Teacher notes

Getting there : Working for efficiency

Description

This topic looks at simplified versions of three different network problems that are encountered in practical logistical planning.

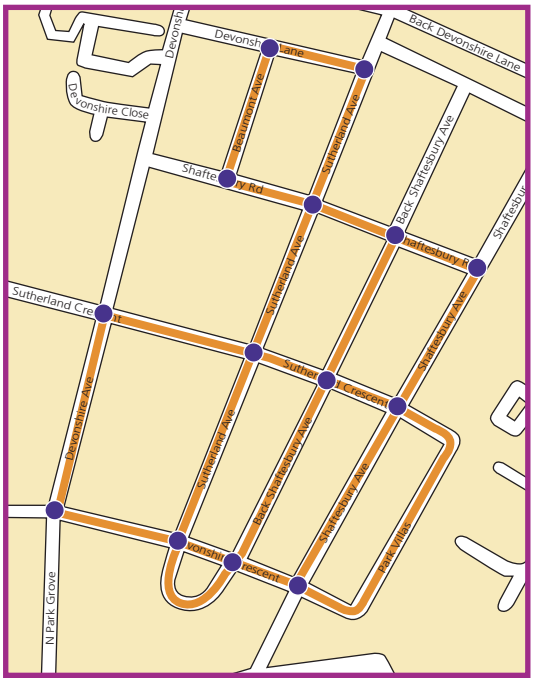
Activity 1: Paper rounds
Activity 2: Cable connections
Activity 3: Deliveries

Edge-traceable graphs are explored in Paper rounds. The starter activity offers a simple version of a practical context in which to explore the concept. The famous mathematician Euler noticed that, to be edge-traceable, a network must have at most two odd nodes.

The worksheet offers opportunities for further exploration and then asks the pupils to search for a rule and to prove their results. At some point it is worth asking: *can you have just one odd node?*

Using tracing paper supports experimenting. For lower attaining pupils, slipping the worksheet into a plastic wallet will allow experimentation using felt tip pens, wiping off incorrect attempts, which may be easier.

Encourage pupils to look for what the edge-traceable networks have in common. Which graphs can be traced whatever starting point is chosen? Which graphs can be traced only if you start, and end, at particular points? Pupils will test out their theories better if they work together.



Cable connections provides a practical example of a minimum connector problem. It is quite possible to find a good or even an optimal solution using trial and improvement methods but the task provides the opportunity to introduce pupils to the key mathematical idea of an algorithm. Prim's algorithm, to find this minimum spanning tree, is quite within the grasp of many pupils, even at key stage 3.

For some of the problems on the worksheet, there is more than one optimum solution.

Prim's algorithm works as follows:

Step 1: *Choose an arbitrary location, say Grimsby, and connect it to the nearest place, in this case, Westgate.* ***Note the distance****.*

Step 2: *Find the nearest place not yet in the solution, in this case either Frenchy place or Wheelsby Road. Whenever you get a choice of connections of equal length, choose either one. Here we have arbitrarily chosen Frenchy place.*

Continue this process until all the locations are connected.

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Deliveries illustrates a very familiar problem in logistics. Finding the shortest route to a set of delivery points will save on both time and money. For a relatively small network like the problem offered here, the shortest route may be found by trial and improvement methods, or by considering all the possible routes. Pupils will develop helpful strategies, for example, they might decide to try to avoid longer stages, like the road which goes directly from Sheffield to Catcliffe.

You can extend this activity for pupils who solve the initial problem by making one or two of the roads one-way, or by making one of the roads impassable. Both of these variations do, of course, represent real features of shortest route problems as applied to road systems.

In this type of problem, the number of calculations needed to exhaust every possibility increases very fast. Because of the very large number of computations required to be sure of the best solution, mathematicians have developed algorithms which give good solutions for problems that are too large to test for every possible solution. Pupils may be intrigued to find out that, to date, there is no known algorithm which can guarantee a best solution for a large number of locations – an unsolved problem in mathematics. If they have derived some helpful strategies, can they find networks where their approach fails to provide the optimal solution?

The mathematics

Paper rounds offers the opportunity for reasoning and proof as the arguments needed to establish Euler's theorem are within their grasp. It also, along with Cable connections and Deliveries, offers opportunities for the mathematical skills of planning, being systematic, recording and logical experiment. The algorithmic thinking developed is picked up in key stage 5 in the decision maths curriculum.