

# SMILE WORKCARDS

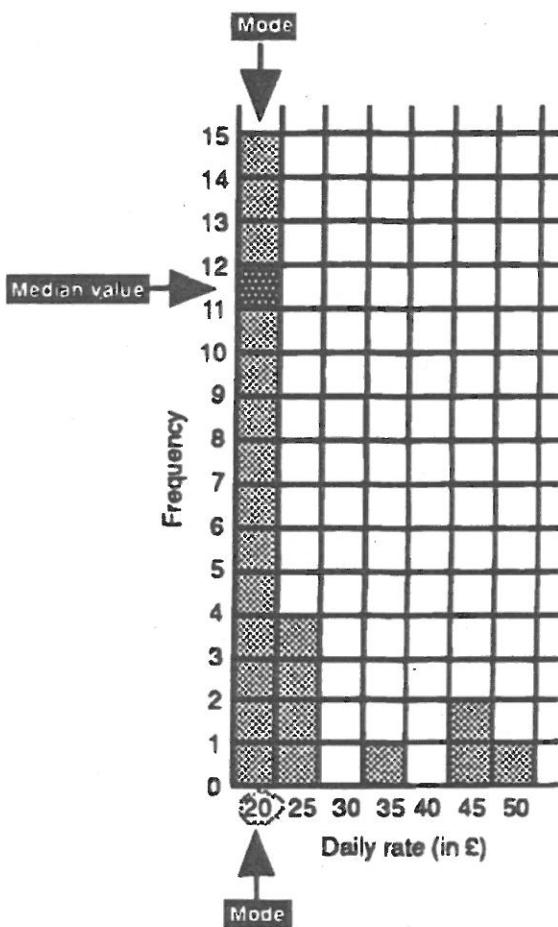
## Analysing and Interpreting Data Pack Two

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# Frequency Graphs

This frequency graph shows the daily rates of pay of the employees of a small company.



The information from the graph is displayed in this table:

Daily rate	Frequency	Daily rate x Frequency	Total
£20	15	£20 x 15	£300
£25	4	£25 x 4	£100
£30	0	£30 x 0	0
£35	1	£35 x 1	£35
£40	0	£40 x 0	0
£45	2	£45 x 2	£90
£50	1	£50 x 1	£50
<b>Total employees</b>	<b>23</b>	<b>Daily total</b>	<b>£575</b>

The mean daily rate is £25 ( $\text{£}575 \div 23$ ).

The median daily rate is £20 (see graph).

The modal daily rate is £20 (see graph).

- Why is the mean misleading?

A survey of the number of students absent from each tutor group in a school is shown below.

Number of absentees in the tutor group	Frequency of tutor groups with this number absent	No. of absentees x Frequency	Total
11	1	$11 \times 1$	11
9	2	$9 \times 2$	18
8	1		
6	7		
4	10		
3	5		
1	1		
Total number of tutor groups.			

Copy and complete the frequency table.

1. Find the total number of absentees.
2. Find the mean number absentees in each tutor group.
3. Find the median number of absentees in each tutor group.
4. Draw a frequency graph to show the above information.
5. Mark the mean and the median on your graph.

Members of a club took part in a sponsored walk. The number of sponsors for each walker was recorded in the following table:

Number of sponsors	Frequency of walkers with this number of sponsors	No. of sponsors x Frequency	Total
46	1	$46 \times 1$	46
35	2		
29	2		
25	9		
24	2		
21	3		
20	9		
18	3		
Total number of walkers			

Copy and complete the frequency table.

1. Make a frequency graph showing this information.
2. Calculate the mean.
3. Find the median.
4. What do you notice about the mean and the median?

# Best Marks

Three classes 9S, 9M and 9L all took the same test.

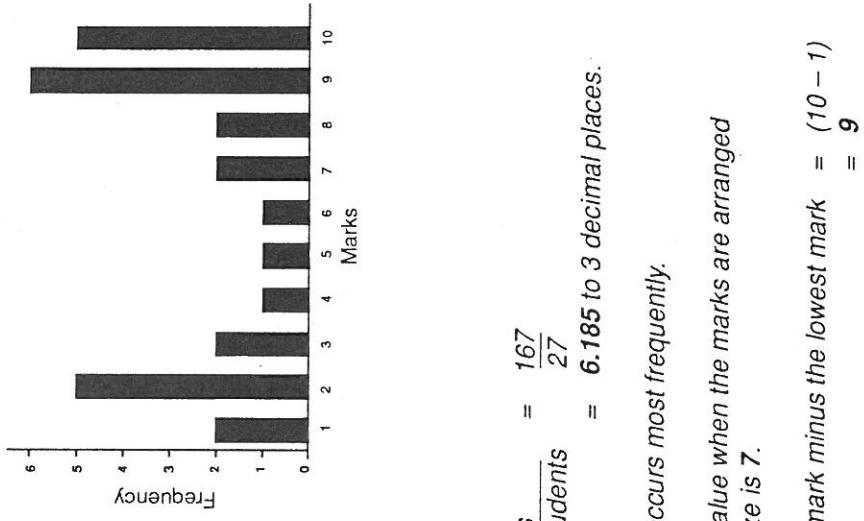
Here are the test marks for each class.

9S	9M	9L
9	7	8
2	3	7
7	2	8
9	9	3
2	9	1
3	10	8
10	1	9
9	6	6
1	9	5
10	6	6
5	3	9
8	10	6
9	9	7
2	1	5
2	3	3
1	1	2
10	2	8
7	8	8
10	3	1
4	5	4
3	6	8
9	10	10
10	10	8
8	10	9
6	8	
2	4	
9		

Tally chart

Here is an analysis of the marks for class 9S.

Mark	Tally	Frequency
1	//	2
2	##	5
3	//	2
4	/	1
5	/	1
6	/	1
7	//	2
8	//	2
9	##/	6
10	##	5



$$\text{Mean} = \frac{\text{total marks}}{\text{number of students}} = \frac{167}{27} = 6.185$$

The mark 9 occurs most frequently.

The middle value when the marks are arranged in order of size is 7.

The highest mark minus the lowest mark =  $(10 - 1) = 9$

1. Make similar analyses for the other two classes.

2. Which class do you think did best in the test? Justify your answer.

## What Does Average Mean?

Smile  
0805A

retribution.—  
**aveng'ress**) one who avenges.  
to seek retribution).

**avenue** (av'e-nü) *n.* principal approach bordered with trees to a mansion; any road in park bordered with trees; a wide street with houses and roads running each side; (*Fig.*) a way or course of action; *fam.* a person's social status.

**aver** (ä'ver) *v.t.* to declare; to assert; to allege.—*pr.p.* averring; *p.p.* averred.—**averment** *n.* the act of averring; a positive assertion; (*Law*) proof of a plea [L. *ad*, to, *verus*, true].

**average** (av'üg'-rä) *a.* containing a mean proportion; ordinary; normal;—*n.* a medial estimate obtained by dividing the sum of a number of quantities by the number of quantities.—*v.t.* to reduce to a mean [O. Fr. *average*, cattle or possessions; fr. L. *habeere*, to have].

**averse** (ä'vers') *a.* reluctant (to do) or disinclined; opposed; unwilling; set against (foll. by *from*) with repugnance.

**avert** (ä'vert') *v.t.* to turn away from or aside; to ward off.—**averted** *a.*—**avertedly** *adv.*—*capable of being avoided* [L. *aversus*, turned away].

The word 'average'  
is used in many  
different ways.....

- (1) Work through cards B to I and look out for different meanings of 'average'.
- (2) Copy out the definitions of the three averages most often used in mathematics (see page 8).
- (3) Look back at cards B to I and decide which of these three averages (if any) is appropriate.

Average Shoe Size (B) :

- (2) You probably chose the mode, possibly the median.
- (4) Explain why the mean is not very useful here.

- (5) Calculate the mode and the median,
- (6) Six more join the class. Their shoe sizes are 6, 8, 9, 6, 8 and 10.  
Have the mode and the median changed ?

How? Comment.

### Average Ability and Average Mark (C, D) :

For both of these you could have chosen the median or the mean — probably the median for Bill's geography mark and the mean for his average mark.

- (7) Explain why the mode is useless in instances such as these.

### Average Breakfast Cereal (E) :

If the data is not in figures the mode is the most appropriate.

- (8) Explain why.

- 4 -

### Average Wage (I) :



We want a  
pay rise!

You already  
earn £5000  
on average.

You're joking! Our average wage  
is only £3500.

Your wage is in a different class than ours. If you don't count your wage, you can use any average you like.

### Average Speed and Average Weekend (F, G) :

Mean, median and mode are all unsuitable here.

- (9) Which average did you use for each of these?

Explain in each case, what the word average does mean.

### Average Weight (H) :

- (10) Can you use the mean, median or mode to enable Bill to be more precise?

- 5 -



We want a  
pay rise!

You already  
earn £5000  
on average.

You're joking! Our average wage  
is only £3500.

- (11) How does it affect the different averages if you do not count the director's salary?
- (12) Which average suits the boss best?
- (13) Which average suits the workers best?

SMILE  
0805 **B**



## Average Shoe Size

In Bill's class there are 25 pupils. Their shoe sizes are:

3, 7, 7, 5, 6, 8, 5, 7, 9,  
7, 4, 4, 6, 9, 7, 6, 5, 6,  
5, 8, 7, 3, 10, 7, 6.

Bill's shoe size is average. What do you think it is?

SMILE  
0805 **C**

## Average Ability

Bill's geography teacher said his mark was average.

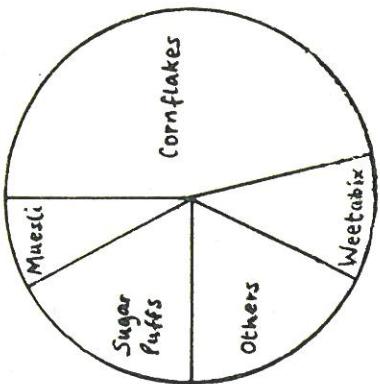
The marks for the whole class were:

84, 77, 77, 76, 76, 70, 69, 64, 60  
55, 52, 52, 51, 50, 49, 49, 47, 46, 45,  
45, 45, 44, 44, 41

SMILE  
0805 **E**

## Average Breakfast Cereal

What is the average?  
Breakfast in Bill's class looks like this:  
of the cereals his classmates had for breakfast and it's  
Bill drew a pie-chart



WILLIAM PETTERS FORM 4	Maths	Home Economics	Art	Geography	French	English	Science	History
28%	88%	24%	65%	66%	65%	65%	73%	52%
Peter	R.E.	Music	Maths	Geography	French	English	Science	History
60	50%	38%	88%	52%	66%	65%	46%	50%

These are Bill's exam marks:

SMILE  
0805 **D**

## Average Mark

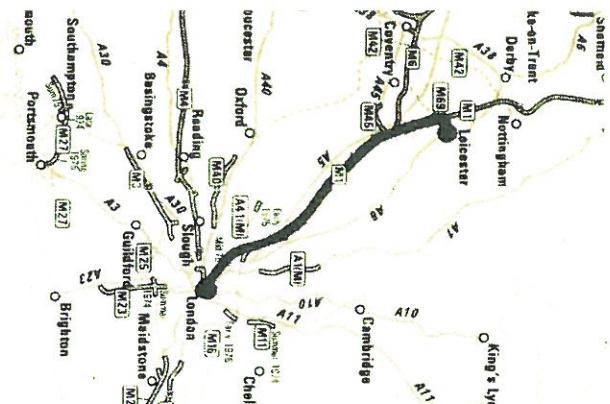
Work out Bill's average mark.



# Average Speed

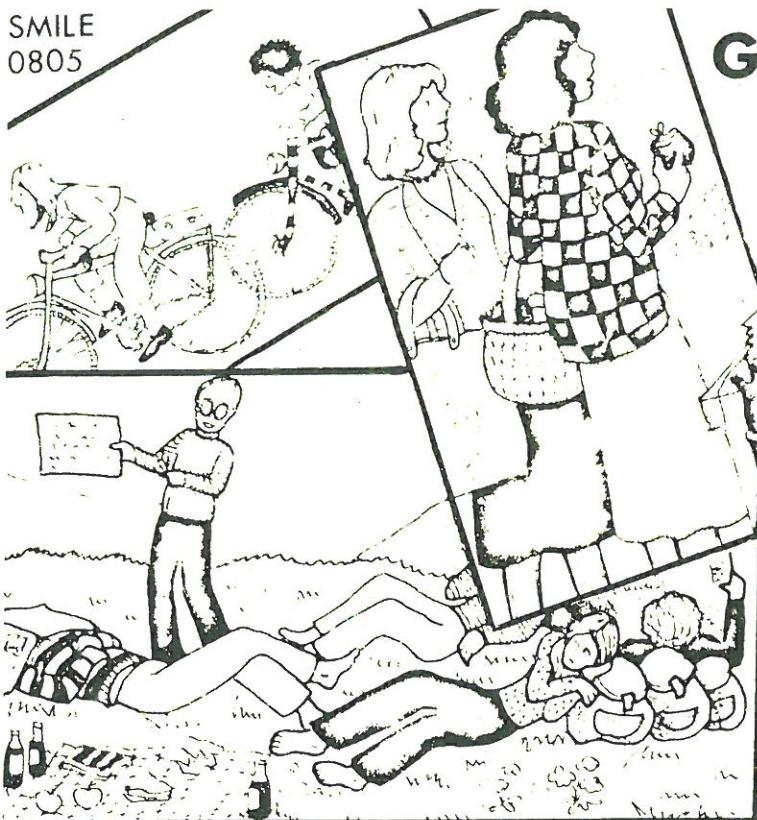
SMILE  
0805

The distance from London to Leicester is 100 miles.



- (1) A car does the journey in 2 hours. What is the average speed?
- (2) The car takes 3 hours to come back. What is the average speed for the return journey?
- (3) What is the average speed for the 200 mile round trip?

SMILE  
0805



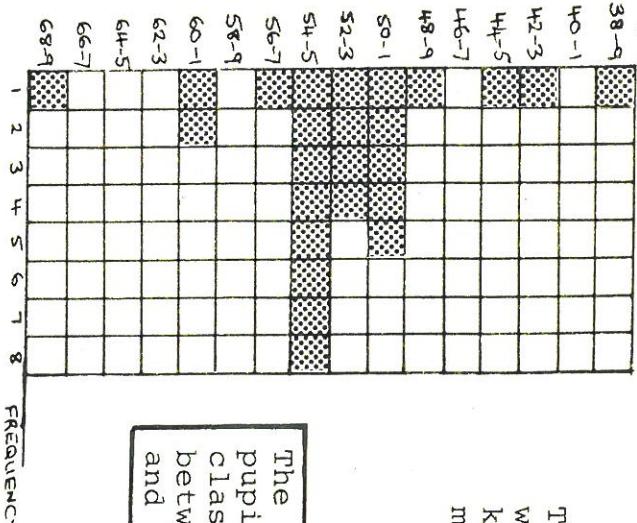
Write down briefly what you do during an average weekend.

## An Average Weekend

# Average Weight

SMILE  
0805

The bar-chart shows the weights (to the nearest kg) of Bill's classmates.



The average pupil in my class weighs between 50 and 55 kg.

SMILE  
0805

# Average Wage

Bill's father is foreman in a factory employing 31 people. The incomes of the director and his 30 staff are below. What is the average wage?

Position in firm	Number of persons	Salary	Position in firm	Number of persons	Salary
Director	1	£26,000	Foreman	1	£5,800
Manager	1	£7,000			
Chief Clerk	1	£5,600	Skilled workers	11	£4,800
Clerk	1	£5,300	Unskilled workers	13	£3,500
Secretary	1	£5,000	Apprentice	1	£2,000

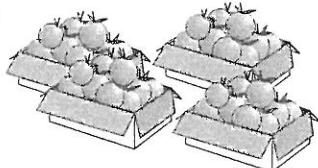
# Average H

Solve these problems.

1. The mean of five consecutive numbers is 12.

What is the mean of the first three of these numbers?

2. The mean mass of four boxes of fruit is 17kg.



A fifth box weighs 12kg.

What is the mean mass of the five boxes?

3. The mean age of a group of 7 students is 13 years.

Another student joins the group, the mean age is still 13 years.

How old is the new student?

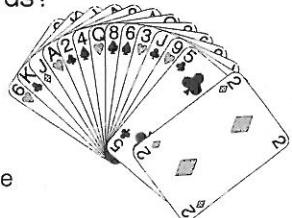
6. The mean weight of nine members of a rugby team is 95kg.

A new player joins the team, the mean weight of the ten players is 94kg.

What is the weight of the new player?

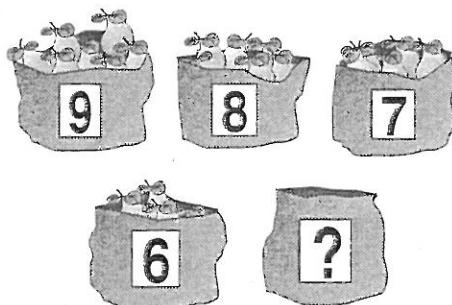
7. Robbie picked five cards from a pack. Two were the same number. The other three were all odd. The mean of the cards was 7.

What were the cards?



There is more than one possible solution.

4. The mean number of pears in these five bags is 8. Four of the bags contain 9, 8, 7 and 6 pears.



How many pears are in the fifth bag?

5. In three maths tests, Joanne scored 72%, 77% and 81%.

She wants her mean mark to be 80% or higher.

What percentage must she get in her next maths test?



Smile 2318

## A Mean Challenge!

The **mean** of a set of values is found by finding the sum of the values and dividing it by the number of values.

$$\text{Mean} = \frac{\text{sum of the values}}{\text{number of values}}$$

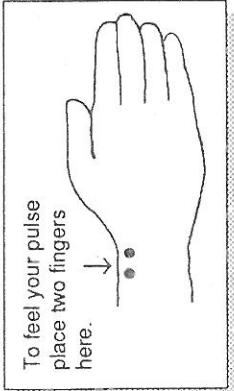
# Grouping Data

Smile 2175

For this set of data:

80, 0, 4, 0, 1, 2, 1, 6, 2, 2, 3, 3, 5, 3, 3, 1, 7, 1, 6, 6, 1, 8, 10, 10, 4, 0, 90

Take your pulse for one minute.  
This is your pulse rate.



**The range** The difference between the highest and the lowest value (*highest value – lowest value*).  
So the **range** of this set of values is 90. (90 – 0.)

There are three main types of **average**:

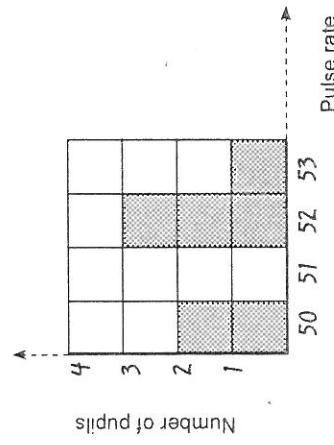
1. Find the pulse rate for each member of your class.

- a) Record this data in a table.\*

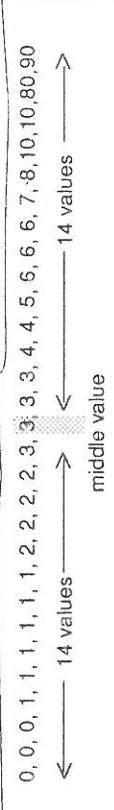
Pulse rate	50	51	52	53
Tally	//	///	/	
No. of pupils	2	0	3	1

\* You will need to collect at least 20 pulse rates. If this is not possible the Answer Book contains data from one year 9 class.

- b) Display this information in a bar chart.

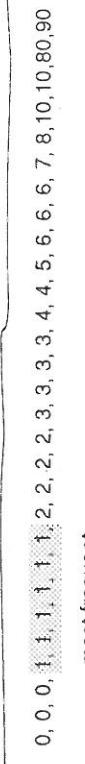


**The median** The middle value after the values have been arranged in order of size.



The **median** of the set of values is 3.

**The mode** The most frequently occurring value.



The **mode** of this set of values is 1.

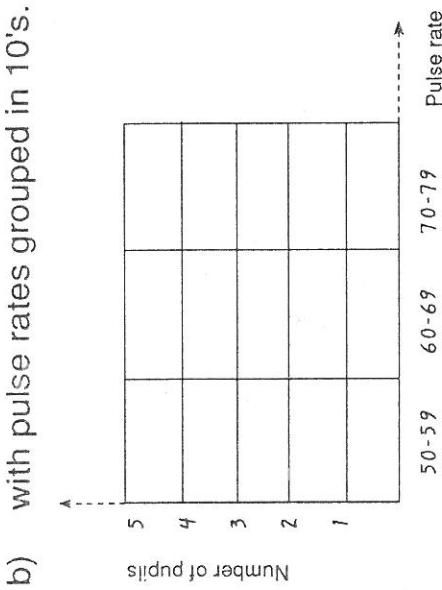
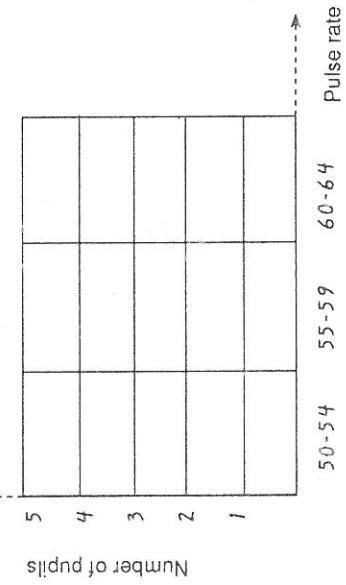
**The mean** The sum of all the values divided by the number of values.

$$\frac{0+0+0+1+1+1+1+1+2+2+2+3+3+3+4+4+5+6+6+7+8+10+10+80+90}{29} = \frac{262}{29}$$

The **mean** of this set of values is 9.03 to 2 d.p.

Look at your pulse rate data.

2. a) What is the **range**?  
b) What is the **median**?  
c) What is the **mode**?  
d) What is the **mean**?
3. Use your original data to draw two further bar charts:
  - a) with pulse rates grouped in 5's
4. Look at your three bar charts.
  - a) with pulse rates grouped in 5's
  - b) with pulse rates grouped in 10's.



Turn over.

Smile 2325

MEAN

Grouped Data, Reviewed

Form 8B		Year 8		Form 8E	
	Number of SMILE activities completed		Frequency		Number of SMILE activities completed
1. All students	41 - 50	41 - 50	6	1. All students	41 - 50
2. English	51 - 60	51 - 60	24	2. English	51 - 60
3. Spanish	61 - 70	61 - 70	18	3. Spanish	61 - 70
4. French	71 - 80	71 - 80	14	4. French	71 - 80
5. German	81 - 90	81 - 90	11	5. German	81 - 90
6. Chinese	91 - 100	91 - 100	29	6. Chinese	91 - 100
7. Japanese	101 - 110	101 - 110	27	7. Japanese	101 - 110
8. Italian	111 - 120	111 - 120	27	8. Italian	111 - 120
9. Other	121 - 130	121 - 130	18	9. Other	121 - 130
Total	131 - 140	131 - 140	5	Total	131 - 140
	T-Total	T-Total	5		T-Total

Year 9	Number of SMILE activities completed	Frequency
51 - 60	5	
61 - 70	12	
71 - 80	21	
81 - 90	28	
91 - 100	32	
101 - 110	40	
111 - 120	25	
121 - 130	17	
131 - 140	4	
141 - 150	1	
Total	185	

To get the  
add the  
and divide

1. a) Copy this table and complete the **Mid-value** and the **Mid-value × Frequency** columns.

Number of SMILE activities completed	Frequency	Total
41 - 50	6	
51 - 60	24	
61 - 70	18	
71 - 80	14	
81 - 90	11	
91 - 100	29	
101 - 110	27	
111 - 120	27	
121 - 130	18	
131 - 140	5	
		179

This table shows the number of SMILE activities completed by Year 8 students, grouped in class intervals of 10 activities.

and complete the **Mid-value** and **Frequency** columns.

It is only possible to calculate an **estimate** for the mean from a set of grouped data. This estimate is based on the **mid-values** of each group.

The estimate for the mean is found by dividing the total of the 'Mid-value x Frequency' values by the total 'Frequency'.

## MODAL GROUP

$$\frac{\text{Total of Mid-value} \times \text{Frequency values}}{\text{Total Frequency}}$$

It is not possible to find the mode from a set of grouped data as the individual data is not available.

- b) Calculate an estimate for the mean number of SMILE activities completed, grouped in class intervals of 10 activities.

$$\frac{\text{[ ]}}{\text{[ ]}} = \boxed{ }$$

The same data is given in this table but this time it is grouped in class intervals of 20 activities.

Number of SMILE activities completed	Frequency
41 - 60	30
61 - 80	32
81 - 100	40
101 - 120	54
121 - 140	23
Total	179

- c) Copy and complete this table.

Number of SMILE activities completed	Mid-value	Frequency	Mid-value x Frequency
41 - 60	50.5	30	1515
61 - 80	70.5	32	
81 - 100			

2. What is the modal group for the number of SMILE activities completed in Year 8 when grouped in 20's?
- d) Calculate an estimate for the mean number of activities completed.
- e) Can you explain why the answers to b) and d) are different?

Number of SMILE activities completed	Frequency
41 - 50	6
51 - 60	24
61 - 70	18
71 - 80	14
81 - 90	11
91 - 100	29
101 - 110	27
111 - 120	27
121 - 130	18
131 - 140	5
Total	179

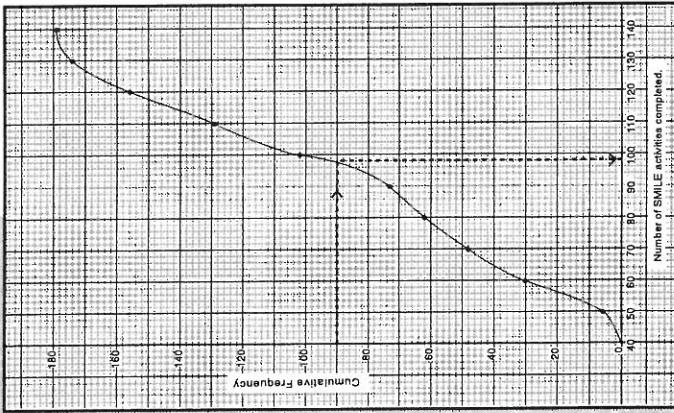
## MEDIAN

## Year 9

It is not possible to find the exact median for a set of grouped data, but it is possible to **estimate** the median by drawing a **cumulative frequency curve** and reading off the **median**.

The cumulative frequency graph is plotted at the end of each interval.

Number of SMILE activities completed	Frequency	Cumulative Frequency
41 - 50	6	6
51 - 60	24	30
61 - 70	18	48
71 - 80	14	62
81 - 90	11	73
91 - 100	29	102
101 - 110	27	129
111 - 120	27	156
121 - 130	18	174
131 - 140	5	179
Total		179



The **median** number of activities will have been completed by the 90th student.

By using the cumulative frequency curve an **estimate** for the **median** value is 98.

4. This set of grouped data shows the numbers of SMILE activities completed by 185 Year 9 students:

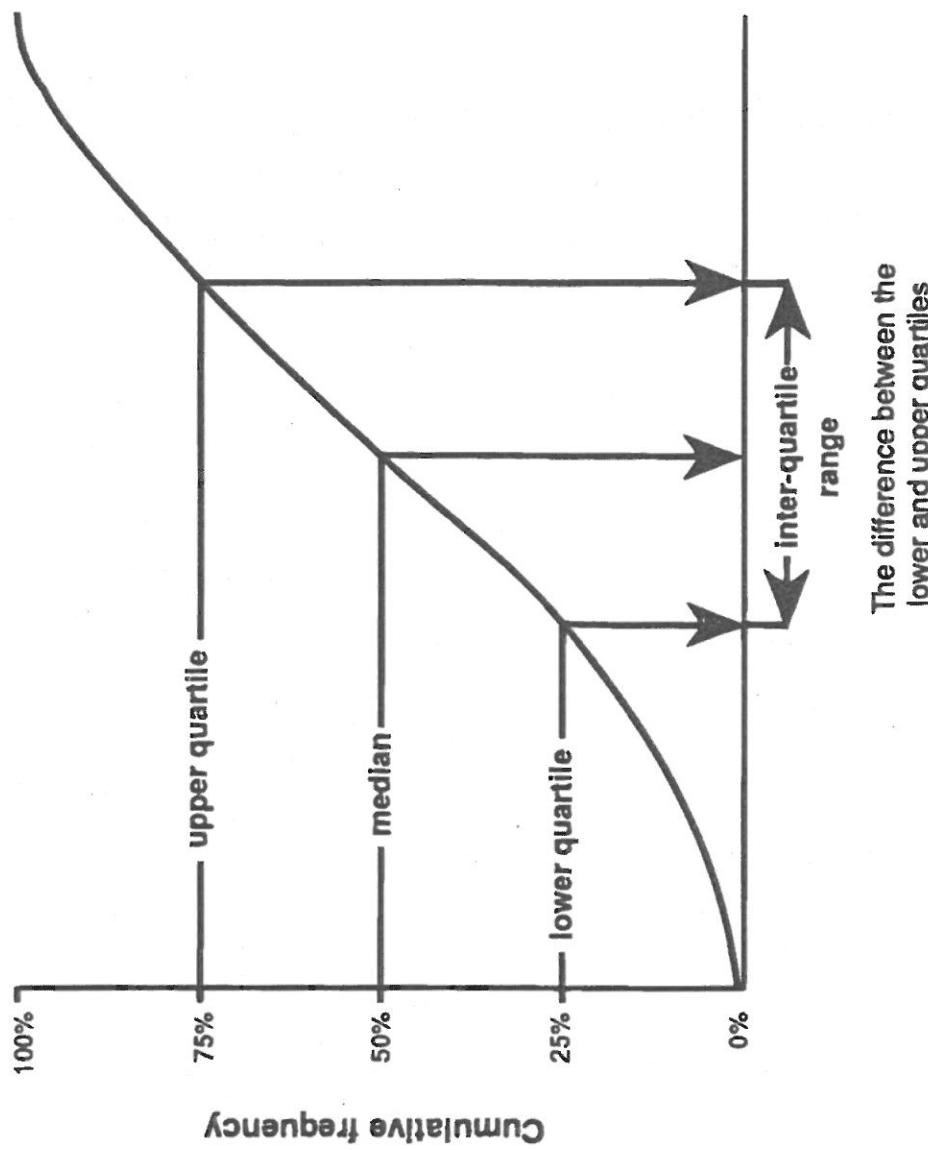
Number of SMILE activities completed	Frequency
51 - 60	5
61 - 70	12
71 - 80	21
81 - 90	28
91 - 100	32
101 - 110	40
111 - 120	25
121 - 130	17
131 - 140	4
141 - 150	1
Total	185

- a) Calculate an estimate for the **mean**.
- b) Find the **modal group**.
- c) Draw a **cumulative frequency curve** and estimate the **median**.
5. Use all the results to compare the number of SMILE activities completed by Year 8 and Year 9 students.
3. a) Complete a cumulative frequency table and graph for the number of SMILE activities completed in Year 8 when grouped in 20's.
- b) Is the estimated median number of SMILE activities the same when the data is grouped in 20's?

# Cumulative Frequency and Quartiles

Smile 1007

This activity is about **cumulative frequency curves** and **quartiles**. When an ordered distribution is divided into four quarters, the quarters are called **quartiles**. Each quartile contains one quarter of the total frequency in the distribution. This information is shown in the diagram below.



The heights of 154 students are measured to the nearest centimetre.

Height(cm)	160	161	162	163	164	165	166	167	168	169	170	171	172
Frequency	4	5	6	9	16	22	27	25	18	11	6	3	2

The upper bound for the height of students in the 164cm group is 164.5cm

- What is the upper bound for the height of students in the 160cm group?

The information can be set out to show the number of students below any given height.  
This is called the **cumulative frequency**.

- Copy and complete the following cumulative frequency table.

Height (cm)	Frequency	Cumulative frequency	Heights of students represented by cumulative frequency
160	4	4	<160.5cm
161	5	9	<161.5cm
162	6	15	<162.5cm
163	9	24	

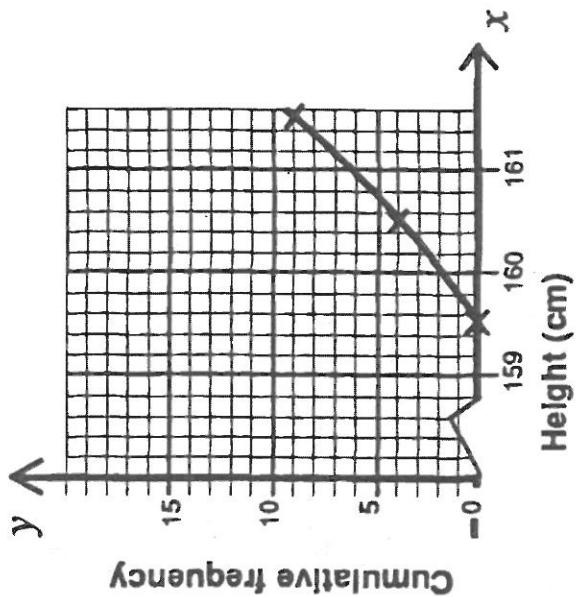
■ What should the last entry in the cumulative frequency column be equal to?

Use this to check your cumulative frequency table.

The information in the cumulative frequency table can be used to draw a **cumulative frequency curve**.

The cumulative frequency is plotted at the upper bound of each group.

3. Why does the point  $(159.5, 0)$  belong to the curve?



4. Draw a pair of axes with  $x$  values from 159cm to 173cm, and  $y$  values from 0 to 154.  
Draw the cumulative frequency curve.

5. Use your cumulative frequency curve to find:
- a) the median height
  - b) the lower quartile
  - c) the upper quartile
  - d) the inter-quartile range.

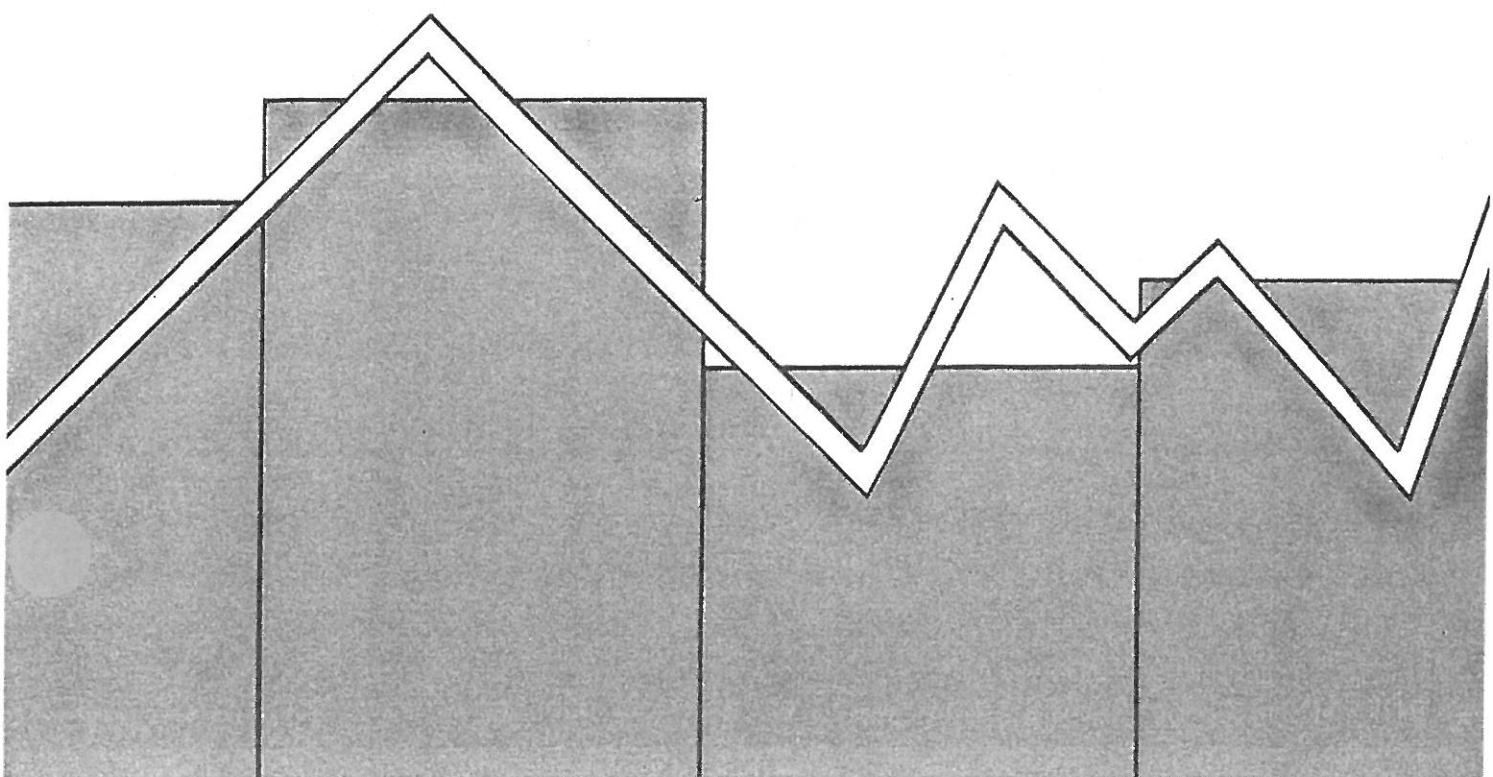
Two machines X and Y produce metal rods of diameter 12mm.  
100 samples are taken from each machine, the measurements to the nearest tenth of a millimetre are:

Diameter(mm)	11.5	11.6	11.7	11.8	11.9	12.0	12.1	12.2	12.3	12.4
Frequency for Machine X	0	0	3	8	11	20	20	18	15	5
Frequency for Machine Y	1	0	4	9	13	31	30	9	2	1

6. a) Construct the cumulative frequency table and draw the cumulative frequency curve for the results from each machine.
- b) Use your graph to obtain a value for the inter-quartile range in each case.
- c) Compare the reliability of the machines.

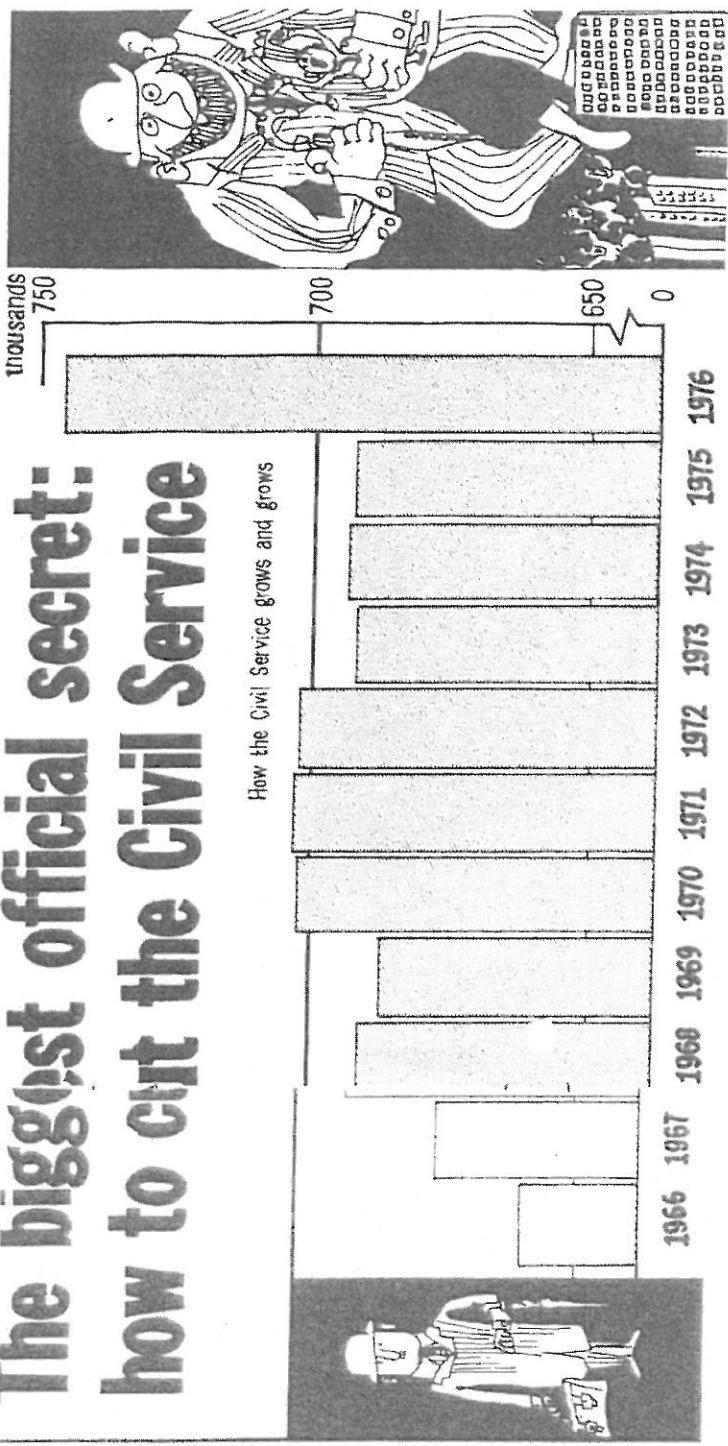
Other ways to describe cumulative frequency curves are:  
**cumulative frequency polygons** or **ogive**.  
The word **ogive** is a term used in architecture.

**“Lies, Damned Lies and Statistics.... ”**



# The biggest official secret: how to cut the Civil Service

- 1) "Look at this! In 1976 the Civil Service has grown 104 times what it was in 1966." Do you agree?
- 2) How many civil servants were there in:
  - a) 1966 b) 1976
- 3) What is the increase over this 10 year period?
- 4) Express this as a percentage.
- 5) Why should someone think the Civil Service had quadrupled in these ten years?
- 6) Redraw the graph so that this misapprehension could not occur.
- 7) Why do you think the graph was presented this way in a well known daily newspaper?



(from a lead in National Daily newspaper)

**Information is often presented in a way that gives a false impression, at least at first glance. It is important not to be mis-led.**

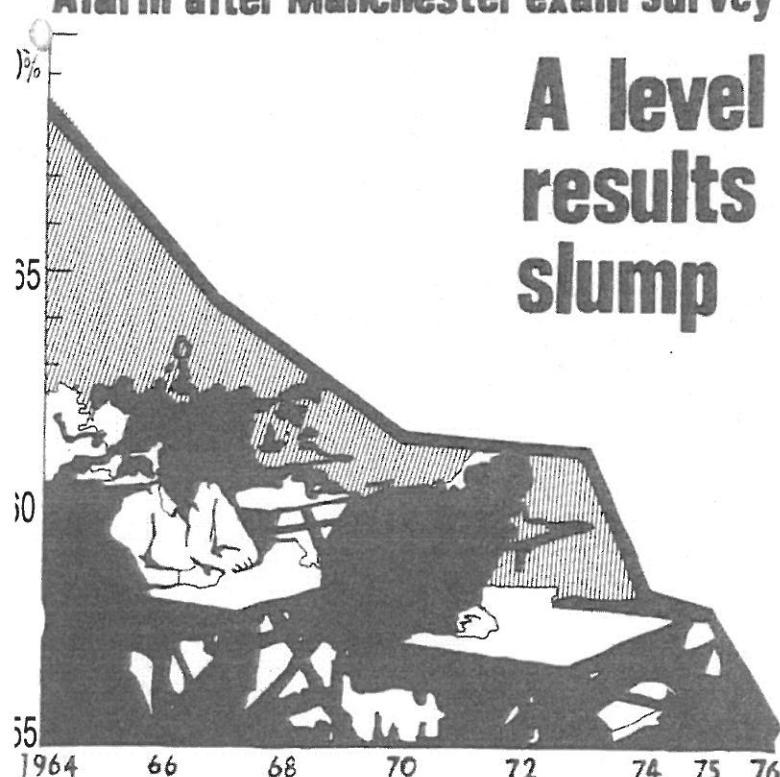
**When you work through the rest of this booklet;**

- 1) Write down your first impressions.
- 2) Read the questions and think again!
- 3) Write down anything important that you notice — you might need to re-draw a graph.
- 4) Compare your results with the hints in the answer book and write more if necessary.

## **Alarm after Manchester exam survey**

# **A level results slump**

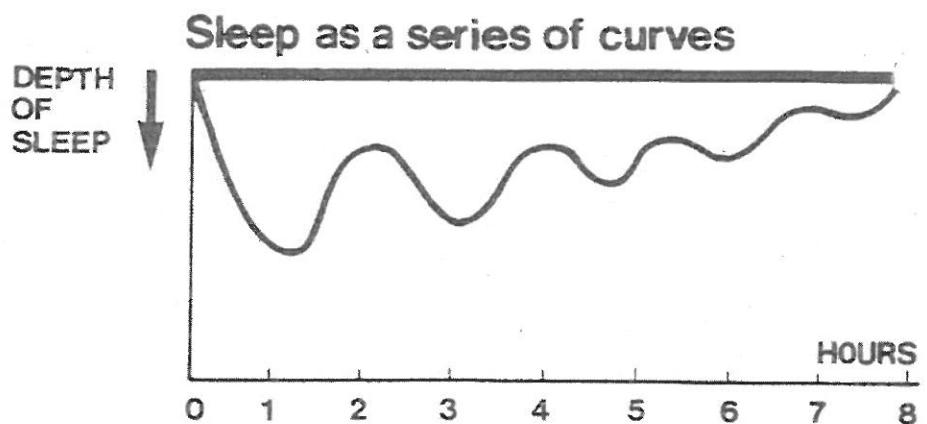
**What's been  
happening in  
Manchester?**



(from a leading local paper)

*Downward trend: A level passes as a percentage of entries. Comprehensive reorganisation in Manchester took place in 1974.*

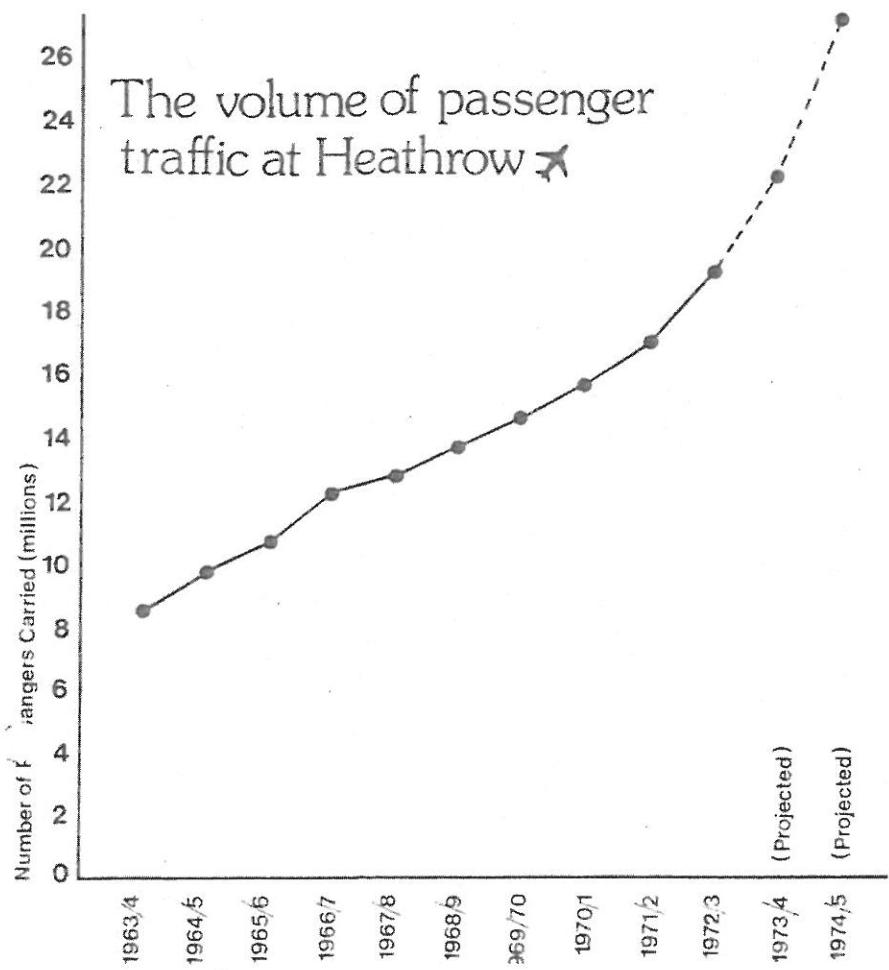
What  
measurements  
have been made?



(from a bed-time drink label)

The volume of passenger traffic at Heathrow ✈

How many passengers would you have predicted for 1974/75 in 1972?

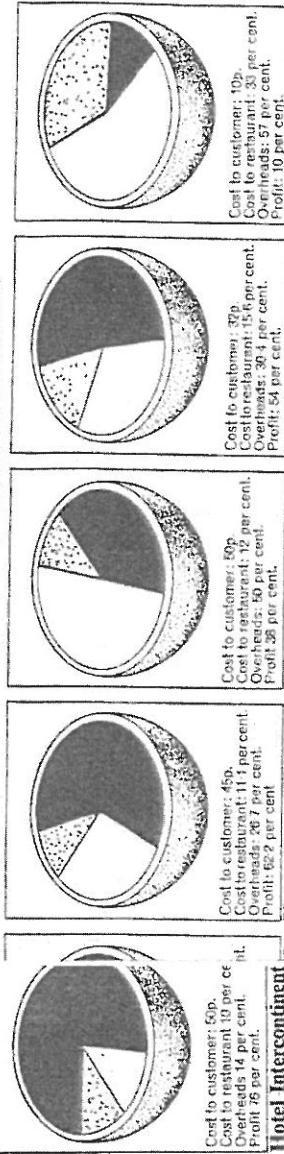


Who makes more profit: the Crown Inn or the Angus Steak House?

## The way the grapefruit slices

in half, sell each half for You expect the cost on the menu to cost the earth, but not the grapefruit. They buy one for 15p or less, cut it up each, and appear to make 300 per cent profit or so. Is that an indication of the way they mark up the rest of the menu, the result? We asked Sandra Hempel to question restaurateurs on their mark-ups of half grapefruits; this is the result.

In the diagram the areas shaded black represent profit; grey is cost; white is overheads



**Hotel Intercontinent, London, W.1**  
Restaurant is 15p. Cost to restaurant 10p. Cost to customer 15p. Cost to restaurant 11p. Overheads 14p. Profit 56p. Proprietor Peter Kronberg, the restaurant chef, is master of the soufflé tart. "Anyone can eat a grapefruit in half at home, so we serve with a 50p item; it's unusually high profit." It's a high mark-up because the low labour cost involved.

**Upper Crust, London, S.W.1**  
Restaurant is 15p. Cost to restaurant 10p. Cost to customer 15p. Cost to restaurant 10p. Overheads 14p. Profit 56p. Proprietor Manny Frankel, cost controler, says: "A cheap grapefruit is very few."

**Crown Inn, near Woking, Surrey**  
Restaurant is 15p. Cost to restaurant 10p. Cost to customer 15p. Cost to restaurant 11p. Overheads 11p. Profit 58p. Owner Anders Lamont: "Normally our profit would be 5p on a 50p item; it's unusually high profit because preparation time on

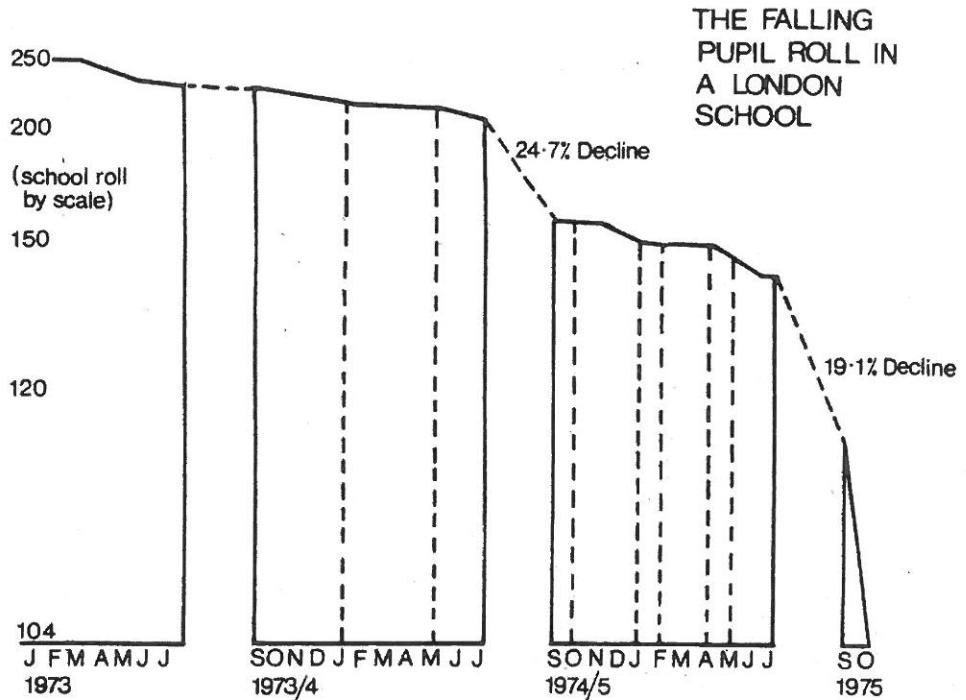
Manager: 20 minutes. Hempel says: "Simple dishes, covers cost of more expensive items on the menu."

**Angus Steak House (typical example)**  
Restaurant is in a pretty pub in lovely Surrey village; classical French food. Owner Anders Lamont: "Normally our profit would be 5p on a 50p item; it's unusually high profit because preparation time on Manager: 20 minutes. Hempel says: "Simple dishes, covers cost of more expensive items on the menu."

**Grand Hotel, Scarborough, Yorks.**  
Restaurant is part of large EMI chain (the record and feature people). Stereotyped menus—standard bland red, black, tartan decor. Strict buying and portion control. New owner, Arthur Turner says: "This is a very large, old building. We work on a lower profit margin than you would in smaller, newer premises."

Hempel says: "Grapefruit appears to be very undervalued. But done on the menu we a whole two months there don't have to

**Describe the state  
of the school in  
the autumn term  
1975.**



(from an educational newspaper)



**What does your  
eye measure?**

**What measure  
represents the  
amount of tea?**