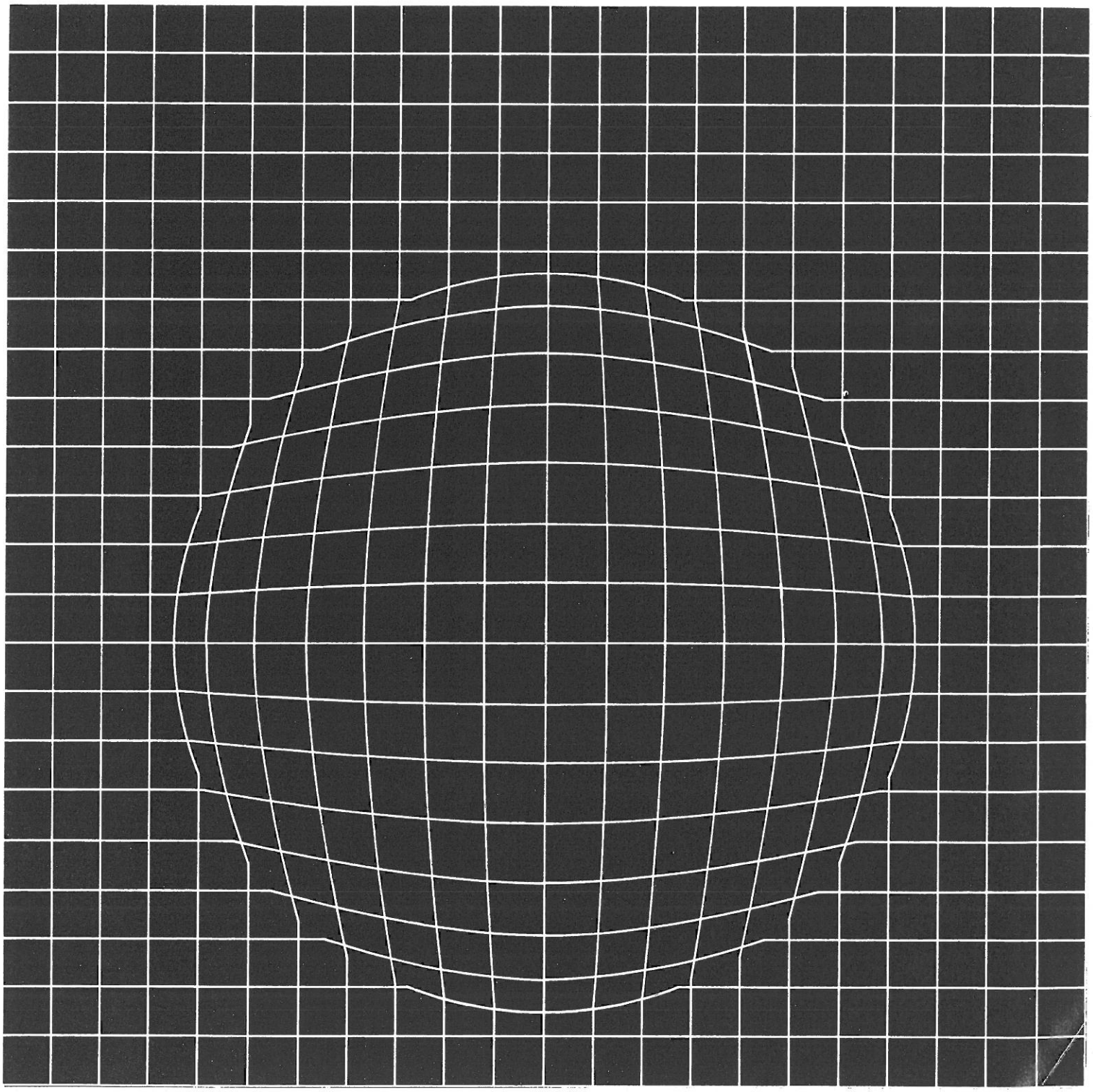
Straight lines

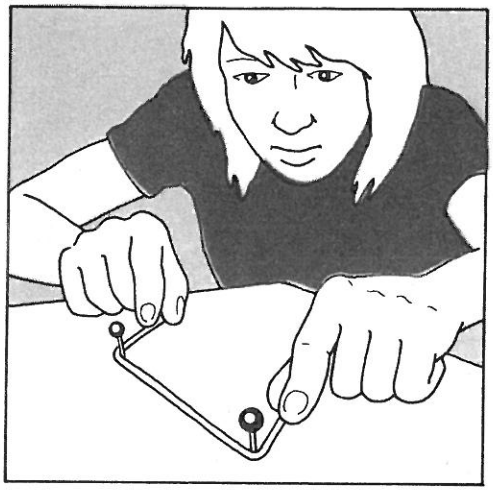
Smile 1679A



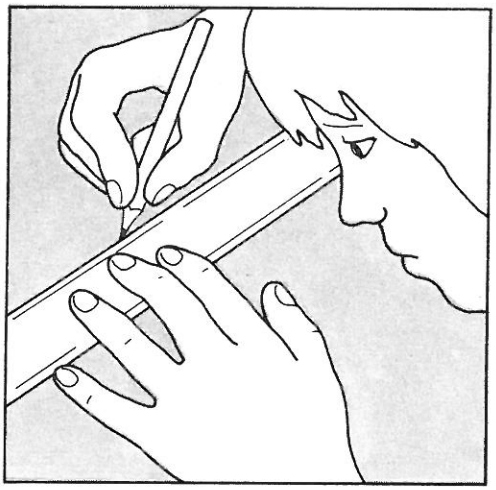
Straight lines

What is a straight line?

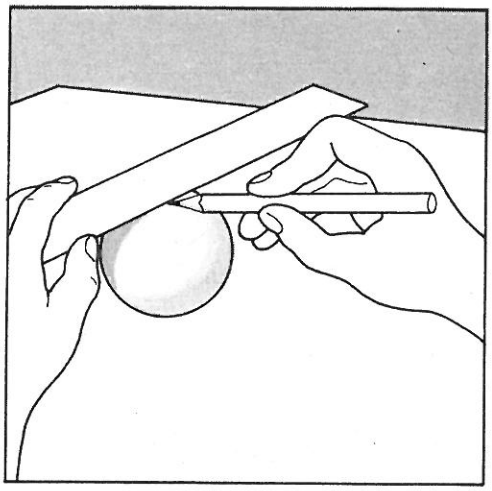
A straight line is the shortest path between two points.



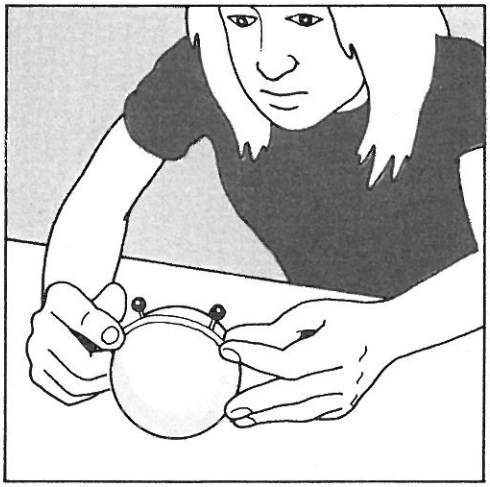
On a piece of paper you can draw straight lines with a ruler.



But you can't use an ordinary ruler on a sphere.



Straight lines on a sphere



Put two pins into the ball.
Take a piece of string or a rubber band and pull it tight between them.

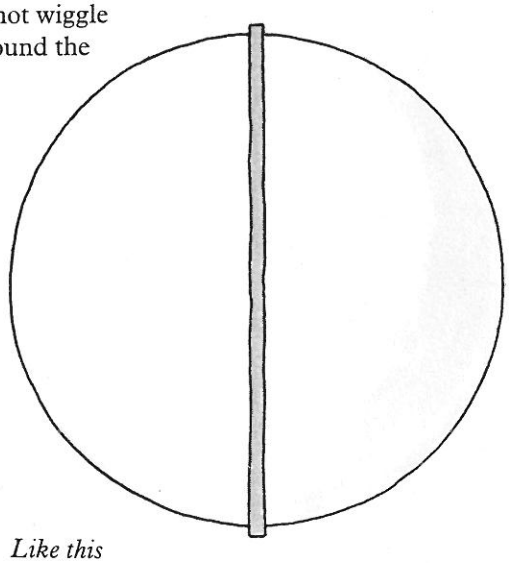
This is the shortest path between the two pins.

We call this a straight line on the sphere.

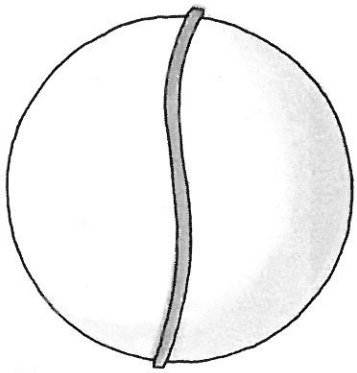
If you continue the straight line round the ball, you find that the ends meet.
Notice that the straight line goes right round the middle of the ball.

Take some thin rubber bands. Put the bands round the ball to show straight lines.

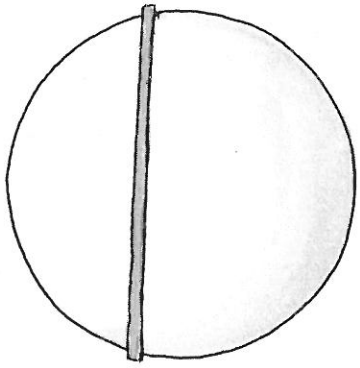
Remember: The bands should not wiggle and the bands should go right round the middle of the ball.



Like this



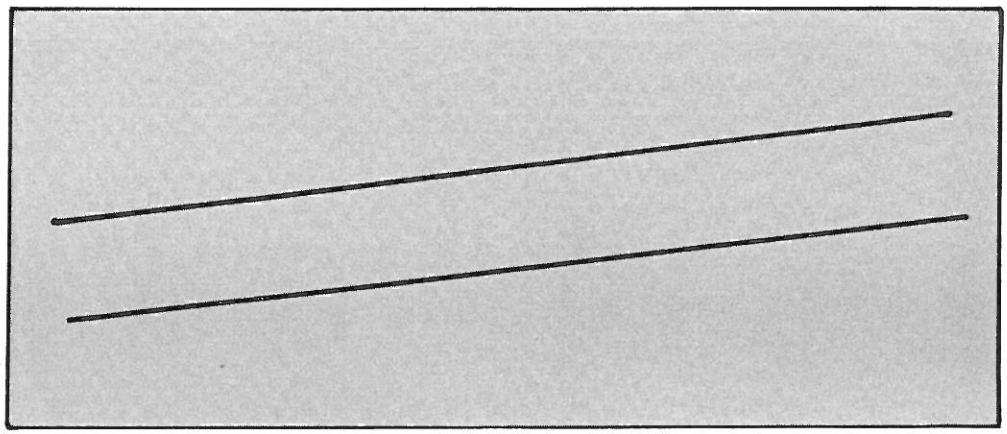
Not like this . . .



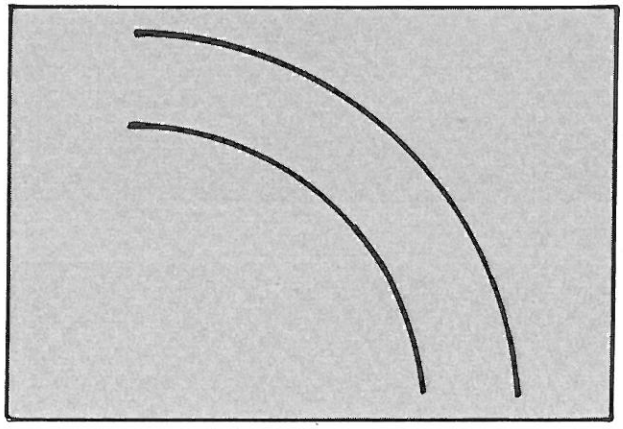
. . . or like this

Parallel lines

These two lines are parallel and straight.



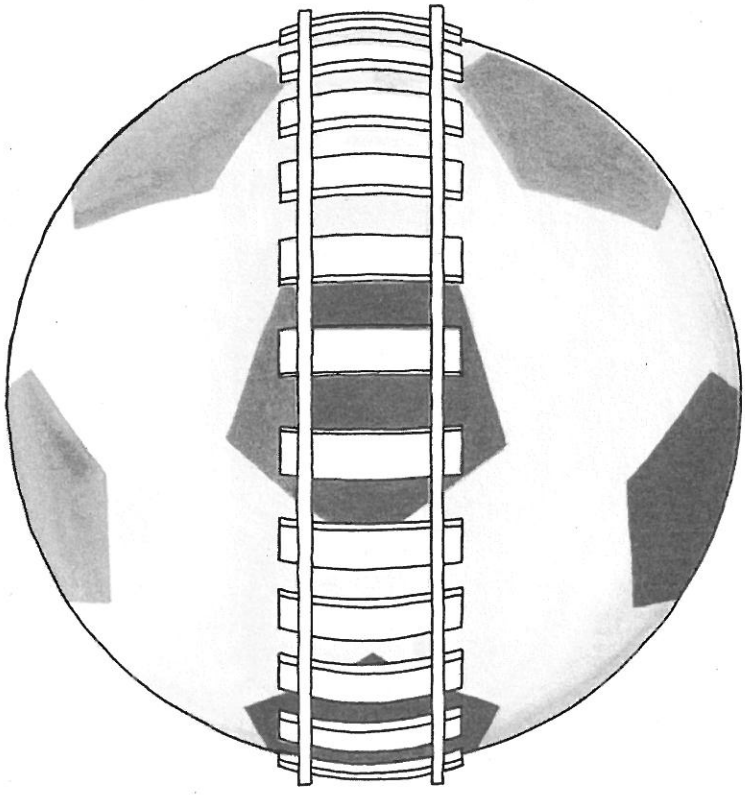
These two lines are parallel, but not straight.



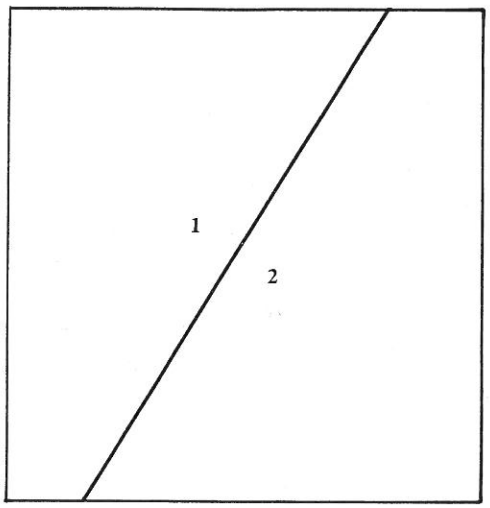
Use rubber bands to try to find parallel straight lines on the sphere.

But take care!
The rails of this track aren't parallel straight lines.

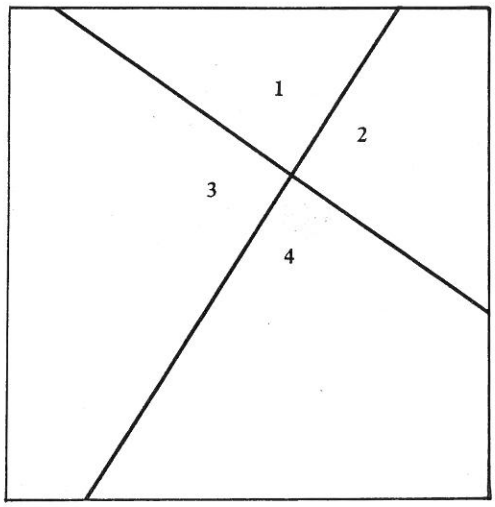
Why not?



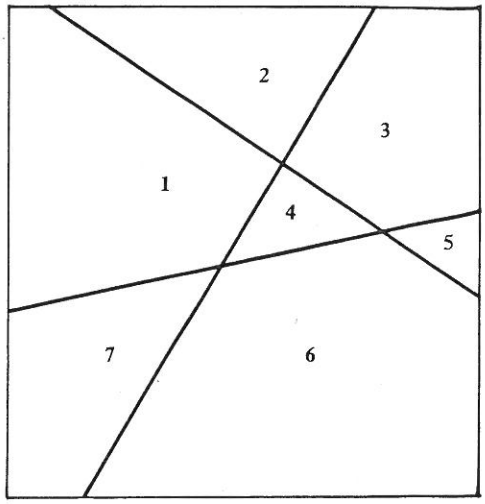
Dividing a flat surface into regions



Draw a line across a sheet of paper.
It divides the sheet into 2 regions.

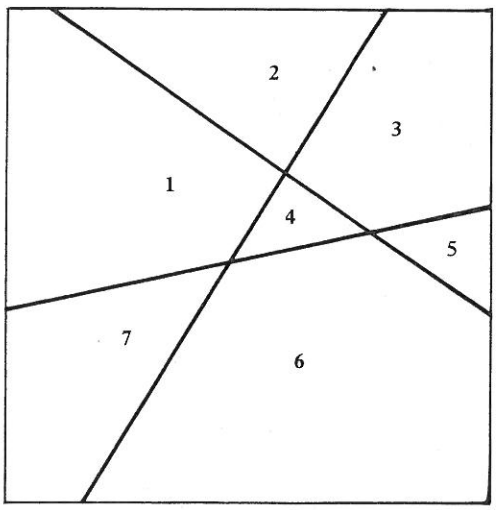


Draw a second line. Make sure it cuts the first line.
Now there are 4 regions.



Draw a third line to make as many new regions as possible. Make sure only 2 lines meet at any point and the new line crosses both the others.

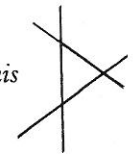
You should have made 7 regions.



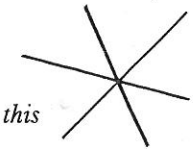
Now **you** draw a fourth line on your drawing to make as many new regions as possible.

Make sure only 2 lines meet at any one point.

Like this

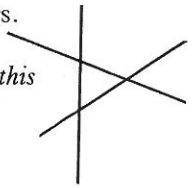


Not like this

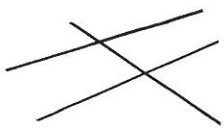


Make sure the new line crosses all the others.

Like this



Not like this



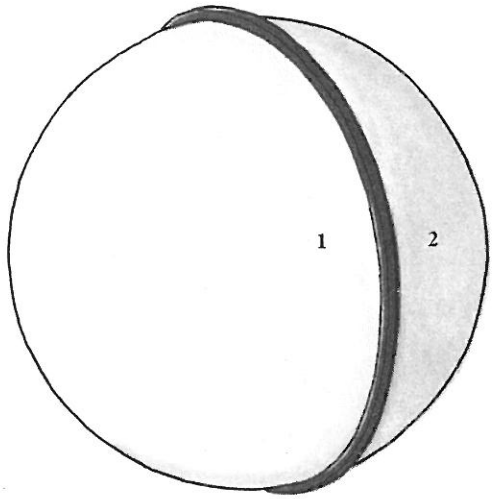
Copy this table. Fill in the number of regions four lines make.

Lines	1	2	3	4	5	6	7	8	9
Regions	2	4	7						

Look at the pattern of numbers.
Predict how many regions 5 lines can make.
Draw a fifth line and count the regions to check.

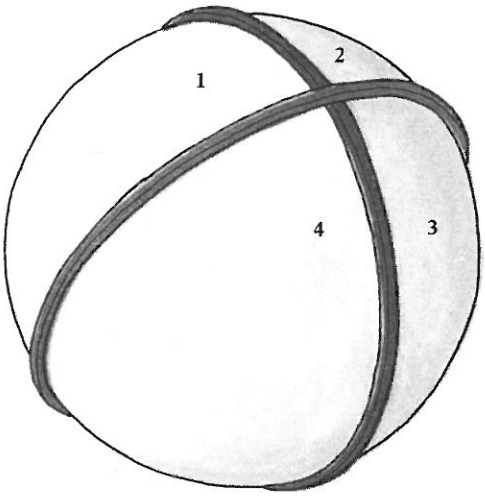
Do the same for more lines. Write down what you find out.

Dividing a sphere into regions



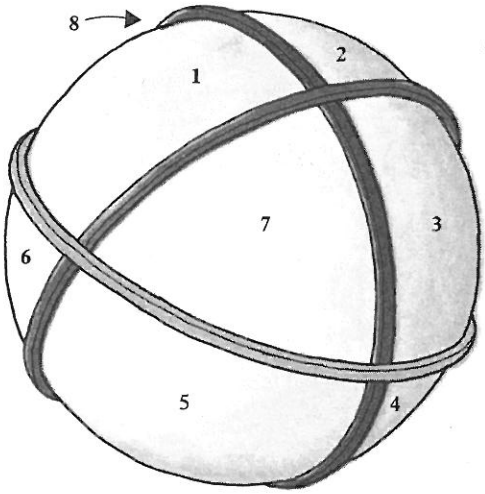
Now take the ball and several rubber bands.

Use a rubber band to put one straight line on the ball.
It divides the surface into 2 regions.



Put on a second straight line.
Now there are 4 regions.

Hint: To count regions, put one pin in each and then count the pins as you take them out.



Put on a third straight line to make as many regions as possible. Make sure it doesn't pass through the points where the first and second lines met.

Now there are 8 regions.

Copy this table.

Lines	1	2	3	4	5	6	7	8	9
Regions	2	4	8						

Predict the number of regions for 4 and more straight lines.
Use rubber bands to check your predictions.
Remember that only 2 lines should meet at any point.

Write down what you find out.

Comparing regions for paper and ball

Look at the table of numbers for the ball.
Compare it with the table of numbers for the paper.

What do you notice?

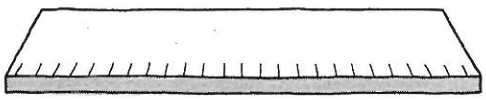
Can you explain why?

How long is a line?

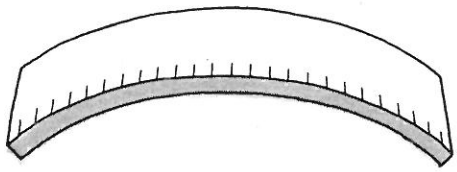
You can leave the work on this page until you have done the other booklets if you wish.

How long is a curved line?

You can measure lengths on a flat surface with a ruler. But if you try to use a ruler on a curved surface you have to bend it.



When the ruler is flat the top and bottom edges are the same length.



But when you bend the ruler, the top becomes longer than the bottom.

We can't tell if the top has been stretched, or if the bottom has been squashed, or both.

Why not?

If the ruler has changed length it will not measure accurately.

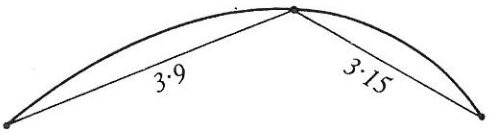
How could you make an improved ruler for curved surfaces?

Would it be absolutely accurate?

The only lines we can measure accurately are straight lines on flat surfaces. So for curves we have to try to approximate them with straight lines. This may mean leaving the surface if the surface is curved.

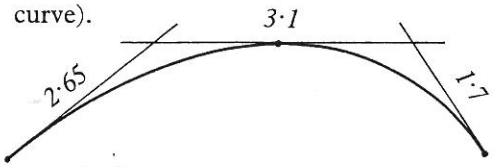
Start with two end points and one in the middle.

First we approximate the curve with chords (which are shorter than the curve).



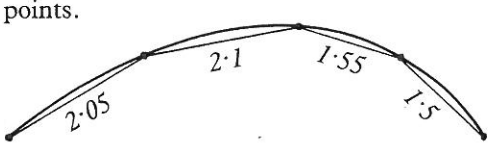
Length of chords = 3·9 + 3·15 =7·05cm

Then we approximate the curve with tangents (which are longer than the curve).

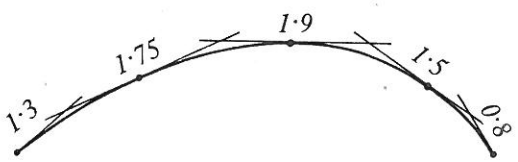


Length of tangents =
2·65 + 3·1 + 1·7 =7·45cm

Put new points between the existing three points.

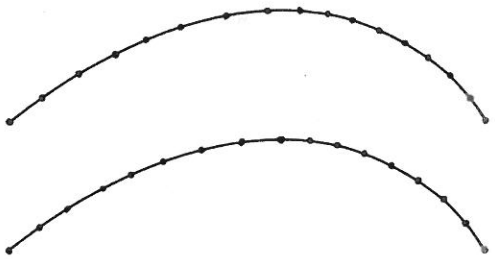


Length of chords =
2·05 + 2·1 + 1·55 + 1·5 =7.2cm



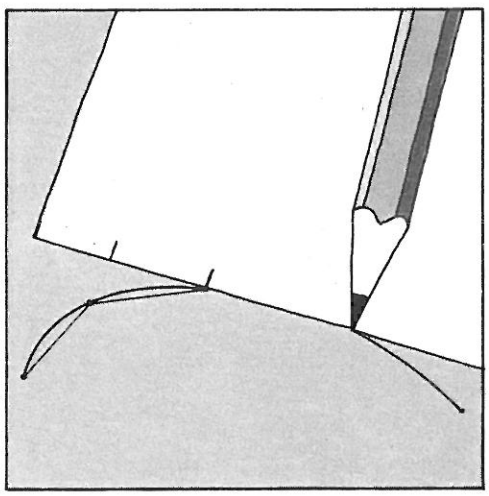
Length of tangents =
1·3 + 1·75 + 1·9 + 1·5 + 0·8 = 7.25cm

We can add new points as often as we wish.

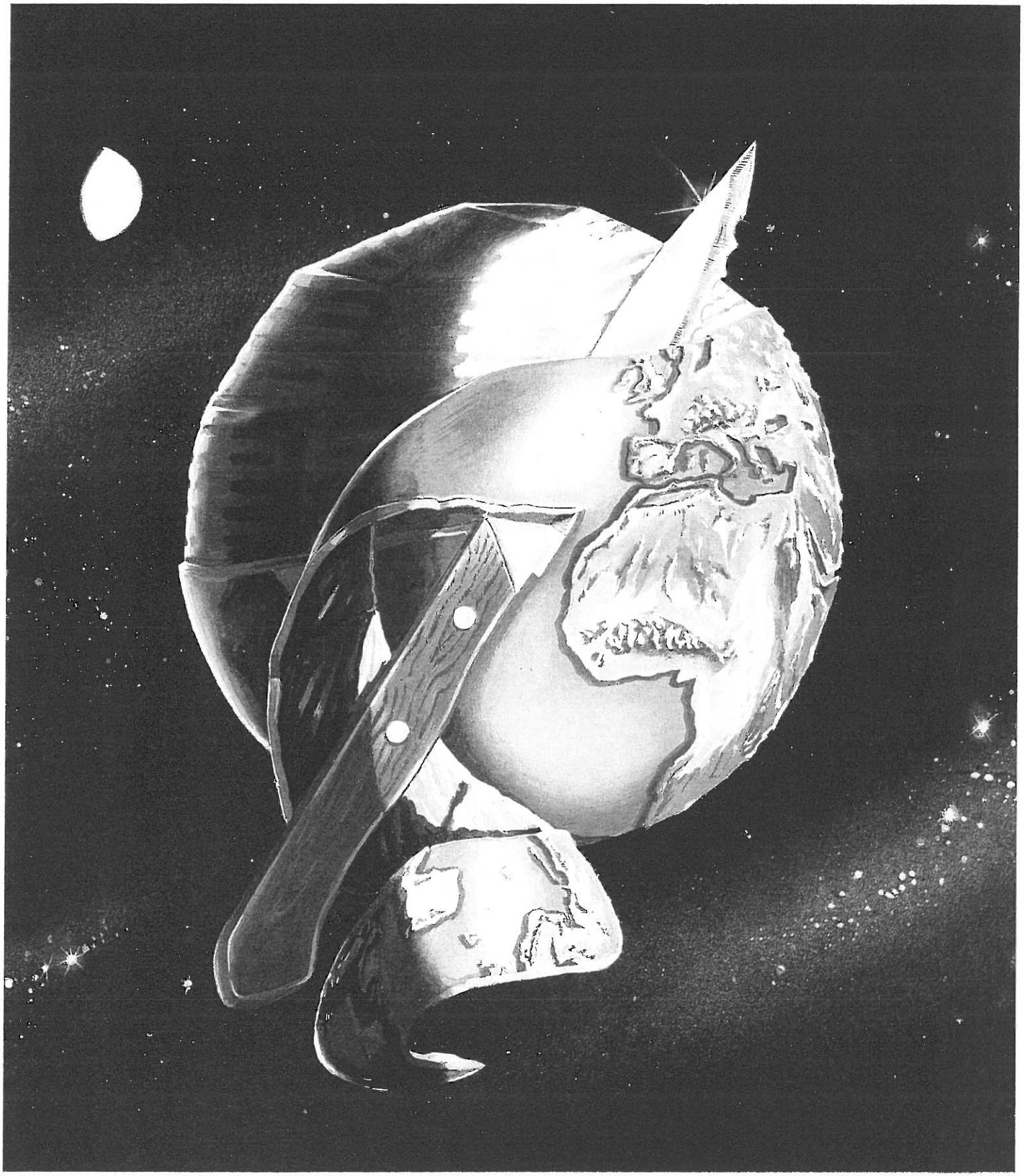


Try finding the length of a curve this way yourself.
Draw a curve on a sheet of paper. Mark the two ends and a point in the middle. Draw the tangents and the chords. Measure their total lengths. Add more points until you have sandwiched the length of the curve to the nearest millimetre.

Hint: Use the edge of a piece of paper to mark off the lengths of tangents or chords. Then you will only need to measure the total lengths with your ruler.



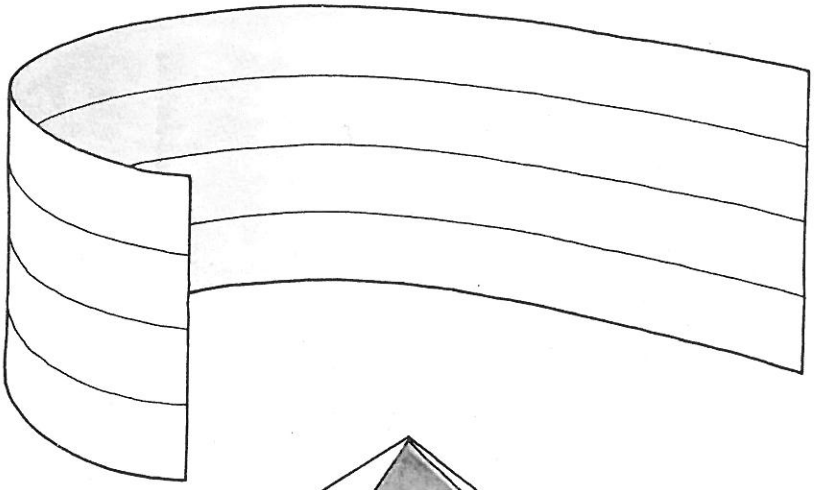
Smile 1679C



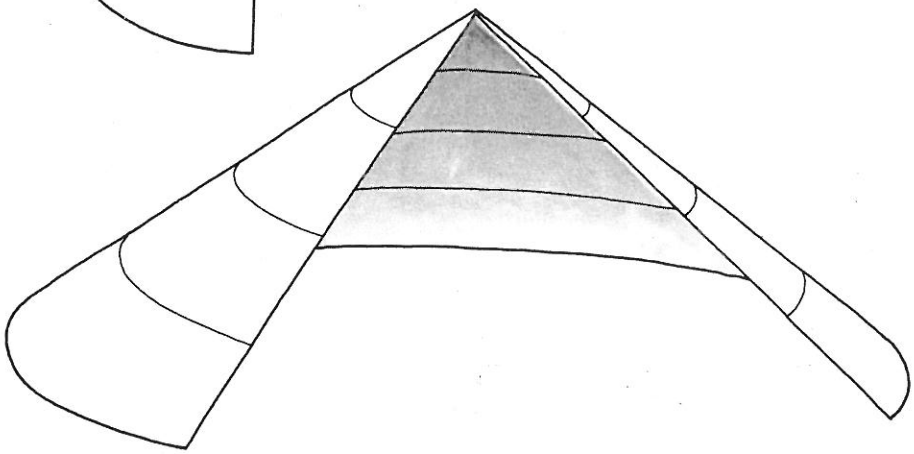
Making solids from flat paper

Take a sheet of paper.

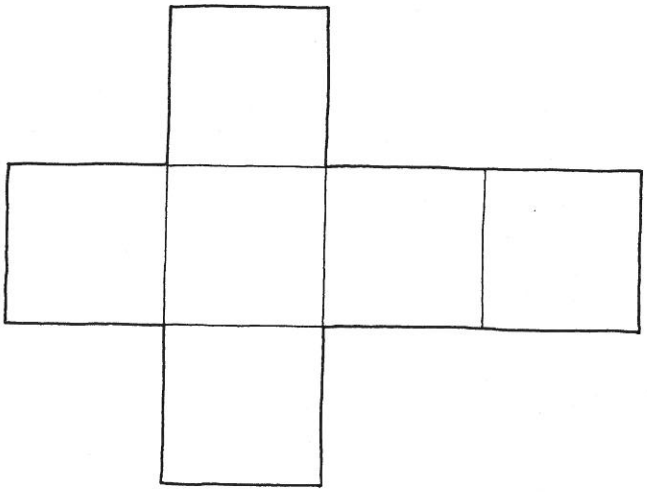
Can you make it into a cylinder?
Do you have to:
bend it
fold it
cut it
stretch it?



Can you make it into a cone?
Do you have to:
bend it
fold it
cut it
stretch it?



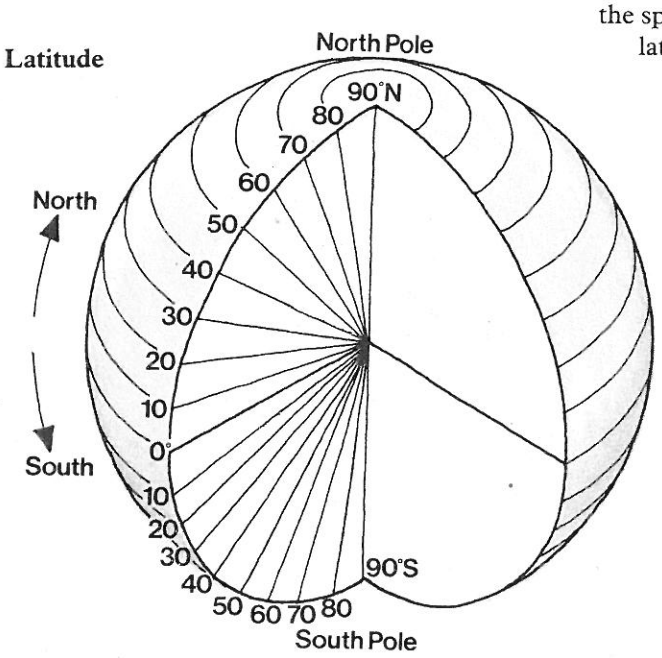
Can you make it into a cube?
Do you have to:
bend it
fold it
cut it
stretch it?



Can you make it into a sphere?
Do you have to:
bend it
fold it
cut it
stretch it?

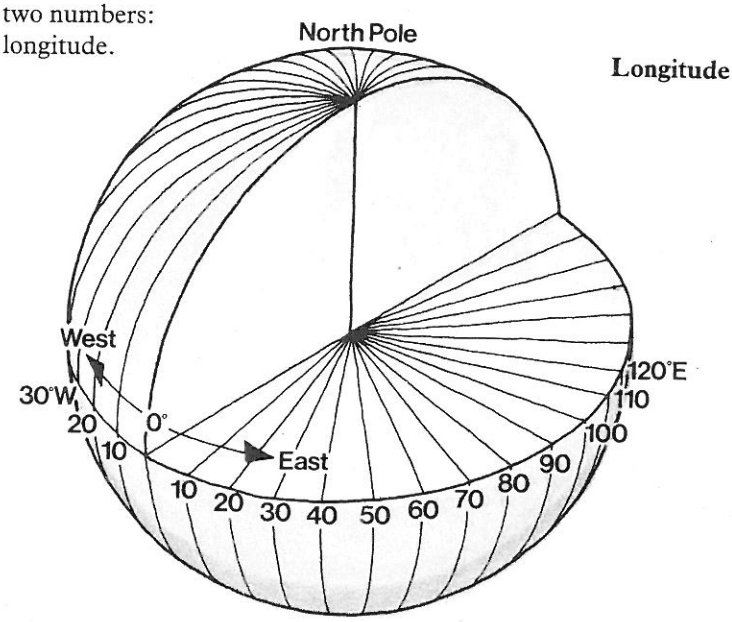
Fixing positions on the sphere

You can fix the positions of any point on the sphere with two numbers: latitude and longitude.



Latitude tells you how far north or south a place is. It is measured from the equator.

The North Pole is 90°N.
The equator is 0°.
The South Pole is 90°S.



Longitude tells you how far east or west a place is. It is measured from the Greenwich meridian, which passes through London.

You need a globe (if your teacher has one).

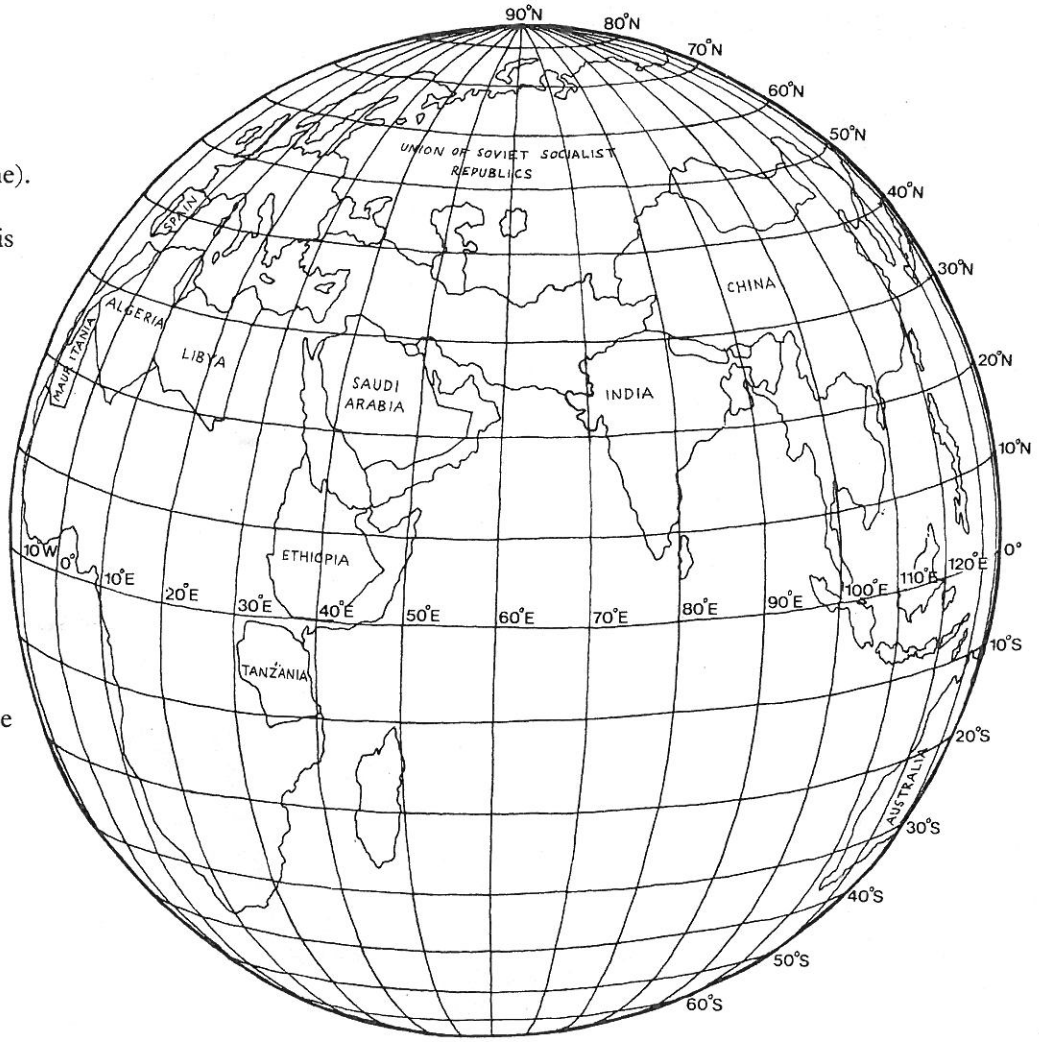
If you don't have a globe you can use this picture to answer these questions.

1 Which countries are these places in?
a) 50°E 20°N
b) 100°E 40°N
c) 35°E 55°N
d) 35°E 5°S
e) 10°W 20°N
(Answers on back page)

2 Are the circles of latitude all the same size?

3 Are the circles of longitude all the same size?

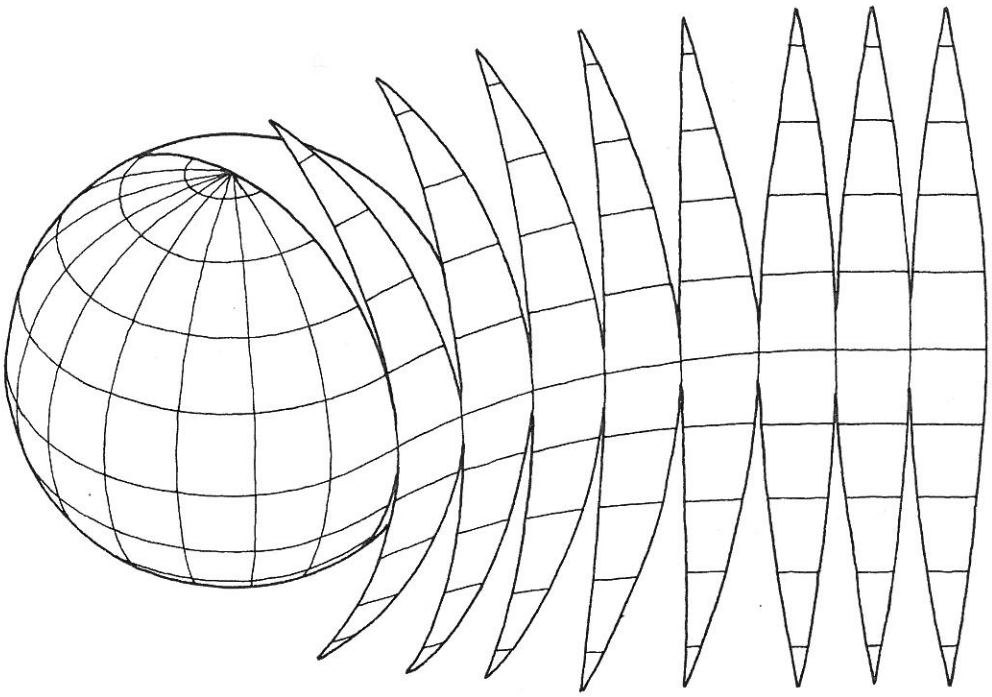
4 Are all the 'squares' on the grid the same size?



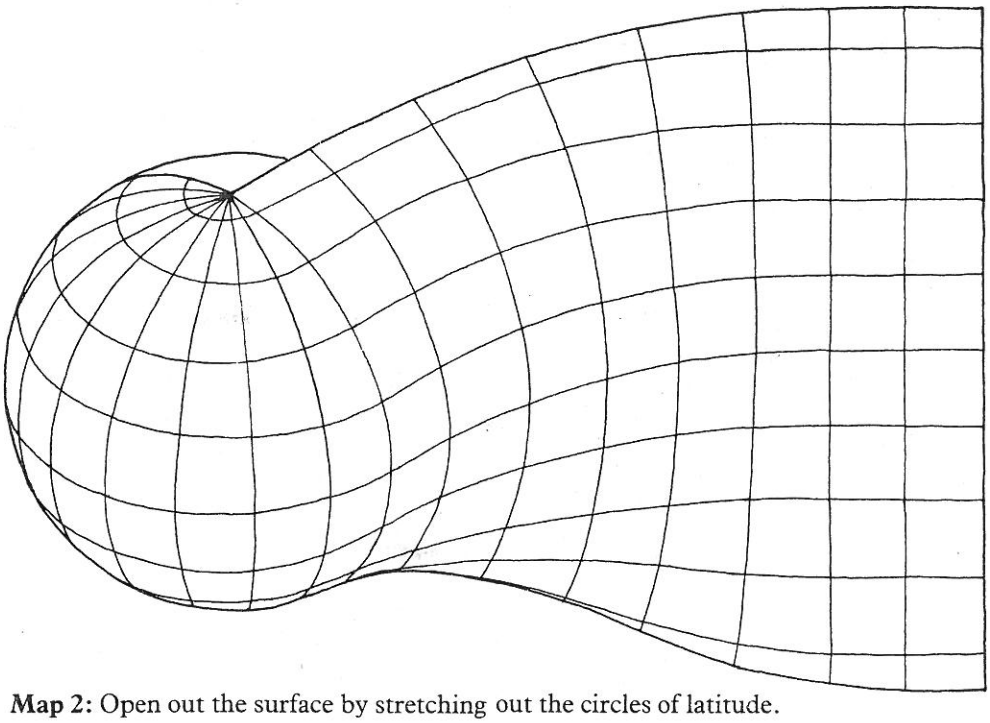
Making Maps

Here are three ways of cutting or stretching the surface of a sphere to make a map.

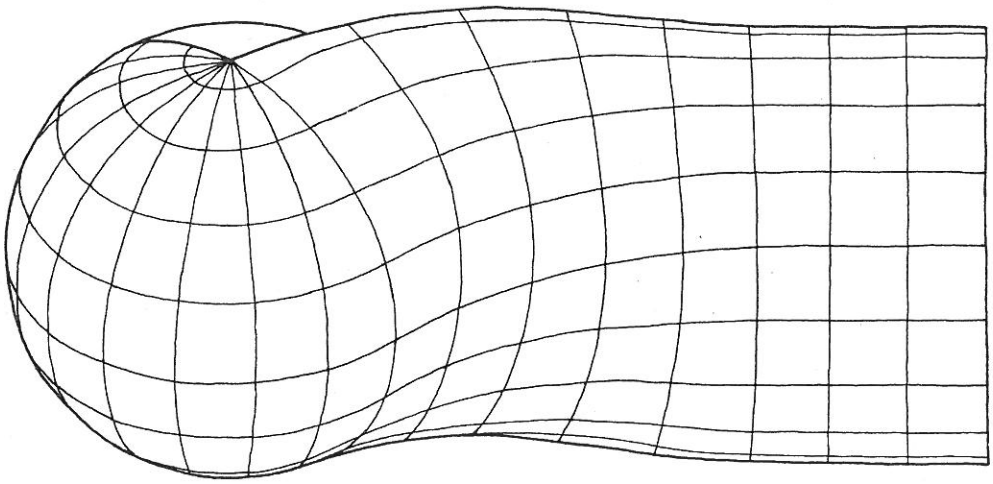
Look at the maps and compare them with a globe if you have one. Write down what you notice.



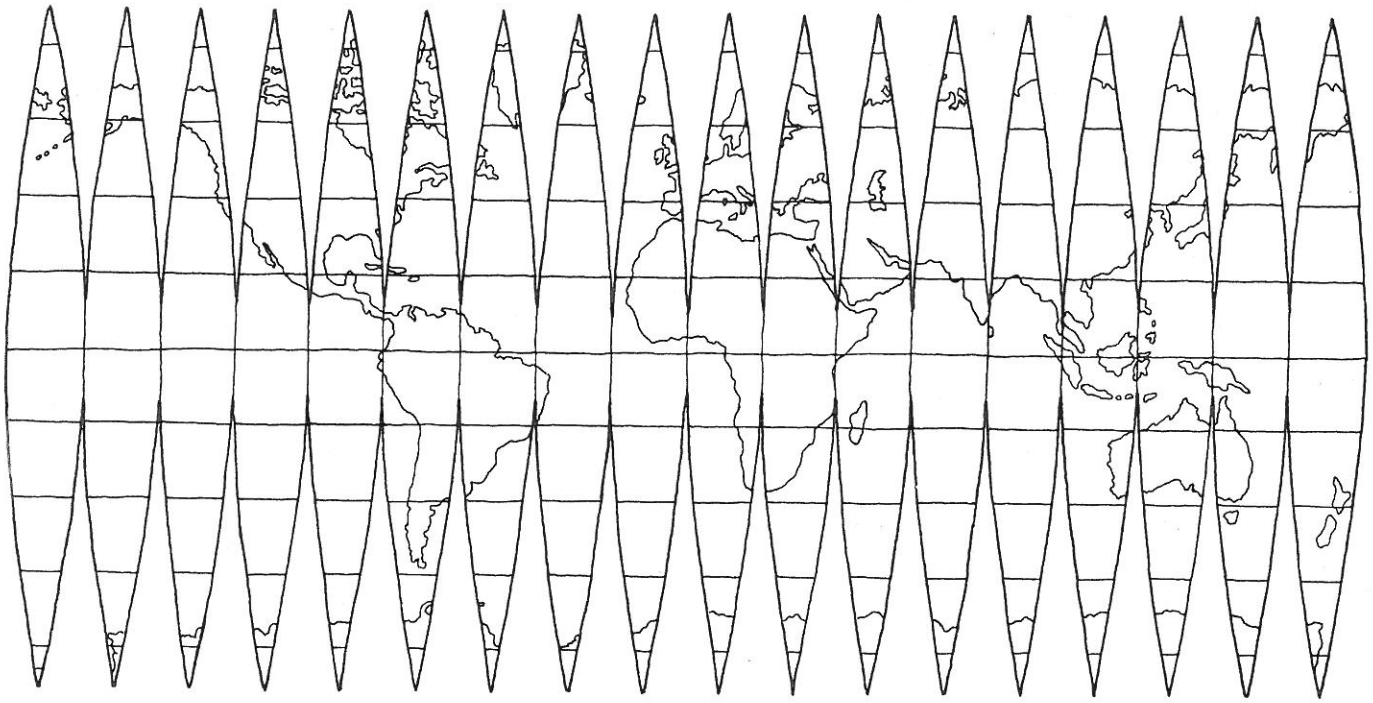
Map 1: Cut down lines of longitude and open out the surface.

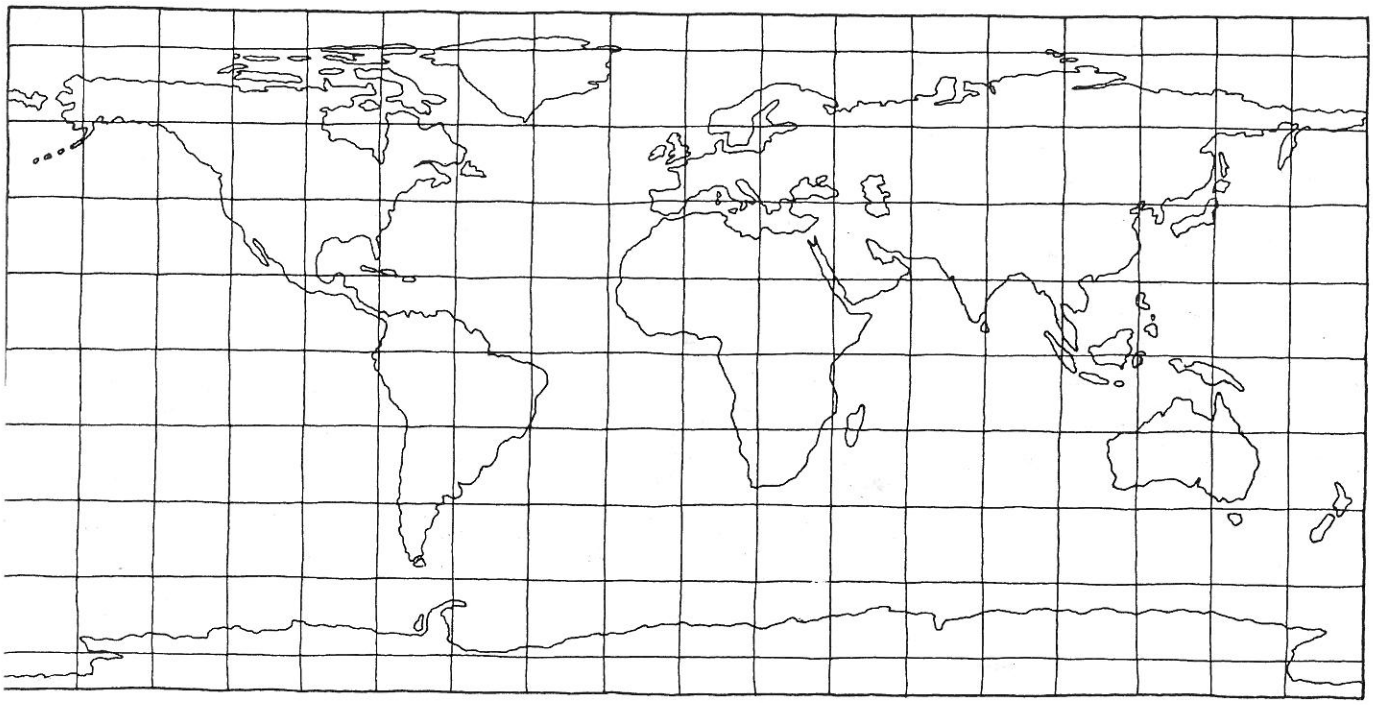


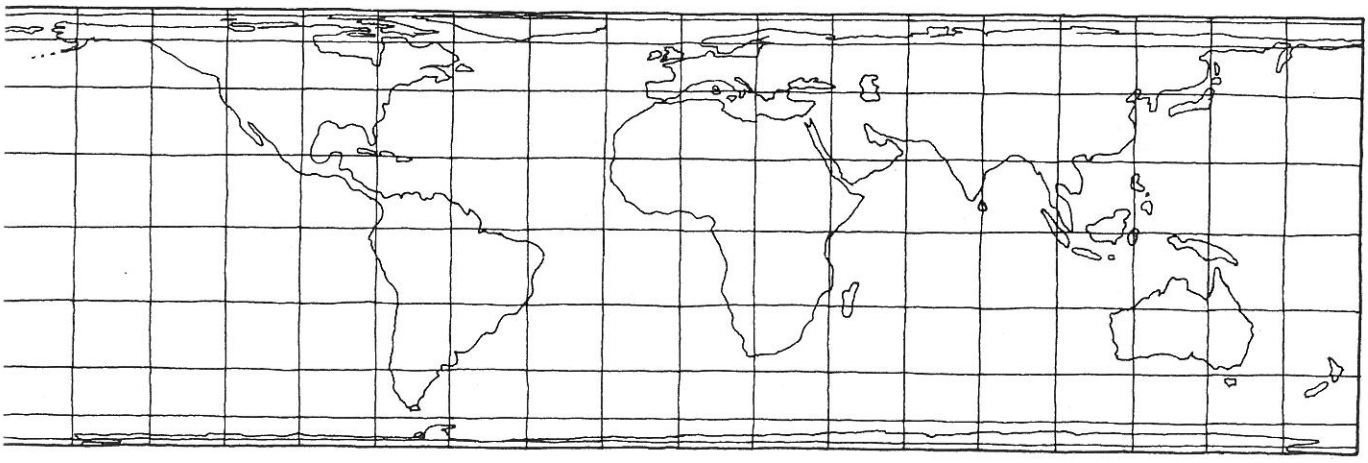
Map 2: Open out the surface by stretching out the circles of latitude.



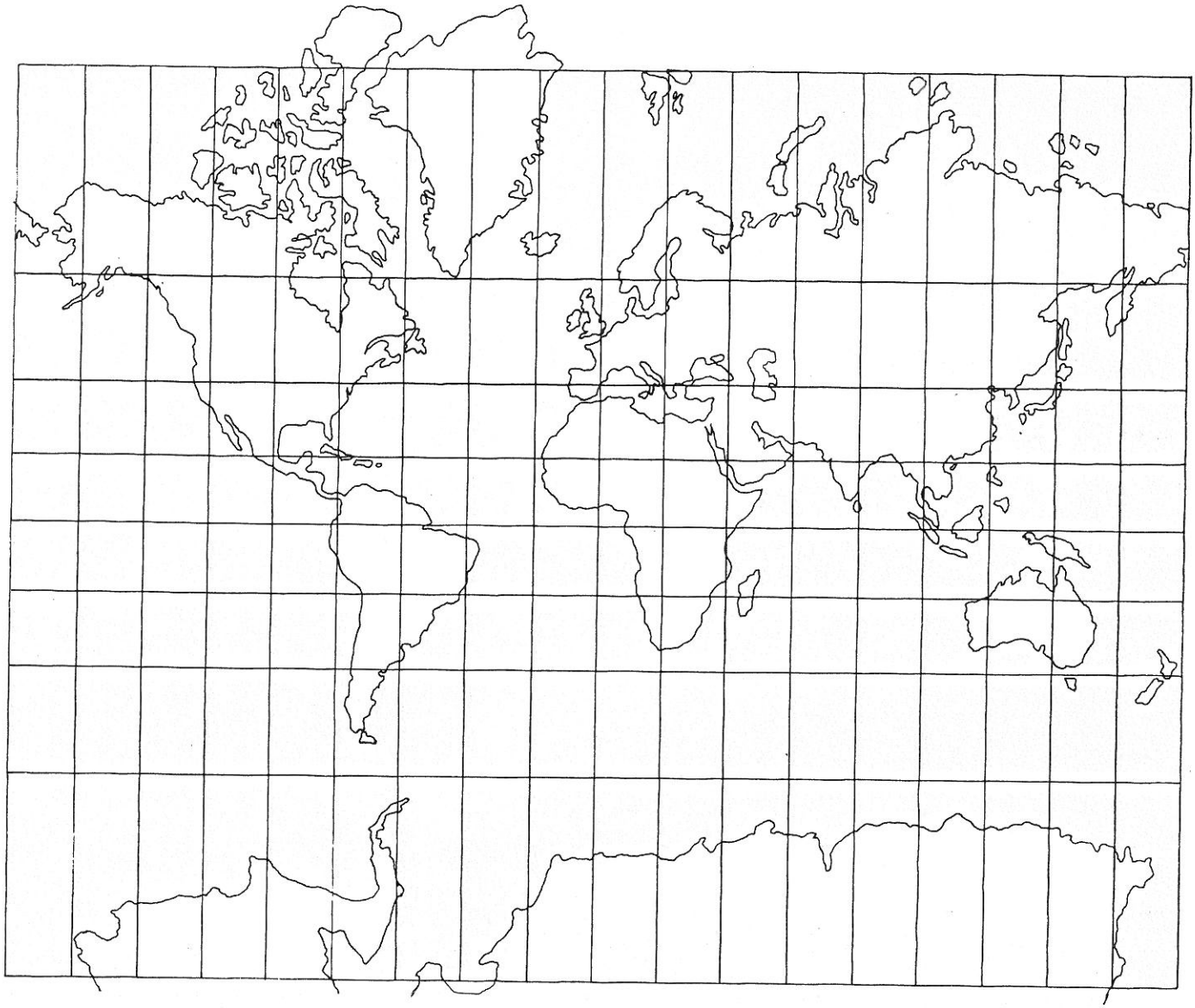
Map 3: Open out the surface by stretching out the circles of latitude and squashing the circles of longitude.



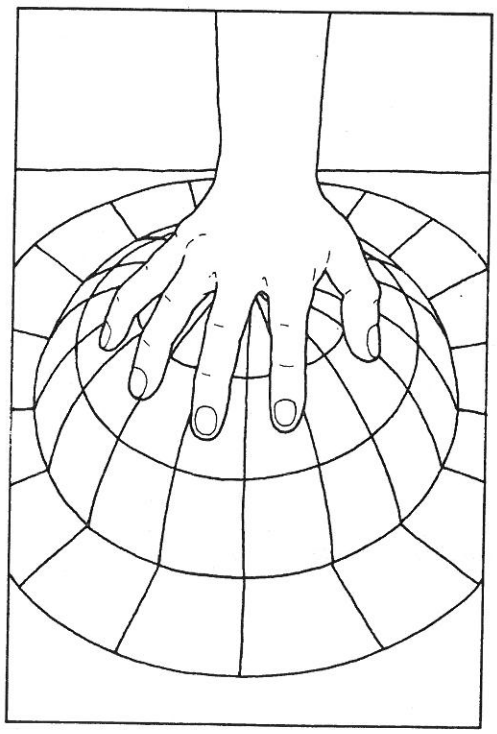


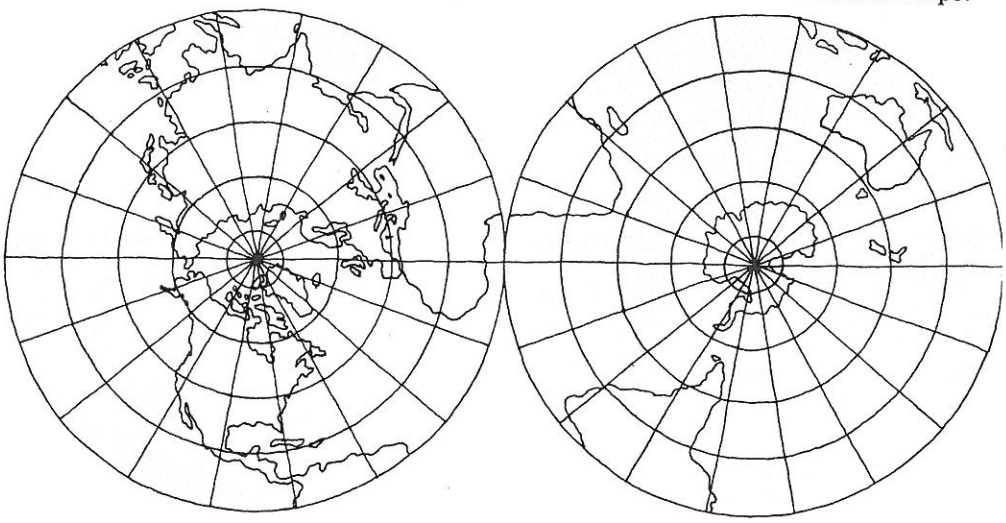


Some other maps



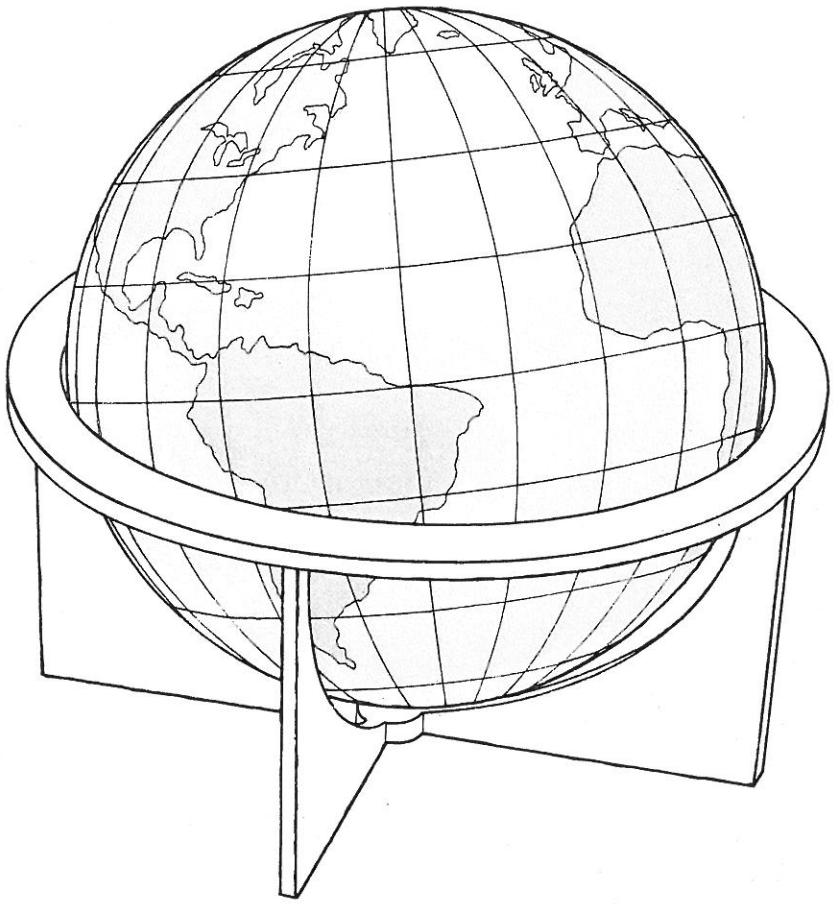
Map 4: This map was designed by a Belgian, Geradus Mercator. He first published his new map of the world in 1569, while he was working in Germany. It was a real breakthrough in maps for sailors. When you've finished this booklet you'll probably see why it is so useful. Mercator was also the first person to use the word 'atlas' for a collection of maps.





Map 5: This map is made by squashing the globe onto a piece of paper at the pole.

How good are the maps?



We had to cut and stretch the surface of the globe to make the maps. So the map is different from the surface of the globe. On this page are several statements which are true.

If you have a globe you can check them.

But because the maps cut and stretch the earth's surface some of the statements are not true on the maps.

Use the maps on worksheet 1679 F to check the statements.

If you are working in a group, share out the work, but don't forget to discuss the work together at the end.

Write two paragraphs for each map saying what its advantages and disadvantages are.

Distance

The shortest routes:
from New York to London
from London to Accra and
from Accra to Mogadishu
are the same length.
(If you have a globe use a piece of string to check.)

Measure the three distances on each of the 5 maps. What do you notice?

Which of these maps can be used to measure distances? Can you explain why?

Shortest Route

The shortest route from London to Tokyo goes over Murmansk. (If you have a globe use a piece of string to check.)

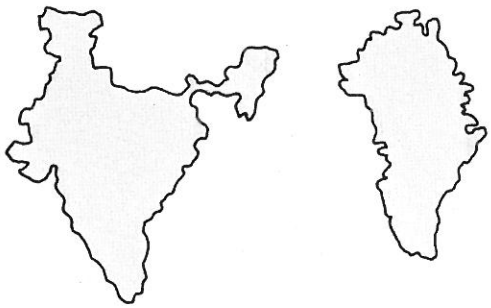
Draw the straight line from London to Tokyo on each of the five maps.

Do any of them go through Murmansk?

Which of these maps can be used to find the shortest route between two places? Can you explain why?

Area and Shape

If you have a globe, trace Greenland and India. Otherwise use the outlines below.



Copy them onto graph paper and find their areas by counting squares.

Which is bigger?

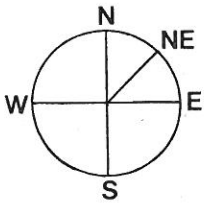
Now find the areas of Greenland and India on each of the five maps. How well does each map show area?

Compare the shape of Greenland and India on the globe with the shape on each of the five maps. How well does each map show shape?

Which of these maps can be used to measure area?
Which of these maps can be used to find shape?
Can you explain why?

Bearings

If you set off in a plane from Georgetown, Guyana keeping a compass bearing of 045° (north east) you will eventually arrive over London.



Draw the line from Georgetown to London on each of the maps.

Measure the angle it makes with the north line.

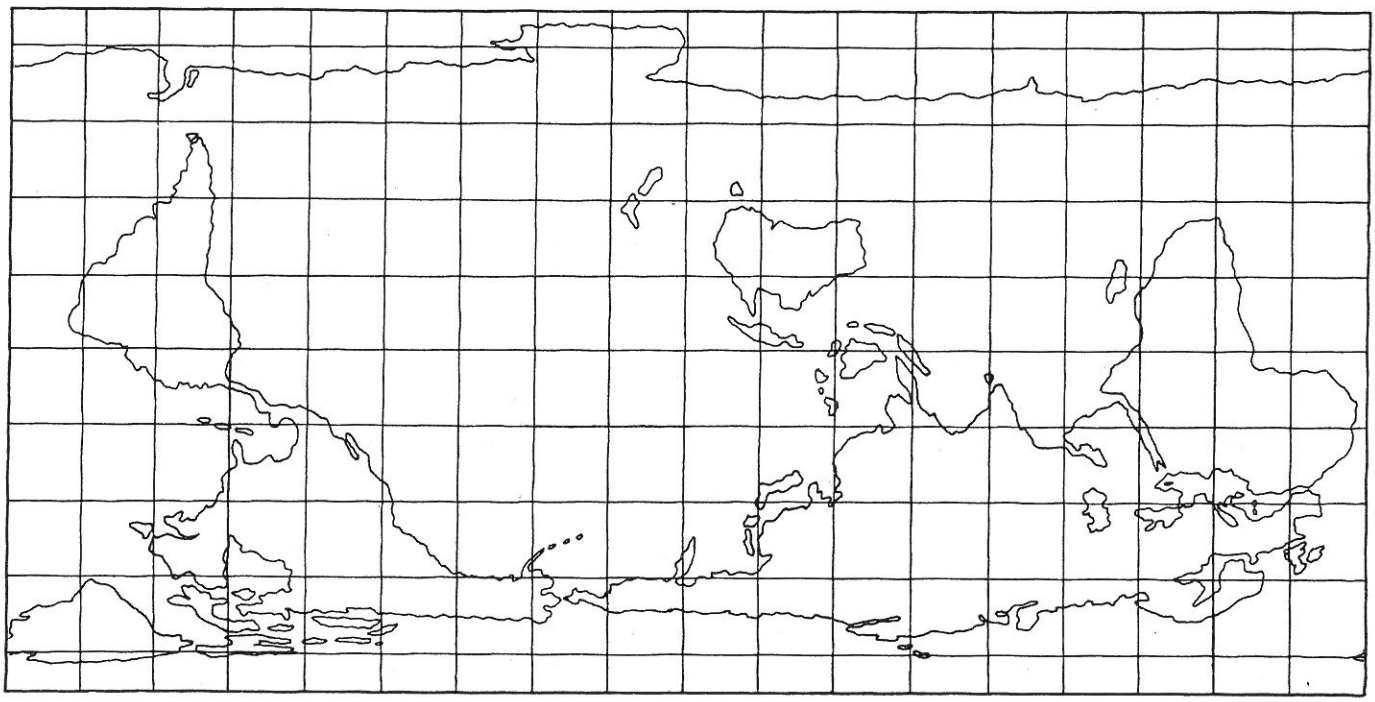
Which of the maps tells you the bearing is 45°?

Other maps

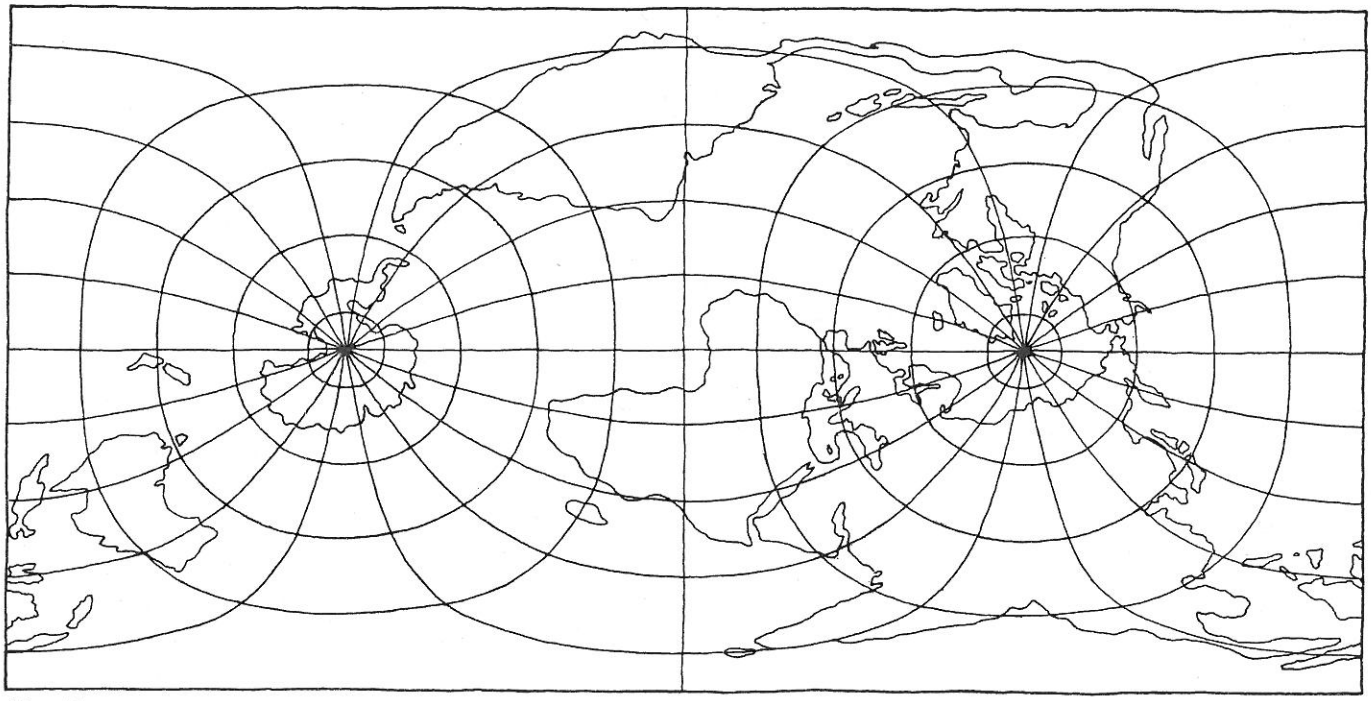
However you draw a map it will distort the surface of the earth.

Even the same type of cutting and stretching will produce different maps if the cuts and stretches are in different places. Both the maps below are made like Map 2.

Compare Maps 2, 6 and 7.



Map 6



Map 7

Fixing positions on the sphere

Answers to question 1.

a) Saudi Arabia b) China

c) Union of Soviet Socialist Republics

d) Tanzania e) Mauritania