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The most accurate thermometer in the world

Key words
temperature
thermometer
speed of sound
kinetic model

Almost every scientific experiment or industrial process requires a measurement of temperature. For example the specification of the length of an object is meaningless without reference to the temperature at which the measurement was made. And all chemical and biological processes are intrinsically temperature-dependent – often critically so.

How do you know what the temperature is?

You use a thermometer.

How do you know your thermometer reads correctly?

You have it calibrated by one of hundreds of laboratories around the country.

How do they know their thermometers read correctly?

Every year they have their thermometers checked at NPL.

How does NPL know its thermometers read correctly?

We compare our thermometers with those in other countries around the world.

And how do we all know our thermometers read correctly?

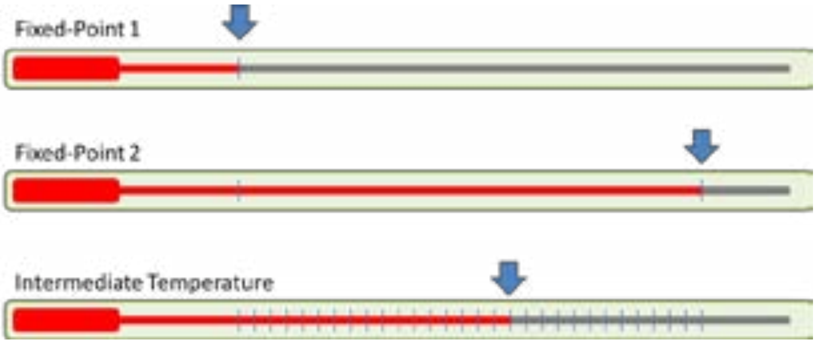
Good question. That's what this article is about.

Every temperature measurement made using a calibrated thermometer is traceable in a series of comparisons back to a standards lab such as the UK's National Physical Laboratory (NPL).

Traditionally

Traditionally we measure temperature on a scale constructed using two fixed points, usually the freezing and melting temperatures of common substances such as pure water. These temperatures are very reproducible and can be marked on the scale of, say, a liquid-in-glass thermometer. We then divide the scale into equal divisions between the two fixed points.

Despite being widely used, there are two fundamental problems with this procedure. Firstly, we cannot be sure that the liquid in the thermometer expands at a steady rate as the temperature rises. And secondly, this procedure makes no link to our microscopic notion of what temperature really is – a measure of the kinetic energy of atoms and molecules.

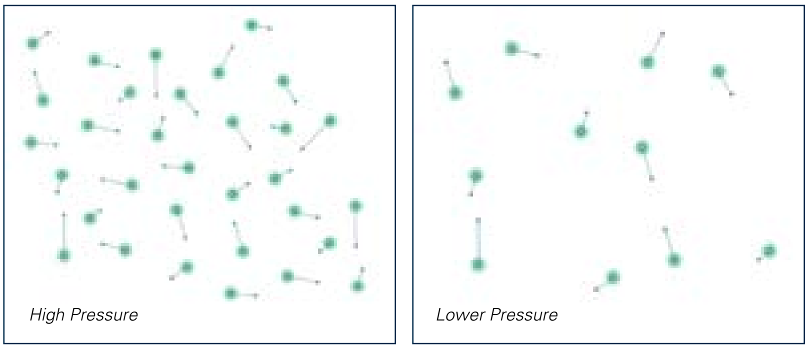


A conventional thermometer is calibrated by recording its reading at each of two temperature fixed-points. Intermediate temperatures are then measured by assuming that the reading varies linearly between the two fixed points. Here I have used the expansion of liquid in a fine tube, but I could have used the variation of electrical resistance or any other measurable property of a substance.

Temperature from first principles

Temperature is a macroscopic (large-scale) property of matter. How does it relate to the behaviour of the microscopic particles of which matter is made?

It is simplest to think about a gas. At low pressures, the molecules of a gas are far apart. They spend most of their time moving at a steady speed in between collisions with other molecules.



In a gas at approximately atmospheric pressure (left), there are large gaps between the molecules, so most of the time each molecule just 'cruises' in between collisions. If the pressure is lowered (right), the fraction of time molecules spend 'cruising' becomes longer and the gas behaves more like an ideal gas. In these diagrams the length of the arrows represents the speed and direction of motion of each molecule.

If you heat a gas, its molecules move faster – they have more kinetic energy. Some molecules move faster than others, but their average kinetic energy increases in proportion to the absolute temperature T. We can write this like this:

$$\frac{1}{2} m v_{\mathrm{ave}}^2 \propto T$$

where v_{ave} is the average speed of a molecule. So if we know the mass of the molecules, m, and their average speed, we can calculate the average KE of a gas molecule. By making measurements at one specific temperature, we can discover how average KE is related to absolute temperature. This is what I have spent the last six years doing.

Making our thermometer

Estimating the average speed of molecules is surprisingly straightforward. The speed of sound c in a gas is directly related to the average speed of molecules. The faster the molecules move, the greater is the speed of sound. For gas molecules such as helium or argon made out of only a single atom the relationship is especially simple:

$$c = \sqrt{\frac{5}{9}}\, v_{\mathrm{ave}}$$

In fact this formula is only approximately true, but it becomes more and more accurate as the gas molecules spend more and more time in between collisions: i.e. as we lower the gas pressure. So in order to get the most accurate results we need to measure the speed of sound versus pressure and extrapolate to the limit of zero pressure. We obviously can't measure the speed of sound at zero pressure because that would be a vacuum!

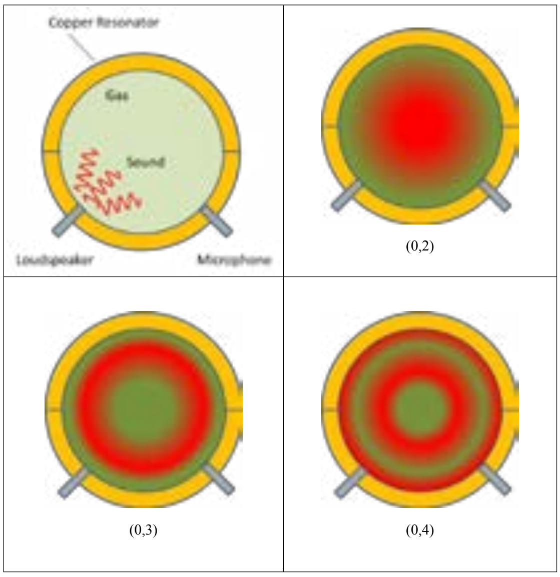
So here is how we made our thermometer, one that is linked fundamentally to the speed of molecules. First we prepared a sample of pure gas made from molecules of known mass; we used argon. The gas was held at a stable temperature – the temperature we wanted to know. And then we measured the speed of sound in the gas. From this we could deduce the average kinetic energy of the gas molecules, and hence the temperature.

Measuring the speed of sound

The simplest way to measure the speed of sound is the time-of-flight method, in which a pulse of sound is sent from a loudspeaker to a microphone across a known distance. However, in order to make this method more accurate one needs to make the known distance very long, and it is hard to keep the temperature uniform over a long distance.

Instead we used a carefully-crafted spherical container called a resonator, fitted with a tiny loudspeaker and microphone lying flush on the inner surface. Instead of a pulse of sound, the loudspeaker emits a continuous sine wave. At certain frequencies, the sound reflected from the walls of the resonator adds in-phase to the sound from the loudspeakers and the sound pressure level inside the resonator increases creating a resonance.

Our acoustic resonator is made from two hemispheres of copper which have been carefully machined and aligned to create an internal spherical hollow with a volume of approximately 1 litre.



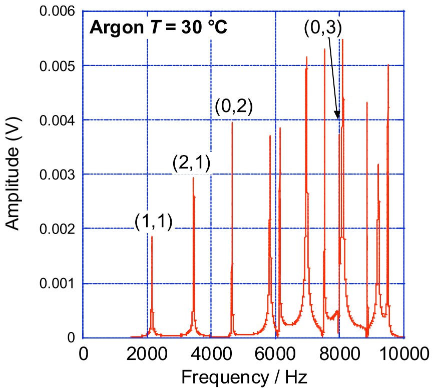
Sound waves emerge from the loudspeaker in the spherical acoustic resonator. At most frequencies the microphone surprisingly detects no sound because the reflected waves cancel each other out. At certain special frequencies the sound reflected from the walls creates standing-wave patterns such as those illustrated here. By measuring the precise frequency at which each resonance occurs, we can estimate the speed of sound. The different patterns are labelled (0,2), (0,3) (0,4) etc…

At resonance, we can find the wavelength of the sound waves from the dimensions of the resonator. We can measure their frequency very precisely and so we can calculate the speed of sound using the equation:

wave speed = frequency × wavelength

This technique sounds complicated but it has several advantages. Firstly, we can vary the frequency of resonance by a factor of 10. The speed of sound estimates inferred from 6 different resonances agreed with a variability of only 9 parts in a billion. Because these different estimates agreed so well, we concluded that it is unlikely that our estimate could be very wrong.

Additionally, because the sound is now contained inside a resonator with a volume of only one litre, it is possible to make the temperature of the resonator very uniform. We estimated the temperature difference between the top and bottom of our resonator to be approximately 0.0001 °C.



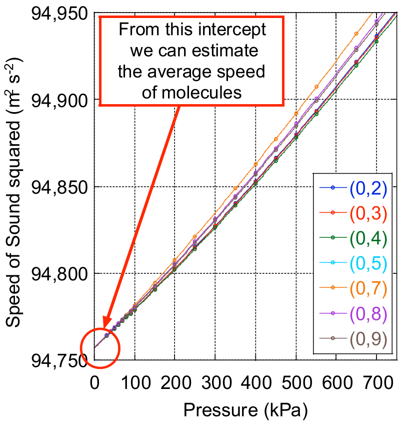
The low-frequency half of the acoustic spectrum of the resonator: On sweeping the loudspeaker frequency from 2 kHz to 10 kHz, the microphone signal increases sharply at each resonant frequency. Each peak corresponds to a distinct standing wave pattern within the resonator. We infer the speed of sound from 7 different resonances over a wide frequency range, and the results have to agree before we believe them. This allows us to eliminate some of the possible ways in which our result might be wrong. We use only the resonances labelled (0,2), (0,3) etc because these correspond to the simplest standing wave patterns.

The most difficult part

The most difficult part of the experiment has been to answer the question: How do we know our answer is correct? The way we answer this is to reverse the question and ask: How wrong could we possibly be?

Every measurement that goes into our experiment has some uncertainty associated with it. For example, the inner surface of the resonator has been very carefully made, but when we measured its radius we established a best estimate, and a range of values within which we were sure the true answer lay. Then we calculated how that uncertainty in the radius could affect our estimate of temperature.

It is very complicated to follow this process through for every component of our measurement to work out how wrong our final estimate could possibly be. But ultimately it is only by showing that our answer could not possibly be very wrong that our actual estimates become valuable.



Experimental results: Each data point on the graph corresponds to a measurement of the speed of sound taken at a different pressure using a different resonance. At low pressures the estimates all converge to a single value from which we estimate the speed of molecules and hence their kinetic energy and hence their temperature.

Why?

The aim of our measurements is to check whether conventional thermometers – the ones you use in class, and the ones used in laboratories around the world – are reading the correct temperature. Our work is not yet completed, but as an example we think that, at 29 °C, all the thermometers in the world are wrong by approximately 0.0045 °C. This is not much, but it is worth knowing. And the only way to find out this kind of information is to use a thermometer that measures temperature directly in terms of the basic physics of molecules.

Meet the team who made the most accurate temperature measurement in history. Michael de Podesta (the Author), Gavin Sutton, Graham Machin and Robin Underwood, with the acoustic resonator in which the measurements were made.

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