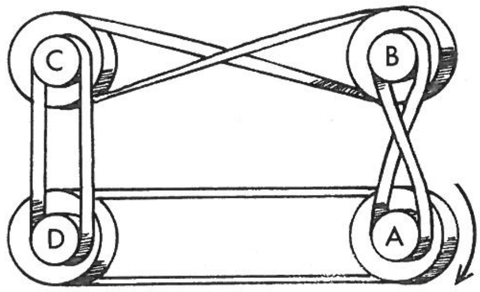
on to the other.

x is the centre of rotation.

Find the centre of rotation for each of these letters.

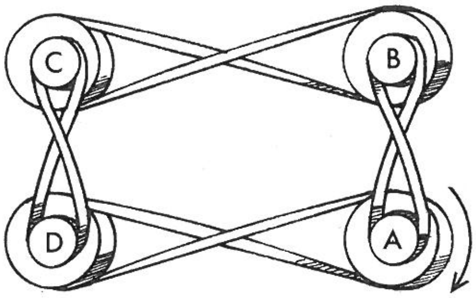


WHEELS



Four wheels are connected by belts. Two of the belts are crossed.

If wheel A turns clockwise, which way do the other wheels turn?



This time all four belts are crossed.

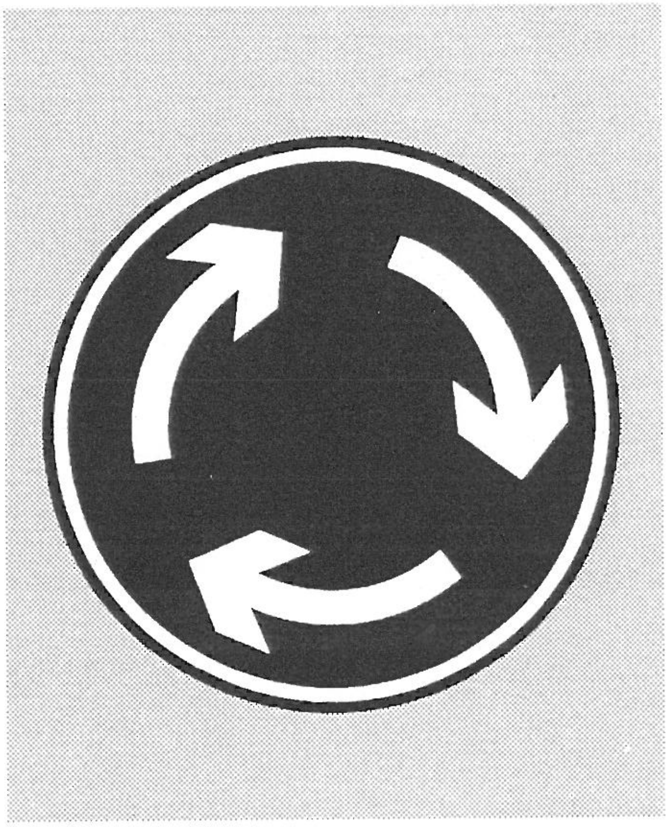
What happens?

Investigate other arrangements of the belts.

How can the belts be arranged so that wheel A turns clockwise and wheels B, C and D all turn anti-clockwise?

Rotational Symmetry

You will need a copy of the Highway Code.



This traffic sign is for a mini-roundabout.

Trace the sign.

How many ways does your tracing fit *exactly* on top of the original?
(You must not turn the tracing paper over.)

The sign has **rotational symmetry** of **order 3.**

Find as many signs as you can which have rotational symmetry.

For each one

- Sketch the sign.
- Write down the order of rotational symmetry.

ROTATION

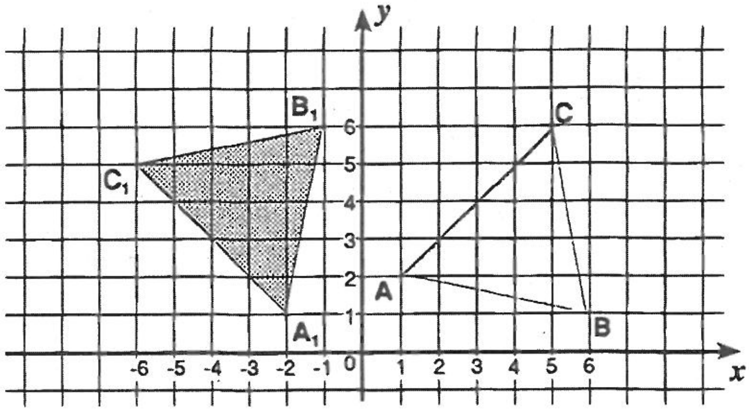
1. a) Draw axes with x values from -6 to 6 and y values from 0 to 6.

   Plot points at **A** (1, 2), **B** (6, 1) and **C** (5, 6). Join them to form the triangle **ABC**.

   Trace the axes and the triangle.

   Rotate the tracing paper through 90° *anti-clockwise*, about (0, 0).

   Draw the rotated triangle, label it $A_1B_1C_1$.



b) Copy and complete the mapping:

A (1, 2)	rotation through 90° about (0, 0) ⟶	A_1 (-2, 1)
B (6, 1)	⟶	B_1 (,)
C (5, 6)	⟶	C_1 (,)

Turn over

2. a) Draw axes, with x and y values from -8 to 8.

Plot the points **P** (3, 2), **Q** (2, 8), **R** (4, 5) and **S** (8, 4). Join them to form **PQRS**.

Use the same method as before to rotate **PQRS** through 90° *anti-clockwise* about (0, 0). Label the rotated shape $P_1Q_1R_1S_1$.

b) Copy and complete the mapping:

rotation through 90° about (0, 0)

P (3, 2) ——————→ P_1 (,)

Q (2, 8) ——————→ Q_1 (,)

R (4, 5) ——————→ R_1 (,)

S (8, 4) ——————→ S_1 (,)

3. a) Rotate **PQRS** through 180° about (0, 0). Label the new shape $P_2Q_2R_2S_2$.

b) Copy and complete the mapping for this rotation.

rotation through 180° about (0, 0)

P (3, 2) ——————→ P_2 (

Q (2, 8) ——————

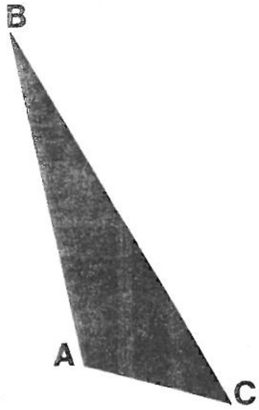
4. a) Rotate **PQRS** through 270° about (0, 0). Label the new shape $P_3Q_3R_3S_3$.

b) Complete a mapping for this rotation.

SMILE WORKSHEET **0839**

ROTATE THIS WAY

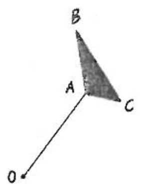
You will need:
compasses and an
angle indicator



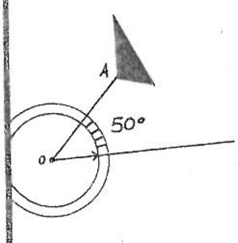
O .

To rotate triangle ABC 50° clockwise about the point O:

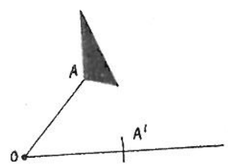
1 JOIN **O** TO **A**



2 USE AN ANGLE INDICATOR TO MARK A LINE 50° CLOCKWISE FROM LINE **OA**

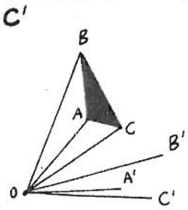


3 USING COMPASSES MEASURE **OA** AND MARK OFF THE SAME DISTANCE FROM **O** ALONG THE 50° LINE

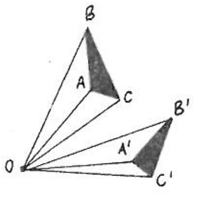


THIS GIVES THE POINT **A'** WHICH IS THE IMAGE OF **A** AFTER A ROTATION OF 50° CLOCKWISE ABOUT **O**

4 REPEAT 1, 2, AND 3 FOR VERTICES **B** AND **C** TO GIVE POINTS **B'** AND **C'**



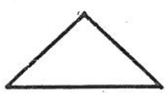
5 DRAW IN TRIANGLE A'B'C'.



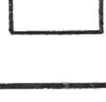
THIS IS **ABC** ROTATED 50° CLOCKWISE ABOUT **O**

6 CHECK THAT A'B'C' AND ABC ARE CONGRUENT

ROTATE THESE SHAPES ABOUT A POINT IN YOUR BOOK:

A)  60° CLOCKWISE

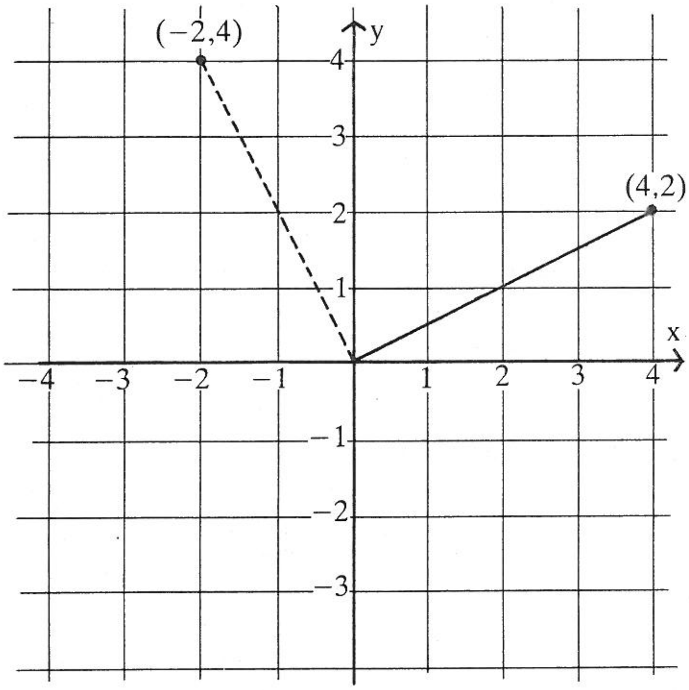
B)  110° ANTI-CLOCKWISE

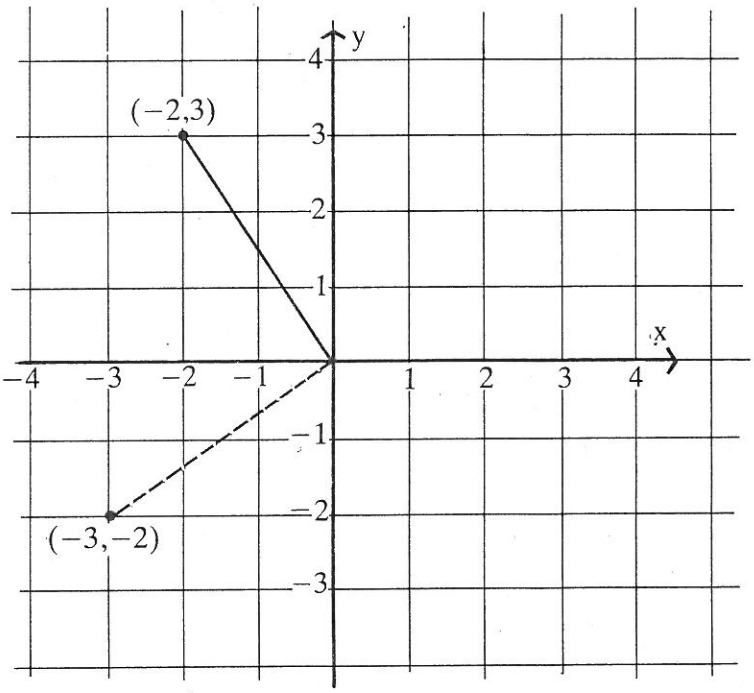
C)  150° ANTI-CLOCKWISE

Matrices for Rotations

This is an activity involving rotations about the origin, and their matrices.

Start by studying a 90° rotation, anti-clockwise.





$$\begin{pmatrix}4\\2\end{pmatrix} \longrightarrow \begin{pmatrix}-2\\4\end{pmatrix} \qquad\qquad \begin{pmatrix}-2\\3\end{pmatrix} \longrightarrow \begin{pmatrix}-3\\-2\end{pmatrix}$$

1. Find the transformation matrix which will rotate both these points 90° anti-clockwise.

$$\begin{pmatrix}\blacksquare & \blacksquare\\ \blacksquare & \blacksquare\end{pmatrix}\begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}-2\\4\end{pmatrix} \qquad \begin{pmatrix}\blacksquare & \blacksquare\\ \blacksquare & \blacksquare\end{pmatrix}\begin{pmatrix}-2\\3\end{pmatrix} = \begin{pmatrix}-3\\-2\end{pmatrix}$$

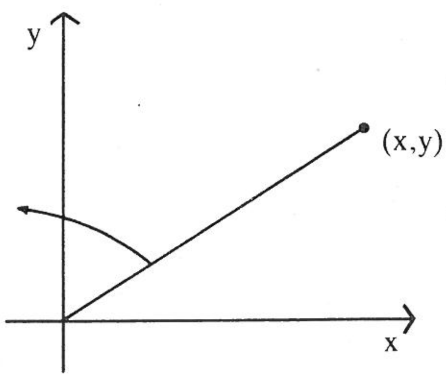
2. Choose four more points. *For each point:*

(a) Draw in its position vector.

(b) Rotate the point 90° anti-clockwise about the origin, and draw in the image vector.

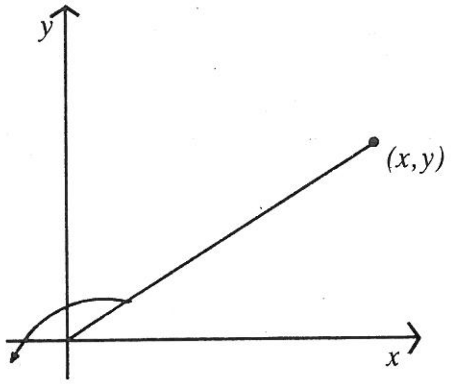
(c) Check that the new point can be obtained using the rotation matrix you found.

3. Repeat the steps of question 2 using the general point (x,y).



turn over

4. (a) Draw the position vector after (x,y) has been rotated **180°** about the origin.



$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} \blacksquare \\ \blacksquare \end{pmatrix}$$

(b) Use this to find the transformation matrix which will rotate 180° about (0,0).

$$\begin{pmatrix} \blacksquare & \blacksquare \\ \blacksquare & \blacksquare \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \blacksquare \\ \blacksquare \end{pmatrix}$$

Choose some points of your own and check by drawing, that this matrix will rotate them through 180°.

5. Square the matrix for a 90° rotation.

   (a) What transformation does the new matrix represent?

   (b) Why?

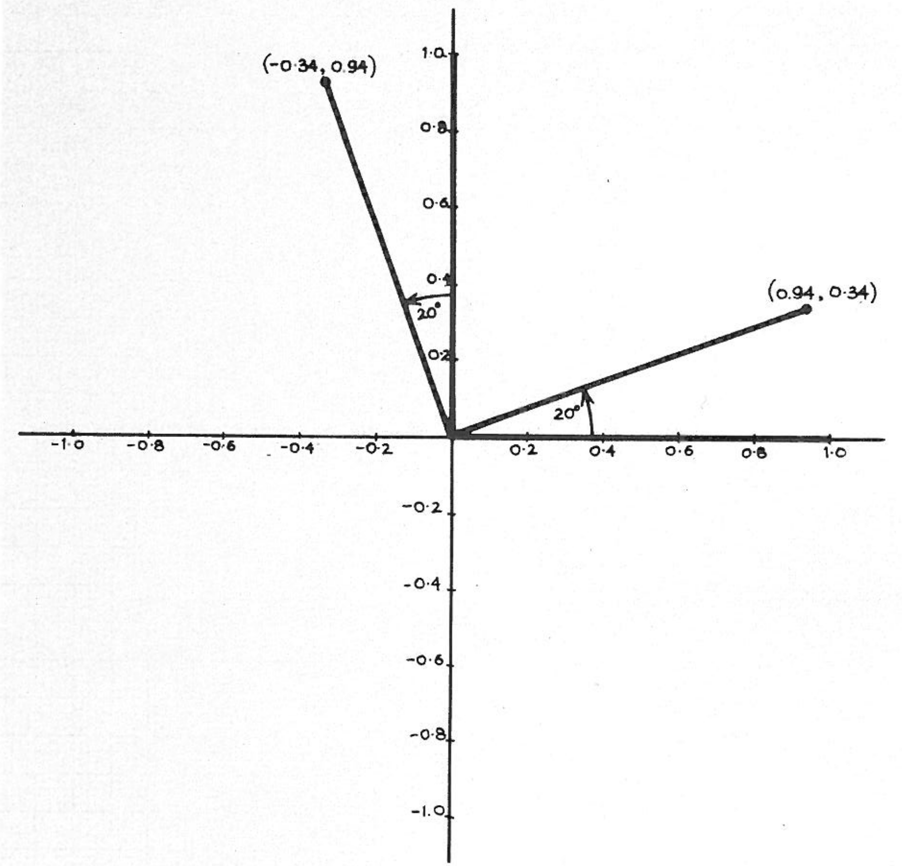
6. Find the matrix which will rotate any point (x,y) 270° anti-clockwise about the origin.

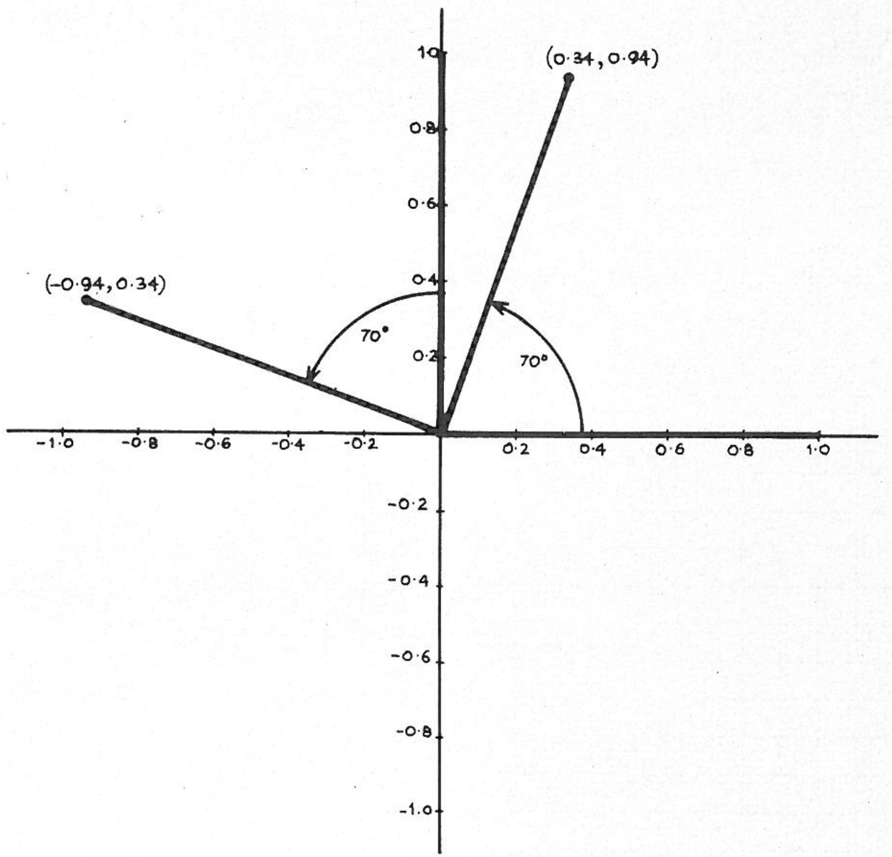
7. What do you think would happen if you multiplied the 90° matrix by the 270° matrix? *Try it and see.*

8. Choose other pairs from the four matrices you found. Multiply them together. *Give reasons for your results.*

You will need an angle indicator and graph paper.

Combining Rotations





For a rotation of 20° the matrix is $\begin{pmatrix} 0.94 & -0.34 \\ 0.34 & 0.94 \end{pmatrix}$

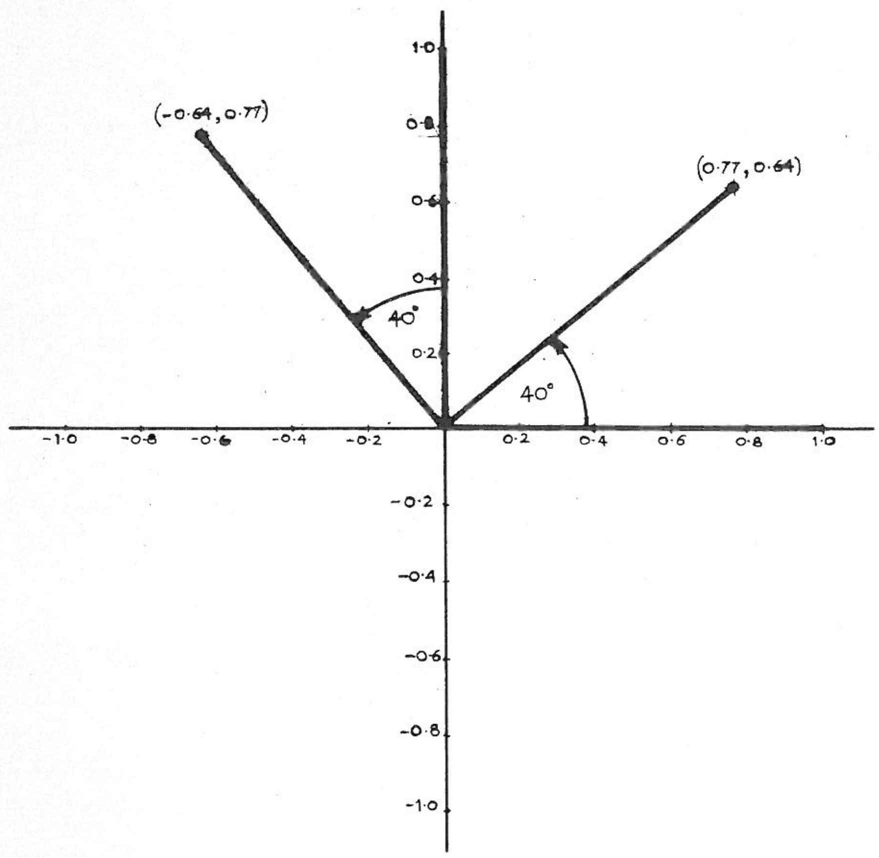
For a rotation of 70° the matrix is $\begin{pmatrix} 0.34 & -0.94 \\ 0.94 & 0.34 \end{pmatrix}$

$20° + 70° = 90°$ and $\begin{pmatrix} 0.94 & -0.34 \\ 0.34 & 0.94 \end{pmatrix} \begin{pmatrix} 0.34 & -0.94 \\ 0.94 & 0.34 \end{pmatrix} = \begin{pmatrix} \blacksquare & \blacksquare \\ \blacksquare & \blacksquare \end{pmatrix}$

Find the matrices for some other rotations. *Turn over if you need a reminder about how to find the matrix for a particular transformation.* Choose pairs of angles which add up to 90°, eg. 50° and 40°. Then multiply the matrices together. *What do you notice?*
Choose pairs of angles which add up to 180° and multiply the matrices.

Repeat the process for pairs of angles which total 270°, and then 360°. *Can you explain the results?*

A useful technique to find the transformation matrix



Look at the unit vectors and see how they have been transformed.

$$\begin{pmatrix}1\\0\end{pmatrix} \longrightarrow \begin{pmatrix}0.77\\0.64\end{pmatrix} \qquad \begin{pmatrix}0\\1\end{pmatrix} \longrightarrow \begin{pmatrix}-0.64\\0.77\end{pmatrix}$$

Use an angle indicator and draw the rotated vectors as accurately as possible. Use graph paper to enable you to read the results accurately.

Use the new vectors to write the matrix:

$$\begin{pmatrix}0.77 & -0.64\\0.64 & 0.77\end{pmatrix}$$

See 1400 A Transformation Technique for a fuller description.